

# Financial Frictions, the Financial Immoderation, and the Great Moderation\*

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## Abstract

Balance-sheet variables of firms have been characterized by greater volatility since the early 1970s. This Financial Immoderation has coexisted with the so-called Great Moderation, which refers to the slowdown in volatility of real and nominal variables since the mid 1980s. In this paper, we examine the divergent patterns in volatility by considering the role played by financial factors. To do so, we use a DSGE model including real, nominal, and financial frictions. We estimate the model allowing for structural breaks in the volatilities of shocks, the monetary policy coefficients, and the average level of financial rigidities. We conclude that (i) the Financial Immoderation is driven by larger financial shocks, (ii) the estimated reduction in the mid 1980s of the average level of financial accounts for the observed decline in investment volatility, (iii) the main drivers of investment volatility are financial and price shocks with technology shocks playing a small role, and (iv) the propagation mechanism of financial shocks has changed significantly since 1984.

**Keywords:** Great Moderation, Financial Immoderation, financial frictions, financial shocks, structural break, Bayesian methods

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# 1 Introduction

The U.S. economy over the 1954-2006 period has been characterized by two empirical regularities. On the one hand, since the mid 1980s, fluctuations at business cycle frequencies for real and nominal variables are milder. This decline in macroeconomic volatility defines the so-called *Great Moderation*. On the other hand, financial variables have become more volatile over time. Jermann and Quadrini (2008) document an increase in the volatility of debt and equity financing in the nonfarm business sector contemporaneous with the slowdown in the amplitude of the real cycle. In this paper, we reconsider the study of the balance-sheet data for the nonfarm business sector along with other financial variables, such as balance-sheet data for households, net private savings, and demand deposits at commercial banks. We document that the widening of the financial cycle starts in 1970. We label this second empirical regularity the *Financial Immoderation*.

We account for those divergent patterns in volatility by means of a structural model. We consider a model featuring a standard set of real and nominal frictions as in Smets and Wouters (2007) extended to accommodate financial rigidities as in Bernanke, Gertler, and Gilchrist (1999). We enrich the theoretical environment by including financial shocks affecting the spillovers of credit market imperfections on the economy. This theoretical framework allows us to quantify the relative role played by financial factors, monetary policy, and economic shocks in shaping the evolution of aggregate volatility. To do so, we estimate our model using a data set containing real, nominal, and financial variables. To account for the breaks in the second moments of the data, we allow for structural breaks in the average level of financial rigidity, coefficients in the monetary policy rule, and the size of shocks. As a byproduct of our analysis, we can not only characterize the propagation mechanism of financial shocks in the US economy, but also study its evolution over the last 50 years.

One of the main objectives of this paper is to quantify the relative role played by financial factors in shaping macroeconomic and financial volatilities. However, the workhorse dynamic stochastic general equilibrium (DSGE) model used in the literature abstracts from interactions between credit markets and the rest of the economy. This benchmark macroeconomic model is based on the capital structure irrelevance theorem by Modigliani and Miller (1958); that is, the composition of agents' balance sheets has no effect on their optimal decisions. Nevertheless, episodes such as the Great Depression or the current financial turmoil stand as compelling evidence of the linkage between the developments in the financial and real sectors. Along these lines, recent contributions to the literature have focused on incorporating credit markets in the workhorse DSGE model. For example, Bernanke, Gertler, and Gilchrist (1999) and Iacoviello (2005) stress the relevance of the balance sheet's condition in determining economic activity. The ability to borrow depends upon borrowers' wealth, which ultimately affects the demand for capital and the level of economic activity they can engage in.

Following Christiano, Motto, and Rostagno (2003), we consider a theoretical framework with real and nominal rigidities as in Smets and Wouters (2007) enriched with frictions in the credit market à la Bernanke, Gertler, and Gilchrist (1999). In this environment, asymmetric information between borrowers and lenders arises because the return to capital depends not only on aggregate but also on idiosyncratic risk. While borrowers freely observe the realization of their idiosyncratic productivity shock, lenders must pay monitoring costs to observe the realized return of a borrower. To minimize monitoring costs, lenders audit borrowers only when they report their inability to pay the loan back under the terms of the contract. In order to be compensated for the risk of default, lenders extend loans at a premium over the risk-free interest rate. The composition of borrowers' balance sheets determines the external finance premium at which the loan is settled. The lower an entrepreneur's net worth (collateral) with respect to her financing needs, the higher the premium required in equilibrium. The external finance premium is at the heart of the mechanics operating in the financial accelerator emphasized by Bernanke, Gertler, and Gilchrist (1999). The financial accelerator hypothesis states that credit market imperfections amplify and propagate economic shocks. For example, in an economic downturn, borrowers' wealth deteriorates because of the decline in asset prices. Such a reduction in the value of collateral translates into a higher premium requested by lenders. Relatively more expensive credit reduces the incentives to engage in investment activities, depressing output production even further. The latter generates an additional drop in asset prices, which feeds the chain again.

In a model à la Bernanke, Gertler, and Gilchrist (1999), the external finance premium is driven by two channels: the *balance-sheet channel* and the *information channel*. The *balance-sheet channel* captures the dependence of external financing opportunities on the composition of firms' balance sheets. The *information channel* implies that the external finance premium is a positive function of the severity of the agency problem. We enrich the DSGE model by introducing financial shocks affecting those two channels. Exogenous shocks to the *balance-sheet channel* are introduced in the form of wealth shocks. Shocks to the *information channel* are modeled as innovations affecting the parameter governing agency costs. In this paper, we study the relative role played by those two shocks in shaping the evolution of aggregate volatility. We also analyze the propagation mechanism of the two financial shocks in the US economy.

We estimate the model economy using Bayesian techniques on a standard data set of real and nominal variables extended to include a series for firms' net worth. We need to take a stand on defining the empirical equivalent to such a model variable. We focus on the data provided by the Flow of Funds Accounts to define net worth as tangible assets minus credit market liabilities for the nonfarm business sector, measured in real per capita terms. As we have stated above, we perform the estimation exercise using the whole data sample, but we allow for structural breaks in the variances of the shocks, the coefficients in the monetary policy rule, and the average size of the financial accelerator. Therefore, we consider three explanations for the *Financial Immoderation*

and the *Great Moderation*: changes in the size of shocks, changes in the conduct of monetary policy, and changes in the US financial system.

The main empirical findings of the paper are the following. Financial factors play a significant role in shaping financial and macroeconomic volatilities. Financial shocks are the main driver of the variance of financial flows. Therefore, the increase in fluctuations at business cycle frequencies for balance-sheet variables is driven by larger financial shocks hitting the US economy. The relevance of financial shocks in accounting for investment volatility is about 38% throughout the sample period which implies that financial shocks are the main driver of the variance of investment. Investment specific technology shocks, however, play no role in the variability of investment which allows us to conclude that failing to include financial shocks in the model economy results in an overstatement of the relative role played by the I-shock. Financial shocks are also an important driver of the business cycle fluctuations of nominal variables accounting for up to 65% of the volatility of the nominal interest rate in the 1970s. increases over time.

We also find that the average level of financial rigidities has been decreasing over time. While in the 1970s the estimated reduction is over 40%, in the mid 1980s the decline exceeds 90%. This latter reduction overestimates the model implied slowdown in investment but exactly deliver the observed moderation. The easier access to credit during the Great Moderation accounts for 78% of the model implied reduction in the volatility of the nominal interest rate and 35% of that in the variability of inflation. The reduction of financial rigidity has important implications for the propagation mechanism of financial shocks. On the one hand, a smaller financial accelerator induces more muted responses to financial innovations; that is, the amplification mechanism linked to imperfections in the credit market gets reduced. On the other hand, a reduction in financial frictions enhances the persistence of the responses to financial shocks in the US economy.

This paper relates to two strands of the empirical macro literature. The first strand addresses the study of the Great Moderation, that is, the evolution of volatilities at business cycle frequencies during the second half of the last century. The second strand considers the estimation of the financial accelerator model.

Since Kim and Nelson (1999) and McConnell and Pérez-Quirós (2000) dated the start of the Great Moderation, there has been a growing literature on dissecting the possible sources of such a mildness in real business cycle fluctuations. Recent contributions have focused on analyzing the link between financial innovations and aggregate volatility. Our paper is along the lines of Jermann and Quadrini (2008) and deBlas (2009), who consider credit market frictions only for firms. In particular, we obtain an estimated reduction in the average level of financial rigidities during the Great Moderation similar to the ones provided by those two papers.

The literature on bringing the financial accelerator by Bernanke, Gertler, and Gilchrist (1999) to the data through an estimation exercise is less vast than the literature on the Great Moderation.

Most of the contributions estimate the theoretical environment using only nominal and real variables and focusing on data from the Volcker-Greenspan era. To the best of my knowledge, besides the study of the Great Depression by Christiano, Motto, and Rostagno (2003), the only reference using pre-1980s data is the recent work by Gilchrist, Ortiz, and Zakrajšek (2009), whose sample spans 1973 to 2008. They do not address, however, the break in second moments of the data observed in the mid 1980s.

The plan of the paper is as follows. Section 2 presents the empirical evidence that motivates the paper. We describe the model in Section 3. Section 4 discusses the choice of parameters allowed to change over time. We describe the estimation procedure and report the estimation results in Section 5. Section 6 analyzes the drivers of the divergent patterns in volatility. In Section 7, we study the relative importance of each shock and the propagation of financial shocks. Section 8 concludes.

## 2 Empirical Motivation

This section presents the empirical evidence that motivates the paper. It characterizes real, nominal, and financial cycles over the period 1954-2006. We do not consider more recent data for reasons of data accuracy. Revisions of NIPA data within a year of publication and of Flow of Funds Accounts within two or three years of publication are often considerable. In addition, at the end of the sample it is difficult to distinguish trend breaks from cycles.

We set the empirical characterization considering two structural breaks in the data: 1970 and 1984. Let us start by motivating the choice of 1984. Since the contributions by Kim and Nelson (1999) and McConnell and Pérez-Quirós (2000), there has been a consensus in the empirical macro literature about the existence of a break in the second moments characterizing real and nominal cycles around 1984. Stock and Watson (2002) popularized it as the starting point of the Great Moderation.

The choice of the break in 1970 is based on several observations. First, analyzing the evolution of the cyclical component of balance-sheet variables such as the debt-to-net-worth ratio reported in Figure B-1, we conclude that the cycle becomes wider in the 1970s. Moreover, both inflation and the federal funds rate are more volatile in the 1970s and early 1980s. The high and volatile inflation over the period has been the subject of careful study by researchers such as Christiano and Gust (1999), Nelson (2005), and Collard and Dellas (2007), among others.

Second, the 1970s are convulsive years in US economic history. There were significant changes not only in the financial system but also in other areas of the economic system. In the financial arena, the 1970s was the decade of the introduction of ATMs, phone transfers for savings balances at commercial banks, NOW (negotiable order of withdraw) accounts, money market certificates

with yields tied to US Treasury securities, IRAs (individual retirement accounts), MMMF (market money mutual funds), incorporation of the NYSE, a partial lifting of Regulation Q, the Securities Protection Act, the Financial Institutions Regulatory and Interest Control Act, the Electronic Fund Transfers Act, the International Banking Act, the Bankruptcy Reform Act, etc. At the same time, the US experienced the collapse of the Bretton Woods currency-exchange mechanism, the appointment of Burns as chairman of the Federal Reserve System after 19 years of Martin, the end of the Vietnam war, the oil crises, the stagflation episode, several government bailouts of the automobile and aviation industries, and the start of the service economy. Therefore, testing for a break at the beginning of the 1970s seems a natural candidate.

Tables A-1 and A-2 report Chow tests on the average squared residuals of regressing an AR(1) with drift for the definition of the variables of interest used in the estimation exercise and their cyclical component, respectively. We reject the null of parameter constancy when testing for a break in 1970 for net worth, inflation, and the federal funds rate for both definitions of the variables. We reject the null for all variables except labor share using the two definitions at hand when the break is set in 1984. Finally, we also reject the null for all variables except labor share when considering the two breaks jointly. In particular, the log-likelihood ratio statistic is larger for this scenario than when considering single breaks. Therefore, we can conclude that the data are best represented by a scenario that allows for two breaks in second moments.

We report in Table A-3 the ratio of standard deviations of the cyclical component for a set of real, nominal, and financial variables. Although the focus of our paper is on financial variables related to the nonfarm business sector, we analyze here a broader data set, including net worth of households, net private savings, and demand deposits at commercial banks. Following Jermann and Quadrini (2008), we report in the first column of Table A-3 the ratio of cyclical standard deviations when only a break in 1984 is considered. All the variables included in our data set deliver the patterns described by Jermann and Quadrini (2008); that is, there is a contemporaneous moderation in the real side of the economy and an exacerbation in the volatility of financial variables. The magnitude of the changes is also along the lines of the results provided by those authors. The novelty of our analysis is the consideration of two breakpoints. The second and third columns of Table A-3 report the relevant statistics to characterize the three subperiods of interest: 1954-1969, 1970-1983, 1984-2006. Therefore, in the remainder of this section we focus our discussion on analyzing the information provided by the last two columns of the table.

Let us start by comparing the standard deviation of the cyclical component in the 1970-1983 sample period with that of the 1954-1969 era. The volatility of real variables is, on average, 50% greater in the 1970s and early 1980s than in the pre-1970 period. Nominal variables are also more volatile in the 1970-1983 sample period, but the increase in their cyclical volatility is greater than the one observed for real variables. In particular, the standard deviation of the cyclical component of both inflation and nominal interest rates more than doubles in the 1970s and early 1980s with

respect to the 1950s and 1960s. Finally, all financial variables are also more volatile over the second sample period. The more dramatic change is the one experienced by demand deposits at commercial banks whose variability triples in the 1970-1983 sample period.

In the last column of Table A-3, we compare the standard deviations of the cyclical components for the post-1984 period with that of the 1970-1983 sample period. The volatility of consumption, investment, and output decreases by about 55%. This result is what characterizes the Great Moderation *per se*. The slowdown in the cyclical variability of hours and labor share is milder. Nominal variables, also in this case, follow the pattern of change of real variables. Financial variables, however, are more volatile in the 1984-2006 sample period. The most significant increases in cyclical variability are the ones for net worth for the nonfarm business sector and net private savings. Both of them are 45% more volatile in the Great Moderation era than in the Great Inflation period (1970-1983). Therefore, we can state that the post-1984 period is characterized by an additional increase in the volatility of financial variables at business cycle frequencies.

We can summarize the empirical regularities present in the US aggregate data over the 1954-2006 period as follows. The first subperiod, 1954-1969, is characterized by relatively stable inflation and interest rates. The 1970-1983 sample period constitutes the first stage of the Financial Immoderation. In this period, fluctuations at business cycle frequencies of real, nominal, and financial variables become wider. The last subperiod expands from 1984 to the end of the sample. It is characterized by the coexistence of the second stage of the Financial Immoderation and the Great Moderation.

### 3 The Model

Our theoretical framework features real and nominal rigidities as in Smets and Wouters (2007) and Christiano, Eichenbaum, and Evans (2005a). However, to assess the role played by financial frictions in the evolution of volatilities in the US economy, we extend the framework including financial rigidities as in Bernanke, Gertler, and Gilchrist (1999). Financial frictions arise because there is asymmetric information between borrowers and lenders. Following Townsend's (1979)'s costly state verification framework, we assume that while borrowers freely observe the realization of their idiosyncratic risk, lenders must pay monitoring costs to observe an individual borrower's realized return.

Since Christiano, Motto, and Rostagno (2003) integrated the financial accelerator mechanism of Bernanke, Gertler, and Gilchrist (1999) in the workhorse DSGE model, several studies have focused on assessing the empirical relevance of the financial accelerator by comparing the model fit with that of the workhorse DSGE model or on studying the propagation of real and nominal shocks. In this paper, we focus the analysis on two issues: the role of financial shocks and the

model's potential to account for breaks in the second moments of the data. We incorporate in the theoretical framework a shock to firms' wealth and a shock to agency costs. While the former has been previously studied, the inclusion of the latter is a major novelty of this paper.

Our model economy is populated by households, financial intermediaries, entrepreneurs, capital producers, intermediate good firms, retailers, labor packers, and government. Entrepreneurs are the only agents able to transform physical capital into capital services to be used in production. They purchase capital from capital producers and rent it to intermediate goods firms. Capital acquisition can be finance using internal financing and external borrowing. Financial intermediaries capture funds from households in the form of deposits and lend them to entrepreneurs. Intermediate goods firms carry out production by combining capital and labor services. Retailers generate the final good of this economy by combining intermediate goods. The government conducts both fiscal and monetary policy. In order to have non-neutrality of monetary policy, we need to include a nominal rigidity in a monopolistically competitive sector. Assuming entrepreneurs have market power would make it more difficult to solve for the debt contract. Hence, we introduce sticky prices in the intermediate good sector instead.

### 3.1 Retailers

The retail sector is populated by infinitely lived and perfectly competitive firms producing final goods,  $Y_t$ , by combining a continuum of intermediate goods,  $Y_t(s)$ . Final goods can be used for consumption and investment. Intermediate goods are transformed into final goods by means of a Dixit and Stiglitz (1977) aggregator.

$$Y_t = \left[ \int_0^1 (Y_t(s))^{\frac{1}{1+\lambda_t^p}} \right]^{1+\lambda_t^p} \quad (1)$$

where  $\lambda_t^p$  is the markup shock and  $\frac{1+\lambda_t^p}{\lambda_t^p}$  measures the elasticity of substitution between differentiated intermediate goods. We assume that the markup evolves as follows

$$\ln(\lambda_t^p) = (1 - \rho_{\lambda^p}) \ln(\lambda_\star^p) + \rho_{\lambda^p} \ln(\lambda_{t-1}^p) + \varepsilon_{\lambda^p,t} \quad (2)$$

where  $\varepsilon_{\lambda^p,t} \sim \mathcal{N}(0, \sigma_{\lambda^p})$  and  $\lambda_\star^p$  stands for the value of the markup at the steady state.

Final goods firms take the prices of intermediate goods as given and choose  $Y_t(s)$  to minimize costs, given by  $\int_0^1 P_t(s)Y_t(s)ds$  subject to the Dixit-Stiglitz aggregator. From the first-order condition, we have that the demand function for the  $s^{th}$  intermediate good is given by

$$Y_t(s) = \left[ \frac{P_t}{P_t(s)} \right]^{\frac{1+\lambda_t^p}{\lambda_t^p}} Y_t \quad (3)$$



Integrating the above and imposing the zero-profit condition, we obtain the following expression for the aggregate price index

$$P_t = \left[ \int_0^1 P_t(s)^{-1/\lambda_t^p} ds \right]^{-\lambda_t^p} \quad (4)$$

### 3.2 Intermediate goods sector

There is a continuum of infinitely lived producers of intermediate goods, indexed by  $s \in [0, 1]$ , operating under monopolistic competition. They produce intermediate inputs,  $Y_t(s)$ , combining labor,  $H_t$ , and capital services,  $k_t$ , using a Cobb-Douglas technology. Labor services are obtained from households and capital services from entrepreneurs.

$$Y_t(s) = [Z_{a,t} H_t(s)]^{1-\alpha} k_t(s)^\alpha - Z_{a,t} \quad (5)$$

where  $Z_{a,t}$  stands for the neutral technology shock. We assume that  $Z_{a,t}$  is such that

$$Z_t \equiv \log(\Delta Z_{a,t}) = (1 - \rho_z) \Upsilon_z + \rho_z Z_{t-1} + \varepsilon_{Z,t}, \quad \text{with} \quad \varepsilon_{Z,t} \sim \mathcal{N}(0, \sigma_Z) \quad (6)$$

Thus, we assume that the growth rate of the neutral technological progress follows an AR(1) process where  $\Upsilon_z$  is the average growth rate of the economy.

