

Model for analysing credit risk in the enterprise sector

Trond Eklund, director, Kai Larsen, adviser and Eivind Berghardsen, student employee, all in the Financial Analysis and Market Structure Department ¹⁾

When banks' overall risk is evaluated, their credit risk exposure to the enterprise sector is a key element. In analyses of banks' credit risk in the enterprise sector, both a macroeconomic and a business economics approach are generally applied, the latter based on corporate earnings, liquidity and financial strength. In this article, we present a new model that predicts enterprise-specific bankruptcy probabilities. On the basis of these probabilities, both aggregate bankruptcy probabilities and the magnitude of accompanying losses for banks can be estimated.

1 Introduction

For many years Norges Bank has used the Sebra model ²⁾ in its analyses of banks' credit risk exposure to the enterprise sector. The new model is based on the same business economics approach and the same data input. Unlike the Sebra model, however, the new model was developed with a view to statistical analysis and therefore represents a quantitative supplement.

The new model predicts individual bankruptcy probabilities as a function of age, size, industry characteristics and accounts variables that can provide an indication of corporate earnings, liquidity and financial strength. An aggregation of individual bankruptcy probabilities provides a picture of the overall risk in the enterprise sector, thereby providing a basis for predicting developments in the near future. It is also possible to predict banks' potential loan losses in NOK.

Section 2 contains a brief explanation of the background for Norges Bank's analyses of credit risk in the enterprise sector and the reasoning underlying the analyses. Section 3 presents the new model, while section 4 evaluates the estimation results. Section 5 discusses the use of the model and a summary follows in section 6. A technical description of the model is presented in the annex.

2 General comments on the analysis of credit risk in the enterprise sector

Many countries have experienced banking crises during the past decade. The experiences of Norway, Finland and Sweden show that the socio-economic costs of banking crises are substantial. In the first half of the 1990s, most major banks in these countries incurred such significant losses that it was not possible to continue operations without government intervention. Problems in parts of the financial sector spread to other parts of the sector, resulting in what can be called a systemic crisis. A very high proportion of banks' losses was ascribable to losses on loans to Norwegian enterprises. The authorities are therefore concerned about this risk.³⁾

Credit risk refers to a credit institution's risk of a borrower's payment default on payment of interest and principal due to the borrower's unwillingness or inability to service the debt. The higher the credit risk an institution is exposed to, the greater the losses may be. For banks and most other credit institutions, credit risk is considered to be the form of risk that can most significantly diminish earnings and financial strength.

Norges Bank uses both microdata and aggregated data from the national accounts in its analyses of credit risk in the enterprise sector. Depending on the source, the analyses are concentrated on enterprises' earnings and debt-servicing capacity. For the central bank, the aim is to monitor developments in credit risk in the enterprise sector at an aggregated level.

The following provides a description of the reasoning underlying the Sebra model and the new quantitative credit risk model. The data input for the analyses is the annual accounts for all limited companies in Norway starting in 1988. In addition to the accounts, Norges Bank has information about industries and geographical locations. This allows us to monitor developments in enterprises' credit risk by industry and geographical area. Most Norwegian banks have similar models. In addition to evaluating credit risk, these are often used for pricing loans, selecting priority areas and assigning priorities for the resources to be used in lending activities.

Key factors in the analysis of credit risk

In the long term, corporate earnings must be reasonable relative to payment obligations. If this is not the case, liquidity will be weakened. Without satisfactory earnings, it will also be difficult for an enterprise to raise other types of capital, such as loan capital and new equity. Our analyses are therefore concentrated on corporate earnings. However, there are many ways to represent earnings in an analytical model. In the Sebra model, we have chosen the variable *annual profit before depreciation and write-downs after tax as a percentage of long-term debt*. The minimum earnings requirement is that it covers dividends, repayments of principal, part of the investment in fixed

¹⁾ We are grateful to Kjersti-Gro Lindquist, Terje Lensberg and Arild Lund for their useful suggestions and comments.

²⁾ See Eklund and Knutsen (1997) and Sæther and Larsen (1999) for a description of the Sebra model.

³⁾ In Norway, the Ministry of Finance, the Banking, Insurance and Securities Commission and Norges Bank are jointly responsible for the authorities' efforts to secure financial stability. The Ministry of Finance has primary responsibility, while the Banking, Insurance and Securities Commission is responsible for the supervision of each market participant. Norges Bank is responsible for fostering robust and efficient payment systems and financial markets, ie promoting financial stability.

assets and any need for increased working capital. In the analysis of each enterprise, the requirement can be set on the basis of dividend policy, repayment schedules for long-term debt and estimated working capital requirements. Eklund and Knutsen (1997) provide such an analysis.

