# Monetary Policy and Asset Prices with Belief-Driven Fluctuations and News Shocks

Marco Airaudo, Roberta Cardani, Kevin J. Lansing<sup>‡</sup>

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#### Abstract

We present a New-Keynesian DSGE model where stock price fluctuations have real effects both via the demand and the supply side. Direct wealth effects on aggregate consumption arise because of a constant turnover between long-time traders and newcomers in financial markets. The presence of credit market frictions and costly loan generation on the supply side implies a direct impact of stock prices on marginal costs and hence inflation.

After calibrating the economy to capture some key features of the 1990-2007 US data, we show that strict inflation targeting induces equilibrium indeterminacy, even if the policy rule satisfies the Taylor principle. Our numerical analysis shows that belief shocks originating from the stock market can account for the observed relative volatilities of some key financial variables in the data.

We show that monetary policy can eliminate the non-fundamental (stock market related) aggregate fluctuations by including a mild response to the stock price index in its policy rule. Furthermore, in addition to restoring determinacy, the policy response to stock prices can also smooth boom-bust cycles generated by new shocks to fundamentals.

Keywords: equilibrium determinacy, asset prices, cost channel, monetary policy, credit spread JEL classification: E4, E5

# 1 Introduction

The recent financial crisis has highlighted the interaction between financial frictions and aggregate fluctuations. Although how to understand and deal with financial crises is likely to remain the subject of debates

<sup>\*</sup>LeBow College of Business, Drexel University, Philadelphia, PA 19146. Email: ma639@drexel.edu.

<sup>&</sup>lt;sup>†</sup>University of Parma, via dell'Università, 12 43100 Parma, Italy. Email: robertacardan@unipr.it.

<sup>&</sup>lt;sup>‡</sup>Federal Reserve Bank of San Francisco, 101 Market Street, San Francisco, 94105 CA. Email: Kevin.J.Lansing@sf.frb.org.

for quite some time, an emerging view calls for putting more emphasis on financial markets and banking in the New Keynesian DSGE model, the current workhorse of monetary policy analysis.

The way the literature has introduced financial issues into the benchmark framework is twofold. On the one hand, researchers have worked on several extentions of the seminal financial accelerator idea of Bernanke and Gertler (1999), Carlstrom and Fuerst (1997) and Kiyotaki and Moore (1997). In these models, potential investors need external financing, but their borrowing capacity is limited by their own net worth. As the value of the latter depends on asset prices, stock market fluctuations have a direct impact on the amount of credit available to firms, and hence on real activity. An exogenous adverse shock to asset prices might then initiate a loss spiral, in which financially constrained firms would be forced to sell some of their assets to fulfil collateral requirements, which would further depress their prices. With the value of own net worth going down, firms would eventually be forced to default on outstanding liabilities, with direct negative consequences for real activity.<sup>1</sup>

On the other hand, a relatively more recent line of research has stressed the role of demand-side wealth effects on real activity. Building on Iacoviello (2005), DSGE models featuring a housing sector can generate spillovers from the housing market to consumer spending through the collateral effects of housing values on private borrowings. A financial accellerator mechanism based on housing can amplify the propagation of real shocks into the macroeconomy.<sup>2</sup> Although the two structures share many common features, the housing-sector DSGE models have been mostly used to capture some of the key macroeconomic trends since 2001, while the financial accellerator models à la Bernanke-Gertler were originally conceived to deal with earlier stock price run-ups in Japan and the U.S.

Despite the remarkable amount of extensions and improvements, in our opinion, the financial accellerator paradigm presents some drawbacks. First, the focus of the analysis has been either on supply side or demand side financial frictions. The complexity of the financial accellerator paradigm makes it rather hard to incorporate both frictions into the same model without loosing trasparency and simple economic intuition. Second, in both cases, the analysis has been mostly concerned with the transmission of real shocks, while there is no lack of empirical evidence showing that asset prices - the key component of these models appear to display volatilities much larger than can be justified by the underlying fundamentals. One of the few exceptions is Bernanke and Gertler (1999), where a non-fundamental (near-rational) exogenous bubble component is added to stock prices. However, because of exogeneity, monetary policy has no means to deflate it.

The aim of this paper is to develop a New-Keynesian DSGE model subject to credit frictions and structural

<sup>&</sup>lt;sup>1</sup>The literature on the financial accellerator is extremely wide and in continous expansion. It would be impossible to cite all the valuable contributions. Among them, the consequences of an explicit policy response to asset prices are studied by Faia and Monacelli (2007) and Singh and Stone (2011).

<sup>&</sup>lt;sup>2</sup>Iacoviello and Neri (2009) extend Iacoviello (2005) to include an explicit housing sector.

linkages between the stock market and the macroeconomy. We present a framework which is capable of preserving sufficient tractability while giving an explicit consideration of stock prices as a non-redundant variable for the business cycle. Our main objective is to assess whether, by setting monetary policy in response to stock price fluctuations, the central bank can completely eliminate or at least smooth expectations-driven fluctuations. The analysis focuses on two types of expectations-driven shocks: non-fundamental belief shocks (of the sunspot type) and anticipated (news) shocks to future fundamentals.

To pursue this, we modify the basic New-Keynesian DSGE model both on the demand and the supply side. On the demand side, we adapt the Blanchard (1985) perpetual-youth model to a discrete-time stochastic environment, as in Nisticò (2006), Airaudo et al. (2009) and Castelnuovo and Nisticò (2010). More specifically, we consider an economy populated by a continuum of overlapping generations with stochastic finite lifetimes who can choose to allocate their savings between a risk-free portfolio of state contingent bonds and a set of risky equities. By interpreting the concepts of "living" and "dying" in Blanchard's model in the economic sense of being "operative" or "not operative" in the markets, our model features a constant turnover between long-time traders (holding assets) and newcomers (entering the market with no wealth at all). This heterogeneity in households' portfolios implies that individual consumption smoothing does not carry over in aggregate terms as the population currently in the market differs from the one that will operate tomorrow. Because of this, for a given stock of wealth, expected aggregate consumption is lower than that implied by the standard infinite-horizon representative agent model.

We show that the wedge between the current and the expected level of aggregate consumption is driven not only by the ex ante real interest rate - as in the standard representative agent model - but also by the stock of wealth accumulated today, since the latter is responsible for the difference between the consumption level of long-time traders and newcomers. Through this mechanism, stock price fluctuations feedback into real activity via their wealth effects on consumption.<sup>3</sup>

On the supply side, monopolistically competitive firms issue equity shares which the households purchase at market prices. These shares entitle them to a future stream of dividends and capital gains. We introduce credit frictions by adding costly loan generation to the simple cost channel set-up of Ravenna and Walsh (2006). Firms borrow from a competitive banking sector in order to finance a fraction of working capital before production and sales take place, implying a direct impact of lending rates (and, in equilibrium, the policy rate) on marginal costs and hence on inflation. Similar to Curdia and Woodford (2010), we assume that issuing loans is a costly activity for banks, in the sense that transforming deposits into loans involves resource costs. However, in contrast to their set-up, we assume that these costs are measured in labor rather than in consumption units, as in Goodfriend and McCallum (2007) and Canzoneri et al. (2008).

 $<sup>^{3}</sup>$ For some empirical evidence on the wealth effects from financial and non-financial assets see Altissimo et al. (2005), Case et al. (2005) and Carrol et al. (2006).

In order to have some supply side impact of stock prices, we also assume that the marginal productivity of the labor used to provide loans is a function of the firm-specific stock price. This collateral-like effect captures the idea that the screening/monitoring activities performed by banks include the acquisition of information on the profitability of the borrowing firms. A public source for this information is the stock market, where share prices presumably reflect the firm's expected future cash-flows and hence are a signal the firm's repayment capacity. Through this mechanism, equilibrium credit spreads depend negatively on the stock price index, which implies that a stock market boom (bust) will reduce (increase) inflation via the cost-channel-augmented Phillips curve. Using data for the U.S. between 1990 and 2007, we find the elasticity of credit spreads to the stock price index to be positive and statistically significant.

Our analysis shows that strict inflation targeting leads to equilibrium indeterminacy, even if the policy rule satisfies the Taylor principle. Belief-driven revisions of expectations are self-fulfilled in equilibrium if the extent of the supply-side credit friction is significant and the policy rule is excessively anti-inflationary. More specifically, the larger the firms' needs for external financing are, the tighter the upper-bound on the response coefficient to inflation in the policy rule. These results are reminiscent of those obtained by Surico (2007) and Llosa and Tuesta (2009) for the standard cost channel model of Ravenna and Walsh (2006). We show that equilibrium determinacy can be restored (and hence non-fundamental belief-driven fluctuations eliminated) by introducing a mild response to stock prices in the policy rule. As our analysis shows, this result is more prominent in economies featuring higher credit frictions and larger demand-side wealth effects from financial holdings.<sup>4</sup>

The benefits of responding to stock prices in our model contrast with the conclusions of Carlstrom and Fuerst (2007) who show that for the benchmark New-Keynesian model an explicit response to stock prices is detrimental for equilibrium determinacy. Although insightful, their result is not surprising given that stock prices in the benchmark model are redundant as they do not feedback into real activity or inflation, making strict inflation targeting sufficient to ensure equilibrium determinacy. Our results are complementary to those of Airaudo et al. (2009) and Pfajfar and Santoro (2011). Airaudo et al. (2009) show that, in a simplified version of the model studied in this paper, an explicit response to stock prices can enlarge (rather than restrict) the determinacy region, therefore ensuring a unique equilibrium even under passive Taylor rules. This occurs when the wealth effects from equity holdings are sufficiently large. Pfajfar and Santoro (2011) show that if the policy rate includes a response to stock price growth (rather than stock price levels) then a unique equilibrium always obtained if the Taylor principle is respected.

Motivated by the strong anti-inflationary stance of central banks since the '90s and the lack of an explicit response to asset prices in a monetary policy setting, we quantify the role of belief-driven shocks originating

<sup>&</sup>lt;sup>4</sup>The issue of whether central banks should or should not respond to stock prices is not new in the literature. For different views see Bernanke and Gertler (2001, Cecchetti et al. (2000, 2002) and Dupor (2005).

from the stock market to explain the volatility of some key financial variables such as the price-dividend ratio, the credit spread, business loans, etc.. over the period 1990-2007. Our impulse response analysis is consistent with the observation by Christiano et al. (2010) that stock price booms are associated with low inflation and low policy rates, which further exacerbate financial instability. However, in our model, the boom-bust cycles are triggered by belief-driven shocks of the sunspot-type, while their analysis considers news shocks. Moreover, the cycles in our model are recurrent, even though the belief-shock is one-time only.

After calibrating the belief-shock to match the volatility of inflation relative to output as in the data, the model can generate volatilities for the price-dividend ratio, real loans, credit spreads and dividends which are considerably larger than the volatilities implied by the fundamental shocks only. model-implied volatilities are close to what we observe in the data. The quantitative performance of the model is rather good irrespective of whether we assume the stock market belief-shocks hit the economy over the whole sample or over a restricted exuberance period, such as the mid-late '90s.

For policy rules that guarantee determinacy, we assess whether being more responsive to stock prices can also smooth the transmission of news shocks about future fundamentals, along the lines of Christiano et al. (2010) and Lambertini et al. (2010). We show that a larger, but still moderate, response to stock price can significantly smooth the boom-bust cycles induced by (unrealized) news on either total factor productivity or bank lending efficiency. A central bank opting for either a more aggressive response to inflation or a positive response to real activity would not be able to achieve the same stabilization results.

The paper is organized as follows. Section 2 develops the model. Section 3 derives the key equilibrium conditions. Section 4 proves the existence of a unique steady-state equilibrium, and defines the log-linearized equilibrium dynamics around it. Section 5 presents the model calibration, together with detailed results on equilibrium determinacy and the role of a policy response to stock prices. We demonstrate how a sunspot-driven stock market belief-shock propagates throughout the economy, and assess the quantitative contribution of this shock in explaining the volatility of some key financial indicators observed in the U.S. data. Section 6 studies the effects of news shocks, and discusses the stabilizing role of a policy response to stock prices. Section 7 concludes and discusses possible extensions.

# 2 The model

### 2.1 Households

The demand-side of the economy is a discrete-time stochastic version of the perpetual youth model introduced by Blanchard (1985) and Yaari (1965), similarly to Nisticó (2005) and Airaudo et al. (2009). The economy is populated by an indefinite number of cohorts of Non-Ricardian agents who survive between any two subsequent periods with constant probability  $1 - \gamma$ . We interpret the concepts of "living" and "dying" in the economic sense of being "operative" or "not operative" in the market, therefore affecting economic activity through the individual decision-making process. In this perspective, the expected life-time  $1/\gamma$  is interpreted as the effective decision horizon of economic agents. Assuming that entry and exit rates are equal, and that total population has size 1, in each period exactly a fraction  $\gamma$  of the population leaves the market and a new cohort of equal size  $\gamma$  enters the economy.<sup>5</sup> In this sense, we can think of our economy as being characterized by a constant turnover  $\gamma$  between newcomers (holding no assets) and long-time traders in financial markets (holding assets). Lifetime utility of the representative agent of the cohort which entered the market at time  $j \leq t$  (from now on, the *j*-th cohort representative agent) is

$$E_t \sum_{k=0}^{\infty} \beta^{k-t} (1-\gamma)^{k-t} \left[ \ln C_{j,t+k} + \iota \ln(1-H_{j,t+k}) + \upsilon \ln(\frac{LB_{j,t+k}}{P_{t+k}}) \right]$$
(1)

where  $\beta, \gamma \in [0, 1], \iota, \upsilon > 0$ . The instantaneous utility is assumed to be log-separable between consumption  $(C_{j,t})$ , leisure time  $(1 - H_{j,t})$  and the real money balances providing liquidity services  $\left(\frac{LB_{j,t}}{P_t}\right)$ .<sup>6</sup> Future utility is discounted because of impatience (through the subjective discount factor  $\beta$ ) and uncertain lifetime (through the probability of remaining active in the market between any two subsequent periods,  $1 - \gamma$ ).

The economy features two sectors: a banking sector and a standard non-financial productive sector. Each household supplies its labor to both sectors, via a perfectly mobile and competitive labor market. Total hours worked  $H_{j,t}$  entering (1) are then given by :  $H_{j,t} = H_{j,t}^p + H_{j,t}^b$ . That is, from the point of view of the household, the two types of labor are perfect substitute in preferences. Hence, in equilibrium, the hourly wage is going to be the same across sectors.

Consumers have access to three types of financial assets: state-contingent bonds, money balances and equity shares. The latter are issued by monopolistically competitive firms, to which the household also supplies labor.<sup>7</sup> At the end of period t, the representative agent of the *j*-th cohort holds a portfolio of contingent claims with one-period ahead stochastic nominal payoff  $B_{j,t+1}$  - which he discounts according to the stochastic discount factor  $\mathcal{F}_{t,t+1}$  - as well as total money balances  $M_{j,t}$  and a set of equity shares issued by each intermediate good-producing firm,  $S_{j,t+1}(i)$ , whose real price at period t is  $Q_t(i)$ . The nominal financial wealth  $A_{j,t}$  carried over from the previous period includes then the nominal pay-offs on the contingent claims,

<sup>&</sup>lt;sup>5</sup>For other recent discrete-time versions of the perpetual youth model see, among others, Annicchiarico et. al. (2004), Chadha and Nolan (2003), Cushing (1999), Leith and Wren-Lewis (2000), Leith and vonThadden (2008), Piergallini (2004) and Smets and Wouters (2002). An alternative approach is given by Weil (1991). See Benassy (2007) for an extensive overview on the latter.

 $<sup>^{6}</sup>LB$  stands for "liquid balances". The assumption that the utility function is logarithmic is necessary in order to retrieve timeinvariant parameters characterizing the equilibrium conditions. See Smets and Wouters (2002) for a non-stochastic framework with CRRA utility.