Intermediate goods producers solve a two-stage problem. First, they decide on the demand schedule for labor and capital services by minimizing total costs subject to (5). the optimal capital-to-labor ratio is given by

$$\frac{k_t(s)}{H_t(s)} = \frac{\alpha}{1 - \alpha} \frac{W_t/P_t}{r_t^k}$$

where  $r_t^k$  is the rental rate of capital. The real marginal cost can be expressed as follows

$$\chi_t(s) = \left( \frac{\alpha}{1 - \alpha} \right)^{1-\alpha} \left( \frac{1}{\alpha} \right)^\alpha \left( \frac{W_t/P_t}{Z_{a,t}} \right)^{1-\alpha} \left( r_t^k \right)^\alpha$$

Given that both the optimal capital-to-labor ratio and the real marginal cost depend only on market prices, common parameters across intermediate producers, and the economy-wide neutral technology shock, we conclude that those two variables are identical for all producers. Hence, we can proceed by assuming a representative agent in the sector.

In the second stage, intermediate goods producers face a pricing problem in a sticky price framework à la Calvo. At any given period, a producer is allowed to reoptimize her price with probability  $(1 - \xi_p)$ . We assume that those firms that do not reoptimize their prices set them using the following indexation rule

$$P_t(i) = P_{t-1}(i) \pi_{t-1}^{\iota_p} \pi_\star^{1-\iota_p} \quad (7)$$

When reoptimization is possible, an intermediate firm  $i$  will set the price  $\tilde{P}_t$  that maximizes the expected value of the firm

$$\max_{\tilde{P}_t(i)} \Lambda_t \left[ \tilde{P}_t - \chi_t \right] Y_t(i) + \mathbb{E}_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \Lambda_{t+s} \left[ \tilde{P}_t \left( \prod_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_{\star}^{1-\iota_p} \right) - \chi_{t+s} \right] Y_{t+s}(i) \quad (8)$$

subject to

$$Y_{t+s}(i) = \left[ \frac{\tilde{P}_t \left( \prod_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_{\star}^{1-\iota_p} \right)}{P_{t+s}} \right]^{-\frac{1+\lambda_{t+s}^p}{\lambda_{t+s}^p}} Y_{t+s} \quad (9)$$

where  $\Lambda_{t+s}$  is the stochastic discount factor between  $t$  and  $t+s$  for households.

Given that not all retailers are allowed to adjust their prices, the aggregate price index is given by the following weighted average

$$P_t = [(1 - \xi_p) \tilde{P}_t^{1/\lambda_t^p} + \xi_p \left( \pi_{t-1}^{\iota_p} \pi_{\star}^{1-\iota_p} \right)^{1/\lambda_t^p}]^{\lambda_t^p} \quad (10)$$

### 3.3 Capital producers

Capital producers are infinitely lived agents operating in a perfectly competitive market. Capital producers produce new physical capital stock,  $K_{t+1}$ , combining final goods,  $I_t$ , with currently installed capital,  $K_t$ , using a constant returns to scale technology. The new capital is sold to entrepreneurs at price  $P_t^k$ . We assume that one unit of time  $t$  investment delivers  $\zeta_t$  units of time  $t+1$  physical capital.  $\zeta_t$  is the investment-specific technology shock along the lines of Greenwood, Hercowitz, and Krusell (2000).

$$\ln(\zeta_t) = \rho_{\zeta,1} \ln(\zeta_{t-1}) + \varepsilon_{\zeta,t} \quad \varepsilon_{\zeta,t} \sim \mathcal{N}(\sigma_{\zeta}, 1) \quad (11)$$

We assume that capital producers repurchase used capital from entrepreneurs. Since previously installed capital is an input for the production of new physical capital, the marginal rate of transformation between old (conveniently depreciated) and new capital is equal to one. This implies that the price of old and new capital is identical.

Bernanke, Gertler, and Gilchrist (1999) assume there are increasing marginal adjustment costs in the production of capital, so that they can obtain time variation in the price of capital. Such a variation contributes to the volatility of entrepreneurial net worth. In our set-up, we can obtain time variation in the price of capital through the investment-specific technology shock. However, we assume adjustment costs to impute some discipline in the volatility of investment. In particular,

we assume that the aggregate capital stock of the economy evolves according to

$$K_{t+1} = (1 - \delta)K_t + \left[1 - \Phi\left(\frac{I_t}{I_{t-1}}\right)\right] \zeta_t I_t \quad (12)$$

where  $\delta$  is the depreciation rate. The function  $\Phi\left(\frac{I_t}{I_{t-1}}\right)$  captures the existence of adjustment costs in investment as in Christiano, Eichenbaum, and Evans (2005b). We assume that at the steady state  $\Phi = \Phi' = 0$  and  $\Phi'' \equiv \xi > 0$ .

The representative capital producer chooses the level of investment that maximizes her future discounted profits, which are given by<sup>1</sup>

$$\max_{I_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{\Lambda_t}{\Lambda_0} \left[ P_t^k (K_{t+1} - (1 - \delta)K_t) - P_t I_t \right] \right\} \quad (13)$$

subject to the capital accumulation equation. Let  $Q_t = \frac{P_t^k}{P_t}$  be the relative price of capital,

$$1 = Q_t \zeta_t \left[ 1 - \Phi - \Phi' \frac{I_t}{I_{t-1}} \right] + \beta \mathbb{E}_t \left[ Q_{t+1} \zeta_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \Phi' \frac{I_{t+1}^2}{I_t^2} \right] \quad (14)$$

which is the standard Tobin's q equation. In the absence of capital adjustment costs, the relative price for capital,  $Q_t$ , is equal to the inverse of the investment-specific shock.

### 3.4 Labor Packers

As in Erceg, Henderson, and Levin (2000), we assume that a representative labor packer or employment agency combines the differentiated labor services provided by households,  $H_t(i)$ , according to

$$H_t = \left[ \int_0^1 H_t(i)^{\frac{1}{1+\lambda_t^w}} \right]^{1+\lambda_t^w} \quad (15)$$

where  $\lambda_{w,t}$  is the wage markup which is assumed to follow the exogenous stochastic process

$$\log(\lambda_t^w) = (1 - \rho_w) \log(\lambda_*^w) + \rho_w \log(\lambda_{t-1}^w) + \varepsilon_t^w \quad (16)$$

with  $\varepsilon_t^w \sim \mathcal{N}(0, \sigma_w)$ .

Profit maximization by the perfectly competitive labor packers implies the following labor demand function

$$H_t(i) = \left[ \frac{W_t(i)}{W_t} \right]^{-\left(\frac{1+\lambda_t^w}{\lambda_t^w}\right)} H_t \quad (17)$$

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<sup>1</sup>Note that one unit of  $t+1$  capital is produced by the following technology  $(1 - \delta)K_t + \zeta_t I_t$ . Old capital is bought at price  $P_t^k$ . Therefore, the cost term cancels out the revenue term.

where  $W_t(i)$  is the wage received from the labor packer by the type  $i$  household. The wage paid by intermediate good producers for their homogenous labor input is given by

$$W_t = \left[ \int_0^1 W_t(i)^{\frac{1}{\lambda_{w,t}}} di \right]^{\lambda_{w,t}}$$

### 3.5 Households

We assume there is a continuum of infinitely lived households, each endowed with a specialized type of labor  $i \in [0, 1]$ . Households consume, set wages when allowed to, invest savings in a financial intermediary in the form of deposits that pay a risk-free rate of return, purchase nominal government bonds, receive dividends from their ownership of firms, pay lump-sum taxes, and obtain (give) wealth transfers from (to) entrepreneurs.

Household  $i$  solves the following optimization problem:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j b_{t+j} \left[ \ln(C_{t+j} - hC_{t+j-1}) - \theta \frac{H_{t+j}(i)^{1+1/\nu}}{1+1/\nu} \right] \quad (18)$$

subject to

$$C_t + \frac{D_{t+1}}{P_t} + \frac{NB_{t+1}}{P_t} \leq \frac{W_t(i)}{P_t} H_t(i) + R_{t-1} \frac{D_t}{P_t} + R_{t-1}^n \frac{NB_t}{P_t} + div_t - T_t - Trans_t \quad (19)$$

where  $C_t$  stands for consumption,  $h$  for the degree of habit formation,  $D_{t+1}$  for today's nominal deposits in the financial intermediary,  $H_t(i)$  for hours worked,  $\nu$  for the Frisch elasticity of labor,  $b_t$  for a shock to the stochastic discount factor,  $\theta_t$  for a labor supply shifter,  $P_t$  for the price level of the final good,  $\frac{W_t(i)}{P_t}$  for real wage paid to household  $i$ ,  $R_t$  for the risk-free interest rate paid on deposits,  $R_t^n$  for the risk-free nominal interest rate paid on government bonds,  $NB_t$  for nominal government bonds,  $T_t$  for real taxes (subsidies) paid to (received from) the government,  $div_t$  for dividends obtained from ownership of firms, and  $Trans_t$  for wealth transfers from/to the entrepreneurial sector. The nature of these transfers is described in section 3.6. All the above variables except hours worked and wages are not indexed by  $i$  since, following Erceg, Henderson, and Levin (2000), we assume complete markets which implies that, in equilibrium, all households make the same choice of consumption, deposit holdings, and nominal bond holdings. Leisure (and, hence, hours worked) and wages differ across households due to the monopolistic labor supply.

The intertemporal preference shock,  $b_t$  aims to capture exogenous fluctuations in preferences due to changes in beliefs or in taste. In particular, the stochastic discount factor fluctuates endogenously with consumption and exogenously with the shock  $b_t$ , which is given by

$$\ln(b_t) = \rho_b \ln(b_{t-1}) + \varepsilon_{b,t} \quad (20)$$

where  $\varepsilon_{b,t} \sim \mathcal{N}(0, \sigma_b)$ .

As usual in the literature, we have assumed log-utility in consumption so that the marginal rate of substitution between consumption and leisure is linear in the former, which is necessary to ensure the existence of a balanced growth path.

Households set nominal wages for specialized labor services by means of staggered contracts. In any period  $t$ , a fraction  $\xi_p$  of households cannot reoptimize their wages, but follows the indexation rule

$$W_t(i) = W_{t-1}(i) (\pi_{t-1} \mathfrak{Z}_{t-1})^{\iota_w} (\pi_{\star} \mathfrak{Z}_{\star})^{1-\iota_w} \quad (21)$$

geometrically weighted average of the steady state increase in nominal wages and of the product of last period's inflation and last period's productivity. A fraction  $(1 - \xi_w)$  of households are allowed to choose an optimal nominal wage  $\bar{W}_t(i)$ , by solving

$$\begin{aligned} & \max \mathbb{E}_t \sum_{s=0}^{\infty} \xi_w^s \beta^s \left[ -b_{t+s} \theta \frac{H_{t+s}(i)^{1+\nu}}{1+\nu} \right] + \Lambda_{t+s} W_t(j) H_{t+s}(j) \\ & s.t. \\ & H_t(j) = \left[ \frac{\bar{W}_t(j)}{W_t} \right]^{-\frac{1+\lambda_t^w}{\lambda_t^w}} H_t \quad \text{for } s = 0, \dots, \infty \\ & W_{t+s}(j) = \left[ \prod_{l=1}^s (\pi_{t+l-1} \mathfrak{Z}_{t+l-1})^{\iota_w} (\pi_{\star} \mathfrak{Z}_{\star})^{1-\iota_w} \right] \bar{W}_t(j) \quad \text{for } s = 1, \dots, \infty \end{aligned}$$

### 3.6 Entrepreneurs and financial intermediaries

Entrepreneurs are finitely lived risk-neutral agents who borrow funds captured by financial intermediaries from households. Borrowing and lending occur in equilibrium because entrepreneurs and households are two different types of agents. As we have stated above, financial rigidities arise because there is asymmetric information between borrowers and lenders. While entrepreneurs can freely observe the realization of their idiosyncratic risk, financial intermediaries must pay an auditing cost to observe it. To minimize monitoring costs, lenders will audit borrowers only when they report their inability to pay the loan back under the terms of the contract. We assume that the auditing technology is such that, when monitoring occurs, the lender perfectly observes the borrower's realized return. Monitoring or bankruptcy costs are associated with accounting and legal fees, asset liquidation, and interruption of business.

Since financial intermediaries may incur these costs in the event of default by a borrower, loans are made at a premium over the risk-free interest rate. Such an external finance premium captures the efficiency of financial intermediation. The external finance premium is affected by two channels: the *balance-sheet channel* and the *information channel*. The *balance-sheet channel* implies that as the share of capital investment funded through external financing increases, the probability of

default also rises. Lenders request compensation for the higher exposure to risk with a higher premium. The *information channel* is linked to the elasticity of the external finance premium with respect to the entrepreneurial leverage ratio. This channel states that the larger the rents generated by asymmetric information, the more sensitive the premium is to the leverage ratio. Therefore, the external finance premium is an increasing function of the level of financial rigidity, which is measured by the agency cost. We enrich the model by introducing financial shocks affecting both the *balance-sheet* and the *information channels* of the external finance premium.

In a costly state verification set-up, entrepreneurs try to avoid the financial constraint by accumulating wealth. However, the assumption of a finite lifetime implies that financial intermediation is necessary; that is, entrepreneurs cannot be fully self-financed. In addition, the deceased fraction,  $\gamma$ , of the population of borrowers transfers wealth to the pool of active entrepreneurs. This transfer of resources guarantees that any active entrepreneur has nonzero wealth so she can gain access to external financing.

### 3.6.1 Individual entrepreneur's problem

Entrepreneurs own the capital stock,  $K_t$ , of the economy. At the beginning of the period, an entrepreneur is hit by an idiosyncratic shock,  $\omega_t^j$ , that affects the productivity of her capital holdings. This idiosyncratic shock is at the center of the informational asymmetry, since it is only freely observed by the entrepreneur. For tractability purposes, we assume  $\omega_t^j$ , for all  $j$ , is *i.i.d* lognormal with *c.d.f.*  $F(\omega)$ , parameters  $\mu_\omega$  and  $\sigma_\omega$ , such that  $\mathbb{E}[\omega^j] = 1$ . After observing the realization of the idiosyncratic shock, entrepreneurs choose the capital utilization rate,  $u_t^j$ , that solves the following optimization problem

$$\max_{u_t^j} \left[ u_t^j r_t^{k,j} - a(u_t^j) \right] \omega_t^j K_t^j \quad (22)$$

where, around the steady state,  $a(\cdot) = 0$ ,  $a'(\cdot) > 0$ ,  $a''(\cdot) > 0$  and  $u^* = 1$ . Therefore, capital services,  $k_t^j$ , rented to intermediate goods producers are given by  $k_t^j = u_t^j \omega_t^j K_t^j$ .

The capital demand for entrepreneur  $j$  is given by the gross returns on holding one unit of capital from  $t$  to  $t + 1$

$$R_{t+1}^{k,j} = \left[ \frac{r_{t+1}^{k,j} u_{t+1}^j + \omega_{t+1}^j (1 - \delta) Q_{t+1}}{Q_t} \right] \frac{P_{t+1}}{P_t} \quad (23)$$

where  $\omega_{t+1}^j (1 - \delta) Q_{t+1}$  is the return to selling the undepreciated capital stock back to capital producers.

As we pointed out before, we can write the equilibrium conditions for intermediate goods producers in terms of aggregate variables. Therefore, we have

$$r_t^{k,j} = \omega_t^j \frac{\alpha \chi_t(s) Y_t(s)}{k_t(s)} = \omega_t^j \frac{\alpha \chi_t Y_t}{k_t} = \omega_t^j r_t^{k}$$

and, hence,

$$R_{t+1}^{k,j} = \omega_{t+1}^j R_{t+1}^k \quad (24)$$

where  $R_{t+1}^k$  is the aggregate gross return on capital.

### 3.6.2 Debt contract

Conditional on survival, an entrepreneur  $j$  purchases physical capital,  $K_{t+1}^j$ , at relative price  $Q_t$ . An entrepreneur can finance the purchasing of new physical capital investing her own net worth,  $N_{t+1}^j$ , and using external financing,  $B_{t+1}^j$ , to leverage her project.

Therefore, she can finance her investment in capital goods as follows:

$$Q_t K_{t+1}^j = B_{t+1}^j + N_{t+1}^j \quad (25)$$

Given that the entrepreneur is risk neutral, she offers a debt contract that ensures the lender a return free of aggregate risk. The lender can diversify idiosyncratic risks by holding a perfectly diversified portfolio. A debt contract is characterized by a triplet consisting of the amount of the loan,  $B_{t+1}^j$ , the contractual rate,  $Z_{t+1}^j$ , and a schedule of state-contingent threshold values of the idiosyncratic shock,  $\bar{\omega}_{n,t+1}^j$ , where  $n$  refers to the state of nature. For values of the idiosyncratic productivity shock above the threshold, the entrepreneur is able to repay the lender at the contractual rate. For values below the threshold, the borrower defaults, and the lender steps in and seizes the firm's assets. A fraction of the realized entrepreneurial revenue is lost in the process of liquidating the firm. In this case, the financial intermediary obtains

$$(1 - \mu_t) \omega_{s,t+1}^j R_{s,t+1}^k Q_t K_{t+1}^j \quad (26)$$

where  $\mu_{t+1}$  stands for the marginal bankruptcy cost. In the literature, the marginal bankruptcy cost is assumed to be a constant parameter. We assume, however, that it is a drifting parameter so that exogenous changes in the level of financial rigidities affect the business cycle properties of the model. In section 3.6.3, we describe in detail the relevance of this assumption and the stochastic specification chosen.

For a given state  $n$ , the threshold value for the idiosyncratic productivity shock is defined as

$$\bar{\omega}_{t+1}^j R_{t+1}^k Q_t K_{t+1}^j = Z_{t+1}^j B_{t+1}^j \quad (27)$$

where  $Z_{t+1}^j$  is the contractual rate whose dynamics, *ceteris paribus*, are governed by those of  $\bar{\omega}_{t+1}^j$ . Hence, we set up the debt contract only in terms of the idiosyncratic productivity threshold. From this equation, we can determine the payoffs for the borrower and lender as a function of

the realized idiosyncratic risk. If  $\omega_{t+1}^j \geq \bar{\omega}_{t+1}^j$ , then the entrepreneur can satisfy the terms of the contract. She pays the lender  $Z_{t+1}^j B_{t+1}^j$  and keeps  $(\omega_{t+1}^j R_{t+1}^k Q_t K_{t+1} - Z_{t+1}^j B_{t+1}^j)$ . If  $\omega_{t+1}^j < \bar{\omega}_{t+1}^j$ , the entrepreneur declares bankruptcy; that is, she defaults on her loans. In this case, the financial intermediary liquidates the firm, obtaining  $(1 - \mu_{t+1}) P_t \omega_{t+1}^j R_{t+1}^k Q_t K_{t+1}$  and leaving the lender with zero wealth.

The terms of the debt contract are chosen to maximize expected entrepreneurial profits conditional on the return of the lender, for each possible state of nature, being equal to the real riskless rate. That is, the participation constraint is given by the zero profit condition for the financial intermediary.

$$\max_{\{\bar{\omega}_{n,t+1}^j, K_{t+1}^j\}} \sum_n \Xi_n \left[ \int_{\bar{\omega}_{n,t+1}^j}^{\infty} \omega dF(\omega) - \left[ 1 - F(\bar{\omega}_{n,t+1}^j) \right] \bar{\omega}_{n,t+1}^j \right] R_{n,t+1}^k Q_t K_{t+1}^j \quad (28)$$

st

$$\left( \left[ 1 - F(\bar{\omega}_{n,t+1}^j) \right] \bar{\omega}_{n,t+1}^j + (1 - \mu_{t+1}) \int_0^{\bar{\omega}_{n,t+1}^j} \omega dF(\omega) \right) R_{n,t+1}^k Q_t K_{t+1}^j = R_t \left( Q_t K_{t+1}^j - N_{t+1}^j \right) \quad (29)$$

where  $\Xi_n$  stands for the probability of reaching state  $n$ ,  $F(\bar{\omega}_{n,t+1}^j)$  is the default probability,  $R_t (Q_t K_{t+1}^j - N_{t+1}^j)$  is the real cost of funds,  $(1 - \mu_{t+1}) \int_0^{\bar{\omega}_{n,t+1}^j} \omega R_{n,t+1}^k Q_t K_{t+1}^j dF(\omega)$  is the payoff if the entrepreneur defaults on the loan, and  $\left[ 1 - F(\bar{\omega}_{n,t+1}^j) \right] \bar{\omega}_{n,t+1}^j R_{n,t+1}^k Q_t K_{t+1}^j$ , which is equal to  $\left[ 1 - F(\bar{\omega}_{n,t+1}^j) \right] Z_{t+1}^j B_{t+1}^j$ , stands for the revenue if the loan pays. Therefore, the left-hand side in equation (29) is the expected gross return on a loan for the financial intermediary.