A shortage of liquidity is often the factor that triggers bankruptcy. One or more variables that can explain the level of and changes in the enterprise's liquidity should therefore be included in a credit risk model. Again, there are a number of variables that can be used. In the Sebra model, we have chosen the variable *liquid assets less short-term debt as a percentage of operating revenues*. This variable has been chosen because a shortage of liquidity may be reflected in either reduced liquid assets or higher short-term debt. Applied to the individual enterprise, the liquidity requirement must be set on the basis of adaptations made by the enterprise in relation to, for example, liquidity reserves, credit period for customers, its inventory policy and the choice of short-term forms of financing.

An enterprise's ability to withstand losses is often assessed on the basis of its financial strength measured by its equity ratio. With a high equity ratio, the enterprise is better equipped to cope with difficult periods, partly because it will be easier to raise capital through the sale of assets without encumbrances and also obtain new loans because better collateral can be offered. Generally, a high equity ratio also implies lower current expenses for interest and principal. However, it is not difficult to find reasons why these elements are not always relevant. The most important reason for representing financial strength in a model is that, in our view, the model accumulates information about the enterprise's historical earnings. An enterprise with a high equity ratio has as a rule procured a substantial portion of its equity through retained earnings in earlier years. It has demonstrated the ability to make a profit, a factor that provides some support for the assumption that it will continue to be able to generate earnings in the period ahead. It should be pointed out that there are several problems associated with measuring an enterprise's financial strength, particularly asset valuation.

An alternative concept to models based on accounts data is to use market information (ie information on equity/ bond prices) in the model. So far, however, Norges Bank has chosen to use models based on accounts data, partly because there are few listed companies in Norway and even fewer companies that are traded regularly. The market information available for analytical purposes is therefore very limited. Analysts and investors also use accounts information in their analyses as a basis for recommendations and trading.

3 The new quantitative credit risk model

Ideally, a credit risk model should estimate bankruptcy/default probabilities for each enterprise. Since individual estimates can be directly linked to enterprises' debt, this

model can be used to predict the risk exposure of the debt. Moreover, the model can be used for pricing commitments and for determining how much capital should be set aside for each commitment. As a result of the desire to produce individual probability estimates, we decided to use a variant of logistical regression (see the annex).

The model is estimated using the entire population of enterprises in Norges Bank's accounts database for the period 1990-1996.⁴⁾ The total database consists of about 400 000 enterprise observations. There are some limitations associated with the accounts database. Originally, we wanted to estimate the probability of default. As a result of data limitations, however, we decided to estimate the probability of bankruptcy. Because banks also incur costs as a result of deferment of payment, default, debt restructuring and winding-up, we cannot capture all costs related to credit risk. Another limitation is that a fairly high proportion (about 15 per cent) of the enterprises disappear from the database without going bankrupt. This may be because they wind up operations (voluntary or compulsory winding-up), fail to submit accounts or merge/are taken over. We have no information as to what has happened to these enterprises. Moreover, some enterprises are temporarily absent from the base for unknown reasons. It is also important to point out that a substantial proportion of enterprises that go bankrupt are newly established enterprises that go bankrupt before they are included in the database.

A key criterion for the choice of model is that it shall be based on the reasoning discussed in section 2. This means, for example, that corporate earnings, liquidity and financial strength shall play a key role. It should be pointed out that it is difficult to capture these elements in a totally satisfactory manner in a model that is only based on accounts data. Moreover, a precondition has been that the model shall be transparent so that others can assess the model's predictive ability and results.

Choice of explanatory variables

In order to reduce the probability of excluding explanatory variables that are both relevant in business economics terms and statistically significant, we carried out an extensive search process.⁵⁾ A large number of explanatory variables and combinations of variables were tested. Against the background of the criteria underlying the choice of model, we selected the following explanatory variables⁶⁾:

Earnings:

- Earnings⁷⁾ as a percentage of total assets (tkr)

Liquidity:

- Liquid assets less short-term debt as a percentage of operating revenues (lik)
- Unpaid indirect taxes as a percentage of total assets (ube)

⁴⁾ The database includes all limited companies in Norway for the period 1988-1999. One criterion for being included in the database is that the enterprise has submitted valid accounts to the Brønnøysund registers and that the accounts have passed the tests of our data supplier, Dun & Bradstreet. We have excluded enterprises with total assets of less than NOK 200 000.

⁵⁾ Among other things, we used a method based on genetic programming (see McKee et al., forthcoming). Two of the explanatory variables in the new model were found with the aid of this method.

⁶⁾ The designation in brackets is the name of the variable used in the estimation process (see annex).

⁷⁾ Profit before extraordinary items plus depreciation and write-downs and minus tax.

