

Norges Bank's management of cash inventories

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In order to fulfil its obligations regarding the supply of notes and coins, Norges Bank needs to hold cash inventories. The level of inventories must be considered with regard to ordinary and extraordinary circumstances. In order to predict demand and reduce the uncertainty concerning inventory requirements, sound models that can enhance the understanding of changes in cash circulation are required. The explanatory variables for cash demand can be divided into three different groups: general macro-economic variables, variables that express the competition between cash and deposits, and variables that may provide insight into the illegal economy. A newly developed model for cash demand shows that demand for actual cash is dependent on real consumption at the point of sale, bank interest rates and a negative linear trend that captures developments in the payment system, in addition to the historical value of real cash. The model-based forecasts show that demand for cash will increase in the next quarters before decreasing towards the end of 2006 and further through 2007.

1 Introduction

According to the Norges Bank Act, Norges Bank is required to issue notes and coins (statutory responsibility for issuing notes and coins). Under the Norges Bank Act, Norges Bank is also responsible for ensuring that cash is available (statutory responsibility for supplying cash). Responsibility for issuing notes and coins and the banknote monopoly entails issuing notes and coins in the amounts implied by demand and ensuring that notes and coins are available to society (see Eklund, Solberg and Veggum, 2005). Norges Bank's goal is to fulfil these obligations in an economical, efficient and secure manner.

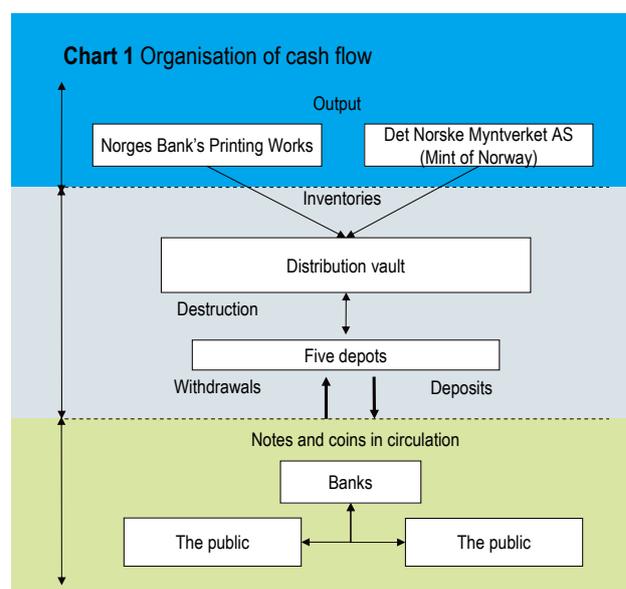
In 2001, Norges Bank outsourced most of the services associated with cash storage and the handling of deposits and withdrawals from the central bank depots. Norges Bank is still responsible for procurement of banknotes and coins, as well as storage and transport associated with the central bank depots. For this reason, analysing future cash needs is an important task.

Section 2 of the article gives a brief introduction to the organisation of cash holdings in Norway. Section 3 explains the necessity of holding inventories of banknotes and coins. Section 4 discusses factors that affect the circulation of cash and section 5 explains the structure of a model for cash demand. The use of the model is then demonstrated in section 6. Section 7 discusses the model's importance in logistical planning.

2 Organisation of cash supply

The physical flow of cash may be divided into three steps: production, storage and circulation (see Chart 1). Production of coins takes place at the Mint of Norway, while Norges Bank is responsible for the production of banknotes. During the course of 2007, the printing works in Norges Bank will be closed, and thereafter

production will take place externally. Storage is organised with a central cash distribution vault and five depots located around the country. The cash distribution vault is operated by Norges Bank, while Norsk Kontantservice AS (NOKAS) operates the depots on behalf of Norges Bank. The quantity of notes and coins in use in the community at any given time is often called the circulation of notes and coins, or cash circulation.² The cash circulation level changes when banks need cash and make withdrawals from Norges Bank, or when they have a surplus of cash and make deposits in Norges Bank. In this context banks operate to all intents and purposes as an intermediary between the public and Norges Bank. In practice, this means that it is the public's demand for notes and coins that determines the level of cash circulation.



