How accurate are credit risk models in their predictions concerning Norwegian enterprises?

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Historically, banks' solvency problems are often due to losses on loans to enterprises. Credit risk associated with loans to enterprises is therefore an important aspect when Norges Bank assesses financial stability. Two different credit risk models are used in the analyses, Norges Bank's SEBRA model and the Moody's KMV Private Firm model. This article compares the quality of predictions made by the two models. The analysis shows that both models are good at selecting bankruptcy candidates among unlisted Norwegian enterprises and that the SEBRA model is somewhat better than the Moody's KMV Private Firm model.

1. Introduction

There are clear methodological differences between the two credit risk models used by Norges Bank. The SEBRA model, which has been developed by Norges Bank, predicts bankruptcy probabilities on the basis of figures from the annual accounts of Norwegian limited companies. The Moody's KMV Private Firm model predicts the probability of default for large unlisted enterprises, based primarily on market information. SEBRA is thus an accounting-based model whereas the Moody's KMV Private Firm model may be characterised as a market-based model. This article compares the quality of the predictions made by these two models on the basis of predictions for Norwegian enterprises made after the financial years 1998 - 2001 and actual bankruptcies in the period 1998 - 2003.

The structure of this article is as follows: Section 2 briefly presents the two models and comments on some methodological differences. Section 3 presents the data underlying the analysis, while Section 4 presents the results. Differences in the two models' treatment of different industries are discussed in Section 5, and a summary follows in Section 6.

2. Credit risk models

2.1 Norges Bank's SEBRA model

The SEBRA model predicts the risk of bankruptcy using 12 explanatory variables connected to figures from the annual accounts and some other enterprise characteristics. The model includes variables for earnings, liquidity, financial strength, industry, size and age.[1] The SEBRA model is based on a database containing annual accounts for all Norwegian limited companies. For the 2002 financial year, the database contains data concerning approximately 140 000 enterprises. The large majority of these enterprises are small. The SEBRA version of 2001 ("SEBRA 01"), which was estimated on the basis of annual accounts for the period 1990-1996, and the SEBRA version of 2003 ("SEBRA 03"), which was estimated on the basis of annual accounts for the period 1990-2000, were estimated on the basis of all enterprises in the database. A SEBRA version ("SEBRA Large") based on enterprises with annual turnover in excess of NOK 40 million was developed in connection with a previous comparison of SEBRA and KMV. The three SEBRA versions are fairly similar since there are only minor differences in the coefficient values of the various variables.

The disadvantage of the SEBRA model is that new information comes in only once a year and that there is a time lag of nine months between the end of the financial year and the time most accounts are available in the database. For example, the bankruptcy predictions in June 2004 were based on annual accounts from 2002.

2.2 The Moody's KMV Private Firm model

The Moody's KMV Private Firm model, a model for unlisted enterprises, is an offshoot of the Moody's KMV Public Firm model, a model for listed enterprises. Whenever the models are discussed in the rest of this article, KMV is used as an abbreviation for Moody's KMV. The fundamental idea in the KMV Public Firm model is that an enterprise will default on its debt obligations if the market value of its assets becomes too low compared with the value of its debt. The level at which an enterprise is assumed to default on its debt obligations is called the default point. On the basis of studies of default statistics, KMV chooses to calculate this level as the value of the enterprise's short-term debt plus a portion of its long-term debt. The default point is thus assumed to be somewhat lower than the value of total debt. The calculation of the default point is based on information from the financial accounts concerning the enterprise's financial position. Market data are used to estimate the market value of the enterprise's assets. On the basis of the share price of the enterprise in question and the volatility of the share price, option pricing the-

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[1] The SEBRA model is described in more detail in Eklund, Larsen and Bernhardsen (2001).

ory is used to estimate the market value of the enterprise's assets. A key variable in the KMV model is the distance to default, which is defined as the difference between the market value of the assets and the default point expressed in standard deviations. Using KMV's database of actual defaults, the distance to default is then converted to expected default probability (EDF). The greater the distance to default, the lower the expected default probability. As standard, the KMV model states the probability of default in the next 12 months for the enterprise in question.[2]

Quoted share prices do not exist for unlisted enterprises. This means that the market value of an enterprise's assets must be determined in some other way. KMV's Private Firm model estimates the market value of an enterprise's assets as the enterprise's EBITDA[3] multiplied by a factor that is a function of share price movements for listed enterprises in the same industry, share price movements for listed enterprises in the same country and the size of the enterprise in question. The methodology used in the KMV Public Firm model is then used to calculate the expected default probability.

