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Documentation of the method used by Norges Bank for estimating implied forward interest rates

by

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Documentation of the method used by Norges Bank for estimating implied forward interest rates

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Introduction

Forward interest rates are an important indicator for monetary policy. They are commonly used, both by central banks and market participants, to gain information about market expectations of future interest rates. Forward rates in the short term are traded directly in the market, but for longer horizons they must be derived implicitly from the yield curve. In this paper we will briefly comment on the method used by Norges Bank for estimating implied forward interest rates. Previously, Kloster (2000) has described this in more detail. However, the Bank has made some adjustments to the procedure described in Kloster's article. Hence, this paper is an update and a summary of the method currently used. The paper is structured as follows: Firstly, we will comment on the selection of instruments. There are in general two alternative types of instruments suitable for estimating implied forward rates in the Norwegian market. One is the market for government securities, while the other is the swap market. Secondly, we will give a brief description of the estimation procedure, which is based on the Svensson method or the extended Nelson and Siegel method. Finally, we will comment on the adjustments we make for risk premia. There are a number of potential risk premia in the estimated forward rates. However, the corrections we make are only minor, and relate to the credit risk element in the forward rates derived from the swap market.

Interest rate data

In the short term it is common to use the market for forward rate agreements (FRAs) when deriving interest rate expectations. This market is regarded as the most liquid part of the money market in Norway. Contracts are traded with settlement on the third Wednesday - the so-called IMM dates – in March, June, September and December. The FRAs are of 3, 6 and 12 months maturity and settle against NIBOR[1] prevailing two days before the IMM date. Combining these contracts provides 3-month FRAs with settlement on the first 7 IMM dates, where the 7th IMM date has settlement 18-21 months ahead.

For longer horizons, however, the forward rates must be estimated using other instruments. There are in general two alternative types of instruments for estimating implied forward interest rates in the Norwegian market. One is the market for government bonds and Treasury bills; the other is the swap market, including the interbank (NIBOR), FRA and interest rate swap market.

The yields on Treasury bills and government bonds contain no credit risk. Implied forward rates derived from these instruments can therefore be interpreted as expectations concerning the future key rate (the sight deposit rate), assuming no other types of risk premia in the forward rates. There are, however, relatively few outstanding Treasury bills and government bonds in the Norwegian market[2], and the volume is by international standards small[3]. As a consequence, the pricing of government securities may be considerably influenced by variation in supply and demand. Historical experience suggests that this may be the case.[4] In addition, Treasury bills are often used for cash management purposes. Their yields may therefore contain less accurate information about expectations of future interest rates in the near term. Since the purpose of estimating implied forward rates is to derive interest rate expectations, this suggests that government securities are not the best suited.

In the estimations we therefore use rates from the swap market. This market is considered to be more liquid than the Norwegian market for government securities. The input data consist of money market rates (NIBOR) with maturities 1, 3, 6 and 12 months and the fixed rates on NIBOR-related interest rate swaps with maturities in the range of 2 to 10 years. Implied forward rates from this market can be interpreted as expectations of future money market rates, again assuming no risk premia in the forward rates. An advantage in using these instruments is that the number of observations along the yield curve increases.[5] Hence, the estimation will be more accurate. A disadvantage is that the instruments contain credit risk. However, since all

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[1] Norwegian Interbank Offered Rate
[2] There are normally four and five outstanding Treasury bills and government bonds respectively. The bills are auctioned at each IMM date with maturity of one year, while the bonds are auctioned with a twoyear frequency. The shortest bond matures in January 2007, while the longest matures in May 2015.
[3] On 12 September the outstanding volume in government securities was NOK 205.7 billion.
[4] See Hein (2003).

instruments are related to NIBOR, they will contain a common credit risk element. Using these instruments will therefore provide a consistent yield curve. The same argument applies when comparing the estimated 3-month forward rates against the FRA rates. As both curves are related to NIBOR, they will normally show the same. Any differences should then be due to the estimation method.

