CAN THE PRICE OF CURRENCY OPTIONS PROVIDE AN INDICATION OF MARKET PERCEPTIONS OF THE UNCERTAINTY ATTACHED TO THE KRONE EXCHANGE RATE?

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Prices in the currency options market can provide an indication of market perceptions of the uncertainty attached to future exchange rates. We have used these option prices to calculate the probability distribution for the krone exchange rate against the Deutsche mark since 1 January 1998. Until August 1998, the market expected relatively low volatility in the krone exchange rate, and the probability of an appreciation or a depreciation of the krone was considered to be virtually the same. The krone depreciated during the autumn of 1998 and uncertainty surrounding future movements of the krone exchange rate increased substantially. At the same time, the prevailing view among market participants seemed to be that a considerable depreciation of the krone was more probable than a corresponding appreciation. This volatility subsided during the spring of 1999 to about the level prior to the depreciation of the krone in the autumn of 1998. The estimated probability distribution for the krone exchange rate at end-May was approximately equal to the corresponding distribution in July of last year, ie the market assessment of krone exchange rate uncertainty seems to be about the same as it was prior to the currency unrest in the autumn of 1998.

Introduction

Financial variables are often employed as an indicator of market expectations. Provided there are no risk premia associated with currency investments, the forward exchange rate is an indicator of market expectations concerning future exchange rates.[2] However, forward rates provide no information about the uncertainty of exchange rate movements. One method of obtaining information concerning uncertainty in the foreign exchange market is to measure the volatility of the exchange rate over a given period. This can be done in several ways: through simple calculations of the standard deviation of changes in the exchange rate or by estimations using advanced models.[3] One disadvantage in using such methods to measure uncertainty in the exchange market is that the historical volatility measured by such means differs from market expectations of future volatility.

A more direct measure of market expectations of future volatility can be obtained by using currency option prices. As shown in this article, it is possible to use market expectations to estimate the implied probability distribution for the future exchange rate. A central bank can make use of this information in many ways: first, such information may be useful in interpreting the evolution of the interest rate differential. The interest rate differential reflects both a risk premium and depreciation expectations. The risk premium depends on the degree of uncertainty attached to the exchange rate, which means that currency options can provide information about the size of the risk premium. Second, the options market can be a source of valuable information on the effect of any exchange market interventions aimed at reducing volatility. Third, the very shape of the probability distribution may provide information on the market assessment

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[2] A risk premium causes the forward rate to deviate from the expected exchange rate (see for example Lewis 1995).

[3] One such model is the GARCH model (Generalized AutoRegressive Conditional Heteroskadasticity), developed by Bollerslev (1986). There are other more complicated exchange rate models which take the form of a combination of a dynamic stochastic process and a process allowing for possible stochastic jumps in the exchange rate ("jump-diffusion" models). (cf Malz 1996). For GARCH estimation of NOK/DEM, see Froyn and Mundaca (1999).

of the probability of different outcomes. For example, "peso problems" can be more easily identified.[4]

This article begins by providing a brief review of the general theory of option prices, followed by a discussion of price determination for the most common type of currency options. Finally, we look at how currency options can be used to estimate the implied probability distribution for the exchange rate. The article is based on developments in the foreign exchange market from January 1998 to May 1999.

What determines the price of an option?

A call option is a contract that confers on one party the right, albeit not the obligation, to purchase an (underlying) asset at a fixed price – the strike price – at or before a designated future date. As payment for this right, a premium must be paid to the option writer. The option writer is under the obligation to sell the underlying asset if the buyer wishes to exercise his right to buy. A put option is a contract whereby the buyer of the option has the right, albeit not the obligation, to sell the underlying asset at a fixed price. The majority of options trading involves equities and bonds, but the market for options with foreign currency as the underlying asset is growing. In this article, we examine currency options.

It may be useful to examine a stylised example of a currency option: Suppose an investor purchases an option at a price of NOK 1.0, which gives the buyer the right to buy 10 euros at a price of NOK 8.30 per euro (the strike price) one month forward. Whether the option will be exercised or not depends on the relationship between the exchange rate at maturity and the strike price on the agreed date. If the krone exchange rate is NOK 8.50 after one month, the option will result in a profit for the buyer. The buyer of the option may then purchase 10 euros at the price of NOK 8.30 per euro and sell them for NOK 8.50 per euro. The profit is 10 x (8.50-8.30) – 1.0 (the option price) = NOK 1.0. On the other hand, if the krone exchange rate against the euro is NOK 8.10, it would not be profitable to exercise the option and it would be worthless at maturity. The loss is limited to the option purchase premium, ie NOK 1.0.

