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Non-linear dynamics in output, real exchange rates and real money balances:
Norway, 1830-2003

by

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Abstract

We characterise the behaviour of Norwegian output, the real exchange rate and real money balances over a period of almost two centuries. The empirical analysis is based on a new annual data set that has recently been compiled and covers the period 1830–2003. We apply multivariate linear and smooth transition regression models proposed by Teräsvirta (1998) to capture broad trends, and take into account non-linear features of the time series. We particularly investigate and characterise the form of the relationship between output and monetary policy variables. It appears that allowance for state-dependent behaviour and response to shocks increases the explanatory powers of the models and helps bring forward new aspects of the dynamic behaviour of output, the real exchange rate and real money balances.

Keywords: *Business cycles, real exchange rates, money demand, non-linear modelling, smooth transition regressions.*

JEL classifications: C51, E32, E41, F31.

1   Introduction

For a long time, linear empirical models of business cycles were the standard tool of trade in analyses of fluctuations in output and its interaction with monetary and fiscal policy and foreign shocks. Accordingly, symmetric behaviour of output in recessions and expansions and symmetric response to positive and negative impulses, irrespective of their sizes, were imposed by the choice of model.

Economic theory, however, has long recognised the presence of real and nominal rigidities in labour and product markets, uncertainty, coordination failure, credit rationing and other constraints facing economic agents that may lead to non-linear demand and supply curves, unemployment hysteresis, multiple equilibria in growth rates, and size and sign dependent response of output to various shocks; see e.g. Akerlof (1973), Stiglitz and Weiss (1981), Diamond (1982), and Murphy et al. (1989). The literature on international trade also notes that large entry costs in a market may lead to size and sign dependent response of imports and exports to real exchange rate shocks; see e.g. Baldwin and Krugman (1989). Real exchange rates themselves are known to undergo different adjustments in the face of small and large shocks; cf. Heckscher (1916) and Sercu et al. (1995).

Moreover, it is well known that the response of output to a shock may even depend on the persistence of the shock as perceived by economic agents. Such persistence dependent response may be ascribed to e.g. the asymmetric response of consumption to transitory versus permanent shocks to income and wealth, or to the effects of transitory versus permanent shocks to pricing, manning, investment and production decisions of firms; see e.g. Taylor (2001), Dixit (1992) and the references therein.

In addition, the literature on time inconsistency of policies has drawn attention to state-dependent commitment of policy makers to announced policies. In particular, the literature on currency crises focuses on trade-offs faced by policy makers when deciding whether or not to honour their commitment to an announced exchange rate target, making their commitment dependent on the state of the business cycle and the size and signs of, e.g., terms of trade shocks; see *inter alia* Dumas and Svensson (1994) and Ozkan and Sutherland (1998). This suggests that not only may the response of output to policy shocks be state-dependent, but even the response of monetary and fiscal authorities can depend on the state of the economy. A number of studies ascribe such state-dependent policy response to preferences of policy makers; see Bec et al. (2002) and the references therein.

Development of non-linear time series and econometric models have vastly increased the scope of empirical analyses. Non-linear autoregressive models have made it possible to investigate and characterise the presence of non-linear behaviour of economic variables and have turned out to be useful forecasting devices. Notable applications of such models to represent the behaviour of key macroeconomic variables include Neftçi (1984), Hamilton (1989), Teräsvirta and Anderson (1992), Dumas (1992), Granger and Teräsvirta (1993), Rothman (1998), and Skalin and Teräsvirta (1999). So far, development and applications of non-linear univariate models seem to dominate the literature on non-linear models relative to multivariate non-linear

models.

Yet, multivariate models are required to examine the response of indicators of business cycles to various shocks and to test non-linear behaviour implied by different kinds of frictions highlighted by economic theories. Important recent contributions to the development of multivariate non-linear models include Engle and Hamilton (1990), Krolzig (1997) and Teräsvirta (1998). These models have been successfully employed in analyses of business cycles, employment, money demand and exchange rates by, *inter alios*, Clements and Krolzig (1998), Burgess (1992), Teräsvirta and Eliasson (2001), Meese and Rose (1991), Michael et al. (1997), and Taylor et al. (2001).

This paper applies both linear and non-linear multivariate models to characterise and explain Norwegian output, the real exchange rate and real money balances. By developing models of the real exchange rate and real money balances, we are able to investigate their interaction with output and with each other. Moreover, they enable us to investigate whether and to what extent the response of monetary policy to domestic and foreign shocks varies with the state of the economy.