The various explanatory variables

In section 2, we discussed the background for our view that a credit risk model should include variables that reflect corporate earnings, liquidity and financial strength. In the new model, we have included some additional explanatory variables. This box provides a brief discussion of why we believe these variables can contribute to explaining bankruptcy.

Unpaid indirect taxes as a percentage of total assets

It is often the tax authorities that file a petition for bankruptcy for an illiquid enterprise. Enterprises are aware of this and are therefore diligent with regard to paying direct and indirect taxes in time. If taxes are not paid in time, and thereby reach a disproportionately high level, it may be an indication that the enterprise's liquidity is weak.

Trade accounts payable as a percentage of total assets

For some enterprises, liquidity problems result in a disproportionately high level of trade accounts payable. The test results indicate that the relative size of trade accounts payable makes a contribution in addition to the other two liquidity variables in the model.

Book equity less than paid-in equity capital (dummy variable)

By looking at the composition of equity it is possible to provide some indication of to what extent a given equity ratio is due to accumulated earnings or paid-in equity capital. If book equity is less than paid-in equity capital, it shows that the enterprise has a book loss, which in turn indicates it has not been run well enough. The opposite is the case if book equity is higher than paid-in equity capital.

Dividend payments the last accounting year (dummy variable)

It is realistic to assume that responsible owners do not take out dividends if the enterprise in some way or another is struggling or has unfavourable future prospects.⁸⁾ If the owners have recently taken out dividends, it may be an indication that the enterprise is solid and that future prospects are favourable.

Industry average for the variable 'equity as a percentage of total assets'

Bankruptcy frequency is normally lower in industries with a high average equity ratio than in industries with a low equity ratio. One possible explanation may be that the former are characterised by relatively little competition and hence relatively high profits. It is not unrealistic to assume that the bankruptcy frequency in such industries is lower than in industries with stronger competition. Moreover, it may be the case that lenders impose stricter equity ratio requirements on enterprises that operate in industries with a high average equity ratio. The threshold for starting up in these industries may therefore be

higher, with an 'elimination' of less serious and weak enterprises before they raise loans.

Industry average for the variable 'trade accounts payable as a percentage of total assets'

It appears that bankruptcy frequency is greater in industries with a high average level of trade accounts payable, such as restaurants and retail trade. It is not inconceivable that these industries feature more 'speculative' activity than other industries. By funding activities with trade accounts payable instead of bank loans, it is easier to avoid credit assessment and follow-up.

Industry standard deviation for the variable 'earnings as a percentage of total assets'

There is reason to assume that there is greater risk associated with operating in an industry that features considerable fluctuations in earnings than in industries with stable earnings. Considerable uncertainty associated with the industry's general earnings may make it difficult for enterprises to plan and initiate necessary measures. It may also make it difficult to gain access to external financing. Moreover, industries with a wide variation in earnings often have a large upside potential. The potential for high earnings may mean that the industry attracts enterprises that are more willing to take risks and/or enterprises that are less serious. A large element of such enterprises will increase bankruptcy frequency in the industry.

Number of years since establishment

Both our test results and studies in a number of countries⁹⁾ show that bankruptcy frequency is greater among newly established enterprises than among established enterprises. One reason may be that it usually takes time to develop relevant expertise in such key areas as financial and cash flow management, organisation, purchasing, sales, production, etc. Moreover, it is often difficult for newly established and young enterprises to gain access to the equity and loan capital market, as well as establishing favourable business ties to suppliers and customers. In some cases, newly established enterprises may not have the 'right to exist', for example because the market is not large enough or it is not possible to produce the products in a sufficiently efficient manner. It is often the case that enterprises do not discover this until one or two years have passed.

Total assets

Bankruptcy frequency is generally higher among small enterprises than among large enterprises. Small enterprises often operate within a limited geographical area and have a limited product range. This means that they have few or perhaps only one leg to stand on and are thus vulnerable to individual events. Moreover, small enterprises are often newly established and hence exposed to many of the

⁸⁾ According to the Companies Act and Accounting Act, dividends may not be distributed to owners if the enterprise's financial strength is weak.

⁹⁾ See, for example, Audretsch (1991) and Audretsch et al. (1994).

same problems facing young enterprises (see above). The test results indicate, however, that bankruptcy frequency¹⁰⁾ among the very smallest enterprises is lower than among the next smallest.¹¹⁾ One reason may be that there is little to be obtained from the bankrupt estate of the smallest enterprises and that they are therefore declared bankrupt to a lesser extent. We have taken this into account by using a variable function that reduces bankruptcy probability if the enterprise's assets are less than NOK 2 million. The variable is included in logarithmic form.

