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The choice of exchange rate assumption in the process of forecasting inflation

by

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# **The choice of exchange rate assumption in the process of forecasting inflation**

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## *Abstract*

In the process of forecasting inflation the central bank needs to make assumptions regarding the future path of the interest rate and the exchange rate. This paper focuses on the exchange rate assumption. In particular, three alternative assumptions are discussed.

First, the random walk hypothesis suggests a constant exchange rate over the forecast horizon. While being theoretically appellant and frequently chosen across central banks, the level of the exchange rate implied by the assumption remains to be determined. Moreover, assuming the exchange rate to be constant may imply a change in the real exchange rate if the inflation target differs from the target among the trading partners.

Second, central banks frequently assume the exchange rate to evolve in line with uncovered interest rate parity. Combined with the assumption of using forward rates for the interest rate, this seems theoretically consistent. However, due to a possible risk premium UIP-predictions may imply a path for the exchange rate far away from market expectations, a path with which the central bank is not necessarily comfortable.

Third, the central bank could base the exchange rate assumption on its best prediction of the exchange rate in the future. This would require some analytical tools for the exchange rate. In this paper we focus on mean-reversion in the real exchange rate as a guide to future nominal exchange rate movements. Exchange rate paths implied by the three alternatives are compared with actual exchange rate outcomes.

## *1. Introduction and background*

Inflation projections in central banks are commonly based on the assumption that future exchange rates and interest rates are in line with market expectations or remain constant over the forecast horizon. These assumptions are frequently referred to as *technical*, meaning that the central bank does not necessarily expect them to materialize. Alternatively, the central bank can base its exchange rate and interest rate assumptions on its best prediction of the future. The purpose of this paper is to discuss these two conceptually different assumptions. Particular focus will be placed on the exchange rate assumption.

Independent of whether the assumptions are technical or reflecting the bank's best prediction of the future, some basic criteria should be satisfied:

- The exchange rate and the interest rate assumptions must be theoretically consistent. This would be satisfied if, for example, both the exchange rate and the interest rate are assumed to follow market expectations.
- The assumptions must be transparent and intelligible.
- In alternative scenarios, the key assumptions must be internally consistent and consistent with those in the main scenario.

Table 1 shows the bank's exchange rate and interest rate assumptions taken from the inflation reports since 1999. In the period 1999-2003 inflation projections were based on a constant future exchange rate. The level of the exchange rate implied by the assumption was calculated as an average of the exchange rate in the past. As from late 2003 inflation projections have been based on UIP-implied forward rates.<sup>2</sup> The interest rate assumption has also changed correspondingly over time. In comparison, Table 2 shows the exchange rate assumption in other selected central banks.

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<sup>2</sup> In Inflation Report 2003/2, two alternative projections were presented, one based on a constant future exchange rate and one based on UIP-implied forward rates.

In a small open economy, the exchange rate plays a paramount role in the inflation process. Figure 1 shows, based on data from the inflation reports in the period 1999–2003, the relationship between the exchange rate assumption error four quarters ahead and the corresponding inflation gap (deviation from the target) eight quarters ahead. The lags reflect the inertia in the exchange rate pass-through. As an example, compared to the exchange rate assumption in Inflation Report 2001/2, one year later the exchange rate turned out to be 8 percent stronger, while the core inflation two years later took the value 1.3 percentage point below target (marked in the figure).<sup>3</sup>

The pairs in the scatter plot should be concentrated in the first (north-west) and the third (south-east) quadrant, as exchange rate outcomes stronger (weaker) than assumed will normally be associated with inflation outcomes lower (higher) than predicted. In fact, out of the 13 dots, 10 are located in the second and the third quadrant. Moreover, apart from one outlier (IR 2002/3) the few observations outside the second and the third quadrant are border line observations, close to the second and the third quadrant. As expected the correlation coefficient is positive, 0.45.

The overall message from the chart is that the exchange rate assumption is of crucial importance in the process of forecasting inflation.<sup>4</sup> Thus, seeking for better methods in forming the exchange rate assumption may reduce the inflation forecast errors significantly. A paramount question is whether we are able to substitute the technical assumption with a reasonable forecast.

Against this background we discuss three alternative principles for determining the exchange rate in the projections, (i) a constant exchange rate, (ii) UIP-implied forward rates and (iii) an assumption reflecting the central bank's best prediction of the future exchange rate, where in particular mean-reversion in the real exchange rate will be emphasized. The ex post paths

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<sup>3</sup> In the Inflation Report 1999/1 the krone exchange rate was assumed to “return to its initial range against European currencies in the course of the next six months. Subsequently, the exchange rate is assumed to remain unchanged through the remainder of the projection period”. To simplify we concentrate on two kinds of assumptions, a constant exchange rate or UIP-implied forward rates. Hence in the following analysis we exclude the data from Inflation Report 1999/1.

<sup>4</sup> Of course, inflation and exchange rate outcomes are interdependent and determined simultaneously in a complex economic environment. Hence the univariate considerations attached to figure 1 are far too simple to explain inflation outcomes as such.

implied by the three different assumptions are compared with actual exchange rate outcomes in the case of Norway.

## 2. A constant exchange rate

The assumption of a constant exchange rate is supported by the random walk hypothesis. If today's exchange rate is the best prediction of the future, a constant exchange rate over the forecast horizon emerges as the best possible assumption. Starting with Meese and Rogoff (1983), the stock of the literature now indicates that a random walk is hard to beat.<sup>5</sup> Figure 2 shows the nominal effective exchange rate for Norway from 1971 (an increase indicates a depreciation). Calculated on the whole sample, formal tests clearly reject mean reversion in the data. However, based on the sub-sample from the beginning of 1990, both eye ball econometrics and formal test indicate that mean reverting has takes place.<sup>6</sup>

Formal tests aside, we will point to two problems regarding the constant exchange rate assumption. First, while the constant exchange rate assumption is frequently based on some average of the exchange rate in recent history, the time span over which to average the exchange rate remains to be determined. This issue will be further discussed below.

Second, as the domestic inflation target may differ from the average of trading partners, a constant nominal exchange rate over the next 2-3 years could imply an expected change of the real exchange rate. In the case of Norway, the inflation target is slightly above corresponding targets abroad. Thus, a constant nominal exchange rate would normally imply a real appreciation over a three years horizon. Though not necessarily a substantial change compared to yearly exchange rate fluctuations, the constant exchange rate assumption does contain a *systematic* – and not necessarily wanted – change in the real exchange rate.

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<sup>5</sup> In the long run, however, there is a tendency for purchasing power parity to hold (Taylor and Taylor, 2004).

<sup>6</sup> As will be shown below, the real exchange rate is stationary. Then, a stationary nominal exchange rate requires that domestic inflation evolves in line with foreign inflation. The tests are based on the unit root test available in EViews.

### *3. Exchange rate assumption based on uncovered interest rate parity*

As from late 2003 Norges Bank has based the exchange rate assumption on UIP. The parity states that due to arbitrage across markets interest rate differentials compensate for expected changes in the exchange rate. More formally, let

$$i_t = i_t^* + E_t[\Delta e] + rp_t,$$

where  $i_t$  and  $i_t^*$  are the domestic and the foreign interest rate in period  $t$  respectively,  $E_t[\Delta e]$  is the expected percentage change in the exchange rate between period  $t$  and  $t+1$  and  $rp$  is a risk premium. The equation holds by definition (of the risk premium), and UIP is defined by setting the risk premium equal to zero. Hence given some assumption of the future interest rate differential, UIP implies a path for the exchange rate which would make domestic and foreign investments equal in terms of expected return. In practice the expected future interest rate differential is calculated on the basis of implied forward rates from the yield curve.