The estimation method

In the estimation of implied forward interest rates we use the estimation method proposed by Svensson (1994, 1995). This is an extended version of the method developed by Nelson and Siegel (1987) and is therefore often referred to as the extended Nelson and Siegel method. The Svensson method is a parametric method, where the entire yield curve is described by a single set of parameters representing the long-run level of interest rates, the slope of the curve and humps in the curve.

The input in the estimation procedure is the yield to maturity on the different instruments mentioned and their corresponding coupon payments and maturity dates. While NIBOR rates can be interpreted as zero-coupon bonds, an interest rate swap can be interpreted as a par bond. The coupon rate will then equal the swap rate.

Svensson's function has six parameters. Once these have been estimated, the function provides continuously compounded zero-coupon rates for all maturities and forward rates for all settlement dates and maturities. Pricing the synthetic bonds based on NIBOR and interest rate swaps with the estimated zero-coupon or forward rates results in a set of estimated yields on these bonds. The parameters in Svensson's function are determined so that the sum of squared yield errors between the estimated yields and the observed yields in the market is minimised. This problem is solved by maximum likelihood estimation.

An alternative to minimising yield errors would be to minimise price errors. As pointed out by Svensson (1995) however, minimising price errors sometimes results in fairly large yield errors for instruments with short maturities. This is due to the fact that prices are very insensitive to yields for short maturities. Minimising yield errors

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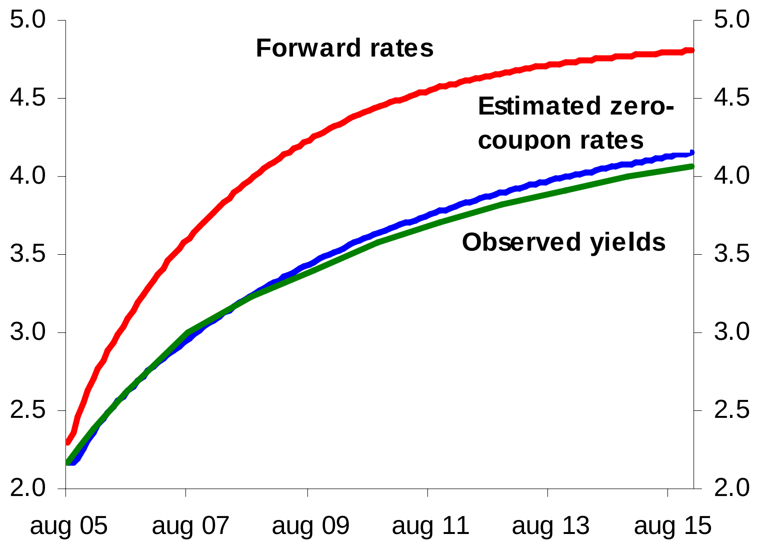
[5] The total number of securities used is 13, compared to a total of 9 available government securities.

gives a better fit for short maturities, while the two procedures seem to perform equally well for long maturities. Also for monetary policy purposes the focus is on interest rates, so it makes sense to minimise errors in the yield dimension rather than in the price dimension. For a more technical description of Svensson's method and the estimation procedure, see the Appendix.

Chart 1 shows the observed yield curve, estimated zero-coupon curve and the implied forward interest rate curve on 10 August 2005.

Chart 1: Observed yields, estimated zero-coupon and implied forward rates. 10 August 2005



There are also other estimation techniques available. Spline-based models are an alternative to the parametric methods developed by Nelson and Siegel and Svensson. For a description of these models and a comparison of parametric and spline-based models, see Anderson and Sleath (1999).

Risk premia in estimated forward rates

The purpose of estimating implied forward interest rates is to derive interest rate expectations in the market. However, the existence of risk premia in forward rates may cause the forward rates to deviate from the true interest rate expectations in the market. The risk premium includes premia for differences in credit quality and liquidity considerations, and a term premium arising from interest rate uncertainty and investor risk aversion. Risk premia are not observable and need therefore to be estimated. However, this is a difficult task as the premium consists of different elements, and may also be time varying.

