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Long-term outlook for fixed income and equity return

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posten@norges-bank.no
or from:Norges Bank, Subscription service,
P.O.Box. 1179 Sentrum
N-0107 Oslo, Norway.
Tel. +47 22 31 63 83, Fax. +47 22 41 31 05

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Norges Bank, Abonnementsservice
Postboks 1179 Sentrum
0107 Oslo
Telefon 22 31 63 83, Telefaks 22 41 31 05

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1 Purpose of the analysis and basis for the current asset allocation

1.1 Introduction

In the management of the Petroleum Fund and the foreign exchange reserves, an organisational distinction is drawn between strategic choices on the one hand and tactical departures from these choices through active management as well as tactical asset allocation on the other. Responsibility for strategic choices for the two funds lies, respectively, with the Ministry of Finance and the Executive Board of Norges Bank, while Norges Bank Investment Management (NBIM) performs the active management.

The basis on which strategic choices are made is wide-ranging, and closely tied up both with the purpose the funds are intended to serve and with the owners' overall wealth situation. Both these elements have a bearing on the owners' perception of relevant risk.

The owners' strategic asset allocation differs from the manager's tactical asset allocation in terms of information relevancy, time horizon, manner of implementation and as a rule also risk perception. Strategic asset allocation aims to describe an optimal allocation of investments relative to the owner's view of long-term market prospects, the purpose of the fund, and the owner's total wealth and attitude to risk. In principle all the above factors may change over time, although, as a rule, it will be changes in long-term market prospects that prompt the investor to revise his strategic allocation. This is normally done by changing the benchmark portfolio or other guidelines contained in the management mandate.

Tactical asset allocation is normally organised as a source of excess return relative to a benchmark portfolio, such that the manager is not required to take account of the investor's overall situation when contemplating active positions. A common time horizon for positions is three to six months. Positions are taken on the basis of current market information usually via futures contracts. Tactical asset allocation resembles a zero-sum game, with just as many losers as winners. Strategic asset allocation is in a different category, since time-varying risk-taking may be seen as an adjustment to the fact that the owner of the fund is not synonymous with the average investor, and thus may have a different personal valuation of a trade than that reflected in the market price.

An important aspect of strategic asset allocation is assessing long-term characteristics of differing broad investment categories or asset classes. Such characteristics are commonly summated by an envisaged probability distribution of returns for the asset classes in question. This distribution is in turn often summated using parameter values for expected return, standard deviation and correlation between asset classes and over time. The owner has to decide whether further parameters need to be considered, such as skewness or tail probabilities. It is important to be aware that a distribution projected on the basis of recent historical experience normally only describes parts of the overall uncertainty: Special events such as wars or other dramatic systemic changes could have a large bearing, especially in case of a long time horizon.

By *market outlook* we mean expected return and risk within a set of relatively normal scenarios based on the current situation. It is difficult to recognise special, important events before they have become history. In the 1900s markets were heavily affected by two world wars, Japan's spectacular entry into the world economy after the Second World War and increased globalisation in general. Examples of important events in recent years are China and India's vigorous economic growth and the size of the USA's accumulated national and foreign debt. How far such events will affect the relationships we are examining in the light of recent history is highly uncertain. The purpose of this report is to consider the long-term market outlook in the broad sense, but confined to markets and asset classes in which the funds are currently invested.

The investment horizon applied to the Petroleum Fund and the foreign exchange reserves is long. Parts of the Petroleum Fund are phased into the Norwegian economy via the fiscal spending rule each year, but the fund's life expectancy is probably unlimited. An average maturity profile can be calculated from the fiscal spending rule which on reasonable, but uncertain, assumptions produces a duration of about 35 years.

In this report we apply an assessment horizon of 10-15 years. This is shorter than the average maturity profile, because we believe information of a nature likely to affect return 10-15 years ahead should be reflected in the strategic asset allocation. This horizon is at the same time substantially longer than that applied for active management purposes.

Dramatic or sudden events of significance for such a long time horizon are likely to be rare. Expectations towards this horizon are more likely to change as a result of the slow development of economic cycles, or the emergence of disproportional market prices (e.g. bubbles).

It is difficult to draw a clear distinction between information and views relevant to long-term market prospects and views relevant to shorter horizons. Moreover, it is not self-evident that a rational basis exists for revising long-term market views over time, i.e., it is controversial whether long-term market returns are predictable or not. One crucial element in this respect is whether financial markets move in cycles around a hypothetical equilibrium, or fundamental value, and how rapid such cycles are. Another reason for revising our perceptions of the market outlook may be a change in the understanding of the way markets work.

However, a number of empirical studies suggest that widely available information about the market's so-called fundamental value has a bearing on expected return in the long term, i.e. a horizon from 5 to 15-20 years. While several studies show stronger results for the longest section of this range, the results are open to many econometric objections.

We believe that asset classes' long-term characteristics probably change over time. Hence a regime of periodic evaluation, and possible revision, of strategic allocation is needed. However, given the substantial uncertainty encumbering such assessments, it is unlikely that the strategic allocation will need to be revised as often as each year.

Norges Bank aims to present strategy reports on an annual basis. They will be two-pronged: one part will discuss the long-term market outlook; another will discuss what implications this has for strategic portfolio choice given the respective purposes of the Petroleum Fund and the foreign exchange reserves. These yearly reports will be confined to asset classes currently included in the funds' investment universe.

Every third year Norges Bank will carry out a comprehensive analysis in which all aspects of the strategic asset allocation will be reviewed. This analysis will be less frequent due to the slower pace of change in information relevant to new asset classes and markets.

The present report is Norges Bank's annual assessment of the market outlook.

1.2 Basis for the current asset allocation

The current basis for strategic allocation decisions cannot be depicted in terms of a close relationship between precise market assumptions, preferences and a portfolio selection model. It is more correct to see choice of allocation as the result of a discretionary assessment, in which a number of factors have been given weight. While market prospects were a part of this picture, these views were not formulated explicitly.

The question of whether a changed market outlook should prompt a changed strategic allocation depends on whether current perceptions differ significantly from the assumptions upon which the current allocation was based. When the significance of such a change in perceptions is assessed, the uncertainty attached to the estimates needs to be taken into account. Next, the characteristics of the new allocation must be better by a margin sufficient to warrant the costs of making the changes.

Although the market outlook on which the current allocation is based is not formulated in explicit terms, an impression of it can be gained by studying various background material. We employ three methods to form our notion of a neutral starting point: 1) studying written sources that discuss the present allocation, 2) attempting to ascertain the market consensus at the time the allocation was decided, which for the Petroleum Fund's part was essentially in 1997, and 3) resolving an optimisation problem in reverse to ascertain what premises would have been consistent with today's allocation being mean-variance efficient while also meeting explicit overall expected return targets.

1.2.1 Written sources for the basis for allocation decisions

The basis for current investment of the Petroleum Fund has been explained in a number of documents. Some important documents describing the purpose and to some extent premises attached to a variety of asset classes are listed below in chronological order:

1. Proposition no. 29 to the Odelsting (1989-90) concerning the Act on the Government Petroleum Fund
2. Letter from Norges Bank to the Ministry of Finance, April – 1997: "Future management of the Government Petroleum Fund" (Calculations taken in part from Olsen, J.P: "How much of the Government Petroleum Fund should be invested in equities", Penger og Kreditt 1997/2 pp.231-237)
3. Revised National Budget 1997
4. Report no. 29 to the Storting (2000-01): Guidelines for economic policy (the fiscal spending rule). March 2001
5. Norges Bank's letter to the Ministry of Finance, March 2001: "An analysis of the Petroleum Fund's equity component"
6. Norges Bank's letter to the Ministry of Finance, March 2001: "Non-government-guaranteed bonds in the reference index"
7. Norges Bank's letter to the Ministry of Finance, April 2002: "An assessment of the Petroleum Fund's regional weightings"
8. Norges Bank's Strategy Report, August 2003
9. Norges Bank's letter to the Ministry of Finance, September 2004: "The Petroleum Fund – Inflation-indexed bonds in the reference portfolio"

The documents explaining the rationale for asset allocation are rather vague about the underlying premises, and they invariably stress the uncertainty of any numerical estimate. The strategic allocation of the foreign exchange reserves has built on the same foundation, and changes have often been made at about the same time.

The most central premise for the strategic allocation is the equity premium, i.e. the expected excess return on equities relative to government bonds. The clearest statements on the size of the equity premium are to be found in the Revised National Budget for 1997 and in Norges Bank's letter of April 1997 to the Ministry of Finance. A historical equity premium of 4% is cited, and the likelihood of negative return on a one-year horizon in a global equity portfolio is put at 16.6%, based on an expected return on equities of 15.5% and a standard deviation of 16%. Moreover, an assumption appears to be made that equities are less risky in the long term than in the short term (with reference to Jeremy Siegel's book "Stocks for the Long Run"). A key reason for considering equities at that juncture was the probable extension of the Petroleum Fund's time horizon. Such an argument implies a belief that there is a cyclical pattern to equity return (mean reversion).

In December 2000, at the ministry's request, a new assessment of the equity component was carried out, c.f. Norges Bank's letter to the Ministry of Finance of March 2001. This letter employs new and enlarged data material, and refers to a historical excess return of 5 – 6%. However, it also cites an average premium of 3.6% over a 200-year period in the USA. The letter discusses valuation methods, and points out that historical mean values do not necessarily provide the best estimates of long-term expectations. It points out that valuation methods indicate a lower expected return than the historical average.

Norges Bank's letter to the Ministry of Finance of March 2001: "Non-government-guaranteed bonds in the benchmark index", contains a section discussing excess return on bonds not backed by a government guarantee. Reference is made to historical figures for the period 1989-1997 showing a realised excess return of 0.5 percentage points on private corporate bonds in the USA. Over a longer period, 1920-1999, a corresponding estimate for a long duration corporate bond is 0.7 percentage points. Expected excess return is however only one of several arguments for introducing this type of bonds in the benchmark portfolio, according to the letter.

Norges Bank's letter from 2002 concerning regional shares contains a brief discussion of rates of return in the various regions. The letter points to high correlation between the return in various regions, taking this as an indication that the expected return is approximately identical across regions. However, it also refers to low realised returns in Japan in recent years, suggesting low expectations of this market, especially in the case of bonds.

The letter about inflation-indexed bonds is also very brief when it comes to expected return and risk. The assumption is that while this asset class will reduce expected return and risk somewhat, it will probably improve the trade-off between expected return and risk for the portfolio as a whole. Hence it will provide a basis for an increased allocation to riskier, higher yielding asset classes.

Norges Bank's Strategy Report from August 2003 draws a clearer distinction between conditional and unconditional[1] expectations. Unconditional expectations are elucidated using

[1] The term "conditional expectation" refers to an expectation where account is taken of the information set which is available at the point in time in question. An unconditional expectation is unaffected by current information. The information set may typically contain price history, current dividend rate, current interest rate, current inflation level etc. A definition of absence of predictability is that there is no difference between a conditional

historical data from Dimson, Marsh and Staunton (2002). The report points out that the average return on a portfolio resembling the Petroleum Fund's benchmark portfolio is 3.6% in the period 1900-2002, measured in local currency. The standard deviation is 10.5 percentage points. Several other data from this material are also shown, but arguments in Dimson, Marsh and Staunton (2002) are cited to the effect that historical means, even over such long periods, may be affected by repricing effects which make them less suited as expectation estimates.

Although the report's valuation chapter does not provide explicit estimates of the expected returns, it emerges that conditional expectations do not differ essentially from those employed when the current asset allocation was decided. Given the regime's high threshold for changes in the strategic allocation, revision of the benchmark portfolio was not recommended. It was however recommended that the Ministry of Finance should take a closer look at the possibility of a more dynamic regime for strategic allocation decisions.

The "fiscal spending rule" was established in March 2001 in Report No. 29 to the Storting (2000-2001). The report premises an expected real rate of return of 4% for the benchmark portfolio, which is the most precise public statement we have on the assumptions underlying the current strategic allocation – assuming that there is a connection between the fiscal rule and the allocation. Table 1 shows combinations of real bond return and equity premium giving a real total return of about 4% for a selection of realistic equity premiums.

Real bond return	Equity premium
2,3	4,0
2,4	3,8 - 4,0
2,5	3,5 - 4,0
2,6	3,3 - 3,7
2,7	3,0 - 3,5
2,8	3,0 - 3,1
2,9	3,0

Table 1.1: *Combinations of real bond returns and equity premiums providing a real rate of return between 3.9 and 4.1 per cent (the real return of equities is the sum of the two components in the table). Computed for a portfolio containing 40% equities and 60% bonds, and given an equity premium between 3 and 4 percent. Source: Norges Bank.*

1.1.2 The average agent's view in 1997

Identifying today's consensus view is difficult enough. Identifying the average perception eight years ago, when the basis for deciding the current allocation was formulated, is even more difficult.

Generally speaking this was a time of great optimism in the equity market, and a discussion was under way on whether the economy had now reached a new juncture where established

and an unconditional expectation for any information set – which will be the case if prices follow a random walk. Conditional expectations are often based on valuation models, while a historical average over a very long period is often used as a proxy for an unconditional expectation.

perceptions of reasonable valuation rates had been rendered obsolete by the impact of new information technology. A year or so previously the American Federal Reserve chairman, Alan Greenspan, had warned against excessive optimism in the equity market, pointing to the disproportion between equity prices and direct return (earnings) at many enterprises. The Federal Reserve's warnings were toned down in the ensuing years, however.

Jeremy Siegel's book, "Stocks for the Long Run", first published in 1994, appears to be a central source in parts of the background material for the current allocation regime (quotes from it appear in the Revised National Budget for 1997). This book, which long remained a best-seller, paints a highly optimistic picture of equity investments. Even so it suggests an equity premium between 2 and 3 per cent (Siegel 1994, p. 20), which would seem to be on the sober side given the historical rates of return cited. Robert Shiller's book, "Irrational Exuberance", was only published in March 2000 and the author was lauded for his warnings against an overvalued market.

A search of The Economist's articles in the period January 1996 to October 1997 brings up 40 or so articles dealing with the equity market. While many describe the market as highly priced, they do not give a clear indication of future expectations. A few articles voice an opinion with titles such as "Crash, dammit"; "Breaking a fall"; "Hot all over", and "When the rain comes".

The period was marked by a long-lasting equity market boom through the nineties, but also by signs of nervousness and speculation about how long the high-return scenario could last.

1.2.3 Model-based analysis of implicit assumptions behind the current strategy

The decision not to employ stringent mean-variance optimisation as a basis for the current allocation was deliberate. This decision was based on the assumption that an analysis of this type would fail to capture all relevant aspects of the portfolio choice for the Petroleum Fund.

Special events, political risk in the country being invested in, and risk associated with the ability of the fund to survive as a fiscal policy instrument are examples of factors not captured in a mean-variance analysis. A discretionary assessment based on extensive background information, mean-variance optimisation being only one of several inputs, was opted for instead.

Hence it is a matter of chance whether the current benchmark portfolio is or is not found to be on the mean-variance efficient frontier. A difference between the assumed return and risk expectations on the one hand, and expectations that would have led to an optimised allocation on the other, may reflect a number of factors. Examples include a possible ad-hoc correction of risk estimates (for presumed weaknesses associated with volatility as a risk measure), or a demand for risk premiums deviating from the average market demand, since the allocation problem at hand only concerns part of Norway's total wealth.

It is similarly unlikely that the market's consensus expectations match the expectations that render the market portfolio optimal in terms of mean-variance, for one thing because supply and demand for securities is probably not guided by global optimisation routines of such simplicity.

Nonetheless it is pertinent to consider the current allocation as though it were the result of an optimisation. This approach provides a consistent set of assumptions within a model, and if these are found to deviate substantially from the assumptions employed, the deviation will be of interest when the current updated market outlook is translated into a portfolio choice.

A portfolio optimisation problem normally includes a set of weights as a function of a co-variance matrix (which includes variance), expected returns and either a required return or a cap on total risk. Similarly, reverse optimisation is finding a set of expectations as a function of a set of known weights, a required overall return for the entire portfolio and a co-variance matrix. The solution to such a problem is particularly simple: Expected return for an asset class equals this class's beta relative to the total portfolio, multiplied by the overall required return.

In our context it is pertinent to take a basis in statements of expected total return for the Petroleum Fund. A real overall required return of 4% for the current allocation of the Petroleum Fund is consistent with the fiscal spending rule. We perform a simple analysis involving six asset classes: equities and government bonds in, respectively, USA, Europe (United Kingdom, France and Germany) and Japan. We employ in the first instance a co-variance matrix calculated on the basis of Dimson, Marsh and Staunton (2002), with annual data in local currency for the period 1950-2003. This is shown in Table 1.

It is difficult to find explicit risk estimates in the background material, although an analysis reproduced in the Revised National Budget for 1997 employs a standard deviation of 4.7% for the bond return and 16.0% for the equity return based on annual, historical data for the period 1950 to 2003.

			Implicit		Data: Dimson, Marsh & Staunton, 1950 - 2003, annual						
			Expected	Risk	Standard	Correlation					
Asset class		Weight	return	premium	deviattion	USA	Europe-3	Japan	USA	Europe-3	Equities
Equities	USA	0.42	**5.65 %**	**3.19 %**	17.76 %						
	Europe-3	0.50	**6.87 %**	**4.27 %**	20.80 %	0.68					
	Japan	0.08	**3.24 %**	**-0.04 %**	30.48 %	0.28	0.32				
Bonds	USA	0.35	**2.47 %**	**-**	11.30 %	0.19	0.13	(0.10)			
	Europe-3	0.55	**2.60 %**	**-**	8.96 %	0.32	0.38	(0.14)	0.61		
	Japan	0.10	**3.28 %**	**-**	19.21 %	0.29	0.18	0.11	0.18	0.30	
Equities	Global	0.40	**6.07 %**	**3.45 %**	17.39 %						
Bonds	Glboal	0.60	**2.62 %**	**-**	8.71 %						0.32
Total	Global	1.00	4.00 %	-	9.93 %						

Table 1.2: *Key figures (in bold) based on a historical co-variance matrix. Numbers in bold are model results, other figures are assumptions. Source: Norges Bank.*

We see that the implicit risk premium for equities over bonds is 3.45%. This is consistent with the most precise statements to be found in the background material, which cites a risk premium of the order of 3 – 4%[2].

However, Table 1 also shows wide variation in implicit risk premiums across regions. In order for the low allocation to Asia to be optimal, given the region's good diversification characteristics, we need to believe in a risk premium in Asia close to zero. We must also believe in a relatively high expected return on Japanese bonds, mainly because historical risk associated with Japanese bonds is relatively high.

[2] While no direct mean estimates are given in the background material, various estimates are referred to on a number of occasions, although their uncertainty is invariably emphasised. In 1997 reference is made to historical estimates of 4%, in 2001 to historical estimates of 5-6% and of 3.6%.

Such statements do not appear consistent with the background material. It is clear that regional shares are not determined on the basis of a differentiated view of expected long-term return and risk, but on the basis of a rationale in which future import patterns, geographical diversification (war and catastrophes) and operational simplification (market weight between Asia and the USA, equities) are key arguments.

If our correlation and risk estimates are not differentiated across regions, an implicit optimisation will produce identical expected return for all regions. In Table 2 we employ equally weighted averages for correlations and an average weighted with regional weights for the standard deviation.