<sup>&</sup>lt;sup>7</sup>Financial intermediaries are directly owned by the households currently active in the market. However, because of perfect competition and free entry, the representative bank will make zero equilibrium profits. As such, how the bank shares are distributed across the active cohorts is irrelevant for our analysis.

 $B_{j,t}$ , total money balances chosen the previous period,  $M_{j,t-1}$ , and the "price plus dividend" on each share of the equity portfolio,  $Q_t(i) + D_t(i)$ :

$$A_{j,t} \equiv \frac{1}{1-\gamma} \left[ B_{j,t} + M_{j,t-1} + P_t \int_0^1 \left( Q_t(i) + D_t(i) \right) S_{j,t}(i) \, di \right].$$
(2)

As in Blanchard (1985), financial wealth  $A_{j,t}$  also pays off the gross return on an insurance contract that redistributes among the agents that have not been replaced (and in proportion to one's current wealth) the financial wealth of the ones who have. Total personal financial wealth is therefore accrued by a factor of  $\frac{1}{1-\gamma}$ .<sup>8</sup>

Similarly to Ravenna and Walsh (2006), the introduction of a cost channel requires the existence of financial intermediaries which transfer resources from the households to the firms. At the beginning of each period t, households transfer part of their total financial wealth to the intermediaries, as money deposits  $M_{j,t}^d$ , and get it back at the end of the same period accrued by the gross nominal interest rate  $R_t^m$ . In modeling households' deposits, we depart from Ravenna and Walsh (2006) in two aspects. First, we adopt a money-in-the-utility function approach whereby liquidity services come from end of period real balances, similarly to Dib (2006) and Atta-Mensah and Dib (2008). Second, while in their cash-in-advance set-up all the money balances held at the intermediary can not be used for transaction purposes - i.e. the intra-period deposits are illiquid - we assume that only a fraction  $\chi \in (0, 1]$  of them do not provide liquidity services.<sup>9</sup> Under these two assumption, total money balances  $M_{j,t}$  and the liquid balances entering into utility,  $LB_{j,t}$ , are defined as:

$$M_{j,t} = M_{j,t}^c + M_{j,t}^d (3)$$

$$LB_{j,t} = M_{j,t}^{c} + (1 - \chi) M_{j,t}^{d}$$
  
=  $M_{j,t} - \chi M_{j,t}^{d}$  (4)

whereby (3) is total money balances (cash plus deposits) and (4) is money balances providing liquidity services. For  $\chi = 1$  deposits do not provide transaction services, similarly to Ravenna and Walsh (2006).

Under this set-up, households can adjust their desired deposits at the beginning of each period, after the shocks are realized, contrary to the limited participation models (which by themselves, introduce some non-neutral effects of monetary policy). In summary, at the beginning of each period t households - entering

<sup>&</sup>lt;sup>8</sup>Perfect competition and free entry into the insurance market imply that for each unit of wealth left at the insurance companies each agent will receive  $\frac{1}{1-\gamma}$  units conditional on his survival. This assumption implies that each new cohort enters the market with zero initial financial wealth.

<sup>&</sup>lt;sup>9</sup>The magnitude of  $\chi$  positively affects the value of total real balances in the economy. But more importantly, it will impact on the degree of pass-through from the policy to the deposits interest rate. We will calibrate  $\chi$  to match the pass-through observed in the data.

with previous period cash balances  $M_{j,t-1}^c$  and deposits  $M_{j,t-1}^d$ -choose new deposits  $M_{j,t}^d$ , which, by the end of the period give them  $R_t^m M_{j,t}^d$ . It follows that at the end of the period, they have  $(R_t^m - 1) M_{j,t}^d$  income coming from deposits and end of period deposits  $M_{j,t}^d$ . Then they choose the new level of cash balances to transfer to the next period,  $M_{j,t}^c$ , which imply a new level of total money balances  $M_{j,t}$  as in (3).

At time t, the *j*-th cohort representative agent seeks to maximize (1) subject to the budget constraint,

$$P_t C_{j,t} + M_{j,t} + E_t \{ \mathcal{F}_{t,t+1} B_{j,t+1} \} + P_t \int_0^1 Q_t(i) S_{j,t+1}(i) \, di \le A_{j,t} + W_t H_{j,t} + P_t T_{j,t} + (R_t^m - 1) \, M_{j,t}^d + V_{j,t}^{FI}$$
(5)

where  $V_{i,t}^{FI}$  are the profits from the financial intermediary, and a No-Ponzi game condition

$$\lim_{k \to \infty} E_t \left\{ \mathcal{F}_{t,t+k} (1-\gamma)^k A_{j,t+k} \right\} = 0.$$
(6)

The first-order conditions for the optimum are given by the budget constraint (5) holding with equality and the following relationships:

$$C_{j,t}^{-1} = \Lambda_{j,t} P_t \tag{7}$$

$$\iota C_{j,t} = \frac{W_t}{P_t} (1 - H_{j,t}) \tag{8}$$

$$\mathcal{F}_{t,t+1}\Lambda_{j,t} = \beta\Lambda_{j,t+1} \tag{9}$$

$$P_t Q_t(i) = E_t \Big\{ \mathcal{F}_{t,t+1} P_{t+1} \Big[ Q_{t+1}(i) + D_{t+1}(i) \Big] \Big\}, \text{ for } i \in [0,1]$$
(10)

$$\frac{M_{j,t}}{P_t} - \chi \frac{M_{j,t}^d}{P_t} = v \frac{R_t}{R_t - 1} C_{j,t}$$
(11)

$$\frac{M_{j,t}}{P_t} - \chi \frac{M_{j,t}^d}{P_t} = v \frac{\chi}{R_t^m - 1} C_{j,t}$$
(12)

where  $\Lambda_{j,t}$  is the Lagrange multiplier and the (nominal) riskless interest rate  $R_t$  is defined by

$$R_t E_t \left\{ \mathcal{F}_{t,t+1} \right\} = 1 \tag{13}$$

From (11)-(12) the no-arbitrage condition between bonds and deposits becomes:

$$R_{t}^{m} = \frac{(1+\chi)R_{t} - \chi}{R_{t}}$$
(14)

From (14), it clearly appears that  $\chi$  will affect the degree of pass-through from the riskless rate (also the policy instrument) and the rate on deposits.<sup>10</sup>

Using the definition of wealth (2), as well as conditions (9) and (10), we can write the budget constraint as follows:

$$P_t C_{j,t} + \frac{R_t - 1}{R_t} M_{j,t} + (1 - \gamma) E_t \{ \mathcal{F}_{t,t+1} A_{j,t+1} \} \le A_{j,t} + W_t H_{j,t} + P_t T_{j,t} + V_{j,t}^{FI} + (R_t^m - 1) M_{j,t}^d + (R_t^m - 1) M_$$

From (11) and (12), we obtain:

$$\frac{R_t - 1}{R_t} M_{j,t} - (R_t^m - 1) M_{j,t}^d = v P_t C_{j,t}$$

Then, the household's budget constraint reduces to:

$$P_t C_{j,t} (1+v) + (1-\gamma) E_t \{ \mathcal{F}_{t,t+1} A_{j,t+1} \} \le A_{j,t} + W_t H_{j,t} + P_t T_{j,t} + V_{j,t}^{FI} \}$$

The latter is a stochastic difference equation with respect to total financial wealth  $A_{j,t}$ . By iterating on  $A_{j,t+1}$  and imposing the No-Ponzi game condition (6), we obtain:

$$P_t C_{j,t} = \frac{1 - \beta \left(1 - \gamma\right)}{1 + \upsilon} \left(A_{j,t} + I_{j,t}\right)$$
(15)

where  $I_{j,t}$  stands for non-financial wealth:

$$I_{j,t} \equiv E_t \sum_{k=0}^{\infty} \mathcal{F}_{t,t+k} \left(1-\gamma\right)^k \left(W_{t+k}H_{j,t+k} + P_{t+k}T_{j,t+k} + V_{j,t+k}^{FI}\right)$$

Equation (15) is the consumption function of the j-th cohort representative agent. It simply states that

nominal consumption, at time t, is proportional to individual financial and non-financial wealth.

#### 2.1.1 Aggregation across cohorts

As stated earlier, in each period, a fraction  $\gamma$  of each cohort is replaced by an equally sized cohort of new market participants (so that population remains constant). The time t size of the cohort which entered the market in period t - j is then  $\gamma (1 - \gamma)^{t-j}$ .<sup>11</sup> Hence, the time t aggregate value of a generic variable X is given by:

<sup>&</sup>lt;sup>10</sup>Note that if  $\chi = 0$  (i. e. all deposits provide liquidity services),  $R^d = 1$ . If this was the case, there would be no impact of the policy rate on real marginal costs of production, and the cost channel would be ineffective. In other words, it is necessary to have some explicit or implicit cost from holding deposits, which, in the case considered here as well as in the cash-in-advance set-up of Ravenna and Walsh (2006), has to do with the forgone liquidity services.

<sup>&</sup>lt;sup>11</sup>This is because each new cohort enters with size  $\gamma$ , and a fraction  $1 - \gamma$  of it leaves the market each period. Hence, the cohort size shrinks to  $\gamma (1 - \gamma)$  after one period in the market, then to  $\gamma (1 - \gamma)^2$  after two periods in the market, and so on.

$$X_t = \sum_{j=-\infty}^t \gamma \left(1-\gamma\right)^{t-j} X_{j,t} \tag{16}$$

for  $X = C, H^p, H^b, B, M^c, M^d, S, A, I$ . For instance, by applying the aggregation formula (16) to the consumption function (15) we obtain:

$$P_t C_t = \frac{1 - \beta (1 - \gamma)}{1 + \upsilon} (A_t + I_t)$$
(17)

where aggregate financial wealth is given by<sup>12</sup>

$$A_t \equiv B_t + M_{t-1} + P_t \int_0^1 \left( Q_t(i) + D_t(i) \right) S_t(i) \, di \tag{18}$$

Equation (17) states that aggregate consumption is proportional to total aggregate wealth. One clear implication of the finite-lifetimes structure is that the shorter the planning horizon is, the larger is the economy's marginal propensity to consume out of financial wealth.<sup>13</sup>

After extensive but straightforward algebra, the aggregate Euler equation becomes.<sup>14</sup>

$$\frac{(1+\upsilon)\,\beta\,(1-\gamma)}{1-\beta\,(1-\gamma)}P_tC_t = \gamma E_t\,(\mathcal{F}_{t,t+1}A_{t+1}) + \frac{(1-\gamma)\,(1+\upsilon)}{1-\beta\,(1-\gamma)}E_t\,(\mathcal{F}_{t+1}P_{t+1}C_{t+1})$$
(19)

Moreover:

$$\frac{M_t}{P_t} - \chi \frac{M_t^d}{P_t} = \upsilon \frac{R_t}{R_t - 1} C_t \tag{20}$$

$$\iota C_t = \frac{W_t}{P_t} (1 - H_t) \tag{21}$$

# 2.2 Productive Sector

The productive sector of our economy consists of two sub-sectors: a retail sector that operates under perfect competition to sell the final goods to households and a wholesale sector which operates under monopolistic competition to produce a continuum of differentiated intermediate goods.

 $<sup>1^2</sup>$  Unless otherwise stated, from now on, individual variables without the *j* subscript imply that the aggregation formula has been applied.

<sup>&</sup>lt;sup>13</sup>Given the available empirical evidence on the marginal impact of financial and non-financial wealth on consumption, equation (17) could be used to calibrate the value of the turnover rate  $\gamma$ .

<sup>&</sup>lt;sup>14</sup>Note that for  $\gamma = 0$  (infinite horizon case), the aggregate Euler equation collapse to its standard form:  $P_tC_t = \beta E_t \left(\mathcal{F}_{t+1}P_{t+1}C_{t+1}\right)$ .

#### 2.2.1 Retail Sector

A perfectly competitive flexible price retail sector produces an aggregate good using a constant returns to scale technology. In the retail sector the final consumption good  $Y_t$  is produced out of the intermediate goods through the following CRS technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} di\right]^{\epsilon/(\epsilon-1)}$$

where  $\epsilon > 1$  is the intratemporal elasticity of substitution between intermediate goods and reflects the degree of competition in the market for inputs.<sup>15</sup> Under perfect competition and flexible prices, the optimal demand for the intermediate good  $Y_t(i)$  and the final good price  $P_t$  are, respectively:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} Y_t,$$
(22)

$$P_t = \left[ \int_0^1 P_t(i)^{1-\epsilon} \, di \right]^{1/(1-\epsilon)}.$$
(23)

#### 2.2.2 Wholesale Sector

The wholesale i-th firm operates under monopolistic competition to produce a continuum of differentiated perishable intermediate goods according to the following linear production function:

$$Y_t(i) = Z_t^y \check{H}_t^p(i) \tag{24}$$

where  $\check{H}_t^p(i)$  is labor hired and  $Z_t^y$  is the aggregate TFP shock. Without loss of generality, we assume that the aggregate TFP shock has mean equal to 1:  $Z^y = 1^{16}$ 

In choosing the optimal level of labor demand, each firm enters a competitive labor market and seeks to minimize total real costs subject to the technological constraint (24). The firm has to pay a fraction  $\alpha$  of its wage bill at the beginning of the period, before production and sales take place. To accomplish that, the firm has to borrow an amount  $L_t(i)$  from the financial intermediary such that:

$$L_t(i) \ge \alpha W_t \check{H}_t^p(i) \tag{25}$$

which they will pay back at the end of the period at the borrowing rate  $R_t^l(i)$ . The borrowing rate may vary across firms because of firm specific features.

<sup>&</sup>lt;sup>15</sup>We assume that a constant elasticity of subtitution across the differentiated goods. Letting  $\epsilon$  to be time-varying would imply an exogenous cost-push shock in the Phillips curve.

<sup>&</sup>lt;sup>16</sup>The productivity shock is labeled  $Z_t^y$  (with the y superscript) to distinguish it from the other shocks.

Total operating costs of production are then:

$$TC_t(i) = \alpha R_t^l(i) W_t \check{H}_t^p(i) + (1 - \alpha) W_t \check{H}_t^p(i)$$

$$= W_t \check{H}_t^p(i) \left[ 1 + \alpha \left( R_t^l(i) - 1 \right) \right]$$
(26)

Given the production technology (24), nominal marginal costs are:

$$MC_t\left(i\right) = \frac{W_t}{Z_t^y} \left[1 + \alpha \left(R_t^l\left(i\right) - 1\right)\right]$$
(27)

For  $\alpha = 0$  in (27), the cost channel disappears and  $MC_t(i) = MC_t = \frac{W_t}{Z_t^y}$ . Define  $\Phi_t(i)$ :

$$\Phi_t(i) \equiv \left[1 + \alpha \left(R_t^l(i) - 1\right)\right] \tag{28}$$

This can be interpreted as the average interest rate paid on borrowings by the *i*-th firm.