The instruments used to estimate the implied forward rates are all based on or related to NIBOR. They will therefore contain a common credit risk element. But it is likely that the credit risk in forward rates increases with maturity. If so, this will contribute to an upward bias in the estimates of the expected interest rates derived from implied forward rates.

A comparison of the forward rates derived from the swap market with the rates derived from the government market is an easy way to gain information about the size of the credit risk premium at different horizons on the forward curve. The forward rates based on government securities contain no credit risk, and all else equal, any difference should then be due to the credit risk in the swap market.

The average[6] difference between these two forward curves indicates that the credit risk seems to increase with settlement up to 2-4 years, and remains fairly constant thereafter. The Bank's approach for correcting for increasing credit risk is based on this average. It implies that the credit risk premium in 3-month forward rates is constant the first six months and increases by 11-14 basis points[7] after 2-4 years. From 4 to 10 years we make no further adjustments. Table 1 shows the adjustments we make for the 3-month implied forward interest rates for different times to settlement.

Table 1: Increasing credit risk in 3-month forward rates for different times to settlement

Time to settlement	6 months	12 months	2 years	4 years
Credit risk	0 bps	4 bps	11 bps	14 bps

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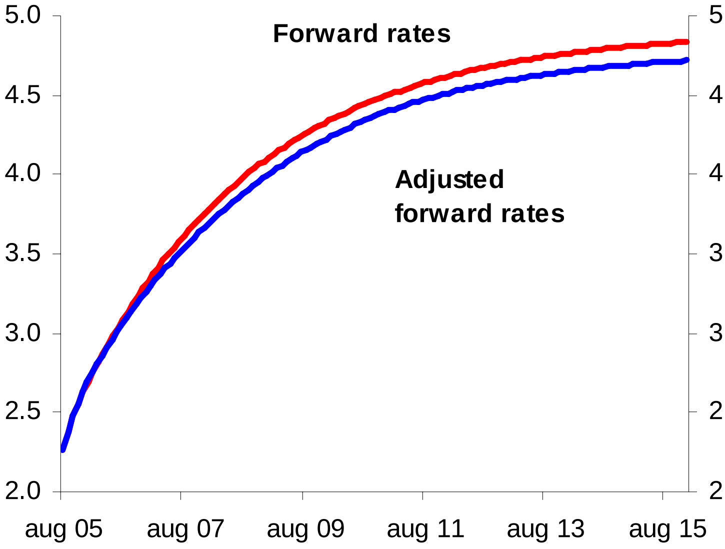
[6] In the period January 2004 - August 2005

Chart 2 shows the implied forward interest rate curve, with and without adjusting for increasing credit risk premia.

Chart 2: Forward interest rates and adjusted rates



This method for adjusting for credit risk is very simple and obviously there are possible sources of error. Firstly, the credit risk premium may vary through time, and using an average can therefore be misleading. However, the period used to estimate the average is fairly short and the calculations are updated on a regular basis. This should therefore reduce some of the potential errors resulting from timevarying credit risk. Secondly, the spread between yields on government bonds and swap rates - the swap spread - will also reflect other factors. As noted earlier, the yields on government bonds may be considerably influenced by variation in supply and demand. Consequently, the swap spread will be affected, and part of it may be due to factors other than differences in credit risk. However, taking an average will probably reduce some of this potential error.

The forward rates will also be affected by a timevarying term premium. As the size of the premium is very uncertain and therefore difficult to estimate, we do not correct for

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[7] A basis point is 0.01 percentage point.

this. [8] The estimated forward rates adjusted for increasing credit risk will therefore normally be interpreted as expected future money market rates.