Many different models for valuing currency options have been developed.[5] A variant of the Black-Scholes model is the one most commonly employed. This model's exchange rate assumptions imply that the return on investments in a given currency has a log-normal distribution with a constant variance. In this model, which is described in Annex A, European-style currency options are determined by five factors:[6]

- The current spot rate
- The difference between domestic and foreign interest rates
- The maturity of the option
- The strike price of the option
- The volatility (standard deviation) of the underlying exchange rate

A higher volatility will – *ceteris paribus* – increase the value of the option. The reason for this is that higher volatility increases the probability that the option will be exercised – ie that it will be "in-the-money" – when the option expires.[7] This is shown in Chart 1 where it is assumed that one option is based on an underlying asset x which is less uncertain than underlying asset y. In both cases, the probability of the option resulting in a profit or of becoming worthless is 50 per cent. The potential loss for the buyer is equal to the option price, whereas there is no upper limit for gains. As shown in the chart, the probability that an option which is based on an asset with wide price variability (option y) will result in a considerable profit is

[4] If there is a given probability of a substantial change in the exchange rate in a particular direction, the mathematical expectation of a change will be significant even if most of the probability density is concentrated on small changes. The peso problem occurred during a period in the 1980s when the interest rate differential between the Mexican peso and the US dollar was substantial in spite of a stable peso/dollar exchange rate.

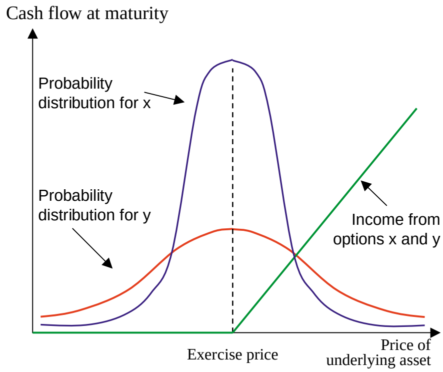
[5] See Brealey *et. al* (1996) for a good introduction to elementary options theory.

[6] European-style options can only be exercised at maturity. American-style options can be exercised at any time until they expire.

[7] In the currency options market, the convention is that the currency option is "at-the-money" when the current forward rate is equal to the strike price. If the forward rate is higher than the strike price at the time the contract is entered into, the option is said to be "in-the-money". If the current forward rate is lower than the strike price, the option is "out-of-the-money". The exchange rate is measured in the number of kroner per unit of foreign currency and, thus, a higher exchange rate implies a depreciation of the Norwegian krone.

greater than for the option for which the price of the underlying instrument is more certain (option x). The price of option y will be higher, reflecting

Chart 1. *Probability distribution and cash flow (at maturity) for two call options, based on underlying assets with differing volatilities*



the higher profit potential.

It is assumed that the two options are based on an underlying asset with the same price and same strike price. The options are assumed to be at-the-money.

Of the variables listed above, only volatility is not directly observable. With an estimate for volatility, the price of the option follows directly from the formula. Similarly, volatility can be calculated if the market price for the option is known. This calculated volatility, known as "implied volatility", is a key element of our discussion of the information content in option prices.[8]

The currency options market

Currency options, and derivatives in general, are traded both on stock exchanges and OTC (over-the-counter) markets. Derivatives traded on the stock exchange are standardised in respect of quality, quantity and terms of delivery, and are settled via a clearing house. Contracts traded in the OTC market are less standardised, and the terms of delivery are set according to the preferences of the contracting parties. Most international currency option trading occurs in the OTC market. Prices are quoted in terms of implied volatility. At the time of settlement, the estimated implied volatility is entered into the Black-Scholes formula to determine the option price. This does not necessarily imply that market participants agree with the assumptions of the Black-Scholes model. As will be shown in this article, there are many indications that the Black-Scholes model's assumption of log-normally distributed relative changes in the exchange rate is oversimplified. One advantage in quoting prices for options in terms of implied volatility is that it eliminates the need to change the currency option price, even if the exchange rate changes.

Turnover in the OTC market is estimated to be close to 50 times greater than for currency options on the stock exchange. According to the triennial international survey of foreign exchange and derivatives markets by the Bank for International Settlements (BIS), average turnover of currency options in the international OTC market amounted to USD 1 650bn in April 1998.[9] This market has more than doubled since April 1995.

In Norway, currency options are traded only in the OTC market. According to the survey for Norway, turnover in the Norwegian currency options market was approximately USD 1bn in April 1998, ie 0.6 per cent of total turnover in the Norwegian foreign exchange market[10], corresponding to an increase of about 20 per cent since April 1995.

The market for Norwegian currency options is relatively liquid in spite of its small size. Some market participants quote indicative prices on screen-based information systems – such as Reuters – so that investors can compare prices. In an efficient market the option price is determined by arbitrage considerations, so that supply and demand for options will not have any direct effect on option prices. However, imperfections such as transaction costs and non-continuous trading can cause the option price to deviate from the theoretically "correct" value.

In the OTC market for currency options, financial institutions quote prices for three products in particular: at-the-money options and two types of option combinations known as risk-reversals and strangles. The prices for these

[8] Formally, implied volatility is the market's estimate of the standard deviation for relative changes in the exchange rate.

[9] See BIS (1998).

[10] See Jacobsen (1999) for a more detailed overview of turnover in the Norwegian foreign exchange market.

products provide information about different characteristics of the probability distribution for future foreign exchange rates. The following section explains what these products are and the information that can be derived from their prices.