Exchange rate assumptions based on UIP are appealing for a variety of reasons. First, it goes without saying that the parity has a strong theoretical foundation and is a cornerstone in financial and macroeconomic literature, linking domestic and foreign financial markets. Second, used in combination with forward interest rates, this seems theoretically consistent as the central bank in this case would build its inflation forecast on market expectations for both the two key variables determined in financial markets. Third, using markets expectations for the exchange rate and the interest rate provides a communication channel between the central bank and market participants. Suppose for instance that the central bank's contingent inflation forecast is higher than the inflation target. This could initiate a correcting mechanism as the bank implicitly conveys the message that it finds current forward rates too expansionary. As the market anticipates that the central bank sooner or later needs to tighten policy, it will – by the logic of rational expectations – adjust its expectations accordingly. In this case, however, the bank does not provide precise information on its preferred interest rate path.

A well-known argument against basing the exchange rate assumption on UIP-predictions is its empirical failure due to the existence of a risk premium. There is a large literature discussing empirical UIP tests and the main result is, overall, that UIP is empirically rejected.



One way to test UIP is to calculate the excess return over time, defined as

$$er = (i_t - i_t^*) - \Delta e_t,$$

where  $er$  denotes excess return. Under the null hypothesis of UIP and rational expectations, excess return will be zero on average. Figure 3 shows quarterly annualized excess returns on Norwegian kroner relative to a composite currency of the trading partners for the period 1991-2004. As an example, the asterisk shows an excess return of around 7 percent in the third quarter of 2000, summing up the interest rate differential in the previous quarter of around 2,5 percentage points and an appreciation of the krone of around 4,5 percent from the second to the third quarter, all variables annualized.

On average, for the whole period 1991-2004 the annualized quarterly excess return turns out to be around 1 percent, while the average excess return is slightly above 3 percent for the period 1999-2004. These figures are, in particular for the latter period, substantial. Truly, due to some well known problems attached to this simple test, notably the replacement of expected exchange rate changes by actual outcomes, the average excess returns may exaggerate the estimate of the risk premium. However, it seems reasonable to assume that a positive risk premium exists, meaning that the UIP-implied forward rates would normally exaggerate depreciation expectations.<sup>7</sup>

This introduces a *policy bias* in the exchange rate assumption: If unadjusted for possible risk premia, a positive interest rate differential would imply a stronger depreciation than what is contained in market expectations and what would on average materialize. This suggests that UIP-implied forward rates should in general be adjusted for a possible risk premium.

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<sup>7</sup> One should note that the test above is based on three-month interest rate data. To obtain a more correct view of how UIP performs on longer horizons, interest rates with longer maturity should be used, preferable horizons of 2-3 years, in line with the horizon for the inflation forecast. However, as we are particularly interested in the recent history of inflation targeting in Norway, a period covering just a few years, the sample period would be too short to conduct the test above.

#### *4. The central bank's best prediction of the future exchange rate*

In one way it is very convenient for a central bank to impose technical assumptions regarding future exchange rate movements, irrespective of whether the exchange rate is assumed to be constant or follow UIP-implied forward rates. Of all key variables in the process of forecasting inflation the exchange rate is perhaps the most difficult one to predict. It is highly volatile and even the direction of movements may frequently deviate from predictions. By disclosing the best guess of the future, observers will scrutinize and assess the central bank's ability to perform exchange rate analysis. Frequent and large errors are inevitable, which in turn could harm the bank's reputation and credibility.

If the bank applies technical assumptions it can't treat the inflation projection as an unconditional expectation. Hence it cannot expect the inflation forecast to materialize, *unless it frequently changes the technical assumptions and aligns them with its best guess of the future*. As table 1 shows, Norges Bank has changed its technical exchange rate assumption frequently by changing the period in the past over which the exchange rate has been averaged. This is related to the fact that in the inflation forecasting process a central bank needs to assess the whole state of the economy in the future, likely exchange rate outcomes included. Hence, in practice it is difficult to choose an exchange rate assumption that deviates substantially from the bank's expectations.

Changing the exchange rate assumption over time to align it with the central bank's overall projections of the economy is beneficial in terms of improving the forecast errors. But then, taking this argument further, the central bank could treat the exchange rate exactly as it treats other key variables in the inflation forecasting process: take stock of the state of the economy and make the best prediction given all available information. The central bank has one instrument at its disposal, the short term interest rate. With this it influences all variables in the economy, the exchange rate included. Then, one could argue, there is no reason to handle the exchange rate differently from other key macroeconomic variables. Very few, if any, central banks apply technical assumptions on e.g. private consumption. Moreover, as technical assumptions on the interest rate in general may deviate from the bank's best prediction, the inflation projection may not be the bank's best forecast. This introduces an additional problem when projections are evaluated ex-post.

Evaluation of contingent forecasts may often be challenging, as discussed by Lohrman (2003):

...The purpose of Norges Bank's projections is to provide a basis for monetary policy decisions. If, for example, the conditional projection for consumer price inflation two years ahead is above the inflation target, the interest rate will normally be increased with a view to achieving the inflation target. In such a situation, the interest rate is changed in order to achieve a different outcome than projected. When the projections are subsequently evaluated, it is important to take into account that the projections do not necessarily represent Norges Bank's view of the most probable outcome...

As the paragraph emphasizes, while technical assumptions may provide a basis for monetary policy decisions, it is difficult to evaluate inflation forecasts *ex post*. Assumptions reflecting the central bank's best prediction of the exchange rate and the interest rate could improve this; it would provide a basis for monetary policy decisions and in addition inflation forecasts could be evaluated in a meaningful way.

There are, however, several problems with an exchange rate assumption based on the best prediction of the future. First, it would require some analytical tools for the exchange rate. Here the technical assumptions discussed above contain some advantages as both the random walk hypothesis and the UIP are well-known and established in the literature. Analytical tools to predict future exchange rate movements could include different kinds of models, ranging from simple econometric models to VAR-models and calibrated models, all of which could be extended by technical analysis and other information from financial markets.

Second, the central bank must resist the temptation of basing the inflation forecast on "wishful expectations", an unrealistic *ad hoc* assumption introduced just to push inflation forecasts towards the target. On the other hand, by systematically basing the inflation forecast on wishful and not realistic assumptions, the central bank would consistently fail to predict inflation.

Third, as the public in general probably perceives the central bank as the most competent institution to predict future exchange rate movements, observers will scrutinize and question the central bank's ability to predict them. With certainty the actual outcomes will deviate from

the bank's prediction, in some cases substantially. Then, the bank may face possible criticism of not understanding what's going on in foreign exchange markets. On the other hand, predicting future exchange rate outcomes is a notoriously difficult task. Other institutions, notably private investment banks, provide predictions of the exchange rate and must frequently explain forecast errors.

Fourth, assumptions reflecting the bank's best prediction of the exchange rate will probably change frequently. In some situations the bank will find a constant rate appropriate. In other situations the exchange rate might be assumed to follow a UIP-adjusted path based on estimates of the risk premium, or the exchange rate paths could be calculated on the basis of some adjustment to PPP in the long run. In any case, as opposed to the technical assumptions, it is a danger that an assumption reflecting the bank's best prediction appears ad-hoc and not transparent as seen from the market. Hence it will be a challenge for the central bank to find a framework for the exchange rate assumptions that is consistent over time.