¹ Thanks to Gunnvald Grønvik and Karsten Gerdrup for useful comments and suggestions. The analysis is based on Aastveit (2005). Especial thanks to Terje Skjerpen (Statistics Norway) for sound guidance. The analysis was performed using PcGive 10.1 (Hendry and Doornik 2001).

² The amount of notes and coins in circulation is defined as the sum of Norwegian notes and coins that is held by banks and money holding sectors (the public and financial enterprises other than banks and state lending institutions).

3 The need to hold cash inventories

One of Norges Bank's goals is to fulfil its statutory responsibility for supplying cash in as secure and cost-efficient a manner as possible. This means that Norges Bank must be able to meet banks' demand for cash, both under normal circumstances and under more extraordinary circumstances. Total inventories required must be assessed in the light of both of these circumstances.

Uncertainty regarding future demand for cash affects estimates of holding levels. It is therefore important to have a sound understanding of which factors affect demand for cash and how these may affect demand in the future.

3.1 Supply capability under normal circumstances

There are various reasons for the need to hold cash inventories. Demand is seasonal, and the purpose of the stock is to help meet these demand fluctuations. The inventories are also held to deal with various types of uncertainty, such as other variations in demand and transport delays or supply delays.

When determining the appropriate level of operating inventories under normal circumstances, two needs are assessed: transaction stock and buffer stock.

- *Transaction stocks* serve to cover normal requirements during the period between one delivery and the next from the producer. On the basis of estimated demand for cash and expected destruction, the transaction stock is determined by minimising the sum of order, transport and storage costs. A large proportion of the production costs for notes and coins are fixed, which means that unit costs are reduced when the volume increases. On the other hand, larger volumes increase the capital tied up in the storage of cash. Transport is often costly due to security requirements and long distances. It will therefore be cost-efficient to exploit certain capacities during transport. By minimising total costs, one can determine the optimal order volume and the resulting size of the transaction stock.
- *Buffer stocks* serve as a buffer against uncertainty, primarily uncertainty regarding demand for cash. Not all uncertainty can be eliminated. This is why it is necessary to decide how much uncertainty the Bank should attempt to cover. We refer to this as choice of supply capability, i.e. the probability of being able to meet demand for a denomination when the transaction stock approaches the level where it needs to be replenished and until a new order arrives. The size of the buffer stock required grows exponentially in step with the supply capability required. The higher the supply capability, the higher the buffer stock. This in turn will affect stock holding costs. The optimal supply capability may be difficult to determine, but is a

balance between increased costs and the consequences of stock depletion. The negative consequences of cash stock depletion are regarded as significant by Norges Bank. Consequently, the Bank aims to have a relatively high supply capability.

Inventory policy during normal circumstances can thus be expressed as the aim of minimising overall order, transport and storage costs in addition to setting targets for supply capability.

3.2 Emergency preparedness

The need to maintain emergency stocks in case of extraordinary circumstances is related to the public's cash requirements in such circumstances. Extraordinary circumstances are said to occur in the case of various forms of failure or disturbances in key public infrastructure, e.g. electronic payment systems. The size of the stocks that Norges Bank needs to hold in order to handle such situations is determined by what the Bank chooses to be prepared for and the degree of preparedness that is chosen.

3.3 Total cash inventories

Norges Bank's operating and emergency inventories may essentially be treated as two separate stocks. Norges Bank has nevertheless chosen to consider them as one due to the small probability of extraordinary circumstances occurring simultaneously with the depletion of the entire stock for ordinary circumstances. This reduces the overall stock requirements. The overall stocks are thus set as cash needs in ordinary circumstances plus a minimum stock that is intended to cover certain extraordinary circumstances.

By obtaining a sound understanding of the factors that influence the circulation of cash, the central bank will be able to reduce the uncertainty and thereby the levels of the overall stock.

3.4 Estimates

There are several methods for estimating demand for cash. Norges Bank has chosen to view future cash demand from both a micro and macro-perspective. From a micro-perspective, the demand for each denomination at each depot is considered. Short-term demand (one to twelve months) is estimated with the aid of historical seasonal variations and trends.

Estimates based on the macro-perspective cover the overall cash demand in the longer term (1–3 years). These estimates are used in the planning of order volumes from producers. In addition, the forecasting process provides an understanding of the mechanisms that affect cash demand. A model based on these assumptions is presented in sections 5 and 6.