The discussion so far has been limited to deriving a path for the expected future money market rates. To derive a similar path for the expected key rate, we need to make an adjustment for the difference between the 3-month money market rate and the key rate. Assuming there are no expectations of changes in the key rate the next three months, this difference is normally around 25 basis points. But as for the other premia discussed, this spread may also vary through time.

Conclusions

In this paper we have described the method used by Norges Bank for estimating implied forward interest rates. We have also described the adjustments we make for risk premia in the estimated forward rates. In short, the procedure is as follows:

- Forward interest rates are calculated on the basis of money market rates (NIBOR) with maturities 1, 3, 6 and 12 months and the fixed rates on NIBOR-related interest rate swaps with maturities in the range of 2 to 10 years.

- The estimation technique used is the parametric method developed by Svensson.

- The estimated implied forward rates are adjusted somewhat downwards due to the fact that the credit risk in implied forward rates is likely to increase with the time to settlement. We make no further corrections for other types of potential risk premia.

- The adjusted forward interest rate curve is normally interpreted as a path for the expected money market rate.

- To derive a path for the expected key rate (the sight deposit rate), we adjust the money market curve downwards by 25 basis points.

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[8] However, the Bank has done some work in this area. See Valseth (2003) and Myklebust (2005) for a detailed description of this work.

Appendix

This appendix describes in further detail the estimation method used by Norges Bank when estimating implied forward interest rates.

We consider the parametric forward interest rate function $f(m,\beta)$ proposed by Svensson (1994,1995) where m denotes the remaining maturity and β the parameter vector to be estimated. The corresponding spot interest rate function can be written as the average of the instantaneous forward rates with settlement between 0 and m.

$$s(m,\beta) = \frac{1}{m} \int_{\tau=0}^{m} f(\tau,\beta) d\tau$$

In the estimation we use NIBOR with maturities 1, 3, 6 and 12 months and the fixed rate on interest rate swaps with maturities in the range of 2 to 10 years. These are converted into synthetic bonds, in the way that NIBOR rates are interpreted as zero-coupon bonds and the interest rate swaps are interpreted as par bonds. The coupon rates will then equal the swap rates.

For a given trading date let there be n synthetic bonds (c_j, m_j, y_j, p_j), j = 1, …….., n represented by their coupon rates c_j, time to maturity m_j, observed yield to maturity y_j and the observed price p_j. The coupon bonds have annual coupon payments, and we index the payments by the sequence τ_{jk}, k = 1, …., K_j, where K_j denotes the number of coupon payments for bond j which equals the maturity m_j. The estimated price of a coupon bond $P_j(\beta)$ can be written as the sum of prices of a sequence of zero-coupon (discount) bonds related to each coupon payment and the face value of the bond (normalised to 1), each priced with the discount function

$$d(m_j,\beta) = \exp\left[-\frac{s(m_j,\beta)}{100} m_j\right].$$

Hence

$$P_j(\beta) = \sum_{k=1}^{K_j} c_j d(\tau_{jk}, \beta) + d(\tau_{jK}, \beta), j = 1, \ldots\ldots, n$$

We note that we can characterise each bond either by the observed triplet (c_j, m_j, p_j) or the triplet (c_j, m_j, y_j,) replacing the price p_j of the bond with the bond's yield to maturity y_j. From the coupons c_j, j = 1, …., n and the indexed sequence of payments τ_{jk}, k = 1, …., K_j we can then use the present value function and estimate a corresponding price p_j of bond j.

$$P_j(c_j, m_j, y_j) = \sum_{k=1}^{K_j} \frac{c_j}{\left(1 + \frac{y_j}{100}\right)^{\tau_{jk}}} + \frac{1}{\left(1 + \frac{y_j}{100}\right)^{\tau_{jK_j}}}, j = 1, \ldots, n$$

Alternatively, when we know the observed price p_j on bond j, we can estimate the yield to maturity by solving for y_j in the above equation using a standard Newton-Raphson algorithm.

