A Bayesian Approach to Optimal Monetary Policy under Parameter and Model Uncertainty

Tim Cogley (NYU), Bianca De Paoli (BoE), Christian Matthes (NYU), Kalin Nikolov (BoE) and Tony Yates (BoE)

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1 What we do

- Compute policy rule coefficients that are optimal in light of uncertainty about shocks, parameters and models.

- Bayesian approach: elicit posterior distributions from data for shocks and parameters conditional on model, and posterior odds for 4 models.

- Compute expected loss for candidate policy rule, expectation taken over distribution of shocks, parameters and models; minimise this expected loss.

- Model Suite consists of: Rudebusch-Svensson; Smets-Wouters, BGG, SOE model.

- Suite chosen to span RE/non-RE, microfounded/non mf, open/closed, with/without financial sector.

- Investigate fault-tolerence of models, disparity of optimal rules across models.

- Describe the influence of components of the suite on the optimal policy.

- Assume policymaker constrained to follow simple Taylor-like rule.

- Use ad-hoc loss criterion involving weighted variance of inflation, output gap, nominal interest rate.

- Rule out learning and experimentation by the policymaker: no feedback from policy rule to distributions.

2 Why are we doing this?

- Live controversies in macro: policymakers confront pervasive uncertainty.

- Approach formalises elements of policymakers' own descriptions of how they deal with uncertainty: eg suite of models philosophy in the Bank of England.

- Bring together in one place elements of a recipe already known and found in other work.

3 Related Literature

- Bayesian estimation of DSGE models

  Schorfheide (2000)

  Smets and Wouters (2003)

- Forecasting literature on model averaging

  Bates and Granger (1969), Clements and Hendry (1998, 2002), Jacobson and Karlsson (2002), Labhard, Kapetanios and Price (2005).

- Our suite also generates forecasts, but our focus is on the additional step of policy design.

- Bayesian optimal policy under model uncertainty

  Brock, Durlauf and West (2003)

- They restrict their attention to non micro-founded models (we have chosen to include some that are and some that are not); they use frequentist methods to estimate each model (we use Bayesian methods).

- Cogley and Sargent (2005)

- This is a positive analysis: can past Fed actions be explained as the outcome of a Bayesian decision problem? Our focus is normative.

- Literature on robustness

  McCallum (1988,....), Levin and Williams (2003), Levin, Wieland and Williams (2003), Levin, Onatski and Williams (2008), Hansen and Sargent (2008)

- Literature seeks to evaluate the variance - or robustness - of policy rules' performance across models.

- Implicit or explicit assumption that we cannot put probability on competing models is distinct from our approach; study performance in worst case out of set of possible models.

- Our policymaker puts zero probability on many models on the table in some of this other work. So we understate the true degree of model uncertainty, despite using suite to span competing model approaches.

4 The suite of models

- Smets-Wouters (2003): large DSGE model with many real and nominal frictions. Fits US and Euro Area data well.

- Rudebusch and Svensson (1997): small backward-looking, non-microfounded model, providing therefore an ideal contrast with SW.

- Bernanke, Gertler and Gilchrist (1999): sticky price RE model with credit frictions in capital production. Adds credit frictions to the 'suite'.

- Gali and Monacelli (2005): small open economy New Keynesian model. The UK is plausibly an SOE, and no other models articulate the external sector.

5 The Data

- UK

- Choose largest set of data articulated by all models: GDP GDP deflator inflation, repo rate.

- Common data set necessary to make model comparisons meaningful.

- Alternative: append time series models for variables not otherwise articulated to the smaller models. Not attractive. Would also render model comparisons less meaningful.

- All data is detrended prior to estimation.

- Sample period (1993 Q1 - 2006 Q3); post IT sample allows us to plausibly assume constant coefficient and constant inflation target policy rules in estimation.

6 Comments on estimation: Rudebusch and Svensson (1997)

- Baseline priors: same modes as Rudebusch and Svensson (1997) US point estimates, with large variances.
- These priors encode high degree of inflation and output gap persistence.
- Posterior: lower persistence in both equations, confirming other econometric studies eg Benati (2008), Levin and Piger (2006)
- Posterior: weaker influence of output gap on inflation.