			Implicit		Data: Dimson, Marsh & Staunton, 1950 - 2003, annual - averages						
			Expected	Risk	Standard	Correlation					
Asset class		Weight	return	premium	deviattion	USA	Europe-3	Japan	USA	Europe-3	Equities
Equities	USA	0.34	**6.10 %**	**3.47 %**	20.55 %						
	Europe-3	0.33	**6.05 %**	**3.44 %**	20.55 %	0.43					
	Japan	0.33	**6.05 %**	**3.44 %**	20.55 %	0.43	0.43				
Bonds	USA	0.34	**2.63 %**	**-**	10.81 %	0.15	0.15	0.15			
	Europe-3	0.33	**2.61 %**	**-**	10.81 %	0.15	0.15	0.15	0.36		
	Japan	0.33	**2.61 %**	**-**	10.81 %	0.15	0.15	0.15	0.36	0.36	
Equities	Global	0.40	**6.07 %**	**3.45 %**	16.16 %						
Bonds	Glboal	0.60	**2.62 %**	**-**	8.20 %						0.25
Total	Global	1.00	4.00 %	-	9.07 %						

Table 1.3: *Key figures (in bold) based on a subjectively adjusted co-variance matrix and equal regional weights. Numbers in bold are model results, other figures are assumptions. Source: Norges Bank.*

We see that this produces an unchanged risk premium of 3.45%, an equity volatility of 16.2% and a bond volatility of 8.2%.

The implicit risk premium in Table 1.3 is of course especially dependent on the difference in risk between equities and bonds. In the current allocation strategy it is assumed that equities, due to mean reversion, are less risky in the long term than over shorter horizons. However, the standard deviation in Table 1.3 is calculated over annual rates of return, and does not reflect a long-term investment horizon.

How much lower the risk can be envisaged to be on a 10-15 year horizon depends on the form of mean reversion that best describes the market. In the simplest model, where return varies around a *constant* level, the standard deviation of horizontal return may fall by 15-20% relative to the standard deviation of the annual return for a horizon of 15 years, given normal parameter values. In a somewhat more realistic model in which return varies around a *time-varying* level, the horizon effect is only this marked in a 30-year perspective, and is virtually zero in a 15-year perspective. See for example Lo and Wang (1995) for an analysis of such models.

If the historical average for equity volatility for each country in table 1.3 is adjusted down by 10%, equity volatility comes to 18.5%. Table 1.4 shows the result of this downward adjustment.

A comparison of tables 1.3 and 1.4 illustrates that mean reversion in the equity market leads to a horizon effect on the risk premium. Table 1.3 can be interpreted as an estimate of the annual equity premium, while table 1.4 reflects an annualised 15-year equity premium.

Furthermore, we see that for a given overall required return of 4%, and risk and co-variance matrix as in Table 1.4, the equity premium must be 2.8% in order for the current equity component of 40% to be risk-minimising. We also see that diversification brings the global equity risk to 15% and fixed income volatility to 8%. The fund's overall level of risk is 8.5%.

			Implicit		Data: Dimson, Marsh & Staunton, 1950 - 2003, annual - adjusted averages						
			Expected	**Risk**	Standard	Correlation					
Asset class		Weight	**return**	**premium**	deviattion	USA	Europe-3	Japan	USA	Europe-3	Equities
Equities	USA	0.34	**5.69 %**	**2.79 %**	18.50 %						
	Europe-3	0.33	**5.65 %**	**2.77 %**	18.50 %	0.43					
	Japan	0.33	**5.65 %**	**2.77 %**	18.50 %	0.43	0.43				
Bonds	USA	0.34	**2.91 %**	**-**	10.81 %	0.15	0.15	0.15			
	Europe-3	0.33	**2.88 %**	**-**	10.81 %	0.15	0.15	0.15	0.36		
	Japan	0.33	**2.88 %**	**-**	10.81 %	0.15	0.15	0.15	0.36	0.36	
Equities	Global	0.40	**5.66 %**	**2.77 %**	14.55 %						
Bonds	Glboal	0.60	**2.89 %**	**-**	8.20 %						0.25
Total	Global	1.00	4.00 %	-	8.53 %						

Table 1.4: *Key figures (in bold) based on equal regional weights, subjectively adjusted co-variance matrix and a downward adjusted equity volatility due to assumed horizon effects from mean-reversion. Numbers in bold are model results, other figures are assumptions. Source: Norges Bank.*

An equity premium of 2.8% is low relative to what was stated, and probably also low relative to average expectations in 1997. Another way of interpreting table 1.4 is: With an equity component as low as 40%, the equity premium does not need to be higher than 2.8% to attain the required return of 4% with a portfolio that is efficient, given the risk picture in table 1.4. All else being equal, a premium of 3.7% would have been required to make a 60% equity component optimal[3]. It is in other words difficult to distinguish between the investor's risk aversion and market expectations when using this method.

Again it is stressed that these are implicit assumptions, reflecting restrictions that were considered relevant when the allocation was decided. Any market-based updating of the estimates of expected returns must undergo similar discretionary corrections before they are compared with the above assumptions.

[3] In this calculation, the overall total return requirement is kept at 4%, despite an increase in the equity component. An alternative is to assume that expected return on bonds remains unchanged at 2.88%. If a 60% equity component were retained exclusively on account of a higher expected equity premium, the equity premium would have to have been as high as 6% and the overall total return would rise from 4% to 6.5%.

1.3 General methods for updating the basis for allocation decisions

There are several ways of approaching an assessment of whether the long-term market outlook, or expected return and risk, have changed in relation to the last time the issue was addressed.

The most usual method, i.e. looking at historical averages over a long period, rarely produces anything new from year to year, unless new relevant data sets have become available. The most recent thorough analysis of historical returns across a number of markets was presented in the latest strategy reports for the Petroleum Fund and the foreign exchange reserves, in August 2003 and April 2004 respectively. Hence historical analyses are more relevant when it comes to analysing the unconditional expectation.

In order to arrive at meaningful conditional expectations it is necessary to believe that the conditional information is not already fully reflected in the value of the asset. A series of valuation indicators are commonly used for equities, bonds and currencies. Such indicators are used to describe so-called fundamental value, and the conditional expectation derives from comparing current value with this assumed equilibrium value.

The third standard method for revising the basis for allocation decisions is also based on the difference between current market value and a fundamental value. However, instead of reasoning on the basis of, say, the dividend-discount model, one describes probable scenarios for the development of important explanatory factors, and seeks to describe a relationship between these factors and expected return. The strength of such scenario analyses is that they are forward looking and do not suffer from problems of possible structural breaks or erroneous model specification. The disadvantage, on the other hand, is that few objective bases are available for selecting correct explanatory variables and scenarios. Hence there is a risk that the analysis will be skewed in the direction of imbalances which, while relevant today, may prove to represent noise in the longer term.

In the previous strategy report on the foreign exchange reserves, from April 2004, two factors were closely discussed in the context of such an approach. It was pointed out that the most likely market consequence of the USA's growing current deficit was a drop in the value of the dollar. It was also pointed out that Japan's economic situation remains problematic and that ways out in the direction of new economic growth probably entail a major downside risk for foreign investors.

A fourth category of methods may be described as identifying the consensus. The object here is to determine the average, or most widespread, expectations held by agents in the market. Interview surveys are the most common method used to bring to light consensus expectations. An alternative is to find those assumptions that turn a market-value weighted portfolio into a mean-variance optimised portfolio, given a supposition of expected return on the entire market portfolio.

Average expectations are of interest as an aid to estimating objective probability distributions for return. The extent to which implicit average expectations are relevant as a neutral basis for strategic allocation is, however, dependent on a number of factors, and is closely related to how far (the global) market portfolio is representative of many investors. Such implicit expectations are probably more relevant for a relative investor with a neutral point of departure, than in a strategic context where differences in investors' total wealth may prompt different allocation decisions.

Long-term, conditional expectations may have implications for a strategic investment choice in several ways. A much-studied theme has been how an agent's own uncertainty as regards model choice and parameter choice should be allowed to influence the implementation phase. This uncertainty is often termed estimation risk.

A promising method in this context is Bayesian updating. This method systematically incorporates the result of a valuation analysis in one's initial assumptions, and provides a probability distribution which takes account of estimation risk. The new information can intuitively be envisaged to shift the average in the distribution, while the estimation risk extends the range of possible outcomes, or the risk in the distribution. It is this predictive distribution that is relevant for the allocation decision, not a distribution which merely incorporates assumed asset characteristics.

In the following we review various methods of assessing probability distributions for bond return and equity return. We incorporate the estimation risk by making a subjective addition to the risk in the distributions.

The next chapter deals with return in the bond market, followed by a chapter on the equity premium.

2 Fixed income markets

2.1 Introduction

This chapter assesses expected return and risk in bond markets with a view to possible changes in the current strategic allocation of the Petroleum Fund and the foreign exchange reserves. Such assessments are attended by great uncertainty. Any implications for strategic allocation will also depend on how certain the owners of the funds are that the parameters in the probability distributions are correctly estimated.

The investment horizon for strategic decisions is normally long, indeed longer than an active investment mandate could be envisaged to capture. We employ an investment horizon of 10-15 years. With such a long horizon, current mean estimates are unlikely to deviate substantially from unconditional expectations, i.e. expectations in a 50-100 year perspective. This analysis is designed to bring to light factors which are sufficiently significant to be envisaged to also have an effect for a 10-15 year investment horizon.

This chapter is structured as follows: It begins with a brief discussion of the individual components of bond return. This is followed by an overview of historical market trends in interest rates and return, inflation, credit margins and term premiums. The ensuing section briefly discusses how an equilibrium level for the interest rate can be estimated. This is followed by a section discussing the relationship between interest rate level and bond return, and what emphasis should be given to, respectively, current interest rate levels and the long-term equilibrium level when estimating future return. This section also discusses historical return and risk. We then turn to the consensus perception in the market. The chapter concludes with a summary leading to a description of probability distributions for bond return in the main markets.

2.2 Decomposition of expected return on bonds

Government bonds provide virtually secure cash flows from coupon payments and principal. A fixed income portfolio can be viewed as the sum of many such cash flows. The market value of a fixed income portfolio is linked to the valuation of these cash flows, which in turn are determined by the shape of the yield curve at the present time and how it develops ahead.

The return on a bond can be decomposed: Direct return is the current interest income on the bond's coupon. A much-used measure of direct return is redemption yield, which is the return produced by the bond if it is held to maturity and current disbursements are reinvested at current interest rates (a constant interest-rate level is assumed).

Capital return approximates to the interest rate change over a period, multiplied by the investment's duration (with the opposite sign)[4]. Over short investment horizons capital return will often be of greater significance than direct return. Over long horizons, on the other hand, direct return will be the most important component, and reinvestment risk is the most

[4] There are two important sources of error in such an approach: The so-called convexity effect derives from the fact that the relationship between an interest-rate change and the change in price of the bond is not linear. For most bonds an interest rate fall will produce a larger price increase than the price decrease resulting from an equally large interest rate increase. Another important source of error is the roll-down effect, which arises if the yield curve generally has a specific slope, e.g. rising. Since a bond's maturity is diminishing, it will be priced at market rates for a shorter horizon, and this will produce a capital gain if the yield curve is normally rising. This effect arises where the investor estimates expected return against a final horizon and he invests in securities which do not coincide precisely with this horizon.

important risk factor. Where interest rates move in long-term trends, however, capital return may be significant even on very long horizons.

An interest rate change has differing significance for an investor using instruments with a maturity shorter than his investment horizon compared with an investor whose investment horizon is shorter than the instruments utilised. For the first investor reinvestment risk, not price risk, is of overriding importance: an interest rate rise will entail higher long-term return on the investment. For the second investor, on the other hand, return will be lower. Macauley duration can be defined as the investment horizon at which the reinvestment effect compensates for the change in capital value resulting from an interest rate change.

The fixed income portfolio's duration is significantly shorter than average investment horizon in the case of both the Petroleum Fund and the foreign exchange reserves. The bond index has a duration of about 5.5 years. In other words, assuming an assessment horizon of around 10-15 years, the reinvestment risk will be substantial. Direct return is thus only partially observable based on the current interest rate level.

On the other hand, the fixed income market offers information on the return that would have materialised after 15 years if one was willing to bind the investment for that period today. Hence the expected return, also in a 15 year perspective, is likely to be affected by the current interest rate level.

It is widely assumed that a long-term equilibrium level exists for the short real interest rate. If this is the case, it must also be assumed that current interest rates are more likely to move in the direction of this equilibrium level[5] than in the opposite direction in the long-term. This has to apply even if it is assumed that the equilibrium level can also change over time[6]. The *return* on a portfolio will depend on how strongly the market interest rate is assumed to be pulled towards the equilibrium rate.

2.3 The market in a historical perspective

In the following we provide a brief overview of the history of nominal interest rates, inflation, estimated real interest rates and credit margins.

Interest rates

Chart 2.1 shows a falling trend in the nominal global interest rate level over the past 25 years, reflecting inter alia a falling trend in inflation. It also shows some degree of convergence among the various countries' interest rates, especially between the USA and the EU member states.

Nominal long-term interest rates can be decomposed into long-term expected inflation, short-term real interest rates and the risk premium for long maturities (the term premium). Ilmanen (2004) shows that all three components have fallen over the past 20 years.

[5] We apply the term neutral interest rate to a long-term equilibrium interest rate on a 15 year horizon.

[6] In this literature it is common to explain changes in the equilibrium interest rate level in terms of variables affected by the business cycle. Hence a pattern of mean reversion in the interest rate could be expected if we indeed observe business *cycles* and not just business *fluctuations*. In other markets, e.g. the stock market, the equilibrium level moves around more randomly. Hence the existence of an equilibrium does not imply mean reversion in prices.

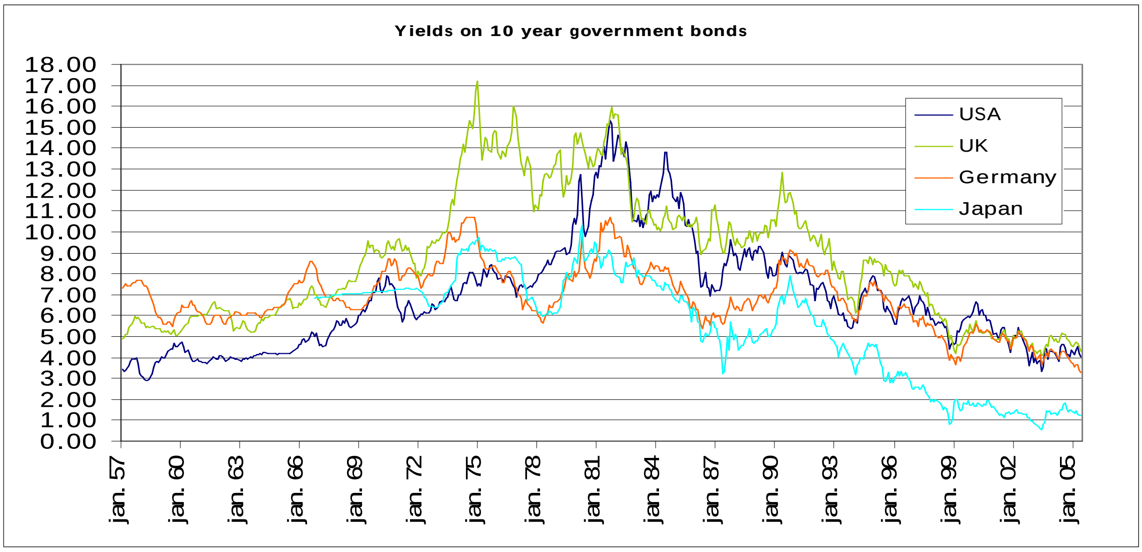

Chart 2.1: *Nominal 10-year government bond rates in the USA, United Kingdom, Germany and Japan. Sources: IMF and Datastream*

Chart 2.2 shows a simple estimate of ex ante real interest rates from 1980 to the present. The estimate is calculated as the interest rates shown in chart 2.1 less a two-year monthly average of inflation rates, which are used here as an estimate of a long-term inflation expectation. Here too, signs of convergence are evident. We see that Japanese real interest rates diverge less than their nominal counterparts.

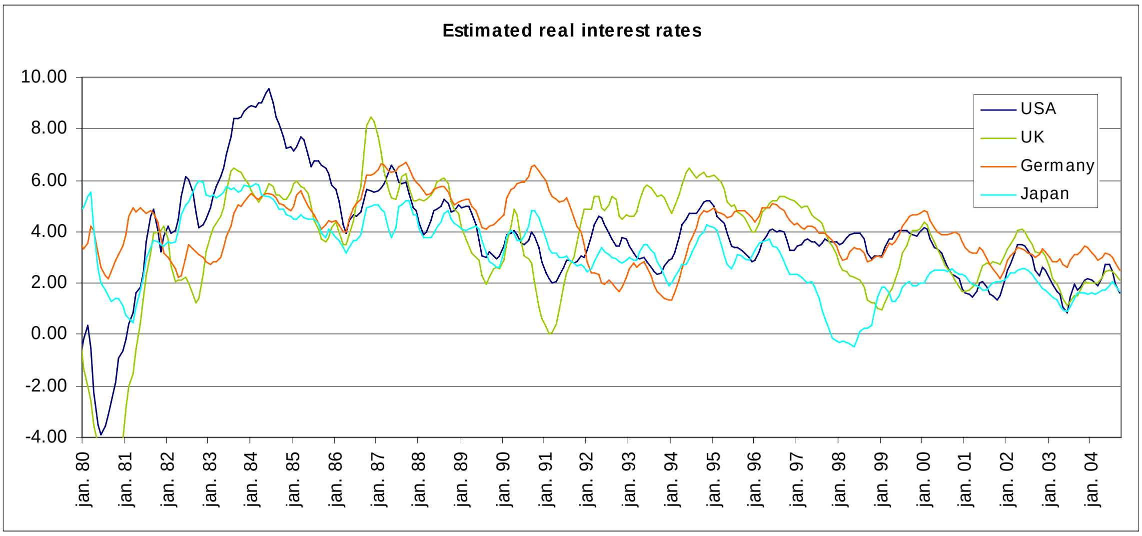

Chart 2.2: *Estimated ex ante real interest rates as the 10-year nominal bond rate less a 2-year moving average of annual inflation rates.*

The historical real return on long government bonds has previously been commented on in the Strategy Report for the Petroleum Fund for 2003. Based on the data material behind Dimson et al. (2002), the following table gives an overview of real return on the main currencies discussed here.

There is particular reason to note the consequence of World War II for Germany and Japan. Government bonds from these countries lost virtually their entire value before 1950, while the subsequent buoyant return was probably influenced by the need for reconstruction. A simple non-weighted average is a meaningful estimate of the equilibrium real interest rate if it is assumed that the relative size of these economies has varied nearly randomly over the past hundred years, and that there is no correlation between a country's size and its level of real interest rates. In the post-war period, the average for the countries in Table 2.1 has been 2.77 per cent.

Annual local real return

	1900 - 2003	1950 - 2003
USA - USD	1.88 %	2.03 %
Germany - DEM	-2.03 %	3.82 %
UK - GBP	1.34 %	1.72 %
Japan - JPY	-1.25 %	3.51 %
Average	-0.01 %	2.77 %

Table 2.1: *Geometric averages of annual real return on nominal government bonds with a long maturity, for various periods and markets. Source: Ibbotson Associates.*

Inflation and inflation expectations

Inflation expectations are not directly observable. One approach to estimating such expectations is by way of interview surveys. Another is to assume that expectations are mainly based on realised inflation in the recent past. A third way is to start out from the difference between the interest rate on inflation-indexed bonds and nominal bonds. This difference will reflect an estimate of market inflation expectations plus the market inflation risk premium.