##### 5. Can we improve the understanding of exchange rate markets?

Substituting the exchange rate assumption for an exchange rate projection will, in addition to the challenges mentioned above, require an appropriate projection method. Given the interest rate assumption the bank's best guess could very well be a constant exchange rate or UIP. In this case, abandoning the term *technical assumption* appears semantic. However, it would require the bank to consider alternative assumptions.

At least over a long-term horizon, stationarity in real exchange rates may give a guidance to exchange rate directions. Figure 4 shows the real exchange rate for Norway, as deviation from the mean since 1970 (deflated by relative consumer prices). First, the chart indicates that the real exchange rate tends to stabilize around a certain level. Second, formal tests indicate mean reversion in the data, with a half life of around two years.<sup>8</sup> This result is also supported by

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<sup>8</sup> The Dickey-Fuller test is equal to -4.2, lower than the critical value of -3.6 at the one percent test level. Hence the null hypothesis of non-stationarity (no mean-reversion) is rejected. Estimating a simple first order autoregressive model for the real exchange rate,  $q(t) = 0.7 q(t-1)$ , the half life turns out to be  $\log(1/2)/\log(0.7) = 1.9$ , around two years.

two IMF studies.<sup>9</sup> Mean-reverting in the real exchange rate could be explained by purchasing power parity (PPP), which Akram (2005) finds to hold on Norwegian data. Hence there are reasons to believe that the real exchange rate's deviation from a historical average level could be helpful in assessing future exchange rate movements.<sup>10</sup>

Based on the assumption that the real exchange rate has a half life of two years and that most of the adjustments within a two years horizon come through the nominal exchange rate, we have calculated 'PPP-paths' for the nominal exchange rate. The paths imply that the nominal exchange rate will approach the average historical level with a half life of two years.<sup>11</sup> Figure 5 shows, for four selected inflation reports (99/2, 01/2, 02/2 and 03/2), the exchange rate assumption undertaken in the report (random walk, blue line), UIP-implied forward rates (green line) and the PPP-implied path (black line). In addition, actual exchange rate outcomes have been included (red line).

Table 3 and 4 summarize the results. Table 3 shows, for each of the three prediction methods, the absolute percentage deviation from actual exchange rate outcomes averaged over the two years after publication of the inflation reports. Figure 6 depicts the same information as table 3, showing yearly averages of the deviations. In 1999 the constant exchange rate assumption and UIP beat the PPP-path. However, for all inflation reports published in 2000 and up to the second quarter of 2002, the PPP-implied path tracks actual exchange rate movements better than the two other alternatives. Finally, in the last inflation report in 2002 UIP turned out to be the assumption closest to actual outcomes. For later reports, published in 2003 and beyond, actual exchange rate outcomes have not yet materialized. For the whole period 1999-2002 in general, the PPP-implied path turns out to be closer to actual outcomes, with an average error of 4.2 percent (with standard deviation equal to 1.7 percent). In contrast, the paths implied by

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<sup>9</sup> See Cashin, P. and McDermott, C. J. (2003, 2004).

<sup>10</sup> The real exchange rate may alternatively be measured by deflating nominal exchange rate by relative wage costs, reflecting developments in competitiveness. This will be further discussed in an appendix.

<sup>11</sup> For all inflation reports from 1999/2 and beyond the average historical level is calculated on data from the beginning of 1993 and up to the publication date of the inflation report. As an example, when Inflation Report 2000/3 was published in the third quarter 2000, the index of the nominal effective exchange rate took the value 103.5. Furthermore, the average level, calculated from 1993 and up to the third quarter of 2000, was 97.5, 6 percent stronger than the average level. Based on a half life assumption of two years, the path implies an appreciation of 3 percent during the next two years. Note that the calculation of the half life of the nominal exchange rate is consistent with the findings above of mean reversion in the nominal exchange rate since 1990.

the constant exchange rate assumption and UIP deviated from actual outcomes, on average, by 4.8 and 6.2 percent, respectively (with standard deviation equal to respectively 2,4 and 3,7 percent).

Table 4 shows the difference between the level of the exchange rate around two years after publication of the reports relative to the levels implied by the three alternative exchange rate paths. The difference between table 3 and 4 is that the former focuses on average errors over the two years future horizon, while the latter focuses on the single numbers two years after the publication of the reports. Figure 7 shows yearly averages of the deviations. Based on this ‘end-point deviation measure’, the PPP-implied path beats the two other paths in 10 out of 13 cases, with an average error of around 5 percent. This is 1.5 percentage point lower than the constant exchange rate assumption and as much as 5 percentage points lower than the UIP assumption.

As discussed above, due to a possible risk premium UIP-projections may deviate considerably from market’s expectations. As an alternative, exchange rate projections could be based on Consensus Forecasts (CF), i.e., forecasts given by analysts in the market. Every month CF provides exchange rate projections two years ahead. By calculating the difference between actual outcomes and predictions, CF-errors can be compared with the errors implied by the other three exchange rate assumptions. Figure 8 shows the results. The blue, green and the black bars are identical to those in figure 7, while the red ones, reflecting CF-errors, have been added.

First, note that the CF-errors are considerably smaller than the UIP-errors. To the extent that the traders’ true expectations coincide with those of the market analysts, the difference between CF-errors and UIP-errors should reflect a risk premium. However, if market analysts and traders expect different paths for future exchange rate, UIP-projections and CF-predictions would simply reflect exchange rate expectations of two different groups. Based on the discussion above, it is reasonable to assume that at least some of the difference between UIP-projections and CF-predictions reflects the existence of a risk premium.

Second, while the CF-errors deviate substantially from the UIP-errors and also to some degree from the errors implied by the constant exchange rate rate assumption, they are close to the ‘PPP-errors’. This supports the view that (i) the difference between the CF-errors and the

UIP-errors is in fact caused by a risk premium, and (ii) the market bases exchange rate projections on some sort of equilibrium value for the exchange rate (nominal or real) and mean reversion towards that value.

## 6. Conclusion

In small open economies with free capital movements the exchange rate plays a significant role in the inflation process. The exchange rate assumptions underlying the inflation projections vary across countries. Typically, the assumptions are constant exchange rate, uncovered interest parity (UIP) or some projection reflecting the bank's best guess on the interest rate assumption. Even though constant exchange rate or UIP may be in accordance with best guess, exchange rate assumptions are frequently referred to as *technical*. Economic theory and empiric studies unambiguously suggest a strong link between the interest rate and the exchange rate. In this respect, the exchange rate does not principally differ from other variables that are endogenous in the projections.

The role of the exchange rate in the inflation process is supported by Norwegian data. During the past four years, part of the inflation forecast errors can be attributed to deviations between the actual exchange rate and the assumption. *Ex post* studies suggest that, in this particular period, a PPP-based approach could have predicted better, based on a gradual move in the exchange rate to restore relative PPP.

Whereas the principle of treating the exchange rate as an endogenous variable seems uncontroversial, the practical implementation of it is far from settled. Even though different equilibrium approaches and estimated models should be explored, forecasting the exchange rate is by nature difficult. Even if the exchange rate is not presented as a best guess, central banks should therefore scrutinize different reasonable exchange rate scenarios and communicate the uncertainty in its projections

## Appendix: The property of real exchange rates across countries

A precondition for basing the exchange rate assumption on deviation from PPP is mean reversion in the real exchange rate. The higher the degree of mean reversion, the more it is plausible to base the exchange rate assumption on deviations from PPP.