4 What affects cash circulation?

When assessing demand for cash, it is appropriate to make use of economic theories concerning demand for money. These theories employ different definitions of the money supply, and various forms of deposits are included.³ Cash in circulation is a small portion of what is usually referred to as money. Cash competes with various forms of deposits⁴ in electronic transactions. The greater the liquidity of such deposits, the lower the demand for cash.⁵ Because cash may be used for anonymous payments on the spot, it is better suited than deposits in the illegal economy. The growth and size of the illegal economy may therefore affect demand for cash.

The explanatory variables for cash demand may therefore be divided into three different groups: general macroeconomic variables in money demand theory, variables that express the competition between cash and deposits and variables that may provide insight into the illegal economy. Each of these groups is discussed below.

4.1 Macroeconomic variables

Money and its function

Macroeconomic explanatory variables include variables from theoretical money demand models. The empirical literature has primarily focused on the demand for broad monetary aggregates.⁶ Broad monetary aggregates have proven to be relatively stable functions over time. In addition, there has proven to be a certain relationship between price trend and growth in broad monetary aggregates. Money demand measured by narrower concepts has a tendency to be more unstable over time and the relationship with price trend is weaker.⁷

Money is often assigned three functions: a medium of exchange in financial transactions, a unit of measurement for value and a store of value (e.g. McCallum (1989)). As we take a closer look at factors that determine demand for cash, we shall concentrate on money's function as a medium of exchange in financial transactions and as a store of value. The latter point is discussed in connection with the illegal economy.⁸

The transaction motive

Cash, as opposed to other financial assets, provides no interest or return. The public does however hold cash, partly because it simplifies transactions. It may be presented as a problem of optimisation to balance the expected gain in the transaction from holding an extra unit of cash against the cost in the form of lost interest. A model for calculating demand for cash should therefore include

both a variable that expresses the transaction gain and one that expresses the loss of interest due to holding cash.

The more payment transactions one wants to carry out, the more cash one wants to hold. There is a close correlation between the number of transactions and disposable income. However, in a modern, highly developed economy such as the Norwegian economy, there will be a number of types of transactions where cash is no longer a feasible means of payment. Cash will typically only be used as a means of payment in transactions that are carried out at the actual point of sale. Using disposable income as an indicator of the amount of the transaction will therefore express a broader range of transactions than is desirable. The transaction motive for holding cash is probably best expressed by using a narrow definition of consumption. We have therefore chosen to express the transaction motive with a variable that includes consumption at the point of sale (cf. Aastveit 2005).

When the public hold cash, they pay an alternative cost in the form of lost interest income. By placing money in interest-bearing financial instruments, one may earn interest income on them. This means that the higher the interest rate is, the higher the cost of holding cash will be. Since cash is mainly used for transactions with settlement at the point of sale, only money in transaction accounts may be viewed as a realistic alternative to the use of cash. Therefore, the alternative cost of holding cash is probably best expressed by a weighted average of the rate of interest offered by banks for deposits in transaction accounts.

4.2 Cash or card?

Developments in the payment system

In the past 10 to 20 years we have seen significant developments in the payment system. In particular, there has been a rapid increase in the use of electronic payment instruments, which has reduced the use of cash.

In empirical economic literature there have been attempts to use different explanatory variables to express this development. However, this has proved difficult due to short and somewhat inadequate data series. Another issue is that developments have taken place so quickly that it may be difficult to identify the effect of each of the new instruments. An example of this is that cheques were a common medium of exchange for much of the 1970s and 1980s, while their use has declined significantly since about 1990. This has led some writers, e.g. Fischer et al. (2004), to argue that technological developments in the payment system are best captured

³ In the money supply statistics published by Norges Bank, the public's liquidity (M2) is defined as the sum of cash, bank demand deposits, deposits and unused bank overdrafts and building loans. Cash comprises only slightly more than 4 per cent of this definition of money.

⁴ Deposits are defined as bank deposits in transaction accounts. Bank deposits in transaction accounts include deposits (in kroner or foreign currency) that may be immediately converted to notes and coins or used as a method of payment without incurring costs other than ordinary transaction and arrangement fees.