One would expect the KMV Public Firm model, which is based on the market's continuous pricing of equity in each enterprise, to be more accurate in predicting default than the KMV Private Firm model. The drawback of the latter model is that the estimated market value of the enterprise's assets is based on average figures for somewhat similar enterprises and not on the market's continuous pricing of enterprise-specific risk factors. The SEBRA model predictions are compared with the predictions of the KMV Private Firm model because there are so few listed enterprises in Norway that it is not meaningful to make a comparison with the KMV Public Firm model.

Moody's KMV has also developed an accounting-based credit risk model for unlisted enterprises called Moody's KMV RiskCalc. We have not tested SEBRA's predictions against this model since one important purpose of the test is to compare SEBRA with a market-based credit risk model.

2.3 Differences between SEBRA and KMV

One important difference between SEBRA and KMV is that SEBRA predicts the probability of bankruptcy during the next three financial years[4] while KMV predicts the probability of default during the next 12 months. These probabilities are somewhat different since an enterprise that defaults on its debt obligations will not necessarily go bankrupt. For example, in the event of default, a creditor may agree to a new repayment plan or to convert debt to equity instead of forcing the enterprise into bankruptcy. Therefore, given the same time horizon for the respective probabilities, the default probability for an enterprise will never be lower than the bankruptcy probability. In practice, the default probability from the KMV model is considerably higher than the bankruptcy probability from the SEBRA model. Both bankruptcy and default probabilities are indicators of the risk exposure associated with credit to enterprises. Thus, there is reason to assume that rankings of enterprises, based on bankruptcy and default probabilities, respectively, are approximately the same. In the comparisons of credit risk models in this article, the ranking of enterprises on the basis of risk exposure plays an important role.

One weakness of rankings is that they only take into account a portion of the information inherent in the magnitude of the predicted bankruptcy and default probabilities. With the SEBRA model, the bankruptcy probability is low for a very large portion of the enterprises.[5] The rankings of these enterprises can therefore easily become quite arbitrary since the bankruptcy probabilities for many enterprises are almost similar. Bankruptcy probabilities for the enterprises with the highest risk exposure normally vary widely, so the ranking of these enterprises should provide a useful picture of the difference in risk. The KMV model truncates the probabilities since default probabilities higher than 20 per cent are set to 20 per cent while all default probabilities lower than 0.02 per cent are set to 0.02 per cent. Thus, the predicted default probabilities are spread over the interval from 0.02 per cent to 20 per cent. In most cases, the difference between default probabilities of different enterprises is larger than the difference between bankruptcy probabilities.

In addition to market data, the KMV model uses a limited selection of accounting data. Whereas SEBRA bases its predictions on data from the company accounts, the KMV model uses data from the consolidated accounts. This difference between the two models is probably not so important in practice since the KMV model uses so few data from the accounts.

3. Underlying data

The SEBRA and KMV models' predictions at various times are used as the basis for the comparison of the two models. The accuracy of these predictions is measured against actual bankruptcies. The reason that bankruptcies are used as the only measure of comparison is that Norges Bank does not have information about defaults. Using bankruptcies as the measure of comparison in spite of the fact that the KMV model predicts default probabilities contributes to a bias in favour of the SEBRA model.

[2] This default probability can be converted fairly easily to a period of more than one year.

[3] EBITDA = Earnings before interest, taxes, depreciation and amortisation.

[4] More precisely, the estimated bankruptcy probability after year t is the probability that the annual accounts for year t are the last ones that the enterprise will deliver and that the enterprise will file for bankruptcy within the next three years.

[5] For example, the bankruptcy probability for 86 per cent of the enterprises in the survey was 1 per cent or less after the 2001 financial year.