Mean reversion in an economic time series is directly related to the concept of stationarity. For a stationary variable an equilibrium value exists and over time the variable will tend to approach the equilibrium (though the speed of the process may be slow or fast). This means that shocks are temporary. A widespread method used to estimate the degree of mean reversion is to run the simple Dickey-Fuller regression,

$$\Delta q_t = \alpha + \beta q_{t-1} + \varepsilon_t,$$

where  $q$  is the variable in question and  $\varepsilon$  is an unsystematic (white noise) error term. If  $\beta$  is less than one, the variable is mean-reverting with a half life given by  $\ln(1/2)/\ln(\beta)$ . The Dickey-Fuller test examines the null hypothesis that the variable in question is non-stationary ( $\beta=0$ ). If the null is rejected, one concludes that mean reversion exists.

A huge literature on stationarity tests exists, and there are several problems attached to the simple test above.<sup>12</sup> In one recent study from IMF (Cashin, P. and J. McDermott, 2003) several methods to estimate the half life of real exchange rates are discussed. Figure A1 shows the half life of real exchanges rates for a selected group of developing countries, for each country calculated as the average of three different methods discussed in Cashin, P. and J. McDermott (2003).<sup>13</sup>

The half life of real exchange rates varies considerably across countries. For some countries, Iceland, Norway, New Zealand and Switzerland the half life is estimated to around 1-2 years. For several countries in the euro area half life varies between 4 and 6 years, for UK it is 5-6 years, while for Australia, Japan and Finland half life is 7-8 years. For some countries, notably Portugal, Sweden, Canada, United States and Belgium the real exchange rate is considered to be non-stationary.<sup>14</sup>

Figures A2-A6 show the real exchange rate and PPP-based exchange rate prediction paths based on the half life in figure 6 for the countries with a half life of 5-6 years and less.<sup>15</sup> To

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<sup>12</sup> See Banerjee, A. et. al (1993) for details.

<sup>13</sup> The authors of the IMF-study go through several methods to estimate the half life of real exchange rates. For our purpose, we have chosen to take the average of three of them, 1) simple and biased Dickey Fuller regressions, 2) Augmented median unbiased Dickey-Fuller regressions and 3) Phillips-Perron median unbiased regressions. The difference between the tests is related to the degree of biasness in the estimate of the regression coefficient in the regression above and to what extent autocorrelation and heteroscedasticity in the error term are accounted for.

<sup>14</sup> One should note that the estimated half life depends strongly on the method used to calculate it. As noted above, we have calculated the half life as the average of three different methods suggested by Cashin, P. and J. McDermott (2003, 2004). By this measure half life of real exchange rates in Portugal, Sweden, Canada, United States and Belgium is estimated to 19, 25, 29, 29 and 35 years respectively, interpreted to be infinite.

<sup>15</sup> For each point in the figure where a new PPP-adjusted path begins, the equilibrium value of the real exchange rate is calculated by the historical average from 1979 (start of IMF data available on the data base EcoWin) and up to the starting point of the PPP-based path. This means that more and more data is used to calculate the



measure the prediction errors in the figures, table A1 shows the percentage deviation between the real exchange rate and the two years “end-point prediction” implied by the PPP-based path. As an example, for Iceland the actual real exchange rate in 1997 deviated from the prediction for 1997, made in 1995, by 10.2 percent. Figure A8 depicts the two last rows of table A1, showing, for each country, the average deviation and the standard deviation.

Overall, the half life for Norway seems to be relatively low compared to most other countries. This result is broadly independent of the method used to calculate the half life.

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equilibrium value as time evolves. For the euro area, half life is calculated as the average numbers for Germany, France and Italy, resulting in a half life of around 5 years.

## **Appendix: The real exchange rate based on relative wage costs**

In the main text the real exchange rate was based on relative consumer prices, reflecting developments in relative purchasing power or cost of living. Alternatively, the real exchange rate could be measured in terms of relative wage costs, reflecting developments in competitiveness.

Figure A9 shows the real exchange rate deflated by relative wage costs, as deviation from the mean since 1970 (similar to figure 4). Based on simple eye-ball econometrics the chart indicates that this measure of the real exchange rate also tends to vary around the average level. However, formal tests like the augmented Dickey-Fuller test do not reject the null hypothesis of non-stationarity. On the other hand, it is well documented in the literature that the power of these standard tests is rather low and that large samples may be necessary to detect mean reversion in a statistical sense. This means that the tests could fail to reject the null even if the series is stationary. Proceeding with the working hypothesis that the real exchange rate is stationary, the simple AR(1) regression turns out to be

$$Q_t = \text{constant} + 0.8 Q_{t-1}$$

where  $Q$  is the real exchange rate based on relative wage costs. The standard deviation of the regression coefficient is equal to 0.12. The equation implies a half life of  $\log(-1/2)/\log(-0.8) = 3.11$  years, a reasonable estimate and somewhat higher than for the real exchange rate deflated by relative consumer prices.

As of mid February 2005 the real exchange rate deflated by relative wage costs is estimated to be around 8 per cent stronger than the average level. A half life of around three years implies that the real value of the Krone should fall by around 4 per cent over a three years period.

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Table 1. Exchange rate and interest rate assumptions at Norges Bank

| Inflation Report | Exchange rate assumption               |               | Interest rate assumption |               |
|------------------|--|---------------|--------------------------|---------------|
|                  | Constant                               | Forward rates | Constant                 | Forward rates |
| <b>1999</b>      |  |               |                          |               |
| <b>1</b>         | X <sup>1</sup>                         |               |                          | X             |
| <b>2</b>         | X (mean last 3 months)                 |               |                          | X             |
| <b>3</b>         | X (last month)                         |               |                          | X             |
| <b>4</b>         | X (mean so far 4th quarter)            |               |                          | X             |
| <b>2000</b>      |  |               |                          |               |
| <b>1</b>         | X (mean so far 2000)                   |               |                          | X             |
| <b>2</b>         | X (mean first 4 months 2000)           |               |                          | X             |
| <b>3</b>         | X (mean so far 2000)                   |               |                          | X             |
| <b>4</b>         | X (mean last 3 months)                 |               |                          | X             |
| <b>2001</b>      |  |               |                          |               |
| <b>1</b>         | X (mean so far 2001)                   |               | X                        |               |
| <b>2</b>         | X (mean last 3 months)                 |               | X                        |               |
| <b>3</b>         | X (mean last 3 months)                 |               | X                        |               |
| <b>2002</b>      |  |               |                          |               |
| <b>1</b>         | X (mean last month)                    |               | X                        |               |
| <b>2</b>         | X (mean last 3 months)                 |               | X                        |               |
| <b>3</b>         | X (mean last 3 months)                 |               | X                        |               |
| <b>2003</b>      |  |               |                          |               |
| <b>1</b>         | X (mean last month)                    |               | X                        |               |
| <b>2</b>         | X <sup>2</sup> (mean 3 - 19 June 2003) | X             | X                        | X             |
| <b>3</b>         |  | X             |                          | X             |
| <b>2004</b>      |  |               |                          |               |
| <b>1</b>         |  | X             |                          | X             |
| <b>2</b>         |  | X             |                          | X             |
| <b>3</b>         |  | X             |                          | X             |
| <b>2005</b>      |  |               |                          |               |
| <b>1</b>         |  |               |                          |               |

<sup>1</sup>In the Inflation Report 1999/1 the krone exchange rate was assumed to “return to its initial range against European currencies in the course of the next six months. Subsequently, the exchange rate is assumed to remain unchanged through the remainder of the projection period”

<sup>2</sup>In Inflation Report 2003/2, two alternative projections were presented, one based on a constant future exchange rate and one based on UIP-implied forward rates.