Extracting information from option prices

The Black-Scholes model assumes that relative price movements in the underlying currency are log-normally distributed and that the expected foreign exchange rate is determined by the forward rate. As a result, uncertainty is expressed in the standard deviation of future foreign exchange rates. In practice, however, it is evident that market participants do not agree with this assumption, and hence more information is needed to describe the characteristics of the implied probability distribution. Under certain circumstances (see Annex B) the prices for at-the-money implied volatility and strangles and risk-reversals, respectively, will describe the entire probability distribution for future exchange rates. The prices for strangles and risk-reversals will provide information on the implied probability distribution's deviation from the log-normal distribution. Before presenting the estimated implied probability distribution, we examine price movements for the three types of currency options in the period January 1998 - May 1999.

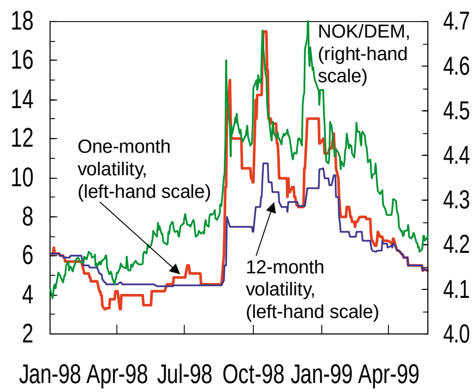
Implied volatility

Implied volatility is a measure of the extent to which the market expects the exchange rate to fluctuate, or more precisely, market participants' estimates of the standard deviation of relative exchange rate changes. Technically, implied volatility is arrived at by extracting it from the Black-Scholes model. As previously mentioned, however, prices for currency options in the OTC market are quoted directly in terms of implied volatility rather than as the option price, which reflects the clear relationship between the price of an option and its implied volatility.

The most common maturities for currency options in the OTC market are one week and one, two, three, six, nine and twelve months. One-week implied volatility reflects market uncertainty about the exchange rate one week forward. Similarly, twelve-month implied volatility expresses exchange rate uncertainty one year forward. Implied volatility is measured as annual standard deviation. By examining implied volatility for each maturity, it is possible to extract information on the uncertainty the market attaches to the underlying currency's movements over time. Implied forward volatility can be estimated in the same way as implied forward rates using the yield curve for bonds.

Chart 2 shows movements in one-month and twelve-month implied volatility respectively and in the krone exchange rate against the Deutsche mark. On 1 January 1999 the Deutsche mark was replaced by the euro. In the chart, the conversion rate between the Deutsche mark and the euro at 1 January 1999 is applied. The chart shows a sharp increase in one-month implied volatility at end-August 1998. Volatility was highest in the last half of October 1998. Twelve-month implied volatility also increased markedly, but much less than volatility for shorter maturities, indicating that the market assumed that one-month implied volatility would gradually decline.

Chart 2. . *Movements in one-month and twelve-month implied volatility and in NOK/DEM. A higher value denotes a weaker krone exchange rate*



Sources: Citibank and Norges Bank

The chart shows a clear correlation between the exchange rate and implied volatility, which may be due to the fact that the krone exchange rate is largely determined by the exchange rate risk market participants attach to their krone investments. However, it is also conceivable that there may be simultaneity between the exchange rate

Common options strategies

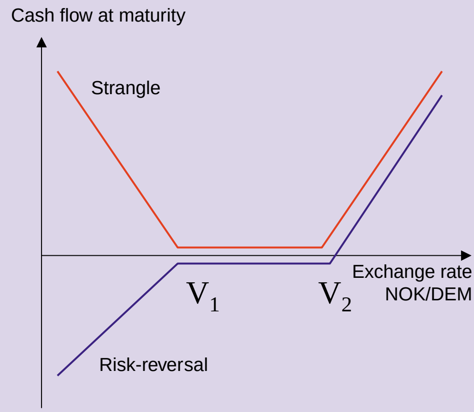
Two common options strategies in the currency options market are so-called strangle and risk-reversal options strategies. These are combinations of an out-of-the money call option and an out-of-the-money put option. These two combinations are traded both internationally and in the Norwegian OTC market. It is normal market convention to quote these prices with a "25 per cent delta". Formally, an option's "delta" expresses how much the price of the option changes as a result of changes in the price of the underlying asset (see Annex A). The delta is also often seen as an indication of how high the probability is that the option will be exercised at the time of maturity. A delta of 25 per cent indicates that there is a 25 per cent chance of the option being exercised.

Strangles

A strangle is a combination of an out-of-the-money put option and an out-of-the-money call option. This strategy involves the purchase of both options. In the chart below, we show how this option's cash flow depends on the exchange rate at the time of maturity. In practice, an investor who buys a strangle will profit from wide fluctuations in the future exchange rate. If the actual exchange rate at the time of the option's maturity is less than V1 or greater than V2, the holder of the strangle will make a profit from the option. If the exchange rate ends up within the range of these two points, the cash flow at the time of maturity will be zero. The price – and implied volatility – for this options strategy will thus reflect the risk of an extreme outcome relative to the market's forecast for future volatility. Statistically, the price of a strangle is linked to the level of kurtosis in the distribution. Relative to a log-normal distribution, a positive kurtosis implies a greater probability of relatively small outcomes as well as a greater probability of extreme outcomes. However, moderate outcomes are less likely (see definition in footnote 2 in Annex B).