⁵ Liquid assets are defined as assets that can be either used directly or may easily be converted in order to make immediate transactions.

⁶ In theoretical models money is defined as a non interest-bearing means of payment. It is often appropriate to interpret the money supply in these models as the monetary aggregate M1. The reason for this is that in many countries the deposit rate on transaction accounts is very low (also historically) and thus nearly interest free. Alternatively, "interest rate" in most theoretical models can also be interpreted as the yield on bonds minus the interest rate on transaction accounts.

⁷ In Norway, narrow monetary aggregates have been more unstable in the short and medium term.

⁸ Money's function as a store of value will also be emphasised by including interest that represents the alternative cost of holding cash. From the perspective of increasing one's return, cash is poorly suited to be a store of value. However, cash may be well suited as a store of value for concealing income/wealth from the authorities. Unfortunately, there is little information about this.

by including a (negative) linear trend. By introducing such a trend, one can capture the effect of the payment system evolving in a direction where more technology-based transactions, and hence less cash, are used. It can therefore be said that a negative trend represents the effect of a gradual substitution away from cash.

It is still relevant to discuss some specific variables that may capture the effect of technological developments on cash demand.

The availability of cash and liquidity of cash deposits

An increase in the number of ATMs will initially lead to lower costs (in the form of time used) for making withdrawals and easier access to cash. According to Baumol (1952) and Tobin (1956), this should reduce the transaction-motivated demand for cash (i.e. cash holdings for transaction purposes). Theoretically, however, it is also conceivable that an increase in the number of ATMs may increase demand for cash because the availability of cash increases. Cash will thereby be easier to use and be a better alternative than other means of payment (see Drehmann and Goodhart (2000)). Theoretically, an increase in the number of ATMs will therefore have an indeterminate effect on cash demand.

The trend in the number of point-of-sale terminals is another variable that may express the effect of developments in the payment system. The more point-of-sale terminals, the easier it is to use payment cards for transactions at points of sale, which viewed in isolation has a negative effect on demand for cash. However, in 1992 the option to withdraw cash (“cashback”) when making purchases was introduced. Theoretically, cashback could have four effects on cash demand. Two of the effects are the same as the effects of an increase in the number of ATMs, i.e. an indeterminate effect. In addition, the introduction of cashback could lead to faster recirculation of cash among the public; in other words, the velocity of cash circulation increases. In isolation, this will have a negative effect on demand for cash from Norges Bank. Second, cashback is free for the account holder. It is therefore cheaper to use cash than other payment instruments. In isolation, this will have a positive effect on demand for cash. Thus, in isolation the introduction of cashback will have an indeterminate effect on demand for cash. The aggregate effect of an increased number of point-of-sale terminals on cash demand would therefore be purely negative until the introduction of cashback in 1992, while in the period following 1992 the effect would be uncertain.

The cost of transactions

When making a payment at a retail outlet (e.g. a grocery), there are in practice two means of payment: cash or payment card. If we choose the latter, a small fee will normally have to be paid to use the payment card.⁹ The size of this fee depends on the terms of the individual bank. According to ordinary market theory, it is reasonable

to assume that a fee for the use of alternative payment instruments promotes the use of cash. As an example of this, a high price on the use of cheques is probably the reason why cheques are currently very rarely used for point-of-sale transactions at present. We have therefore constructed a variable that indicates the fee for using various payment instruments (cf. Aastveit (2005)).

4.3 The illegal economy

Cash is unique in the sense that it may be used for anonymous point-of-sale transactions. Whereas the use of deposits in transaction accounts is registered, the use of cash cannot be traced. Neither the payer nor the receiver can be identified by information in the settlement. The properties of cash therefore make it difficult to gain an overview of how often and in what type of transactions it is used. This makes cash a suitable means of payment in the illegal economy. It has gradually become recognised that the illegal economy has a considerable effect on cash demand (cf. e.g. Dotsey (1988)).

There are mainly two different types of motives behind the use of cash in the illegal economy. It may therefore be appropriate to distinguish between them and their effect on demand for cash. First, it is well known that cash is the primary means of payment in criminal circles. We have very little information regarding the amount and prevalence of crime. It is also difficult to find suitable variables that detect the effect of this type of illegal economy on cash demand.