Table 1. Number of enterprises present in the databases of both the SEBRA and KMV models after different financial years, and the number of these enterprises that went bankrupt in subsequent years

Financial year	Number of enterprises		Number of bankrupt enterprises (KMV March in brackets if different)				
	SEBRA and KMV (September)	KMV (March)	1999	2000	2001	2002	2003
1998	3 414	3 399	3	12	18	37	30
1999	3 482	3 439	0	6	18	39	31
2000	3 502	3 055	0	0	8 (6)	44 (39)	32 (26)
2001	3 182	2 931	0	0	0	20 (16)	26 (24)

3.1 Basis of comparison

The comparison of the SEBRA and KMV models is based on Norwegian non-financial enterprises, excluding enterprises in the oil and gas industry, that are present in the databases for both the Moody's KMV Private Firm model and the SEBRA model. KMV's database is limited to enterprises with annual turnover of more than NOK 70 million. The KMV database contains monthly observations of expected default probabilities for a period of up to 5 years (60 months), whereas the SEBRA database contains annual accounts data and estimated bankruptcy probabilities for virtually all Norwegian limited companies since the 1988 financial year.

Predictions made by all three SEBRA versions are included in the comparison with the KMV model. While there is only one prediction (bankruptcy probability) per enterprise per financial year for each SEBRA version, the KMV model provides 12 predictions (default probabilities) per enterprise per year. Therefore, one must decide which KMV predictions to include in the comparison. Since the SEBRA predictions for most enterprises are not available until September, nine months after the end of the financial year, the KMV default predictions as per September have been selected for use in the comparison. At this time, the KMV model also includes accounting data for the last financial year.[6] In order to assess the KMV model's ability to extract information from market data, the KMV predictions as per March are also included in the comparison. KMV's September prediction (9 months after the end of the financial year) and March prediction (15 months after the end of the financial year) are based on the same accounting data, but the March prediction is based on newer market data.

The SEBRA and KMV models are compared on the basis of predictions made after the financial years 1998-2001 and actual bankruptcies in the three subsequent years.[7] For example, bankruptcies in the years 1999-2001 are used to assess the quality of the predictions made after the 1998 financial year. For each of the financial years in question, the combined database for the SEBRA and the KMV models include somewhat more than 3000 non-financial enterprises excluding enterprises in the oil and gas industry (see Table 1). March predictions do not exist for all enterprises for which KMV had September predictions. The number of enterprises that have disappeared is highest following the 2000 financial year, i.e. from September 2001 to March 2002 (see Table 1). When calculating the key figures for KMV's March predictions, adjustments have been made for the effect of the enterprises that have disappeared from the database.

Table 1 also shows how many of the enterprises went bankrupt in subsequent years. Due to a cyclical downturn, the number of bankruptcies in 2002 and 2003 were considerably higher than in the previous years. The decline in the number of bankruptcies in 2002 from the row for the 2000 financial year to the row for the 2001 financial year means that many of the enterprises that went bankrupt in 2002 and were included in both databases in September 2001 had disappeared from one or both of the databases in the period to September 2002.

4. Comparison of the quality of the predictions

We base our comparison of the quality of the predictions on power curves and accuracy ratios. Power curves and accuracy ratios are frequently used when comparing the accuracy of credit risk models (see Sobehart, Keenan and Stein (2000) and Engelmann, Hayden and Tasche (2003)). These two methods are closely related and are based on ranking enterprises by risk exposure.

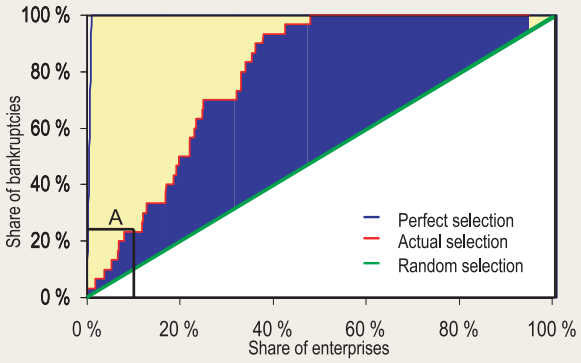
4.1 Power curves and accuracy ratio

A power curve is constructed as follows: Enterprises are ranked from the one with the highest risk exposure to the one with the lowest risk exposure based on the risk exposure measure being used. The power curve for the selection of bankruptcy candidates is obtained by presenting the share of accurately picked bankrupt enterprises as a function of the share of enterprises (in ranked order) (see Chart 1). For example, point A in the chart shows that 23 per cent of the enterprises that subsequently went bankrupt were among the 10 per cent of