Let  $\varrho_{t+1}^j = \frac{B_{t+1}^j}{N_{t+1}^j}$  be the debt-to-wealth ratio,  $\Gamma(\bar{\omega}_{t+1}^j) = \int_0^{\bar{\omega}_{t+1}^j} \omega f(\omega) d\omega + \bar{\omega}_t \int_{\bar{\omega}_{t+1}^j}^{\infty} f(\omega) d\omega$ , the expected share of gross entrepreneurial earnings going to the lender,  $1 - \Gamma(\bar{\omega}_{t+1}^j)$ , the share of gross entrepreneurial earnings retained by borrowers, and  $\mu_{t+1} G(\bar{\omega}_{t+1}^j) = \mu_{t+1} \int_0^{\bar{\omega}_{t+1}^j} \omega f(\omega) d\omega$ , the expected monitoring costs. Then we can rewrite the standard debt contract problem as

$$\max_{\{\bar{\omega}_{n,t+1}^j, \varrho_{t+1}^j\}} \sum_n \Xi_n \left\{ \left[ 1 - \Gamma(\bar{\omega}_{n,t+1}^j) \right] \frac{R_{n,t+1}^k}{R_t} (1 + \varrho_{t+1}^j) + \Psi(\bar{\omega}_{n,t+1}^j) \left[ \frac{R_{n,t+1}^k}{R_t} \left[ \Gamma(\bar{\omega}_{n,t+1}^j) - \mu_{t+1} G(\bar{\omega}_{n,t+1}^j) \right] (1 + \varrho_{t+1}^j) - \varrho_{t+1}^j \right] \right\}$$

where  $\Psi(\bar{\omega}_{n,t+1}^j)$  is the Lagrange multiplier linked to the participation constraint<sup>2</sup>. From the

<sup>2</sup>We can explicitly derive the expression for the Lagrange multiplier from the first order condition with respect to the schedule  $\bar{\omega}_{t+1}$



first-order condition with respect to the debt-to-wealth ratio

$$0 = \mathbb{E}_t \left[ \left( 1 - \Gamma(\bar{\omega}_{t+1}^j) \right) \frac{R_{t+1}^k}{R_t} + \Psi(\bar{\omega}_{t+1}^j) \left( \left[ \Gamma(\bar{\omega}_{t+1}^j) - \mu_{t+1} G(\bar{\omega}_{t+1}^j) \right] \frac{R_{t+1}^k}{R_t} - 1 \right) \right],$$

we can conclude that the schedule of threshold values for the idiosyncratic productivity shock depends upon aggregate variables so that it is common for all entrepreneurs. We can proceed, hence eliminating the superscript in  $\omega_{t+1}$ . From the participation constraint for the financial intermediary, it directly follows that the debt-to-wealth ratio,  $\varrho_{t+1}^j$ , is identical for all  $j$ . Therefore, we perform the remainder of the analysis dropping all superscripts.

We derive the supply for loans from the zero profit condition for the financial intermediary

$$\frac{R_{t+1}^k}{R_t} [\Gamma(\bar{\omega}_{t+1}) - \mu_{t+1} G(\bar{\omega}_{t+1})] = \left( \frac{Q_t K_{t+1} - N_{t+1}}{Q_t K_{t+1}} \right) \quad (30)$$

The above states that the external finance premium,  $\left[ \frac{R_{t+1}^k}{R_t} \right]$ , is an increasing function of the debt-to-assets ratio and of the severity of the agency problem between borrowers and lenders. Equation (30) provides one of the foundations of the financial accelerator mechanism: a linkage between the entrepreneur's financial position and the cost of external funds, which ultimately affects the demand for capital.

The other main component of the financial accelerator is the evolution of entrepreneurial net worth. Note that the return on capital and, hence, the demand for capital by entrepreneurs depends on the dynamics of net worth. Let  $V_t$  be entrepreneurial equity and  $W_t^e$  be the wealth transfers made by exiting firms to the pool of active firms. Then, aggregate entrepreneurial net worth (average net worth across entrepreneurs) is given by the following differential equation

$$\begin{aligned} N_{t+1} &= x_t \gamma V_t + W_t^e \\ &= x_t \gamma \left[ R_t^k Q_{t-1} K_t - R_{t-1} B_t - \mu_t R_t^k Q_{t-1} K_t \int_0^{\bar{\omega}_t} \omega f(\omega) d\omega \right] + W_t^e \\ &= x_t \gamma \left[ R_t^k Q_{t-1} K_t - R_{t-1} B_t - \mu_t G(\bar{\omega}_t) R_t^k Q_{t-1} K_t \right] + W_t^e \end{aligned}$$

where  $x_t$  is a wealth shock,  $\left[ R_t^k Q_{t-1} K_t - R_{t-1} B_t \right]$  is the nominal gross return on capital net of repayment of loans in the nondefault case, and  $\mu_t G(\bar{\omega}_t) R_t^k Q_{t-1} K_t$  is the gross return lost in case of bankruptcy. Therefore, equity stakes for entrepreneurs that survive to period  $t$  are given by the aggregate return on capital net of repayment of loans.

Wealth shocks can be interpreted as shocks to the stock market that generate asset price movements that cannot be accounted for by fundamentals. Christiano, Motto, and Rostagno (2003) suggest that shocks to entrepreneurial wealth capture the so-called irrational exuberance. We can

also consider wealth shocks as a reduced form for changes in fiscal policy that have redistributive effects between firms and households. Exogenously driven changes in the valuation of entrepreneurial equity need to be financed by another sector of our model economy. We assume that the household sector receives (provides) wealth transfers from (to) the entrepreneurial sector, which are defined as

$$Trans_t = N_{t+1} - \gamma V_t - W_t^e = \gamma V_t (x_t - 1) \quad (31)$$

where  $\gamma V_t + W_t^e$  is the value that entrepreneurial equity would have taken if there were no wealth shocks.

### 3.6.3 Financial shocks

In a model with informational asymmetries, financing capital acquisitions with internally generated funds is preferred to external borrowing since it is less costly. The difference between external and internal financing is the so-called external finance premium. In our environment, we obtain this premium from the zero profit condition in the debt contracting problem

$$\frac{R_{t+1}^k}{R_t} = \left[ \frac{1}{\Gamma(\bar{\omega}_{t+1}) - \mu_{t+1}G(\bar{\omega}_{t+1})} \right] \left[ \frac{Q_t K_{t+1} - N_{t+1}}{Q_t K_{t+1}} \right] \quad (32)$$

The external finance premium is determined by two channels: the *balance-sheet channel*, through the debt-to-assets ratio

$$\frac{Q_t K_{t+1} - N_{t+1}}{Q_t K_{t+1}},$$

and the *information channel*, through the elasticity of the external finance premium with respect to the leverage ratio, which is given by

$$\frac{1}{\Gamma(\bar{\omega}_{t+1}) - \mu_{t+1}G(\bar{\omega}_{t+1})}$$

The external finance premium is the key relationship of the financial accelerator, since it determines the efficiency of the contractual relationship between borrowers and lenders. We enrich the theoretical framework by assuming that this essential mechanism is affected exogenously by two financial shocks: a wealth shock and a shock to the marginal bankruptcy cost.

The *balance-sheet channel* states the negative dependence of the premium on the amount of collateralized net worth,  $N_{t+1}$ . The higher the stake of a borrower in the project, the lower the premium over the risk-free rate required by the intermediary. We introduce shocks to this channel through an entrepreneurial equity shifter. These types of wealth shocks were first introduced by Gilchrist and Leahy (2002). Recently, they have been explored by Christiano, Motto, and Rostagno (2009), Nolan and Thoenissen (2009), and Gilchrist, Ortiz, and Zakrajšek (2009).

Recently, Dib (2009) has explored shocks to the elasticity of the risk premium with respect to the entrepreneurial leverage ratio. He solves the model discarding the contribution of the dynamics of the idiosyncratic productivity threshold to the dynamics of the remaining variables.<sup>3</sup> Hence, those shocks can refer to shocks to the standard deviation of the entrepreneurial distribution, to agency costs paid by financial intermediaries to monitor entrepreneurs, and/or to the entrepreneurial default threshold. He cannot, however, discriminate among the sources of the shock. Christiano, Motto, and Rostagno (2009) solve the model completely so that they can introduce a specific type of shock affecting the efficiency of the lending activity. In particular, they propose riskiness shocks affecting the standard deviation of the entrepreneurial distribution. A positive shock to the volatility of the idiosyncratic productivity shock widens the distribution so that financial intermediaries find it more difficult to distinguish the quality of entrepreneurs.

We introduce exogenous disturbances affecting the elasticity of the premium with respect to the leverage ratio by assuming the marginal bankruptcy cost is time-variant. The information channel, therefore, establishes that the external finance premium is a positive function of the severity of the agency problem measured by the marginal bankruptcy cost,  $\mu_t$ . An increase in the level of financial rigidity implies an enlargement of the informational asymmetry rents which translates into a higher premium on external funds. To the best of my knowledge, only Levin, Natalucci, and Zakrajšek (2004) have explored time variation along this margin. They estimate a partial equilibrium version of the BGG model using a panel of 900 US nonfinancial firms over the period 1997:1 to 2003:3. They find evidence of significant time variation in the marginal bankruptcy cost. In particular, they conclude that time variation in the parameter of interest is the main driver of the swings in the model-implied external finance premium.

We assume that the shock to entrepreneurial wealth follows the following process

$$\ln(x_t) = \rho_x \ln(x_{t-1}) + \varepsilon_{x,t}, \quad \varepsilon_{x,t} \sim \mathcal{N}(0, \sigma_x) \quad (33)$$

and the shock to the marginal bankruptcy cost

$$\ln(\mu_t) = (1 - \rho_\mu) \ln(\mu^*) + \rho_\mu \ln(\mu_{t-1}) + \varepsilon_{\mu,t}, \quad \varepsilon_{\mu,t} \sim \mathcal{N}(0, \sigma_\mu) \quad (34)$$

The unconditional mean of the process governing the agency problem between borrowers and lenders,  $\mu^*$ , determines the average level of financial rigidity in the model economy. This parameter governs, then, the size of the financial accelerator. In particular,  $\mu^*$  stands for the steady-state level of the marginal bankruptcy cost.

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<sup>3</sup>Bernanke, Gertler, and Gilchrist (1999) perform simulation exercises under a parameterization that implied a negligible contribution of the dynamics of the cutoff. However, most of the contributions to the financial accelerator literature have adopted this result as a feature of the model. Therefore, they proceed by setting those dynamics to zero.

### 3.7 Government

Government spending is financed by government nominal bonds sold to households and by lump-sum taxes.

$$NB_{t+1} + P_t T_t = P_t G_t + R_{t-1}^n NB_t \quad (35)$$

where the process for public spending  $G_t$  is given by  $G_t = gY_t$ , with the government spending-to-output ratio,  $g$ , being constant.

The monetary authority follows a Taylor-type interest rate rule. We assume the authority adjusts the short-term nominal interest rate responding to deviations of inflation and output growth from the target, i.e., their steady-state values.

$$\left( \frac{R_t^n}{R^{n*}} \right) = \left( \frac{R_{t-1}^n}{R^{n*}} \right)^{\rho_R} \left( \frac{\pi_t}{\pi^*} \right)^{(1-\rho_R)\psi_\pi} \left( \frac{\Delta Y_t}{\Upsilon_z} \right)^{(1-\rho_R)\psi_y} e^{\varepsilon_{R,t}} \quad (36)$$

with  $\rho_R > 0$ ,  $(1 - \rho_R)\psi_\pi > 0$ ,  $(1 - \rho_R)\psi_y > 0$ , and  $\varepsilon_{R,t} \sim \mathcal{N}(0, \sigma_R)$ . If  $\psi_\pi > 1$ , then monetary policy is consistent with stabilizing inflation. If  $\psi_y > 0$ , then monetary policy is consistent with stabilizing output growth.

### 3.8 Competitive equilibrium

**Definition 1** *A competitive equilibrium is defined by a sequence of prices*

$$\{P_t, P_t(s), W_t, R_t, R_t^k, R_t^n, Q_t, Z_{t+1}\}_{t=0}^\infty,$$

*decisions rules for*

$$\{C_t, NB_t, D_{t+1}, H_t, I_t, Y_t(s), Y_t, u_t, B_{t+1}, K_{t+1}\}_{t=0}^\infty,$$

*and laws of motion for  $\{N_{t+1}, K_{t+1}\}_{t=0}^\infty$  such that all of the above optimality conditions are satisfied, the monetary authority follows its policy rule, and all markets clear.*

Let us state here the final goods market clearing condition (total resources constraint)

$$Y_t = C_t + I_t + G_t + a(u_t)K_t + \mu_t G(\bar{\omega}_t)R_t^k Q_{t-1} K_t \quad (37)$$

and the credit market clearing condition

$$D_{t+1} = B_{t+1} = Q_t K_{t+1} - N_{t+1} \quad (38)$$

## 4 Structural Breaks in Parameters

Traditional approaches to the Great Moderation have focused on two explanations for the slowdown in real and nominal volatilities: smaller shocks hitting the US economy and tougher reaction to inflation by the monetary authority. Those two hypotheses are, however, insufficient to account for the empirical evidence since the mid 1980s. On the one hand, smaller shocks cannot account for more volatile financial cycles. On the other hand, it is hard to reconcile that better monetary policy translates into more stable real and nominal cycles and a destabilization of the financial cycle. Among other researchers, Jermann and Quadrini (2008) highlight the potential relevance of changes in the US financial system to account for the contemporaneous divergence of volatility patterns.

In this paper, we test the relative role that changes in the size of the shocks hitting the economy, changes in monetary policy, and changes in the financial system played in the two stages of the Financial Immoderation and the Great Moderation. To do so, we allow for structural breaks in three sets of parameters intimately linked to each of these potential explanations: variance of the innovations, monetary policy coefficients, and the average level of financial rigidity. We use, however, a relatively naïve approach in treating structural breaks. We assume economic agents do not face an inference problem to learn endogenously about the regimes. When forming rational expectations about the dynamic economy, they take regime changes as completely exogenous events and assume that the current regime will last forever. Thus, once a structural break in parameters happens, agents learn about it immediately and conveniently readjust their choices. This simplifying assumption facilitates the estimation when, as in our case, breaks in the steady state of the economy are allowed.

In this section, we first discuss how breaks in parameters affect the system matrices of the state space representation of the solution to the linear rational expectations (LRE) model. The system of log-linearized equilibrium conditions can be represented as

$$\Gamma_0(\varrho) \tilde{s}_t = \Gamma_1(\varrho) \tilde{s}_{t-1} + \Psi(\varrho) \varepsilon_t + \Pi(\varrho) \eta_t \quad (39)$$

where  $\tilde{s}_t$  is a vector of model variables expressed in deviations from steady state,  $\varepsilon_t$  is a vector of exogenous shocks,  $\eta_t$  is a vector of rational expectations errors with elements  $\eta_t^x = \tilde{x}_t - \mathbb{E}_{t-1}[\tilde{x}_t]$ , and  $\varrho$  is the vector of structural parameters. The solution to the LRE model can be cast in state space form as

$$\textit{Transition equations} : \quad s_t = [I - \Phi(\varrho)] \bar{s} + \Phi(\varrho) s_{t-1} + \Phi_\varepsilon(\varrho) \varepsilon_t \quad (40)$$

$$\textit{Measurement equations} : \quad y_t = B(\varrho) s_t \quad (41)$$

where  $s_t = \tilde{s}_t + \ln(\bar{s})$  and  $\bar{s}$  is the state vector evaluated in the steady state. Breaks in any

parameter affect  $\varrho$ . However, while changes in monetary policy affect  $\Phi(\varrho)$  and variations in the size of exogenous shocks shift  $\Phi_\varepsilon(\varrho)$ , structural breaks in the average level of financial rigidities have an impact on  $\Phi(\varrho)$  and  $\bar{s}$ . That is, changes in  $\mu^*$  not only affect the coefficient matrices but also the steady state of the economy. This poses a challenge in the estimation exercise, since we need to conveniently adapt the filter used to evaluate the likelihood of the data.

In the remainder of the section, we discuss how structural breaks in the parameters of our choice help the model to account for the empirical evidence. For example, in our theoretical framework, an increase (decrease) in the size of a disturbance generates a nonnegative (nonpositive) change in the volatility of all model variables. Therefore, an enlargement in the variability of the shocks hitting the economy could account for the empirical evidence of the 1970s and early 1980s since the volatility of all variables of interest moved in the same direction.

Recent US economic history highlights the relevance of monetary policy to the level and stability of inflation. That is, changes in the degree of response to objectives by the monetary authority will have a larger impact in shaping nominal cycles. In particular, we should expect a loosening of the monetary authority's reaction to deviations of inflation from the target during the Burns-Miller era and a tightening in the Volcker-Greenspan era.

Changes in the average level of bankruptcy costs imply changes in the level of financial frictions due to asymmetric information. An increase (decrease) in the average marginal bankruptcy cost enhances (weakens) the transmission of exogenous shocks to entrepreneurial wealth and costs of capital. Consequently, the responses of investment and output to shocks are more active (muted), since the sensitivity of borrowing costs to leverage increases (decreases). Given that the 1960s, 1970s, and early 1980s were years of profound changes in the US financial system, we should expect a decrease in the unconditional average of the level of financial rigidities in the model economy.

## 5 Parameter Estimates

We estimate the model with standard Bayesian estimation techniques using eight macroeconomic quarterly US time series as observable variables: the growth rate of real per capita net worth in the nonfarm business sector, the growth rate of real per capita gross value added (GVA) by the nonfarm business sector, the growth rate of real per capita consumption defined as nondurable consumption and services, the growth rate of real per capita investment defined as gross private investment, log hours worked, the growth rate of real wage, the log difference of the GVA deflator, and the federal funds rate. A complete description of the data set is given in Appendix C. The model is estimated over the full sample period from 1954.4 to 2006.4.

All the series enumerated above except net worth in the nonfarm business sector are standard in the data sets used in the empirical macro literature. We discuss in further detail the inclusion of

such financial variable in our set of observable variables. Our theoretical framework describes the evolution of three financial series: entrepreneurial wealth, debt, and the external finance premium. Therefore, the estimation exercise could aim to match the behavior of all of those. Net worth for a firm is generally defined as total assets minus total liabilities. However, in order to be consistent with the model, we define net worth as tangible assets minus credit market liabilities. First, the model is a model of tangible assets purchased by firms so that it has nothing to say about financial assets held by entrepreneurs. Second, external financing in the model relates only to that obtained in credit markets. Hence, we do not consider trade and taxes payable nor miscellaneous liabilities provided in the Flow of Funds Accounts. An alternative measure for entrepreneurial wealth used by Christiano, Motto, and Rostagno (2009) is stock market data. This measure contains information only for publicly traded firms, which are a smaller set of firms than the one linked to the aggregate macroeconomic variables of our data set. In addition, in our model there is no role for equity finance.

Following the reasoning provided in the previous paragraph, our definition of debt is given by credit market liabilities in the nonfarm business sector. This information is contained in the series for entrepreneurial wealth. Therefore, if we are to consider only one financial variable in our empirical analysis, it seems reasonable to include net worth, since its informational content includes that of the dynamics of debt.