Once one leaves the realm of linear models, however, one is faced with a choice between various types of non-linear models. Such models generally differ in the extent and form of implicit and explicit restrictions on model formulation and estimation algorithms. We have, however, limited our choice to the smooth transition regression (STR) class of models; see Granger and Teräsvirta (1993) and Teräsvirta (1998). These models are quite flexible and enable one to represent many forms of non-linear behaviour. In particular, they allow for both smooth and abrupt transitions between different states.

We employ a new data set that covers a period of more than 170 years, from 1830 to 2003. The time series for the Norwegian economy has recently been extended so far back to the 19th century.

The paper is organised as follows. Section 2 outlines the modelling and evaluation framework of STR models. Section 3 provides a brief description of the time series that we use in the empirical analysis. Section 4 develops linear and non-linear multivariate models of output (real gross domestic product, GDP), the real exchange rate against pound sterling and narrow money balances (M0) in real terms over the full sample. The UK represents the foreign sector in our study as it has been among Norway's major trading partners over the whole sample period and because of its technological lead, which may potentially account for a stochastic trend in the Norwegian time series.

More specifically, in Section 4 we first develop linear equilibrium correction models of GDP, the real exchange rate and real narrow money balances and undertake a comprehensive investigation of potentially neglected non-linear effects of various foreign and domestic variables in these models. In light of these results, we develop non-linear equilibrium correction models that improve on the corresponding linear models in terms of explanatory power and bring forward new aspects of the behaviour of GDP, the real exchange rate and real money.

Section 5 is devoted to the specification and evaluation of the non-linear models. Section 6 concludes and finally, an appendix provides a detailed account of

deterministic variables used in this study.

2   STR models

A smooth transition regression (STR) model of variable x can be formulated as follows:

$$x_t = z'_t(\varphi_0 + \varphi_1 \times F(\gamma, c; s_t)) + \varepsilon_t, \tag{1}$$

where z_t is a vector of explanatory variables, which may include lags of x_t. φ_0 and φ_1 are vectors of the associated coefficients and $F(\gamma, c; s_t)$ is a transition function (hereafter denoted as F), which is characterised by two parameters γ and c, and a variable s_t that governs the transition function.[1]

In STR models, F is assumed to increase monotonically with the level of s_t and it is bounded. It can either be specified as a logistic function (LSTR) or as an exponential function (ESTR). These specifications of F define LSTR and ESTR models, respectively.

The logistic function is specified as:

$$F(\gamma, c; s_t) = (1 + \exp\{-\gamma(s_t - c)\})^{-1}, \qquad \gamma > 0. \tag{2}$$

The STR model allows the coefficients to change with the value of F. Accordingly, the process determining x_t changes with the state variable s_t. Specifically, the LSTR specification allows the process for x_t to vary between $z'_t\varphi_0 + \epsilon_t$ and $z'_t(\varphi_0 + \varphi_1) + \epsilon_t$ as $(s_t - c) \to -\infty$ and $(s_t - c) \to \infty$, respectively. The parameter γ determines the speed of transition between these two extreme regimes, for a given deviation s_t from a presumably constant threshold value c. In general, LSTR models allow one to take into account effects of both the size and sign of s_t on the x_t-process.

The exponential smooth transition function (ESTR) is specified as:

$$F(\gamma, c; s_t) = 1 - \exp\{-\gamma(s_t - c)^2\}. \tag{3}$$

In this case F rises symmetrically when s_t deviates from c. Moreover, small deviations have smaller effects on x_t than large deviations due to the quadratic term in the transition function. The parameter γ determines the speed of transition between regimes when s_t deviates from c. The exponential specification of F allows the process determining x_t to shift between $z'_t\varphi_0 + \varepsilon_t$ and $z'_t(\varphi_0 + \varphi_1) + \varepsilon_t$ depending on the size of the deviation $(s_t - c)$. In general, ESTR models are well suited to capture size-dependent effects of s_t on the x_t-process.