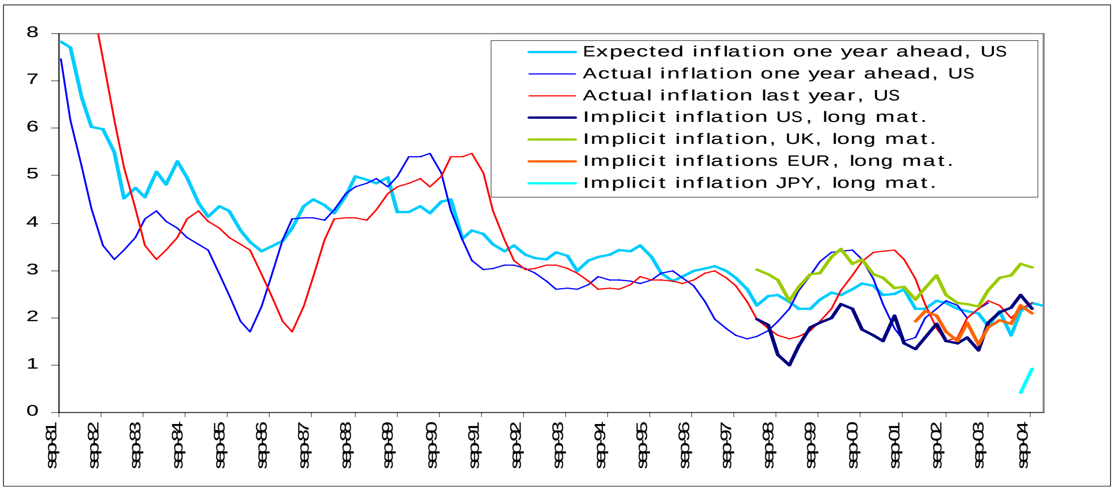

Chart 2.3: *Inflation expectations. Comparison of survey-based expected inflation one year ahead (produced by the Federal Reserve Bank of Philadelphia), realised (actual) inflation for the same period, and realised (actual) inflation in the past year. Here the difference between the interest rate on the nominal and real bonds with a long maturity (more than 5 years) is termed "implicit inflation". Sources: IMF, Federal Reserve Bank of Philadelphia, Datastream, Barclays Bank and Bloomberg.*

Chart 2.3 shows all three types of estimates for the USA, compared with actual realised inflation. It will be seen that inflation expectations are less volatile than realised inflation, although recent years' inflation rates seem to be about as good a predictor of next year's inflation as the expectations gleaned from interview surveys. A current average of realised inflation could seem to be a useful estimate of expected inflation.

Chart 2.4 shows the path of annual, realised inflation for a selection of countries since 1957. Viewed in a longer historical perspective, inflation has been above normal in this fifty-year period. In the century prior to this period, inflation seems mostly to be associated with war situations.

The past decade has seen a growing spread of inflation targets as the guiding element of monetary policy for a number of countries. This may explain the falling trend in inflation. Judging from recent years, inflation in the USA and the EU member states could seem to be converging, while inflation in Japan is on a more independent path.

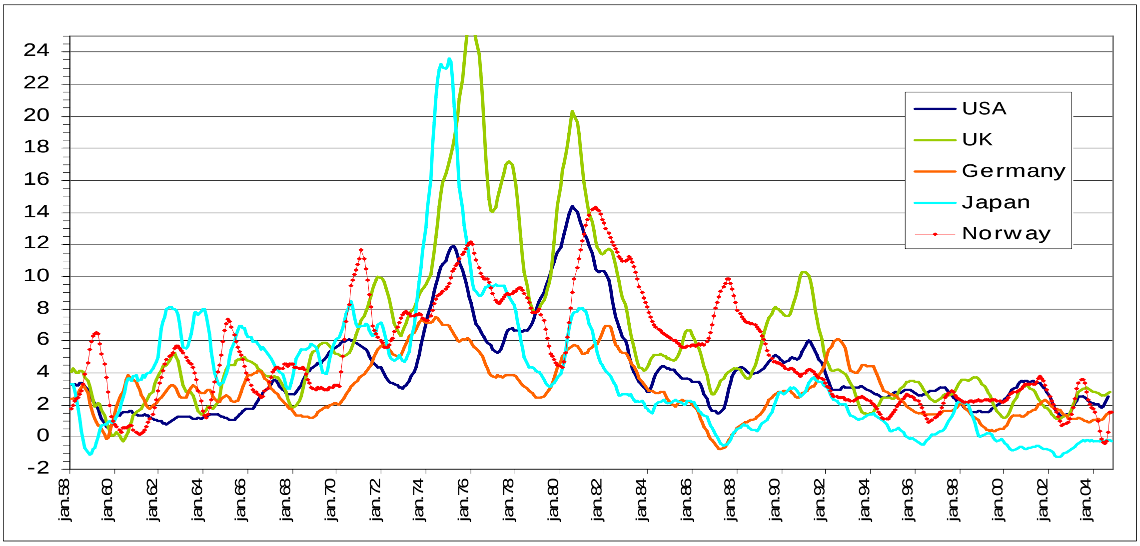

Chart 2.4: *Realised inflation. Monthly observations of annual inflation rates (consumer prices) presented as six-month moving averages. Source: IMF and Datastream.*

Term premiums

The term premium is often measured as the difference between a short interest rate and a long interest rate. Chart 2.5 shows the trend in the differential between a 10-year interest rate and a monthly interest rate on government paper. The data are taken from the IMF, and the length of available time series varies somewhat.

Three components can be envisaged to explain the term premium: One component ensures consistency between interest rates for various maturities, pre-empting profitable arbitrage on positions in different segments of the yield curve. The other two components relate to the risk premiums demanded by a risk-adverse, representative agent in order to assume real interest rate risk (or forward risk) and inflation risk, respectively. It is difficult to distinguish empirically between the individual components.

The chart shows that the total term spread varies widely over time, from -4 to +6 percentage points. Average spread is respectively 1.5; 1.0; 0.6 and 0.6 percentage points for the USA, United Kingdom, France and Japan.

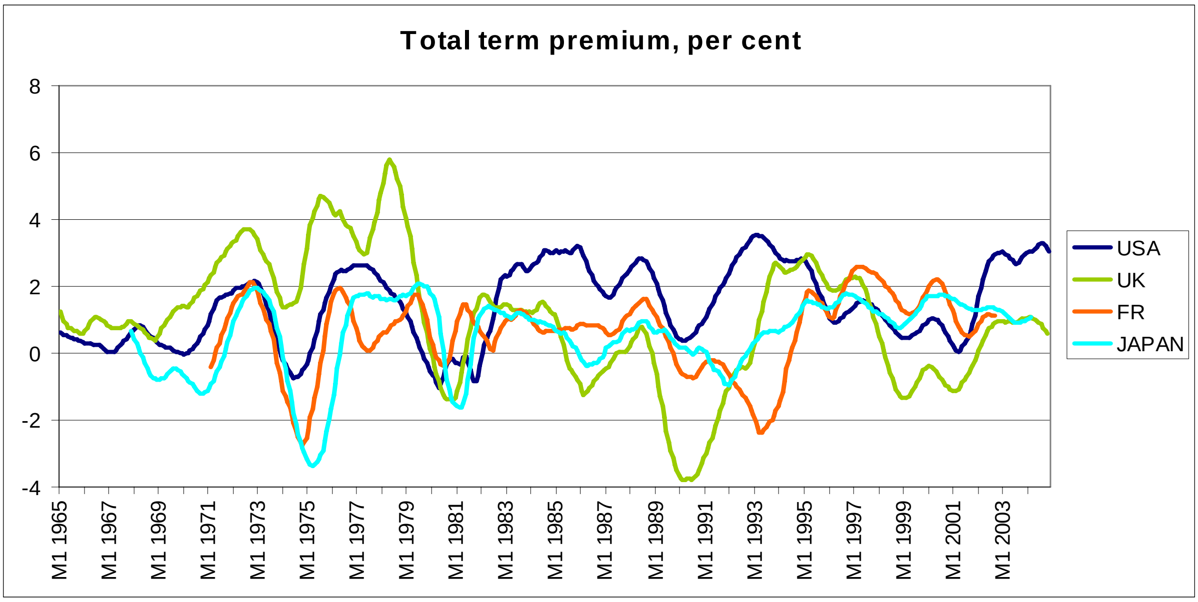

Chart 2.5: *Development of the term premium for a selection of countries. Source: IMF.*

Credit bonds

The credit margin, i.e. the difference between a government bond yield and a corporate bond yield with the same maturity, is normally larger than warranted by the higher risk of bankruptcy. For this reason credit securities normally have a higher expected return than government bonds.

The observed credit margin may reflect a number of differences between government securities and credit bonds, such as differing tax treatment, liquidity differences, compensation for expected loss resulting from bankruptcy, risk premium, and possibly implicit option contracts. Estimates of expected excess return generally control for the expected bankruptcy loss and for the value of any options. However, they rarely control for tax, liquidity and volatility effects (systematic credit risk) which are not necessarily related to security-specific credit quality, but which may nonetheless increase the credit margin.

Ilmanen (2004) finds that the average, options-adjusted spread in the market for securities with an A and BBB rating was, respectively, 95 and 160 bp (basis points) in the USA in the period 1985-2004. Studies based on earlier data give somewhat higher estimates, although the quality of these data is poorer.

Elton et al. (2001) find that the part of the options-adjusted spread needed to compensate the investor for bankruptcy risk averages 14 basis points for credit quality with an A rating (S&P rating) and 40 bp for BBB paper. Another study by Verloot (2002) finds estimates of respectively 25 bp and 40 bp. Both studies employ US data. These figures must be deducted from the observed options-adjusted spread when looking for the expected excess return on securities. Hence the expected excess return is about 80 basis points for A-rated securities, based on Elton et al. (2001).

Table 2.2 shows average interest rates and spreads for the Lehman index at the end of 2004. We see that the average options-adjusted spread at this point was somewhat higher for high credit quality and lower for low credit quality than shown by Ilmanen (2004).

Credit margins vary over time and influence expected return in the same way as an interest rate change. Elton et al. (2001) control for bankruptcy risk and tax effects and find that the residual premium appears to be clearly explained by changes in the equity market, i.e. there appears to be compensation for systematic risk. Fundamentals affecting credit risk, such as enterprise earnings and bankruptcy rates, are generally correlated with cyclical conditions.

	AAA-A Treasuries	AAA Corporate Credit	AA Corporate Credit	A Corporate Credit	BAA Corporate Credit
Yield, EUR	3.45	3.93	3.89	4.10	4.15
Yield, USD	3.56	4.12	4.11	4.31	4.82
Yield, JPY	1.09	-	-	-	-
Share of Corporate Credit	-	20.18 %	13.82 %	36.22 %	29.79 %
OAS, EUR	0.02	0.52	0.48	0.57	0.61
OAS, USD	0.00	0.72	0.66	0.75	1.16
OAS, Global	-	0.58	0.54	0.68	1.00

Table 2.2: *Yield to maturity, market value shares and "options-adjusted spread" (OAS) for a selection of categories in the Lehman Global Aggregate Index as per 31 October 2004. Source: Lehman Brothers.*

From Chart 2.6 it will be seen that the level of spread reflected in Table 2.2 is just under the normal level. The average spread on A-rated paper is about 1 percentage point in the USA, compared with the current level of 0.75%.

Dimson et al. (2002) find that the realised return on long-term credit bonds in the USA was 0.5 percentage points higher than for government paper in the period 1900-2000, although this estimate suffers from uncertain data quality and differences in credit quality over time.

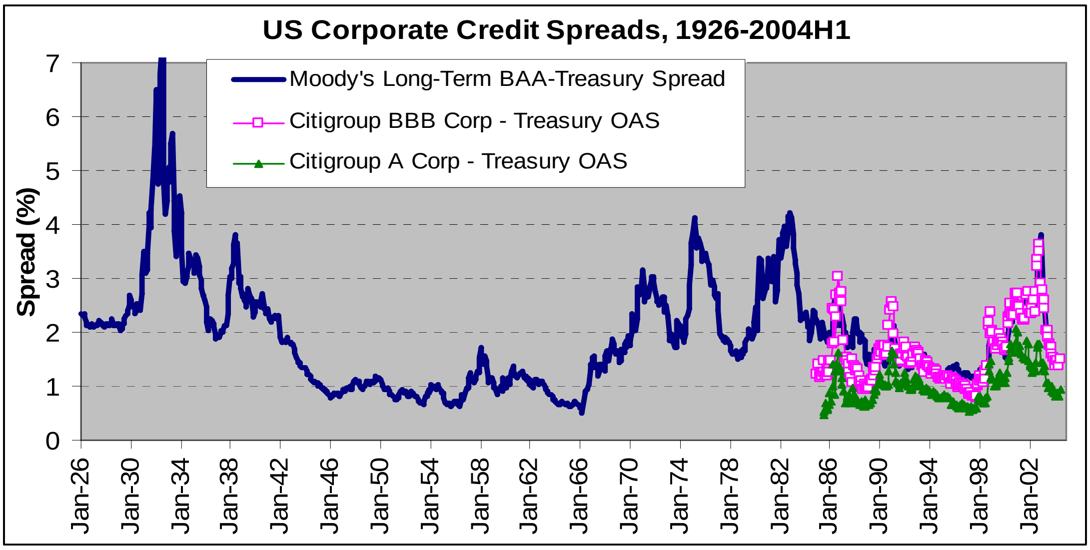

Chart 2.6: *Historical trend in credit spreads. Source: Ilmanen (2004).*

2.4 Assessment of the equilibrium level for short real interest rates

An equilibrium real interest rate is often defined as an interest rate that balances aggregate demand and production under stable inflation. The term has roots going far back and it has recently seen something of a renaissance. It is often associated with a real interest rate for a short maturity, and it is also commonly assumed that the equilibrium level for the real interest rate can change over time.

Applying this type of terminology is however questionable since the equilibrium level is not observable and because current real interest rates are observable only up to a point. In this section we discuss ways of assessing the equilibrium interest rate. This is followed by a discussion of whether we have sufficient knowledge to enable any observed deviations from this level to be utilised to shape expectations of future return.

It is difficult to draw clear-cut conclusions about the equilibrium level for the real interest rate on the basis of theoretical models of the functioning of the economy (cf for example Wicksell, Solow, Ramsey, IS-LM). The majority of such models posit a link between real interest rates and economic growth via technological assumptions and the population's preferences as regards saving and consumption. Other explanatory factors which are normally regarded as important, such as risk premiums, institutional conditions and the relationship between global and local factors, are not included in such models.

In broad terms, five approaches can be envisaged for assessing the equilibrium level for real interest rates:

- An average of historical real interest rates
- An average of historical real interest rates, corrected for specific economic shocks
- Time series models
- Structural models
- The market interest rate on inflation-linked bonds

When calculated over a sufficiently long period, short-term deviations between actual interest rates and the natural interest rate might be expected, on average, to even out. Long-term historical estimates based on data material from 1800 to 1990 for the USA and the United Kingdom are analysed in Siegel (1992). The analysis shows the likelihood of large, long-lasting deviations from the average for the entire period. In most countries average interest rates in 1980-1990 were higher than earlier in the century. Ex ante real interest rates (based on inflation expectations gleaned from interview surveys) for long-term bonds in the USA have typically ranged from 2 to 4 per cent, with the exception of the first half of the 1980s when this real interest rate was unusually high.

The so-called Taylor rule (see Taylor 1993) is a method much used to correct long-term averages for economic shocks such as unexpected inflation and deviations from potential production. In other words, situations where we know that the interest rate level has *not* balanced aggregate demand and production. The equilibrium interest rate or the natural interest rate is the constant term in a regression of the interest rate level to these variables. Gerdsmeier and Roffia (2003) seek by various means to estimate the natural interest rate level for the Euro area. By far the majority of the estimates are between 2 and 3 per cent. Laubach and Williams (2001) conduct a similar analysis for the USA, finding a slightly wider range.

Time series models can be used to estimate a time-varying equilibrium interest rate where account is taken of any lasting pattern of this time-variation, such as "mean reversion". This type of study seeks to decompose interest rate series into underlying trend movements and more high-frequency and temporary fluctuations around the trend. The estimate of the equilibrium interest rate corresponds to a non-stationary component in such models. A similar study for the euro area, by Crespo-Cuaresma et al. (2003), indicates that the equilibrium interest rate is now just over 2 per cent. However, this type of model does not seek to utilise assumed causal relationships in the economy to explain the interest rate level. Structural models have such aspirations, but their disadvantage is that the argumentation tends to be highly model-dependent.

There is commonly assumed to be a connection between the real interest rate in the economy and economic growth. However, Ilmanen (2004) shows that the *direct* empirical relationship between many macro variables and real interest rates is weak. He presents simple empirical estimates for the explanatory power of GDP growth, expected GDP growth, productivity growth, unemployment, short interest rates, capacity utilisation, private saving, and demographic composition. However, he finds significant predictive power related only to government saving and inflation.

Ilmanen (2004) finds a strong empirical connection between inflation level and the level of long real interest rates, and there are indications suggesting a common trend for inflation and real interest rates (co-integration). Two possible reasons for this are discussed. First, the inflation risk premium for nominal bonds is probably higher when inflation is high, since market participants expect high inflation levels to be unstable, whereas low inflation is likely to persist over time. The second reason may be more vigilant central bank behaviour in periods of high inflation, and a will to permit lower real interest rates when inflation has been low for some time. Chart 2.7 shows the historical relationship between expected inflation and ex ante real interest rates for a selection of countries.

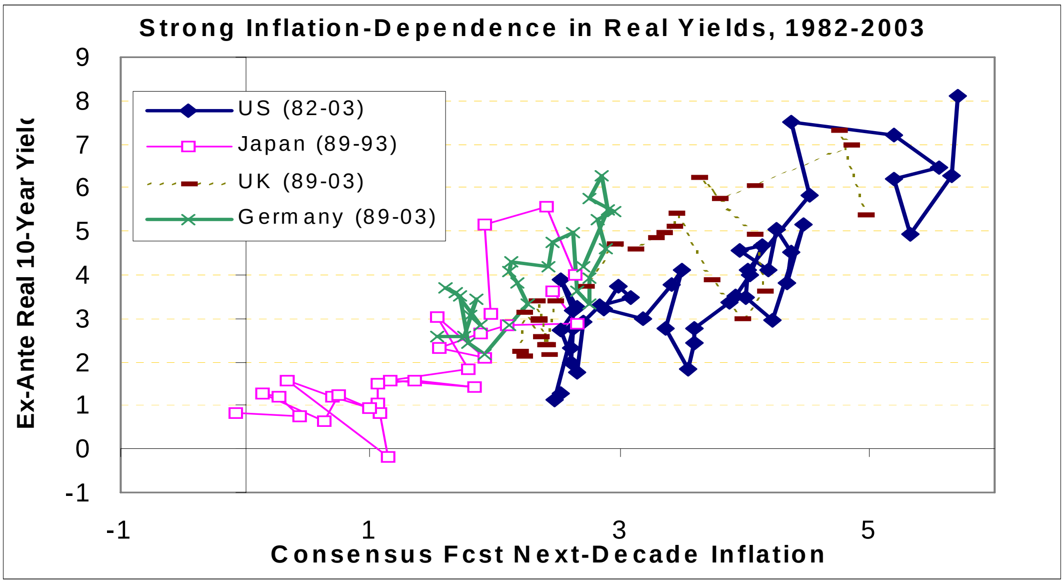

Chart 2.7: *Relationship between 10-year real interest rates (estimated yields) and expected inflation for the USA, United Kingdom, Germany and Japan. Source: Ilmanen (2004).*

Ilmanen gives an estimate of 2.5-3.0 per cent for expected average real interest rates on long nominal government bonds over the next 10-20 years. The inflation risk premium incorporated in this estimate is put at about 0.5 per cent. The estimate is not differentiated between regions. This estimate is at the lower end of estimates of the natural interest rate since he attaches much importance to the role of low inflation in guiding low real interest rates.

Ang and Piazzesi (2003) estimate a structural VAR model for nominal interest rates' term structure, with restrictions ensuring that the term structure does not provide arbitrage opportunities. They find that macroeconomic factors are important explanatory variables for the variation in long bond rates when the prediction horizon is long. Factors utilised include growth in manufacturing production, growth in unemployment, demand for labour and inflation estimates. Out-of-sample testing indicates that predictive ability increases as a result of including no-arbitrage restrictions and macro variables.