The external finance premium is essentially an unobservable variable. Hence, any empirical counterpart to be used in the estimation exercise is a proxy for the model concept of interest. Bernanke, Gertler, and Gilchrist (1999) suggest considering the prime lending rate and the 6-month Treasury bill rate in defining the external finance premium. Christiano, Motto, and Rostagno (2003) define the external finance premium as the premium on Baa bonds. Recently, Gilchrist, Ortiz, and Zakrajšek (2009) have used individual security-level data to construct a corporate credit spread index. They use such a credit spread as a proxy for the fluctuations in the unobservable external finance premium. We refrain from using a proxy for the external finance premium for several reasons. First, constructing a measure for such a variable using individual firm data is beyond the scope of this paper. Second, we focus on the analysis of the nonfarm business sector, which includes both corporate and noncorporate US firms. Therefore, using corporate credit or bond spreads and real variables for the nonfarm business sector introduces a discrepancy between financial and macroeconomic variables that would make it harder to evaluate the goodness of fit of our analysis. Third, in our theoretical framework, external financing is modeled using a simple debt contract. However, as long as it is the only form of external financing, the external finance premium is interpreted in the literature (see De Graeve, 2008) as pertaining to all forms of external finance. Therefore, there is no choice of approximation to this model variable that is free of controversy.

## 5.1 Prior distribution of the parameters

In this section, we discuss the prior information on the parameters used in the estimation exercise (see Table A-4). First, we provide a thorough description of our prior choice for the parameters linked to the financial accelerator. Then, we discuss the priors on the remaining parameters. Our prior choice for these parameters is fairly standard in the literature. We use identical priors across subsamples for those parameters subject to structural breaks. We let the data speak about the size of the structural break without imposing any additional a priori information.

Let us discuss the prior choice for the parameters linked to the financial accelerator. As is standard in the literature, we use degenerate priors on the default probability,  $F(\bar{\omega})$ , and the survival probability,  $\gamma$ . Altman and Pasternack (2006) report historical default rates for US bonds over the period 1971-2005 and deliver an average equal to 3%. This is the value for the annual default rate widely used in the literature on the financial accelerator to pin down the quarterly default probability. We obtain the survival probability,  $\gamma$ , from the steady state of the economy given that we set the debt-to-wealth ratio to its historical average. The value for  $\gamma$  is 98.54%, which implies that firms live, on average, 17 years. This tenure is close to the median tenure reported by Levin, Natalucci, and Zakrajšek (2004) from a panel of 900 nonfinancial firms.

Conversely, we use an informative prior for the unconditional average of financial rigidity,  $\mu^*$ . Such a parameter captures the steady state value of the marginal bankruptcy cost. Therefore, it must lie inside the unit interval. A beta distribution guarantees that the parameter of interest belongs to the 0-1 interval. In order to determine the location parameter of the beta prior distribution, we consider micro evidence on bankruptcy costs. Altman (1984), using data from 26 firms, concludes that bankruptcy costs are about 20% of the firm's value prior to bankruptcy and in the range 11-17% of firm's value up to three years prior to bankruptcy. Alderson and Betker (1995) analyze 201 firms that completed Chapter 11 bankruptcies during the period 1982-1993 to determine that the mean liquidation costs are 36.5%. Using those two results, Carlstrom and Fuerst (1997) conclude that the interval empirically relevant for the marginal bankruptcy cost parameter is  $[0.20, 0.37]$ . Levin, Natalucci, and Zakrajšek (2004) estimate a partial equilibrium version of the model by Bernanke, Gertler, and Gilchrist (1999) using panel data over the period 1997 to 2003. As a byproduct of their estimation, they obtain the model implied time series for the marginal bankruptcy cost. Their estimates lie in the range of 7% to 45%. Therefore, we assume the beta distribution for the unconditional average level of financial rigidity is centered at 0.28. We choose the diffusion parameter to be equal to 0.1 so that the 95% credible set encompasses the values provided in the literature.



## 5.2 Posterior estimates of the parameters

The estimation procedure is as follows. First, we obtain the posterior mode by maximizing the posterior distribution, which combines the prior distribution of the structural parameters with the likelihood of the data. By assuming  $\varepsilon_t \sim_{iid} \mathcal{N}(\mathbf{0}, \Sigma_\varepsilon)$  in equation 40, we can use the Kalman filter to evaluate the likelihood function. We modify the Kalman filter to accommodate for changes in the system matrices. A full description of the modification used in the estimation exercise is given in Appendix D.2. Second, we use the random walk Metropolis-Hastings algorithm to obtain draws from the posterior distribution. In particular, we run 3 chains of 250,000 draws using a burn-in period of 20% of the draws.

Tables A-6 and A-7 report the posterior median and the 95% credible intervals obtained by the Metropolis-Hastings algorithm. Let us first analyze Table A-6, which contains those parameters not allowed to change over time. Some of the estimates are fairly standard, such as the inflation rate in the steady state, log hours in the steady state, the average growth rate, the elasticity of capital utilization costs, the price and wage markup in the steady state, the habit parameter, the backward looking parameter of the monetary policy rule, and the autoregressive coefficients. The first three parameters of the previous enumeration are close to their historical averages. The remaining ones are close enough to the widely accepted values in the literature so we do not discuss them further. We just highlight here that while the wealth shock is quite persistent,  $\rho_x = 0.85$ , the shock to the information channel of the external finance premium is much less persistent,  $\rho_\mu = 0.33$ .

The posterior median estimate for the Frisch elasticity,  $1/\nu = 0.35$ , is inside the bounds found in the RBC literature and along the lines of the estimates provided by Justiniano, Primiceri, and Tambalotti (2010). The estimate for the adjustment cost parameter,  $\xi = 0.14$ , is low compared to the estimates available in the literature. This is due to the fact that real rigidities such as investment adjustment costs are not as relevant to account for the dynamics of real variables when financial rigidities are at play.

Table A-7 reports the estimates for those parameters allowed to change in 1970 and 1984. The first group of parameters is formed by the average level of financial rigidity,  $\mu^*$ . The size of the financial friction has decreased over time. In particular, there is over a 40% reduction in the early 1970s and a 90% reduction in the Great Moderation. The latter result is along the lines of Jermann and Quadrini (2008), who obtain that after the mid 1980s, the model economy is in a virtually frictionless environment, and deBlas (2009) who estimates an 80% reduction in monitoring costs in a model based on Carlstrom and Fuerst (1997). The reduction in the average level of financial rigidities accounts not only for the decrease in bankruptcy costs linked to the Bankruptcy Reform Act of 1978 (see White, 1983) but also for other changes in the US financial system. The decades under analysis are characterized by the IT revolution, waves of regulation and deregulation, development of new products, and improvements in the assessment of risk. All these

factors define the level of financial rigidity in terms of the model economy. Therefore, the Great Moderation period is characterized by easier access to credit, which accounted for a reduction in  $\mu^*$ . We should discuss further the estimated values for  $\mu^*$ . For the 1954-1969 period, the estimated marginal bankruptcy cost parameter is 65%. That is, in the event of bankruptcy of an entrepreneur, the financial intermediary will only be able to recover 35% of the value of the firm after liquidation. This estimate is above the values provided in the micro-evidence reported above. It is, however, below the value used by Christiano, Motto, and Rostagno (2009) who suggest that the scrap value is below 10%. The estimate for the Great Inflation period,  $\mu_\star = 0.37$  is equal to the mean liquidation costs estimated by Alderson and Betker (1995).

In the second group of parameters, we have the size of the shock to the marginal bankruptcy cost,  $\sigma_\mu$ , the size of the wealth shock,  $\sigma_x$ , and the response of the monetary authority to deviations of output growth from the sample average,  $\psi_y$ . These parameters are characterized by the fact that their values increase over time. The response of the monetary authority increases a 14% in the 1970s and a 6% in the mid 1980s. The size of the shock to the *balance-sheet channel* of the external finance premium is 43% larger in the 1970s and more than doubles in the mid 1980s. Larger balance-sheet shocks affecting the model economy reflect the increasing sensitivity of the system to asset price movements. This result does not come as a surprise, since the US data have been characterized by several price "bubbles" over the last few decades: the dramatic rise in US stock prices during the late 1990s or the housing bubble during the early 2000s, for example. One possible interpretation of wealth shocks is that they stand for asset price changes not driven by fundamentals.

The enlargement of the shock to the marginal bankruptcy cost is more dramatic. Therefore, given that  $\mu_\star$  is smaller over time, the unconditional average of the process governing the level of financial rigidity is smaller but the variability of the disturbance to the process is larger. We can reconcile these two results by noting that a reduction in  $\mu^*$  increases the average recovery rate for financial intermediaries. Hence, intermediaries are willing to enlarge their exposure to risk, which is captured by the increase in  $\sigma_\mu$ .

The remaining standard deviations of innovations increase in the 1970s and decrease in the last sample period. The neutral, the investment-specific, and the intertemporal preference shocks are smaller, in the post-1984 than in the pre-1970 sample period. The post-1984 value of the monetary policy shock is, however, identical to that taken in the pre-1970 sample period. That is, the 1970s and early 1980s were an "exception," in the sense of Blanchard and Simon (2001), for these parameters. The value during the Great Moderation of the size of the markup shocks is larger than the value taken in the 1950s and 1960s.

Finally, we describe the results for the response of the monetary authority to deviations of inflation from the target. As pointed out elsewhere in the literature, the monetary authority

chooses a looser reaction to inflation in the 1970s. Post-1984, however, there is a tightening in the response to inflation. As long as the reaction to inflation post-1984 is similar to the one pre-1970, we can say, in simplistic terms, that it seems Volcker overcame Burns-Miller's will in terms of inflation by reusing Martin's recipes.

### 5.3 Model evaluation

*[include here MDD comparison]*

We study the model fit of the data using the posterior distribution. In particular, we compare model-implied statistics with those as in the data. We generate samples of the same length as the data (after a burn-in period of 100 observations) from the model economy using 1000 posterior draws. We focus on analyzing the performance of the model at replicating the observed swings in volatility. To do so we report in Table A-8 the model-implied volatility ratios and the 90% credible intervals for raw data. In the literature characterizing the business cycle, model fit is performed using the moments of the cyclical component of the variables. Therefore, we compute the cyclical component of the observable variables in log-levels and the model-implied series using the Hodrick-Prescott filter. Table A-9 reports the volatility ratios for the cyclical component.

Both Table A-8 and Table A-9 show the big success of the model which is being able to deliver not only the increase in the volatility of all variables in the 1970s, but delivering the divergents patterns in volatility characterizing the mid 1980s. The model does a better job reproducing the magnitude of the immoderation in net worth of the mid 1980s for the cyclical component than for the raw data. The size of the real and nominal moderation is of similar magnitude both for raw and HP-filtered data. The relative sizes of the divergent changes in volatility fall short with respect to the observed ones but the average gap is of about 10%. It is also remarkable the performance of the model at reproducing the so called Great Inflation period for both inflation and the nominal interest rate and the consequent large drop in volatilities. The main failure of the model is the inability to reproduce the increase in the volatility of wages of the mid 1980s. In the estimation exercise, the tension between the slowdown in volatilities of all of the other real variables and the increase in the volatility of the real compensation of labor translates into the data pushing parameter estimates into a region of the parameter space that generates a smaller moderation in real variables.

Given that the model delivers moments that are in consonance with the data both for the raw and filtered series, we can conclude that the model proposed in this paper fits the data fairly well. Therefore, our model is a good candidate for analyzing the US business cycle.

## 6 Assessing the Drivers of the Financial Immoderation and the Great Moderation

In this section, we analyze the contribution of each of the potential candidates, size of the shocks, monetary policy stance, and severity of financial rigidities, to the model-implied changes in business cycle properties. To do so, we perform two sets of counterfactual exercises: one for the first stage of the Financial Immoderation and another for the second stage and the Great Moderation.

Counterfactuals 1-4 refer to the first stage of the Financial Immoderation. We perform simulations using the following procedure:

1. Simulate the model economy for 200 periods (after a burn-in of 100 observations) using the parameter vector characterizing the 1954-1969 sample period.
2. Simulate the model economy for 200 periods (after a burn-in of 100 observations) using the parameter vector characterizing the 1970-1983 sample period.
3. Compute the ratio of standard deviations.
4. Simulate the model economy for 200 periods (after a burn-in of 100 observations) using the parameter vector of the counterfactual.
5. Compute the ratio of standard deviations with respect to those obtained in step 1.
6. Compute the percentage of the ratio obtained in step 3 attributable to the counterfactual.
7. Repeat the above 10,000 times.
8. Compute 90% credible intervals.

Table A-10 delivers the percentage of the total increase or decrease in standard deviation generated by the model that can be accounted for by the corresponding counterfactual. We focus on the performance of the model when accounting for the evolution of the cyclical volatility.

In *Counterfactual 1*, we analyze the role played by the estimated changes in 1970 in the response of the monetary authority to deviations of inflation and output growth from the target. In particular, we simulate the model economy as described above, using a parameter vector with the same entries as the one characterizing the 1954-1969 sample period but with the monetary policy coefficients of the 1970-1983 parameter vector. The contemporaneous loosening in the response to inflation and the tightening in the response to output observed in the 1970s and early 1980s account for the following percentages of the model-implied increase in cyclical volatility: 34% for inflation, 11% for net worth, 8% for hours worked, 4% for output and investment, and 2% for wages.

In *Counterfactual 2*, we study the relative significance of the estimated 38% decrease in the level of financial rigidity. Such a decrease in agency costs accounts for an average of 16% of the model-implied increase in the volatility of the cyclical component of consumption and for a 1% and

2% of the increase in the volatility of hours and wages respectively. A reduction in the average size of the financial rigidity present in the model economy translates into a slowdown in the volatility of the other variables.

We analyze the role played by the financial shocks in the immoderation of the 1970s and early 1980s in *Counterfactual 3*. The change in the size of financial shocks overestimates the increase in the cyclical volatility of net worth and investment. It also accounts for the following percentages of the widening of business cycle fluctuations: 21% for output, 48% for consumption, 21% for hours, 5% for wages, 28% for inflation, and 80% for the nominal interest rate.

*Counterfactual 4* assesses the relative importance of changes in the remaining shocks of the economy. The estimated changes in the size of the shocks account for 41% of the increase in the cyclical volatility of net worth, 78% of that in investment variability, 85% of output, 50% of consumption, 81% of hours, 95% of wages, 46% of inflation, and 40% of the nominal interest rate.

We conclude that the change in behavior of the monetary authority explains a significant fraction of the increase in the variability of nominal variables observed in the 1970s and early 1980s. The immoderation observed in real and financial variables is driven by larger shocks hitting the US economy. It is remarkable the role played by financial shocks in delivering the immoderation characterizing the 1970s.

In *Counterfactuals 5-8*, we study the drivers of the empirical evidence of the post-1984 sample period, which is characterized by a contemporaneous enlargement of the financial cycle and a smoothing of real and nominal cycles. We proceed as described above but the baseline parameter vector is the one linked to the 1970-1983 period and the parameter vector used in step 2 of the procedure is the one for the 1984-2006 sample period.

In *Counterfactual 5*, we study the relative contribution of the tightening of monetary policy in response to inflation to the Great Moderation and the widening of the financial cycle. Stricter monetary policy accounts for 36% of the model-implied reduction in the cyclical volatility of inflation, 23% of the decrease in the variability of the nominal interest rate, 17% of the slowdown in the volatility of hours, 4% of the one in wages, 12% of that of output, and 6% of investment. It has, however, a negligible effect on the variability at business cycle frequencies of consumption.

We analyze the role played by the reduction in the unconditional average level of financial rigidity in *Counterfactual 6*. A model with a smoother financial sector overpredicts the model-implied slowdown in investment. In particular, easier access to credit delivers 111% of the model-implied reduction in the cyclical volatility of investment. It also accounts for 78% of the reduction in the volatility of nominal interest rate, 35% of that of inflation, and 4% of the slowdown in output volatility. The effect on the remaining real variables is almost negligible.

We study the effect of the estimated increase in the size of financial shocks in the mid 1980s in *Counterfactual 7*. It generates an increase in the magnitude of the cyclical variation for all

variables generating an overprediction of the immoderation implied by the model (and observed) for net worth. In particular, it generates an immoderation 10 times bigger than the model implied increase in volatility for net worth.

*Counterfactual 8* analyzes the effect of the decrease in the size of all the remaining shocks in the model economy. We obtain the result that smaller real and nominal shocks overpredict the slowdown in output and the volatility of hours worked. These changes in the size of shocks account for the fraction of the reduction in the amplitude of the nominal cycle not accounted for by the tightening of monetary policy and the relaxation of financial rigidity.

From the counterfactual exercises, we conclude that the behavior of the monetary authority has a significant impact on shaping the nominal cycle and a secondary role in the generating the Great Moderation for real variables in the model economy. Changes in the financial system are relevant for the variability of investment, inflation, and nominal interest rates. The remaining swings in the amplitude of fluctuations at business cycle frequencies are driven by changes in the size of shocks hitting the economy.

## 7 Shocks: Relative Importance and Propagation Dynamics

In this section, we focus on the study of the two financial shocks introduced in the model economy. To do so, we analyze the variance decomposition and the impulse response functions.

### 7.1 Variance decomposition

Table A-11 indicates the variance decomposition at business cycle frequencies for output, investment, consumption, net worth, hours worked, labor share, inflation, and the nominal interest rate. We compute the spectral density of the observable variables implied by the DSGE model evaluated at the posterior median and use an inverse difference filter to obtain the spectrum for the level of output, investment, consumption, and net worth (see Appendix D.3). We define business cycle fluctuations as those corresponding to cycles between 6 and 32 quarters.

The main driver of output variance is the neutral technology shock. The relative significance of this shock decreases over time from 57% to 21%. The wage markup shock becomes more relevant over time accounting for 60% of the volatility in the post-1984 period. Since the wage markup shock is the main driver of the variance in hours, we can conclude that the dynamics of output have shifted from being determined by capital to being determined by labor services. Wages are mainly driven by the neutral technology shock in pre-1970 period. After 1970, we observe the same behavior as with output: the relative contribution of the neutral technology shocks decreases 30%

every subperiod. In this case, however, the protagonism lost by the technology is captured not only by the wage markup shock but also the price markup shock.

The main drivers of the variance of consumption are the neutral technology shock, the wealth shock, and the shock to the intertemporal discount factor. While the neutral technology shock accounts for 46% of the variance of consumption at the business cycle frequency in the 1950s and 1960, the relative contribution of this shock declines by 50% in the 1970s and another 50% during the Great Moderation. The wealth shocks accounts for about 28% of the volatility of consumption in the pre-1984 era but only for about 17% post-1984. The former contribution of the wealth shock to the variance of consumption shows a channel of interaction between the financial and real sides of the model economy. The significant drop in the relative contribution of the wealth shock to the variance of consumption is linked to the smaller size of the financial rigidity which translates into a lighter interaction between the financial and the real sides. It is remarkable that post-1984 the main driver of the variance of consumption at the business cycle frequency is the intertemporal preference shock followed by the price markup shock. Therefore, consumption is driven by demand type shocks during the Great Moderation era.

Nominal interest rate variance is driven mainly by financial shocks pre-1984. During the Great Moderation, financial shocks and the investment-specific technology shock account for 42% and 33% of the variance of the risk-free interest rate respectively. While before the mid 1980s the wealth shock is the soloist in orchestrating the variance of the federal funds rate; in the following decades, it is the shock to the bankruptcy cost the most relevant financial shock. A similar evolution for the relative importance of financial shocks characterizes the variance decomposition of inflation. The wealth shock accounts, in average, for about 60% of the variance of inflation during the pre-moderation era. The contribution of such a shock is only 4% in the post-1984 period, but the relative importance of the shock to the bankruptcy shock goes from 3% to 38%.

Our estimates provide a large role for monetary policy shocks as drivers of the business cycle fluctuations of inflation. Monetary surprises account for about 25% of the volatility of inflation in the 1960s and the post-1984 era. They are responsible for 40% of the total variance of inflation at business cycle frequencies in the 1970s and early 1980s. These results are at odds with the ones available in the literature in which the contribution of the monetary policy shock is small. One of the usual main contributors to the variance of inflation is the price markup shock. But, in our environment, the price markup shock shapes mainly the evolution of wages. Therefore, the model faces a tradeoff when using the realization of the price markup shock to match up the dynamics of either wages or the inflation rate. In our set-up, the model solves this issue by drawing the dynamics of wages through the price markup shock and the dynamics of the inflation rate using the monetary shock, which is almost irrelevant for any other variable.