[6] Bureau Van Dijk provides accounts data to KMV. KMV states that these data are available in June of the year after the financial year.

[7] Only in the two subsequent years after the 2001 financial year.

Chart 1 Illustration of power curves and accuracy ratio



the enterprises with highest risk according to the model. The expected power curve for a random selection will be the 45 degree line, whereas the perfect selection is that all bankrupt enterprises were ranked ahead of all other enterprises. This means that if 1 per cent of the enterprises go bankrupt, the power curve for the perfect selection includes 100 per cent of the bankruptcies after having gone through the top 1 per cent of the ranking list of all enterprises.

$$\text{Accuracy ratio} = \frac{\text{Area below the power curve for the actual selection} - \text{Area below the power curve for the random selection}}{\text{Area below the power curve for the perfect selection} - \text{Area below the power curve for the random selection}}$$

The accuracy ratio is a quantitative measure of how accurate the model is at selecting bankruptcy candidates. The accuracy ratio is defined as:

By definition, a perfect selection has an accuracy ratio of 100 per cent, while a selection whose quality is in line with a random selection has an accuracy ratio of 0 per cent. Although this is not the case in Chart 1, the power curve for the actual selection may be entirely or partly below the power curve for the random selection. In the case where the accuracy ratio is negative, the accuracy of the prediction method is lower than what one would have expected with a random selection. One should expect that any method that is called a credit risk model is considerably better in its selection than a random selection.

4.2 Results

After each financial year, five predictions are made, three with different versions of SEBRA (SEBRA 01, SEBRA 03 and SEBRA Large) and two with KMV (September and March predictions). The accuracy ratios are calculated on the basis of the power curves after the different financial years (see Table 2).

The table shows that both credit risk models' predictions are considerably better than a random selection. Since SEBRA Large was developed for large enterprises, one would expect that this model was more accurate than the other SEBRA versions for the enterprises in this comparison. Surprisingly, the quality of the SEBRA Large predictions is poorer than the quality of the other two SEBRA versions' predictions in both 1998 and 1999. The accuracy ratios for all SEBRA versions are particularly high after the 2001 financial year. This indicates that the key figures on which the SEBRA model's predictions are based are more informative when the economy is facing a cyclical downturn than at other times.

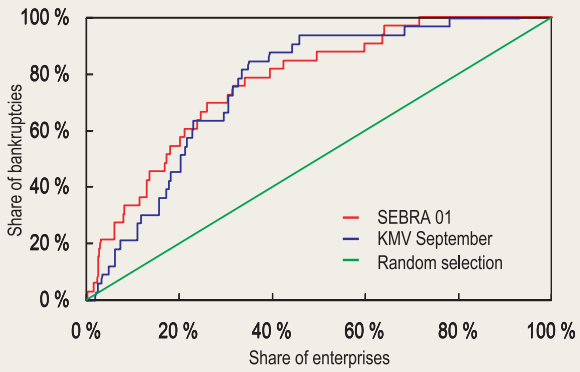
Table 2. Accuracy ratios for the credit risk models after the different financial years

Financial-year	SEBRA 01	SEBRA 03	SEBRA large	KMV September	KMV March
1998	55.2 %	55.8 %	50.9 %	53.2 %	51.8 %
1999	57.2 %	58.5 %	55.2 %	50.2 %	49.4 %
2000	54.1 %	54.6 %	54.6 %	40.7 %	49.1 %
2001	74.7 %	75.3 %	78.3 %	40.9 %	46.2 %

On the basis of the accuracy ratios, SEBRA 01's predictions are better than the September predictions from KMV every year. The difference is small in 1998, but in 2001 the difference is substantial. This is also reflected in the power curves from these two years (see Charts 2 and 3). When evaluating these results, one must bear in mind that the measure of comparison is bankruptcies, which is advantageous for the SEBRA model since the KMV model predicts defaults.