STR models are quite general and allow for both smooth and abrupt transitions between two regimes $z'_t\varphi_0 + \varepsilon_t$ and $z'_t(\varphi_0 + \varphi_1) + \varepsilon_t$ for a process x_t. Sufficiently large values of γ may lead to an abrupt transition from one regime to another upon a typical deviation $s_t - c$, while small values lead to smooth transitions. In the former case, STR models resemble threshold models where even small deviations between s_t

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[1]A smooth transition autoregressive (STAR) model is obtained when z_t only contains an intercept and lags of x_t, and $s_t = x_{t-l}$, where the integer $l > 0$.

and c make F shift from one extreme value to another. Furthermore, a linear model is nested in a STR model. Specifically, if $\gamma \to 0$, F converges towards a constant, and hence F becomes independent of s_t. The generality of STR models make them suitable for allowing state dependent responses of a variable to changes in other variables, e.g. for allowing sign- and size-dependencies in the adjustment towards equilibrium, or for representing asymmetric responses of a variable to various shocks.

2.1 Testing for non-linearity and its form

In this sub-section, we outline the STR modelling strategy which is described in detail in Teräsvirta (1998). This modelling strategy consists of three stages. In the first stage, we test linear dynamic model specifications against non-linear STR alternatives. If the null hypothesis of linearity is rejected, we conclude that a non-linear modelling approach is warranted and the next two stages consist of specification and evaluation of non-linear STR models until a set of model design criteria is met.

In the first stage, residuals from a linear model of x, say (4), are subjected to tests for neglected state-dependent (non-linear) effects of a set of variables z:

$$x_t = z_t'\varphi_0 + u_t. \tag{4}$$

The potentially neglected non-linear effects of STR form are approximated by cross products of z_t and a state variable s raised to the power of 1–3. The relevance of these terms is thereafter tested in an auxiliary regression model, such as:

$$\widehat{u}_t = z_t'\beta_0 + (z_t s_t)'\beta_1 + (z_t s_t^2)'\beta_2 + (z_t s_t^3)'\beta_3 + v_t, \tag{5}$$

where $\widehat{u}_t$ is a residual from model (4) and v_t is an error term.

The test of a linear model against a ST(A)R model characterised by a state variable s is equivalent to conducting a joint test of:

$$H_0\colon \beta_1 = \beta_2 = \beta_3 = 0.$$

Empirically, s can be determined by conducting this test for several variables in e.g. the vector z. If linearity is rejected for more than one variable, the variable causing the strongest rejection of the null hypothesis, i.e. the variable corresponding to the lowest p-value of the joint test, is likely to be an appropriate state variable s.

If the linear model is rejected in this test, one needs to test the appropriateness of a logistic specification of F against an exponential specification. For this purpose, the following sequence of tests within the auxiliary regression has been suggested:

$$
\begin{aligned}
H_{04} &: \quad \beta_3 = 0, \\
H_{03} &: \quad \beta_2 = 0 \mid \beta_3 = 0, \\
H_{02} &: \quad \beta_1 = 0 \mid \beta_2 = \beta_3 = 0.
\end{aligned}
$$

An LSTR model is chosen if H_{04} or H_{02} is rejected, but an ESTR model is chosen if H_{03} is rejected for the chosen s_t; see Teräsvirta (1998). If all hypotheses are rejected, an LSTR (ESTR) specification of F is chosen if H_{04} or H_{02} is rejected more (less) strongly than H_{03}. When testing H_{03} and H_{02}, β_2 and β_1 are tested by prior imposition of $\beta_3 = 0$ and $\beta_2 = \beta_3 = 0$, respectively.

2.2 Evaluation of STR models

After deriving a certain specification of an STR model for $s_t = s_t^*$, say (6), it remains to be seen whether it adequately characterises the non-linearity of STR form:

$$x_t = z_t' \beta_0 + z_t' \theta \times F(\gamma, \ c; \ s_t^*) \ + \epsilon_t. \tag{6}$$

For this purpose we formulate the following auxiliary regression:

$$
\begin{aligned}
\widehat{\epsilon}_t \ = \ & z_t' \theta_0 + z_t' \times F(\widehat{\gamma}, \ \widehat{c}; \ s_t^*)\theta_1 \ + z_t'\widehat{\varphi}_1 \times (\partial F(.) \ /\partial\widehat{\gamma}) \, \theta_2 + z_t'\widehat{\varphi}_1 \times (\partial F(.) \ /\partial\widehat{c}) \, \theta_3 \\
& + (z_t s_t)' \widetilde{\beta}_1 + (z_t s_t^2)' \widetilde{\beta}_2 + (z_t s_t^3)' \widetilde{\beta}_3 + w_t, \tag{7}
\end{aligned}
$$

where $\widehat{\epsilon}_t$ is the residual from the non-linear model (6), w_t is an error term and "$\widehat{}$" indicates the estimated value of a parameter or an error term.