Table 2. Exchange rate assumptions in selected central banks

| <b>Technical assumptions</b>  | <b>Best Forecast</b>   | <b>Other</b>  |
|---|--|---|
| <b>New Zealand</b><br>Gradual return to an estimated real exchange rate | <b>Sweden</b><br>Uses a variety of models to predict the most like outcome (conditioned on the interest rate assumption) | <b>Canada</b><br>Not publicly available   |
| <b>Australia</b><br>Constant exchange rate                              | <b>Czech Republic</b><br>Forecast of the koruna/euro rate based on the real exchange rate's deviation from equilibrium   | <b>USA</b><br>FED publishes the central tendency of the FOMC participants' individual forecasts for key macroeconomic variables. The exchange rate assumption is not discussed. |
| <b>ECB</b><br>Constant exchange rate                                    |  |   |
| <b>UK</b><br>Average of constant exchange rate and UIP                  |  |   |
| <b>Norway</b><br>UIP  |  |   |

Table 3. Deviation from exchange rate outcomes, averaged over 2 years

|                | <b>Random Walk</b> | <b>UIP</b>  | <b>PPP</b>  |
|----------------|--------------------|-------------|-------------|
| <b>IR 2/99</b> | 2.30               | 1.18        | 3.57        |
| <b>IR3/99</b>  | 2.02               | 1.70        | 2.95        |
| <b>IR4/99</b>  | 1.94               | 2.44        | 2.28        |
| <b>IR1/00</b>  | 2.24               | 3.44        | 1.83        |
| <b>IR2/00</b>  | 3.14               | 7.18        | 2.60        |
| <b>IR3/00</b>  | 5.43               | 8.60        | 4.01        |
| <b>IR4/00</b>  | 7.73               | 10.03       | 5.84        |
| <b>IR1/01</b>  | 8.16               | 10.79       | 6.58        |
| <b>IR2/01</b>  | 7.84               | 11.05       | 6.75        |
| <b>IR3/01</b>  | 6.91               | 9.56        | 6.11        |
| <b>IR1/02</b>  | 5.36               | 7.95        | 5.29        |
| <b>IR2/02</b>  | 3.28               | 3.54        | 2.66        |
| <b>IR3/02</b>  | 5.42               | 3.46        | 4.43        |
| <b>Average</b> | <b>4.75</b>        | <b>6.23</b> | <b>4.22</b> |
| <b>St.dev.</b> | <b>2.39</b>        | <b>3.68</b> | <b>1.73</b> |

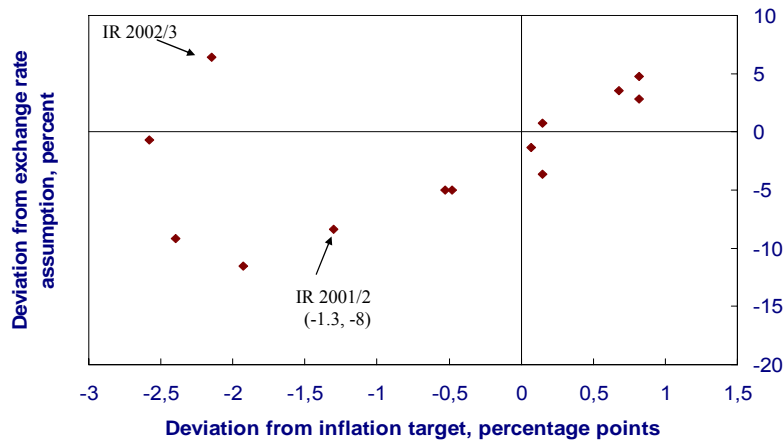
Table 4. Deviation from exchange rate outcomes, 2 years after publication of IR

|                | <b>Random Walk</b> | <b>UIP</b>   | <b>PPP</b>  |
|----------------|--------------------|--------------|-------------|
| <b>IR 2/99</b> | 0.91               | 3.45         | 2.80        |
| <b>IR3/99</b>  | 0.70               | 4.84         | 1.16        |
| <b>IR4/99</b>  | 2.19               | 5.95         | 0.20        |
| <b>IR1/00</b>  | 4.03               | 7.69         | 1.85        |
| <b>IR2/00</b>  | 9.56               | 15.98        | 8.08        |
| <b>IR3/00</b>  | 14.14              | 19.26        | 11.26       |
| <b>IR4/00</b>  | 15.95              | 19.72        | 12.70       |
| <b>IR1/01</b>  | 13.80              | 18.46        | 11.25       |
| <b>IR2/01</b>  | 9.08               | 14.85        | 7.36        |
| <b>IR3/01</b>  | 4.38               | 9.29         | 3.47        |
| <b>IR1/02</b>  | 0.61               | 3.97         | 0.51        |
| <b>IR2/02</b>  | 2.49               | 3.41         | 0.02        |
| <b>IR3/02</b>  | 7.10               | 3.32         | 4.61        |
| <b>Average</b> | <b>6.53</b>        | <b>10.01</b> | <b>5.02</b> |
| <b>St.dev.</b> | <b>5.49</b>        | <b>6.63</b>  | <b>4.59</b> |

Table A1. The percentage deviation between the real exchange rate and the two years “end-point prediction” implied by the PPP-based path.

|                 | Switzerland | Norway      | Iceland     | Euro area   | UK          | New Zealand  |
|-----------------|-------------|-------------|-------------|-------------|-------------|--------------|
| 1997            | 1.46        | 3.22        | 10.20       | 0.13        | 7.86        | 16.57        |
| 1998            | 1.64        | 0.66        | 5.81        | 8.13        | 25.49       | 4.69         |
| 1999            | 4.07        | 7.05        | 4.93        | 1.07        | 8.13        | 12.51        |
| 2000            | 3.15        | 3.67        | 1.79        | 9.03        | 7.39        | 14.51        |
| 2001            | 1.04        | 1.94        | 5.19        | 12.16       | 6.68        | 10.60        |
| 2002            | 6.21        | 0.70        | 11.98       | 2.93        | 2.06        | 7.17         |
| 2003            | 8.10        | 16.34       | 0.88        | 9.39        | 3.92        | 10.02        |
| 2004            | 1.65        | 2.00        | 3.59        | 18.33       | 2.44        | 19.61        |
| <i>Average</i>  | <i>3.42</i> | <i>4.45</i> | <i>5.55</i> | <i>7.65</i> | <i>8.00</i> | <i>11.96</i> |
| <i>Std.dev.</i> | <i>2.56</i> | <i>5.22</i> | <i>3.84</i> | <i>6.11</i> | <i>7.47</i> | <i>4.90</i>  |

**Chart 1. Deviation from inflation target against deviation from exchange rate assumption**  
 $(\pi_{t+8} - \pi^{IR})$  against  $(V_{t+4} - V^{IR})$