Price rigidities are modelled as in Ireland (2003), which adopts the Rotemberg (1982) adjustment cost framework: each period t, firms face a quadratic resource cost to price changes given by  $AC_t(i) = \frac{\theta}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 P_t Y_t$ . The *i*-th firm solves the following intertemporal maximization problem:<sup>17</sup>

$$\max E_{t} \sum_{k=0}^{\infty} \mathcal{F}_{t,t+k} \left\{ (P_{t}(i) - MC_{t}(i)) Y_{t}(i) - \frac{\theta}{2} \left( \frac{P_{t}(i)}{P_{t-1}(i)} - 1 \right)^{2} P_{t} Y_{t} \right\}$$

subject to (22), (24) and (27). <sup>1819</sup> Taking first order conditions, we obtain the optimal price setting rule

$$\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t \left(1 - \epsilon + \epsilon \frac{MC_t(i)}{P_t(i)}\right) - \theta \left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 \frac{P_t}{P_t(i)} Y_t$$

$$= \theta E_t \left[\mathcal{F}_{t,t+1} \left(\frac{P_{t+1}(i)}{P_t(i)} - 1\right)^2 \frac{P_{t+1}P_{t+1}(i)Y_t}{(P_t(i))^2}\right]$$
(29)

# 2.3 Banking Sector

A continuum of financial intermediaries (banks), operating under perfect competition, conveys resources from the household sectors to the wholeseale sector through loans. Similar to Curdia and Woodford (2010),

<sup>&</sup>lt;sup>17</sup>Implicitly, we are assuming a zero inflation steady state. For the case of a non-zero inflation, the adjustment cost should be modified as follows:  $AC_t(i) = \frac{\theta}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - \bar{\pi}\right)^2 P_t Y_t$ , for  $\bar{\pi} > 1$ . <sup>18</sup>Notice that we express the resource cost of price adjustments with respect to aggregate activity,  $P_t Y_t$ , rather than individual

<sup>&</sup>lt;sup>18</sup>Notice that we express the resource cost of price adjustments with respect to aggregate activity,  $P_tY_t$ , rather than individual output. This approach is similar to Monacelli (2009), but results would not change if we went the other way.

<sup>&</sup>lt;sup>19</sup>The are pros and cons of having the Rotemberg's adjustment cost with respect to the Calvo pricing. Calvo allows for price dispersion, which is a key friction that the policymaker might want to eliminate. Rotemberg's approach would allows us to deal with heterogeneity across firms (for instance, if we allowed for the possibility of different financing needs - i.e. different  $\alpha$ s- or different lending rates), since we can focus on a symmetric equilibrium. In the current verions of the model, both approaches are equivalent.

we assume that issuing loans is a costly activity, in the sense that transforming the deposits gathered from the household sector into loans to productive firms involves resource costs. However, in contrast with them, we assume that these costs are measured in labor rather than consumption units. Banks need to employ workers from the household sector to manage their branches and provide the standard financial services. When approached by productive firms looking for new loans, banks have to engage in monitoring and screening activities to verify the profitability of the firms' projects, the quality of their management, etc...We interpret these monitoring costs as working hours.<sup>20</sup> Given perfect competition and free entry in the banking sector, without loss of generality, we consider a representative bank.

The bank issues loans to the i-th wholesale firm according to the following technology:

$$\frac{L_t(i)}{P_t} = K_t(i)\,\check{H}_t^b(i) \tag{30}$$

 $\check{H}_{t}^{p}(i)$  is the amount of labor employed to issue loans to the *i*-th firm and  $K_{t}$  is a bank efficiently factor. We assume that the latter takes the following form:

$$K_t(i) = \Upsilon Z_t^b \left(\frac{Q_t(i)}{Q(i)}\right)^{\sigma}$$
(31)

where  $Z_t^b$  is a bank lending shock,  $\frac{Q_t(i)}{Q(i)}$  is the *i*-th firm stock price relative to its long run mean and  $\Upsilon$  is a scaling factor. The term  $\left(\frac{Q_t(i)}{Q(i)}\right)^{\sigma}$  captures, in reduced form, a collateral-like effect. Both the screening and the monitoring activities performed by the banks' employees consist in the acquisition of information on the profitability of the borrowing firm. A publicly source of information is the stock market, where share prices reflect the future dividends expected by the market's participants. Assuming that financial intermediaries look at market prices when screening among different borrowers, an increase in  $\frac{Q_t(i)}{Q(i)}$  signals higher profitability of the *i*-th firm, which would then lower the working hours required to issue a certain amount of loans, or, equivalently, increase the amount of loans issued per hour worked. The parameter  $\sigma$ is the elasticity of lending activities to the stock market.<sup>2122</sup> The assumed loan technology in (30)-(31) will imply procyclical equilibrium loans, i.e higher (respectively, lower) lending activities during stock market booms (respectively, bust), with an elasticity equal to  $\sigma \geq 0$ . Although we do not derive the loans' technology from first principles, our specification find strong support in the data and can be see as a hybrid combination of the loan technologies adopted by Canzoneri et al. (2008) and by Goodfriend and McCallum (2007).<sup>23</sup>

 $<sup>^{20}</sup>$ Demirel (2010) introduces costly loan generation by assuming that banks face a quadratic adjustment cost in terms of "lost" deposits.

<sup>&</sup>lt;sup>21</sup>We are implicitly assuming that the financial intermediaries do not have any impact on the individual firms's as well as on the aggregate stock price indexes. When deciding on how many loans to issues they take the stock price index  $Q_t(i)$  as given.

<sup>&</sup>lt;sup>22</sup>We could have formulated the productivity  $K_t$  with respect to the stock price level (undemeaned), i.e.  $K_t = Z_t^b (Q_t(i))^{\sigma}$ .

Although the equilibrium dynamics would be exactly identical, the model might feature multiple steady states. Our formulation in terms of  $\frac{Q_t(i)}{Q(i)}$  guarantees that the steady state is unique. See the Appendix.

 $<sup>^{23}</sup>$ Canzoneri et al. (2008) assume that the production of loans is simply linear in the hours worked in the banking sector.

The sequence of events is as follows. At the beginning of period t, the bank gets  $M_t^d$  deposits from the households. The bank's balance sheets are simple:

$$L_{t} = \int_{0}^{1} L_{t}(i) \, di = M_{t}^{d} \tag{32}$$

i.e., total loans have to equal total deposits. For each of the borrowing firms, the bank observes the stock price indexes  $Q_t(i)$ , and then hires employees in order to generate loans according to the technology (30)-(31). The revenues from loans repayments of each borrowing firm are pooled together (there is no default) and used to pay back depositors at the rate  $R_t^m$  and employees at the competitive wage  $W_t$ .

The bank's profit maximization problem is:

$$\max_{\check{H}_{t}^{b}(i), i \in [0,1]} \int_{0}^{1} \left[ \left( R_{t}^{l}(i) - R_{t}^{m} \right) P_{t} \Upsilon Z_{t}^{b} \left( \frac{Q_{t}(i)}{Q(i)} \right)^{\sigma} \check{H}_{t}^{b}(i) \right] di - W_{t} \int_{0}^{1} \check{H}_{t}^{b}(i) di$$

given  $R_t^l(i)$  and  $Q_t(i)$  for  $i \in [0,1]$ , and given  $R_t^d$  and  $W_t$ .<sup>24</sup> The first order condition gives:

$$R_t^l(i) - R_t^m = \frac{\frac{W_t}{P_t}}{\Upsilon Z_t^b \left(\frac{Q_t(i)}{Q(i)}\right)^{\sigma}} \text{ for every } i \in [0, 1]$$
(33)

that is, the credit spread  $R_t^l(i) - R_t^d$  depends positively on the real wage  $\frac{W_t}{P_t}$ , but negatively on the banking shock  $Z_t^b$  and the firm's own stock price index  $Q_t(i)$ . Notice that if  $\sigma = 0$  the equilibrium credit spread of each borrowing firm would inherit the procyclicality of the real wage. By allowing for  $\sigma > 0$ , our model can generate counter-cyclical credit spread, in line with the empirical evidence documented by Aliga-Diaz and Olivero (2010).

### 2.4 Monetary and Fiscal Policy

Unlike Ravenna and Walsh (2006), we do not consider government spending shocks.<sup>25</sup> In our economcy, the government simply transfers newly created money to the households via lump-sum transfers. Its budget is given by:

Goodfriend and McCallum (2007) assume instead a Cobb-Douglas production function in hours and the (real) market value of the collateral pledged by the borrowing firm. The de-meaned stock price index in (31) plays a similar role to the value of the collateral in McCallum and Goodfriend (2007).

 $<sup>^{24}</sup>$ Lending and deposit rates are taken as given because of the assumption of perfect competition in banking. The continuum of banks compete with each other to supply credit to each firm in the productive sector.

<sup>&</sup>lt;sup>25</sup>In Ravenna and Walsh (2006) fiscal shocks are an additional source of distortions, since government spending is a stochastic fraction of real GDP and agents do not internalize this proportionality. They show that the output gap entering the central bank's objective should be corrected for such shock.

$$\frac{M_t - M_{t-1}}{P_t} = T_t$$

Monetary policy takes the form of a simple Taylor-type interest rate rule. That is, it fixes the gross nominal interest rate  $R_t$  according to the following non-linear rule:

$$R_t = RZ_t^r E_t \left[ \left( \frac{\Pi_{t+1}}{\Pi} \right)^{\phi_\pi} \left( \frac{Y_{t+1}}{Y} \right)^{\phi_y} \left( \frac{Q_{t+1}}{Q} \right)^{\phi_q} \right]$$
(34)

where R,  $\Pi$ , Y and Q are, respectively, the steady state values for the gross nominal interest rate, gross inflation, real ouput and the real stock price index (to be defined in the next Section), while  $Z_t^r$  is a stochastic interest rate shock. The coefficients  $\phi_{\pi}$ ,  $\phi_y$  and  $\phi_q$  are the responses of the policy rate to the deviations of the endogenous variables from their respective steady state value.

It is worth stressing that our analysis does not focus on optimal policy rules. This is the reason why the rule (34) is not specified with respect to the deviation of each endogenous variable from its efficient level. The heterogeneity coming from the stochastic finite-lifetime structure complicates the definition of an efficient equilibrium and the derivation of a welfare-based criterion a'-la' Woodford (2003).<sup>26</sup> For what concerns the existence of sunspot-driven equilibria, whether the rule is written in the levels or in the gaps is irrelavant. This is because the latter would just depend on the stochastic shocks hitting the economy, while, as it is well-known, the equilibrium determinacy analysis is independent from the structure of intrinsic uncertainty.

Our restriction to forward-looking policy rules is motivated on the following grounds. First, rules responding to expectations better capture the forward-lookingness of policy-making. Second, as McCallum (1999) argues, contemporaneous rules are not operational, since they imply a response to endogenous variables whose values are yet to be determined in equilibrium. Third, as shown by Cogley and Sargent (2005), forward-looking rules find strong empirical support from the data.

# 3 Equilibrium

As standard for the Rotemberg's price rigidity set-up, we consider a symmetric equilibrium whereby all monopolistically competitive firms act identically. Along this equilibrium, firm set the same price,  $P_t(i) = P_t$ , hire the same amount of labor,  $\check{H}_t^p(i) = \check{H}_t^p$ , borrow the same amount from the banking system,  $L_t(i) = L_t$ , generate the same dividends,  $D_t(i) = D_t$ , as well as they feature identical equity share prices,  $Q_t(i) = Q_t$ , and borrow at the same rate,  $R_t^l(i) = R_t^l$  for every  $i \in [0, 1]$ . Accordingly, the representative bank employs the same amount of labor to issue loans to any of the borrowing firms:  $\check{H}_t^b(i) = \check{H}_t^b$  for every  $i \in [0, 1]$ .

 $<sup>^{26}</sup>$ Nistico' (2011) has made some progress in this direction for a simplified version of our model, without real money balances in utility and the credit market friction. He shows that the existence of intergenerational distributional issues introduces a motive for stock price stabilization.

Market clearing in our economy requires that

$$\check{H}_t^p = H_t^p \text{ and } H_t^b = H_t^b \tag{35}$$

$$B_t = 0 \tag{36}$$

$$S_t(i) = 1 \text{ for every } i \in [0, 1]$$

$$(37)$$

$$Y_t \left[ 1 - \frac{\theta}{2} \left( \Pi_t - 1 \right)^2 \right] = C_t \tag{38}$$

$$Y_t = Z_t^y H_t^p \tag{39}$$

Equation (35) requires that labor demand is equal to labor supply. Equation (36) states that state contingent bonds in our economy are in zero net supply. Equation (37) assumes that the supply of equite shares by any of the monopolistically competitive firms is constant, and, without loss of generality, we set it equal to 1. Equation (38) is the resource constraint of our economy: output, net of the price adjustment costs, equals consumption. Finally, equation (39) states the technology constraint of our economy.

Aggregate consumption is going to be driven by the aggregate Euler equation (19). Given the definition of aggregate wealth, we can rewrite the term  $E_t (\mathcal{F}_{t,t+1}A_{t+1})$  appearing in (19) as follows:

$$E_{t}\left(\mathcal{F}_{t,t+1}A_{t+1}\right) = E_{t}\left\{\mathcal{F}_{t,t+1}\left[B_{t+1} + M_{t} + P_{t+1}\int_{0}^{1}\left(Q_{t+1}(i) + D_{t+1}(i)\right)S_{t+1}(i)\,di\right]\right\}$$
  
$$= E_{t}\left(\mathcal{F}_{t,t+1}M_{t}\right) + E_{t}\int_{0}^{1}\mathcal{F}_{t,t+1}P_{t+1}\left(Q_{t+1}(i) + D_{t+1}(i)\right)\,di$$
  
$$= \frac{M_{t}}{R_{t}} + P_{t}Q_{t}$$
(40)

That is, aggregate financial wealth is given by the money balances and the market value of the equity portfolio. This last term is obtained by imposing symmetry on the individual firm's stock price in equation (10), and letting  $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$  be gross inflation:

$$Q_t = E_t \left[ \mathcal{F}_{t,t+1} \prod_{t+1} \left( Q_{t+1} + D_{t+1} \right) \right]$$
(41)

Plugging (40) ino (19), we obtain a new espression for the aggregate Euler equation:

$$\frac{(1+\upsilon)\,\beta\,(1-\gamma)}{1-\beta\,(1-\gamma)}C_t = \gamma\left(\frac{M_t}{P_t R_t} + Q_t\right) + \frac{(1-\gamma)\,(1+\upsilon)}{1-\beta\,(1-\gamma)}E_t\,(\mathcal{F}_{t+1}\Pi_{t+1}C_{t+1}) \tag{42}$$

By the definition of real balances (20), the bank's balance sheet (32), the borrowing requirement (at the equilibrium)  $L_t = \alpha W_t H_t^p$ , and the technology (39), we can write total real balances entering (42) as:

$$\frac{M_t}{P_t} = v \frac{R_t}{R_t - 1} C_t + \chi \alpha \frac{W_t}{P_t} \frac{Y_t}{Z_t^y}$$

$$\tag{43}$$

where the real wage,  $\frac{W_t}{P}$ , is determined by the consumption-leisure trade-off condition:

$$\iota C_t = \frac{W_t}{P_t} (1 - H_t) \tag{44}$$

Under symmetry, from (29), we obtain the non-linear Phillips curve describing the dynamics of aggregate inflation:

$$\Pi_t \left( \Pi_t - 1 \right) = E_t \left[ \Pi_{t+1} \left( \Pi_{t+1} - 1 \right) \mathcal{F}_{t,t+1} \frac{P_{t+1} Y_{t+1}}{P_t Y_t} \right] + \epsilon \left( M C_t^r - \frac{\epsilon - 1}{\epsilon} \right)$$
(45)

where

$$MC_t^r \equiv \frac{MC_t}{P_t} = \frac{W_t}{P_t Z_t^y} \left[ 1 + \alpha \left( R_t^l - 1 \right) \right]$$
(46)

are the real marginal costs.

Equilibrium real dividends (or profits) are given by:

$$D_t = Y_t - \frac{W_t}{P_t} H_t^p \Phi_t \tag{47}$$

where  $\Phi_t \equiv [1 + \alpha (R_t^l - 1)]$  is the lending rate factor due to the cost channel. By the productive technology (39), (47) can then be written as:

$$D_t = Y_t \left( 1 - MC_t^r \right) \tag{48}$$

Hours worked in the banking sector are such that total loans issued are equal to the total demand of loans by the productive sector:

$$H_t^b = \frac{L_t}{K_t P_t} \text{ for } K_t = \Upsilon Z_t^b \left(\frac{Q_t}{Q}\right)^{\sigma}$$
(49)

Finally, the gross nominal interest rate is set according to the policy rule (34).