The null hypothesis of no remaining non-linearity dependent on s is tested by conducting the joint test of H_0: $\widetilde{\beta}_1 = \widetilde{\beta}_2 = \widetilde{\beta}_3 = 0$. If this null hypothesis is rejected for a transition variable s including s^*, the form of the remaining non-linearity can be determined by undertaking the test sequence specified above with $\widetilde{\beta}_i$ replacing β_i where $i = 1, \ 2, \ 3$. The model is then respecified accordingly to obtain a satisfactory characterisation of the remaining non-linearity. The adequacy of the respecified model is examined by testing for remaining non-linearity within a new auxiliary regression analogous to (7), where the estimated first derivatives of the terms defining the additional non-linearity with respect to their parameters are added to the auxiliary regression.

Evaluation of a non-linear model also includes tests for parameter non-constancy, residual autocorrelation, heteroscedasticity of different form and alternative tests of model misspecification. These tests may be conducted in the following way.

Tests for parameter constancy with respect to, e.g., the initial parameters defining the linear model, can be performed by testing the null hypothesis of non-linearity with $s_t = t$, which denotes a deterministic trend. If this null hypothesis is rejected, one can characterise the non-constancy of being either of the LSTR or the ESTR form; see Lin and Teräsvirta (1994) for an elaboration.

A test for residual autocorrelation of order p can be conducted by replacing the regressors in the second row of the auxiliary regression (7) with lagged residuals up to order p and testing their significance; see Eitrheim and Teräsvirta (1996). The test for heteroscedasticity is based on the regressors and their cross products and can be undertaken by replacing the regressors in the second row of (7) with the squares of the regressors in the first row and testing their significance; cf. White (1980). A test for autoregressive conditional heteroscedasticiy (ARCH) up to order p can be performed by regressing squares of the residuals ($\widehat{\epsilon}_t^2$) on a constant and their lagged values up to order p and testing for their significance.

Finally, model specification can be examined by conducting a RESET (Regression Specification Error Test) by replacing the regressors in the second row of (7) with the square and/or the third power of the fitted value of x, i.e. $\widehat{x}^2$ and $\widehat{x}^3$, from the non-linear model and testing whether they become significant in the model. A

RESET, however, tests for general model misspecifications (non-linearity and omitted variables) and may have low power against specific forms of non-linearities such as STR forms; see e.g. Teräsvirta (1996).

3   Data and its properties

We use annual observations for the period 1830–2000 for estimation of models. Many of the time series for Norway have only recently been compiled for such a long period, particularly for most of the 19th century and the early 20th century. Some time series span a longer period than 1830–2003, but a common sample of all time series is only available for the period 1830–2000. In particular, annual observations of Norwegian and UK GDP are available only from 1830.

The time series that have been extended include Norwegian GDP public consumption; all measured in fixed prices. In addition, the extension covers time series of a number of nominal variables including the index of consumer prices, narrow money balances, the nominal spot exchange rate of pound sterling in terms of the Norwegian krone and government bond yields.

The new figures for Norwegian GDP (Y) and public consumption (CO) cover the period 1830–1865. They are spliced with Norwegian official statistics from 1865 onwards; see Grytten (2004b) for details.

The index of consumer prices for Norway (CPI) has been extended backward for the period 1516–1870; see Grytten (2004a) for details. For the years afterwards, the new estimates are linked with consumer price indices provided by Norwegian official sources.

Narrow money balances ($M0$) are defined as total currency in circulation (notes and coins) plus total demand deposits at Norges Bank, and measures the total amount of liquid claims on the central bank held by the private sector including all banks. The data for $M0$ excludes amounts due to the treasury and various public sectors; see Klovland (2004c).

Annual nominal yield on long-term bonds issued by the Norwegian government (R) is based on monthly data for market quotations on Norwegian bonds traded on several European bourses (until 1920) and in Christiania/Oslo (from 1881); see Klovland (2004a) for details.

The real exchange rate (REX) is defined as $REX = S \times CPIUK/CPI$ where S is the nominal spot exchange rate, while CPI and $CPIUK$ are consumer price indices for Norway and the UK , respectively; see Klovland (2004b) for details.

The UK index of consumer prices ($CPIUK$) is based on the cost of living indices for the UK constructed by Feinstein (1991, 1998) for the century before WWI; the official cost of living after 1914; Mitchell (1998)'s record of the UK cost of living index for the years 1914 to 1988; and the official consumer price index for the years afterwards.