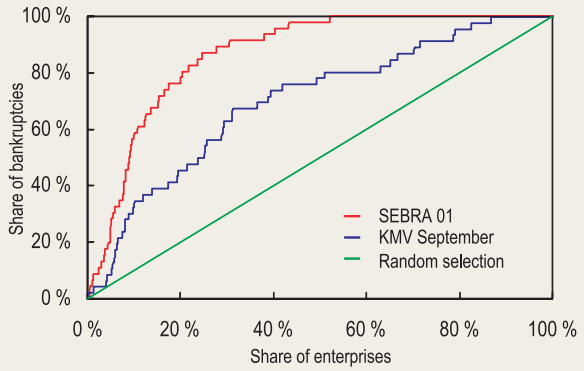
Due to more recent market information, and given the same accounting information, one would expect that KMV's March predictions are better than the September predictions. This is the case for the predictions after the 2000 and 2001 financial years, whereas the March predictions are actually somewhat worse than the September predictions after the 1998 and 1999 financial years.[8]

Chart 2 Power curves for the SEBRA and KMV predictions after the 1998 financial year based on bankruptcies in the period 1999–2001



[8] Stock price movements are important for developments in expected default probabilities. In the periods October 1999 to March 2000 and October 2001 to March 2002, the stock market picked up markedly, while it declined in the period October 2000 to March 2001.

Chart 3 Power curves for the SEBRA and KMV predictions after the 2001 financial year based on bankruptcies in the period 2002–2003



5. Industry differences between the models

Since the SEBRA and KMV accuracy ratios are different, it is of interest to study differences in the models' assessments of industry risk exposure. Industry differences between KMV's September predictions and the SEBRA 01 predictions are analysed below. We divide the enterprises into 18 industries. Retail trade, with roughly 39 per cent of the enterprises, and manufacturing, with approximately 26 per cent of the enterprises, are clearly the largest industries. Five of the industries each have less than 1 per cent of the enterprises. The analyses are limited to industries with a minimum number of selected enterprises over the period 1998-2001. Approximately 10 industries fill this requirement in each of the analyses below.

The analyses of industry differences are based on the same enterprise rankings that were used in the calculation of the power curves. The KMV and SEBRA rankings for each enterprise are juxtaposed as a value pair.

Chart 4 Ranking of all enterprises on the basis of risk exposure after the 2000 financial year. Delimitation of outliers

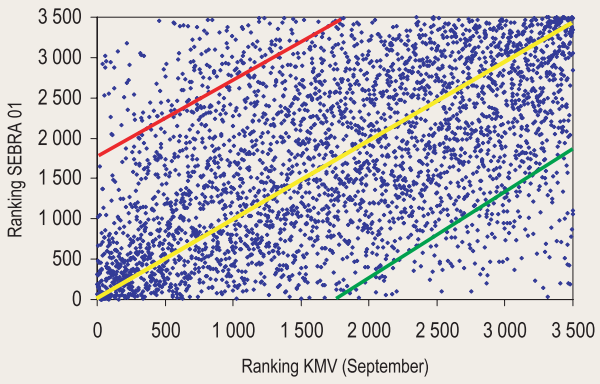


Table 3. Share of enterprises classified as outliers

Financial year	South-east corner KMV: Low risk SEBRA: High risk	North-west corner KMV: High risk SEBRA: Low risk	Total
1998	2.5 %	2.3 %	4.8 %
1999	3.8 %	3.9 %	7.8 %
2000	3.8 %	4.6 %	8.4 %
2001	3.9 %	4.0 %	7.9 %

The value pairs for all enterprises are then set down as points in a two-dimensional diagram (see Chart 4). If the two models had been completely in agreement in their risk assessments, the value pairs would have formed a straight line from the southwest corner to the northeast corner, like the yellow line in the chart. The further the value pair is from the yellow line, the greater the divergence between the two model's assessments. The largest density of value pairs is in the southwest corner. This means that the two models more or less concur in their assessments of which enterprises represent the highest risk.