Let us look more carefully to the contribution of the investment specific technology shock to

the variance of inflation and the nominal interest rate. While the role played by this shock is minimal before 1984, its contribution to the variance of inflation and the nominal interest rate is 19% and 33% respectively in the recent decades. Note that the I-shock accounts for 53% of the variance of net worth in the 1950s and 1960s and 30% during the Great Inflation era. Post 1984, the relative importance of the I-shock drops to 3%. Therefore, the investment specific technology shock shapes the business cycle fluctuations of the financial variable in the data set until the mid 1980s and it shapes the volatility of the nominal variables afterward. The relative role played by this shock in the variance of all other variables is, however, almost negligible. These results are in contrast with the standard results in the literature incorporating investment technology shocks to macro models. We conclude that the relative contribution of the investment-specific technology shock to the variance of real variables is overstated in the literature. Once financial shocks are at play, the I-shock is just a shifter of the relative price of capital goods but not a reduction-form proxy for changes in the financial sector. Therefore, its role as driver of business cycle fluctuations of investment and consumption is captured by financial shocks. The only role for I-shocks when financial shocks are incorporated into the model is limited to shaping nominal and financial cycles.

The cyclical variance of net worth is mainly driven by financial shocks. The contribution of the wealth shock, which is a net worth valuation shifter, grows from 31% in the 1950s and 1960s to 93% in the Great Moderation. As discussed above, the relative role of the I-shock is quite large in the years before the Great Inflation. Finally, we consider the variance decomposition of investment. The most remarkable observation has been pointed out already: the lack of significance of the contribution of the investment specific technology shock. Across subsamples, the main drivers of the variance of investment are the financial shocks which account, on average, for 38% of the total variance. That is, investment is no longer technology driven, but driven by financial factors. Regarding the relative contribution of each of the financial shocks, we observe that pre-1984 the wealth shock is the most relevant one. But the contribution of the shock to the bankruptcy cost has increased steadily over time. In particular, the contribution of this shock to the variance of investment more than doubled in each of the subperiods under analysis. Given that, post-1984, the bankruptcy cost accounts for about 40% of the variance of investment and the nominal interest rate, we can conclude that the model uses such a shock to match the observed dynamics of these two variables.

## 7.2 Impulse response functions

The propagation of real and nominal shocks in the context of a model of the financial accelerator has already been studied in the literature. Therefore, in this section, we focus only on the study of the propagation dynamics of financial shocks. For both the wealth shock and the innovation to the marginal bankruptcy cost, we plot the responses in the first 40 quarters in terms of percentage



deviations with respect to the steady state. Each plot contains three impulse response functions (IRFs). The solid line is the IRF computed using the parameter vector characterizing the 1954-1969 sample period. The dashed line is the IRF for the 1970s and early 1980s. The starred line is the IRF of the post-1984 period. We analyze the impulse response functions for all observable variables and the model implied external finance premium since it facilitates the explanation of the propagation mechanism of financial shocks.

### 7.2.1 Wealth shock

Figure B-2 reports the impulse response functions following a wealth shock that, upon impact, induces an increase in entrepreneurial net worth equal to a 1% deviation from its steady-state value. The size of shock generating such a response upon impact is 0.43 in the 1954-1969 sample period, 0.45 in the 1970-1983 period, and 0.97 in the post-1984 period. The main messages from the figure are: *(i)* the impulse response functions in the 1954-1969 and 1970-1983 are quite similar, in particular, for the nominal interest rate, wages, and consumption the responses upon impact are identical across subsamples; *(ii)* responses upon impact of the external finance premium, and investment, are a positive function of the size of the financial rigidity; and *(iii)* the responses become more persistent post-1984. Regarding the larger persistence, figure B-3 reports the IRFs for net worth, investment, and output for 200 quarters. For example, while the effect of a wealth shock on investment dies out after 50 quarters before 1984, investment does not come back to its steady state value until after quarter 100 in the most recent decades. For the sake of comparison, we report in figure (B-4) the impulse response functions following a wealth shock of size 0.43 for all sub-periods. The only remarkable differences when comparing figures (B-2)-(B-4) are: *(i)* the response upon impact for net worth post-1984 is smaller but it still peaks at a higher value and several quarters after the pre-1984 responses and it is more persistent than those; and *(ii)* the responses upon impact post-1984 for all variables are smaller.

Let us first analyze the impulse response functions for the pre-1984 sample periods. The response of net worth is very persistent, which is the source of the large contribution of the wealth shock to the low frequency fluctuations of entrepreneurial wealth. A positive wealth shock that increases the value of collateral reduces the probability of default so that financial intermediaries are willing to lend at a lower premium. Therefore, the response of the external finance premium upon impact is negative. This immediate improvement in credit markets has a significant amplification effect on investment so that the response of investment upon impact more than doubles the initial response of net worth. The initial response of output is positive but smaller than the boost in investment because consumption decreases upon impact and the total resources constraint needs to be satisfied. The negative response of consumption upon impact is linked to the general equilibrium effects of our model. A nonfundamental increase in entrepreneurial wealth is financed through a

reduction in household wealth. The reduction in total disposable income is not large enough to generate a decrease in consumption of the same magnitude as the increase in entrepreneurial wealth. This is due to the fact that other sources of household wealth, such as labor income, react positively to the wealth shock, since hours worked increase upon impact. The positive response of inflation and the nominal interest rate suggests that the wealth shock displays the features of a standard demand shock: quantities and prices move in the same direction, leading to a tightening of monetary policy.

The larger response of inflation upon impact in 1970s-early 1980s is due to a less active response to deviations of inflation from the target by the monetary authority. That is, inflation is left to vary more *ad libitum*.

In the Great Moderation era, the response of net worth to the same wealth shock peaks at a higher value and several quarters later. From the second quarter onward, the response function post-1984 always lies above those for the pre-1984 sample periods. This can be easily reconcilable from the definition of aggregate net worth. Lower average agency costs alleviate the deadweight loss associated with bankruptcy,  $\mu_t G(\bar{\omega}_t) R_t^k Q_{t-1} K_t$ , which implies that for the same initial increase in wealth, the effects are more long-lasting, since more resources are accumulated from period to period. Higher persistence induced by the lower dependence on the financial accelerator mechanism translates into more persistent responses for all variables. For example, it takes to net worth more than 200 periods to return to its steady state value. Therefore, the persistence implied by the financial accelerator is a negative function of the size of financial rigidity. The responses for all variables except net worth are also characterized by a significantly smaller response upon impact. This is driven by the smaller size of the financial accelerator mechanism. Lower levels of credit market imperfections reduce the elasticity of the external finance premium with respect to the leverage ratio. Therefore, the amplification effect linked to the improvement in credit market conditions is more muted.

### 7.2.2 Shock to the marginal bankruptcy cost

Figure B-5 reports the impulse response functions to shocks to the marginal bankruptcy cost. We focus on a negative shock that generates a reduction upon impact in the external financial premium of 0.13% in the 1954-1969 sample period. Such a shock generates a response upon impact of similar magnitude to the one generated by the wealth shock for the same sample period. We define the innovation under analysis so as to make the comparison with the impulse response functions to a wealth shock reported in the previous section easier. To better illustrate the smaller effect in the economy of shocks to the marginal bankruptcy cost, we have computed the size of the shock needed to generate a 1% in net worth for all sample periods. For the 1950s-1960s, the size of the

expansionary shock<sup>4</sup> would be 68, for the 1970-1983 sample period, it would be 111, and for the post-1984, 3710. This evidences the role played by the smaller size of the financial rigidity in the propagation of financial shocks. This also shows how the large size of shocks to the bankruptcy cost post-1984 have a minimal impact in the model economy. In particular, figure B-5 reports the transmission of a shock to the bankruptcy cost as size  $-16$ .

A negative shock to agency costs creates an incentive for entrepreneurs to select contractual terms with a larger debt-to-net worth ratio, since the deadweight loss linked to bankruptcy is smaller. There are two opposing effects operating as a result of higher debt-to-net worth ratios. On the one hand, both the default probability and the default productivity threshold increase, offsetting the effect of lower bankruptcy costs in determining entrepreneurial net worth. We label this effect the *default effect*. On the other hand, there is a *mass effect* that stays for the increase in capital investment linked to a larger set of resources available. Larger amounts of capital holdings imply a larger equity value through an increase in total capital returns. Given that the response upon impact of entrepreneurial net worth is positive, the *mass effect* dominates the *default effect*.

The response of investment upon impact is smaller than the response we obtained to a wealth shock. However, the positive response upon impact of investment also exceeds the response upon impact of net worth due to the *mass effect* explained above. Irrespective of the relative dominance of this effect in terms of shaping the response of entrepreneurial wealth, the increase in the pool of resources available for purchasing capital enhances investment activity in the economy.

Consumption responds to the expansionary shock positively. Hours worked, however, decrease upon impact. The enlargement in investment raises future productivity and, hence, future wages. Households perform intertemporal substitution by decreasing current hours and increasing them in the future when wages are higher. As a consequence of the reduction of hours upon impact, output responds negatively to an expansionary shock to agency costs. However, as with net worth, after a few periods output's response is positive and long-lasting.

Given the expansionary effect on investment of the shock at hand, inflation increases upon impact. A positive response of inflation requires a contractionary monetary policy but the negative response upon impact of output suggests an expansionary policy. The increase in inflation dominates the reduction in output growth forcing the monetary authority to use a contractionary monetary policy. Therefore, the risk-free nominal interest rate increases upon impact. The peak of the response of the nominal interest rate coincides with the peak in the response of output.

Given the significant decline in the size of the financial accelerator, the post-1984 impulse response functions are all characterized by smaller responses for all variables. Given the size of the shock, the responses post-1984 are almost negligible.

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<sup>4</sup>A expansionary shock to the marginal bankruptcy cost is a shock that decreases the realization of the marginal agency cost.

### 7.2.3 Comparison of impulse response functions

In this section, we compare the impulse response functions to the two expansionary financial shocks. Let us focus on the responses for the 1954-1969 sample period. A similar rationale follows for the other sample periods of interest. Both financial shocks generate the same response upon impact for the external finance premium. The responses upon impact of consumption, hours worked, wages, and output reported in Figure B-6, are very different. Therefore, we can learn about the source of the responses. While consumption and wages response positively to a shock affecting the monitoring technology, their response upon impact after an expansionary wealth shock is negative. Therefore, consumption responds to an expansionary financial shock as to any expansionary shock hitting the economy whenever the financial shock affects the marginal bankruptcy cost. Output and hours worked, however, respond positively to an expansionary wealth shock but negatively to a shock to the marginal bankruptcy cost.

## 8 Conclusions

We have studied two empirical regularities characterizing the US aggregate data over the last 55 years. The *Great Moderation* is related to the significant slowdown in the amplitude of the real and nominal cycles since the mid 1980s. The *Financial Immoderation* refers to the enlargement of the cyclical volatility of financial variables present since 1970. In this paper, we have made inference on the size of the structural breaks in parameters needed to account for the evolution of the second moments of the data in a model featuring nominal, real, and financial frictions. In particular, we have focused on breaks in the size of shocks, monetary policy coefficients, and the average size of the financial accelerator to disentangle the role played by changes in luck, in the conduct of monetary policy, and in the financial system respectively.

We conclude that while changes in the conduct of monetary policy account for a relevant proportion of the changes in the volatility of nominal variables, they play a smaller role in accounting for the evolution of financial and real cycles. Financial factors are not only relevant in shaping the business cycle properties of financial variables, but also nominal variables and investment. Financial shocks the main drivers of the variance of net worth, investment, and the nominal interest rate. Financial shocks joint with monetary policy shocks are the main drivers of the variance of inflation. The estimated reduction in the size of the financial accelerator has two effects. On the one hand, it allows the model to generate a slowdown in the volatility of investment of the same magnitude as the observed moderation and to account for over 75% of the smoothing in the business cycle fluctuations of the nominal interest rate and 35% of that of inflation. On the other hand, a smaller level of financial rigidity changes the propagation mechanism of financial shocks to the economy.

The responses upon impact are smaller for both financial shocks and the responses to a wealth shock are more persistent.

Our study reaffirms the growing convention in the literature on integrating credit market imperfections in otherwise standard macroeconomic models. We have documented the importance of including financial shocks in the analysis. Moreover, we highlight the relevance of taking into account structural breaks in the data, since our , in terms of assessing the main drivers of the cycle or characterizing the propagation dynamics of shocks, may differ significantly.

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## A Tables

Table A-1: Chow's Breakpoint Test: AR(1) with drift

|                      | 1970   |        | 1984     |          | 1970-1984 |          |
|----------------------|--------|--------|----------|----------|-----------|----------|
|                      | F      | LR     | F        | LR       | F         | LR       |
| $\Delta$ Net worth   | 6.51** | 6.47** | 10.18*** | 10.03*** | 5.52***   | 10.90*** |
| $\Delta$ Output      | 3.29*  | 3.30*  | 24.20*** | 23.11*** | 12.69***  | 24.28*** |
| $\Delta$ Investment  | 2.87*  | 2.87*  | 15.32*** | 14.92*** | 7.80***   | 15.25*** |
| $\Delta$ Consumption | 1.47   | 1.48   | 10.75*** | 10.58*** | 5.65***   | 11.16*** |
| Hours                | 0.11   | 0.11   | 7.59***  | 7.53***  | 4.94***   | 9.80***  |
| $\Delta$ Wage        | 3.03*  | 3.03*  | 4.79**   | 4.79**   | 2.58*     | 5.17*    |
| Inflation            | 2.92*  | 2.93*  | 10.99*** | 10.81*** | 16.66***  | 31.33*** |
| Federal Funds Rate   | 3.54*  | 3.55*  | 6.24***  | 6.21***  | 12.10***  | 23.21*** |

Notes: F refers to the F-statistic, which is distributed as  $F(k, T - 2k)$  where  $k$  is the number of parameters to be tested and  $T$  the total number of observations. The critical values are 2.73 at 10% significance level, 3.89 at 5%, and 6.76 at 1% when we test for one break. For two breaks, the critical values are 2.33 at 10%, 3.04 at 5%, and 4.71 at 1%. LR refers to the log-likelihood ratio statistic, which is distributed as  $\chi^2$  with  $(m - 1)k$  degrees of freedom, where  $m$  is the number of subsamples. The critical values when there is only one break are 2.71 at 10% significance level, 3.84 at 5%, and 6.64 at 1%. For two breaks, the critical values are 4.61 at 10%, 5.99 at 5%, and 9.21 at 1%. If the statistic is above the critical value, the null hypothesis of no structural change can be rejected. The symbol \* indicates we can reject the null of parameter constancy at 10%, \*\* at 5%, and \*\*\* at 1%.

Table A-2: Chow's Breakpoint Test: Cyclical component. AR(1) with drift

|                    | 1970   |        | 1984     |          | 1970-1984 |          |
|--------------------|--------|--------|----------|----------|-----------|----------|
|                    | F      | LR     | F        | LR       | F         | LR       |
| Net worth          | 4.01** | 4.01** | 6.03**   | 6.00**   | 3.29**    | 6.58*    |
| Output             | 2.66   | 2.67   | 20.45*** | 19.69*** | 10.79***  | 20.82*** |
| Investment         | 1.92   | 1.93   | 13.54*** | 13.24*** | 7.08***   | 13.89*** |
| Consumption        | 0.86   | 0.86   | 14.50*** | 14.14*** | 8.36***   | 16.31*** |
| Hours              | 0.54   | 0.54   | 11.23*** | 11.04*** | 6.62***   | 13.02*** |
| Wage               | 4.34** | 4.34** | 6.68**   | 6.64***  | 3.63**    | 7.24**   |
| Inflation          | 3.68*  | 3.69*  | 11.50*** | 11.30*** | 18.77***  | 34.98*** |
| Federal Funds Rate | 3.88** | 3.88** | 7.47***  | 7.41***  | 14.17***  | 29.93*** |



Table A-3: Ratio post- to pre- standard deviation: **Cyclical component**

|                      | $\frac{1984-2006}{1954-1983}$ | $\frac{1970-1983}{1954-1969}$ | $\frac{1984-2006}{1970-1983}$ |
|----------------------|-------------------------------|-------------------------------|-------------------------------|
| Output               | 0.49                          | 1.55                          | 0.41                          |
| Investment           | 0.60                          | 1.47                          | 0.51                          |
| Consumption          | 0.54                          | 1.74                          | 0.44                          |
| Hours                | 0.74                          | 1.47                          | 0.63                          |
| Wage                 | 1.43                          | 1.48                          | 1.21                          |
| Inflation            | 0.46                          | 2.58                          | 0.34                          |
| Federal funds rate   | 0.65                          | 2.81                          | 0.47                          |
| Net worth business   | 1.67                          | 1.32                          | 1.47                          |
| Debt business        | 1.41                          | 1.71                          | 1.13                          |
| Net worth households | 1.23                          | 1.94                          | 1.04                          |
| Net private savings  | 1.55                          | 1.16                          | 1.44                          |
| Demand deposits      | 1.57                          | 3.09                          | 1.14                          |

*Notes:* The cyclical component is extracted using the Hodrick-Prescott filter for the quarterly frequency ( $\lambda = 1600$ ).

Table A-4: Prior

| Name                  | Description  | Density        | Para 1        | Para 2 |
|-----------------------|--|----------------|---------------|--------|
| $\beta$               | Discount rate  | Fixed          | 0.9988        | –      |
| $\delta$              | Depreciation rate                                    | Fixed          | 0.025         | –      |
| $\alpha$              | Capital share  | Fixed          | 0.3           | –      |
| $(\frac{G}{Y})^*$     | Public spending share                                | Fixed          | 0.20          | –      |
| $\gamma$              | Survival probability                                 | Fixed          | 0.9854        | –      |
| $[F(\bar{\omega})]^*$ | Default probability                                  | Fixed          | 0.0075        | –      |
| $\sigma_\omega$       | Var of $\mathcal{LN}$ for idiosyncratic productivity | Fixed          | $\sqrt{0.19}$ | –      |
| $\lambda_p$           | Price markup in the steady state                     | Gamma          | 0.15          | 0.02   |
| $\lambda_w$           | Wage markup in the steady state                      | Gamma          | 0.15          | 0.05   |
| $\iota_p$             | Price indexation                                     | Beta           | 0.75          | 0.15   |
| $\iota_w$             | Wage indexation                                      | Beta           | 0.75          | 0.15   |
| $\xi_p$               | Calvo prices   | Beta           | 0.8           | 0.05   |
| $\xi_w$               | Calvo wages  | Beta           | 0.75          | 0.05   |
| $h$                   | Degree of habit formation                            | Beta           | 0.7           | 0.1    |
| $\pi_n^*$             | Inflation in steady state                            | $\mathcal{N}$  | 3             | 1      |
| $\ln(H^*)$            | Log hours at the steady state                        | $\mathcal{N}$  | 0.026         | 0.01   |
| $\xi$                 | Adjustment cost                                      | Gamma          | 4             | 2      |
| $\nu$                 | Inverse Frisch elasticity                            | Gamma          | 2             | 0.75   |
| $(\mu^*)_j$           | Level of financial friction                          | Beta           | 0.28          | 0.1    |
| $a''$                 | Elasticity of capital utilization costs              | Gamma          | 1             | 0.5    |
| $\rho_r$              | Degree of backward looking in MP                     | Beta           | 0.6           | 0.2    |
| $(\psi_\pi)_j$        | MP reaction to inflation                             | $\mathcal{N}$  | 1.7           | 0.3    |
| $(\psi_y)_j$          | MP reaction to output growth                         | $\mathcal{N}$  | 0.2           | 0.05   |
| $\rho_i$              | Persistence of shocks                                | Beta           | 0.60          | 0.1    |
| $\Upsilon_z$          | Drift in the neutral technology process              | $\mathcal{N}$  | 0.005         | 0.01   |
| $(\sigma_i)_j$        | Std i shock, subsample j                             | $\mathcal{IG}$ | 0.01          | 4      |
| $(\sigma_\mu)_j$      | Std i shock, subsample j                             | $\mathcal{IG}$ | 1             | 4      |

Notes:  $j = 1, 2, 3$ . Para 1 and Para 1 list the means and the standard deviations for Beta, Gamma, and Normal distributions; the upper and lower bound of the support for the Uniform distribution;  $s$  and  $\nu$  for the Inverse Gamma distribution, where  $p_{IG}(\sigma|\nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$ . The effective prior is truncated at the boundary of the determinacy region. The value for households' discount rate,  $\beta$ , is chosen so that, in the steady state, the nominal risk-free interest rate matches the historical quarterly gross federal funds rate.