The time series for UK real GDP (YUK) has been obtained from Officer (2003) who offers a continuous annual time series of UK GDP for the period 1830–2000. For earlier years of the 19th century, only decennial observations on UK GDP seem

to be available.

A time series for government consumption in the UK ($COUK$) since 1830 has been obtained by linking Mitchell (1998)'s estimates of UK civil government total expenditures for the period 1830–1980 with the series of UK government consumption expenditures from the IMF's International Financial Statistics data base (IMF-IFS hereafter) from 1981 onwards.

Similarly, estimates of narrow money balances for the UK ($M0UK$) for the full sample period rely on two sources. For the period 1830–1968, we define UK M0 as the sum of notes issued by Bank of England and by other UK banks as recorded by Mitchell (1975). For the years afterwards, we chain the derived series with statistics from the IMF-IFS database on $M0UK$.

We also allow for effects of foreign interests rates, i.e. UK government bond yield, on domestic variables. The time series for the UK government bond yield refers to the yield on UK consols in the period 1830–1968 and to the yield on UK government bonds afterwards. Data for the UK consols was obtained from Mitchell (1975), while that on the government bond yield has been extracted from the IMF-IFS database.[2]

[Figure 1 about here.]

[Figure 2 about here.]

[Figure 3 about here.]

Figures 1–3 display observations of our main variables and their transformations over the period 1830–2003. The shaded areas designate the Crimean war (1854–1856), WWI (1914–1918), WWII (1940–1945) and the Korean war (1950–1953). Table 1 presents summary statistics for some of the variables over different subperiods.

[Table 1 about here.]

The period 1914–1945 stands out as quite turbulent because of WWI and WWII and large volatility during the interwar period. Almost all of the time series undergo large fluctuations during these periods; see Figures 2 and 3. In comparison with previous subperiods, the period 1990–2003 has been a relatively tranquil period. Table 1 shows small standard deviations for most variables in 1990–2003 with one

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[2]In addition to the variables presented below, we have also investigated to what extent measures of temperature and rainfall could account for GDP fluctuations, especially in the 19th and early 20th century. However, our preliminary inquiry did not suggest any systematic relationship between the employed measures and GDP fluctuations (not reported).

notable exception: the real exchange rate has been quite volatile, which partly reflects sizeable nominal exchange rate fluctuations in the value of the Norwegian krone against pound sterling over the period.

Figure 2 shows that GDP growth varies in the range between 0% and 5–6%. The exceptions are in the earliest part of the sample (before 1870) and around the two World Wars. The mean growth rate of GDP is 2.9 % over the period 1831–2003 with a standard deviation of 3.7 per cent. However, Table 1 suggests considerable variation in mean growth and volatility over different subperiods.[3]

The real exchange rate appreciated by about 60% over the period, 1830–2003, see Figure 1. The appreciation has however not been uniform over the sample period. Roughly, the real exchange rate appreciated markedly (ca. 30%) until the Crimean War, before it started fluctuating around almost the same level until the beginning of WWI. It displayed particularly large fluctuations from about the start of WWI to WWII. During WWII, it again appreciated (ca. 20%) relative to its pre-WWII level and remained fairly stable at the new level until about the mid 1960s. Thereafter it appreciated substantially (ca. 30%) until about the mid 1970s.

Most of the notable fluctuations in the real exchange rate can be mainly associated with the development in domestic prices relative to the foreign prices as the nominal exchange rate was kept stable over several periods. It was quite stable during: the silver parity regime (1842–1873); the gold parity regime (1873–1914); and most part of the Bretton Woods system, i.e. until the pound was devalued in the late 1960s. The appreciation until the mid 1970s can be partly associated with this devaluation, but mainly to the discovery of Norwegian offshore petroleum resources in the late 1960s and Norway's emergence as a net oil exporter around 1970s; see Akram (2004) for an elaboration. The petroleum resources raised Norwegian GDP growth substantially relative to growth in mainland GDP and to that of its main trading partners, cf. Table 1.