In the options market a strangle is quoted as the difference between the call and put options' average volatility and at-the-money volatility. If the call option has a volatility of 6.9 per cent and the put option a volatility of 6.5 per cent, and the at-the-money option has a volatility of 6.3 per cent, the strangle will be quoted at 0.4 percentage point in implied volatility.

Chart 3. *Illustration of risk-reversals and strangles*



Risk-reversals

A risk-reversal is a combination of a purchase of an out-of-the-money call option and the sale of an out-of-the-money put option. The chart above shows the income derived from this combination as a function of the future exchange rate. As shown, an investor who purchases a risk-reversal will make money if the exchange rate is equal to or less than V2.

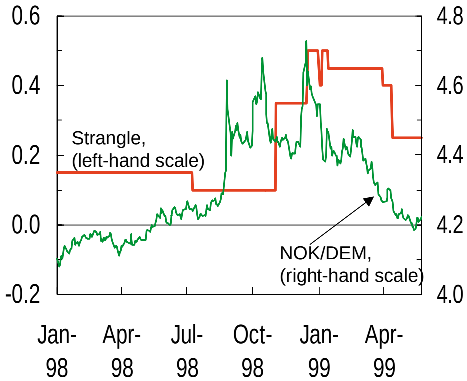
The price of a risk-reversal is set at the difference between the implied volatilities of the call and put options respectively. If the call option has an implied volatility of 6.7 per cent and the put option has an implied volatility of 6.3 per cent, the risk-reversal will be valued at 0.4 percentage point in implied volatility. If one assumes that it is more probable that the call option will be in-the-money rather than the put option, it will be more profitable to purchase a risk-reversal. In our analysis of the foreign exchange market, this implies that the investor believes that it is more likely that the exchange rate will weaken than strengthen. For this reason, a risk-reversal reflects market expectations of the direction of uncertainty regarding the future exchange rate. Statistically, a risk-reversal is an indicator of the degree of skewness in the distribution (see definition in footnote 2 in Annex B). A positive value means that there is a positive skewness in the probability distribution of the underlying asset, ie a greater probability density on the right side of the distribution.

and the risk premium, so that they have a mutual influence. Other factors may also affect both the exchange rate and volatility, entailing that the observed relationship between the exchange rate and implied volatility is spurious. In practice, it may be difficult to determine the source of the above-mentioned correlation. The nature of the causal relationship may, however, have implications for the optimal use of monetary policy instruments. If it is purely a matter of portfolio adjustments, then interventions may be justified. However, if fundamentals are influencing both the exchange rate and volatility, it may be preferable to use interest rates instead of interventions. Information on such factors requires econometric analyses, and do not fall within the scope of this article.

Strangles and risk-reversals – indicators of deviation from the normal distribution

Strangles and risk-reversals are two different combinations of currency options. Participants who believe that the exchange rate will fluctuate considerably will prefer to buy a strangle. Participants who believe a substantial weakening of the exchange rate is more likely than a substantial strengthening will be more interested in purchasing a risk-reversal.

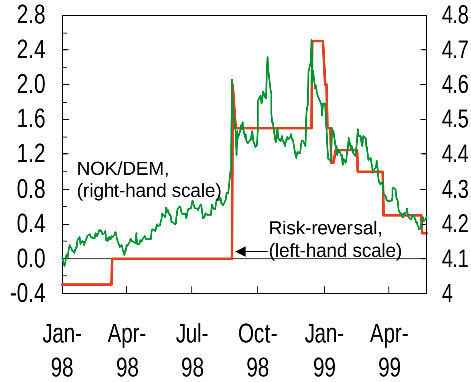
Chart 4. *Strangles and NOK/DEM. A higher value denotes a weaker krone exchange rate*



Sources: Citibank and Norges Bank

As shown in Chart 4, the strangle price rose sharply at the end of 1998, indicating that market participants assumed that wide fluctuations in the exchange rate were more likely than indicated by the Black-Scholes model (for a given standard

Chart 5. *Risk-reversals and NOK/DEM. A higher value denotes a weaker krone exchange rate*



Sources: Citibank and Norges Bank

deviation). The strangle price has dropped substantially recently, but remains slightly higher than the price in the period to November of last year.

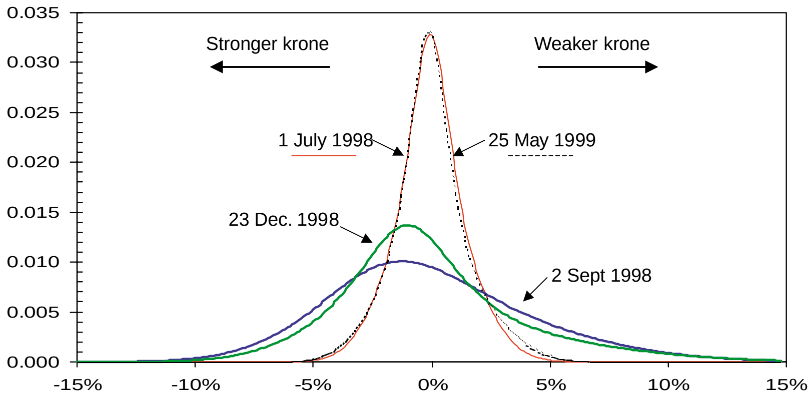
The correlation with the exchange rate seems particularly strong for the price for risk-reversals (see Chart 5). In the autumn of 1998, weak exchange rates tended to be seen as an indication that the probability of a considerable weakening of the krone was greater than for a corresponding appreciation. This also seemed to be the case in the UK and in Sweden during this period.[11] The price for risk-reversals fell sharply towards the end of the year, implying that market participants no longer believed that the probability distribution was significantly asymmetrical.