Our definition of Rational Expectations Equilibrium is standard.

**Definition 1** Given the exogenous processes  $\{Z_t^y, Z_t^b, Z_t^r\}_{t=0}^{\infty}$ , an equilibrium is a set of stochastic processes for all endogenous variables that satisfy the conditions (35)-(49) together with the policy rule (34).

# 4 Steady State and Log-Linearized Equilibrium

We are interested in the equilibrium dynamics of the economy, under Rational Expectations, around the non-stochastic steady state. The next Proposition shows that the economy features a unique steady state equilibrium, whereby, absent all shocks, all endogenous variables remain constant.

**Proposition 1** The economy displays a unique non-stochastic steady state equilibrium

### **Proof.** See Appendix $\blacksquare$

Using standard techniques, we log-linearize the equilibrium conditions around the steady state defined by Proposition 1, and denote the related percentage deviations by lower case letters. Although our economy departures from the benchmark New-Keynesian model along different dimensions, the characterization of the (local) equilibrium dynamics boils down to solving a stochastic linear system made of the linearized version of the policy rule (34),

$$r_t = \phi_\pi E_t \pi_{t+1} + \phi_y E_t y_{t+1} + \phi_q E_t q_{t+1}$$
(50)

and the following first-order difference equations

$$y_t = \Psi_y \underbrace{\left[ E_t y_{t+1} - \left( r_t - E_t \pi_{t+1} \right) \right]}_{\text{standard NK model terms}} + \underbrace{\Psi_q q_t - \Psi_r r_t}_{\text{wealth effect from OLG structure}} - \Psi_z z_t^y - \Psi_b z_t^b \tag{51}$$

$$\pi_t = \underbrace{\tilde{\beta} E_t \pi_{t+1} + \Theta_y \kappa \left( y_t - z_t^y \right)}_{\text{standard NK model terms}} + \underbrace{\Theta_r r_t}_{\text{cost channel}} - \underbrace{\Theta_q q_t}_{\text{endogenous spread}} - \Theta_z z_t^y - \Theta_b z_t^b \tag{52}$$

$$q_t = \underbrace{\tilde{\beta}E_t q_{t+1} + \Gamma_y E_t y_{t+1} - (r_t - E_t \pi_{t+1})}_{\text{standard NK model terms}} + \underbrace{\Gamma_q E_t q_{t+1}}_{\text{endogenous spread}} - \underbrace{\Gamma_r E_t r_{t+1}}_{\text{cost channel}} + \Gamma_z z_t^y + \Gamma_b z_t^b$$
(53)

where

$$\widetilde{\beta} \equiv \frac{\beta}{1+\psi}, \qquad \psi \equiv \gamma \frac{1-\beta(1-\gamma)}{(1+\nu)(1-\gamma)} \frac{A}{PC}$$

$$\varphi \equiv \frac{H}{1-H}, \qquad \kappa \equiv \frac{\epsilon-1}{\theta} (1+\varphi)$$
(54)

while the remaining coefficients  $\Psi_i$  (for  $i = y, \pi, q, z, b$ ),  $\Theta_i$  (for i = y, r, q, z, b) and  $\Gamma_i$  (for i = q, y, r, z, b) are convoluted espressions of the underlying structural parameters and of the steady state values, whose analytical forms are reported in the Appendix.<sup>27</sup>

 $<sup>2^{7}</sup>$  The value of  $\psi$  is closely related to the computation of the steady state (real) interst rate. We refer the reader to the Appendix for a full derivation of its value.

As highlighted in (51)-(53), our reduced form equilibrium system features terms which are common to the basic New-Keynesian model, as well as terms that derive from the three key additional elements of our model: the turnover rate in market activities (due to the OLG structure), the standard cost channel a-la Ravenna and Walsh and the endogenous credit spread due to the costly loan generation.

Equation (51) is the Euler equation (or IS curve). It departs from the benchmark model in two key aspects. First, because of  $\Psi_y \in (0, 1)$ , current output  $y_t$  is less affected by the standard New-Keynesian terms. Namely, it depends less on future output expectations and on the ex-ante real interest rate  $(r_t - E_t \pi_{t+1})$ . In particular, the higher the turn-over rate  $\gamma$  in the market, the smaller  $\Psi_y$ , i.e. the shorter is the effective planning horizon of the economic agents, the lower the impact of future expectations.<sup>28</sup> Second, because of  $\Psi_q > 0$  and  $\Psi_r > 0$ , current activity is positively affected by the stock price index,  $q_t$ , and negatively by the nominal interest rate,  $r_t$ . The presence of these two terms is entirely due to the finite market lifetime of the economic agents. As evident from equation (42), a higher nominal interest rate and/or a higher stock price index have, respectively, a negative and a positive impact on financial wealth. From the same equation, one should immediately infer that for  $\gamma = 0$  the term  $\Psi_q q_t - \Psi_r r_t$  would be equal to zero. But more importantly, a higher turnover rate increases  $\Psi_q$ , thus strengthening the structural linkage between the stock market and real activity.

Equation (52) is the Phillips curve regulating the dynamics of inflation. Similarly to the benchmark model, current inflation depends on its expected one-period ahead value and current real activity. However, because of  $\psi$  being strictly increasing in  $\gamma$ , the higher the turnover rate in markets the smaller the impact of future inflation (that is, future marginal costs) on current inflation. At the same time, current inflation depends more on current output since  $\Theta_y > 1.^{29}$  The nominal interest rate  $r_t$  appears due to its impact on real marginal costs, as in Ravenna and Walsh (2006). The quantitative importance of the cost channel is captured by the coefficient  $\Theta_r$ , which depends positively on a) the external finance needs (as measured by the parameter  $\alpha$ ); b) the pass-through from the deposit to the lending rate; c) the pass-through from the riskless rate to the deposit rate. Finally, because of the collateral-like effect embedded into the loan production technology, the stock price index negatively affects current inflation: a stock market boom lowers the monitoring costs in financial intermediation, bringing down the credit spread and hence the marginal costs faced by the monopolistically producing firms. The coefficient  $\Theta_q$  on  $q_t$  is increasing in the elasticity parameter  $\sigma$  appearing in the loan technology (30), with  $\Theta_q = 0$  if  $\sigma = 0.^{30}$ 

<sup>&</sup>lt;sup>28</sup>As a matter of comparison, under the logarithmic preferences assumed in (1), the benchmark infinitively-lived agent economy would have  $\Psi_y = 1$ .

<sup>&</sup>lt;sup>29</sup>This result holds for sure if  $\varphi \equiv \frac{H}{1-H} < 1$ , which simply requires steady state hours worked to be below 50% of the time endownment. The coefficient  $\Theta_y$  would instead collapse to one if issuing loans was costless. In such case, all labor would be employed by the productive sector and the only constraint faced by the banking sector would be its balance sheets:  $L_t = M_t^d$ .

 $<sup>^{30}</sup>$ Even if  $\sigma = 0$ , we would still have a positive credit spread (issuing loans still requires hiring workers), but it would not depend on the stock price index.

Finally, equation (53) is the equilibrium stock price equation. By log-linearizing equation (41), we obtain:

$$q_t = \tilde{\beta} E_t q_{t+1} + \left(1 - \tilde{\beta}\right) E_t d_{t+1} - (r_t - E_t \pi_{t+1})$$
(55)

which states that the stock price index is the weighted average of the expectation of its one period ahead value and of the related dividends, minus the ex-ante real interest rate. From the definition of  $\tilde{\beta}$  in (54), it is immediate that the finite lifetime implies a lower weight on the expected stock price but a higher weight on dividends. The shorter the agents' planning horizon is, the more the stock prices respond to next period dividends, but the less to those related to the more distant future. Equation (53) is obtained from (55) by writing equilibrium dividends in terms of the endogenous variables of the model. The first three terms in (53) are common to the benchmark model: the current stock price depend on its one-period ahead expectations, on next period output and negatively on the real interest rate. The additional term  $E_tq_{t+1}$  comes from the collateral-like effect in the loan technology: a higher (expected) stock price lowers the expected marginal costs, and hence increases the expected future dividends. On the other hand, by the cost channel, the expectation of higher policy rate lowers future dividends, putting downward preassure on the current stock price index.

The system (50)-(53) can be written in compact form as follows:

$$x_t = \Omega E_t x_{t+1} + \Xi z_t \tag{56}$$

where  $x_t = [y_t, \pi_t, q_t, r_t]'$ ,  $z_t = [z_t^y, z_t^b, z_t^r]'$ , and  $\Omega$  and  $\Xi$  are comformable matrices, whose entries depend on the structural parameters of the model and the policy coefficients in (50).

For the time being, we assume that the three shocks  $z_t^y$ ,  $z_t^b$  and  $z_t^r$  are simple AR(1) processes with *iid* innovations:

$$z_t^k = \rho_k z_{t-1}^k + u_{k,t} \text{ for } k = y, b, r$$
(57)

where  $\rho_k \in [0, 1)$  and  $u_{k,t} \sim iid(0, \sigma_k^2)$  for k = y, b, r. Later, when considering the possibility of anticipated news shocks, we will relax the *iid* assumption on the innovation term  $u_{k,t}$ .

We are interested in assessing under what conditions on the policy parameters  $\phi_{\pi}$ ,  $\phi_{y}$  and  $\phi_{q}$  a simple expectations-based interest rate rule can shield the economy against fluctuations that are entirely expectations-driven. For this purpose, we are going to consider two types of expectations related shocks: non-fundamental belief shocks (of the sunspot type) and anticipated (news) shocks to future fundamentals.

The former are related to the self-fulfilling prophecies literature pionereed by Azariadis (1981), Farmer and Guo (1994) and Benhabib and Farmer (1994) for the RBC model.<sup>31</sup> The extension of the analysis to

<sup>&</sup>lt;sup>31</sup>See Benhabib and Farmer (1999) for an extensive review of the literature.

the benchmark New-Keynesian framework has produced the following benchmark result: in order to induce a unique (locally determinate) Rational Expectations Equilibrium (REE), the nominal interest rate should be raised more than one-to-one with respect to (current or expected) inflation.<sup>32</sup> A policy following this simple advice is said to be active, or, equivalently, to satisfy the Taylor principle. While the literature has extensively investigated how robust the Taylor principle is once the benchmark model is amended to include additional features, to the best of our knowledge, there have been very few attempts to assess whether these policy-induced belief shocks can explain the aggregate volatility in the data.<sup>33</sup>

Differently from sunspot shocks, news shock do not require the equilibrium to be indeterminate. They are essentially public signals about future fundamentals (such as TFP or policy shocks), which the agents come to observe periods ahead of the actual possible realization. Essentially, they are anticipated shocks which may or may not materialize. As Christiano et al. (2010) show, unrealized news shocks in a New-Keynesian framework can generate boom-bust cycles similar to those observed in the U.S. data during the recent financial turmoils.<sup>34</sup>

# 5 Equilibrium Determinacy and Belief-Driven Fluctuations

This section examines the conditions for the determinacy of the Rational Expectation Equilibrium (REE) in our economy. Two are the main questions. First: is responding to inflation with a coefficient above one, i.e.  $\phi_{\pi} > 1$ , necessary and (effectively) sufficient to guarantee equilibrium determinacy and hence rule out expectations-driven fluctuations of the sunspot type? Second: does an explicit response to stock prices improve or worsen the equilibrium determinacy conditions obtained for a standard interest rate rule?

The determinacy of equilibrium analysis employs the standard procedure of Blanchard and Khan (1980). Since none of the four endogenous variables is predetermined, under Rational Expectations the equilibrium is (locally) determinate if and only if all eigenvalues of the Jacobian  $\Omega$  in (56) lie within the unit circle in the complex plane.<sup>35</sup> Unfortunately, due to the system's dimensions, it is not possible to obtain analytical conditions for equilibrium determinacy. Hence, we resort to numerical methods for a calibrated version of our economy.

 $<sup>^{32}</sup>$ See Bullard and Mitra (2004) for a summary of the main results for the benchmark New-Keynesian model.

<sup>&</sup>lt;sup>33</sup>The most notable exceptions are Lubik and Schorfheide (2003,2004) which test for the possibility of indeterminacy in the US data, under the benchmarket New-Keynesian model. However, as the Taylor principle applies in their case, the economy would be driven by non-fundamental belief shock only if the policy rule was passive.

<sup>&</sup>lt;sup>34</sup>The pioneers of the news shock literature are Beaudry and Portier (2004, 2006). Lambertini et al. (2010) study news shock in a DSGE model with housing.

 $<sup>^{35}</sup>$ See also Farmer (1999).

### 5.1 Calibration

All parameters related to the benchmark New-Keynesian framework are fixed at standard values. For what concerns the log-utility specification in (1), we set the subjective discount factor  $\beta$  equal to 0.99, the parameter  $\iota$  on the leisure term in order to obtain total hours worked in the economy equal to 1/3 of the time endownment, and the parameter  $\upsilon$  equal to 0.01, consistent with a small role of money in providing liquidity services.<sup>36</sup> We set intratemporal elasticity of substitution  $\epsilon$  equal to 6, implying a steady state gross mark-up  $\frac{\epsilon}{\epsilon-1} = 1.2$  (20% net mark-up).<sup>37</sup> The price adjustment cost coefficient  $\theta$  is set equal to 58.25. Under our calibration, this value provides a reduced form New-Keynesian Phillips Curve whose coefficient on the real marginal cost matches what one would obtain under the Calvo price setting with a probability of not resetting the price equal to 0.75.

The coefficient  $\chi$  appearing in (4) - which we have referred to as the degree of illiquidity of deposits (with  $\chi = 1$  meaning completely illiquid) - is set equal to 0.75. For this value, we obtain a degree of pass-through  $\eta$  from the policy rate  $r_t$  to the deposit rate  $r_t^d$  equal to 0.67. This is degree of pass-through estimated by Karagiannis et al. (2010) for the US economy between 1994 and 2007.

Our calibration of the loan technology in (30)-(31) is the following. We fix the scaling factor  $\Upsilon$  such that the share of hours worked in the banking sector over total hours worked is equal to 1.5%, similarly to what reported by both McCallum and Goofriend (2007) and Canzoneri et al. (2008). This gives us  $\Upsilon = 50$ . The elasticity of the interest rate spread to the stock price index,  $\sigma$ , is obtained by estimating the log-linearized version of equation (33) on US data by simple OLS regression over the period 1997 to 2010. For a regression based on HP filtered logged series we obtain  $\sigma = 1.6$ .<sup>38</sup>

In the model, the extent of the credit friction (or cost channel) is given by  $\alpha$ , i.e. the share of the wage bill that firms have to pay upfront by borrowing from the banking sector. At the two extremes of the spectrum we have the benchmark New-Keynesian model (where the cost channel is absent) featuring  $\alpha = 0$  and the original cost-channel model of Ravenna and Walsh, featuring  $\alpha = 1$ . We fix  $\alpha$  to match the average net business loans to GDP ratio observed in the US, between 1990 and 2010. We measure total loans as the difference between total credit market debt owed by non-farm non-financial businesses and the asset they hold, using FRED data. This ratio is about 0.18 (at yearly frequency), which correspond to 0.72 in our

<sup>&</sup>lt;sup>36</sup>See for instance Stoltemberg and Paustian (2008).

 $<sup>^{37}</sup>$ In the related literature, calibrations for the intratemporal elasticity  $\epsilon$  vary between 6 and 11. We have chosen 6 to reflect the high profitability occurred in the corporate sector after the mid-90s. We will consider alternative calibrations in the sensitivity analysis. However, the value assigned to  $\epsilon$  will not qualitatively affect our result. Within the range considered, it will only (slightly) affect the slope of the determinacy/indeterminacy frontiers.