Comments on the industry variables

The industry classification is based on Statistics Norway's five-digit industry code. We have, however, chosen a rougher classification in which each industry group contains at least 1000 observations. We nevertheless operate with a large number of industry groups and have therefore confined the study to the calculation of aspects of the model's key variables for these groups.

- Trade accounts payable as a percentage of total assets (lev)

Financial strength:

- Equity as a percentage of total assets (eka)
- Dummy variable for book equity less than paid-in equity capital (taptek)
- Dummy variable for dividend payments the last accounting year (div)

Industry:

- Industry average for the variable 'equity as a percentage of total assets' (meaneka)
- Industry average for the variable 'trade accounts payable as a percentage of total assets' (meanlev)
- Industry standard deviation for the variable 'earnings as a percentage of total assets' (sdtkr)

Age:

- Dummy variable for number of years since establishment (a1,a2,...a8)

Size:

- Total assets (size)

Model structure

Analyses of the data set show that there is a considerable lag for a high percentage of bankrupt companies between the time the last accounts were submitted and the time at which bankruptcy proceedings are initiated.¹²⁾ Against this background, we deemed it most appropriate to define the variable we model as the event 'last year with submitted accounts and bankruptcy is registered within three years'.

We attempted to estimate time-specific effects directly in the model in order to capture cyclical effects. This was not successful, partly because we have a limited number of years in the estimation sample and partly because the data set is influenced by time-specific sample problems. Accounts data and bankruptcy data have different sources, and it is likely that the quality of the bankruptcy data varies somewhat over the years. The relationship between the explanatory variables and the bankruptcy event

is assumed to be constant over time, and the coefficients therefore represent 'average effects' over the business cycle. It appears that a high portion of the cyclical variation in bankruptcy risk is captured in the explanatory variables (see Table 1), which shows that there is a relatively stable relationship between predicted and actual probability of bankruptcy, irrespective of the cyclical phase. Cyclical variation can also be captured as shown in Chart 1a, where a variable aggregated over the predicted bankruptcy probabilities is used to explain banks' loan losses, or as shown in Chart 1b, where the variable based on predicted bankruptcy probabilities is supplemented by a macro-variable.

The model structure permits non-linear transformations of individual variables (see Annex). This makes the model more flexible, as the marginal effect of a variable is explicitly permitted to depend on the level of the variable. With this structure, the compensation rate between the two variables will not necessarily be constant.¹³⁾ This is a useful property for the model. For example: to what extent earnings must be increased in order to keep the risk unchanged when liquidity falls marginally should depend on the initial level of earnings and liquidity. The model structure implies that the marginal effect of a given variable gradually approaches zero as the variable takes on extreme values. This means that the predictions are to a lesser degree marked by extreme observations.

All the variables are included with a level of significance of at least 0.1 per cent. With stepwise inclusion, all the variables make significant contributions to the model's explanatory power. See the Annex and Bernhardsen (2001) for a more detailed description of the model.

4 Evaluation of the estimation results

In Table 1, the enterprises are divided into groups on the basis of predicted bankruptcy probability. By looking at the percentage of enterprises in the various groups that actually went bankrupt, we gain an impression of the model's predictive ability. There is close accord between predicted probabilities and actual bankruptcy frequencies. For example, the average predicted bankruptcy probability

¹⁰⁾ But not necessarily the frequency of winding up in another way.

¹¹⁾ Statistical tests show that bankruptcy frequency decreases when the assets are less than NOK 2 million.

¹²⁾ About 25 per cent of the enterprises that go bankrupt are registered as bankrupt the year after the last accounts were submitted, while about 55 and 20 per cent are registered as bankrupt two and three years, respectively, after the last accounts were submitted.

¹³⁾ This problem is discussed in Laitinen and Laitinen (2000). The model structure proposed in this article, however, is not the same as that presented here.

Table 1. Predicted bankruptcy probabilities and actual bankruptcy frequencies¹⁾

	Grouped according to predicted bankruptcy probabilities					
	Group 1 p>20%	Group 2 10%<p<=20%	Group 3 5%<p<=10%	Group 4 2%<p<=5%	Group 5 1%<p<=2%	Group 6 p<1%
1990	27.8	16.9	11.7	6.3	3.9	1.1
1991	27.0	15.6	9.8	5.1	3.0	0.9
1992	23.7	11.8	7.2	4.0	2.5	0.6
1993	26.0	13.2	7.8	4.1	2.4	0.7
1994	23.4	11.8	7.0	3.2	2.1	0.5
1995	23.0	12.8	8.0	4.1	2.2	0.5
1996	26.7	14.5	9.5	5.2	3.3	1.0
Average for actual bankruptcy frequencies ²⁾ (standard deviation)	25.4 (1.96)	13.8 (1.94)	8.7 (1.69)	4.6 (1.03)	2.8 (0.64)	0.8 (0.25)
Average for model's predicted bankruptcy probabilities ³⁾	26.7	14.1	7.1	3.2	1.4	0.2

1) Bankruptcy within 3 years. The figure for Group 1 in 1990, for example, shows that on the basis of the accounts for 1990, 27.8 per cent of the enterprises estimated to have a bankruptcy probability of more than 20 per cent went bankrupt in the period 1991-1993.