Table A-5: Marginal Data Densities Comparison

Table A-6: Posterior estimates

| Name            |        |              | Name            |        |              |
|-----------------|--------|--------------|-----------------|--------|--------------|
|                 | Median | 95%C.I.      |                 | Median | 95%C.I.      |
| $\pi_*$         | 3.40   | [2.68, 4.13] | $\ln(H^*)$      | 0.05   | [0.03, 0.07] |
| $\xi$           | 0.14   | [0.09, 0.19] | $a''$           | 0.47   | [0.23, 0.83] |
| $\nu$           | 2.82   | [1.50, 4.20] | $h$             | 0.65   | [0.58, 0.71] |
| $\lambda_*^p$   | 0.13   | [0.11, 0.16] | $\lambda_*^w$   | 0.35   | [0.22, 0.50] |
| $\xi_p$         | 0.54   | [0.50, 0.59] | $\xi_w$         | 0.40   | [0.32, 0.48] |
| $\iota_p$       | 0.06   | [0.02, 0.11] | $\iota_w$       | 0.16   | [0.09, 0.23] |
| $100Y_z$        | 0.54   | [0.37, 0.73] | $\rho_R$        | 0.74   | [0.70, 0.78] |
| $\rho_z$        | 0.26   | [0.19, 0.34] | $\rho_\zeta$    | 0.87   | [0.83, 0.91] |
| $\rho_\mu$      | 0.33   | [0.28, 0.38] | $\rho_x$        | 0.85   | [0.78, 0.91] |
| $\rho\lambda^p$ | 0.99   | [0.99, 0.99] | $\rho\lambda^w$ | 0.93   | [0.88, 0.98] |
| $\rho_b$        | 0.82   | [0.78, 0.91] |                 |        |              |

Table A-7: Posterior estimates

| Name                    | Pre 1970 |               | 1970-1983 |               | Post 1984 |               |
|-------------------------|----------|---------------|-----------|---------------|-----------|---------------|
|                         | Median   | 95%C.I.       | Median    | 95%C.I.       | Mean      | 95%C.I.       |
| $\mu^*$                 | 0.65     | [0.55, 0.75]  | 0.41      | [0.30, 0.51]  | 0.01      | [0.01, 0.02]  |
| $\sigma_\mu$            | 0.70     | [0.54, 0.89]  | 1.93      | [1.38, 2.67]  | 67.6      | [40.3, 107.2] |
| $100\sigma_x$           | 0.26     | [0.20, 0.32]  | 0.45      | [0.34, 0.58]  | 1.29      | [1.06, 1.53]  |
| $\psi_y$                | 0.28     | [0.21, 0.36]  | 0.32      | [0.23, 0.41]  | 0.34      | [0.28, 0.41]  |
| $\psi_\pi$              | 2.05     | [1.77, 2.33]  | 1.61      | [1.38, 1.91]  | 1.93      | [1.57, 2.30]  |
| $100\sigma_Z$           | 1.52     | [1.26, 1.80]  | 1.57      | [1.30, 1.86]  | 0.87      | [0.76, 1.01]  |
| $100\sigma_\zeta$       | 0.95     | [0.72, 1.20]  | 1.06      | [0.82, 1.35]  | 0.72      | [0.57, 0.90]  |
| $100\sigma_b$           | 1.94     | [1.36, 2.70]  | 2.86      | [2.13, 3.82]  | 1.64      | [1.26, 2.10]  |
| $100\sigma_R$           | 0.17     | [0.13, 0.21]  | 0.42      | [0.34, 0.51]  | 0.19      | [0.16, 0.24]  |
| $100\sigma_{\lambda^p}$ | 7.09     | [5.35, 8.97]  | 12.33     | [9.10, 16.21] | 8.98      | [6.94, 11.26] |
| $100\sigma_{\lambda^w}$ | 6.74     | [3.92, 10.93] | 10.14     | [5.21, 16.82] | 8.82      | [4.73, 14.67] |

Table A-8: Ratio of standard deviations. Levels.

| Series                | 1970-1983<br>1954-1969 |        | 1984-2006<br>1970-1983 |        |      |              |
|-----------------------|------------------------|--------|------------------------|--------|------|--------------|
|                       | Data                   | Model  | Data                   | Model  |      |              |
|                       |                        | Median | 90%                    | Median | 90%  |              |
| $\Delta$ Net worth    | 1.42                   | 1.52   | [1.32, 1.69]           | 1.42   | 1.11 | [0.95, 1.26] |
| $\Delta$ Output       | 1.10                   | 1.50   | [1.30, 1.69]           | 0.43   | 0.70 | [0.62, 0.79] |
| $\Delta$ Investment   | 1.08                   | 1.51   | [1.33, 1.71]           | 0.51   | 0.73 | [0.65, 0.82] |
| $\Delta$ Consumption  | 1.15                   | 1.36   | [1.27, 1.58]           | 0.55   | 0.63 | [0.53, 0.74] |
| Hours                 | 1.10                   | 1.63   | [0.96, 2.24]           | 1.20   | 0.87 | [0.52, 1.20] |
| $\Delta$ Wage         | 1.19                   | 1.27   | [1.07, 1.49]           | 1.26   | 0.68 | [0.58, 0.79] |
| Inflation             | 2.05                   | 2.22   | [1.80, 2.61]           | 0.36   | 0.42 | [0.34, 0.49] |
| Nominal interest rate | 2.16                   | 1.78   | [1.68, 2.42]           | 0.64   | 0.48 | [0.31, 0.64] |

Table A-9: Ratio of standard deviations. Cyclical component using the HP filter.

| Series                | 1970-1983<br>1954-1969 |        | 1984-2006<br>1970-1983 |        |      |              |
|-----------------------|------------------------|--------|------------------------|--------|------|--------------|
|                       | Data                   | Model  | Data                   | Model  |      |              |
|                       |                        | Median | 90%                    | Median | 90%  |              |
| Net worth             | 1.32                   | 1.27   | [0.99, 1.52]           | 1.47   | 1.37 | [1.01, 1.73] |
| Output                | 1.55                   | 1.52   | [1.20, 1.82]           | 0.41   | 0.75 | [0.58, 0.91] |
| Investment            | 1.47                   | 1.45   | [1.12, 1.75]           | 0.51   | 0.64 | [0.51, 0.77] |
| Consumption           | 1.74                   | 1.44   | [1.08, 1.81]           | 0.44   | 0.65 | [0.47, 0.80] |
| Hours                 | 1.47                   | 1.72   | [1.32, 2.11]           | 0.63   | 0.77 | [0.60, 0.95] |
| Wage                  | 1.48                   | 1.43   | [1.12, 1.76]           | 1.21   | 0.72 | [0.55, 0.90] |
| Inflation             | 2.58                   | 2.16   | [1.83, 2.50]           | 0.34   | 0.45 | [0.38, 0.52] |
| Nominal interest rate | 2.81                   | 1.70   | [1.25, 2.08]           | 0.47   | 0.53 | [0.42, 0.65] |

Table A-10: Counterfactuals: Percentage of the model-implied change in cyclical standard deviations.

| Counterfactual   | Net worth | Output | Investment | Consumption | Hours | Wage | Inflation | Nominal interest rate |
|------------------|-----------|--------|------------|-------------|-------|------|-----------|-----------------------|
| Counterfactual 1 | 11        | 4      | 4          | 0           | 8     | 2    | 34        | 27                    |
| Counterfactual 2 | -         | 0      | -          | 16          | 1     | 2    | -         | -                     |
| Counterfactual 3 | 137       | 21     | 131        | 48          | 21    | 4    | 28        | 80                    |
| Counterfactual 4 | 41        | 85     | 78         | 50          | 81    | 95   | 46        | 40                    |
| Counterfactual 5 | 0         | 12     | 6          | -           | 17    | 4    | 36        | 23                    |
| Counterfactual 6 | -         | 4      | 111        | -           | 0     | 0    | 35        | 78                    |
| Counterfactual 7 | 1000      | -      | -          | -           | -     | -    | -         | -                     |
| Counterfactual 8 | -         | 92     | 44         | 66          | 91    | 96   | 55        | 47                    |

Notes: We include a dash (-) when the direction of the counterfactual implied change is at odds with the data.

Table A-11: Variance decomposition at business cycle frequencies

| Output       |            |        |         |         |          |          |           |              |
|--------------|------------|--------|---------|---------|----------|----------|-----------|--------------|
|              | Bank. cost | Wealth | Neutral | I-shock | P-Markup | W-markup | Intertemp | Monet Policy |
| Pre-1970     | 0          | 2      | 57      | 0       | 7        | 31       | 0         | 2            |
| 1970-1984    | 1          | 4      | 35      | 1       | 12       | 38       | 1         | 8            |
| Post-1984    | 3          | 1      | 21      | 2       | 10       | 60       | 1         | 2            |
| Investment   |            |        |         |         |          |          |           |              |
|              | Bank. cost | Wealth | Neutral | I-shock | P-Markup | W-markup | Intertemp | Monet Policy |
| Pre-1970     | 8          | 26     | 22      | 1       | 16       | 18       | 8         | 2            |
| 1970-1984    | 13         | 26     | 11      | 1       | 17       | 16       | 8         | 7            |
| Post-1984    | 38         | 4      | 8       | 3       | 11       | 28       | 8         | 4            |
| Consumption  |            |        |         |         |          |          |           |              |
|              | Bank. cost | Wealth | Neutral | I-shock | P-Markup | W-markup | Intertemp | Monet Policy |
| Pre-1970     | 0          | 22     | 46      | 0       | 4        | 11       | 16        | 0            |
| 1970-1984    | 1          | 34     | 21      | 1       | 6        | 12       | 25        | 0            |
| Post-1984    | 2          | 17     | 10      | 5       | 13       | 8        | 45        | 0            |
| Net Worth    |            |        |         |         |          |          |           |              |
|              | Bank. cost | Wealth | Neutral | I-shock | P-Markup | W-markup | Intertemp | Monet Policy |
| Pre-1970     | 7          | 31     | 2       | 53      | 4        | 2        | 0         | 2            |
| 1970-1984    | 10         | 48     | 1       | 30      | 4        | 2        | 0         | 6            |
| Post-1984    | 3          | 93     | 0       | 3       | 0        | 0        | 0         | 0            |
| Hours        |            |        |         |         |          |          |           |              |
|              | Bank. cost | Wealth | Neutral | I-shock | P-Markup | W-markup | Intertemp | Monet Policy |
| Pre-1970     | 1          | 6      | 4       | 1       | 16       | 69       | 0         | 4            |
| 1970-1984    | 2          | 7      | 1       | 1       | 18       | 57       | 0         | 13           |
| Post-1984    | 4          | 1      | 1       | 2       | 13       | 76       | 1         | 2            |
| Wage         |            |        |         |         |          |          |           |              |
|              | Bank. cost | Wealth | Neutral | I-shock | P-Markup | W-markup | Intertemp | Monet Policy |
| Pre-1970     | 0          | 1      | 71      | 0       | 19       | 9        | 0         | 0            |
| 1970-1984    | 0          | 1      | 47      | 0       | 36       | 14       | 1         | 1            |
| Post-1984    | 1          | 0      | 31      | 0       | 39       | 28       | 1         | 0            |
| Inflation    |            |        |         |         |          |          |           |              |
|              | Bank. cost | Wealth | Neutral | I-shock | P-Markup | W-markup | Intertemp | Monet Policy |
| Pre-1970     | 3          | 36     | 14      | 4       | 6        | 3        | 15        | 20           |
| 1970-1984    | 3          | 32     | 4       | 4       | 6        | 3        | 9         | 40           |
| Post-1984    | 25         | 2      | 6       | 19      | 7        | 11       | 1         | 28           |
| Nominal rate |            |        |         |         |          |          |           |              |
|              | Bank. cost | Wealth | Neutral | I-shock | P-Markup | W-markup | Intertemp | Monet Policy |
| Pre-1970     | 3          | 58     | 3       | 5       | 9        | 1        | 20        | 1            |
| 1970-1984    | 4          | 61     | 2       | 6       | 12       | 1        | 12        | 2            |
| Post-1984    | 38         | 4      | 4       | 33      | 11       | 9        | 0         | 1            |

*Notes:* It corresponds to periodic components of cycles between 6 and 32 quarters.

B figures

Figure B-1: Debt to net worth ratio. Cyclical component.

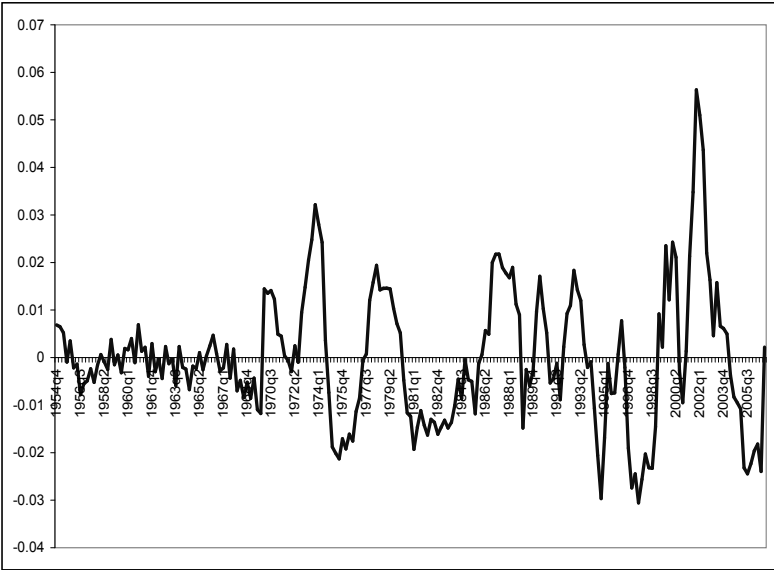


Figure B-2: Impulse Response Functions with respect to a wealth shock. Identical response upon impact of net worth across sub-samples. The solid line is the IRF for the 1954-1969 period, the dashed line is the IRF for 1970-1983, and the starred line is the IRF for the post-1984 period.

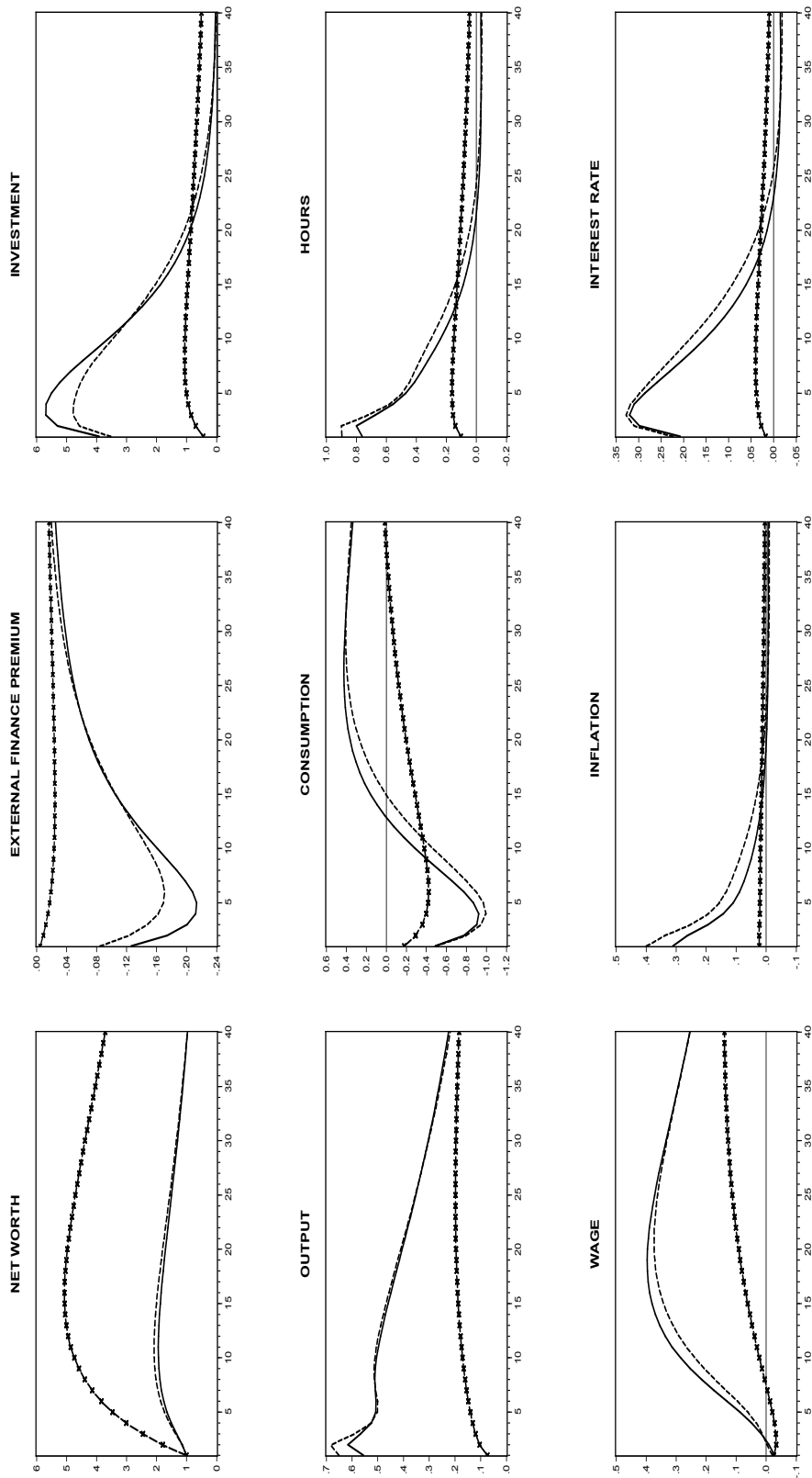




Figure B-3: Impulse Response Functions with respect to a wealth shock. Identical response upon impact of net worth across sub-samples. The solid line is the IRF for the 1954-1969 period, the dashed line is the IRF for 1970-1983, and the starred line is the IRF for the post-1984 period.