Estimating implied probability distributions

In the previous section, we showed that option prices for at-the-money volatility, strangles and risk-reversals reflect different aspects of market expectations of future exchange rate movements. However, it is often useful to have information about the probabilities market participants assign to various exchange rate outcomes. Breeden and Litzenberger (1978) have shown how option prices can be used to derive implied probabilities for various exchange rate outcomes, provided that participants are risk neutral. If it were possible to observe option prices with a continuum of different strike prices, it would also be

[11] See Cooper and Talbot (1999) and Aguilar and Hördahl (1999).

Chart 6. *Implied probability functions for NOK/DEM*



The horizontal axis measures changes in the strike price relative to the forward rate. A higher value denotes a depreciation of the krone, as an annualised percentage rate.

Sources: Citibank and Norges Bank

possible, in principle, to derive the entire implied probability distribution. As mentioned in the previous section, only three prices in the OTC market for currency options are quoted: at-the-money implied volatility, strangles and risk-reversals. Malz (1997) has developed a method for estimating the implied probability distribution on the basis of these three prices. The method is described in Annex B.

In order to estimate the implied probability distribution it is assumed that participants are risk neutral. In practice, this assumption hardly holds true. Even if participants were risk averse, there is still reason to believe that the shape of the probability distribution would not change substantially (see for example Rubinstein (1994)). However, the location of the distribution will depend on the degree of risk aversion and the size of the risk premium. For this reason, we have chosen to estimate the distribution over the relative deviation from the forward exchange rate instead of different exchange rate levels.

The implied probability distribution for the Norwegian krone against the Deutsche mark (the euro since 1 January 1999) is shown in Chart 6. The horizontal axis measures the deviation in per cent between the strike price and the forward rate. A value of 15 per cent means that the strike price one month forward will be 15 per cent weaker than the forward exchange rate at the contract date.[12] We illustrate developments on the basis of option prices on four different dates: 1 July 1998, 2 September 1998, 23 December 1998 and 25 May 1999. The chart shows that the probability distribution was relatively symmetrical at the beginning of July 1998, indicating that market participants did not expect any significant movements in the exchange rate in a particular direction. The distribution was also concentrated around the expected value, which indicates that market participants considered the uncertainty to be small. The area below the curve within the interval -0.5 and 0.5 contains most of the probability density, which indicates that market participants were almost certain that the exchange rate would not appreciate or depreciate by more than 5 per cent relative to the forward exchange rate. The first column of Table 1 shows the properties[13] of the distribution on that day. As shown, the expected standard deviation in the exchange rate that day was slightly over 5 per cent.

Table 1. *Estimated properties for NOK/DEM (NOK/EUR after 1 January 1999)*

	1 July 99	2 Sept. 99	23 Dec. 99	25 May 99
Standard deviation	0.051	0.150	0.135	0.056
Skewness	0.052	0.412	0.706	0.264
Kurtosis	0.563	0.183	1.013	0.969

Sources: Citibank and Norges Bank

12 The horisontal axis is formally defined by $\ln(\frac{X}{F_r})$, where FT represents the forward exchange rate at-the-money and X represents the strike price. The exchange rate is measured in NOK per unit of foreign currency, ie a higher value denotes a weaker krone exchange rate.

13 The properties are defined in more detail in Annex B. See also the box for an intuitive explanation of kurtosis.

From the beginning of July to September, the Norwegian krone weakened by 4.5 per cent against the Deutsche mark. Norges Bank raised its key rates by 3.5 percentage points in four steps during this period in order to limit exchange rate fluctuations. The depreciation of the krone was partly due to domestic factors, but international financial turbulence also played an important role. Past experience has shown that international investors shift their portfolios from minor currencies to major and presumably safer currencies during periods of unrest – referred to as a "flight to quality". This may explain why many investors reduced their holdings in the Norwegian market during the autumn of 1998. This probably resulted in a substantial increase in the uncertainty attached to future exchange rates. The standard deviation increased to 15 per cent (see Table 1). At the same time, there seemed to be a tendency among market participants to view a substantial weakening of the krone as more probable than a marked appreciation. As shown in the chart, the expected exchange rate's probability distribution at 2 September 1998 was positively skewed.

The krone exchange rate continued to weaken to end-1998. The uncertainty surrounding the future exchange rate abated somewhat, but the distribution for the expected exchange rate became more skewed than in September.