A second motive for using cash is the need to conceal income and thereby evade paying taxes and duties to the authorities. Here too, there is very little information on how widespread this type of illegal economy is. In an attempt to capture the effect of tax evasion on demand for cash, we have looked at various tax variables. Tax variables that have been tested are: the average tax rate for the household sector, the average tax rate for wage-earners, and tax (and pension contribution) as a percentage of gross domestic product (GDP). Tanzi (1982) and later Rogoff (1998) argue that these variables should have a positive effect on cash demand.¹⁰ They maintain that the higher marginal tax is, or the higher the percentage of tax (and pension contributions) as a share of gross domestic product, the greater the incentive will be for participants in the economy to attempt to evade tax by transferring part of their financial activity to the illegal economy. Since cash is the most common payment instrument in the illegal economy, this will probably lead to an increased demand for cash.

A theoretical motive for tax evasion that no one has attempted to model concerns the effect of inflation and tax on net worth. Low inflation and low bank interest rates combined with wealth tax may lead to a loss after taxes on bank deposits, while the profit on cash that is not declared for wealth taxation will be close to nil. An increase in real wealth taxation as a result of lower

⁹ It is worth noting that the user only pays such fees if debit cards are used. If credit cards are used, the shop pays the fee.

¹⁰ In Rogoff (1998) a theoretical model of demand for cash is also presented. Among other things, he argues here that a variable for marginal tax may capture the effect of this type of illegal economy.

inflation could thereby lead to an increase in tax evasion and increased demand for cash.

5 A cash demand model

We model cash demand deflated by prices (real cash).¹¹ One reason for this is that it is primarily real consumption of goods and services that is relevant to the public. A behavioural context for the public's adaptation should therefore relate the demand for real cash holdings to planned real transactions.¹²

We started modelling with a flexible, dynamic model that took into consideration the effects of households' consumption at retail outlets, banks' deposit rates, the number of ATMs, the number of point-of-sale terminals, the price of using alternative means of payment, various tax variables and lagged values of the cash itself. See Charts 2 to 7 for an illustration of the data series. In addition, we included a linear trend. This was included in an attempt to capture the aggregate effect on demand for cash of developments in the payment system. A large number of explanatory variables and combinations of variables have been tested, where quarterly data from the first quarter of 1980 up to and including the second quarter of 2004 have been used.¹³

The series for numbers of ATMs and numbers of point-of-sale terminals proved to be strongly correlated.¹⁴ In order to avoid multicollinearity problems, we chose to include only one of these variables at a time as an explanatory variable in the estimated equation. However, it turned out that none of them had a significant effect on demand for real cash.

Nor did the price of using alternative payment instruments have a significant effect on cash demand. One reason for this may be measurement errors associated with the variable, mainly due to a lack of data (cf. Aastveit (2005)).

A general problem with regard to the analysis and modelling of demand for cash has been the way in which the illegal economy should be treated. As mentioned, we have tried to include various tax variables in an attempt to detect the part of the illegal economy that is associated with tax evasion and its effect on demand for cash. However, it turns out that none of the variables are significant.

The preferred model that we are left with is specified in the appendix. The model is a so-called error-correction model for the logarithm of the demand for real cash.¹⁵ The model shows that demand for real cash

depends on real consumption at retail outlets, banks' deposit rates and a negative linear trend that is intended to capture developments in the payment system, in addition to lagged values of the cash itself.¹⁶ The expression in brackets measures the deviation from an estimated long-term relationship between real cash, real consumption at retail outlets and banks' deposit rates. The coefficient of -0.41 indicates that demand for real cash increases (decreases) by 0.41 per cent in quarter t if the demand for real cash is one per cent below (above) the estimated long-term relationship in quarter $t-1$ (all else being constant).

According to the model, demand for real cash will increase by 0.53 in the long-term if real consumption at retail outlets increases by one per cent and the other explanatory factors remain constant. The long-term effect on real cash of a change in interest rates is slightly weaker. According to the model, the demand for real cash will be reduced by 0.02 per cent in the long term if banks' deposit rate increases by one percentage point and the other explanatory factors remain constant.

6 Forecasts and use of the model

As mentioned, the purpose of a cash demand model is to underpin management of the purchase and storage of notes and coins. In order for Norges Bank to order cash in as efficient a way as possible in the future, we are dependent on accurate forecasts.