Another way of estimating the natural real interest rate level is to calculate historical averages of market interest rates on inflation-indexed bonds. This method premises that market participants' inflation expectations are the best estimate of future inflation, and that these expectations are best expressed through the supply of and demand for real interest rate bonds of this type.

The advantage of this method is that it is simple and forward-looking, and that data for inflation-indexed bonds are readily available. On the other hand market-related factors, such as liquidity premiums and tax effects, may influence such estimates, and premiums of this type may vary over time. Moreover, the estimate is simply the market participants' expected interest rate, which may be influenced by extraordinary optimism or pessimism.

	Maturity		
	2 year	10 year	20 year
USA	1.1932	1.6100	1.8000
UK	1.8198	1.6020	1.5405
FRANCE	-	1.1920	1.5940
JAPAN	-	0.5070	-

Bloomberg: 28.06.2005

Table 2.3: *Yield to maturity for a selection of inflation-indexed bonds on 12 January 2005. Maturities are approximately 2, 10 and 20 years. The French bonds are indexed to EUR inflation, for 10 and 30 years. Source: Bloomberg.*

Inflation-indexed bonds have existed for only a short period in Europe and Japan, somewhat longer in the USA and the United Kingdom. Table 2.3 shows relevant interest rates for a selection of inflation-indexed bonds with about 2, 10 and 20 years' residual maturity.

Chart 2.8 shows the trend in Barclays' inflation-indexed bond indices; the chart shows average interest rates for bonds with a maturity longer than five years, apart from in Japan where the maturity is almost 10 years.

A high degree of correlation is evident between all four markets, possibly with the exception of the initial years. We also see that real interest rates in the USA, United Kingdom and the Euro area have recently converged. The implicit real interest rate in the most mature market, i.e. the United Kingdom, has remained relatively stable at around 2 per cent for long-term bonds.

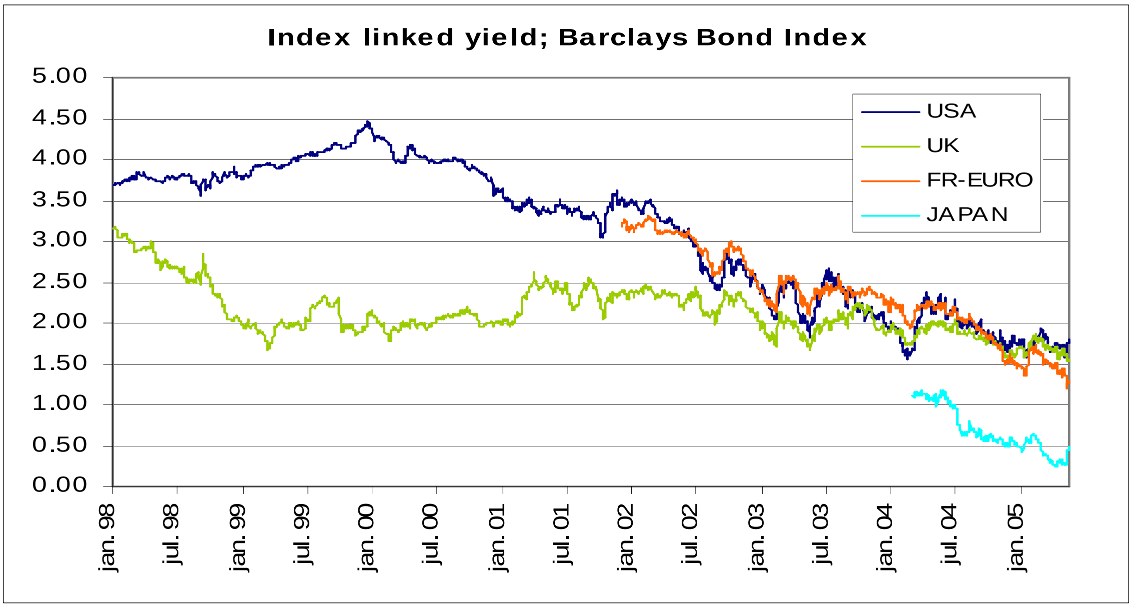

Chart 2.8: *Barclays Capital Bond Indices. Average yield of inflation-indexed bonds with more than five years' residual maturity in the USA, United Kingdom and France (EUR). In Japan three bonds have been issued with about 10 years' residual maturity. Source: Bloomberg.*

2.5 From interest rate level to return

The difference between interest rates and return is also important when judging the risk associated with bond investments. An interest rate is a return that can be agreed today for an investment with an agreed maturity. Hence an interest rate is forward-looking, while the realised return is backward-looking and contains both expected and unexpected components. Expected return may deviate from the interest rate both on account of incorrect pricing (e.g. of liquidity and credit risk), and on account of reinvestment risk and capital risk if the horizon for the expectation does not match the maturity for the investment.

The random (stochastic) characteristics of an interest rate will differ greatly from the characteristics of the return on an individual bond, whose residual maturity continually diminishes. However, in our context the alternative is an investment in a bond index, in which a series of securities with varying maturities and coupon payments are aggregated in a portfolio. The return on a bond index whose duration is kept more or less constant will be more likely to resemble an interest rate for the corresponding average maturity.

Two methods for analysing expected return and risk for a bond portfolio are particularly relevant. One method is to calculate the mean and the standard deviation of realised return on a bond index and to make an assumption regarding the shape of the probability distribution. An alternative method is to estimate a stochastic process for the interest rate path, simulate future interest rate developments, derive bond prices for varying maturities via a term structure model, find the return on a (simplified) portfolio of bonds, and in this way arrive at a simulated probability distribution for future return. The estimated interest rate process can be corrected subjectively, for example to match our estimate of the equilibrium level for the real interest rate.

Both methods have strengths and weaknesses. If the equilibrium interest rate is highly important as an anchor, in other words if current interest rates cycle rapidly around this level, and the level stands relatively firm, a long historical return series will probably produce better mean estimates than if actual interest rates are likely to deviate from the equilibrium level for a long period. On the other hand, the present interest rate level may have a large bearing on expected return even over long horizons if interest rates move only slowly towards an equilibrium level. There may be reason to believe that both the inflation premium and interest rate risk are dependent on the level of inflation, producing a time-varying level of risk.

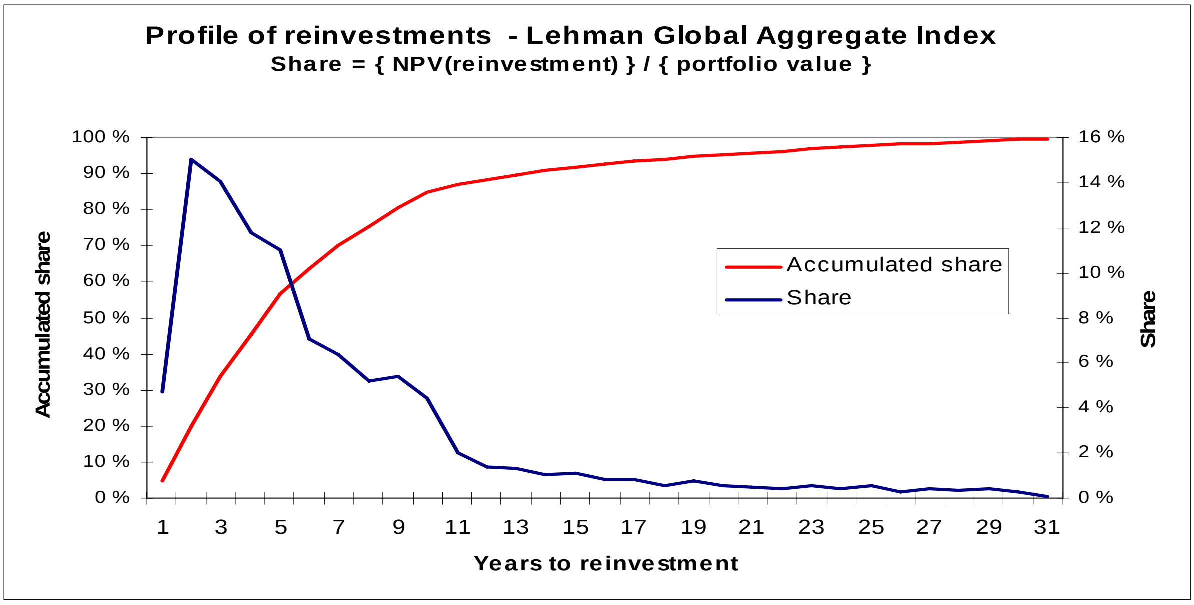

Chart 2.9: *Approximate reinvestment profile for the portfolios' bond index: Lehman Global Aggregate. The chart shows the current value of future coupon and maturity payments as a share of current portfolio value. All payments are assumed to fall due at mid-year, and discounts are at a fixed rate producing a total current value equal to portfolio value. Source: Lehman Brothers and Norges Bank.*

The second method lends itself to finding return expectations conditioned by the current interest rate level. However, model risk is present since interest rates have to be translated into bond prices via a term structure model; while this makes our description more precise, we could be describing a model somewhat removed from reality.

Relatively short time series are available for bond indices (except for the USA), while interest rate series are more readily available for other countries too. Estimates of return aggregated from monthly data build on (implicit) assumptions regarding auto-correlation in the return, whereas such factors are directly estimated when the interest rate process is specified.

Chart 2.9 shows the maturity profile of the Lehman index. This profile shows portfolio components and maturities for a portfolio of zero coupon bonds that replicates the index. It shows little movement over time. The duration of the index is determined by the market weights of the securities included and also varies little over time.

We see that about 60% of the portfolio will be reinvested within six years. Index duration is about five years. Hence, while the return on a 10-15 year horizon is affected by the interest rate at which reinvestment can take place, part of the return is already locked in now and previously.

In the following we describe, first, historical return and risk in selected markets. We then analyse demand from a simulated short interest rate process.

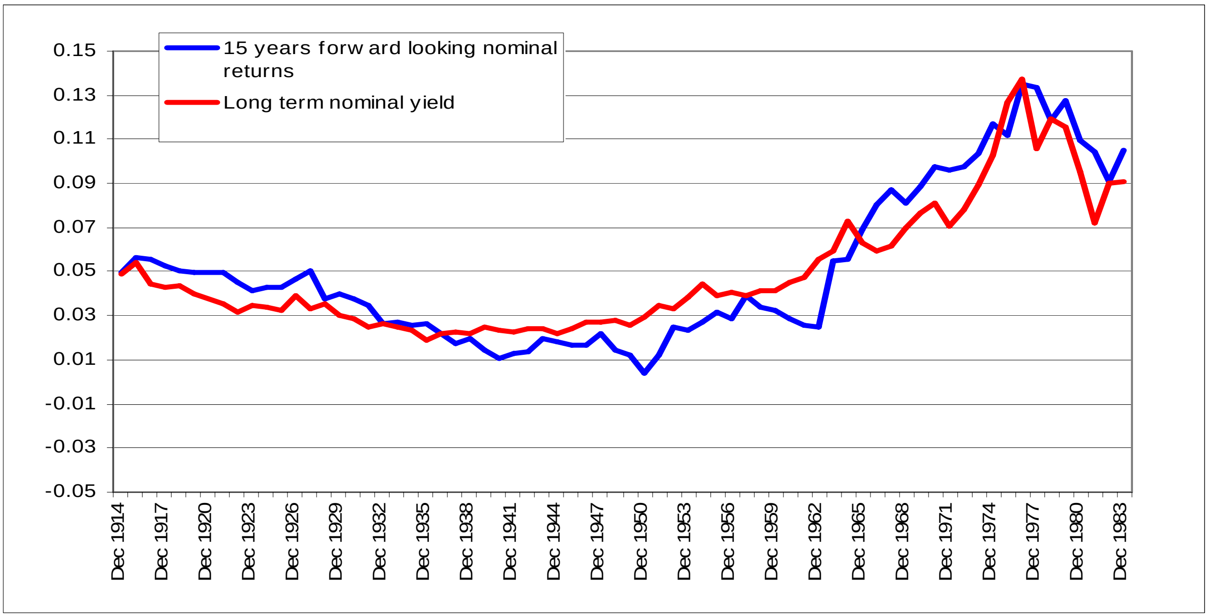

Chart 2.10: *History of annualised (geometric) 15-year returns in overlapping windows and long yields at the start of each 15-year period. USA. Source: Ibbotson Associates (returns), and NBER and the Board of Governors of the Federal Reserve System (interest rates).*

Historical return and risk

The Strategy Report for the Petroleum Fund 2003 contains a table showing historical return and risk for the period 1900 to 2003 for a number of markets. The same data material is shown earlier in this report for the first and second half of this 100-year period. Average geometric real return since 1950 has been 2.77% for the four main markets.

In Chart 2.10 we have plotted annualised 15-year return against the yield on long bonds at the start of each 15-year period, in the period 1919 to 1988. The impression gained from the chart is that the current level of long interest rates is a possible predictor for realised return 15 years later. The return shown is a result of a strategy involving virtually constant duration and annual reinvestment in bonds with a long maturity.

If one sought, in a regression, to explain or predict 15-year return using the long yield at the start of each 15-year period, a goodness of fit (R^2) of about 45% would be obtained for the US data in Chart 2.10. A similar result will be found for the United Kingdom, France and Japan, albeit with somewhat varying R^2 (77%, 23% and 36% for the three markets respectively). Data from the same sources were used in the period 1957 to 1988 for the United Kingdom and France, and from 1966 to 1988 for Japan.

Chart 2.11 shows the trend in standard deviation calculated over 15-year windows for, respectively, annual nominal, 15-year nominal and 15-year real return in the USA. The standard deviation of annual return for the entire period 1900-2003 is 10% for real return and 8.28% for nominal return. The USA is in the lower half of the range in this period; many European countries, and Japan in particular, have shown higher volatility. This may be related to the world wars, which created less turbulence in financial markets in the USA than in many other countries.

The average of the annualised standard deviations of 15-year return in the USA in Chart 2.11 is about 2% in real terms and 1.1% in nominal terms. Similarly, the standard deviation of the six non-overlapping nominal 15-year returns in the period is 2.9%, compared with 3.2% when overlapping periods are included.

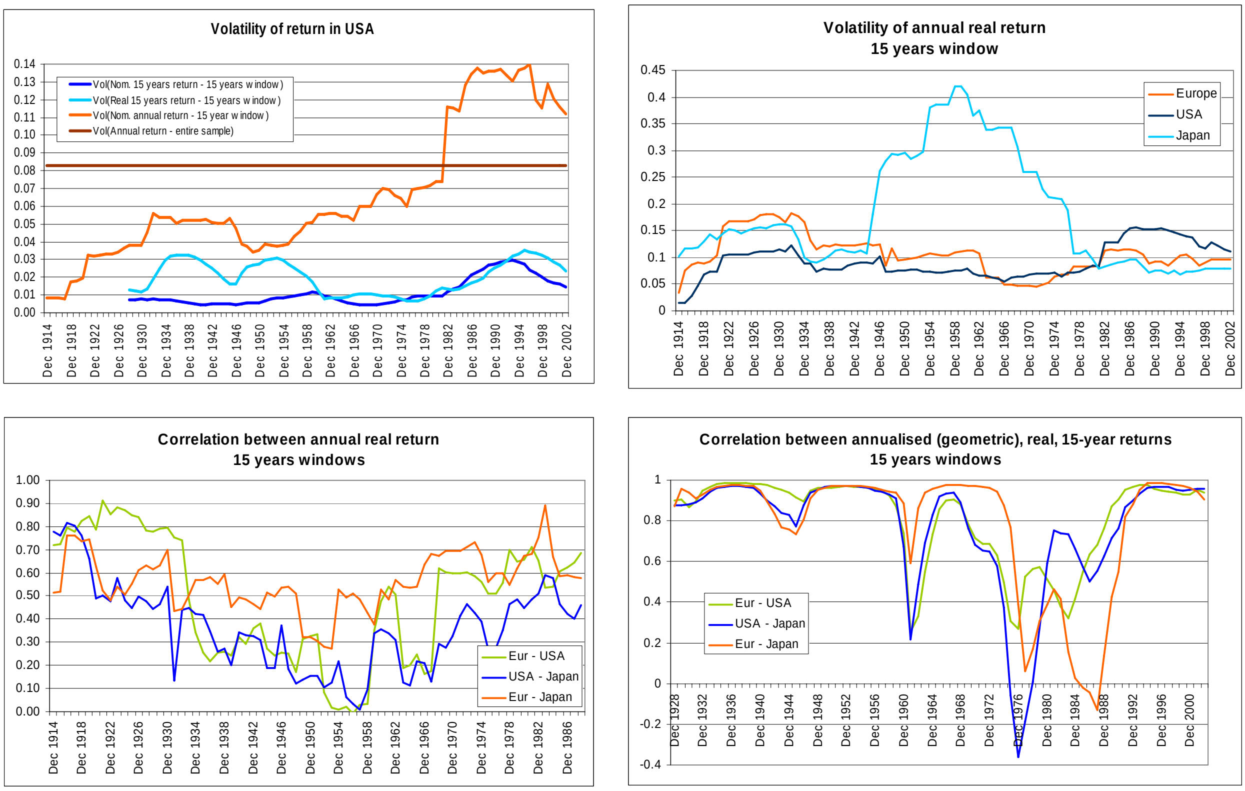

Chart 2.11: *Various measures of risk in selected markets. The risk measures are estimated on 15-year windows. Source: Ibbotson Associates.*

Volatility is usually measured as the standard deviation of the annual, logarithmic return on an asset. If this is σ, then the volatility of accumulated return t years ahead in time will be $\sigma\sqrt{t}$, and the annualised volatility will be $\frac{1}{t}\sigma\sqrt{t}$, in the absence of auto-correlation in the return series. Based on an estimate of 8.28% for the standard deviation of annual nominal return, one would expect 15-year annualised return to show a standard deviation of 2.14% in the absence of auto-correlation. In the case of a negative auto-correlation or mean reversion, the annualised standard deviation will be smaller.

When the empirical volatility of the 15-year return is compared with the above, the data do not appear to confirm negative auto-correlation since the estimates for volatility are higher than 2.14%. However, measurement errors may be substantial since the long-term risk estimate is calculated on the basis of a small number of non-overlapping periods.
Chart 2.11 also shows the path for correlation between 15-year annualised returns in the main markets. The main picture here is very high correlation, interrupted by some periods of turbulence. The first low points can broadly be explained by the effect of World War II, particularly for Japan; they correspond to points in time when 1945-46 enter and exit the estimation windows for the correlation. Realised 15-year real returns show divergent development between the three countries in the period 1965 to 1981, but after 1981 the trends are fairly parallel and appear to converge. This is reflected in markedly higher correlation towards the end of the period.

Otherwise we see that the annual volatility is to some extent time-varying, while annualised volatility is more stable in the long term. Note the dramatic effect of World War II on volatility for Japan.

Simulated return from the estimated interest rate process
An active field of research of late has been the relationship between dynamic term structure models, mainly developed for derivative pricing purposes, and stylised empirical facts for the return on a bond. Key references in this context are Dai and Singleton (2000, 2002).

Chan et al. (1992) compare empirical results for a number of term structure models. The Merton, Vasicek, Dothan, Brennan-Schwartz, GBM and Cox-Ingersoll-Ross models are all based on a process for short interest rates and are special versions of the process

$$dr = (\alpha + \beta r)dt + \sigma r^{\gamma} dZ$$

They estimate this process applying various restrictions to the parameters used in the models mentioned. They find empirical support for time-varying volatility, in other words a γ greater than zero. Based on a Chi-square test, they find that models with $\gamma < 1$ can be rejected (e.g. the Vasicek model), whereas models with $\gamma > 1$ cannot be rejected with a 90% level of confidence, implying that the volatility of the process is highly sensitive to the level of r. Moreover, they find only a weak degree of mean reversion, i.e. a non-significant β in the process without restrictions (however, other tests have problems rejecting the hypothesis that β differs from zero). None of the models appear to be good at predicting interest rate changes, while models with time-varying volatility do better in terms of explaining volatility.