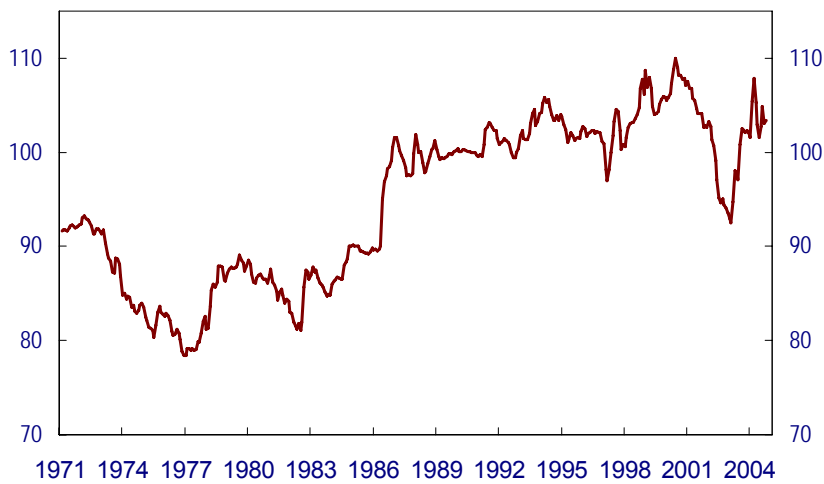


Source: Norges Bank

Correlation = 0.45

1

**Chart 2. Nominal effective exchange rate**  
 Increase indicates depreciation

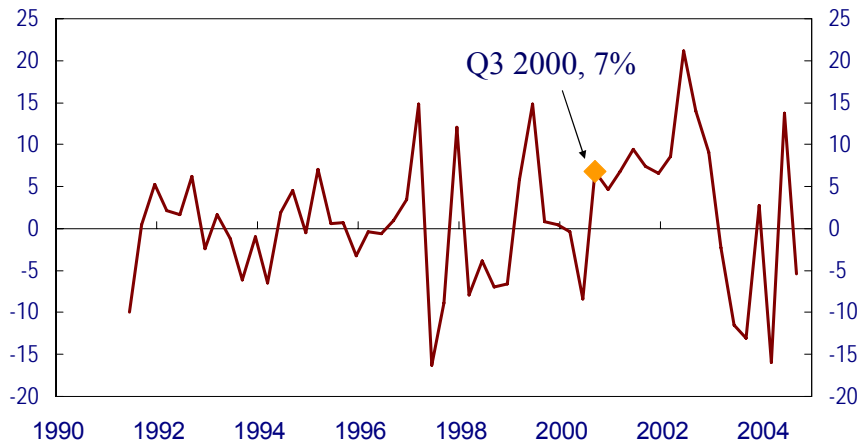


Source: Norges Bank

2



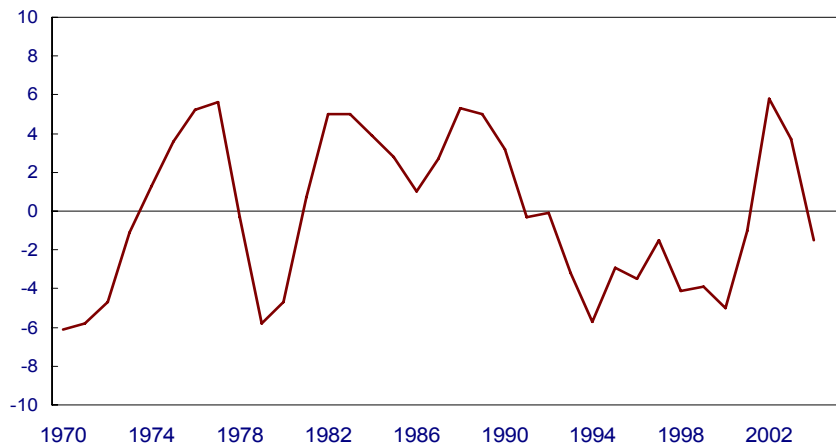
**Chart 3. Excess return on Norwegian kroner relative to a composite foreign currency**  
 Quarterly, annualised. Percent



Source: Norges Bank

3

**Chart 4. Real exchange rate (deflated by relative consumer prices). Deviation from the average from 1970<sup>1)</sup>**

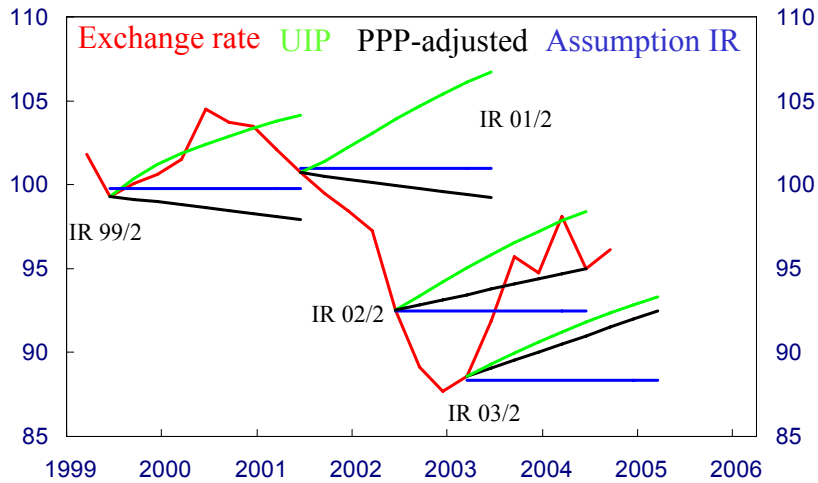


<sup>1)</sup> For 2004 the prediction is based on the average nominal exchange rate so far in 2004 and wage and inflation projections in Inflation Report 2004/2.

Source: Norges Bank

4

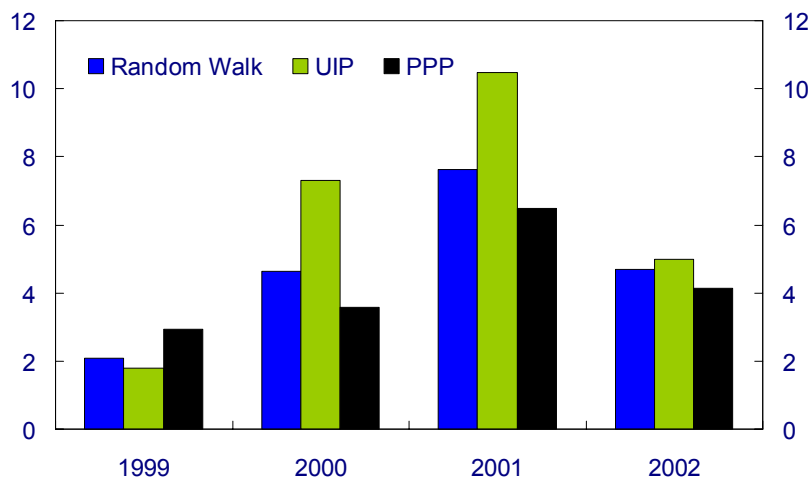
**Chart 5.** Exchange rate, exchange rate assumptions in inflation reports, UIP-paths and PPP-adjusted paths