<sup>&</sup>lt;sup>38</sup>The result does not change much if we regress (unfiltered) first differences, for which we obtain  $\sigma = 1.3$ . In both cases, the estimates a highly significant, and the  $R^2$  is around 0.5. We have used data from the FRED dataset of the Federal Reserve Bank of St. Louis. As a measure of the spread we have used the US Corporate 7-10 Year Option-Adjusted Spread. The real stock price index has been computed by deflating the S&P 500 by the CPI index.

quartely model. This procedure gives us  $\alpha = 0.9$ .

Another key parameter of our model is the turnover rate  $\gamma$ . As a benchmark, we set the probability  $\gamma = 0.1$ , which is slightly below what Castelnuovo and Nistico' (2010) obtain by the Bayesian estimation of a New-Keynesian DSGE model with the Blanchard-Yaari's stochastic OLG structure on US data. This value corresponds to a planning horizon of about 2.5 years. Although this might appear rather short, it is consistent with the empirical evidence on the investment horizon by financial investors over the last 10-15 years. Kozora (2010) reports an average investment horizon for institutional investors ranging between 19 and 25 months, for the period 1990-2007 in the US. Cella et al. (2010) find that, on average, institutional investor turn over about 17% of their portfolio each quarter, which implies an investment horizon of about 6 quarters. By looking at ownership duration (i.e. the length of equity investment) for companies listed at the Oslo Stock Exchange between 1989 and 1999, Bohren et al. (2009) report financial investment horizons ranging between 1 and 2.5 years, depending on the size of portfolio.

We parametrize the technology shock as in Schmitt-Grohe and Uribe (2005, 2007), setting  $\rho_y = 0.85$  and  $\sigma_y = 0.006$ . Consistent with the literature, we let  $z_t^r$  be a simple *iid* process with standard deviation equal to 0.003. Using the loan technology (30)-(31) and the spread equation (33), we write an expression for the lending shock  $Z_t^b$ :

$$Z_t^b = \frac{W_t}{P_t \Upsilon S_t} \left(\frac{Q_t}{Q}\right)^{-\sigma}$$

where  $S_t \equiv (R_t^l - R_t^d)$  is the credit spread. Taking logs and first-differencing, we obtain that:

$$\Delta z_t^b = \Delta \left( w_t - p_t \right) - \Delta s_t - \sigma \Delta q_t \tag{58}$$

Given empirical observations on the real wage, the credit spread and the real stock price index, we use (58) to construct a lending shock series consistent with our model, on which we estimate the AR(1) process (57) by simple OLS regression. Our analysis generates a shock series  $z_t^b$  with autoregressive coefficient equal to 0.89 and a standard deviation for the *iid* innovation equal to 0.13. This simple analysis provides some immediate evidence on the existence of persistent and volatile bank-specific disturbances in the U.S. data, even before the subprimce crisis.

TABLE 1: BENCHMARK CALIBRATION									
β	$\gamma$	ι	v	χ	$\epsilon$	θ	α	Υ	σ
0.99	0.1	1.8	0.01	0.75	6	58.25	0.9	50	1.6

### 5.2 Equilibrium Determinacy

Figure 1 displays the result of the equilibrium determinacy analysis for the benchmark calibration of Table 1. To highlight the role of an explicit response to stock prices, we have set the response to output equal to zero,  $\phi_y = 0$ . The key result is that, in our economy, strict inflation targeting induces equilibrium indeterminacy.<sup>39</sup> This result is related to what obtained analytically by Surico (2008) and Llosa and Tuesta (2009) for the Ravenna and Walsh (2006) model. Because of the simpler structure, they are able to analytically show that under a full cost channel,  $\alpha = 1$ , the equilibrium is always indeterminate for any  $\phi_{\pi} \geq 0$ . Our numerical analysis shows that the same result holds in a model with real effects from financial wealth and endogenous credit spreads.

An explicit response to the stock market significantly expands the determinary region, and hence makes a forward-looking interest rate rule less prone to induce sunspot-driven endogenous fluctuations. For instance, a mild response to stock prices, e.g.  $\phi_q = 0.1$ , makes equilibrium determinacy occur for any  $\phi_{\pi} \in (1.1, 3.56)$ .<sup>40</sup>

From the same figure, one can also see that for a given  $\phi_{\pi} > 1$ , hence obeying the Taylor principle, equilibrium determinacy is attained for intermediate responses to the stock market. For instance, if we set  $\phi_{\pi} = 2$  - which is at the lower end of time-varying estimates of the degree of activism by the Fed during the '90s, as reported by Cogley and Sargent (2005) - determinacy occurs for  $\phi_q \in (0.05, 0.48)$ . A policy rule that responds too aggressively to the stock market might actually bring back the expectations-driven fluctuations that it was trying to eliminate. As Figure 1 shows, there exists a  $\phi_q$  upper bound (which in the figue is about 1.9) above which the equilibrium is indeterminate for any degree of activism towards inflation.<sup>41</sup>

It is useful to compare our equilibrium determinacy results with those obtained by Carlstrom and Fuerst (2007) for the benchmark New-Keynesian model without credit frictions. As they show, an explicit response to stock prices is detrimental for equilibrium determinacy. More specifically, a positive  $\phi_q$  makes the Taylor principle insufficient to rule out sunspot equilibria: the larger  $\phi_q$  the more aggressive should the central bank be towards inflation in order to induce a unique REE. This is not surprising since in the bechmark New-Keynsian model the stock price dynamics do not feedback into the real side, making strict inflation targeting sufficient to ensure equilibrium determinacy.

In a related paper, Airaudo et al. (2009) determine analytical conditions for equilibrium determinacy in

<sup>&</sup>lt;sup>39</sup>To gain even more perspective on this, in the benchmark New-Keynesian model without the cost channel ( $\alpha = 0$ ), the upper bound on  $\phi_{\pi}$  would be around 30, making the Taylor principle *de facto* sufficient for equilibrium determinacy.

<sup>&</sup>lt;sup>40</sup>On the negative side, a positive response to stock prices raises the minimum response to inflation necessary for determinacy. This is displayed in the Figure by the upward-sloping lower bound between the determinacy and the indeterminacy areas. However, it also clearly appears that, as long as the response to stock price does not become excessive, such lower bound increases at a much smaller rate than the upper bound.

<sup>&</sup>lt;sup>41</sup>This upper bound on the response coefficient  $\phi_q$  remains high under any reasonable calibration. In this respect, it constitutes more a theoretical curiosity rather than a practical constraint. No central bank would ever respond that aggressively to the stock market.



Figure 1: Equilibrium Determinacy. The Figure displays the regions of determinacy (white) and indeterminacy (grey) under different combinations of  $\phi_{\pi}$  and  $\phi_{q}$ , with  $\phi_{y} = 0$ . Structural parameters are set at the values displayed in Table 1.

a simplified version of the model we are studying here. Namely, they consider a New-Keynesian economy which departs from the benchmark only because of the stochastic finite lifetimes, leaving aside real money balances and the credit friction. They show that an explicit response to the stock price index can enlarge (rather than restrict) the determinacy region, by making the equilibrium (locally) determinate also under passive Taylor rules. As they show this occurs when the wealth effects from equity holdings are sufficiently large, which positively depends on the turnover rate  $\gamma$  and the size of steady state dividends.

In order to build an intution for why a positive response to stock prices results in equilibrium determinacy, it is useful to take a closer look at the type of sunspot-driven fluctuations that would occur under strict inflation targeting.<sup>42</sup> Suppose there are no fundamental shocks hitting the economy. Then, absent any other source of uncertainty, the economy would be stable at its steady state equilibrium, whereby  $\pi_t = y_t = q_t = 0$ . Now, suppose that the agents, due to a sunspot shock, believe that current inflation will jump to a positive value,  $\pi$ , and then slowly revert back to the steady state. From equation (53), under an active rule, the stock price  $q_t$  drops below zero, with the size of the drop depending on the value of  $\phi_{\pi}$ . By equation (51),

<sup>&</sup>lt;sup>42</sup>From now on, we are going to refer to inflation, output and the stock price, why what we actually mean is their deviations from the respective steady state.

this effect combined with the higher real interst rate makes equilibrium output move below the steady state, creating a contraction. To be a REE, this sunspot-driven path has to be consistent with what implied by the New-Keynesian Phillips curve in equation (52). It is straightforward to see that, under the same sunspotdriven inflation, equation (52) implies a drop in output if the extent of the cost channel (captured by the term  $\Theta_r$ ) is sufficiently large. The initial upward revision in expected inflation is self-fulfilled in equilibrium.

Now, consider the same situation but allow for a positive response to the expected stock price,  $\phi_q > 0$ . Similarly to the no-response case, the stock price  $q_t$  initially drops below the steady state, and so does its next period expectation.<sup>43</sup> While the New-Keynesian Phillips curve still implies a contraction under a sizable cost channel, the Euler equation (51) may imply the oppositive effect (an expansion) if the central bank grants a sufficiently positive response to the stock market. With the stock price and its expectation dropping, the extended Tayor rule might in fact induce a decrease rather than an increase in the real interest rate. If this was the case, the conjectured sunspot-driven inflationary path would not be self-fulfilled.

#### Sensitivity Analysis

Figure 2 shows how the extent of the cost channel (or the degree of credit market friction) affects the equilibrium determinacy results. Two things are worth noticing. First, for  $\phi_q = 0$  - a case that basically corresponds to a standard forward-looking interest rate rule - the stronger the credit friction is, the tighter the equilibrium determinacy upper bound for the response to inflation. Second, as  $\alpha$  increases, the lower determinacy/indeterminacy frontier becomes slightly steeper. From both these observations, we can clearly conclude that a higher credit friction make the determinacy region shrink.

So far, our analysis has not considered rules that respond to real activity. This is motivated by some recent empirical evidence showing that, over the last 10 years or so, most central banks shifted towards stricter inflation targeting regimes, hence giving less weight to real activity in their policy action. Hamilton et al. (2010), for instance, estimate a Taylor rule for the US over the period 1994-2007 using market expectations. Their estimates are consistent with a rule that since 2000 has responded more aggressively to inflation, with an insignificant concern for output.

Two key features make our model depart from the benchmark New-Keynesian framework: the credit friction (indexed by the cost channel parameter  $\alpha$ ) and the financial wealth effects coming from holding equities (indexed by the turnover rate  $\gamma$ ). Although our benchmark calibration is consistent with previous literature and, to some extent, it captures some recent evidence on trading in financial markets, it is interesting to assess how an extended Taylor rule performs under alternative calibrations of both parameters. In this exercise, we fix  $\phi_{\pi} = 2$  and  $\phi_q = 0.05$ : that is, a rule that grants an active response to inflation and a mild response to the stock market. From Figure 3, we can infer that the impact of the turnover rate  $\gamma$  (or,

 $<sup>^{43}</sup>$ The same would apply to the no-response case. But it was irrelevant for our discussion, since the central bank was not responding to it.



Figure 2: Equilibrium Determinacy and The Credit Friction. The Figure displays the regions of determinacy (white) and indeterminacy (grey) under different combinations of  $\phi_{\pi}$  and  $\phi_{q}$ , with  $\phi_{y} = 0$ , for different extents of the cost channel:  $\alpha = 0.25$  (top panel),  $\alpha = 0.5$  (medium panel) and  $\alpha = 1$  (bottom panel). All remaining structural parameters are fixed at benchmark values reported, as in Table 1.



Figure 3: Equilibrium Determinacy: Wealth Effects and the Cost Channel. The figure displays the determinacy (white) and intedeterminacy (grey) regions with respect to the turnover rate  $\gamma$  and the extent of the cost channel, indexed by  $\alpha$ . The interest rate rule has  $\phi_{\pi} = 2$  and  $\phi_q = 0.05$ . All remaining structural parameters are as in Table 1.

equivalently, of the planning horizon) on equilibrium determinacy is non-monotonic. At first, an increase in the turnover rate  $\gamma$  away from zero raises the minimum degree of credit friction above which indeterminacy occurs. But, as  $\gamma$  goes above 0.2, the effect is reversed: a higher turnover rate makes the economy more prone to multiple equilibria. From this, one could conclude that the equilibrium determinacy benefits of granting an explicit response to the stock market increase as the planning horizon gets shorter, but diminish once it passess a certain threshold.

A possible explanation for this non-monotonicity is the following. A higher turnover rate  $\gamma$  (or, equivalently, a shorted planning horizon) increases the direct wealth effects from holding equities, which reinforce the contractionary effect on output due to the initial sunspot-driven raise in inflation. This makes a rule responding to the stock market even more desirable. However, at the same time, a shorter planning horizon makes future expectations matter less, as the agents are much more affected by current developments in the economy rather than those related to the next period. Hence, expectation based policy rule might eventually become destabilizing.

Figure 4 displays the equilibrium determinacy region with respect to the response to the stock price  $\phi_q$ 



Figure 4: Equilibrium Determinacy: The Credit Friction and The Stock Price Response. The figure displays the determinacy (white) and intedeterminacy (grey) regions with respect to the response to the stock price,  $\phi_q$ , and the extent of the cost channel, indexed by  $\alpha$ . The interest rate rule has  $\phi_{\pi} = 2$ . All remaining structural parameters are as in Table 1.

and the extent of the credit friction, indexed by  $\alpha$ , fixing the response to inflation to  $\phi_{\pi} = 2$ . It appears that as long as  $\alpha$  remains below 0.4 a standard forward-looking Taylor rule does not induce sunspot fluctuations. An increase in the credit friction above 0.4 calls for a positive response to the stock market, with more credit constrained economies requiring a higher  $\phi_q$ .

### 5.3 Belief-Driven Fluctuations

We study the propagation of belief-driven shocks, that is, non-fundamental (extrinsic) uncertainty that arises from the agents'self-fullfilled expectations. As show in the previous sub-section, in our economy, these fluctuations are possible if the extent of the credit friction is sufficiently high and the central bank adopts a standard inflation-based interest rate rule. The aim of this quantitative exercise is similar in spirit to Benhabib and Wen (2004) and Schmitt-Grohé (200) who assess whether equilibrium indeterminacy (and the induced sunspot shocks) can account for the excessive volatility and endogenous persistence observed in the data.<sup>44</sup>

The numerical implementation of the sunspot equilibria follows the procedure by Lubik and Schorfheide (2003,2004). In particular, we focus on the propagation of a belief-driven revision of the time t - 1 forecast of the stock price index. That is, we write the stock price index  $q_t$  as follows:

$$q_t = E_{t-1}q_t + \nu_t^q$$

where  $E_{t-1}q_t$  is the t-1 forecast of the current stock price index (based on information occurring before the realization of the current shocks) and  $\nu_t^q$  is the *iid* forecast error. We assume that, because of a sunspot shock, the stock price expectation  $E_{t-1}q_t$  is revised by  $\zeta_t^q$ , such that  $q_t = (E_{t-1}q_t + \zeta_t^q) + \tilde{\nu}_t^q$ , where  $\tilde{\nu}_t^q$  is the new forecast error. As Lubik and Schorfheide (2003) show, under indeterminacy, the overall forecast error  $\zeta_t^q + \tilde{\nu}_t^q$  can be written as a linear combination of the fundamental shocks and the belief shock, making the equilibrium allocation and prices depend on extrinsic uncertainty.<sup>45</sup>

Figure 5 displays the impact and the propagation of a 1% stock price belief shock on output, inflation and the stock price index, for a simple Taylor rule - i.e.  $\phi_q = 0$  - under three alternative parametrizations of the response coefficient to inflation:  $\phi_{\pi} = 2$  (thick bold line),  $\phi_{\pi} = 3$  (thin bold line) and  $\phi_{\pi} = 5$  (dotted line). The first thing to notice is that both output and the stock price raise on impact. The impact on output is essentially due to the wealth effect coming from the stochastic OLG structure. Because of the short(er) planning horizon, the agents respond to the (non-fundamental) stock market boom by consuming more. On the other hand, inflation drops on impact. This comes from the negative impact of stock prices on marginal costs, implied by the costly loan generation technology of Section 2.3. Second, for all variables, the propagation of the belief shock is oscillatory. This is a consequence of the complexity of the roots of the system in the indeterminacy region. It is worth pointing out that the same non-monotonic sunspot dynamics occur in the benchmark New-Keynesian model for Taylor rules that assign a very high value to the response coefficient  $\phi_{\pi}$ , typically above 30 for standard calibrations. Interestingly, due to the credit friction, similar dynamics occur in our economy at much lower values.<sup>46</sup> Third, under the milder active rule (i.e.  $\phi_{\pi} = 2$ ), the belief shock has a rather prolonged impact on the economy. As the figure shows, a more aggressive response to inflation can effectively mitigate the dynamic propagation of the belief shock, but it can not eliminate it

<sup>&</sup>lt;sup>44</sup>Although our simulation exercise does not consider the possibility of news shocks (i.e. the innovation to the TFP is unpredictable), we would like to clarify that the presence of news shocks is completely independent from the issue of equilibrium determinacy/indeterminacy. Karnizova (2010) compares the dynamic responses to news shocks and to sunspot shocks, under indeterminacy, for the benchmark New-Keynesian model.