We have previously discussed the natural interest rate level, and the related implication that market interest rates fluctuate around a long-term equilibrium interest rate. One reason why it is difficult to detect mean reversion using the models mentioned may be their assumption of a constant equilibrium level. In practice this equilibrium level is commonly envisaged as changing over time.

Interest rate models are usually applied to value interest rate derivatives. In such a context it is crucial that the model represents future volatility in a satisfactory manner. In our case, however, what we are seeking is a connection between the present interest rate level, the

natural interest rate level and the long-term return on a bond portfolio, along with the risk inherent in this portfolio. A stylised model such as the Vasicek model may be of some use in illuminating this issue, despite its limited empirical tenability. The advantage of the model is its ease of implementation, and its consistency with the simplest form of mean reversion around an equilibrium interest rate level. The model lends itself to developing intuition as regards the significance of an equilibrium level for future return and risk. Conclusions should, however, be collated with other information, given the model's many known weaknesses.
In the Vasicek model γ is bound to be zero, producing constant volatility and normal distribution of interest rates. A weakness of the model is that it allows negative interest rates, although this is of little practical significance if the initial interest rate and the equilibrium level of the interest rate is sufficiently high.

Chan et al. (1992) estimate all the models, including the Vasicek model, on yields for US Treasury Bills with one month's maturity. An average of bid and ask prices is used. Monthly data from June 1964 to December 1989 are used. These data are virtually identical to the IMF's Treasury Bill Rate[7] in the period. We estimate the model anew using the same method,[8] with data from June 1964 to November 2004.

Period: 1964.6 : 1989.12:	α	β	σ^2	γ	Long term mean $\theta=\alpha/\kappa$	Reversion speed $\kappa=-\beta$	Volatility σ	Half-life H
General model	0.0419	-0.6077	1.7788	1.5081	0.0689	0.6077	1.3337	1.1406
t-value	2.64	-2.25	0.61	4.81				
Vasicek	0.0301	-0.4485	0.0004	0	0.0671	0.4485	0.0200	1.5455
t-value	2.13	-1.82	4.00	-				
Period: 1964.6 : 2004.11:								
General model	0.0120	-0.2080	3.4730	1.7834	0.0577	0.2080	1.8636	3.3324
t-value	1.38	-1.22	0.85	8.03				
Vasicek	0.0056	-0.1052	0.0001	0	0.0532	0.1052	0.0100	6.5889
t-value	0.70	-0.65	-	-				

Table 2.4: *Estimated parameter values for the Vasicek model. Source: Norges Bank*

Table 2.4 shows that an extended historical data set further exacerbates the fit of the Vasicek model. Moreover, the significance of interest rate level-dependent volatility seems to be even more important in the extended data set. This could be related to the historical interest rate path, featuring low interest rates early and late in the extended data set, and high and volatile interest rates at the start of the 1980s.

We see from Table 2.4 that the long-term, average interest rate level is estimated at just below 6 per cent. We have previously argued that under current inflation regimes this is probably higher than the current equilibrium level. In the simulation we therefore override this parameter and set the long-term nominal interest rate level for short maturities at 4-4.5 per cent (2-2.5% real return plus 2% inflation).

[7] From International Financial Statistics, series code 11160C..ZF...
[8] The process is estimated using the generalized method of moments (GMM). Most of the analysis in Chan et al. (1992) is reproduced by Cliff (2003), and we apply his estimation algorithm. The algorithm requires "James LaSage's Econometrics Toolbox", from http://www.econ.utoledo.edu/matlab_gallery.

The reversal velocity towards equilibrium, or the half-life of the interest differential, is an important parameter with a large bearing on what importance should be attached to the current interest rate level in relation to the equilibrium level. The longer the half-life, the smaller is the bearing of the equilibrium interest rate, even for long valuation horizons.

Note that we have estimated the parameters in the time series model for the short interest rate using time series analysis and historical data for a one-month interest rate. In other words we have estimated the process under an objective, or real-world probability measure. We could alternatively have found those parameters which in the given model produce prices on zero-coupon bonds that are consistent with the current observed term structure (prices). We would in that case have estimated the interest rate process under a risk-adjusted probability measure, cf. standard asset pricing theory.

The strength of our approach is that it lends itself to building on studies of the equilibrium interest rate. These studies are largely applicable to a short-maturity interest rate, abstracting from the term structures of the real world. Moreover, while the current term structure would be highly preferable as a basis for the usual application of a term structure model, i.e., pricing derivatives, our aim is to assess the long-term expected return on bonds. The current term-structure may reflect specific supply-demand features that are less relevant for the long term. Given our belief in the relevance of the Taylor-rule approach we accordingly focus on the dynamics of the short interest rate.

Hence, in the following we apply the Vasicek model, with parameters obtained through time series analysis, to find prices of bonds with the relevant maturities. From these estimated prices we analyse the return of a trading strategy; we buy a 5.5 year zero coupon bond, hold it for one year, sell the 4.5 bond and reinvest the proceeds in a 5.5 year bond. This amounts to a constant (almost) duration strategy which produces a duration close to that of the Lehman Global Aggregate Bond Index.

From standard asset pricing theory, i.a. the Girsanov theorem, we know that in principle the diffusion term ("the riskiness") of a stochastic process is the same under the real-world probability measure as under a risk-adjusted measure. However, the trend-part of the process differs. In fact, our procedure implicitly assumes that the market price of risk is zero, in other words that the representative market participant is risk-neutral. This is the only context in which it would be appropriate to use the time-series approach as opposed to the currently observed term structure of interest rates. Since the implicit assumption of our approach is plainly incorrect, we adjust for this by adding an ad-hoc, exogenous risk premium.

The Vasicek model exhibits an endogenous term premium. The size of this premium depends on the parameters of the model, in particular the distance between the current short rate and the equilibrium rate and the half-life of the short rate process. The curve is steeper the lower the current rate is relative to the equilibrium rate. The curve would be declining if the current short rate were above the equilibrium rate, and approximately flat when in equilibrium. It is the no-arbitrage restriction in the model that produces this effect through the consistent pricing of bonds with different maturities.

In table 2.5 below we show the expected return for combinations of two different estimates for the equilibrium interest rate and two different half-live estimates.

Half-life	Equilibrium rate 4.0 %	4.5 %
3.5 year	3.5 %	3.9 %
1.5 year	3.8 %	4.3 %

Table 2.5: *Simulated expected annualised return on a 15-year horizon of an investment strategy with a fixed duration of about 5 years, for different parameter choices in a Vasicek model. An interest rate volatility of 1% is assumed, and the initial interest rate is 2.5%. Source: Norges Bank.*

The table shows expected annualised return on a 15-year horizon given the respective assumptions, from the vantage point of a risk-neutral agent. Hence, in this table we have not added our ad-hoc risk premium. The starting point for all four alternatives is the current short interest rate of about 2.5% (in the USA and the Euro area; in the United Kingdom it is closer to 4.7%). The risk estimate is an annual volatility of 1% for the interest rate. The return is the result of an investment strategy in which the entire portfolio is reinvested annually in a 5.5-year zero coupon bond. This strategy is simulated 3000 times.

From Chart 2.5 we see that the average term premium across USA and Europe has been slightly above 1% over the last 40 years. By adding an exogenous risk premium of 1% to the underlying paths for the 5-year rates in table 2.5 we get approximately 1 per cent higher expected return than the values reported in table 2.5. Note that by doing this we only assume that the term premium is 1% when the market is in equilibrium, i.e., when the current short rate is equal to the equilibrium short rate. The implicit assumptions in the four cases for the total (end ogenous and exogenous) term premium and their paths towards equilibrium are given in Chart 2.12 - left panel.

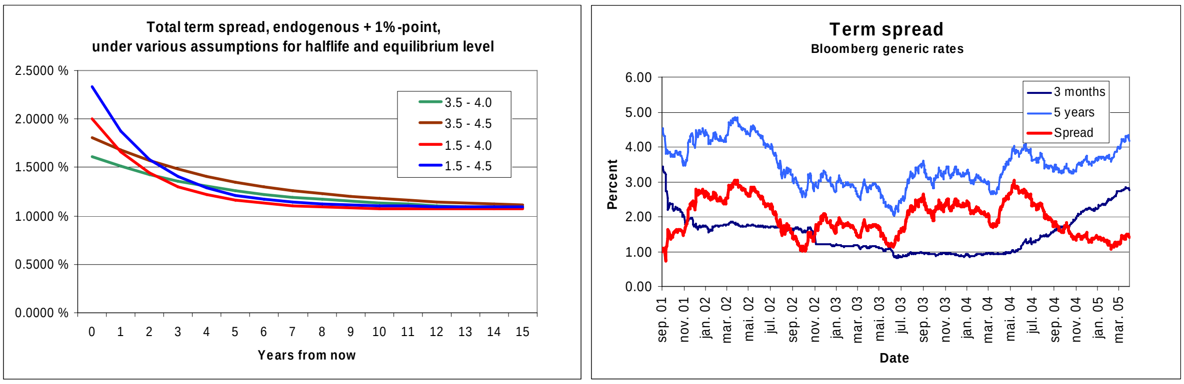

Chart 2.12: *Left panel: The path of the total term premium as a result of the expected path for the sort rate towards its equilibrium level, for four different combinations of half-life and equilibrium short rate. The total premium is the sum of the endogenous premium and an exogenous premium of 1%. Right panel: Historical, daily development of the term premium between a 3 months rate and a 5 year rate. Bloomberg generic rates. Source Norges Bank and Bloomberg.*

By calibrating the model to both the current short rate and an exogenous equilibrium term premium, we imply a current 5-year rate that is different from the currently observed 5-year rate. The rationale for this is that the exact shape of the curve varies greatly over relatively short periods of time, and anchoring the short rate rather than the 5-year rate provides a better link to discussions about the long term equilibrium rate.

Chart 2.12 - right panel shows the daily development in the term premium from August 2001 through March 2005. We see that the term premium has varied between 1 and 3 per cent in this period. Looking at the current premium, the two scenarios with a short half-life seem to produce too high a term premium, maybe indicating that the current low interest-rate environment is somewhat persistent. We attach an equal probability to our four scenarios, however, reflecting less emphasis on the exact shape of the current yield curve for assessing the expected return over the next 15 years. The average return in Table 2.5 is about 3.8%.

Chart 2.13 shows the trend in the simulated short interest rate for one of the four cases. We see that with a half-life as long as 3.5 years, it will take a long time (20 years) for the expected interest rate to reach 4% given an initial interest rate level as low as 2.5%. However, we also see a reasonably high likelihood of it reaching 4% as early as next year. With a half-life of 1.5 years the interest rate can be expected to reach its equilibrium level after about 10 years.

To reconcile Chart 2.13 with the upper left corner of Table 2.5, we have to translate the expected path for the short rate given in Chart 2.13 to a 5.5 year rate and a 4.5 year rate via the Vasicek model. These are the rates that are relevant for our trading strategy.

The problem in choosing a half-life as low as 1.5 years is that the risk attending a bond investment when the interest rate has reached its equilibrium level is unrealistically low. This probably reflects weaknesses in the Vasicek model where the reversal velocity is constant, the equilibrium interest rate is constant and volatility is constant. All these parameters are likely to be time-varying in a more realistic model which would have to take into account the possibility of external shocks.

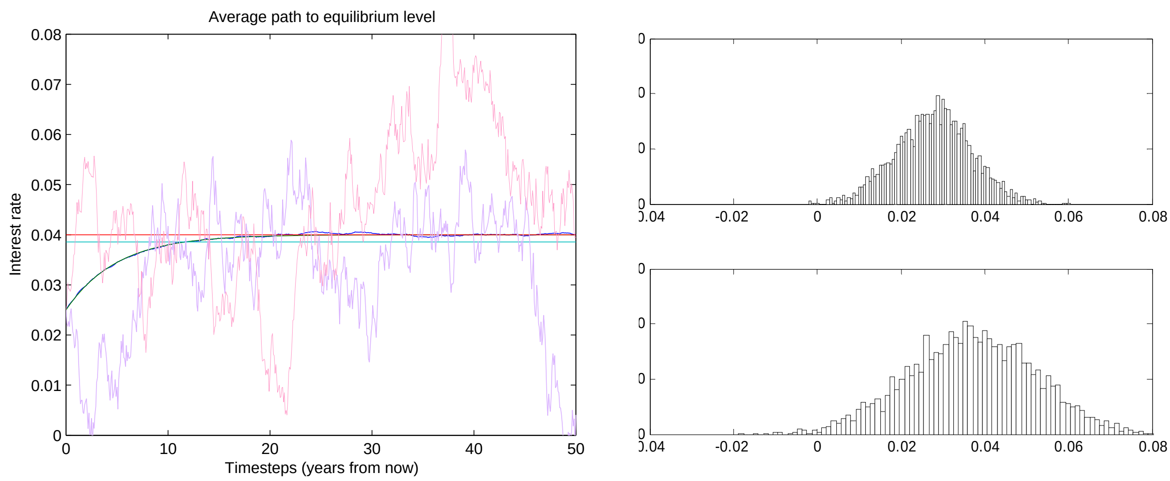

Chart 2.13: *The expected path of nominal interest rates given an initial interest rate of 2.5%, a neutral interest rate of 4%, interest rate volatility of 1% and a half-life of 3.5 years. Pink and purple lines represent two sample paths for the interest rate. The charts to the right show histograms of the interest rate 1 and 10 years from now respectively. The simulation model is not corrected for negative interest rates, but this is insignificant given the above parameters. Source: Norges Bank.*

Chart 2.14 shows the distribution of returns on a 3, 10 and 15 horizon in the Vasicek model. The neutral interest rate is here 4%, half-life is 3.5 years, and risk in the interest rate is based on an estimate of 1% from table 2.4. The risk in relation to annual return is about 3.2%, corresponding to the historical volatility measured over 15-year periods in the period 1900-1950, but lower than recent historical estimates.

The Vasicek model underestimates risk if the equilibrium interest rate is time-varying, if volatility varies over time (for example due to shocks in the economy), and if total risk is significantly affected by volatility in the term premium for interest rate and inflation risk. As mentioned, the Vasicek model, as applied here, produces prices and return that do not include risk premiums.

The Vasicek model is chosen because of its ease of implementation. Despite its limited realism, some intuition can be gleaned from the model in terms of how much weight should be given to the current interest rate level in relation to the assumed neutral interest rate level when shaping expectations of return on a 15-year horizon. Our chief impression is that the current low interest rate level has a large bearing on expected return even on a 15-year horizon, provided parameters are determined in such a way as to preserve more or less realistic risk estimates in the model.

We have so far discussed a Vasicek model with parameter values which can be assumed to be realistic for Europe and the USA. For Japan, on the other hand, the current nominal level of short interest rates is close to zero. The equilibrium real interest rate is possibly below its European and US counterpart. In saying this, we stress the importance of the inflation level as an explanatory factor for the real interest rate, and we expect inflation to remain lower in Japan than in the Western world.

If we premise an equilibrium real interest rate of 1.5% and an inflation rate of 0.5%, the nominal, neutral interest rate will be in the vicinity of 2%. With such parameters there is a high likelihood of negative interest rates in the Vasicek model, and ad hoc correction of this will affect model-developed expectations. We therefore choose to omit this type of analysis where Japan is concerned.

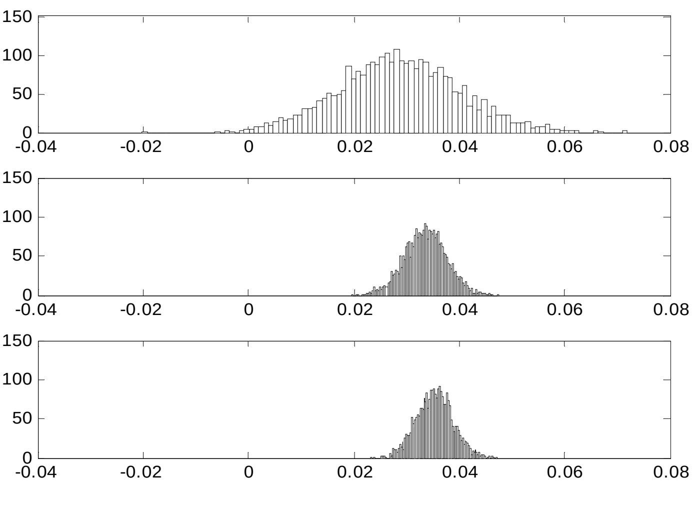

Chart 2.14: *Distribution of nominal annualised return on, respectively, 3 (top), 10 (middle) and 15 (bottom) year horizons. Parameter values as in Chart 2.12. Source: Norges Bank.*

2.6 Consensus perceptions

We have opted to set the relevant valuation horizon for future market developments at about 10-15 years for the purpose of assessing strategic decisions. Consensus perceptions are often produced as an average of market participants' statements, but often with a relatively short horizon. Two categories of perceptions are presented below compiled, respectively, by large investment management companies and major investment banks. Selection of both groups is influenced by the availability of explicit predictions. Many management companies do not publish explicit estimates. However, their estimates are of particular interest due to their long time horizon. Estimates are more readily available from investment banks, but are mostly prepared for short horizons.

Investment management companies

The Strategy Report for the Petroleum Fund from 2003 gives an overview of the strategic allocation in 19 large funds. A review of their respective web pages shows that only a few of them publish the assumptions on which their strategic allocation is based. The table below presents expectations from OTPP, GE Asset Management, PGGM and AP4. These statements are for the most part taken from annual reports for 2003 and are therefore from early 2004.

In most cases we have little precise information about these estimates. We do not know whether they are estimates of geometric or arithmetic averages, or whether they are conditional or unconditional estimates. Whether they are seen as conditional, i.e. expected to change over time, or unconditional depends on the way the various funds have organised the need to make changes in the strategic asset allocation. There is reason to believe that the estimates are mainly geometric averages and that they are very long term – i.e. virtually unconditional estimates. The exception is OTPP's estimate which appears to be conditioned to a larger extent by current economic conditions.

	AP4	GE	PGGM	OTTP
Publication	Ann.report 03	Article	Ann.report 03	Ann.report 03
Date for estimate	ca Febr. 2004	spring 2003	ca Febr. 2004	ca Febr. 2004
Conditional / Unconditional	na	na	na	na
Region	global	USA?	global	North America
Time horizon	long	long	long	10 years
Short real rate	1.25			
Risk premium for real interest rate risk	1.50			
Expected real return, index-linked bonds	2.75	2.50		~< 3.00
Risk premium for inflation risk	0.25	0.50		
Expected real return, nominal bonds	3.00			
Expected inflation	2.25	2.50		
Expected nominal return, nominal bonds	5.25	5.50	5.30	
Risk premium for credit risk, A-rating	0.50			
Expected nominal return for A-rated credit	5.75			<< CPI + 4.00
Equity premium over government bonds	2.50		2.20	2.50
Expected nominal return, equities	7.75		7.50	<< CPI + 6.50

Table 2.6: *Overview of assumptions used for long-term strategic allocation at some pension funds. The decomposition is derived from source material in cases where detailed statements are available. Source: The funds' home pages on the Internet.*

Investment banks
In the following we reproduce a small selection of statements from major investment banks. The statements were selected from long-term analyses made available to institutional investors. The overview aims to give a rough-and-ready indication of the average perception among investment banks, although the sample is too small to provide a precise picture.