Source: Norges Bank

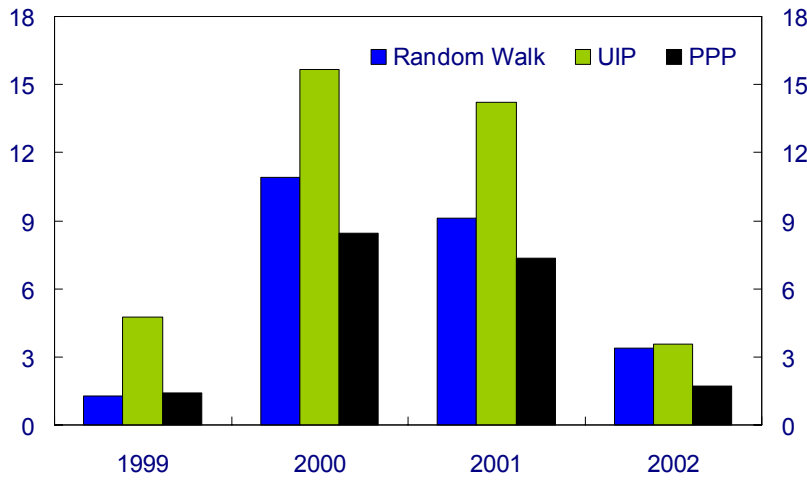
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**Chart 6.** Deviation from exchange rate outcomes, averaged over 2 years after publication of the inflation reports



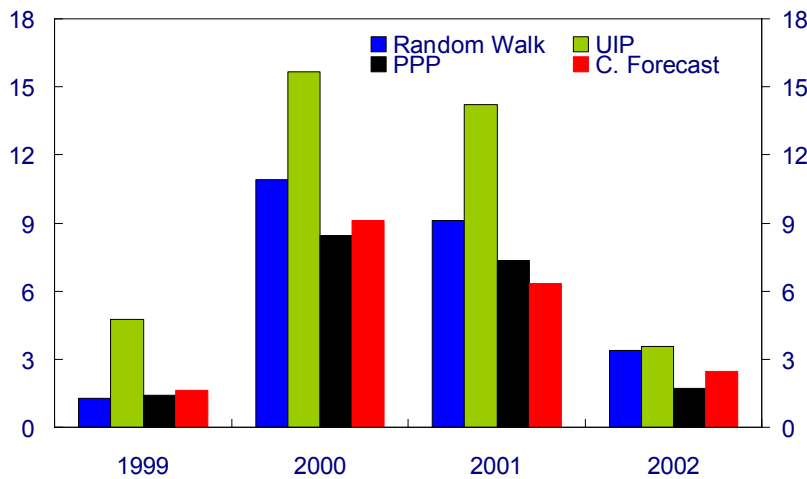
6

Chart 7. Deviation from exchange rate outcomes, 2 years after publication of the inflation reports



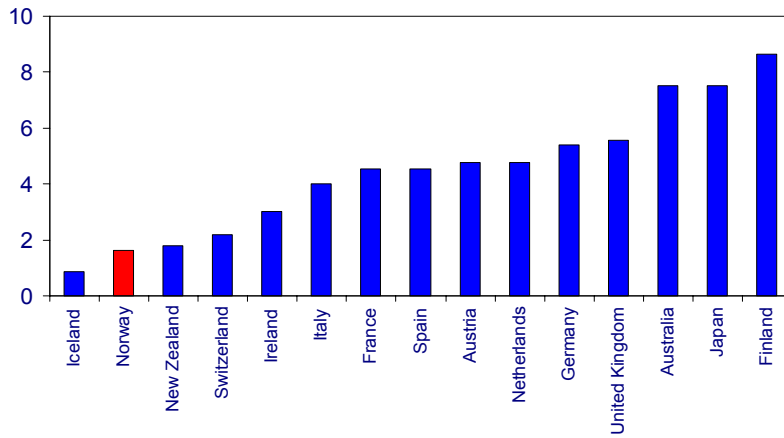
7

Chart 8. Deviation from exchange rate outcomes, 2 years after publication of the inflation reports. Consensus Forecasts included



8

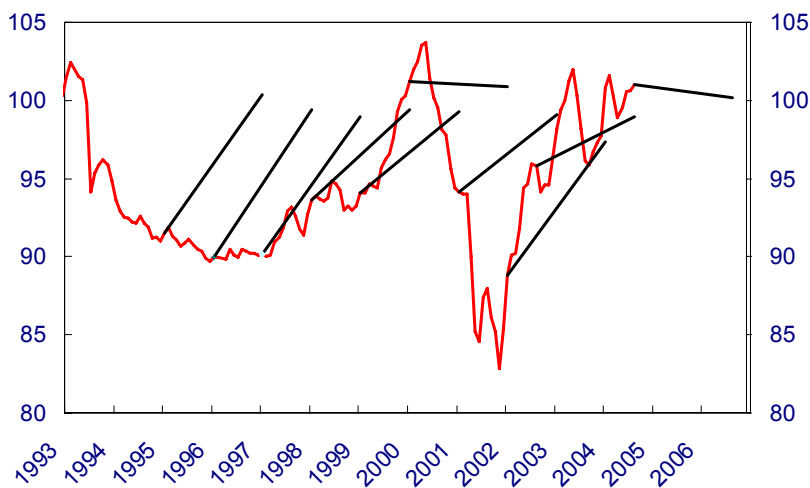
### A1. Half life of real exchange rates. Years



Source: IMF

8

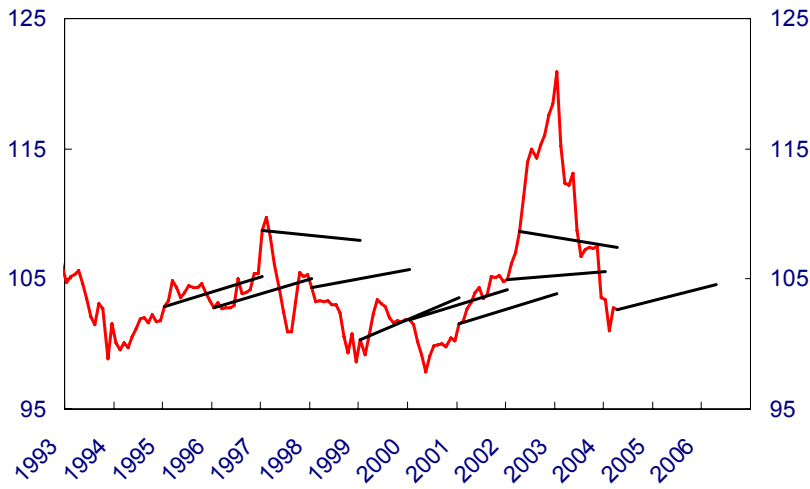
### A2. Real exchange rates and PPP-adjusted paths for Iceland (half life ≈ one year)



Source: IMF

9

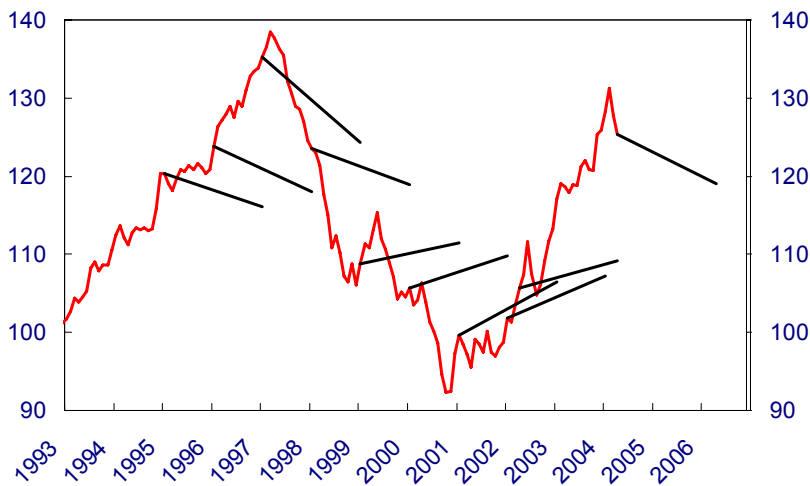
**A3. Real exchange rates and PPP-adjusted paths for Norway (half life ≈ 2 years)**