<sup>&</sup>lt;sup>45</sup>This analysis can also be pursued for belief-driven revision to the inflation and the output forecasts. Our focus on stock prices is motivated by the ongoing discussion on non-fundamental asset price components and their impact on real activity. A more detailed description of the methodology is reported in the Appendix.

<sup>&</sup>lt;sup>46</sup>Oscillatory convergence and dynamic cycles occour also in indeterminate RBC models, like Benhabib and Wen (2004) and Schmitt-Grohe (2000).



Figure 5: Impulse Responses to *iid* Belief Shock to Stock Price Expectation: Variables in the Reduced-From System. Impulse responses to a belief-driven revision of the t - 1 conditional forecast of the stock price index, under a standard inflation-based interest rate rule ( $\phi_q = 0$ ).

completely.

Figure 6 shows the responses of the remaining financial variables. The most striking effect of the belief shock is on the price-dividend ratio, which increases on impact by about 1.5%, and then significantly oscillates for the first 40 quarters. As expected, the positive belief shock makes the credit spread drop significantly (by almost 0.8%), which induces a positive response of the loans to GDP ratio. The riskless policy rate drops by about 0.4%. This is due to the fact that monetary policy responds to expected inflation, which, as shown in Figure 5, drops below the steady state. The lower policy rate exacerbates the on-impact response of aggregate output to the stock price belief shock.

Indeed, according to the impulse responses of Figure 5 and 6, a stock price belief shock can have sizable consequences for aggregate volatility. But can it account for the the financial instability observed since the '90s? To quantify that, we simulate 1000 time series made of 70 observations, which corresponds to the lenght of the period 1990(1)-2007(2), at quarterly frequency.<sup>47</sup> Consistent with the raising degree of activism of

<sup>&</sup>lt;sup>47</sup>We restrict our analysis to the pre subprime crisis years.



Figure 6: Impulse Responses to *iid* Belief Shock to Stock Price Expectation: Financial Variables. Impulse responses to a belief-driven revision of the t-1 conditional forecast of the stock price index, under a standard inflation-based interest rate rule ( $\phi_q = 0$ ).

the '90s documented by Cogley and Sargent (2005), we fix  $\phi_{\pi} = 2$ . To generate equilibrium indeterminacy, we assume no response to the stock market,  $\phi_q = 0$ . This is consistent with the common wisdom that the Fed should not have responded to stock market fluctuations occurred during the IT revolution started in the early '90s.

TABLE 2: STOCHASTIC PROCESSES						
$\rho_y$	$\sigma_y$	$\rho_b$	$\sigma_b$	$\rho_r$	$\sigma_r$	
0.85	0.006	0.89	0.13	0	0.03	

We report the standard deviations relative to output of selected variables, considering two alternative solutions: the fundamental solution and the belief-driven solution. The fundamental solution is obtained by computing the fundamental Minimum State Variable (MSV) solution of the model, which is well-defined also under indeterminacy, assuming that the only fundamental shock is aggregate TFP. The latter takes the form of a simple AR(1) process as in (57), with autoregressive coefficient and standard deviation as in Table 2. This has been taken from Schmitt-Grohé and Uribe (2005). The belief-driven solution instead allows for a non-fundamental *iid* belief shock hitting the stock market. Given the policy parametrization, we have set the standard deviation of this belief shock in order to capture the relative volatility of inflation displayed in the data. This procedure implies a standard deviation for the stock price belief shock equal to 0.012.  $^{48}$ 

TABLE 3: RELATIVE VOLATILITIES				
Variables	U.S. Data	TFP Only	TFP + Belief Shock	
Output	1	1	1	
Inflation	0.32	0.15	0.32	
Price-Dividend Ratio	8.4	1.42	6.28	
Real Loans	2.66	0.82	2.21	
Credit Spread	23.7	1.12	1.6	
Real Dividends	5.64	2.7	5.73	

The empirical evidence for the U.S. over the period 1990(1)-2007(2) is reported in the second column of Table 3. While inflation appears significantly less volatile than real GDP - a sign of the strong antiinflationary concern by the Fed - all other variables are considerably more volatile, in particular, as expected, the price-dividend ratio and the credit spread. With the exception of the latter, the indeterminate model does a rather good job at capturing all the remaining relative volatilities. The *iid* stock price belief-shock clearly dominates the fundamental (TFP driven) solution. These results together with the reported evidence of increasing anti-inflationary stance witnessed in the '90s show that the surge of financial/credit market instability could have resulted from non-fundamental (but still rational) stock market related belief shocks.

 $<sup>^{48}\</sup>mathrm{The}$  volatilities for the US data are computed on the HP-filtered logged series.

Table 4 pursue the same analysis by allows for the other two fundamental shocks of our economy: the lending shock,  $z_t^b$ , and the policy rate shock,  $z_t^r$ . Two key results emerge. On the one hand, the addition of the lending and the policy shock significantly improves the performance of the indeterminate model with respect to the relative volatility of the credit spread. The model can now explain about 50% of what observed in the data, without any significant impact on the remaining variables. Despite the additional shocks, the performance of the fundamental-based model remains poor, with the exception of the credit spread.

TABLE 4 - RELATIVE VOLATILITIES						
Variables U.S. Data		Fundamental Shocks Only	Fundamental Shocks + Belief Shock			
Output	1	1	1			
Inflation	0.32	0.15	0.32			
Price-Dividend Ratio	8.4	1.85	6.21			
Real Loans	2.66	1.23	2.23			
Credit Spread	23.7	24.47	11.9			
Real Dividends	5.64	2.71	5.65			

One could object that having the economic agents hit by non-fundamental belief shocks every quarter is a rather unrealistic scenario. This is probably not what policy-makers and market participants had and still have in mind as the (ir)rational exuberance of the mid '90s. Stock prices indeed grew much faster than the underlying fundamentals during those years, but the true accelleration was really confined to a limited period of time. We fit this idea into our model by assuming that the indeterminate economy experiences a restricted period of "stock market exuberance". Within this period, we let the economic agents significantly revise upwards their stock price expectations, while for the rest of the sample we make the belief shocks negligible.<sup>49</sup> More specifically, we let the exuberance period start at t = 21 (that is, after 5 years) and last for either 4,8 or 16 quarters. This corresponds, roughly, to having exuberance starting in 1995. For each of the three horizons, we assume that the agents revise upwards their stock price expectations by a constant percentage per quarter. In each case, we pick the constant percentage to match the relative volatility of inflation, as in the previous experiments. Table 5 reports our results, assuming that the economy is also hit by the three fundamental shocks, parametrized according to Table 2.

<sup>&</sup>lt;sup>49</sup>Technically, because of equilibrium indeterminacy, belief shocks hit the economy in every period. While we can not rule them out completely, we can arbitrarily assume that, outside the exuberance period, they are generated by a normal distribution highly concentrated around the zero mean.

TABLE 5 - RELATIVE VOLATILITIES					
Variables	$h = 4 \\ {}_{(\zeta^q = 0.02)}$	$h = 8 \\ {}_{(\zeta^q = 0.01)}$	$h = 16 \ (\zeta^q = 0.008)$		
Output	1	1	1		
Inflation	0.32	0.32	0.32		
Price-Dividend Ratio	6.36	6.3	6.12		
Real Loans	2.28	2.25	2.20		
Credit Spread	10.44	11.83	12.43		
Real Dividends	5.76	5.75	5.60		

The analysis confirms that the model performs equivalently well under the assumption of short-term exuberance in the stock market. For a 1-year long exuberance period (h = 4), a belief-driven upward revision of the stock price expectations of 2% per quarter, generates relative volatilities comparable to what displayed in Tables 4. As the horizong is lengthned, similar results are obtained for smaller quarterly revisions of stock price expectations.

We evaluate the fundamental and the belief-driven solutions also with respect to the correlation of the endogenous variables with real output, as well as their serial correlation. As displayed in Table 6, the fundamental solution fails with respect to inflation and the price-dividend ratio, where, contrary to the data, it predicts counter-cyclicality. The belief-driven solution does a better job with respect to these two (although for inflation, it basically predicts a-cyclicality), but miserably fails on dividends: they are pro-cyclical in the data and according to the fundamental solution, but counter-cyclical according to the belief-driven solution. Both solution capture the counter-cyclicality of the credit spread, with the belief-driven performing slightly better.

TABLE 6 - CORRELATION WITH REAL OUTPUT (1990:1-2007:2)						
Variables	U.S. Data	Fundamental Shocks + Belief Shock				
Inflation	0.25	-0.87	-0.03			
Price-Dividend Ratio	0.85	-0.42	0.74			
Real Loans	0.45	0.89	0.94			
Credit Spread	-0.27	-0.35	-0.21			
Real Dividends	0.43	0.74	-0.60			

Table 7 reports the first order autocorrelation coefficients. Because of the lack of endogenous state variables, by no surprise the fundamental solution can not generate enough persistence for some variables, most notably, for the price-dividend ratio and real loans. It also predicts excess persistence for inflation, which, over the period considered, appears to be a-cyclical. Indeterminacy is an endogenous source of persistence, as the solution features lagged inflation, output, stock prices and interest rates. Except for inflation, the belief-driven solution slightly outperforms the fundamental solution also with respect to serial correlations.

	TABLE 7 - AUTOCORRELATION					
Variables	U.S. Data	Fundamental Shocks Only	Fundamental Shocks + Belief Shock			
Output	0.82	0.78	0.86			
Inflation	0.21	0.81	0.85			
Price-Dividend Ratio	0.83	0.39	0.82			
Real Loans	0.93	0.52	0.82			
Credit Spread	0.83	0.81	0.82			
Real Dividends	0.71	0.71	0.82			

# 6 News Shocks

The analysis of Section 5.2 has shown that a mild response to stock prices by the policy rate can rule out expectation-driven fluctuation of the sunspot-type. This section introduces an alternative source of aggregate volatility which also has to do with people expectations: public news about future fundamentals. News and sunspot shocks are clearly different objects. First, sunspot shocks are restricted to the case of equilibrium indeterminacy, while news shocks can occur both under determinacy and indeterminacy. Second, sunspot shocks affect the agent's expectations about endogenous variables, while news shocks affect the expectations on future exogeneous but still fundamental shocks. As in Section 5.2, we are going to assess whether the policy rule (34) with  $\phi_q > 0$  can eliminate or at least smooth the news-related fluctuations.

We study the effect of news on the technology shock (TFP)  $Z_t^y$  and the bank lending shock  $Z_t^b$ . We assume that  $z_t^k = \ln Z_t^k$  follows the process:

$$z_t^k = \rho_k z_{t-1}^k + u_{k,t} \text{ for } k = y, b$$
(59)

The innovation  $u_{k,t}$  is now made of two components:

$$u_{k,t} = u_{k,t}^0 + u_{k,t-T}^T \tag{60}$$

where  $u_{k,t}^0$  is a contemporaneous innovation to  $z_t^k$  while  $u_{k,t-T}^T$  is the T- period anticipated change in  $z_t^k$ . Although agents come to observe the "news" at time t - T, the related innovation may get realized only T periods later. For instance, suppose that at time t the agents observe  $u_{y,t}^T > 0$ , that is, they anticipate a positive innovation to TFP to happen at t + T.<sup>50</sup> When time t + T comes, two events are possible: 1) the

<sup>&</sup>lt;sup>50</sup>The case of an anticipated negative innovation is symmetric.

news are realized: if this is the case,  $u_{y,t+T}^0 = 0$  and hence  $u_{y,t+T} = u_{y,t}^T$ ; 2) the news are not realized (the anticipated innovation does not pan out): in this case,  $u_{y,t+T}^0 = -u_{y,t}^T$  and  $u_{y,t+T} = 0.51$ 

As standard in the related literature, we assume that the news shock  $u_{k,t}^T$  has mean zero and standard deviation  $\sigma_k$ , and that it is orthogonal to the contemporaneous innovation  $u_{k,t}^0$ , for any t. This implies that the innovation  $u_{k,t}$  is unconditionally mean zero and serially uncorrelated:  $Eu_{k,t} = 0$  and  $E(u_{k,t}u_{k,t-m}) = 0$  for m > 0 for k = y, b.

We perform the following numerical experiment. We assume that at time t the agent receive a signal (news) that there will be a positive innovation to either  $z_t^y$  or  $z_t^b$  at time t + 4 (one year ahead). Under our notation, this means that  $u_{k,t}^4 > 0$ . The agents are assumed to have full confidence in the signal, meaning that at time t, t + 1, t + 2 and t + 3 they believe that the news will materialize in an actual innovation at time t + 4. In other words, we assume that between time t (when the news become public) and time t + 4 (when they can materialize) there is no further useful information for the agents that might lead them to revise their expectations.

Figure 7 displays the response of selected variables to realized and unrealized news on a four-quarter ahead 1% innovation to total factor productivity. From period 1 to period 4, under both the "realized news" and the "unrealized news" scenarios, the dynamic response to the news are identical. This is because all that matters to the agents, up to period 4, is what they expect. The dynamics are completely different from period 5 on. If the news turn out to be true, there is no need for the agents to revise their expectations. No additional unexpected innovation occurs and the fundamental shock gradually reverts to its steady state value.

If, on the other hand, in period 5 the agents realize that the news were false, the fundamental shock immediately jumps back to its steady state value, and so do all endogenous variables. Under this scenatio, the economy diplays a boom-bust cycle which is entirely driven by (unrealized) expectations on future fundamentals.

Let's focus on the "unrealized news" case, which corresponds to the bold lines in Figure 7. Output, the stock price index and real loans display the typical boom-bust cycle. Anticipating future improvements, the agents immediately increase consumption, which drives up output. Although mildly, the stock market booms in anticipation of future higher dividends. This effect combined with the credit friction and the pro-cyclicality of wages, makes real loans increase on impact. The dynamics of inflation are due to the fact that the marginal cost reduction - due to the expected technological innovation - might materialize only four quarters later. At the beginning of the transition, aggregate demand is strong (and so are wages) and next period inflation is expected to be high. This combined with a higher nominal interest rate (coming from a positive response to

 $<sup>^{51}</sup>$ In other words, the contemporaneous innovations plays the following role: it corrects for the unrealized news, or it takes a zero value if the news were correct.