5.1 Analyses of enterprises for which the models disagree strongly

One way to utilise the rankings in Chart 4 is to study the enterprises that have been ranked very differently by the two models. These are the enterprises for which the absolute value of the difference between the SEBRA and KMV rankings is greater than a predefined limit. We choose to set this limit at the number that corresponds to 50 per cent of the total number of enterprises. Enterprises are considered to be outliers if the difference is higher than this limit. Disagreements between SEBRA and KMV may be manifested in two ways. KMV may consider an enterprise to be considerably more high-risk than SEBRA, or the opposite may be the case. These two cases are represented by observations in the northwest corner (above the red line) and the southeast corner (below the green line) respectively in Chart 4. The share of enterprises that are classified as outliers, given the chosen limit, is lowest in 1998 and relatively stable the other years (see Table 3). The share of outliers in the two corners is fairly similar.

What is most interesting about the outliers is to study whether there are any industry differences between the two corners. Therefore, we have calculated each industry's share of outliers in one corner in relation to the total number of outliers for the industry. In the southeast corner, the share of outliers from the hotel and restaurant industry, construction and tourism is very high (79 per cent or higher). This indicates that KMV regards enterprises in these industries to be less risky than SEBRA does. In the northwest corner, the share of outliers from the property management industry is very high (77 per cent). This indicates that SEBRA regards enterprises in

this industry to be less risky than KMV does. The comparison for property management enterprises is not very meaningful, however, since the number of such enterprises in the joint database is very limited.[9]

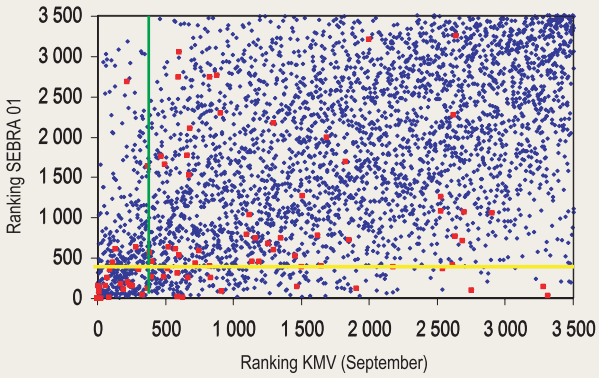
5.2 Analyses of the 10 per cent of enterprises classified as very high-risk

The analyses in this section are based on the two models' selections of the 10 per cent of enterprises with the highest risk. These enterprises are classified as very high-risk.[10] These selections consist of all value pairs that are located below the yellow line and/or to the left of the green line in Chart 5. Both models concur that the enterprises represented by value pairs that are both below the yellow line and to the left of the green line are very high-risk. Only one of the models regards the enterprises represented by value pairs that are either below the yellow line or to the left of the green line, but not both, as very high-risk. The first analysis compares the industry mix of the selected enterprises, while the second analysis evaluates the selection of high-risk enterprises against actual bankruptcies.

Agreement between the two models concerning the selection of very high-risk enterprises is strongest in the telecommunications industry, and weakest in the shipbuilding industry, shipping and commercial services. However, with the exception of commercial services, these industries have few enterprises represented in the study. For a period covering all years and all industries, the models agree in their classification of enterprises as very high-risk for approximately 48 per cent of the selected enterprises.[11] By comparison, with a completely random distribution of value pairs, one would expect to find only 1 per cent (10 per cent multiplied by 10 per cent) of the observations in this area.

As an extension of the analysis above, we have studied industry imbalances in the models' risk classifications. When only one of the two models has classified an enterprise as very high-risk, this is described as a "one-model selection". The two industries with the largest proportion of one-model selections are telecommunications, where KMV accounts for close to three-quarters

Chart 5 Ranking of all enterprises on the basis of risk exposure after the 2000 financial year. Delimitation of the 10 per cent of the enterprises classified as very high-risk. Actual bankrupt enterprises in the period 2001–2003 (red marks)



of the one-model selections, and the hotel and restaurant industry, where KMV accounts for less than one-third of the one-model selections. Not surprisingly, the industries with relatively few enterprises show the largest deviations from the average, with regard to both agreement and imbalances. Therefore, the results for these industries may be partly due to chance.