Table 2.7 shows available numerical estimates from respectively *Citigroup Global Markets (2004), JP Morgan (2004),* and *Lehman Brothers (2004).* These estimates are conditional on various types of information deemed relevant by these three investment banks. The table illustrates the great uncertainty attending such predictions. Examples can be seen showing that differences in assumed GDP growth between the participants are not necessarily reflected in consistent differences in interest rate expectations.

It is also apparent that these factors show a greater spread in terms of macro variables such as GDP growth and inflation, than in terms of market variables. For example Lehman expects almost one percentage point lower inflation in Europe in 2006 than the other two, but nevertheless foresees a stronger rise in nominal interest rates. Similarly, while JP Morgan has a far more optimistic view of growth prospects for Japan, this appears to have limited consequences for the fixed income market.

		Global			Europe			USA			Japan		
		Citigroup	JPMorgan	Lehman	Citigroup	JPMorgan	Lehman	Citigroup	JPMorgan	Lehman	Citigroup	JPMorgan	Lehman
	2004	4.2	3.8	3.2	1.8 / 3.2	1.8 / 3.0	1.8 / 3.2	4.4	4.4	4.4	2.7	2.9	2.8
GDP growth	2005	3.6	3.1	2.3	1.7 / 3.2	1.9 / 2.5	1.5 / 2.3	4.0	3.7	3.6	1.1	1.5	0.6
	2006	3.6	3.2	2.3	2.3 / 3.0	2.2 / 2.5	1.5 / 2.2	3.9	3.4	3.5	0.9	2.1	0.8
	2004		42.9	40.4									
Oil	2005		38	38.9									
	2006			37.7									
	2004	3.0	2.8	1.9	2.1 / 1.3	2.3 / 1.4	2.1 / 1.3	2.7	3.3	2.7	-0.2	-0.2	0.0
Inflation	2005	2.9	2.0	1.7	1.7 / 1.6	1.5 / 1.6	1.5 / 1.5	2.3	1.9	2.7	-0.2	0.0	-0.2
	2006	2.5	2.3	1.5	1.8 / 2.2	1.7 / 1.7	1.0 / 1.8	1.8	2.5	2.5	0.2	0.1	-0.2
	2004		1.36 / 103	1.32 / 103									
Currency	2005	1.40 / 95	1.34 / 100	1.40 / 90									
	2006			1.35 / 85									
	2004				2.00 / 4.75	-	2.0 / 4.75	2.0	-	2.3	0.0	-	0.0
Short rate	2005				2.50 / 5.25	2.50 / 4.75	2.0 / 4.75	3.8	4.3	3.3	0.0	0.0	0.0
	2006				2.50 / 5.25	3.25 / 4.75	2.0 / 4.75	4.0	4.5	3.8	0.0	0.0	0.0
	2004				3.78 / 4.61	3.68 / 4.46	3.7 / 4.61	4.3	4.3	4.4	1.4	1.4	1.5
10-year rate	2005				4.20 / 5.23	4.65 / 4.80	4.1 / 4.84	4.9	5.8	5.0	1.8	1.9	1.5
	2006				4.25 / 5.28	-	4.4 / 4.98	5.1	-	5.5	1.8	-	1.8

Table 2.7: *Model predictions from three investment banks. Figures for Europe refer to the Euro area / United Kingdom. Currency figures refer to USD/EUR and JPY/USD exchange rates. The estimates refer to the end of each year. The line marked 2004 is for various points in December for market data and estimates for macro data. Sources: Publications from the respective investment banks, see footnotes in the text.*

The table illustrates shared expectations of small changes in inflation in the three regions (a slight reduction in Europe and the USA, a slight increase in Japan), expectations of a larger increase in the 10-year interest rate in the USA than in Europe and only a marginal interest rate increase in Japan. However, the publications referred to show that the arguments for this path differ somewhat between these three market participants.

2.7 Summary

This chapter has discussed central distributional characteristics for the return on bonds. The mean estimate in the distribution was discussed first. The return on a 10 – 15 year horizon will probably be coloured both by the current interest rate level and interest rate developments ahead. We have started out from estimates of a long-term equilibrium interest rate, and have sought to balance the significance of current interest rates against this supposed neutral level.

We have also looked at the trend in risk estimates such as standard deviation and correlation between main markets. We have described the significance of a meaningful concept of a long-term equilibrium interest rate for risk in the long-term, but have also pointed to reasons why the risk estimates produced by the Vasicek model probably underestimate market risk.

The estimate of expected return
The following main points have been mentioned:

Equilibrium level for the short real interest rate
Substantial uncertainty attaches to the estimate. Most empirical studies are in the range 2 – 3 per cent for Europe and the USA. Too little information is at hand to differentiate between the two regions. Very few studies are available for Japan. We believe the equilibrium interest rate for Japan in the very long-term is of the same order of magnitude, but that it means much less for the return in a 15-year perspective given the economy's current deflation scenario. We envisage a low, stable inflation scenario ahead for all regions, and therefore opt for a short equilibrium real interest rate in the area of 2-2.5% in the western world and 1.5% in Japan.

Inflation
We believe Europe and the USA will experience low, stable inflation, close to the central banks' inflation target of 2%. However we believe Japan will continue to show very low inflation for most of the coming 15-year period; the implicit inflation estimate from inflation-indexed bonds in Japan is just over 0.5%.

Term premium
We believe the total term premium for interest-rate and inflation risk will on average be just below its historical average of about 1% for Europe and the USA based on an expectation of stable inflation ahead, and close to its historical average of 0.6% for Japan.

Credit premium
The average spread for A-rated securities is about 1%. We accept the estimate of expected excess return of 0.80% from Elton et al. (2001).

A summary of interest rate assumptions in a 15-year perspective is therefore:

	Europe	USA	Japan
Equilibrium interest rate, real, short maturity	2.00	2.00	1.50
Risk premium for real interest rate risk	0.50	0.50	0.30
Risk premium for inflation risk	0.50	0.50	0.30
Real interest rate, long maturities	**3.00**	**3.00**	**2.10**
Expected inflation	2.00	2.00	0.50
Nominal interest rate, long maturities	**5.00**	**5.00**	**2.60**
Credit spread, A-rating	1.00	1.00	-
Nominal yield on A-rated credit bonds	**6.00**	**6.00**	**-**

Table 2.8: *Summary of our interest rate assumptions. Source: Norges Bank.*

Given these assumptions, the consequence for expected return depends on how rapidly the interest rate level is expected to move towards the neutral level from the current level. We believe half-life is in the range 1.5 to 3.5 years for Europe and USA and longer for Japan.

We used the Vasicek model to translate the expected path from the current short interest rate to its assumed equilibrium level into returns from a constant duration (about 5 years) trading strategy, resembling an investment in our benchmark index.

Various estimates of expected return based on neutral-interest-rate and half-life assumptions were given in Table 2.5. With allowance for estimation risk on these two components, the average return in Table 2.5 is close to 3.8%. This is similar to the case with a 2% real equilibrium rate and a half-life of 1.5 years. Adding the average risk premium of 1% to this risk neutral number leads us to an expected nominal return of 4.8%. A simple decomposition of our estimates of expected return, consistent with our assumptions is then:

	Europe	USA	Japan
Long term, risk neutral expected real return	1.80	1.80	0.50
Risk premium for real interest rate risk	0.50	0.50	0.30
Risk premium for inflation risk	0.50	0.50	0.30
Expected real return, nominal bonds	**2.80**	**2.80**	**1.10**
Expected inflation	2.00	2.00	0.50
Expected nominal return	**4.80**	**4.80**	**1.60**
Risk premium for credit risk, A-rating	0.80	0.80	-
Expected nominal return for A-rated credit	**5.60**	**5.60**	**-**

Table 2.9: *Estimates of expected, geometric mean return on a 15-year horizon. Source: Norges Bank.*

The estimates for Japan are discretionarily adjusted in relation to Table 2.8 to take greater account of current interest rate levels in Japan than in Europe and the USA.

We have previously shown that the current long interest rate appears to be a possible predictor of future 15-year return. As a reality check, it may be noted that the estimates for expected nominal return in Table 2.9 are close to, but slightly higher than, the current 15-year interest rate in Europe and the USA and more or less equal to the current 15-year interest rate in Japan.

There is often assumed to be a positive yield curve effect on return, due to a normal rising yield curve and to the fact that bond lifetimes diminish as time passes. In the case of a very long valuation horizon, this effect will probably be negligible since it will only have an effect towards the end of the valuation horizon, after the final reallocation of the portfolio. Whereas the effect will have a bearing on an arithmetic average of return over the period, the effect over 15 years will be limited.

The expected credit premium applies to A-rated securities, which are the largest category of corporate credit in the benchmark portfolios for the Petroleum Fund and the foreign exchange reserves. As above, we do not differentiate between the USA and Europe in this context, despite the current spread differentials. In the long-term the differentials will probably narrow as and when the credit market becomes more established in Europe. We do not provide an estimate for credit bonds in Japan since the funds have no investments in such bonds in that country.

The risk estimate

The relevant probability distribution in a decision-making situation is the one that reflects expectation and uncertainty in our market outlook. The literature generally draws a distinction between an objective distribution which is as precise an estimate as possible of the market distribution, and a predictive distribution which also takes into account the risk of erroneous perceptions, so-called estimation risk for the market distribution.

We have developed our estimate of expected return based on a belief that interest rates are moving towards a long-term equilibrium level. This mean reversion of interest rates should be reflected in a lower risk estimate for return in the long term than in the short term. On the other hand, the estimate risk for the expected return will have the opposite effect; estimate risk makes investments more risky the longer the horizon.

Risk for credit bonds is higher than for government bonds due both to market risk and bankruptcy risk. We employ a common risk estimate here, however; we believe the error this involves is acceptable for our purpose, which is to model the probability distribution on a 10-15 year horizon.

Our projection for estimation risk on the expected value of real return is 0.25% for Europe and the USA and 0.5% for Japan. Our estimate for market risk is based on historical estimates for the volatility of real return. We put the standard deviation of 15-year of annualised return at about 2%. Including a mark-up for estimation risk, this can be translated into estimates for the volatility of annual return of about 9% for Europe and the USA and about 10% for Japan. The two horizon effects are assumed to counterbalance each other.

Overall probability distribution

The Lehman index comprises about 55% government bonds and about 20% corporate bonds, while the remainder consists of various bonds falling mainly within an intermediate category of creditworthiness, for example because of mortgage backing or government guarantee. A large part of this group could be expected to produce return equal to that on government bonds, while a small portion could be expected to produce substantially higher expected return. For our purpose the error involved in not taking explicit account of these groups when describing the probability distribution is probably slight.

The proportion of corporate bonds is approximately identical in Europe and the USA, while in Japan all fixed-income investments are in government bonds. In Table 2.10 we have weighed together the estimates for expected return on government and corporate bonds from Table 2.9. The table summarises our market outlook for the Lehman index by region.

	Expected real return	Estimation and market risk	Correlations	
			Europe	USA
Europe	3.00 %	9.00 %		
USA	3.00 %	9.00 %	0.50	
Japan	1.10 %	10.00 %	0.40	0.40
Total	2.81 %	7.58 %		

Table 2.10: *Summary of the assumptions underlying the probability distribution for real bond returns on a 15-year horizon. The assumptions are based on an absence of auto-correlation. Source: Norges Bank.*

The correlation estimates are based on an average of historical correlations between annual real returns. We have, however, made a slight downward adjustment in the correlation between Japan and the other regions, in keeping with our belief that the trend in this market may be influenced by factors other than those in the United States and Europe.

It should be emphasised that the time horizon of 15 years is an important premise for these estimates. Distribution on shorter horizons tends to vary widely, and the expected return is probably lower.

3 Equity markets

3.1 Introduction

The expected real return in the equity market is equal to the expected risk-free real return in the fixed income market plus an expected equity premium (also called risk premium) that will be required by investors to hold the equity market portfolio. In order to quantify expectations it is first necessary to specify the relevant risk-free return: the equity premium is the expected excess return over the risk-free rate.

The literature defines risk-free return as the return either on government bonds or on money market instruments, and the risk premium may be discussed in relation to either of these returns. What is deemed to be a relevant “risk-free” return will normally depend on the investment horizon and possible liabilities of the portfolio. In this report a natural starting point is the return on government bonds, given the long investment horizon of the Petroleum Fund and the foreign exchange reserves. Thus the equity premium is taken to be the difference between the return on, respectively, equities and government bonds.

Hence this chapter discusses the real return that can be expected in equity markets over and above the *expected real return on government bonds.* We start by assessing the size of the equity premium in the very long-term, a type of unconditional equilibrium premium. We then discuss whether there is a basis for estimating a conditional equity premium over the next 15 years, and what the estimates might be over this horizon.

Future equity premium cannot be quantified with any degree of certainty. The historical (realised) equity premium cannot automatically be extrapolated into the future. The expected equity premium is probably a function of many variables, such as risk aversion and risk assessment over differing horizons. Realistic estimates of this premium and the uncertainty involved therefore require the use of various analytical approaches. We base our estimates on a combination of recent research findings, published consensus views and our own assessments.

The equity premium can be measured either as an arithmetic or geometric average of accrued excess rates of return (realised or expected). A geometric average is a growth rate, and is lower than an arithmetic average (which states average return) when returns are volatile; the difference is about half the variance in the case of a log-normal return distribution. Which average is best suited to expressing *expected* annual equity return in the future depends on the investment horizon. We are interested in accumulated return over long horizons and will focus on *geometric* average equity premium (c.f. Diamond 2004). Given an expected annual volatility, the corresponding arithmetic equity premium can be estimated by adding half the variance.

3.2 Historical (realised) equity premium

The realised equity premium has varied both over time and from country to country. Chart 3.1 and table 3.1 show the geometric average equity premium from 1900 to 2004 in the USA, Japan, ten European countries, Europe as a whole, and in Europe apart from the United Kingdom[9]. The chart also shows averages for the period 1945-2004. The calculations employ total return data from Dimson, Marsh and Staunton (2004).

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[9] In this context the equity premium in Europe is a weighted average of average equity premiums in the ten selected European countries, where the weights are for simplicity’s sake equal to the market capitalisation

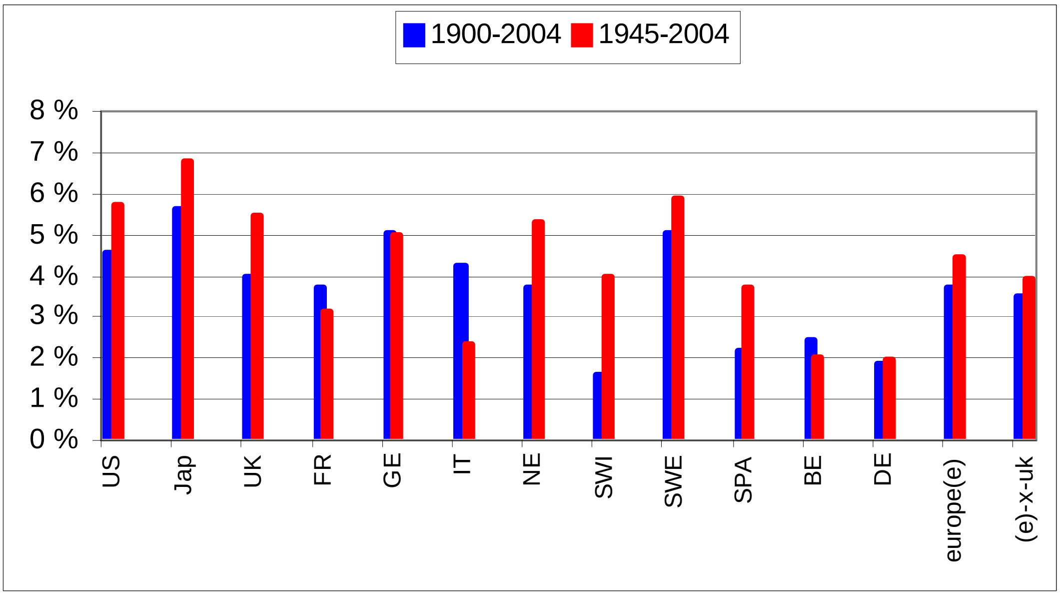

Chart 3.1: *Realised annual risk premiums (geometric averages) in per cent in the USA, Japan, 10 European countries, Europe as a whole, and Europe apart from United Kingdom, in the periods 1900-2004 and 1945-2004. The values are calculated from total return data from Dimson, Marsh and Staunton (2004).*

	US	**JP**	UK	FR	GE	IT	NE	SWI	SWE	SPA	BE	DEN	**EUR-OPE**	**EUR-ex-UK**
1900 - 2004	**4,6**	**5,6**	4,0	3,7	5,0	4,2	3,7	1,6	5,0	2,2	2,4	1,8	**3,7**	**3,5**
1945 - 2004	**5,7**	**6,8**	5,4	3,2	5,0	2,3	5,3	4,0	5,9	3,7	2,0	1,9	**4,5**	**3,9**

Table 3.1: *The figures behind Chart 3.1.*

The figures show wide variations between the countries, from 1.6% in Switzerland to 5.6% in Japan (1900-2004). Since 1945 the range has been from 1.9% in Denmark to 6.8% in Japan. We also see that, with three exceptions (France, Italy, Belgium), the equity premium was higher in the period after 1945 than in the period from 1900[10]. This does *not*, however, reflect a steadily rising trend in recent times. Chart 3.2 shows 15-year rolling (geometric) averages of annual equity premiums in the USA, Japan, the United Kingdom and the Netherlands since 1900. The increase in average realised equity premiums since 1945 is largely due to very high equity premiums from the end of World War II up to about 1960. After this period of recovery the equity premium has not been particularly high. Since 1960 it has been 3.0% in the USA, -0.3% in Japan, 3.5% in the United Kingdom and 3.7% in the Netherlands. In

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weights in 2004; the estimate is therefore not precise. Similarly, the equity premium in Europe apart from the United Kingdom is a weighted average of the equity premium is in the nine selected European countries apart from the United Kingdom (market capitalisation weights from 2004).

[10] The reason why France, Italy and Belgium are exceptions is principally the fall in the bond market in these countries early in the last century.

France and Germany it has been -0.3% and 0.3% respectively, while the six other European countries in table 3.1 are in the range -1.1% (Italy) to 4.3% (Sweden).

Siegel (1998) has calculated the (geometric) equity premium in the USA for the periods 1802-1998 and 1871-1998, finding 4.1% and 4.3% respectively. This is yet another indication that the high equity premiums in the years after World War II were atypical.

Dimson, Marsh and Staunton (2004) find that the global equity premium (weighted average of 18 countries) was 4.0% (geometric) in the period 1900-2004.