The model is based on quarterly data and will be periodically re-estimated when information from new quarters becomes available. New forecasts will then be made.

The model presented in the appendix is a single-equation model. This means that attempts to make forecasts with the aid of this model must be based on assumptions as to how the explanatory variables will develop. Ordinarily, Norges Bank will base its assumptions regarding private consumption at retail outlets and banks' deposit rates on the projections for private consumption and interest rates published in the *Inflation Report*.

Chart 9 shows the model-based forecasts for cash demand up to and including 2007.¹⁷ The projections were prepared using data up to and including the second quarter of 2004. The chart also shows actual developments in demand for cash in the period after the forecasts were made.

The chart shows that the model-based forecasts were accurate during the period from the third quarter of 2004 up to and including the third quarter of 2005. The only

11 The price variable that is used to deflate demand for cash is related to the variable consumption at retail outlets. The price variable is calculated as the ratio of consumption at retail outlets in current prices to consumption at retail outlets in fixed prices. This means that the individual price indices for each sub-component in the consumption term 'consumption at retail outlets' will be weighted by the percentage the respective sub-component constitutes of the total value of consumption at retail outlets.

12 Another reason that it is more appropriate to model demand for real cash is that the series for real cash is integrated of order 1. This means that the series for percentage changes in real cash is stationary. The series for nominal cash is neither integrated of order 0 nor integrated of order 1.

13 We have used a so-called "general-to-specific" approach as a basis for choice of model. See, for example, Hendry and Krolzig (2001) for a more detailed description of this method.

14 The variables had a correlation coefficient of 0.94.

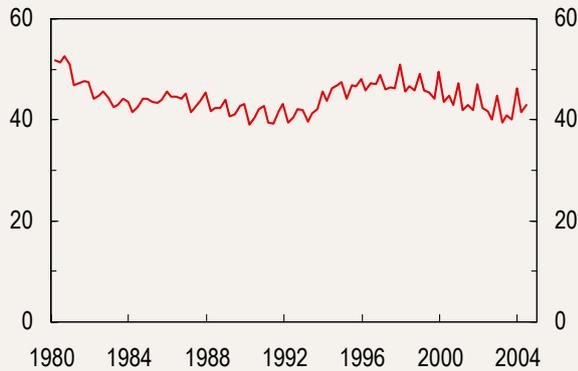
15 This type of model makes it very easy to interpret both short-term and long-term effects on demand for real cash of a change in one of the explanatory variables. For a more detailed discussion and interpretation of such a cash demand model, see Aastveit (2005).

16 Lagged values of cash itself are included in order to correct the model for autocorrelation. At the same time, such lags will to a certain degree capture any seasonal effects.

17 In order to make the forecasts more robust, a so-called constant adjustment has been added so that the model hits the mark exactly in the final observation. For a thorough explanation of constant adjustment, see Clements and Hendry (1998).

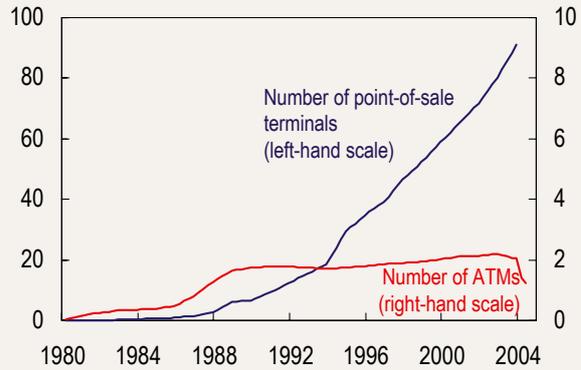
Chart 2-7 Illustration of data series

Chart 2 Actual cash. In billions of 2001 NOK



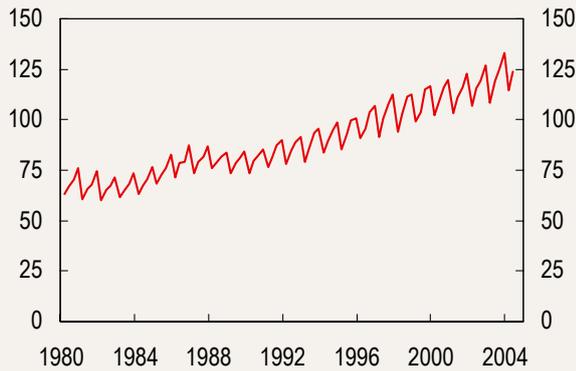
Source: Statistics Norway and Norges Bank