6.1 Alternative priors for RS model

- Finding of low persistence, and weak effect of output gap on inflation is important for overall conclusions about optimal policy in the suite, so test robustness.

- Use 2 alternative priors

- Tighter priors based on the original RS point estimates: posterior delivers higher persistence, but still struggle to force it on the data, which is informative about persistence.

- Tighter prior centred on low persistence in inflation and output.

6.2 Comments on estimation: Smets-Wouters model

- Retain only 4 shocks (mark-up, tfp, mon pol, govt exp); and remove MA components.

- Priors standard...

- Except: diffuse priors for price and wage indexation centred on 0.5.

- Tight priors centred around low persistence for the mon pol shock, and high persistence for the govt spending shock.

- Posteriors generally = prior: small number of data series worsens identification.

- Nominal rigidities badly identified with exception of degree of price indexation (mode $= 0.16$).

  Other parameters about which data are informative: eg elasticity of capital utilisation costs, where posterior mode greater than SW prior.

7 Comments on estimation: Bernanke, Gertler and Gilchrist (1999) model

- Calibrate parameters in the contracting block to BGG values.

- Posterior suggests frequency of price adjustment 1.5 quarters, shorter than in SW model.

8 Comments on estimation: Gali and Monacelli (2005) SOE model

- Priors taken from Lubik and Schorfheide (2007)

- Posterior mode for degree of price stickiness suggests something in between SW and BGG

9 Welfare and model uncertainty

$$l_i(\phi) = \int l_i(\phi, \theta_i) p(\theta_i | Y, M_i) d\theta_i. \tag{1}$$

$$l(\phi) = \sum_{i=1}^{m} l_i(\phi) p(M_i | Y). \tag{2}$$

$$p(M_i | Y) \propto p(Y | M_i) p(M_i), \tag{3}$$

where

$$p(Y | M_i) = \int p(Y | \theta_i, M_i) p(\theta_i | M_i) d\theta_i \tag{4}$$

10 Period loss, policy rule

- For a given policy ϕ, and a given model j with parameterization θ_{jk}, the period loss function is

$$l_j(\phi, \theta_{jk}) = E\left[var\,(4\pi_t) + \lambda_y var\,(y_t - y_t^*) + \lambda_i var(4i_t)|\phi, \theta_{jk}\right]. \tag{5}$$

- Simple rule for policy given by:

$$i_t = \phi_i i_{t-1} + (1 - \phi_i)\,(\phi_\pi \pi_t + \phi_y y_t) + \phi_{dy}\,(y_t - y_{t-1}) \tag{6}$$

11 Optimal policy in individual models

Coefficients	**SW**	**BGG**	**SOE**	**RS1**	**RS2**	**RS3**
Smoothing	0.99	0.03	0.61	0.06	0.81	0.05
Inflation	65.3	100.0	42.19	0.01	1.01	0.01
Output	7.71	-0.06	-0.20	0.03	0.08	0.05
Output Growth	1.71	-0.20	4.10	0.00	0.10	0.00
Loss	5.62	0.035	0.83	3.45	6.75	3.28

- Optimal rule for individual models differs substantially across model
  - Best estimate of RS model$\rightarrow$ little intrinsic inflation inertia$\rightarrow$ passive optimal rule, mainly to minimize interest-rate volatility
  - Policy optimal for the BGG model close to inflation-only Taylor rule (model encodes low inflation inertia)
  - SW-optimal rule close to first-difference rule for the nominal interest rate + high long-run response coefficients on inflation and output (model features bigger policy trade-off)
  - GM $\rightarrow$ policy can stabilize the output gap and producer prices (though not consumer prices)$\rightarrow$ welfare lower than SW model. Forward-looking model $\rightarrow$ high long-run coefficient on inflation.