2) Average bankruptcy frequency and standard deviation over the period 1990-1996.

3) Average predicted bankruptcy probability (registered bankruptcy within 3 years) over the period 1990-1996.

Note to the table:

Whereas the model predicts the probability of the composite event: 'last year of submitted accounts and bankruptcy proceedings initiated within 3 years', actual bankruptcy frequencies show the event 'bankruptcy proceedings initiated within 3 years'. The difference between the concepts is greatest for the least risk-exposed enterprises, with the result that the predicted bankruptcy probability is reduced somewhat. The concept was chosen because (for bankrupt enterprises) it is easier to find a date for the last submitted accounts before bankruptcy than to predict the exact date for the initiation of bankruptcy proceedings. The choice of this definition also provides a quantity that can be added up over a period of years in an appropriate manner, which is important when the macro picture is to be analysed.

Source: Norges Bank

for the group that was most exposed to risk was 26.7 per cent in the period 1990-1996, while the average actual bankruptcy frequency was 25.4 per cent.

Bankruptcy frequency in the various groups varies relatively little in the 1990s. This indicates that a high proportion of cyclical developments was captured in the enterprises' accounts. In other words, a given set of accounts does not appear to be associated with particularly greater risk during a cyclical downturn than during an upturn. Cyclical developments are reflected in changes in the percentage of enterprises in the various risk categories.

Another way to evaluate the estimated model is to look at the percentage of bankrupt enterprises for which bankruptcy was predicted and the percentage of non-bankrupt enterprises that are predicted non-bankrupt at various levels of bankruptcy probability. With an optimal level of bankruptcy probability, we achieve a rate of accuracy of about 83 in both categories. If we had estimated the model for a designed sample, it would probably have been possible to achieve a higher rate of accuracy. In order to construct a model that is as stable and robust as possible, however, we have chosen to estimate the model for the entire population of enterprises.

We have also tested the model by estimating it on data for the years 1990-1993 and then used it to predict bankruptcy in 1996. By choosing an optimal level of bankruptcy probability, we achieve an accuracy rate of about 82 for both bankrupt and non-bankrupt enterprises. This rate of accuracy is almost as high as when the model is estimated and tested on the same sample.

We have also divided the data set into two equally large sets (random drawing) and estimated the model on the one set and tested it on the other. The results show

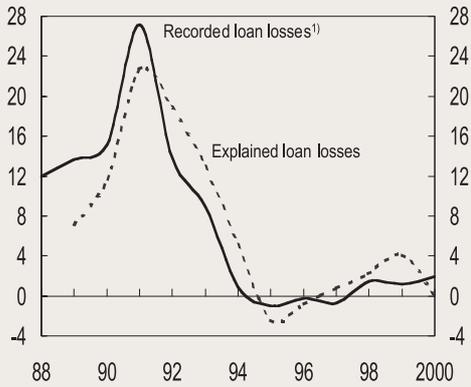
that the model is just as accurate for this test sample as for the estimation sample. Moreover, there are no significant differences in the coefficient estimates. See the annex for a further evaluation of the model.

5 Use of the model

Because the model generates individual probability estimates, it can be used in a number of areas related to credit risk analysis. Multiplying the debt of individual enterprises by the bankruptcy probability and adding up the figures for all enterprises provides an estimate of risk-weighted debt. This variable may be considered an estimate of banks' expected loan losses, given the absence of realised collateral. By including one or more variables that can provide some information about the value of banks' collateral, it is possible to provide an estimate of the level of loan losses in the short run. An attempt is made in Charts 1a and 1b to explain loan losses by means of the previous year's estimate of risk-weighted debt alone and the previous year's estimate of risk-weighted debt and the change in an index variable¹⁴⁾, which attempts to capture changes in expectations associated with the realisation value of collateral. The fit for these two simple models is relatively good (see annex for a further description). The exact distribution of recorded loan losses over time will largely depend on banks' expectations. In particular, the distribution may have been influenced by changes in procedures for bank's assessment of credit risk in the period surrounding the banking crisis. Here, it must be pointed out that there is a difference between being able to explain loan losses in retrospect and being able to provide estimates of future losses.

¹⁴⁾ The change in real house prices between September and September, measured by the ECON index, is used as an index variable.