Chart 5 Number of ATMs and point-of-sale terminals. In 1000s



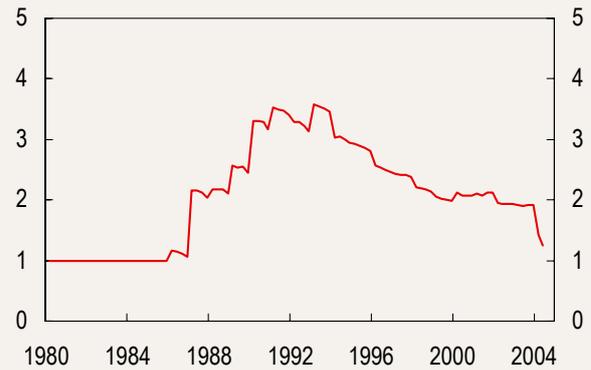
Source: Norges Bank

Chart 3 Real consumption at point of sale. In billions of 2001 NOK



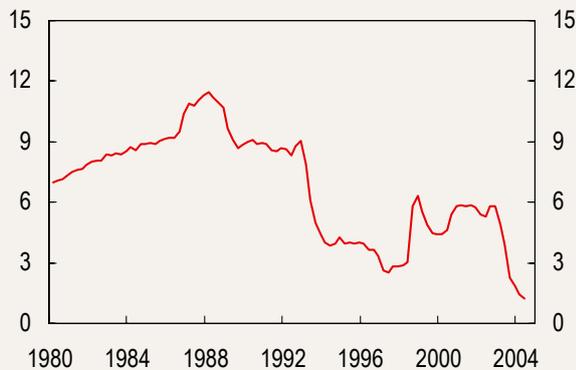
Sources: Statistics Norway and Norges Bank

Chart 6 Price for use of alternative payment instruments. NOK



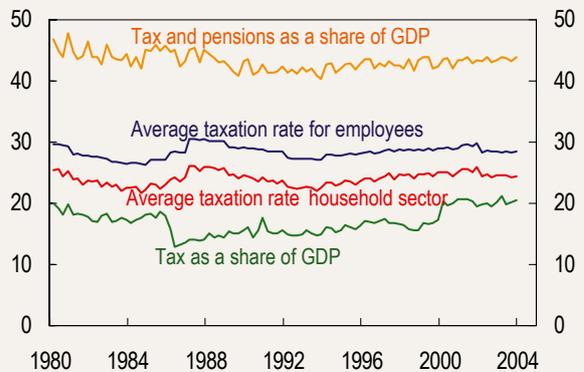
Source: Norges Bank

Chart 4 Banks' deposit rate. Percentage points



Source: Norges Bank

Chart 7 Average taxation rate and tax as a share of GDP
Percentage points

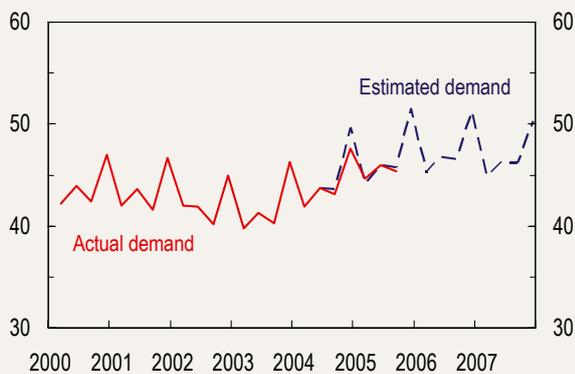


Source: Statistics Norway

exception is the fourth quarter of 2004, where the forecasts overshoot actual demand for cash by 4 per cent.

With regard to developments over the next two years, the model forecasts that demand for cash will increase during the next quarters before decreasing towards the end of 2006 and further through 2007.