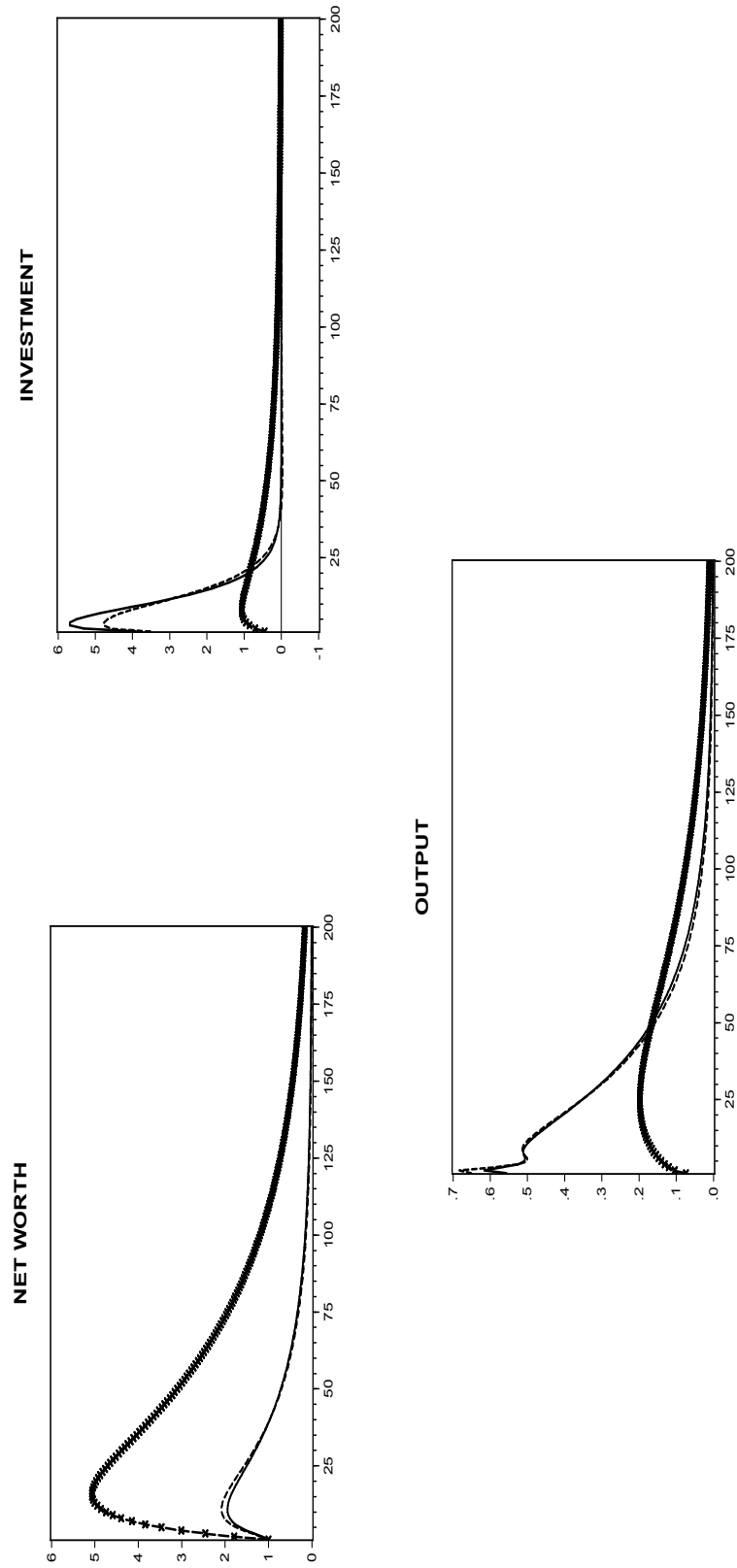


Figure B-4: **Impulse Response Functions with respect to a wealth shock. Identical shock across sub-samples..** The solid line is the IRF for the 1954-1969 period, the dashed line is the IRF for 1970-1983, and the starred line is the IRF for the post-1984 period.

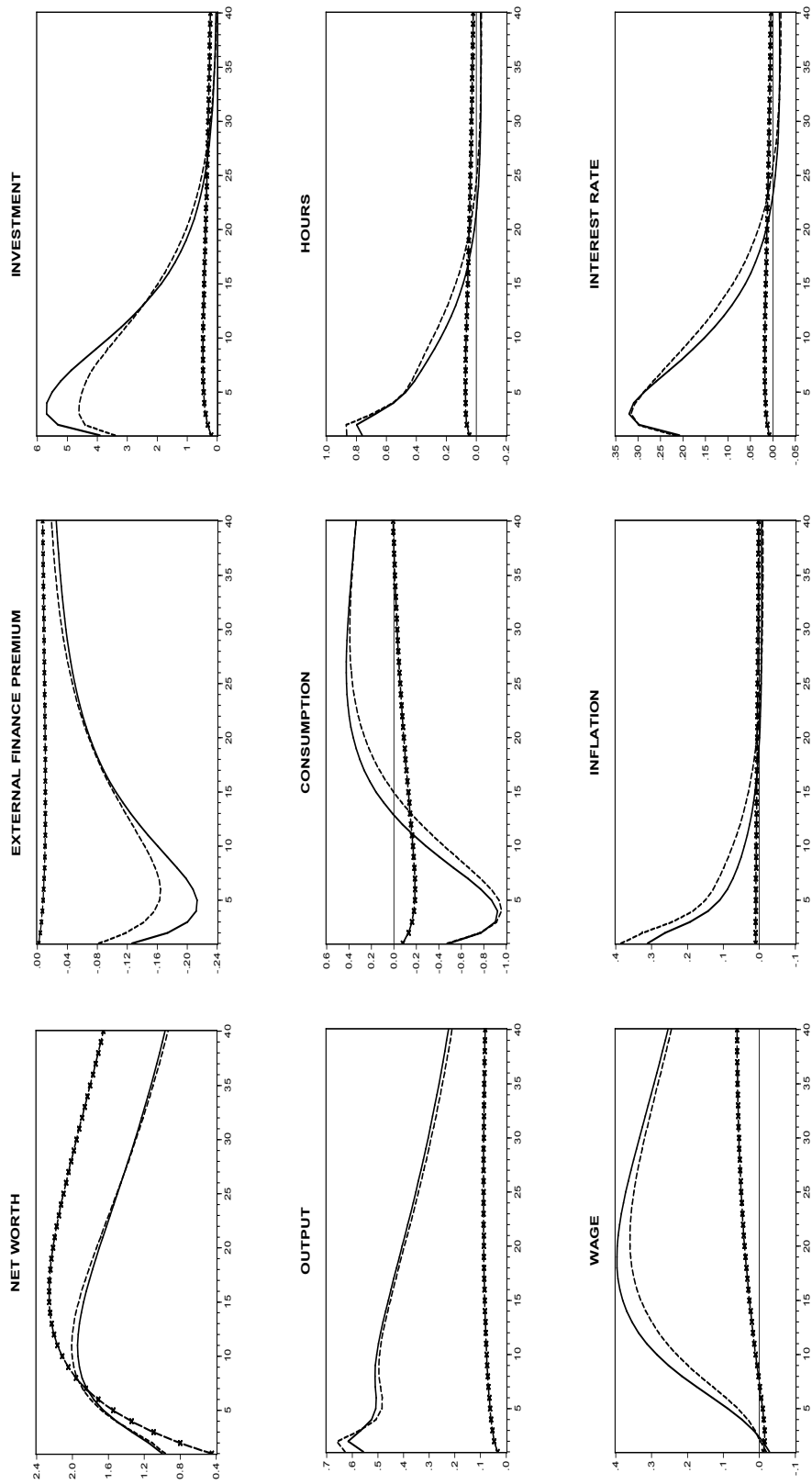


Figure B-5: Impulse Response Functions with respect to the marginal bankruptcy cost. The solid line is the IRF for the 1954-1969 period, the dashed line is the IRF for 1970-1983, and the starred line is the IRF for the post-1984 period.

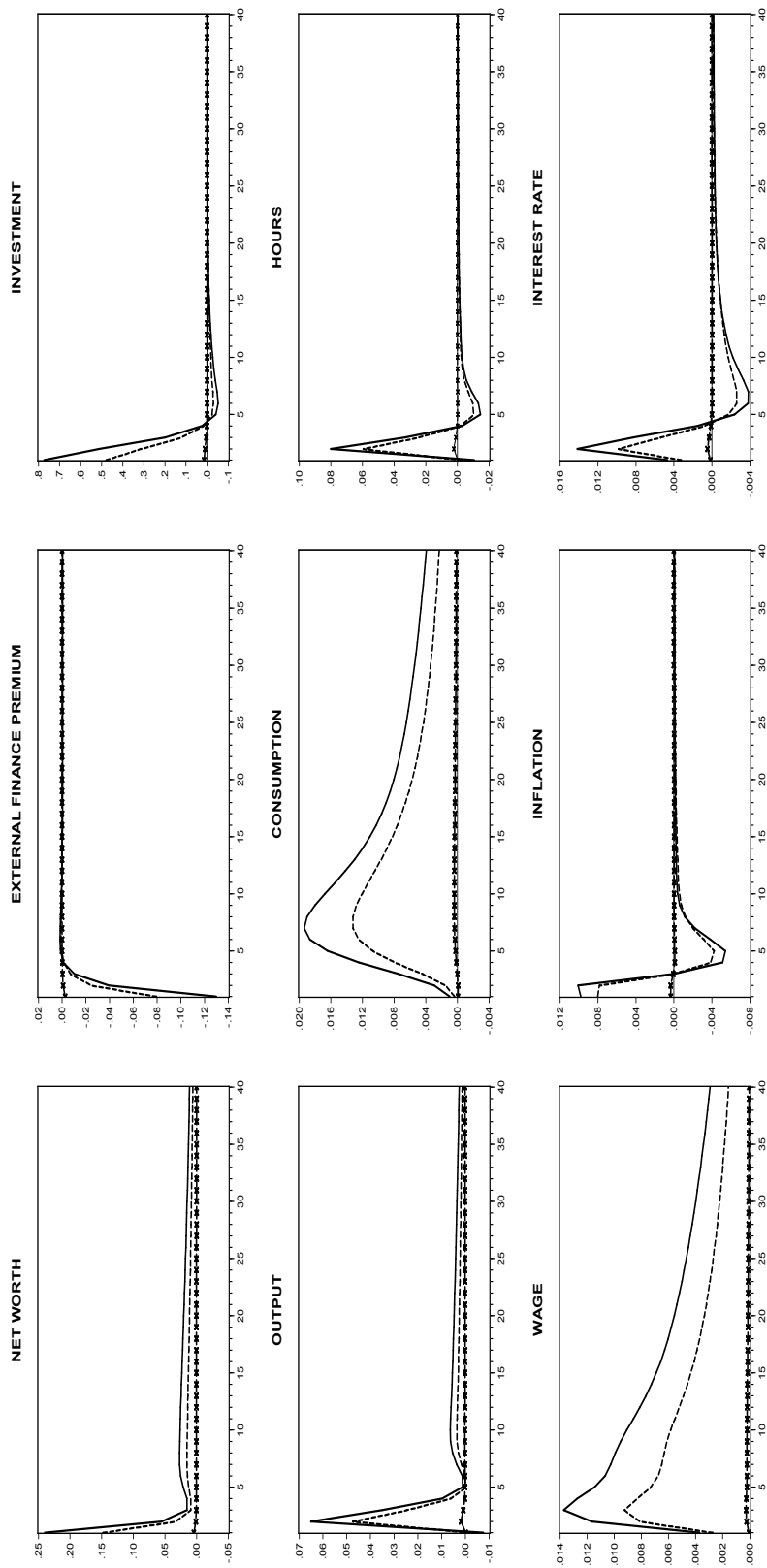
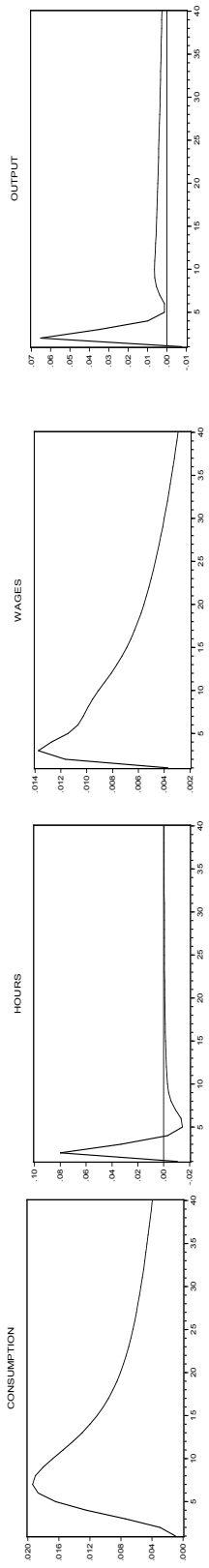
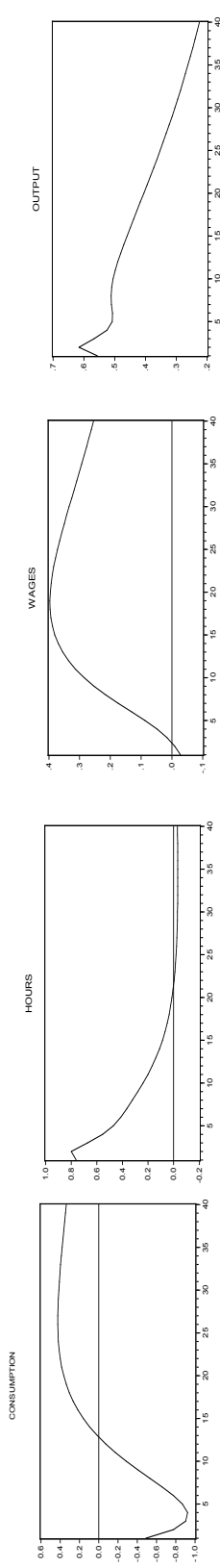


Figure B-6: Impulse Response Functions: A comparison

SHOCK TO BANKRUPTCY COST



WEALTH SHOCK



## C Data

We use US data from NIPA-BEA, CPS-BLS, the FRED database, and the Flow of Funds Accounts from the Federal Reserve Board for the period 1954.4-2006.4.

### C.1 Data used in estimation

- *Growth rate of real per capita gross value added by the nonfarm business sector.* Data on nominal gross value added are available in NIPA Table 1.3.5. We have deflated such a series using the the implicit price index from table 1.3.4. We divide the new series by the Civilian Noninstitutional +16 (BLS ID LNU00000000) series to obtain per capita variables. The data provided by the BEA are annualized so we divide by 4 to obtain quarterly values for the measures of interest.
- *Growth rate of real per capita investment.* Investment is defined as gross private domestic investment from NIPA Table 1.1.5. We deflate the nominal variables using the GDP deflator provided by NIPA Table 1.1.4. We weight the resulting series using the relative significance of the nonfarm business sector in total GDP. Finally, we do the same correction described above to render the investment series in per capita quarterly terms.
- *Growth rate of real per capita consumption.* Consumption is defined as the sum of personal consumption expenditures of nondurables and services from NIPA Table 1.1.5. We deflate the nominal variables using the GDP deflator provided by NIPA Table 1.1.4. We weight the resulting series using the relative significance of the non-farm business sector in total GDP. Finally, we do the same correction described above to have the series in per capita quarterly terms.
- *Growth rate of net worth.* We define net worth as the real per capita weighted average of net worth for the corporate and noncorporate nonfarm business sector. To ensure the measure of net worth from the data is close enough to the series the model can actually account for, we define net worth as tangible assets minus credit market instruments at market value. On the one hand, we use tangible assets only as a measure for assets because, in our model, collateral is related only to physical capital and inventories; that is, there is no role for financial capital. On the other hand, we evaluate net worth at current (market) prices, since such a variable in our theoretical framework stands for the value of the collateral perceived by lenders. Credit market liabilities from the Flow of Funds Accounts (the weighted sum of series FL104104005.Q from Table B.102 and series FL114102005.Q from Table B.103) stand for entrepreneurial debt. Tangible assets are given by the weighted sum of series FL102010005.Q from Table B.102 and series FL112010005.Q from Table B.103.

- *Hours worked* is defined, following Smets and Wouters (2007), as the log level of the BLS series PRS85006023 divided by 100 and multiplied by the ratio of civilian population over 16 (CE16OV) to a population index. The population index is equal to the ratio of population at the corresponding quarter divided by the population in the third quarter of 1992. This transformation is necessary, since the series on hours is an index with 1992=100.
- *Growth rate of real wages* . Real wages are defined as the real compensation per hour in the nonfarm business sector (COMPRNFB) provided by the BLS.
- *Inflation* is defined as the log difference of the price index for gross value added by the nonfarm business sector (NIPA Table 1.3.4).
- The *Federal funds rate* is taken from the Federal Reserve Economic Data (FRED).

## C.2 Data used in the empirical evidence section

In addition to the series described above, we also consider the following ones

- *Net private savings*: Data on nominal net private savings are available in the NIPA Table 5.1. We have deflated such a series using the implicit price index from Table 1.3.4. We divide the new series by the Civilian Noninstitutional +16 (BLS ID LNU00000000) series to obtain per capita variables. The data provided by the BEA are annualized, so we divide by 4 to obtain quarterly values for the measures of interest. We weight the resulting series using the relative significance of the nonfarm business sector in total GDP.
- *Debt in the nonfarm business sector*: We define debt as the real per capita weighted average of credit market liabilities for the corporate and noncorporate nonfarm business sector. Debt is defined as the weighted sum of series FL104104005.Q from Table B.102 and series FL114102005.Q from Table B.103.
- *Net worth of households (and nonprofit organizations)*: It is given by the real per capita transformation of the series FL152090005 from Table B.100 from the Flow of Funds Accounts.
- *Demand deposits*: It stands for real per capita demand deposits at commercial banks provided by the series DEMDEPSL in the FRED database. Data are available from 1959.

## D Methodology

### D.1 MCMC Algorithm

1. **Posterior Maximization:** The aim of this step is to obtain the parameter vector to initialize our posterior simulator. To obtain the posterior mode,  $\tilde{\varrho}$ , we iterate over the following steps:
  - (a) Fix a vector of structural parameters  $\varrho'$ .
  - (b) Solve the DSGE model conditional on  $\varrho'$  and compute the system matrices. We restrict ourselves to the determinacy region of the parameter space.
  - (c) Use the Kalman filter to compute the likelihood of the parameter vector  $\varrho'$ ,  $p(Y^T|\varrho')$ .
  - (d) Combine the likelihood function with the prior distribution.
2. Compute the **numerical Hessian** at the posterior mode. Let  $\tilde{\Sigma}$  be the inverse of such a numerical hessian.
3. Draw the initial parameter vector,  $\varrho^{(0)}$ , from  $\mathcal{N}(\tilde{\varrho}^{(0)}, c_0^2 \tilde{\Sigma})$  where  $c_0$  is a scaling parameter. Otherwise, directly specify a starting value for the posterior simulator.
4. **Posterior Simulator:** for  $s = 1, \dots, n_{sim}$ , draw  $\vartheta$  from the proposal distribution  $\mathcal{N}(\varrho^{(s-1)}, c^2 \tilde{\Sigma})$ , where  $c$  is a scaling parameter<sup>5</sup>. The jump from  $\varrho^{(s-1)}$  is accepted with probability

$$\min\{1, r(\varrho^{(s-1)}, \vartheta|Y)\}$$

and rejected otherwise. Note that

$$r(\varrho^{(s-1)}, \vartheta|Y) = \frac{\mathcal{L}(\vartheta|Y)p(\vartheta)}{\mathcal{L}(\varrho^{(s-1)}|Y)p(\varrho^{(s-1)})} \quad (\text{A.1})$$

5. Approximate the expected value of a function  $h(\varrho)$  by

$$\frac{1}{n_{sim}} \sum_{s=1}^{n_{sim}} h(\varrho^{(s)})$$

### D.2 Kalman Filter

Let us cast the log-linearized dynamic system in state-space form:

---

<sup>5</sup>The scale factor is set to obtain efficient algorithms. Gelman, Carlin, Stern, and Rubin (2004) argue that the scale coefficient should be set  $c \approx 2.4\sqrt{d}$ , where  $d$  is the number of parameters to be estimated. However, we will fine tune the scale factor to obtain a rejection rate of about 25%

- Transition equation:

$$\begin{aligned} s_t &= (I - T) \ln(\bar{s}) + T s_{t-1} + R \epsilon_t \\ &= J + T s_{t-1} + R \epsilon_t \end{aligned}$$

where  $T = \Phi(\rho)$ ,  $R = \Phi_\epsilon$ ,  $\epsilon \sim (\mathbf{0}, Q)$ ,  $s_t$  stand for the vector of DSGE state variables in log levels, and  $\ln(\bar{s})$  is the vector of steady-state log-values of these state variables. Let  $\mathfrak{s} = \text{rows}(s_t)$ .

- Measurement equation:

$$y_t = Z s_t$$

where  $Z = B(\rho)$  and we have imposed the assumption of zero measurement error in the system.

Linearity and Gaussian errors allow us to use the Kalman filter to evaluate the likelihood function. We give an overview here of such a filter; for a complete description, please see Chapter 13 in Hamilton (1994).

1. **Initialization:** The filter is initialized with the unconditional distribution of the state vector.