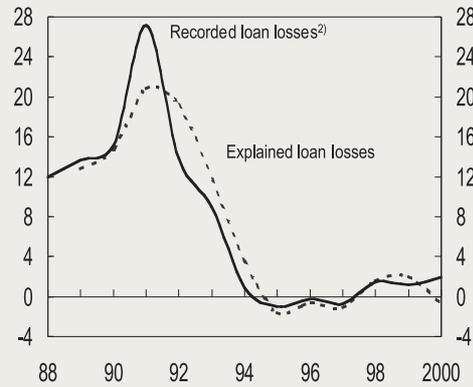
Chart 1a Loan losses explained through previous year's estimate of risk-weighted debt. Recorded loan losses¹⁾. In billions of 1999 NOK



¹⁾ Recorded losses and loss provisions adjusted for reversal of previous years' loss provisions. The figures cover all types of loans

Source: Norges Bank

Chart 1b Loan losses explained through previous year's estimate for risk-weighted debt and changes in house prices¹⁾. Recorded loan losses²⁾. In billions of 1999 NOK



¹⁾ Current year's change. Source: ECON/NEF and Norges Bank

²⁾ Recorded losses and loss provisions adjusted for reversal of previous years' loss provisions. The figures cover all types of loans

Source: Norges Bank

By projecting the model's variables, it is possible to provide some indication of developments in risk-weighted debt in the longer term. At the moment, Norges Bank is evaluating the possibility of linking the model's key variables to projections of selected macroeconomic variables in the Rimini model¹⁵⁾ in order to be able to provide some information about developments in credit risk and loan losses in the future based on forecasts for key macroeconomic variables.

The risk-weighted debt can also be summed up across industries and regions. It is thereby possible to shed light on potential diversification gains by investing in different industries or regions. The model indicates that most of the main industries in Norway move in tandem with regard to credit risk (see Charts 2a and 2b).

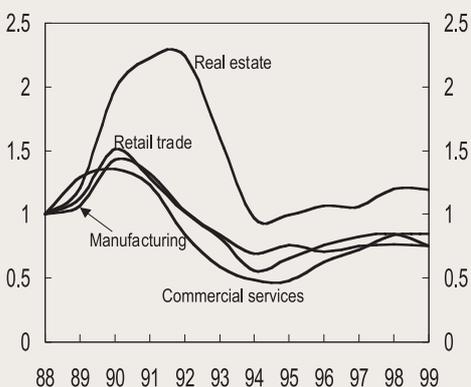
The model can also be used to study the movement between different risk groups over time. This allows us to provide more general information about developments in the enterprise sector. A cyclical upturn, for example, will be marked by net migration from the groups that are most exposed to risk to less exposed groups, and vice versa during a cyclical downturn.

Moreover, the model can be used for sensitivity analyses. By looking at various scenarios for the model's key variables, it is possible to indicate the factors required for credit risk to rise to a 'critical' level (for example, to the level just before the banking crisis). As it is very difficult to provide indications of developments in the future, these 'what if' analyses may make a useful contribution to the analysis of financial stability. We would also refer to Norges Bank's report Financial Stability 1/2001, in which the model is used in the assessment of credit risk for banks' exposure to the enterprise sector.

6 Summary

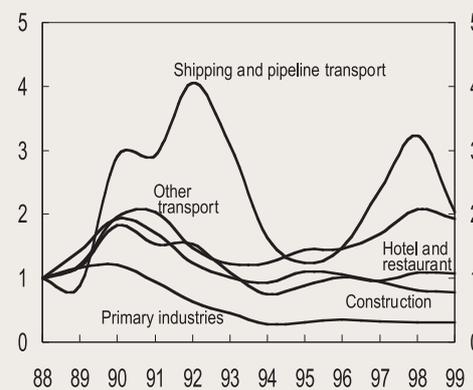
Norges Bank has developed a new quantitative model for analysing banks' credit risk in the enterprise sector. The new model predicts individual bankruptcy probabilities as a function of age, size, industry characteristics and accounts variables that can provide an indication of corporate earnings, liquidity and financial strength. The estimation results show that there is close accord between

Chart 2a Risk-weighted debt in selected industries. Index: 1988=1



Source: Norges Bank

Chart 2b Risk-weighted debt in selected industries. Index: 1988=1



Source: Norges Bank

¹⁵⁾ Norges Bank's macroeconomic model (see Eklund and Gulbrandsen, 2000).

predicted probabilities and actual bankruptcy frequencies. By aggregating individual bankruptcy probabilities, we obtain an indication of overall risk in the enterprise sector. The model also allows us to shed light on the level of banks' loan losses in the short run. The Bank is currently considering the possibility of linking the model's key variables to projections of selected macroeconomic variables. If this is successful, it will be possible to provide an indication of developments in credit risk and loan losses in the longer term.