It is also interesting to study whether the selection of the 10 per cent of enterprises classified as very high-risk tallies with the enterprises that actually went bankrupt. Note that the share of bankrupt enterprises among the 10 per cent classified as very high-risk may be read directly from the power curves. Therefore, the analysis here focuses on evaluating how much the credit risk models missed the mark in their predictions. For this purpose, we still use Chart 5 which shows the rankings of the enterprises represented by value pairs, as well as which of these enterprises went bankrupt in the three subsequent years. The analysis focuses on those cases where either SEBRA or KMV classifies an enterprise as low risk, while the other model classifies the same enterprise as very high-risk and the enterprise goes bankrupt. This is unsatisfactory for the credit risk model that predicted that the credit risk associated with this enterprise was low. An enterprise's risk exposure is regarded as low if

Table 4. Bankrupt enterprises within selected risk categories after different financial years. Number and share of the total number of bankrupt enterprises

Financial year	Risk classification						Total	
	SEBRA: Very high risk KMV: Very high risk		SEBRA: Very high risk KMV: Low risk		SEBRA: Low risk KMV: Very high risk			
1998	5	15 %	0	0 %	0	0 %	33	100 %
1999	8	13 %	1	2 %	0	0 %	63	100 %
2000	14	17 %	4	5 %	1	1 %	84	100 %
2001	11	24 %	4	9 %	0	0 %	46	100 %

[9] KMV places most of the property management enterprises in a different database than the one used in the comparison.

[10] Other limits are of course possible. With the limit of 10 per cent, the enterprises that are selected may be characterised as "very high-risk".

[11] Calculated as: $\frac{\text{Number of enterprises classified by both models as very high risk}}{0.1 \times \text{Total number of enterprises}}$

the enterprise is among the 50 per cent of enterprises with the lowest risk. The analysis shows that KMV missed the mark far more often than SEBRA (see Table 4). A total of nine enterprises that KMV classified as low risk and SEBRA classified as very high-risk went bankrupt. The only case of bankruptcy among the enterprises that SEBRA classified as low risk and KMV classified as very high-risk occurred after the 2000 financial year (the enterprise is in the northwest corner of Chart 5).

Ideally, all bankrupt enterprises should have been classified by both SEBRA and KMV as very high-risk. Following the four financial years in question, between 13 and 24 per cent of the bankrupt enterprises are in this category.

The most obvious conclusion emerging from the analyses in this section is that SEBRA classifies enterprises in the hotel and restaurant industry as more high-risk than KMV does. Unlike SEBRA, KMV has not been specially developed for Norway. One possible explanation for the results for the hotel and restaurant industry may therefore be that they are due to special conditions regarding this industry in Norway.

6. Summary

The comparison of the SEBRA model and the KMV Private Firm model shows that both models are good at selecting bankruptcy candidates among large unlisted Norwegian limited companies. On the basis of accuracy ratios, the SEBRA model's predictions are somewhat better than the predictions of the KMV model. This means that the SEBRA model's use of a larger number of accounting variables more than compensates for the KMV model's advantage of using updated market information. A further development of the SEBRA model may be to include some market indicators. The industry comparisons show some differences in the two models' assessments. The most prominent difference is that the SEBRA model considers the hotel and restaurant industry to be considerably more high-risk than the KMV model does.

The fact that overall the accounting-based SEBRA model provides more accurate predictions than the KMV Private Firm model does not necessarily mean that accounting-based credit risk models are better than market-based credit risk models. The reason for the difference in the quality of predictions may be that attempts are made to use the market-based model in an area (unlisted enterprises) where this type of model has some drawbacks due to the lack of market prices. When evaluating the results of the comparison, it is also important to be aware that the comparison is based on a limited time period and that the event the models are measured against, namely bankruptcies, contribute to a bias in favour of the SEBRA model.

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