Figure 7: News on Technology Shock. Anticipated four quarter-ahead 1% innovation to the TFP shock. Bold line: unrealized news. Dashed line: realized news. Interest rate rule parametrization:  $\phi_{\pi} = 2$ ,  $\phi_q = 0.1$  (determinate equilibrium).

stock prices and inflation) makes firms set higher prices. As period 5 approaches, inflation drops below the steady state: firms expect next period inflation to be low (as the positive TFP innovation is approaching), which, by the term  $E_t \pi_{t+1}$  in Phillips curve, the credit friction and the forward-looking policy rule, makes them set lower prices. Along this dynamic transition, the nominal interest rate starts dropping because of the strong anti-inflationary stance by the central bank. A similar pattern is followed by the credit spread: it initially increases (as real activity, and hence wages, expands while the stock market's boom is less marked), but then moves downward as stock prices pick up.

The responses to the news on the bank lending shock are qualitatively similar to what seen for TFP. As Figure 8 shows though, these are much milder. The transmission comes mostly from the stock market wealth effect on consumption and the collateral-like effect on loan generation. The positive news on bank lending make the agents expect lower future spreads, and hence higher dividends. This puts upward preassure on stock prices, which, because of the wealth effect, push up consumption and then output. The boom in real activity drives up the wage bill, and hence the amount of loans subscribed by firms. Similarly to the TFP case, the fact that inflation, the nominal interest rate and the credit spreads rise on impact, and decline only later, has to do with the news being related to an innovation which is still a few periods away.

Does responding more aggressively to stock prices stabilize against news shocks? Figures 9 and 10 display, respectively, the responses to news shocks to TFP and bank lending for some selected variables.<sup>52</sup> On the

<sup>&</sup>lt;sup>52</sup>Output and inflation are the standard targets of stabilization policies in the benchmark New-Keynesian framework. The price-dividend ratio and loans to GDP are often used as indicator of financial/credit market turmoil.



Figure 8: News on Lending Shock. Anticipated four quarter-ahead 1% innovation to the lending shock. Bold line: unrealized news. Dashed line: realized news. Interest rate rule parametrization:  $\phi_{\pi} = 2$ ,  $\phi_q = 0.1$ (determinate equilibrium).

one hand, for both cases, a higher  $\phi_q$  in the policy rule significantly smooth the dynamic responses to the news shocks for output, inflation and the price-dividend ratio. As stock prices boom after the news become public, a policy rule with a larger  $\phi_q$  induces a higher nominal interest rate which slows down real activity - thus lowering the wage bill and the required borrowings - and deflates stock prices. On the other hand, a higher  $\phi_q$  does not seem able to smooth the dynamics of inflation. It just turns the news shocks from inflationary to deflationary.

Finally, we compare the stabilizing properties of rules granting a higher response to stock prices with rules being either more aggressive on inflation or granting a positive response to output. The bold lines in Figure 11 display the transmission of a TFP news shock for a rule with  $\phi_{\pi} = 2$  and  $\phi_q = 0.25$ . Under our calibration, this identifies a point in the determinacy region (white area) of Figure 1. We then consider two options: 1) raise the response to inflation, while keeping  $\phi_q = 0.25$ ; 2) raise the response to stock prices, while keeping  $\phi_{\pi} = 2.5^3$  As in Figure 9, raising  $\phi_q$  (from 0.25 to 0.4) significantly stabilizes output, the price-dividend ratio and loans to GDP. On the other hand, a more anti-inflationary (but still determinate) policy rule ( $\phi_{\pi}$  goes from 2 to 3.5) can amplify the boom-bust cycle induced by the unrealized news: while the on-impact response is reduced (and almost muted for output and loans), the period 4 peak and the gradient of the transition towards it are significantly hampered.

Figure 12 displays the dynamic responses of output and the price-dividend ratio to (unrealized) TFP  $5^{3}$ The reason why we have picked  $\phi_q = 0.25$  and not lower is because it allows us to consider more anti-inflationary rules without necessarily inducing indeterminacy.



Figure 9: News on Technology Shock (Unrealized) and Stabilization. Impulse responses to news on four-quarter ahead TFP shock (unrealized), for different responses to stock prices. The response to inflation is fixed at  $\phi_{\pi} = 2$ .



Figure 10: News on Lending Shock (Unrealized) and Stabilization. Impulse responses to news on four-quarter ahead bank lending shock (unrealized), for different responses to stock prices. The response to inflation is fixed at  $\phi_{\pi} = 2$ .



Figure 11: News on Technology Shock (Unrealized) and Stabilization: Inflation versus Stock Price Targeting. Impulse responses to news on four-quarter ahead TFP shock (unrealized).

related news, assessing whether an explicit response to output can attain the same stabilizing properties of responding to stock prices.<sup>54</sup> The bold lines correspond to a rule that sets  $\phi_q$  to the lowest value compatible with equilibrium determinacy. Keeping the response to output equal to zero,  $\phi_y = 0$ , we increase  $\phi_q$  to make the on-impact response of output as low as possible, while preserving determinacy. In our numerical exercise, this corresponds to  $\phi_q = 0.42$ . Under this policy, the TFP news effects are greatly smoothed for both variables. Next, keeping  $\phi_q = 0.05$ , we set  $\phi_y$  to the higher value compatible with determinacy (that is,  $\phi_y = 2$ ). As it clearly appears, responding to stock prices outperforms responding to real activity.

# 7 Conclusions

We present a New-Keynesian DSGE model where stock price fluctuations have real effects both via the demand and the supply side. Direct wealth effects on aggregate consumption arise because of a constant turnover between long-time traders and newcomers in financial markets. The presence of credit frictions, costly loan generation and collateral-like effects on the supply side implies a direct impact of stock prices on

 $<sup>^{54}</sup>$ For what concerns inflation, responding to stock prices or to output gives very similar results: in both cases, the news shocks can not be effectively smoother. The results for the loans to GDP ratio are qualitatively similar to those obtained for output and the price-dividend ratio.



Figure 12: News on Technology Shock (Unrealized) and Stabilization: Ouput versus Stock Price Targeting. Impulse responses to news on four-quarter ahead TFP shock (unrealized).

marginal costs and hence inflation.

After calibrating the economy to capture some key features of the 1990-2007 U.S. data, we show that strict inflation targeting induces equilibrium indeterminacy, even if the policy rule satisfies the Taylor principle. Belief-driven revisions of expectations are self-fulfilled in equilibrium if the extent of the supply-side credit friction is significant and the policy rule is excessively anti-inflationary.

We quantify the role of belief-driven shocks originating from the stock market to explain the volatility of some key financial variables - such as the price-dividend ratio, the credit spread, business loans, etc... - relative to real output observed in the U.S. over the period 1990-2007. Our impulse response analysis is consistent with the observation by Christiano et al. (2010) according to whom stock price booms are associated with low inflation and low policy rates, which further exacerbate financial instability. However, in our model, the boom-bust are triggered by belief-driven shocks of the sunspot-type (while, their analysis considers news shocks) and are recurrent, even though the shock is one-time only.

For a reasonable calibration of the belief-shock standard deviation, the model can generate volatilities for the price-dividend ratio, real loans, credit spreads and dividends which are considerably larger than the volatilities implied by the fundamental shocks only. model-implied volatilities are close to what we observe in the data. The quantitative performace of the model is rather good irrespective of whether we assume the stock market belief-shocks hit the economy over the whole sample or over a restricted exuberance period, such as the mid-late '90s.

We show that monetary policy can eliminate the non-fundamental beliefs-driven fluctuations by including a mild response to stock prices in the policy rule. As the analysis shows, the related benefits of responding to stock prices are more prominent in economies featuring higher credit frictions and larger demand-side wealth effects from financial holdings.

For interest rate rules that guarantee determinacy, we assess whether being more responsive to stock prices can also smooth the transmission of news on fundamental shocks, following the line of Christiano et al. (2010) and Lambertini et al. (2010). We show that, except for inflation, a larger, but still moderate, response to stock price can significantly smooth the boom-bust cycles induced by (unrealized) news on either total factor productivity or bank lending efficiency. A central bank opting for either a more aggressive response to inflation or a positive response to real activity would not be able to achieve the same stabilization results.

Our analysis has been intentionally restricted to instrumental (Taylor-type) interest rate rules. A natural extension would be to study the implication of optimized policy rules. The main difficulty with that is the definition of an appropriate welfare measure for the policy-maker. Because of the Blanchard (1985) OLG structure, our model does not feature an economy-wide representative agent, but only a representative agent at each cohort level. In principle, a benevolent policy-maker sitting at a generic time t should consider the welfare of all the cohorts who entered the market somewhere in the past (and that are still active), together with the welfare of the current newcomers and of all possible future cohorts which are not in the market yet. Nisticò (2011) has made some progress in this respect for a model without credit frictions, showing that, due to the turnover rate in markets, the central bank's objective should include an explicit concern for stock price stabilization. It would be interesting to extend the analysis to our environment. We conjecture that Nisticò's result would carry over to our environment, although, because of the credit friction and the real balances in utility, the relevant objective might include a concern for interest rate smoothing and a different definition of efficient output.

# A Appendix

## A.1 Steady State

### A.1.1 Proof to Proposition 1

This section provides a proof to Proposition ??, which states that the non-stochastic steady state is always unique.

Consider a non-stochastic zero inflation steady state, that is gross inflation  $\Pi$  is equal to 1. Without loss of generality, we assume that both the technology and the banking sector-specific shock have an unconditional mean equal to one: i.e.  $Z^y = Z^b = 1$ .

First of all, notice that from (38), since at the steady state adjustment costs to price changes are zero, and given technology, we have that Y = C = H. Without loss of generality,

From the Phillips curve (45) and  $MC^r = \frac{MC}{P}$ , at the steady state  $MC^r = \frac{\epsilon-1}{\epsilon}$ , from which (using (27)) the real wage is:

$$\frac{W}{P} = \frac{1}{\mu\Phi\left(R^l\right)} \tag{61}$$

where  $\Phi(R^l) \equiv (1 - \alpha + \alpha R^l)$  is (28) at the symmetric steady state and

$$\mu \equiv \frac{\epsilon}{\epsilon - 1} : \text{ steady state mark-up}$$
(62)

Using the consumption-leisure trade-off equation (21), technology and market clearing, we get that steady state hours H, as we all C and Y:

$$H = \frac{1}{[1 + \iota \mu \Phi(R^l)]} = C = Y$$
(63)

that is, steady state hours depend on the steady state lending rate  $R^l$ .

From (19) at the steady state:

$$\beta \frac{\left(1+\upsilon\right)\left(1-\gamma\right)}{1-\beta\left(1-\gamma\right)}PC = \gamma \frac{A}{R} + \frac{\left(1-\gamma\right)\left(1+\upsilon\right)}{1-\beta\left(1-\gamma\right)}\frac{PC}{R}$$

which, is equivalent to:

$$\beta R = 1 + \gamma \frac{1 - \beta \left(1 - \gamma\right)}{\left(1 + \upsilon\right) \left(1 - \gamma\right)} \frac{A}{PC}$$

We want to write an expression for  $\frac{A}{PC}$  entering (??). From (18):

$$A = M + P\left(Q + D\right) \tag{64}$$

while from (??)

$$Q = \frac{Q+D}{R} \tag{65}$$

Putting the latter two together:

$$\frac{A}{PC} = \frac{M}{PC} + R\frac{Q}{C} \tag{66}$$

At the steady state, from (45) and the definition in (62):  $MC^r = \frac{1}{\mu}$  (the inverse of the gross mark-up). Hence, using (48), steady state dividends are  $D = Y \frac{\mu - 1}{\mu}$ . Using the stock price equation (65), we have that  $Q = \frac{D}{R-1} = \frac{Y}{R-1} \frac{\mu - 1}{\mu}$ . By inserting the latter and the market clearing condition Y = C into (66), we obtain  $\frac{A}{PC} = \frac{M}{PC} + \frac{R}{(R-1)} \frac{\mu - 1}{\mu}$ . We are left with the computation of the ratio  $\frac{M}{PC}$ .

From money demand (43) and market clearing C = Y:

$$\frac{M}{CP} = v\frac{R}{R-1} + \chi\frac{M^d}{CP} \tag{67}$$

Notice that even if the role of liquid balances in utility was very small  $(v \to 0)$ , we could still have a substantial amount of total real balances to consumption at the steady state due to the presence of deposits. This second term would also vanish if deposits were very liquid,  $\chi \to 0$ . Substituting the expressions  $\frac{A}{PC} = \frac{M}{PC} + \frac{R}{(R-1)} \frac{\mu-1}{\mu} \text{ and } \frac{M}{CP} \text{ from (67) into (??), we obtain:}$ 

$$\beta R = 1 + \gamma \frac{1 - \beta (1 - \gamma)}{(1 + v) (1 - \gamma)} \left[ \left( v + \frac{\mu - 1}{\mu} \right) \frac{R}{R - 1} + \chi \frac{M^d}{CP} \right]$$
(68)

The last term we have to write out is the steady state expression for real deposits to consumption  $\frac{M^a}{CP}$ . Going back to the balance sheets of the financial intermediary (??), under the market clearing condition C = Y = H:

$$\frac{M^d}{CP} = \frac{\alpha}{\mu \Phi\left(R^l\right)} \tag{69}$$

From the credit spread (33), the steady state real wage (61) and the espression for the deposit rate  $R_t^d$  (14), we obtain that:

$$R^{l} - \frac{(1+\chi)R - \chi}{R} = \frac{1}{\mu\Upsilon\Phi(R^{l})} \text{ for every } i \in [0,1]$$

$$(70)$$

Equation (70) defines an implicit relationship between the riskless rate R and the lending rate  $R^{l}$ . The following Lemma states and proves that at the steady state the lending rate is a strictly increasing function of the risk-less rate.

**Lemma 1** At the steady state,  $R^{l} = g(R)$  with g'(R) > 0 for any R > 1.

**Proof.** Consider equation (70). Let  $LHS(R^l)$  and  $RHS(R^l)$  be respectively its left and right-hand sides, expressed as function of  $R^l$ . The following properties are strightforward to show: a)  $LHS(R^l)$  is strictly increasing in  $R^l$  with  $\lim_{R^l \to 1} LHS(R^l) = \frac{\chi(1-R)}{R} < 0$ , as R > 1; b) given the definition  $\Phi(R^l) \equiv (1 - \alpha + \alpha R^l)$ ,  $RHS'(R^l) < 0$  with  $\lim_{R^l \to 1} RHS(R^l) = \frac{1}{\mu \Upsilon \Phi(R^l)} > 0$  and  $\lim_{R^l \to +\infty} RHS(R^l) = 0$ . It then easily follows that, for any R > 1, there exists a unique  $R^l > 1$  that solves equation (70). By a straightforward application of the Implicif Function Theorem, we obtain that  $\frac{\partial R^l}{\partial R} > 0$ .

By this Lemma and equation (69), it is immediate that real deposits to consumption are strictly decreasing in the steady state interest rate R. Given the result in the Lemma, let  $\tilde{\Phi}(R) \equiv \Phi(g(R))$ . Combining this with equation (68) and equation (69), we obtain an equation which implicitly defines the steady state real interest rate R:

$$\beta R = 1 + \gamma \frac{1 - \beta \left(1 - \gamma\right)}{\left(1 + \upsilon\right) \left(1 - \gamma\right)} \left[ \left(\upsilon + \frac{\mu - 1}{\mu}\right) \frac{R}{R - 1} + \chi \frac{\alpha}{\mu \tilde{\Phi}\left(R\right)} \right]$$
(71)

Let F(R) be the right hand sides of (71). We can immediately notice that F'(R) < 0,  $\lim_{R \to 1} F(R) = +\infty$  and  $\lim_{R \to +\infty} F(R) = 1$ , which implies that there exists a unique steady state value R solving equation (71).

The following coefficient will be extensively used in what follows:

$$\psi \equiv \gamma \frac{1 - \beta \left(1 - \gamma\right)}{\left(1 + \upsilon\right) \left(1 - \gamma\right)} \frac{A}{PC}$$
(72)

### A.2 Log-Linearization

Notation: for a generic variable  $X_t$ , we denote  $x_t = \ln\left(\frac{X_t}{X}\right)$ , where X is the steady state value.