12 Models' 'fault tolerance'

- Relative Loss in Model i (rows) Under a Policy Optimized for Model j (columns)

	SW	BGG	SOE	RS1	RS2	RS3
SW	1	∞	5.37	∞	1.35	∞
BGG	334	1	3.82	∞	3339	∞
SOE	5.98	1.40	1	∞	50	∞
RS1	2.77	∞	45	1	1.02	1.00
RS2	3.30	∞	∞	∞	1	∞
RS3	1.94	∞	44	1.00	1.02	1

- Rules optimized for variants of RS model bad for forward-looking economies:
  - Rules for RS1 and RS3 do not satisfy Taylor principle$\longrightarrow$ indeterminacy in forward-looking models
  - Rule for RS2 satisfies Taylor principle, but low long-run inflation response bad for models with little nominal inertia (BGG and SOE)
- Inflation-only Taylor rules (BGG rule)
  - Good when there is little nominal inertia (BGG and SOE)
  - Bad for other models
    - Explosive in the backward-looking models

  * SW model also unstable$\longrightarrow$ backward-looking indexation calls for a high degree of interest-rate smoothing in order to stabilise inflation and the output gap

- RS1 and RS3$\longrightarrow$ high fault tolerance when rule does not have an enormous short-run response coefficient on inflation (BGG rule)

- SW rule performs reasonably well in all models

13 Posterior model weights

(Prior weight=1/4 each model)

- Baseline RS model

  SW = 0.80, BGG = 0.175, RS = 0.020, GM = 0.003

- Tight RS prior with high persistence

  SW = 0.82, BGG = 0.18, RS = 0.00, GM = 0.003

- Tight RS prior with low persistence

  SW = 0.163, BGG = 0.036, RS = 0.801, GM = 0.001

14 Policy analysis for the suite

14.1 Optimal policy with Bayesian model weights

Coefficients	Bayes 1	Bayes 2	Bayes 3
Smoothing	0.97	0.97	0.51
Inflation	39.5	48.81	1.53
Output	4.60	4.92	0.07
Output growth	1.60	1.85	-0.01
Loss	5.59	5.42	4.09

- 1st and 2nd columns: high degree of interest smoothing, large long-run responses to inflation and real activity$\longrightarrow$ similar to SW-optimal rule...
  - but a bit less interest smoothing than SW rule, lower long-run inflation and output responses and stronger short-run responses to inflation and output (in the direction of the BGG- and SOE-optimal rules).
  - Reflects the high probability weight on the SW model, relatively good performance of SW rule in other models, backward-looking models have low probability weight and their rules perform badly elsewhere

- Relative Loss Under Bayesian Policies

	Bayes 1	Bayes 2	Bayes 3
SW	1.04	1.07	1.11
BGG	106	84.5	142.1
SOE	3.01	2.66	7.67
RS1	3.57	–	–
RS2	–	5.69	–
RS3	–	–	1.09

- By going in the direction of BGG- and SOE-optimal policies, Bayesian policy-maker mitigate losses in these models while still achieving good performance in the SW model. (relative to the SW-optimal rule, these policies inflation volatility)

- Outcomes in RS models are worse under the Bayesian policies than under the SW-optimal rule (nominal interest rate is enormously volatile under Bayesian policies 1 and 2 but these models have low probability weights in suites 1 and 2).

- Suite 3: combines forward-looking models with RS3 $\rightarrow$ Bayesian policy resembles a conventional Taylor rule with interest smoothing.
  - Bayesian policy differs significantly from the optimal policy of its most probable member. RS3-optimal rule: the best a central bank can do is to minimize nominal interest volatility. But RS3-optimal policy cannot be optimal for the suite because it violates the Taylor principle

14.2 Optimal policy with equal weights

Coefficients	Suite 1	Suite 2	Suite 3
Smoothing	0.37	0.27	0.37
Inflation	2.55	2.17	2.57
Output	0.04	0.01	0.03
Output growth	0.63	0.57	0.64
Loss	3.58	4.68	3.53

- Policies resemble speed-limit versions of the Taylor rule + modest degree of interest smoothing (ϕ_y close to zero, ϕ_{dy} around 0.6, long-run response to inflation of 2.2 - 2.6, ϕ_r around 0.35).