Figure 1 shows that real narrow money balances generally increase until the end of WWII; they are relatively stable until early 1990s, but increase thereafter. The following details are notable. The growth in real money balances is fairly stable until WWI. In particular, they seem to grow quite steady in the period after the Crimean War and WWI. During WWI and WWII they increase relatively sharp, despite high inflation during these wars. In the interwar period, real money balances fluctuate around the high level established during WWI, before shifting swiftly to an even higher level during WWII. Real money balances fluctuates around this post-WWII level until the early 1990s, when they start increasing again. The latter increase can be partly associated with the relatively low and falling inflation rate from the early

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[3]Notably, while mean GDP growth varied around the same level in the two subperiods 1831–1870 and 1870–1914 (2.4 and 2.2 % respectively), growth volatility Stdev was substantially lower in the latter period, 4.0 % versus 1.9 %. This reduction in the volatility of GDP growth over time may be ascribed to shifts in the sectoral composition of the economy over time; specifically, to the substantial increase in the share of the secondary sector at the expense of the primary sector (agriculture, forestry, fishing and mining) over time. In the first half of the 19th century, the primary sector accounted for about 45% of GDP, and the secondary sector (manufacturing) for about 15%. In addition, an increase in the livestock production relative to arable production may also have contributed to lower volatility in total agricultural output and hence in GDP.

1990s onwards.

Most of the major fluctuations in the (ex-post) real interest rate ($RR \equiv R - \Delta cpi$) over the sample period can be associated with periods of high inflation and deflation. Nominal interest rates have mostly fluctuated around 4–5%. Large deviations from these levels can be mainly associated with periods after wars. In addition, they climbed to their highest levels ever to about 13% during the 1980s before falling to their apparently normal levels of 4–5% in in the early years of 2000. The sharp increase in the nominal interest rate was not equally reflected in the real interest rates owing to the relatively high inflation rate during the late 1970s and 1980s. However, the increase in the real interest rate was substantial as they rose from about -3% to around 7%.

Figure 3 shows the growth and log level of Norwegian GDP, narrow money balances and public consumption relative to those in the UK. Norwegian GDP relative to UK GDP (Y/YUK) displays distinctively different behaviour before and after WWI. Prior to the war, it grows slowly over time, especially in the period 1830–1870, as it is fairly stable afterwards until WWI. After the war however, it grows remarkably over the remaining sample period, except during WWII. In contrast with the case during WWI, Norwegian GDP falls relatively much during WWII, but recovers swiftly after WWII and continues it upward path, without exhibiting large fluctuations.

The ratio between Norwegian and UK public consumption seems to be fairly stable until after WWI when it drops to a lower level where it remains until after the end of WWII. Thereafter it displays a downward trend throughout the remaining sample period, suggesting that public consumption in the UK has grown at a higher rate than in Norway.

Norwegian narrow money balances relative to those in the UK seem to display largely the opposite pattern relative to that of the ratio between the public consumptions. The Norwegian money balances grow faster than those in the UK until the end of WWI, and especially during WWI. They become relatively stable after WWII around a weak negative trend. During the interwar years however, the Norwegian money balances relative to the UK money balances fall substantially below their WWI level and even below their post-WWII level.

[Table 2 about here]

Table 2 presents test of time series properties of the key variables. It appears that the levels of the variables can be treated as integrated of order one. It is worth noting that the real exchange rate appears to be integrated of order one, and hence not consistent with the purchasing power parity (PPP) hypothesis. This finding does not conform with studies that have reported evidence of stationary real exchange rates on particularly large samples, but it is not surprising in light of the time series behaviour of the real exchange rate over the sample, see Figure 1.

Apparently, the nominal interest rate is non-stationary and integrated of order one while the real interest rate seems to be stationary. This results is puzzling

as annual inflation (Δcpi) seems to be stationary. One explanation could be that nominal interest rates is actually a stationary time series but the transitory increase in especially the latter part of the sample induces the non-rejection of the null hypothesis by the ADF-tests, see Figure 1. It is well known that the ADF test has low power when there are breaks in a time series. However, when inflation is extracted from the nominal interest rate to construct the series of real interest rates, relatively high inflation rates that often coincide with nominal interest rates make the real interest rate series relatively more stable. Thus, the null hypothesis is easily rejected by the ADF-test in the case of the real interest rate.

4   Multivariate linear models

In this section, we develop multivariate models of Norwegian GDP, the real exchange rate and narrow real money balances. We aim to characterise main trends in GDP and variation in the growth rate over the rather long sample period. We would especially like to investigate the role of monetary and fiscal policies and foreign shocks on output in the short-run while taking into account possible effects of relevant political events and technological changes. In addition, we would like to test for and characterise possible asymmetries in the response of output to changes in terms of trade and monetary and fiscal policies.