- Initial mean:

$$\hat{s}_{0|0} = \ln(\bar{s})$$

- Initial variance:  $P_{0|0}$  is given by the solution to the following discrete Lyapunov equation

$$P_{0|0} = T P_{0|0} T' + R Q R'$$

2. **Forecasting step**

$$\begin{aligned} \hat{s}_{t|t-1} &= J + T \hat{s}_{t-1|t-1} \\ P_{t|t-1} &= T P_{t-1|t-1} T' + R Q R' \\ \hat{y}_{t|t-1} &= Z \hat{s}_{t|t-1} \\ F_{t|t-1} &= Z P_{t|t-1} Z' \end{aligned}$$

3. **Evaluation of the log-likelihood**



#### 4. Updating step

$$\begin{aligned}\widehat{s}_{t|t} &= \widehat{s}_{t|t-1} + P_{t|t-1} Z' F_{t|t-1}^{-1} (y_t^{obs} - \widehat{y}_{t|t-1}) \\ P_{t|t} &= P_{t-1|t-1} - P_{t-1|t-1} Z' F_{t|t-1}^{-1} Z' P_{t-1|t-1}\end{aligned}$$

So far, we have assumed that the system matrices were all constant. The Kalman filter, however, is also suitable for state-space models in which those matrices vary over time. The filter needs simply to be modified so that the appropriate matrix is used at each  $t$ . Given that the state-space system under analysis is a reduced-form representation of a structural model, we should be careful when extending the filter to allow for breaks in the system matrices. Note that if we allow for structural breaks in the size of the shocks and/or the monetary policy coefficients, the system matrices vary but there is no effect on the definition of the steady state of the economy. However, if there is a break in a parameter defining the steady state of our model economy, the econometrician needs to make sure she is using the same information set as the economic agents.

Let us assume there is a shift in the steady state of the economy so that we go from  $\bar{s}_1$  to  $\bar{s}_2$ . This implies a shift in the entries of  $T$  and, hence,  $J$ . We need to introduce the following modification in the forecasting step

- If  $t < t^*$ ,

$$\begin{aligned}\widehat{s}_{t|t-1} &= J_1 + T_1 \widehat{s}_{t-1|t-1} \\ P_{t|t-1} &= T_1 P_{t-1|t-1} T_1' + R Q_1 R'\end{aligned}$$

- If  $t = t^*$ ,

$$\begin{aligned}\widehat{s}_{t|t-1} &= J_2 + T_2 (\ln(\bar{s}_2) - \ln(\bar{s}_1)) + T_2 \widehat{s}_{t-1|t-1} \\ P_{t|t-1} &= T_2 P_{t-1|t-1} T_2' + R Q_2 R'\end{aligned}$$

- If  $t > t^*$ ,

$$\begin{aligned}\widehat{s}_{t|t-1} &= J_2 + T_2 \widehat{s}_{t-1|t-1} \\ P_{t|t-1} &= T_2 P_{t-1|t-1} T_2' + R Q_2 R'\end{aligned}$$

### D.3 Variance Decomposition

Our data set contains the following series

$$\{\Delta Y, \Delta I, \Delta C, \Delta N, \log(H), \log(LS), \log\left(1 + \frac{\pi}{400}\right), \log\left(1 + \frac{R^n}{400}\right)\}$$

We are interested, however, in the second moments and dynamic properties of

$$\log(Y), \log(I), \log(C), \log(N), \log(H), \log(LS), \log\left(1 + \frac{\pi}{400}\right), \log\left(1 + \frac{R^n}{400}\right)\}$$

Therefore, we use an inverse difference filter for the first four components on the spectrum implied by the DSGE model. The spectral density is obtained using the state-space representation of the DSGE model and 500 bins for frequencies in the range of periodicities of interest. In particular, we compute the variance decomposition at business cycle frequencies, that is, we focus on those periodic components with cycles between 6 and 32 quarters.

#### Inverse difference filter

Let  $X_t$  be univariate data in log-levels and  $Y_t = (1 - L) X_t$ . Note that

$$\begin{aligned} X_t &= (1 - L)^{-1} Y_t \\ &= \sum_{h=0}^{\infty} L^h Y_{t-h} \\ &= \sum_{h=0}^{\infty} \exp(-i\omega j h) Y_{t-h} \end{aligned}$$

Then, the spectral density of  $X_t$  is given by

$$s_X(\omega) = \left| \sum_{h=0}^{\infty} \exp(-i\omega j h) \right|^2 s_Y(\omega)$$

which can be approximated by

$$s_X(\omega) = \left| \frac{1}{1 - \exp(-i\omega j h)} \right|^2 s_Y(\omega)$$

at any frequency by 0.

## E Log-linearized equilibrium conditions

Let  $\tilde{Y}_t = \frac{Y_t}{Z_{a,t}}$  for  $C, I, K, G, W/P, B_{t+1}/P_t, NB_{t+1}/P_t, D_{t+1}/P_t, div, T, N_{t+1}$ . Let  $\hat{\varsigma} = \log\left(\frac{\varsigma}{\varsigma^*}\right)$  where  $\varsigma^*$  is the steady state value of the variable  $\varsigma$ .

The following equations can be solved for the 27 variables  $w_t, \bar{w}_t, \pi_t, H_t, \Lambda_t, \lambda_t^w, \lambda_t^p, \mathfrak{Z}_t, R_t, C_t, b_t, Q_t, I_t, K_t, k_t, \zeta_t, G_t, R_t^k, r_t^k, Y_t, u_t, \chi_t, B_{t+1}, \bar{w}_t, N_{t+1}, \mu_t, x_t$

### 1. Exogenous stochastic processes

#### (a) Inter-temporal preference shock

$$\hat{b}_t = \rho_b \hat{b}_{t-1} + \varepsilon_{b,t}$$

#### (b) Neutral technology shock

$$\hat{\mathfrak{Z}}_t = \rho_z \hat{\mathfrak{Z}}_{t-1} + \varepsilon_{a,t}$$

#### (c) Investment specific technology shock

$$\hat{\zeta}_t = \rho_\zeta \hat{\zeta}_{t-1} + \varepsilon_{\zeta,t}$$

#### (d) Price markup shock

$$\hat{\lambda}_t^p = \rho_{\lambda^p} \hat{\lambda}_{t-1}^p + \varepsilon_{\lambda^p,t}$$

#### (e) Wage markup shock

$$\hat{\lambda}_t^w = \rho_{\lambda^w} \hat{\lambda}_{t-1}^w + \varepsilon_{\lambda^w,t}$$

#### (f) Monetary policy shock

$$\varepsilon_{R,t}$$

#### (g) Shock to the marginal bankruptcy cost

$$\hat{\mu}_t = \rho_\mu \hat{\mu}_{t-1} + \varepsilon_{\mu,t}$$

#### (h) Wealth shock

$$\hat{x}_t = \rho_x \hat{x}_{t-1} + \varepsilon_{x,t}$$

2. Optimal wage set by households

$$\begin{aligned}\widehat{w}_t &= \frac{1}{1+\beta}\widehat{w}_{t-1} + \frac{\beta}{1+\beta}\mathbb{E}_t\widehat{w}_{t+1} - \kappa_w \left[ \widehat{w}_t - \left( \nu\widehat{H}_t + \widehat{b}_t - \widehat{\Lambda}_t + \widehat{\lambda}_{w,t} \right) \right] \\ &\quad + \frac{\iota_w}{1+\beta}\widehat{\pi}_{t-1} - \frac{1+\beta\iota_w}{1+\beta}\widehat{\pi}_t + \frac{\beta}{1+\beta}\mathbb{E}_t\widehat{\pi}_{t+1} + \frac{\iota_w}{1+\beta}\widehat{\mathfrak{Z}}_{t-1} - \frac{1+\beta\iota_w - \rho_z\beta}{1+\beta}\widehat{\mathfrak{Z}}_t\end{aligned}$$

where

$$\kappa_w = \frac{(1 - \xi_w\beta)(1 - \xi_w)}{\xi_w(1 + \beta) \left( 1 + \nu \left( 1 + \frac{1}{\lambda_w} \right) \right)}$$

3. Euler equation for deposits

$$\widehat{\Lambda}_t = \widehat{R}_t + \mathbb{E}_t \left[ \widehat{\Lambda}_{t+1} - \widehat{\pi}_{t+1} - \widehat{\mathfrak{Z}}_{t+1} \right]$$

4. Euler equation for nominal bonds

$$\widehat{\Lambda}_t = \widehat{R}_t^n + \mathbb{E}_t \left[ \widehat{\Lambda}_{t+1} - \widehat{\pi}_{t+1} - \widehat{\mathfrak{Z}}_{t+1} \right]$$

5. Household's FOC with respect to  $C_t$

$$\begin{aligned}\widehat{\Lambda}_t &= \left( \frac{\rho_z\beta h\mathfrak{Z}_* - \mathfrak{Z}_*h}{(\mathfrak{Z}_* - \beta h)(\mathfrak{Z}_* - h)} \right) \widehat{\mathfrak{Z}}_t + \left( \frac{\mathfrak{Z}_* - \beta h\rho_b}{\mathfrak{Z}_* - \beta h} \right) \widehat{b}_t - \left( \frac{\mathfrak{Z}_*^2 + \beta h^2}{(\mathfrak{Z}_* - \beta h)(\mathfrak{Z}_* - h)} \right) \widehat{C}_t \\ &\quad + \left( \frac{\mathfrak{Z}_*h}{(\mathfrak{Z}_* - \beta h)(\mathfrak{Z}_* - h)} \right) \widehat{C}_{t-1} + \left( \frac{\beta h\mathfrak{Z}_*}{(\mathfrak{Z}_* - \beta h)(\mathfrak{Z}_* - h)} \right) \mathbb{E}_t\widehat{C}_{t+1}\end{aligned}$$

6. Price of capital (from capital producers)

$$\widehat{Q}_t + \widehat{\zeta}_t = \mathfrak{Z}_*^2\xi \left[ \widehat{I}_t - \widehat{I}_{t-1} + \widehat{\mathfrak{Z}}_t \right] - \beta\mathfrak{Z}_*^2\xi\mathbb{E}_t \left[ \widehat{I}_{t+1} - \widehat{I}_t + \widehat{\mathfrak{Z}}_{t+1} \right]$$

7. Capital accumulation

$$\widehat{K}_{t+1} = \frac{1 - \delta}{\mathfrak{Z}_*} \left( \widehat{K}_t - \widehat{\mathfrak{Z}}_t \right) + \frac{\widetilde{I}^*}{\widetilde{K}^*} \left( \widehat{\zeta}_t + \widehat{I}_t \right)$$

8. New Keynesian Phillips curve

$$\widehat{\pi}_t = \frac{(1 - \xi_p\beta)(1 - \xi_p)}{(1 + \iota_p\beta)\xi_p} \left[ \widehat{\chi}_t + \frac{\lambda^p}{1 + \lambda^p}\widehat{\lambda}_t^p \right] + \frac{\iota_p}{1 + \iota_p\beta}\widehat{\pi}_{t-1} + \frac{\beta}{1 + \iota_p\beta}\mathbb{E}_t\widehat{\pi}_{t+1}$$

9. Government constraint

$$\widehat{G}_t = \widehat{Y}_t$$

10. Taylor rule

$$\widehat{R}_t^n = \rho_R \widehat{R}_{t-1}^n + (1 - \rho_R) \rho_\pi \widehat{\pi}_t + (1 - \rho_R) \rho_Y \left( \widehat{Y}_t - \widehat{Y}_{t-1} + \widehat{\mathfrak{Z}}_t \right) + \varepsilon_{R,t}$$

11. Definition of effective capital

$$\widehat{k}_t = \widehat{u}_t + \widehat{K}_t - \widehat{\mathfrak{Z}}_t$$

12. Optimal capital utilization

$$\widehat{r}_t^k = \frac{a_\star''}{r_\star^k} \widehat{u}_t$$

13. Production technology

$$\widehat{Y}_t = \alpha \widehat{k}_t + (1 - \alpha) \widehat{H}_t$$

14. Optimal capital-to-labor ratio for intermediate goods producers

$$\widehat{k}_t = \widehat{w}_t - \widehat{r}_t^k + \widehat{H}_t$$

15. Real marginal cost

$$\widehat{\chi}_t = (1 - \alpha) \widehat{w}_t + \alpha \widehat{r}_t^k$$

16. Average return on capital

$$\widehat{R}_t^k = \frac{r_\star^k}{R_\star^k} \widehat{r}_t^k + \frac{r_\star^k}{R_\star^k} \widehat{u}_t + \frac{(1 - \delta)}{R_\star^k} \widehat{Q}_t - \widehat{Q}_{t-1}$$

17. Amount of the loan

$$\widehat{Q}_t + \widehat{K}_{t+1} = \frac{\widetilde{B}_\star}{\widetilde{K}_\star} \widehat{B}_{t+1} + \frac{\widetilde{N}_\star}{\widetilde{K}_\star} \widehat{N}_{t+1}$$

18. Supply for loans or external finance premium

$$\widehat{R}_t^k - \widehat{R}_{t-1} = \widehat{B}_t - \widehat{Q}_{t-1} - \widehat{K}_t - \frac{\Gamma_\omega(\bar{\omega}_\star) - \mu_\star G_\omega(\bar{\omega}_\star)}{[\Gamma(\bar{\omega}_\star) - \mu_\star G(\bar{\omega}_\star)]} \bar{\omega}_\star \widehat{\omega}_t - \frac{G(\bar{\omega}_\star)}{[\Gamma(\bar{\omega}_\star) + \mu_\star G(\bar{\omega}_\star)]} \mu_\star \widehat{\mu}_t$$

19. Net worth

$$\begin{aligned}\widehat{N}_{t+1} = & \widehat{x}_t + \frac{\gamma R_\star^k (1 - \mu_\star G(\bar{\omega}_\star)) \widetilde{K}_\star}{\mathfrak{Z}_\star \widetilde{N}_\star} \left[ \widehat{R}_t^k + \widehat{K}_t - \widehat{\mathfrak{Z}}_t + \widehat{Q}_{t-1} \right] \\ & - \frac{\gamma R_\star \widetilde{B}_\star}{\mathfrak{Z}_\star \widetilde{N}_\star} \left[ \widehat{R}_{t-1} + \widehat{B}_t - \widehat{\mathfrak{Z}}_t \right] - \frac{\gamma \mu_\star G(\bar{\omega}_\star) R_\star^k \widetilde{K}_\star}{\mathfrak{Z}_\star \widetilde{N}_\star} \widehat{\mu}_t \\ & - \frac{\gamma \mu_\star G_\omega(\bar{\omega}_\star) R_\star^k \widetilde{K}_\star}{\mathfrak{Z}_\star \widetilde{N}_\star} \bar{\omega}_\star \widehat{\omega}_t\end{aligned}$$

20. First-order condition with respect to the debt-to-wealth ratio

$$\Phi_r \left[ \mathbb{E}_t \widehat{R}_{t+1}^k - \widehat{R}_t \right] = \Phi_\omega \bar{\omega}_\star \mathbb{E}_t \widehat{\omega}_{t+1} + \Phi_\mu \mu_\star \mathbb{E}_t \widehat{\mu}_{t+1}$$

where

$$\begin{aligned}\Phi_r &= [(1 - \Gamma(\bar{\omega}_\star)) + \Psi(\bar{\omega}_\star, \mu_\star) (\Gamma(\bar{\omega}_\star) - \mu_\star G(\bar{\omega}_\star))] \frac{R_\star^k}{R_\star} \\ \Phi_\omega &= \frac{R_\star^k}{R_\star} [\Gamma_\omega(\bar{\omega}_\star) (1 - \Psi(\bar{\omega}_\star, \mu_\star)) - \Psi_\omega(\bar{\omega}_\star, \mu_\star) (\Gamma(\bar{\omega}_\star) - \mu_\star G(\bar{\omega}_\star)) + \mu_\star \Psi(\bar{\omega}_\star, \mu_\star) G_\omega(\bar{\omega}_\star)] \\ &\quad + \psi_\omega(\bar{\omega}_\star, \mu_\star) \\ \Phi_\mu &= \Psi_\mu(\bar{\omega}_\star, \mu_\star) - \frac{R_\star^k}{R_\star} (\Psi_\mu(\bar{\omega}_\star, \mu_\star) (\Gamma(\bar{\omega}_\star) - \mu_\star G(\bar{\omega}_\star)) - \Psi(\bar{\omega}_\star, \mu_\star) G(\bar{\omega}_\star))\end{aligned}$$

21. Market clearing conditions

(a) Credit market

$$\widehat{B}_{t+1} = \widehat{D}_{t+1}$$

(b) Total Resources

$$\begin{aligned}\widehat{Y}_t = & \frac{\widetilde{C}_\star}{\widetilde{Y}_\star} \widehat{C}_t + \frac{\widetilde{I}_\star}{\widetilde{Y}_\star} \widehat{I}_t + \frac{\widetilde{G}_\star}{\widetilde{Y}_\star} \widehat{G}_t \\ & + \frac{\mu_\star G(\bar{\omega}_\star) R_\star^k \widetilde{K}_\star}{\widetilde{Y}_\star \mathfrak{Z}_\star} \left[ \widehat{R}_t^k + \widehat{Q}_{t-1} + \widehat{K}_t - \widehat{\mathfrak{Z}}_t + \frac{G_\omega(\bar{\omega}_\star)}{G(\bar{\omega}_\star)} \bar{\omega}_\star \widehat{\omega}_t \right] \\ & + \mu_\star R_\star^k \frac{G(\bar{\omega}_\star) \widetilde{K}_\star}{\widetilde{Y}_\star \mathfrak{Z}_\star} \widehat{\mu}_t + r_\star^k \frac{\widetilde{K}_\star}{\widetilde{Y}_\star} \widehat{u}_t\end{aligned}$$

22. Some definitions

$$\begin{aligned}
 F(\bar{\omega}) &= \int_0^{\bar{\omega}} \frac{1}{\omega \sigma_\omega \sqrt{2\pi}} e^{-\frac{(\ln(\omega)+0.5\sigma_\omega^2)^2}{2\sigma_\omega^2}} d\omega = \Phi\left(\frac{\ln(\bar{\omega}) + 0.5\sigma_\omega^2}{\sigma_\omega}\right) \\
 F_\omega(\bar{\omega}) &= \frac{1}{\bar{\omega} \sigma_\omega \sqrt{2\pi}} e^{-\frac{(\ln(\bar{\omega})+0.5\sigma_\omega^2)^2}{2\sigma_\omega^2}} \\
 F_{\omega\omega}(\bar{\omega}) &= -\frac{1}{\bar{\omega}} F_\omega(\bar{\omega}) \left[1 + \frac{\ln(\bar{\omega}) + 0.5\sigma_\omega^2}{\sigma_\omega^2}\right] \\
 G(\bar{\omega}) &= \int_0^{\bar{\omega}} \omega f(\omega) d\omega = 1 - \Phi\left(\frac{0.5\sigma_\omega^2 - \ln(\bar{\omega})}{\sigma_\omega}\right) \\
 G_\omega(\bar{\omega}) &= \bar{\omega} F_\omega(\bar{\omega}) \\
 \Gamma(\bar{\omega}) &= \int_0^{\bar{\omega}} \omega f(\omega) d\omega + \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega = \bar{\omega} (1 - F(\bar{\omega})) + G(\bar{\omega}) \\
 \Gamma_\omega(\bar{\omega}) &= 1 - F(\bar{\omega}) \\
 \Psi(\bar{\omega}, \mu) &= \frac{\Gamma_\omega(\bar{\omega})}{\Gamma_\omega(\bar{\omega}) - \mu G_\omega(\bar{\omega})} \\
 \Psi_\omega(\bar{\omega}, \mu) &= \frac{-F_\omega(\bar{\omega}) [1 - F(\bar{\omega}) - \mu \bar{\omega} F_\omega(\bar{\omega})] - [1 - F(\bar{\omega})] [-F_\omega(\bar{\omega}) - \mu F_\omega(\bar{\omega}) - \mu \bar{\omega} F_{\omega\omega}(\bar{\omega})]}{(1 - F(\bar{\omega}) - \mu \bar{\omega} F_\omega(\bar{\omega}))^2} \\
 \Psi_\mu(\bar{\omega}, \mu) &= \frac{G_\omega(\bar{\omega}) \Psi(\bar{\omega}, \mu)}{\Gamma_\omega(\bar{\omega}) - \mu G_\omega(\bar{\omega})}
 \end{aligned}$$