- Differences in the prior over RS-model have little impact on optimal policy

- Suites 1 and 2: rule deviate more from the SW-optimal rule than the Bayesian policies (less interest smoothing, smaller response to long-run inflation, output and output growth) given higher weight on BGG, SOE, and RS

- Suite 3: closer to Bayesian-weighted policy (given bigger weight of RS3)

- Pros and cons of using equal model weights:

  - Pros: difficulties in managing model set and estimating Bayesian probabilities

  - Favors poor-fitting models at the expense of good-fitting models.

15 Conclusions

- Compute policy rules optimised wrt uncertainty about shocks, parameters and models, using Bayesian estimates of 4 models on UK data from inflation targeting period

- Forward-looking models $\rightarrow$ low fault tolerance to policies designed for backward-looking models (these either violate/or barely satisfy Taylor principle)

- But backward-looking RS model $\rightarrow$ high fault tolerance to policies designed for forward-looking models

- In suites in which backward looking model has low weight, optimal policy entails aggressive response to inflation fluctuations + high degree of interest rate smoothing

- When RS model has high weight, optimal policy still far from rule optimal in RS model, since that rule violates Taylor principle.

16 Other tables

Table 4: Volatility under Model-Specific Policies

	Inflation	Output	Nominal Interest
SW	4.31	1.18	1.34
BGG	0.0002	0.003	0.30
SOE	0.20	1.05	2.64
RS1	3.33	0.10	0.17
RS2	6.17	0.27	3.19
RS3	3.14	0.12	0.17

Table 6: Volatility in Model i under a Policy Optimized for Model j

	SW	BGG	SOE	RS1	RS2	RS3
SW	4.31 1.18 1.34	–	3.45 13.1 136	–	4.57 2.93 0.77	–
BGG	11.09 0.03 0.96	0.0002 0.003 0.3	0.09 0.003 0.35	–	108 0.23 38.3	–
SOE	4.43 0.88 1.58	0.001 1.14 6.82	0.20 1.05 2.64	–	38.7 1.11 15.12	–
RS1	3.32 0.17 60.5	–	3.28 1.60 1507	3.33 0.10 0.17	3.32 0.10 1.051	3.33 0.10 0.17
RS2	5.90 1.13 153	–	–	–	6.17 0.27 3.19	–
RS3	3.14 0.19 30.3	–	3.17 2.36 1385	3.14 0.12 0.17	3.14 0.12 0.95	3.14 0.12 0.17

Table 8: Relative Loss Under Bayesian Policies

	Bayes 1	Bayes 2	Bayes 3
SW	1.04	1.07	1.11
BGG	106	84.5	142.1
SOE	3.01	2.66	7.67
RS1	3.57	–	–
RS2	–	5.69	–
RS3	–	–	1.09

Note: Losses are reported relative to the policy that is optimal in each model.

Table 9: Volatility Under Bayesian Policies

	Bayes 1			Bayes 2			Bayes 3		
SW	4.21	1.39	2.70	4.17	1.49	3.36	4.47	1.35	4.49
BGG	3.57	0.01	0.79	2.76	0.01	0.71	4.56	0.01	2.04
SOE	2.06	0.93	1.39	1.80	0.93	1.26	5.34	0.99	8.05
RS1	3.31	0.21	87.7		–			–	
RS2		–		5.66	1.94	308		–	
RS3		–			–		3.14	0.12	3.06

Table 11: Volatility under equal-weight policies

	Suite 1			Suite 2			Suite 3		
SW	4.14	1.92	8.09	4.18	1.91	8.14	4.13	2.00	8.10
BGG	0.99	0.01	0.84	1.15	0.01	0.99	1.05	0.01	0.82
SOE	1.21	1.02	2.66	1.36	1.02	3.20	1.17	1.02	2.58
RS1	3.32	0.10	11.8		–			–	
RS2		–		5.82	0.30	21.9		–	
RS3		–			–		3.14	0.12	11.4