We proceed in the following way. First, we develop long-run models of GDP, the real exchange rate and money balances by using the two-step procedure proposed by Engle and Granger (1987). [4] Thereafter, we develop and evaluate linear equilibrium correction models (ECMs) of these variables based on the estimated long-run relationships in Section 4.1.

[Table 3 about here.]

Table 3 presents the long-run relationships for GDP, real exchange rate and narrow real money together with augmented Dickey-Fuller (ADF) tests of their validity. Panel I suggests that Norwegian GDP has followed foreign GDP over time, which is represented by UK GDP, and a deterministic trend. This indicates that Norwegian GDP contains both a stochastic and a deterministic trend. The stochastic trend seems to be accounted for by UK GDP and can be associated with the stochastic nature of technological changes that may be stemming from the

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[4]This procedure may be potentially inefficient as the three long-run relationships are estimated separately. Alternatively, we could have employed Johansen's method for deriving multivariate long-run relationships; see Johansen (1995). However, analysis within Johansen's framework turned out to be quite demanding in the light of the large number of presumably conditioning variables and influential/extreme observations requiring a number of deterministic variables. It is well known that valid inference within Johansen's framework places considerable demands on the specification of the models and that derivation of interpretable long-run relationships may not be straightforward when the analysis includes several variables. Given the mainly explorative nature of this study and for convenience, we chose to employ the two-step procedure.

relatively advanced economy of the UK during most of the sample period. The deterministic trend is included to account for the evolution of the labour force and physical capital over time.

From the 1950s, however, the relationship between Norwegian GDP, UK GDP and the trend appears to break down, unless one controls for the growing size of public consumption, which can be associated with the growing size of the public sector since the 1950s. Accordingly, developments in Norwegian GDP beyond the level mainly accounted for by the public sector in the post-WWII era can still be ascribed to foreign GDP and a deterministic trend.

In the GDP equation, we have also allowed for level shifts by including a number of step dummies that may be associated with technological and political regimes and the two World Wars. The allowance for separate intercept terms M1830to13, M14to49 and SD50 for the three main periods 1830–1913, 1914–1949 and 1950 onwards, respectively, can be associated with the chronology of technological regimes for industrialised countries proposed by Maddison (1991).[5] Shifts in GDP level owing to the World Wars have been allowed for by two step dummies W1 and W2, respectively.

It turns out that possible shifts in the level of GDP prior to the 1950s are negligible. In particular, the intercept term until the end of WWI remains the same, as the coefficient of M1830to13 is almost equal to the coefficient of $M14to49_t$ once we subtract the effect of WWI. Afterwards, there is a slight upward shift in the level, which is partly reversed during WWII. The intercept term since the 1950s is smaller, indicating a downward shift in GDP. Another interpretation is that the intercept term in the previous periods partly accounts for the relatively stable share of the public sector in those periods and that the decline in the intercept term after 1950 partly reflects the explicit account of the public sector through public consumption.

The ADF test rejects the null hypothesis that the estimated relationship for GDP does not constitute a valid long-run relationship at about the 5% level. The t-ADF value is -4.17, while the critical value suggested by a standard ADF test is -3.50. However, if we take into account that the long-run estimates have been estimated and use the critical values suggested by MacKinnon (1991) for three integrated variables, a constant and a trend, the 5% critical value is about -4.23. Still, the results must be considered indicative given that we include more deterministic terms than just a trend and an intercept as supposed by MacKinnon (1991).

Panel II presents a long-run relationship between the real exchange rate (in logs) and the difference between logs of Norwegian GDP and UK GDP. Accordingly, Norwegian growth in excess of that in the UK leads to a real appreciation

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[5]Actually, Maddison (1991) divides the period from 1950 onwards into two subperiods, 1950–1973 and 1974 onwards. This division is associated with OPEC I in 1973, which presumably affected the growth rates in the industrialised countries. However, oil price shocks had an ambiguous net effect on the growth of the Norwegian economy. One explanation is that the negative shocks to the Norwegian economy owing to recessions in its trading partners were largely counteracted by the increased oil revenues from Norwegian oil exports. Furthermore, the oil revenues were mainly used to finance the public sector. Their effect on Norwegian GDP may therefore be accounted for by the growth in public consumption.