### A.2.1 The Reduced Form System

#### New-Keynesian Phillips Curve

From the balance sheets of the financial intermediaries,  $\frac{M_t^d}{P_t} = \frac{L_t}{P_t}$ , the loan technology  $\frac{L_t}{P_t} = \Upsilon Z_t^b \left(\frac{Q_t}{Q}\right)^{\sigma} H_t^b$ , and the external financing constraint  $\frac{L_t}{P_t} = \alpha \frac{W_t}{P_t} H_t^p$ , we have that  $\Upsilon Z_t^b \left(\frac{Q_t}{Q}\right)^{\sigma} H_t^p = \alpha \frac{W_t}{P_t} H_t^p$ . Its linearization gives:

$$w_t - p_t + h_t^p = z_t^b + \sigma q_t + h_t^p \tag{73}$$

From the consumption leisure trade-off condition (21):

$$c_t + \varphi h_t = w_t - p_t \tag{74}$$

where  $\varphi \equiv \frac{H}{1-H}$  is the inverse of the Frisch elasticity of labor supply. Combining (74) with the log-linearized versions of the market clearing condition  $C_t = Y_t$ , the production technology  $Y_t = Z_t H_t^p$  and the definition of total hours worked,  $H_t = H_t^b + H_t^p$ , we obtain:

$$w_t - p_t = y_t + \varphi \left[ \omega_h h_t^p + (1 - \omega_h) h_t^b \right]$$
(75)

where  $\omega_h = \frac{H^p}{H}$  is the share of total hours worked employed in the productive sector. Combining (73) and (75), we obtain an espression for the real wage:

$$w_t - p_t = w_y y_t - w_q q_t - w_z z_t^y - w_b z_t^b$$
(76)

$$w_y \equiv \frac{(1+\varphi)}{1-\varphi(1-\omega_h)}$$
$$w_q \equiv \frac{(1-\omega_h)\,\sigma\varphi}{1-\varphi(1-\omega_h)}$$
$$w_z \equiv \frac{\varphi}{1-\varphi(1-\omega_h)}$$
$$w_b \equiv \frac{(1-\omega_h)\,\varphi}{1-\varphi(1-\omega_h)}$$

From the credit spread equation (33):

$$r_t^l = \omega_r r_t^m + (1 - \omega_r) \left[ w_t - p_t - z_t^b - \sigma q_t \right]$$
(77)

where  $\omega_r \equiv \frac{R^m}{R^l}$ . From (14), we have that  $r_t^d = \eta r_t$ , where  $\eta \equiv \frac{\chi}{(1+\chi)R-\chi}$  is the degree of pass-through from the policy to the deposit rate. From the expression for real marginal costs (46), we have:

$$mc_t^r = w_t - p_t - z_t^y + \varsigma r_t^l \tag{78}$$

where  $\varsigma \equiv \frac{\alpha R^{l}}{[1+\alpha(R^{l}-1)]}$  identifies the degree of pass-through from the lending rate to real marginal costs. Plugging (76) and (77), together with  $r_{t}^{d} = \eta r_{t}$ , into (78), and rearranging, we obtain:

$$mc_t^r = \tau_y y_t + \tau_r r_t - \tau_q q_t - \tau_z z_t^y - \tau_b z_t^b \tag{79}$$

$$\begin{aligned} \tau_y &\equiv w_y \left[ 1 + \varsigma \left( 1 - \omega_r \right) \right] \\ \tau_r &\equiv w_r \varsigma \eta \\ \tau_q &\equiv w_q \left[ 1 + \varsigma \left( 1 - \omega_r \right) \right] + \varsigma \left( 1 - \omega_r \right) \sigma \\ \tau_z &\equiv 1 + \left[ 1 + \varsigma \left( 1 - \omega_r \right) \right] w_z \\ \tau_b &\equiv \varsigma \left( 1 - \omega_r \right) + \left[ 1 + \varsigma \left( 1 - \omega_r \right) \right] w_b \end{aligned}$$

In order to make our Phillips curve comparable to the benchmark New-Keynesian counterpart, we can write the real marginal costs as follows:

$$mc_t^r = \frac{1+\varsigma(1-\omega_r)}{1-\varphi(1-\omega_h)} (1+\varphi) (y_t - z_t) + \tau_r r_t$$
$$-\tau_q q_t - \frac{\varsigma(1-\omega_r) - \varphi(1-\omega_h)}{1-\varphi(1-\omega_h)} z_t^y - \tau_b z_t^b$$
(80)

By log-linearizing the non-linear Phillips curve (45), we obtain:<sup>55</sup>

$$\pi_t = \tilde{\beta} E_t \pi_{t+1} + \frac{\epsilon - 1}{\theta} m c_t^r \tag{81}$$

Combining (80) and (81), we obtain:

$$\pi_t = \tilde{\beta} E_t \pi_{t+1} + \Theta_y \kappa \left( y_t - z_t^y \right) + \Theta_r r_t - \Theta_q q_t + \Theta_z z_t^y - \Theta_b z_t^b$$

where  $\kappa \equiv \frac{\epsilon - 1}{\theta} \left( 1 + \varphi \right)$ , and

$$\Theta_y \equiv \frac{1+\varsigma (1-\omega_r)}{1-\varphi (1-\omega_h)}$$
$$\Theta_z \equiv \kappa \frac{\varsigma (1-\omega_r)+\varphi (1-\omega_h)}{1-\varphi (1-\omega_h)}$$
$$\Theta_i = \kappa \tau_i \text{ for } i=r,q,b$$

### Euler Equation

From the linearization of (42), we get:

$$c_{t} = \frac{1}{R\beta} E_{t} \left( c_{t+1} - r_{t} + \pi_{t+1} \right) + \gamma \frac{1 - \beta \left( 1 - \gamma \right)}{\beta \left( 1 - \gamma \right) 1 + \upsilon} \left[ \frac{Q}{C} q_{t} + \frac{M}{RPC} \left( m_{t} - p_{t} - r_{t} \right) \right]$$
(82)

<sup>55</sup>If we had positive steady state inflation, the coefficient on marginal costs would be  $\frac{\epsilon - 1}{\pi^2 \theta}$ .

Notice that the coefficient multiplying expected inflation is now  $\tilde{\beta} \equiv \frac{\beta}{1+\psi}$  and not simply  $\beta$ . The reason is simple: independently from the value of the turnover probability  $\gamma$ , the coefficient on expected inflation is simply the inverse of the gross (steady state) real interest rate,  $R^{-1}$ . This is clearly equal to  $\beta$  in the standard model, but instead equal to  $\frac{\beta}{1+\psi}$  here.

Now consider the coefficient multiplying  $q_t$  in (??). Using (66) and simple algebra:

$$\begin{split} \gamma \frac{1 - \beta \left(1 - \gamma\right)}{\beta \left(1 - \gamma\right) 1 + v} \frac{Q}{C} &= \frac{\gamma}{\beta \left(1 - \gamma\right) \Xi} \left(\frac{A}{RPC} - \frac{M}{RPC}\right) \\ &= \frac{\psi - \zeta}{1 + \psi} \end{split}$$

where

$$\zeta \equiv \frac{\gamma}{1-\gamma} \frac{1-\beta\left(1-\gamma\right)}{1+\upsilon} \frac{M}{CP}$$
(83)

and  $\frac{M}{CP}$  comes from (67) evaluated at the steady state. Notice that the introduction of real balances into the model (which is needed to introduce a cost channel) tends to reduce the role for the stock price index into the Euler Equation.

Following similar steps, we can also write the coefficient on real money balances and the interest rate in (??) as follows:

$$\gamma \frac{1 - \beta \left(1 - \gamma\right)}{\beta \left(1 - \gamma\right) 1 + \upsilon} \frac{M}{RPC} = \frac{\zeta}{1 + \psi}$$

Hence, (??) becomes:

$$c_{t} = \frac{1}{1+\psi} E_{t} \left( c_{t+1} - r_{t} + \pi_{t+1} \right) + \frac{\psi - \zeta}{1+\psi} q_{t} + \frac{\zeta}{1+\psi} \left( m_{t} - p_{t} - r_{t} \right)$$
(84)

From money demand (43):

$$m_t - p_t = \omega_d \left( m_t^d - p_t \right) + (1 - \omega_d) \left( c_t - \nu r_t \right)$$
(85)

$$\omega_d \equiv \chi \frac{M^d}{M}$$
: share of deposits in total money (86)

$$\nu \equiv (R-1)^{-1}$$
: SS interest rate elasticity of money (87)

From the banks' balance sheet:

$$m_t^d - p_t = l_t - p_t$$

$$= w_t - p_t + y_t - z_t^y$$
(88)

Combining the latter with (85) and (76), we obtain that:

$$m_t - p_t = m_y y_t - m_q \left( q_t - q_{t-1} \right) - m_r r_t - m_z z_t^y - m_b z_t^b \tag{89}$$

where

$$m_y \equiv 1 - \omega_d + \omega_d (1 + w_y)$$

$$m_q \equiv \omega_d w_q$$

$$m_r \equiv (1 - \omega_d) \nu$$

$$m_z \equiv \omega_d (1 + w_z)$$

$$m_b \equiv \omega_d w_b$$

Plugging the latter into the Euler equation (84), together with the market clearing condition  $c_t = y_t$  and rearranging:

$$y_{t} = \frac{1}{1+\psi^{m}} E_{t} y_{t+1} - \frac{1}{1+\psi^{m}} \left[ \left( 1 + \zeta \left( 1 + m_{r} \right) \right) r_{t} - E_{t} \pi_{t+1} \right] \\ + \frac{\psi - \zeta \left( 1 + m_{q} \right)}{1+\psi^{m}} q_{t} - \frac{\zeta m_{z}}{1+\psi^{m}} z_{t}^{y} - \frac{\zeta m_{b}}{1+\psi^{m}} z_{t}^{b}$$

$$\psi^m = \psi - \zeta m_y \tag{90}$$

In a more compact form:

$$y_t = \Psi_y \left[ E_t y_{t+1} - (r_t - E_t \pi_{t+1}) \right] - \Psi_r r_t + \Psi_q q_t - \Psi_z z_t^y - \Psi_b z_t^b$$
(91)

where

$$\begin{split} \Psi_y &\equiv \frac{1}{1+\psi^m} \\ \Psi_r &\equiv \frac{1+\zeta \left(1+m_r\right)}{1+\psi^m} \\ \Psi_q &\equiv \frac{\psi-\zeta \left(1+m_q\right)}{1+\psi^m} \\ \Psi_z &\equiv \frac{\zeta m_z}{1+\psi^m} \\ \Psi_b &\equiv \frac{\zeta m_b}{1+\psi^m} \end{split}$$

# **Stock Price Equation**

From (??) we get:

$$q_t = \tilde{\beta} E_t q_{t+1} + \left(1 - \tilde{\beta}\right) E_t d_{t+1} - (r_t - E_t \pi_{t+1})$$
(92)

From dividends (48) and (80):

$$d_t = y_t - \frac{mc_t^r}{\mu - 1}$$

Using (79):

$$d_t = y_t \left( 1 - \frac{\tau_y}{(\mu - 1)} \right) + \frac{\tau_q}{(\mu - 1)} q_t - \frac{\tau_r}{(\mu - 1)} r_t + \frac{\tau_z}{(\mu - 1)} z_t^y + \frac{\tau_b}{(\mu - 1)} z_t^b$$

Moving this one period forward and inserting it into (92):

$$q_t = \tilde{\beta} E_t q_{t+1} + \Gamma_y E_t y_{t+1} - (r_t - E_t \pi_{t+1}) + \Gamma_q E_t q_{t+1} - \Gamma_r E_t r_{t+1} + \Gamma_z z_t^y + \Gamma_b z_t^b$$
(93)

$$\begin{split} \Gamma_{q} &= \left(1 - \tilde{\beta}\right) \frac{\tau_{q}}{(\mu - 1)} \\ \Gamma_{y} &= \left(1 - \tilde{\beta}\right) \left[1 - \frac{\tau_{y}}{(\mu - 1)}\right] \\ \Gamma_{r} &= \left(1 - \tilde{\beta}\right) \frac{\tau_{r}}{(\mu - 1)} \\ \Gamma_{z} &= \rho_{y} \left(1 - \tilde{\beta}\right) \frac{\tau_{z}}{(\mu - 1)} \\ \Gamma_{b} &= \rho_{b} \left(1 - \tilde{\beta}\right) \frac{\tau_{b}}{(\mu - 1)} \end{split}$$

# A.3 News Shocks

The reduced-form model can be written in compact form:

$$x_t = \Omega E_t x_{t+1} + \Gamma z_t$$

where  $x_t = [y_t, \pi_t, q_t, r_t]'$ ,  $z_t = [z_{y,t}, z_{b,t}, z_{r,t}]'$ , while  $\Omega$  and  $\Gamma$  are comformable matrices depending on the structural parameters of the model and the policy rule. In particular:

$$\Gamma_{y,y} \quad \Gamma_{y,b} \quad \Gamma_{y,r}$$

$$\Gamma = \quad \Gamma_{\pi,y} \quad \Gamma_{\pi,b} \quad \Gamma_{\pi,r}$$

$$\Gamma_{q,y} \quad \Gamma_{q,b} \quad \Gamma_{q,r}$$

where  $\Gamma_{i,j}$  is the response of variable *i* to the shock  $z_{j,t}$ , for  $i = y, \pi, q$  and j = y, b, r.

Although news shocks are present, each of the three shocks can be given a simple recursive representation:

$$\tilde{z}_{k,t} = U_k \tilde{z}_{k,t-1} + \tilde{u}_{k,t}$$

where  $\tilde{z}_{k,t} \equiv [z_{k,t}, u_{y,t}^4, u_{y,t-1}^4, u_{y,t-2}^4, u_{y,t-3}^4]', \ \tilde{u}_{k,t} \equiv [u_{k,t}^0, u_{k,t}^4, 0, 0, 0]'$  and

$$U_k = \begin{bmatrix} \rho_k & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

for k = y, b, r. Unde this new notation, the reduced form linearized system can be written as:

$$x_t = \Omega E_t x_{t+1} + \tilde{\Gamma}_y \tilde{z}_{y,t} + \tilde{\Gamma}_b \tilde{z}_{b,t} + \tilde{\Gamma}_r \tilde{z}_{r,t}$$
(94)

where

$$\tilde{\Gamma}_{y} = \begin{bmatrix} \Gamma_{y,y} & 0 & 0 & 0 & 0 \\ \Gamma_{\pi,y} & 0 & 0 & 0 & 0 \\ \Gamma_{q,y} & 0 & 0 & 0 & \lambda_{z} \end{bmatrix}, \quad \tilde{\Gamma}_{b} = \begin{bmatrix} \Gamma_{y,b} & 0 & 0 & 0 & 0 \\ \Gamma_{\pi,b} & 0 & 0 & 0 & 0 \\ \Gamma_{q,b} & 0 & 0 & 0 & \lambda_{b} \end{bmatrix}, \quad \tilde{\Gamma}_{r} = \begin{bmatrix} \Gamma_{y,r} & 0 & 0 & 0 & 0 \\ \Gamma_{\pi,r} & 0 & 0 & 0 & 0 \\ \Gamma_{q,r} & 0 & 0 & 0 & -\lambda_{r} \end{bmatrix}$$

The latter can be writte more compactly as:

$$x_t = \Omega E_t x_{t+1} + \tilde{\Gamma} \tilde{z}_t \tag{95}$$

where  $\tilde{z}_t \equiv vec[\tilde{z}_{y,t}, \tilde{z}_{b,t}, \tilde{z}_{r,t}]$  (i.e. the three vectors stacked on top of each other) and  $\tilde{\Gamma} = \left[\tilde{\Gamma}_y, \tilde{\Gamma}_b, \tilde{\Gamma}_r\right]$ .

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