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Annex:

The predicted bankruptcy probabilities can be expressed:

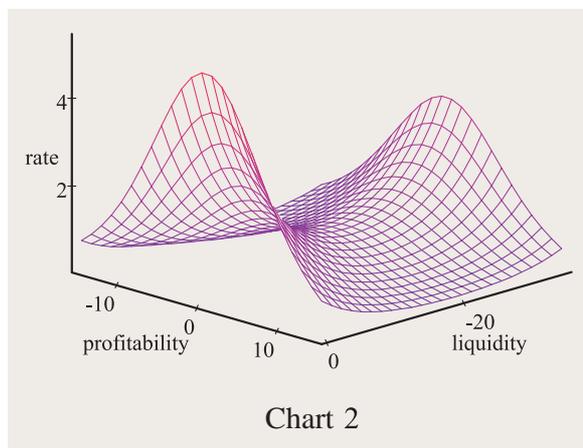
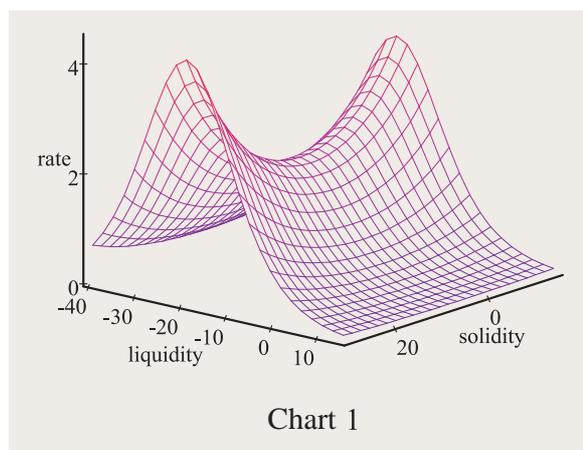
$$(1) \quad \hat{\beta} = \frac{1}{1 + e^{-\hat{v}}}$$

$$(2) \quad \hat{v} = \hat{\beta}_0 + \hat{\beta}_1 T_1(x_1) + \hat{\beta}_2 T_2(x_2) + \dots + \hat{\beta}_k T_k(x_k)$$

$$(3) \quad T_i(x_i) = \frac{1}{1 + e^{-\left(\frac{x_i - \alpha_i}{\delta_i}\right)}}$$

where the variables x_1, x_2, \dots, x_k are financial key figures calculated from the enterprises' annual accounts. The coefficient vector β and the variable-specific scaling parameters α and δ are estimated alternately by means of an iterative maximum likelihood procedure¹⁾. Given the scaling parameters, the structure results in a logit model in the transformed variables $T_i(x_i)$, $i = 1, 2, \dots, k$. If the equation (3) is replaced by $T_i(x_i) = x_i$, the compensation rate between the two variables x_i and x_j will be constant.

$$(4) \quad \frac{\partial x_i}{\partial x_j \mid dp=0} = -\frac{\beta_j}{\beta_i}$$



¹⁾ We have used $\alpha_i = 0$, $\delta_i = 1$, $i=1,2,\dots,k$ as initial values.