Chart 9 Estimated (our model) demand for cash.
In billions of NOK



Source: Norges Bank

7 Conclusion

Increased focus on improving cash supply efficiency has resulted in the modernisation of inventory policies and the development of a cash demand model. The model only provides information concerning aggregate developments in cash circulation. Further work on the model will therefore include testing of how well it works with different denominations or groups of denominations, such as ATM notes or coins.

So far, we have little experience with the new policy and use of the model. However, we have gained greater knowledge about the logistics processes and an improved understanding of the factors that affect cash circulation.

In the future, the model will play a key role in long-term planning with regard to the procurement of notes and coins from external suppliers. In the case of actual orders, however, the model must be combined with micro-models, where the distribution among different regions and denominations is included.

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Appendix: A model of cash demand

$$\begin{aligned} \Delta cu_t = & 2.10 - 0.002 trend_t + 0.409 \Delta cu_{t-4} + 0.308 \Delta cu_{t-8} + 0.162 \Delta c_t - 0.012 \Delta i_t \\ & (3.14) \quad (4.37) \quad (5.35) \quad (3.80) \quad (3.39) \quad (3.03) \\ & - 0.410 [cu_{t-1} - 0.527 c_{t-1} + 0.022 i_{t-3}] - 0.044 D_{1991.2,t} + 0.047 D_{1993.4,t} + 0.053 D_{1997.4,t} \\ & (6.50) \quad (4.21) \quad (2.98) \quad (2.45) \quad (2.58) \quad (2.86) \\ & + 0.0468 D_{1999.4,t} + \varepsilon_t \\ & (2.53) \end{aligned}$$

$$\begin{aligned} R^2 = 0.92 \quad \sigma = 0.0173 \quad AR_{1-5}: F(5.71) = 0.84 \quad ARCH_{1-4}: F(4.68) = 0.95 \\ NORM: \chi^2(2) = 1.86 \quad HET: F(20.55) = 1.22 \quad RESET: F(1.75) = 2.43 \end{aligned}$$

Estimation period: 1980 Q1 – 2004 Q2.

Estimation method: Least square method

Absolute t-values are provided in brackets below the estimates. In the long-term context, long-term t-values are provided.¹ The equation fulfils requirements (diagnostic tests) that are relevant for a well specified model. It also passes (recursive) Chow tests for structural breaks at one per cent significance level during the last ten years. The explanatory variables (consumption at retail outlets and interest rate) have weak exogeneity with regard to all of the parameters in the structural equation for real cash.²

Δ is a differential operator: $\Delta X_t = (X_t - X_{t-1})$.

cu	= The logarithm for real cash. Source: Norges Bank, Statistics Norway (SN).
c	= The logarithm for real consumption at retail outlets. Source: SN.
i	= Weighted average of the banks' deposit rate for transaction accounts. Source: Norges Bank.
$D_{1991.2}$	= Dummy variable for 1991 Q1. Introduction of a new 500 krone note at the same time as Series V of the 1000-krone note is withdrawn from circulation. We assume that when the public turn in their old 1000-krone notes, many choose to deposit them in an account instead of exchanging them for new notes.
$D_{1993.4}$	= Dummy variable for 1993 Q4. 1993 and the first half of 1994 are an unstable period in the Norwegian economy. The model has problems with reproducing the trend in demand for cash during this period. We have therefore chosen to introduce this dummy variable.
$D_{1997.4}$	= Dummy variable for 1997 Q4, due to exceptionally large outstanding holdings of cash among the public at the year end.
$D_{1999.4}$	= Dummy variable for 1999 Q4 due to an exceptional demand for cash at the turn of the millennium.
ε	= Regression residuals (unexplained variation in the left-hand variable).
R^2	= The percentage of variation in the left-hand variable that is explained by the model
σ	= Standard deviation of regression residuals.
AR_{1-5}	= A test of 5th order autocorrelation in the residuals.
$ARCH_{1-4}$	= A test for 4th order ARCH residuals.
NORM	= A test for whether the residuals have a normal distribution.
HET	= A test for heteroscedasticity.
RESET	= A test of the model's functional form.

The expression in brackets measures deviation from an estimated long-term relationship between demand for real cash and real consumption at retail outlets and the banks' deposit rates.

¹ These are calculated using the same method as in Kmenta (1997, p. 486).

² Test for weak exogeneity has been performed as suggested by Boswijk and Urbain (1997).