Implied volatility and skewness have declined considerably to end-May this year. The estimated probability distribution for the krone exchange rate at end-May is approximately equal to the corresponding distribution at 1 July last year, ie market participants interpret the uncertainty of the krone exchange rate to be approximately the same as it was prior to the period of currency unrest in autumn 1998.

In principle, it is possible to go beyond estimating market perceptions of exchange rate uncertainty. It is possible, for example, to analyse whether these perceptions express uncertainty related to the reaction pattern of the central bank or uncertainty related to external factors of importance to the exchange rate. In order to obtain more knowledge about this, it is necessary to analyse information on currency options together with other information, such as oil options and options for other financial assets.

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Annex A: The Black-Scholes model

The Black-Scholes model applied to currency options assumes that the exchange rate, S_t, follows a geometric Brownian motion given by:

$$dS_t = (R - R^*)S_t dt + \sigma S_t dB \tag{A.1}$$

where dt is the time change, dB is the growth in a standard Brownian motion ("random walk in continuous time"), R and R^* are domestic and foreign risk-free interest rates respectively, and σ denotes volatility (the standard deviation of the logarithm of S_t). Assuming no arbitrage opportunities in financial markets, the value of a European-style call option, given that the exchange rate follows the process in $(A.1)$, can be written as:

$$v(S_t, \tau, X, \sigma, R, R^*) \quad = \quad S_t e^{-R^*\tau}\Phi(d_1) - Xe^{-R\tau}\Phi(d_2) \tag{A.2}$$

where τ is the options's maturity, X the strike price, $\Phi(.)$ the cumulative log-normal distribution function and d_1 and d_2 are given by:

$$d_1 \equiv \frac{\ln(\frac{S_t}{X}) + (R - R^* + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}, \quad d_2 \equiv \frac{\ln(\frac{S_t}{X}) + (R - R^* - \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}$$

There is a unique correspondence between volatility, σ, and the value of an option, v, based on the Black-Scholes formula for given values of the other parameters in the formula. Thus, the market price of an option may be given either in units of volatility or currency. The price as a unit of volatility is called *implied volatility.*

The extent to which the currency option is either *in-the-money* or *out-of-the-money* is measured using the option's delta value, which expresses how the value of an option changes when the exchange rate changes. The delta function is obtained in the following way:

$$\delta_c(S_t, \tau, X, \sigma, R, R^*) \quad \equiv \quad \frac{\partial v(S_t, \tau, X, \sigma, R, R^*)}{\partial S_t} = e^{-R_t^*\tau}\Phi(d_1) \tag{A.3}$$

The delta value for a put option can be expressed using the delta value of a call option with the same maturity and strike price:

$$\delta_p(S_t, \tau, X, \sigma, R, R^*) = 1 - \delta_c(S_t, \tau, X, \sigma, R, R^*)$$

Currency options for a series of different strike prices X are not quoted in the OTC market, but the price can be derived implicitly from quotations for certain delta values. For example, the strike price for a 25-delta call option is found by defining $X^{25\delta}$ as the solution of $\{X : \delta_c(S_t, \tau, X, \sigma_t, R_t, R_t^*) = 0.25\}$. It can be shown that a 25-delta call option and a 25-delta put option have a strike price with the same relative distance to the current forward rate $F_{t,T}$, allowing us to write $X^{75\delta}/F_{t,T} = F_{t,T}/X^{25\delta}$. The delta value for an *at-the-money* call option is approximately 50% (or $\delta_c(S_t, \tau, X, \sigma, R, R^*) = 0.5$).

Annex B: Estimating the risk-neutral probability distribution

The price of currency options may contain information that can be used to determine the risk-neutral probability distribution of future exchange rates. The following is a technical description of the method developed by Malz (1997) for estimating the probability distribution of the exchange rate, S_T, at a future time, T, on the basis of the price of currency options at time t. The residual maturity, τ, of the options is given by $\tau = T - t$. The method is based on the existence of a well developed OTC market for standardised currency options which are either *at-the-money*, valued at the forward exchange rate $F_{t,T}$ on a contract with the same maturity, or can be combinations of two *out-of-the-money* options in the form of a *risk reversal* or a *strangle*. It will be seen that observations of the associated option prices (atm_t, rr_t, str_t) at time t can be used to estimate the risk-neutral probability distribution of future exchange rates $\pi(S_T)$.

The prices of the three options can be expressed as follows (Malz, 1997) as functions of the implied volatility $\sigma_t^{(\delta)}$, where δ measures the extent to which the option is *in-the-money*:

$$atm_t = \sigma_t^{(0,5)} \tag{B.4}$$
$$rr_t = \sigma_t^{(0,25)} - \sigma_t^{(0,75)} \tag{B.5}$$
$$str_t = \frac{\sigma_t^{(0,25)} + \sigma_t^{(0,75)}}{2} - atm_t \tag{B.6}$$

Generally speaking, $\delta = 0,5$ for an *at-the-money* option. If the volatility σ_t is independent of the delta value of the option (Black-Scholes), then $rr_t = 0$ and $str_t = 0$. From the equations above it follows that

$$\sigma_t^{(0,25)} = atm_t + str_t + 0,5 rr_t \tag{B.7}$$
$$\sigma_t^{(0,75)} = atm_t + str_t - 0,5 rr_t \tag{B.8}$$