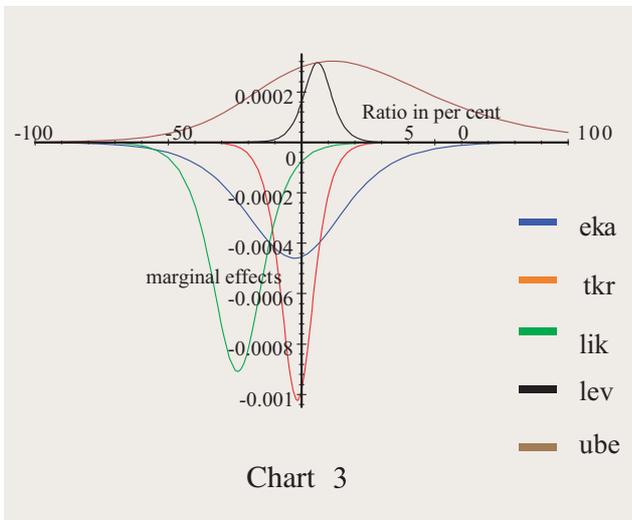


Chart 3

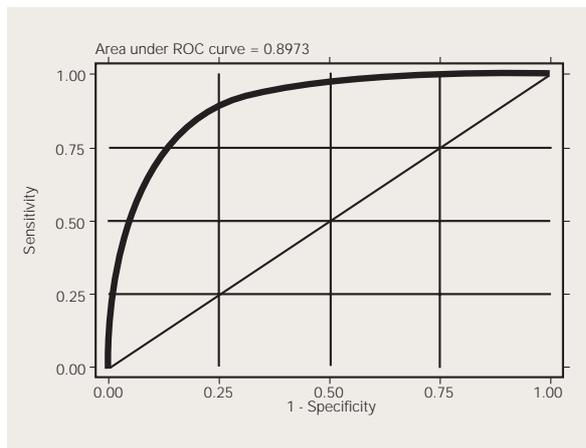


Chart 4: Ability to discriminate

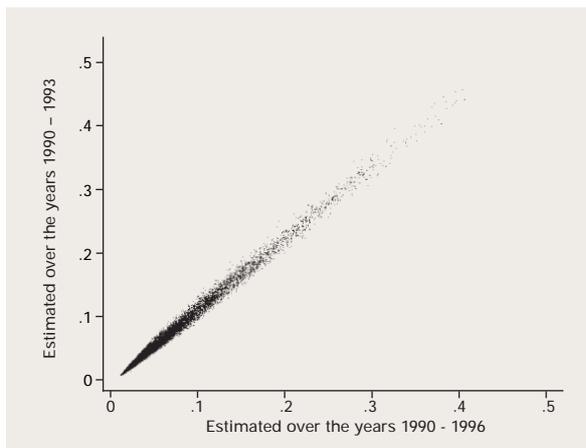


Chart 5: The stability of the model

The change required in variable x_i to maintain constant risk when variable x_j increases marginally is thus assumed to be independent of the levels of variables x_i and x_j . In the structure given by equations (1)-(3) the compensation rate will generally vary:

(5)

$$\frac{\partial x_i}{\partial x_j} \Big|_{dp=0} = -\frac{\beta_j T_j(x_j)(1-T_j(x_j))\delta_i}{\beta_i T_i(x_i)(1-T_i(x_i))\delta_j} \equiv -g_{ij}(x_i, x_j) \frac{\beta_j \delta_i}{\beta_i \delta_j}$$

Table 1. Model estimates

variable	β	s.e.	α/δ	s.e.	$1/\delta$	s.e.
eka	-1.4459	0.0604	0.4464	0.0977	0.0782	0.0049
tkr	-1.0948	0.0386	0.1216	0.1274	0.2096	0.0190
lik	-1.4925	0.0421	-2.9618	0.1977	0.1529	0.0087
lev	0.4968	0.0486	1.5224	0.4142	0.2895	0.0660
ube	6.8069	0.2019	-1.1474	0.0243	0.0362	0.0017
a1	0.8380	0.0438
a2	0.9707	0.0382
a3	0.8310	0.0398
a4	0.6729	0.0429
a5	0.5282	0.0468
a6	0.3189	0.0528
a7	0.2689	0.0575
a8	0.2076	0.0638
div	-1.0639	0.0742
taptek	0.5386	0.0419
size	-0.0543	0.0064
meanlev	1.0404	0.1692
meanek	-3.9690	0.2273
sdtkr	1.8229	0.3319
konstant	-7.0131	0.2786

The more x_i deviates from a_i and the less x_j deviates from a_j , the larger the function $g_y(x_i, x_j)$ will be. Otherwise the compensation rate between x_i and x_j is independent of all x_r , $r \neq i, j$. Charts 1 and 2 below show the compensation rates for liquidity/solidity and earnings/liquidity as estimated in the model.

If the parameter δ_i is large enough, $T(x_i)$ will be virtually linear over a given variation range for x_i . The model estimates are shown in Table 1. The variables are measured as percentages. The coefficient estimates cannot be considered in isolation from the scaling parameters. In particular, a high δ_i (high degree of linearity) will scale up β , all else being equal. Chart 3 shows partial simulations of the marginal effects of the variables. In each graph, all variables other than the plotted one are kept constant at their average values. The chart has been plotted for an enterprise three years old which has not paid a dividend in the current year and has not lost any equity since its formation.

The curve in Chart 4 provides a range of choices between the percentage of correct predictions for bankrupt firms and the percentage of incorrect predictions for non-bankrupt firms. The area under the curve is regarded as a measure of the model's ability to discriminate. This measure will lie between 0.5 and 1.

It is difficult to draw any definite conclusions about the model's stability over time. In Chart 5, two sets of predictions for 1996 are plotted against one another. The model is estimated on the basis of accounts up to and including 1993 and up to and including 1996. The predictions generally, and the ranking of enterprises in particular, do not appear to depend much on this broadening of the range of estimates. Because the bankruptcy data are more strongly influenced by time-specific registration errors than the annual accounts, however, it is an advantage to estimate the model over a number of years.