Source: IMF

10

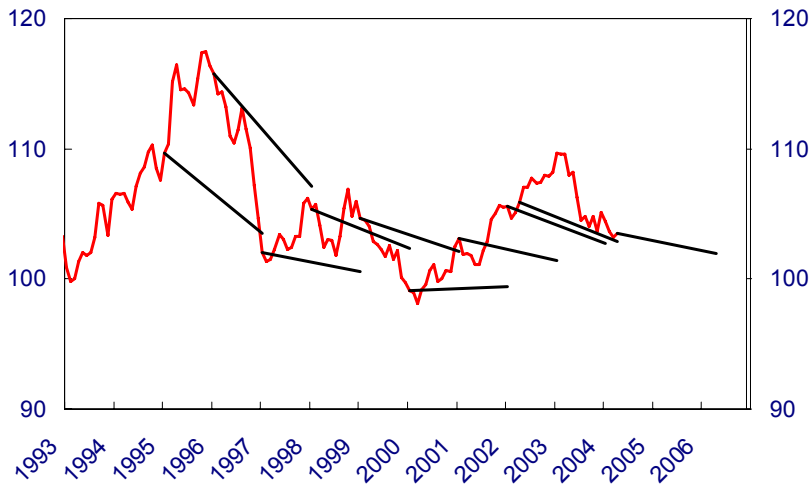
**A4. Real exchange rates and PPP-adjusted paths for New Zealand (half life ≈ 2 years)**



Source: IMF

11

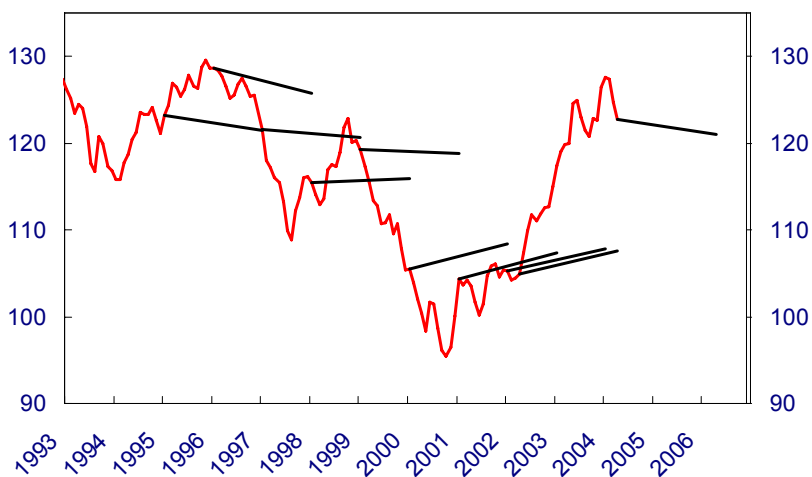
**A5. Real exchange rates and PPP-adjusted paths for Switzerland (half life ≈ 2 years)**



Source: IMF

12

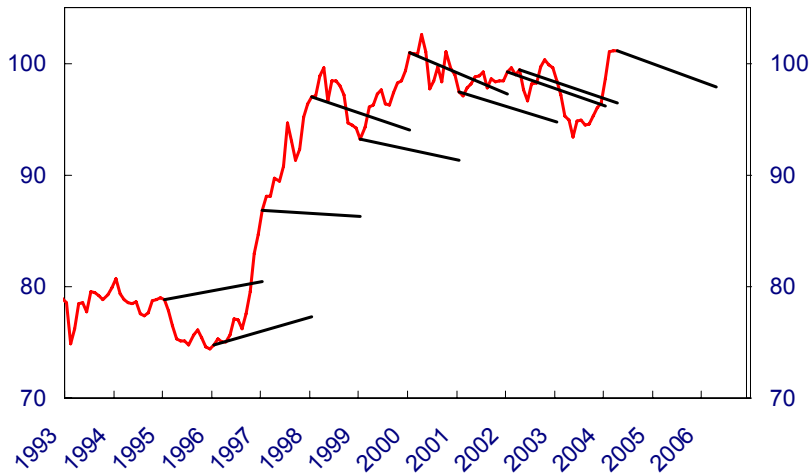
**A6. Real exchange rates and PPP-adjusted paths for the euro area (average of Germany, France and Italy, half life ≈ 5 years)**



Source: IMF

13

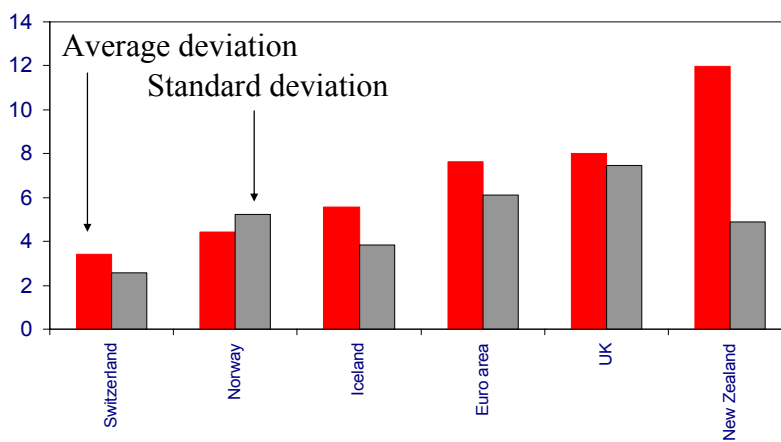
### A7. Real exchange rates and PPP-adjusted paths for UK (half time ≈ 5-6 years)



Source: IMF

14

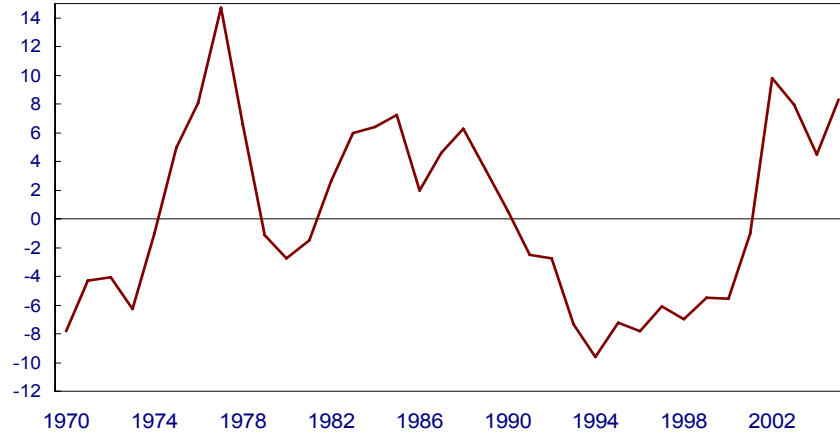
### A8. Average deviation and standard deviation between the real exchange rate and the prediction implied by the PPP-based path exactly two years after the prediction took place



Source: IMF

15

**Chart A9.** Real exchange rate (deflated by relative wage costs). Deviation from the average from 1970



Source: Norges Bank