From the delta function $\delta_c(S_t, \tau, X, \sigma_X(t, X, T), R, R^*) = \delta$ it follows that the implied volatility can be calculated as a function of the option's delta value, i.e. as $\sigma_t^{(\delta)} = \sigma_X(t, X_t^{(\delta)}, T)$, which follows from the implicit function

$$\delta = \delta_c(S_t, \tau, X_t^{(\delta)}, \sigma_t^{(\delta)}, R, R^*)$$

We can then use the expression for implied volatility as a function of the option's delta value, $\sigma_t^{(\delta)}$, to find the value of the option in the usual way as $c(t, X_t^{(\delta)}, T) = v(S_t, \tau, X_t^{(\delta)}, \sigma_t^{(\delta)}, R, R^*)$.

A key assumption in Malz (1997) is that the volatility $\sigma_t^{(\delta)}$ can be expressed by means of a second derivative Taylor approximation around the volatility of an option that is *at-the-money* at maturity ($\delta = 0,5$):

$$\sigma_t^{(\delta)}(\delta) = \beta_0 atm_t + \beta_1 rr_t(\delta - 0,5) + \beta_2 str_t(\delta - 0,5)^2 \tag{B.9}$$

It follows from (B.4)-(B.6) and (B.9) that the parameter vector $(\beta_0, \beta_1, \beta_2)$ is given by the values $(1, -2, 16)$. The relationship between the volatility and the delta value of the option in (B.9) is generally called the "volatility smile" in the literature (cf. the example in Chart B.1(a).

For currency options, we can simplify the expression for the delta function (A.3) in Annex A. We define the degree of the option's *in-the-moneyness* by means of the relative strike price, Q, measured relative to the forward rate, i.e. as $Q = X/F_{t,T}$, where $F_{t,T} = S_t e^{(R-R^*)\tau}$. To find the associated delta function, the following equation is used:

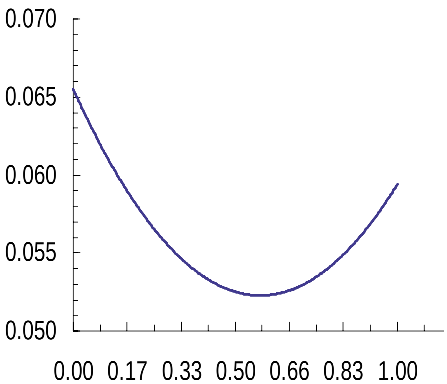
$$\delta_c(S_t,\tau,X,\sigma,R,R^*) = \frac{\partial v(S_t,\tau,X,\sigma,R,R^*)}{\partial S_t} = e^{(R-R^*)\tau}\frac{\partial v(F_{t,T},\tau,X,\sigma,R,R^*)}{\partial F_{t,T}} = \delta_v(Q,\tau,\sigma,R^*)$$

The fact that volatility can be expressed as a function of the relative strike price is also used, and $\sigma_Q(Q)$ is substituted for σ in the delta function:

$$\delta_v(Q,\tau,\sigma_Q(Q),R^*) = e^{-R^*\tau}\Phi\left(-\frac{\ln(Q) - \frac{\sigma_Q(Q,t)^2}{2}\tau}{\sigma_Q(Q,t)\sqrt{\tau}}\right) \qquad \text{(B.10)}$$

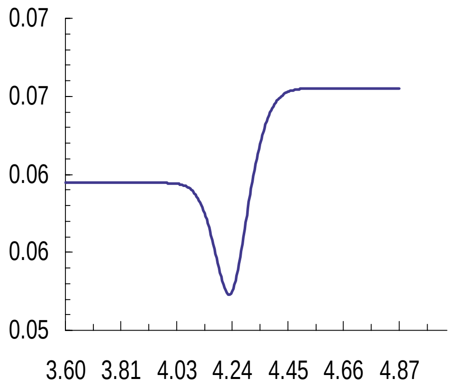
Chart B.1: *The"volatility smile" and the risk-neutral probability distribution*

(a) Implied volatility as a function of the delta value of an NOK/DEM currency option, 25 May 1999



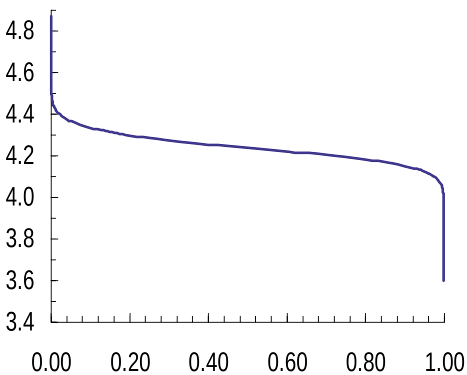
Sources: Citibank and Norges Bank

(b) Implied volatility as a function of the strike price of an NOK/DEM currency option 25 May 1999