Likewise, this relationship between y_j and p_j can be used in the parametric case when we derive the discount function from the forward interest rate function $f(m, \beta)$. Hence the estimated yield to maturity for bond j denoted $Y_j(\beta)$ can then be computed from the present value function

$$P_j(\beta) = \sum_{k=1}^{K_j} \frac{c_j}{\left(1 + \frac{Y_j(\beta)}{100}\right)^{\tau_{jk}}} + \frac{1}{\left(1 + \frac{Y_j(\beta)}{100}\right)^{\tau_{jK_j}}}, j = 1, \ldots, n$$

also using a standard Newton-Raphson algorithm.

The observed yield to maturity y_j is assumed to differ from the estimated yield to maturity $Y_j(\beta)$ by a normally distributed error term $\varepsilon_j \sim Niid(0, \sigma_\varepsilon), \forall_j$, *i.e.*

$$y_j = Y_j(\beta) + \varepsilon_j, j = 1,....,n$$

We use the method proposed in Svensson (1994, 1995) and estimate the following forward rate function, with parameters $\beta = (\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2)$:

$$f(m,\beta) = \beta_0 + \beta_1 \exp\left(-\frac{m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1} \exp\left(-\frac{m}{\tau_1}\right) + \beta_3 \frac{m}{\tau_2} \exp\left(-\frac{m}{\tau_2}\right)$$

This relationship is also denoted as the extended Nelson-Siegel forward rate function. The parameters β_0, τ_1 and τ_2 must be positive. Svensson's function is a sum of four components. The first component is a constant, β_0, and is the horizontal asymptote of the forward rate function. It may be interpreted as the constant level the forward rates converge towards in the long term. The second component, $\beta_1\exp(-m/\ \tau_1)$, is monotonically decreasing (or increasing, if β_1 is negative) towards zero when the term to maturity m is increasing. When the term to maturity approaches zero, the forward rate approaches the constant $(\beta_0 + \beta_1)$. This must obviously be non-negative to ensure non-negative forward interest rates. The third component generates a hump (or a U shape, if β_2 is negative) on the curve as a function of the term to maturity. Finally, the fourth component generates an additional hump or U shape. Thus, the function allows for two humps or U shape. The second additional hump or U shape is also the difference between the Svensson method and the Nelson and Siegel method.

It can be shown that the corresponding spot interest rate function can be expressed as:

$$s(m,\beta) = \beta_0 + \beta_1 \frac{1-\exp\left(-\frac{m}{\tau_1}\right)}{\frac{m}{\tau_1}} + \beta_2 \left[\frac{1-\exp\left(-\frac{m}{\tau_1}\right)}{\frac{m}{\tau_1}} - \exp\left(-\frac{m}{\tau_1}\right)\right] + \beta_3 \left[\frac{1-\exp\left(-\frac{m}{\tau_2}\right)}{\frac{m}{\tau_2}} - \exp\left(-\frac{m}{\tau_2}\right)\right]$$

The parameters in the forward rate function β are estimated by solving the following maximum likelihood estimation problem:

$$\hat{\beta} : \underset{\beta}{Max}\left[-\frac{n}{2}\ln(2\pi\sigma_{\varepsilon}) - \frac{1}{2}\sum_{j=1}^{n}\left(\frac{y_j - Y_j(\beta)}{\sigma_{\varepsilon}}\right)^2 \right]$$

inserting the following MLE: $\hat{\sigma}_{\varepsilon} = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(y_j - Y_j(\beta))^2}$ for σ_{ε}.

The continuously compounded spot and forward interest rates that are derived from the equations above for a given $\hat{\beta}$, are finally transformed into annually compounded interest rates, i.e.

$$s_a(m,\hat{\beta}) = 100\left[\exp\left(\frac{s(m,\hat{\beta})}{100}\right) - 1 \right]$$

$$f_a(m,\hat{\beta}) = 100\left[\exp\left(\frac{f(m,\hat{\beta})}{100}\right) - 1 \right]$$

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