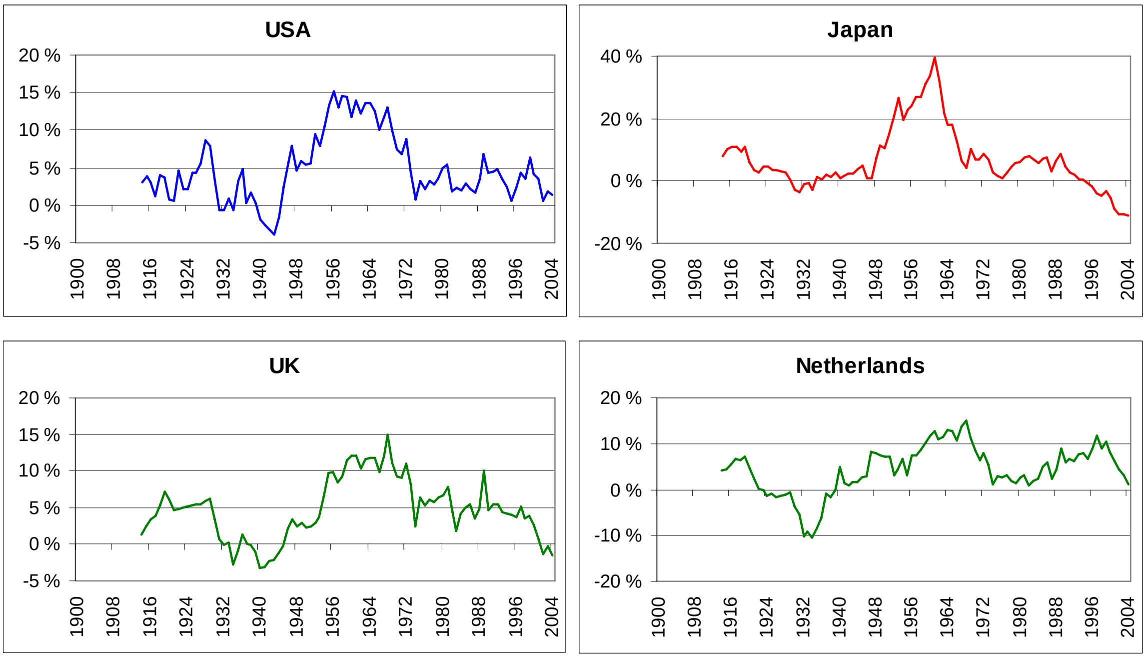

Chart 3.2 *15-year rolling geometric average equity premiums since 1900 in the USA, Japan, United Kingdom and the Netherlands (local currency). Data from Dimson, Marsh and Staunton (2004).*

3.3 Expected equity premium

Being long-term investors we are interested in learning what equity premium we can currently expect over long time horizons. As mentioned by way of introduction, this is a non-trivial question. There are several arguments, both quantitative and qualitative, in favour of the notion that the historic realised equity premium does not represent a good estimate of the future equity premium.

First, theoretical equilibrium models do not provide precise estimates of the equity premium. Even so, Mehra and Prescott (1985) have pointed out that consumption-based equilibrium theory for price formation in the equity market (the consumption based CAPM) indicates an expected equity premium which at most is 1 percentage point above the risk-free return, given realistic estimates of investors' risk aversion. The high realised equity premium throughout the last century (and also through the 19th century in the USA, cf Siegel 1998) has therefore been termed the equity premium puzzle. The model has subsequently been refined, partly in light of this puzzle. Recent work by Mehra, Constantinides and Donaldson (2002) shows that the expected equity premium rises with the introduction of heterogeneous market participants and borrowing restrictions. This may indicate that the equity premium puzzle is to some extent rooted in a lack of realism in the original model. Kurz, Jin and Motolese (2003) have recently developed an equilibrium model featuring heterogeneous investors with a variety of rational market views, which they believe can explain the realised equity premium, thereby "solving" the equity premium puzzle. Other models, such as the international CAPM in its unconditional form (Solnik 1974, Adler and Dumas 1983) can be fairly readily calibrated to the realised equity premium of about 4%, without risk aversion becoming unnaturally large. The equity premium's variation over time and across countries has however been more difficult to explain within conditional versions of these models. An important and recurrent result of this type of conditional model estimation is that *expected equity premium is probably time-varying* and conditioned by state variables related to economic conditions (Dumas and Solnik, 1995; De Santis and Gerard, 1997). This empirical result appears to tell against simple extrapolation of historic realised equity premium, at any rate over short and medium-term horizons (economic cycles).

A weightier argument builds on the so-called dividend discount model, or Gordon model (Gordon 1962). The model assumes rational pricing of future dividend flows. In its simplest static form, in which all variables are constants, expected real equity return E[R] is given by the formula:

$$E[R] = D/P + GD$$

where D/P is the dividend-price ratio (dividend yield) and GD is expected real annual dividend growth. The intuition behind the formula is that total return on equities has two components: dividend gain and capital gain (price return). Since equity return equals the risk-free real bond return (RF) plus an equity premium, the expected equity premium is expressed as E[RP] (risk premium):

$$E[RP] = D/P - RF + GD$$

The Gordon model has recently been used to estimate the size of the expected equity premium through the last century. Such analyses required a model of expected dividend growth *GD*, which of course is a non-observable variable. Another problem is that the dividend-price ratio may have been pushed down artificially by companies buying back their own stocks, an effect that many analyses seek to correct for[11]. A third problem is that the expected real return on government bonds needs to be estimated. These factors introduce extra uncertainty into this type of analysis. But the analyses that have been carried out all appear to point in the same direction.

Fama and French (2002) focus on the unconditional expected return. Given specific assumptions for expected dividend growth, they estimate that the expected equity premium (geometric) in the USA in the period 1872-2000 must have been around 2% (3.5% arithmetic), which is less than half the realised equity premium (4.3% in the period 1871-1998, according to Siegel 1998). Since the expectation is unconditional, 2% is also their estimate for the expected equity premium today.

Arnott and Bernstein (2002) analyse equity return over the last 200 years in the USA. They assume that expected dividend growth is given by expected real per capita growth in GDP,

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[11] Liang and Sharpe (1999) have calculated that the observed dividend yield D/P must be adjusted up by 0.75-1 percentage point to take account of companies' buybacks.

minus a dilution effect which takes into account the fact that a large proportion of growth in the economy derives from new (unlisted) companies[12]. The analysis, which gives time-varying estimates of expected equity premium, indicates that the expected equity premium must have averaged 2.4%, and that annual expectations can rarely have exceeded 5%. Expected equity premium today is very low in their model.

Dimson, Marsh and Staunton (2002, 2004) find that real dividend growth in the USA in the period 1900-2000 was unexpectedly large. This annual "surprise" is estimated at around 0.2 percentage points. In the Gordon model the expected equity return is pushed down by a corresponding margin. In addition the authors assume that an observed fall in the dividend-price ratio (since the mid-1980s) reflects falling expected equity premium. The outcome of their analysis is that the expected equity premium through the last century must have been lower than the realised level, and that it has currently fallen to around 2.5% geometric (4% arithmetic).

Hence the above analyses find that the expected equity premium cannot have been higher than 2-2.5% through the last century. The high realised equity premium is mainly explained in two ways: as *positive surprises* (surprisingly high economic growth, dividend growth, liberalisation of trade and finance, etc.); and *falling expected equity premium* (entailing rising realised equity premium). The analyses also indicate a low expected equity premium today.

A feature common to the above analyses (and many similar ones) is that they take the static Gordon model as their basis. This could be problematic, since the model describes a steady state in which expectations and the dividend-price ratio are constant. Even so, the model is estimated on historical data, where the dividend-price ratio shows wide variation, and conditional analyses result in time-varying expectations. Little has been done to control for this inconsistency in the literature. A more consistent and demanding approach would be to use a dynamic version of the Gordon model, as proposed by Campbell and Shiller (1988 a,b) and Vuolteenaho (2000).

Other arguments against extrapolating the realised equity premium into the future are as follows:

i) Statistical uncertainty: Even with a hundred years of historical data, the statistical uncertainty surrounding the equity premium's historical expectation value is about 2 percentage points (given that the expectation value has remained constant). This means that when the historical average is measured at 4%, there is an approximately 30% likelihood that the expected equity premium has remained outside the range 2 to 6%. In the simplest model featuring random annual variations, 400 years of history are required to contemplate narrowing this range to 3 to 5%.

ii) Difference between expected and realised equity premium in the case of time variation: If the realised equity premium rises, it may be because the expected equity premium is falling (and vice versa). It would in this case be quite wrong to extrapolate the realised equity premium into the future; the expected equity premium should have been adjusted *down*, not up.

[12] Arnott and Bernstein (2002) find that the dilution effect reduces expected real growth in dividend by about 0.8 percentage points.

iii) Lower transaction and information costs and greater diversification opportunities: Transaction and information costs are now lower than during large parts of the last century. Similarly, diversification and hedging opportunities are wider, both nationally and internationally, thanks partly to liberalisation of capital markets and the emergence of new instruments (for example derivatives) and fund constructions (for example cost-effective index funds). It is reasonable to assume that investors have reduced their excess return requirement (the equity premium) in step with this development. A downward adjustment of this type will be only partly captured by looking at a long-term historical average.

3.4 What is the consensus expectation for the long-term equity premium today?

As the above discussion has illustrated, there are both quantitative and qualitative arguments for the notion that the expected long-term equity premium is now lower than the average historic equity premium of around 4% (geometric). As already mentioned, the realised equity premium has indeed been lower than 4% since 1960 in most developed economies. Moreover, we have seen that statistical uncertainty attached to a point estimate for the expectation value is substantial, of the order of 2 percentage points.

Two surveys by Welch (2000, 2001) show that a large number of financial academics in the USA had revised down their equity premium expectations (for the USA) in August 2001 in relation to their expectations at the end of 1997. In 1997 the average expectation was just under 6% (geometric), with a standard deviation of 2 percentage points. By August 2001 the average expectation had fallen to about 4% (standard deviations not reported). It could seem that the dramatic falls in equity markets triggered in the autumn of 2000 led to a reconsideration of equity markets' valuation and a new belief in traditional valuation models (variants of the Gordon model).

Clarke and da Silva (2003) have more recently compiled a number of estimates for the expected equity premium in the USA provided by academics and professional market participants and consultants (among them the already mentioned results from Fama and French 2002, Arnott and Bernstein 2002, Dimson, Marsh and Staunton 2002, and Welch 2000, 2001, as well as consultants and market participants such as Frank Russell, Goldman Sachs, Ibbotson, and Warren Buffett). The average expected equity premium in this sample is about 2.5% (geometric), and the standard deviation is about 1.5%[13].

This average is very close to the long-term expectations stated by some pension funds in their annual reports and other publications in the period 2003-2004. These pension funds include Ontario Teachers' (2.5%), General Electric Asset Management (2.5% over a 10-year horizon), PGGM (2.2%), AP4 (2.5%), ABP (3%) and Universities Superannuation Scheme (2%).

Thus it appears that the average expectation for the long-term equity premium may now be of the order of 2.5%, and that the estimate uncertainty (standard deviation) around this value is about 1.5 percentage points.

We are not aware of similar thoroughgoing surveys of the expected equity premium in other equity markets (Europe, Asia). In the very long term (50 years or more) there is, however, little basis for differentiating between developed economies, with a possible exception in the case of differing demographic developments, which might perhaps influence long-term expectations.

[13] Many of the expectations are stated as an arithmetic average. We have subtracted 1.5 percentage points to arrive at corresponding geometric expectations.

3.5 Distinction between long and medium term?

Our expectations of the future equity premium should be consistent with traditional valuation models, such as the Gordon model. As we have seen, the consensus expectation of around 2.5% is largely a result of such valuation analyses. In this section we ask the following question: Is an expected equity premium of 2.5% (geometric) currently in line with the Gordon model in the long term?

As historical data suggest, real dividend growth cannot be expected to be as high as real growth in the economy (GDP), since the latter is driven by both listed and unlisted companies (dilution effect)[14]. Arnott and Bernstein (2002), for example, find that real growth of dividend has been as low as about 1% in the USA since 1810, substantially lower than real GDP growth (and also lower than real per capita growth). They argue that even under optimistic growth estimates for the US economy (2.5 – 3% real growth), future real growth of dividend will be in the area 1 – 2%. A realistic basis would seem to be 1.5%.

According to the Gordon model, expected real return on equities equals the current dividend-price ratio (D/P) plus expected real growth of dividend. The current dividend-price ratio in the USA is around 1.75%, while the average over the past 12 months has been 1.7%. If we take this 12-month average, and add an expected real dividend growth of 1.5%, we obtain an expected real return of 3.2%. Assuming an expected real growth of 2.8% on long government bonds over the next 15 years (see Chapter 2 on expected bond return), there is not much scope for a significant equity premium in this period. The dividend-price ratio should however be corrected for companies' buybacks of their own stocks. As already mentioned, Liang and Sharpe (1999) find that this justifies an upward revision of the dividend-price ratio by 0.75-1 percentage point. Adding 1 percentage point makes room for an expected equity premium of around 1.4 percentage points (4.2% - 2.8%).

Similar calculations can be done for the other major equity markets. Expected real dividend growth is again assumed to be 1.5%. Correction for companies' buybacks of their own shares is however reduced to 0.75% since the scale of such buybacks is assumed to be smaller outside the USA[15]. Given average dividend-price ratios over the past 12 months, the Gordon model produces the following expected real return rates on equities over the next 15 years:

> 4.8% in Europe apart from the United Kingdom (expected equity premium 2%);
> 5.5% in the United Kingdom (expected equity premium 2.7%);
> 3.2% in Japan (expected equity premium 2.1%);
> 5.3% in Asia apart from Japan (expected equity premium 2.5%).

According to these calculations, the expected equity premium in the above equity markets is between 2 and 2.7%, tallying reasonably well with the long-term consensus expectation of 2.5%.

A market-capitalisation weighted average of expected real equity returns in the United Kingdom and Europe apart from United Kingdom gives an expected real equity return of 5% in all Europe (expected equity premium 2.2%). Corresponding figures for Asia are 4% (equity

[14] US data from Robert Shiller (2004), covering the period 1871-2004, show that the (geometric) average real growth of dividends has been about 1.2% annually, while real earnings growth has been 1.7%.
[15] The buyback correction may possibly be even smaller than 0.75% outside the USA, in which case the expected equity premium would be correspondingly lower.

premium 2.3%). These figures represent our best estimates of expected real return on equities in Europe and Asia over the 15 years ahead. Estimation uncertainty (standard deviation) is put at about 1.5 percentage points.

The USA, on the other hand, singles itself out in as much as the expected equity premium is about one percentage point below the long-term consensus expectation of 2.5%. Assuming it is correct that the expected equity premium in the USA is now as low as 1.4%, it represents a substantial fall, even in relation to the relatively low expectations that several scholars now believe investors must have had during much of the past century (2-2.5%, see above). It would seem unlikely that the expected equity premium has fallen by such a wide margin, even if risk tolerance most probably varies and may at times be quite high. Another possible explanation is that the *equity market in the USA is currently in disequilibrium,* or *overpriced*, such that D/P has fallen too far. Several scholars find this explanation both plausible and probable (see for example Campbell 2001, Diamond 2001, Reichenstein 2002). If this is the case, equity prices need to fall or, at least, rise more slowly than the dividend level (i.e. D/P must rise) before the expected equity premium can reach a new long-term level of around 2.5%. According to this view the correction of equity prices started in the autumn of 2000, and is still not over with. How long time remains will among other things depend on the economic climate in which the equity market operates.

Hence in this correction scenario the *realised* equity premium in the USA in the years immediately ahead will be lower than the long-term expected equity premium of 2.5%. Assuming a rather long correction period of, say, 15 years, a simplified calculation shows that the realised annual equity premium may prove to be around 1 – 2 percentage points lower. Expected real return on US equities will in this case be about 3.3 – 4.3%. We let the midpoint of this range (3.8%, or around 4%) represent our best estimate for the real return in the USA over the next 15 years. Estimation uncertainty is put at 1.5 – 2 percentage points.

One way to assess the possibility of such a correction scenario is to look at historic rolling (geometric) averages of the equity premium and real equity return. As already seen, Chart 3.2 reveals sizeable fluctuations in realised equity premium, which has at times been close to zero or even negative (moving average 10-year equity premiums show a similar picture). The fluctuations illustrate that the possibility that the average realised equity premium may prove very low or negative over 15-year periods *cannot be ruled out*. However, the charts say little about the *probability* of such outcomes.

Similarly, Chart 3.3 shows 15-year moving geometric averages for the real equity return in the USA, Japan, the United Kingdom and Germany since 1900. The chart shows that realised real return has been very low at times. Hence the real equity return may well be low also in some future 15-year periods.

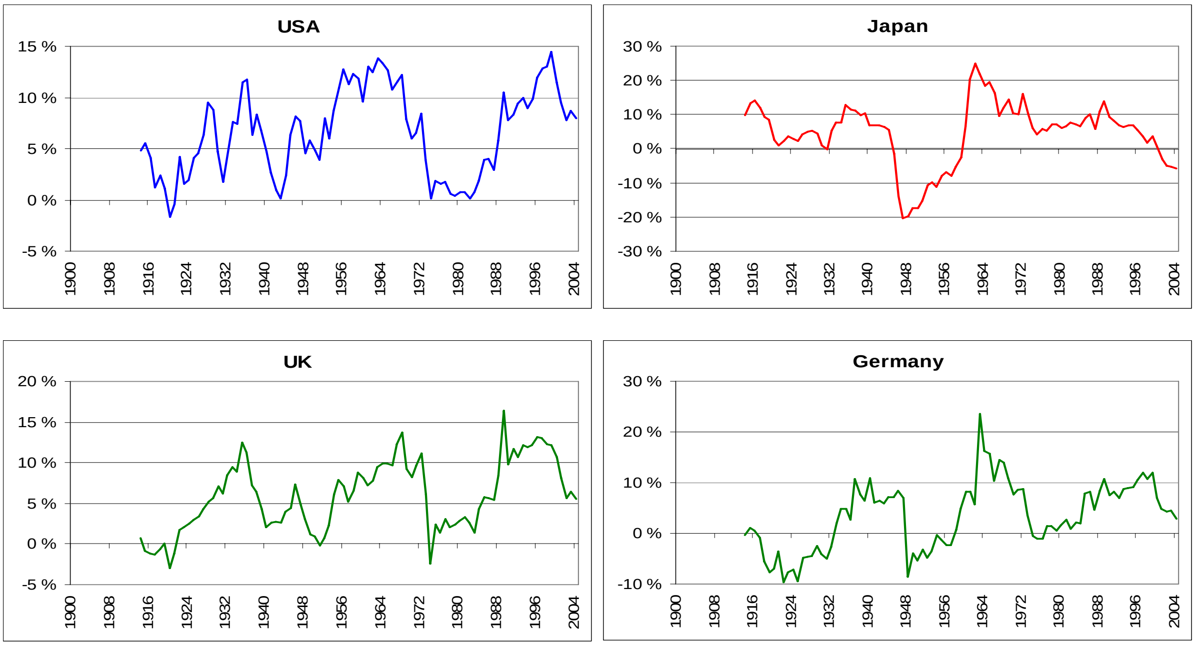

Chart 3.3: *15-year moving geometric average real return on equities since 1900 in the USA, Japan, the United Kingdom and Germany (local currency). Data from Dimson, Marsh and Staunton (2004).*

Another way to assess the likelihood of this scenario is to look at dividend-price and earnings-price ratios. Long-term historical data are only available for the USA (data from Robert Shiller). Chart 3.4 shows that both these valuation indicators are still very low in a historical context. This is especially true of the dividend-price ratio, which may to some extent be due to increased buybacks of equities. However, the corrected dividend-price ratio (adjusted up by 1 percentage point) is also relatively low in historical terms. A gradual increase of D/P (repricing) in the USA which, all else equal, will make a negative contribution to real return, cannot be ruled out and appears reasonable given the historical development shown in chart 3.4. How *probable* it is depends on the valuation indicators' ability to *predict* future real return over differing horizons.