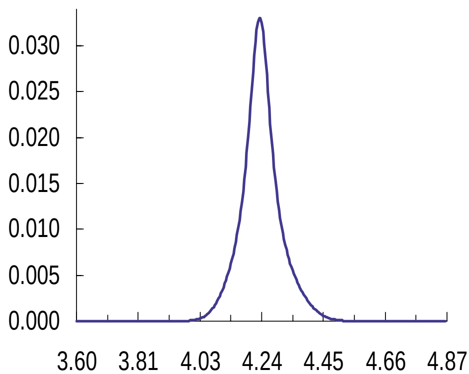
Sources: Citibank and Norges Bank

(c) Estimated relationship between the strike price and the delta value of an NOK/DEM currency option, 25 May 1999



Sources: Citibank and Norges Bank

(d) The risk-neutral probability distribution of the NOK/DEM exchange rate as a function of estimated strike prices, 25 May 1999



Sources: Citibank and Norges Bank

Finally, the expression for $\delta_v(Q, \tau, \sigma_Q(Q,t), R^*)$ is inserted into the volatility function (B.9) and it is clear that implied volatility $\sigma_Q(Q,t)$ can be determined as a function of the degree of the option's *in-the-moneyness*, Q, from this equation once we know the option prices (atm_t, rr_t, str_t).

$$\begin{aligned}\sigma_Q(Q,t) &= atm_t - 2rr_t\left[e^{-R^*\tau}\Phi\left(-\frac{\ln(Q)-\frac{\sigma_Q(Q,t)^2}{2}\tau}{\sigma_Q(Q,t)\sqrt{\tau}}\right)-0,5\right] \\ &\quad +16str_t\left[e^{-R^*\tau}\Phi\left(-\frac{\ln(Q)-\frac{\sigma_Q(Q,t)^2}{2}\tau}{\sigma_Q(Q,t)\sqrt{\tau}}\right)-0,5\right]^2\end{aligned} \tag{B.11}$$

In practice we take as our starting point a suitable sequence of relative strike prices Q for options with varying degree of *in-the-moneyness* (e.g. $Q \in [0,85,\ldots,1,15]$), and calculate implied volatility $\hat{\sigma}_Q(Q,t)$ from (B.11) by numerical methods. In the present example, the Gauss[1] OPTMUM optimisation routine is used. See Chart B.1(b), where volatility $\hat{\sigma}_Q(Q,t)$ is plotted as a function of the strike price X (instead of Q), centred around the *at-the-money* option with a strike price equal to the forward exchange rate $F_{t,T}$.

The values calculated for implied volatility, $\hat{\sigma}_Q(Q,t)$, are then inserted into the expression for the option value $v(Q,\tau,\hat{\sigma}_Q(Q,t))$ (see (B.12) below).

$$\begin{aligned}\hat{v}(Q,t) &= v(Q,\tau,\hat{\sigma}_Q(Q,t)) \\ &= \frac{e^{R\tau}}{F_{t,T}}v(S_t,\tau,X,\hat{\sigma}_X(X,t,T),R,R^*) \\ &= \Phi\left(-\frac{\ln(Q)-\frac{\hat{\sigma}_Q(Q,t)^2}{2}\tau}{\hat{\sigma}_Q(Q,t)\sqrt{\tau}}\right) - Q\Phi\left(-\frac{\ln(Q)+\frac{\hat{\sigma}_Q(Q,t)^2}{2}\tau}{\hat{\sigma}_Q(Q,t)\sqrt{\tau}}\right)\end{aligned} \tag{B.12}$$

Finally, the cumulative distribution function and the associated density function of the risk-neutral probability distribution of future exchange rates, $\hat{\Pi}(Q,t)$ and $\hat{\pi}(Q,t)$[2], are calculated from (B.13) and (B.14) (see Chart B.1(d)).

$$\hat{\Pi}(Q,t) = 1+\frac{\partial\hat{v}(Q,t)}{\partial Q} = 1+\frac{\partial v(Q,\tau,\sigma)}{\partial\sigma}\frac{\partial\hat{\sigma}_Q(Q,t)}{\partial Q}+\frac{\partial v(Q,\tau,\sigma)}{\partial Q} \tag{B.13}$$

$$\hat{\pi}(Q,t) = \frac{\partial^2\hat{v}(Q,t)}{\partial Q^2} \tag{B.14}$$

[1] Gauss was developed in the US, and is marketed by Aptech Systems Inc. in Seattle Wa.

[2] The risk-neutral expectation is defined by the forward exchange rate $F_{t,T}$. The rth order central moment of the probability distribution around $F_{t,T}$ can generally be expressed as

$$\mu_t^{(r)} = \int_{-\infty}^{\infty}(X-F_{t,T})^r\pi(X)dX$$

This yields the following expression for the moments of the risk-neutral probability distribution (Malz, 1997). The standard deviation of the distribution is defined by $\sigma_t = \sqrt{\mu_t^{(2)}}$. Calculated per annum, we find the standard deviation $\sigma_t^{\text{pa}} = \sqrt{\frac{\mu_t^{(2)}}{\tau}}$. The skewness is $sk = \frac{\mu_t^{(3)}}{[\mu_t^{(2)}]^{\frac{3}{2}}}$ and excess kurtosis (over and above that occurring with a normal distribution) is defined by $ek = \frac{\mu_t^{(4)}}{[\mu_t^{(2)}]^2} - 3$.