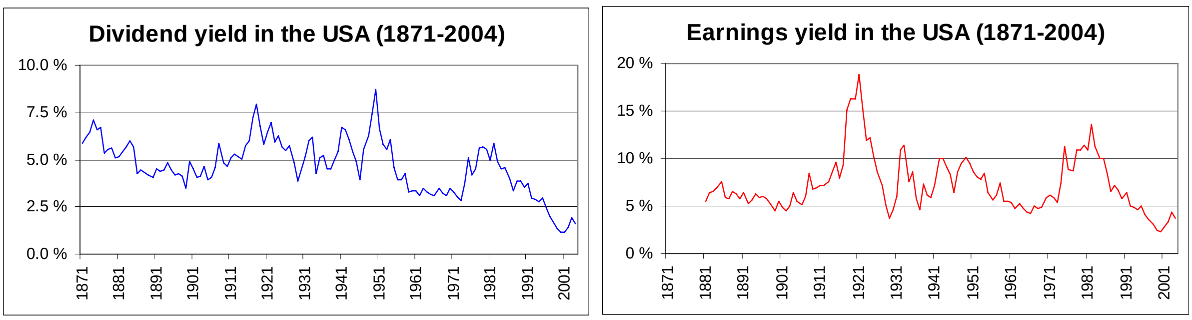

Chart 3.4: *Historical dividend-price and earnings-price ratios in the USA. The earnings-price ratio is not corrected for buybacks of equities, while the dividend-price ratio is corrected for cyclical variations in earnings. Data from Robert Shiller (2004).*

A number of analyses have indicated that valuation indicators such as D/P and E/P to some extent predict real return in the medium term (10-15 years) (see for example Campbell and

Schiller, 1988a,b; Fama and French, 1988a; Cochrane, 2001). This result has had a significant impact on both theoretical and empirical research, not least in relation to dynamic asset allocation. The Gordon model establishes a theoretical link between expected real return and valuation indicators. However, recent studies have cast doubt on the robustness of these conclusions. Bosseart and Hillion (1999), for example, find the explanatory power of these valuation indicators to be relatively good "in sample", but not "out of sample", possibly because the model's parameters are not stationary. Goyal and Welch (2004) arrive at a similar result for a larger selection of state variables (valuation indicators). Ang and Bekaert (2004) make a similar analysis and extend the database to include more countries. They find the prediction ability of the dividend-price ratio to be neither statistically significant, robust across countries, nor stable for various sub-periods. Campbell and Yogo (2003) develop new, more robust statistical tests to evaluate state variables' prediction ability. They find that D/P and E/P have some prediction ability also "out of sample", but that it is far weaker than previously assumed, and that prediction ability has diminished since 1952.

The question of predictability is key and highly controversial in empirical finance. We have assumed that a slight degree of predictability over 10-15 year horizons is credible, but realize that the issue remains unsettled. Hence the current low values for D/P and E/P are indicative of low real return on equities over the next 10-15 years, but this is far from certain. As mentioned, the alternative is that the low values represent a new long-term equilibrium level, and that the expected equity premium is very low (around 1.4%, which also gives low expected real return; see section 3.5). Which scenario is most likely must to some extent be a subjective assessment.

3.6 Assessments

We have discussed several arguments, both quantitative and qualitative, to the effect that the expected long-term equity premium may now be lower than the realised equity premium through the last century. Although *uncertainty is very substantial*, two scenarios appear to be most likely.

One is that the expected long-term equity premium is 2.5%, which appears to be the average expectation amongst financial scholars and various market participants. Estimate uncertainty is put at 1.5 – 2 percentage points, which represents the spread in their views, as well as the statistical uncertainty around the equity premium's historical expectation value. However, the realised equity premium in the USA will in this scenario be lower in a transitional period of uncertain duration, due to correction of current overpricing (repricing via rising D/P). How low it falls depends partly on the duration of the transitional period. If this period is set, for example, at 15 years, the annualised realised equity premium will be around 1% in the USA in the transitional period (realised real return is about 4%).

The other scenario is that the US equity market has already reached a new long-term equilibrium (no transitional period), and that the expected *long-term* equity premium is very low. *However, we believe that this scenario is less likely than the previous one*[16], for several reasons. First, it seems unlikely that the equity premium should have fallen by such a wide margin in relation to the expectations that investors apparently have held through the previous century (2 – 2.5%), despite cogent arguments to the effect that the excess return requirement

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[16] We are not alone in holding this view. Several scholars, for example Campbell (2001), Diamond (2001) and Reichenstein (2002) believe that some repricing will occur before the expected equity premium can attain its long-term level (first scenario above). How much repricing can be expected, and how long it will take, is a moot question.

must have been falling (lower transaction and information costs, improved diversification opportunities, etc). Second, the scenario entails a substantial difference in the long-term equity premium between countries. Third, it seems likely that the market has, to some extent, started pricing in some new economic, demographic and geopolitical risk factors (see below). An expected long-term equity premium as low as 1.4 percentage points in the USA appears unreasonable given this set of risk factors.

3.7 Economic and geopolitical risk factors in the medium and long term

We consider it likely that the market will over time focus more on specific fundamental risk factors, which will have an impact over medium-term and long horizons. Current valuations of equity markets, particularly in the USA, could indicate that these risk factors have yet to be fully priced. All else equal, an increased focus on these risk factors will in time push up the expected equity premium, and reduce the realised (ex post) equity premium. It is also possible that the expected *long-term* equity premium will be affected, in which case it could possibly reach higher than the current consensus of 2.5%. Since these effects are very hard to quantify, they do not modify our forecasts. However, they clearly contribute to estimation uncertainty, and could be captured by skewness in the probability distribution.

Significant risk factors with a possible impact over the next 10-15 years are:

- The USA's twin deficits.
- Japan's uncertain growth.
- The effect of China's and India's growth and integration in the world economy.
- The oil price (and commodity prices in general).
- Inflation outlook.
- Pension obligations (private and public).
- International bond and real estate markets.
- Geopolitical risk (terrorism, Middle East, + +).

In the long term (30-50 years) the market may increasingly focus attention on the following structural risk factors (among others):

- The effect of China and other new large economies on today's developed markets.
- Demographic trend (ageing).
- Scarcity of important natural resources (particularly oil).
- Environmental problems (particularly global warming).

3.8 Expected risk (volatility) and correlations in the equity markets

We have discussed the expected equity premium and the estimation uncertainty involved. Estimate uncertainty must not be confused with volatility. The former represents the uncertainty surrounding the expectation value proper, while volatility expresses the annual standard deviation around a given expectation value. In a simulation of a portfolio's market value, these uncertainties can be dealt with separately (for example, as stochastic drift and diffusion).

In the following we discuss estimates for equity market volatility and co-variation in the long term, and how far auto-correlation in long-term return (mean reversion) can be expected to reduce risk in the long term.

3.8.1 Volatility and auto-correlation (mean reversion)

Long-term volatility and correlation projections are generally based on average historical values. When annual rates of return are not auto-correlated, annual volatility also describes the long-term risk (annualised volatility)[17]. If, on the other hand, returns show negative auto-correlation (mean reversion), annual volatility will be reduced over time.

Fama and French (1988b) and Poterba and Summers (1988) find negative auto-correlation in equity returns over long horizons. Campbell and Viceira (2002) have calibrated a VAR model (Vector Auto Regression) for equity and bond returns to US data from 1890 to 1998. They find signs of long-term negative auto-correlation in the equity market (mean reversion) and long-term positive auto-correlation (mean aversion) in the bond and money market. However, as in the case of the closely related issue of predictability, the question of mean reversion remains unresolved. The reliability of empirical studies is limited by the fact that historical time series can only provide a small number of non-overlapping period returns over long horizons. And there is no saying that "history will repeat itself".

We have opted to take a basis in Campbell and Viceira's (2002) VAR analysis, which gives an indication of how rapidly the annualised volatility of equity return may fall over time in the presence of negative auto-correlation. Their results show a reduction by approximately one-sixth over a 15-year period (from 18% to 15%), and about one-fourth over a 50-year period. Although these results refer to the US equity market, we will for simplicity assume the same reduction of annualised volatility in other markets (the same degree of mean reversion).

Chart 3.5 shows volatility of real return on equities for rolling 15-year windows in the USA, Japan, the United Kingdom, and Europe apart from United Kingdom[18] since 1900 (local currency).

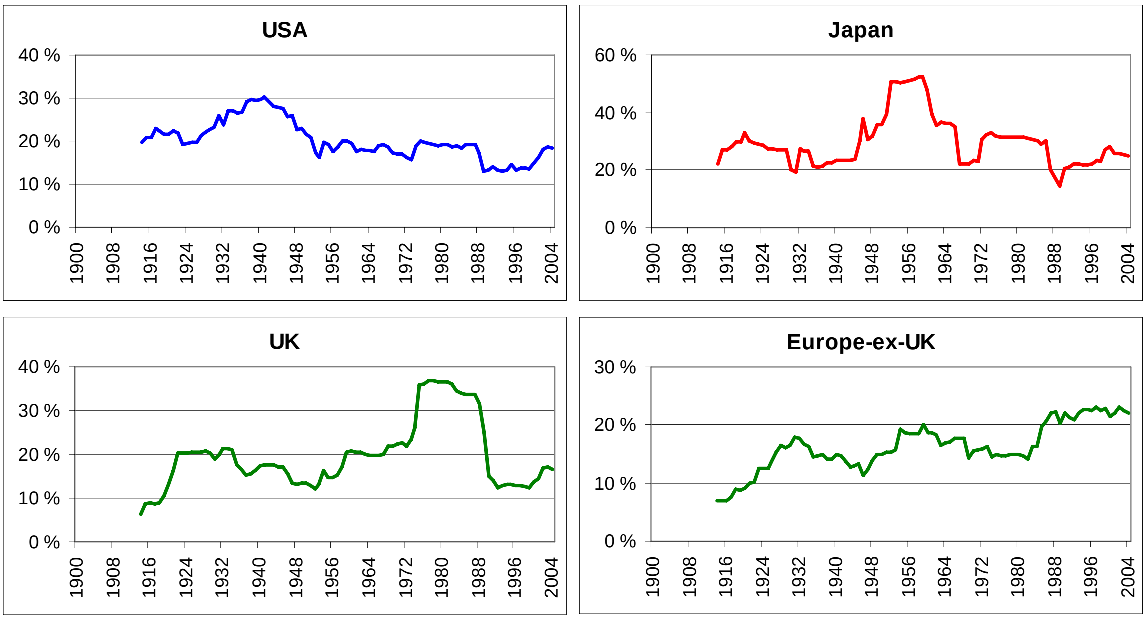


Chart 3.5: *15-year rolling volatility windows of real return on equities since 1900 (local currency) in the USA, Japan, the United Kingdom and Europe apart from United Kingdom (portfolio of nine European countries). Data from Dimson, Marsh and Staunton (2004).*

[17] Under the random walk assumption, the standard deviation in relation to return over a period of T years equals the standard deviation multiplied by the square root of T.

[18] Here "Europe apart from United Kingdom" is a portfolio of equity markets in nine West European countries.

In the USA volatility has averaged around 20% since 1900, and 18% since 1945. The latest 15-year window's volatility is now close to the average for the post-war period, after a long period of relatively low volatility. An approximate estimate for future annual volatility is 20%. An assumption of mean reversion reduces annualised volatility by about one-sixth over a 15-year period, to 17%.

In Japan annual volatility has been 30% since 1900, and 34% since 1945. Several factors may explain why volatility has been higher here than in the USA: a turbulent market in the years during and after World War II, Japan's sharp growth in the post-war period, and a thinner market. However, volatility has been high (28%) even after 1970. Whether volatility will remain equally high in the future is a moot point, depending among other things on Japan's economic fate. Japan will probably remain a thinner market than the American and European markets. However, this is not a sufficient basis for asserting that volatility in Japan will *differ significantly* from volatility in other markets *in the long term.* Volatility is therefore envisaged to be about 20% in the long term, but higher over the next 15 years (25%). Here too we assume that mean reversion will reduce annualised volatility risk by about one-sixth over a 15-year period, to 21%.

In the United Kingdom volatility was 20% after 1900, and somewhat higher after 1945 (23%). Economic turbulence in the 1970s brought a strong, but transient, increase in volatility. A reasonable estimate for future volatility is 20%, dampened by about one-sixth by mean reversion over a 15-year period (to 17%).

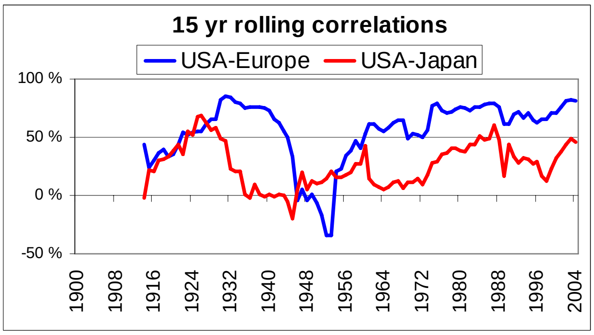
In Europe apart from United Kingdom, volatility was 17% after 1900 and 20% after 1945. A possible explanation for the increase is stronger co-variation (correlation) between national markets. A reasonable estimate of future volatility is 20% (again dampened to 17% over a 15-year period).

In Europe as a whole (including the United Kingdom), historical volatility was 16% from 1900 and 18% after 1945. A relatively low correlation between the United Kingdom and the rest of Europe (respectively 54% and 56% after 1900 and 1945) means that volatility in Europe as a whole has been lower than in either of the two regions separately. This diversification gain may diminish as the two markets become more integrated. An approximate estimate of future volatility is 20% (dampened to 17% over a 15-year period).

3.8.2 Correlations between equity markets

The liberalisation of capital markets in recent decades, along with general globalisation, has brought some increase in co-variation between equity markets. It is therefore pertinent to estimate future correlations on the basis of correlations in the recent past. Since we are also interested in exchange rate correlations, a basis can suitably be taken in market correlations since the collapse (in 1971) of Bretton Woods. We have chosen the period 1973-2004.

In historical terms the USA and Europe have shown high correlation when World War II is disregarded (Chart 3.6). Since 1973 correlation has been about 80%, which is our estimate of the future correlation between real return on US and European equities.



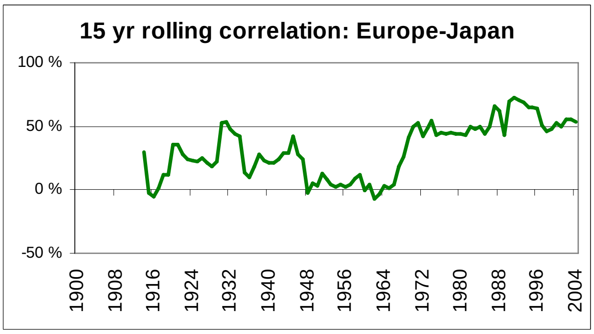


Chart 3.6: *Correlations between annual real return on equities in rolling 15-year windows since 1900 in the USA, Japan and Europe (portfolio of ten European countries, including the United Kingdom). Annual returns in local currency. Data from Dimson, Marsh and Staunton (2004).*

In historical terms the USA and Japan have shown far lower and more volatile correlation (Chart 3.6). Since 1973 the correlation has been about 48%. Our best estimate of future correlation is therefore 50%.

The correlation between Europe and Japan has been 59% since 1973. A reasonable estimate of future correlation is assumed to be 60%.

Table 3.2 summarises our estimates of correlations between equity markets.

	Japan	Europe
USA	50%	80%
Japan		60%

Table 3.2: *Estimate of future correlations between real return on equities in the USA, Japan and Europe (including the United Kingdom). Annual returns in local currency.*

3.8.3 Correlations between equities and government bonds

Here we are looking at historical correlations between real return on equities and government bonds within countries (regions)[19]. We have again opted to base our estimates of future correlations on historical correlations in the period 1973-2004.

In the USA, rolling 15-year correlations have shown sizeable fluctuations (Chart 3.7). Our estimate of future correlation is the value in effect since 1973, which is around 35%.

In Japan the rolling 15-year correlation has fluctuated between 2% and 90%, and is now in the low end. As previously, our estimate for the future is the historical correlation since 1973, which is about 25%.

In the United Kingdom the rolling 15-year correlation has been somewhat less unstable (varying between 30% and 80%), and is now 54%. The correlation since 1973 has been 57%. In Europe apart from United Kingdom, the 15-year correlation fell dramatically through the

[19] To save space we omit discussion of correlations between equities in one country and government bonds in another country.

thirties and forties, but recovered and stabilised in the second half of the century. The correlation since 1973 has been 40%. In Europe as a whole (including United Kingdom) the correlation has been around 50% since 1973, which represents our forecast for the future.

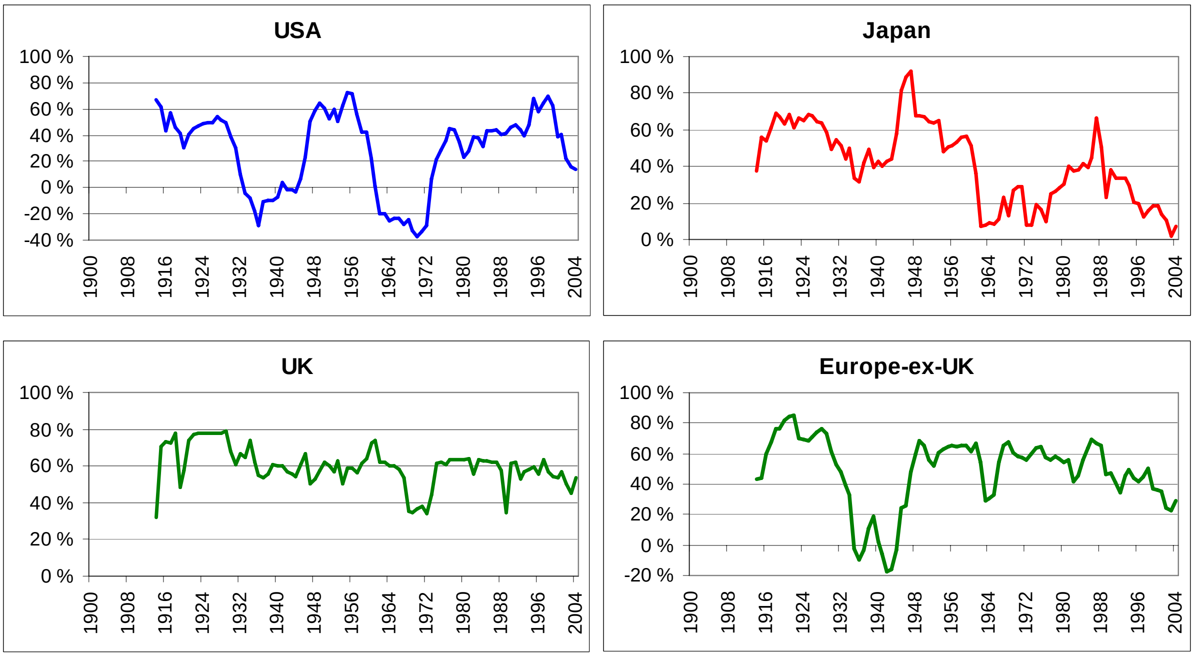


Chart 3.7: *Correlations between annual real return on equities and government bonds in 15-year windows in the USA, Japan, United Kingdom and Europe apart from United Kingdom (a portfolio of nine European countries) since 1900. Annual returns in local currency. Data from Dimson, Marsh and Staunton (2004).*

Table 3.3 summarises our estimates of correlations between returns on equities and government bonds.

USA	Japan	Europe
35%	25%	50%

***Table 3.3*:** *Estimates of future correlations between real return on equities and government bonds in the USA, Japan and Europe (including the United Kingdom). Annual returns in local currency.*

3.9 Summary

Our estimates of the expected (geometric) equity premium, real return and annualised volatility in the long term and over the next 15 years are summarised in Table 3.4. Estimate uncertainty in relation to the expected equity premium is also shown (in percentage points). The expected real equity return incorporates the expected real bond return calculated in Chapter 2.

	USA	***Europe***	***Asia***
Long-term expected equity premium (%)	2.5	2.5	2.5
Estimate uncertainty in relation to long-term expected equity premium (%-points)	1.5	1.5	1.5
Expected equity premium next 15 years (%)	1	2.5	2.5
Estimate uncertainty in relation to expected equity premium next 15 years (% points)	1.5	1.5	1.5
Expected real return next 15 years (%)	4	5	4
Expected effective volatility next 15 years, if weak mean reversion (%)	17	17	21

Table 3.4: *Summary of our estimates of expected (geometric) equity premium, estimate uncertainty, real return and volatility, long-term and over the next 15 years.*

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