($\Delta rex_t < 0$) and a real depreciation in the opposite case. It also follows that the real exchange rate remains constant in the face of equal growth rates at home and abroad. These implications are consistent with the Balassa-Samuelson hypothesis; see Balassa (1964) and Samuelson (1964). Similar results were obtained by Edison and Klovland (1987) who examined the behaviour of the real exchange rate between Norway and the UK over the period 1874–1971. Notably, these results differ from studies that support PPP on long spans of data; see Sarno and Taylor (2002) and the references therein.

The t-value for the cointegration test is about -5.40. Thus, the null hypothesis of no cointegration between the real exchange rate and the GDP ratio can be clearly rejected at the 5% level, even when we compare against MacKinnon's critical values.

Finally, Panel III first presents the unrestricted estimate of narrow money demand. The estimated income, price and nominal interest rate effects are consistent with standard models of money demand. Numerically, the long-run income and price elasticities are close to one, as implied by e.g. the quantity theory of money. Accordingly, ($m0-cpi-y$) can be interpreted as the inverse of the velocity of money, which is often assumed to rise with nominal interest rates. Hence, the negative interest rate effect could be proxying the inverse of the velocity of money.

Statistically, however, there is only weak evidence in support of this constituting a valid long-run relationship. The t-value is just -2.35 while the Dickey-Fuller and MacKinnon critical values are as above, –2.90 and –3.40.

Nevertheless, preliminary analysis indicates that if one controls for relatively large shocks to the money balances over the sample period, it is possible to find statistical support for the suggested long-run relationship for narrow money. This becomes evident in Table 4, which presents a linear dynamic model of the real money demand. Also, if the relationship investigated is characterised by non-linear dynamics, the cointegration test may have low power.

4.1 Linear dynamic models

Table 4 presents a vector equilibrium correction model (VECM) of Norwegian GDP, real exchange rate and real money. The three equations are treated as a system of simultaneous equations and estimated by the method of full information maximum likelihood (FIML) over a common sample period 1834–2000. These equations were developed by following a "general to specific" model specification strategy, cf. Hendry (1995). The general versions of the equations initially allowed for three lags of each of the explanatory variables, except for the equilibrium correction terms (and dummy variables). Thereafter, statistically insignificant variables were sequentially left out for the sake of parsimony.

[Table 4 about here.]

The VECM characterises the short-run behaviour of these variables and their adjustment towards their long-run relationships. In this model we have allowed for

short-run effects of variables that have long-run effects, but also of those that are only short-run determinants. We assume that the domestic real interest rate, RR_t, public consumption at home, co_t, public consumption and narrow money in the UK, $couk_t$ and muk_t, are valid conditioning variables for inference purposes, i.e., they are weakly exogenous variables with respect to parameters of interest. A test of this assumption requires that we develop models of these variables which are beyond the scope of this study.

We control for relatively large shocks that remain unexplained by our information set by using impulse dummies. It appears that these impulse dummies can be mainly associated with relatively extreme movements in GDP, real exchange rate and money balances during and between the two World Wars and other well known economic and financial crises.[6]

We note that the left-hand-side variables respond such that they partly correct past deviations from their long-run relationships, $\widehat{u}_y$, $\widehat{u}_{rex}$ and $\widehat{u}_{rm}$, respectively. The t-values associated with the deviation terms are –2.94, –4.71 and –5.23, respectively. This implies that the null hypotheses of no response to lagged deviations can be rejected at the standard 5% level of significance.

Broadly, the VECM suggests many interactions between the modelled variables and strong influence of foreign shocks on the domestic economy. More specifically, money growth and public consumption affect output and the real exchange rate in the short-run. It appears that real interest rates and the real exchange rate have also short-run effects on output. Domestic output follow the foreign output in the long-run and a deterministic trend representing evolution of physical capital and labour force over time. These variables also influence the course of output in the short-run through the equilibrium reversion process. Furthermore, foreign output together with domestic output appear as important short-run and long-run factors in the VECM. They determine the real exchange rate in the long-run and have substantial effects on it in the short-run (owing to the equilibrium reversion process). Moreover, domestic output has strong influence on money balances in both the long-run and the short-run.

In more details, the equation of Δy_t shows that terms of trade shocks (as represented by changes in the real exchange rate) and monetary and fiscal policies are among the main determinants of GDP in the short-run. As expected, an increase in real interest rates and in the growth of real money tend to have negative and positive short-run effects on GDP, respectively. Their effects are of almost equal magnitude on GDP growth. Higher growth in public consumption also tends to boost the activity level. This effect as well as the long-run effects of public consumption appear explicitly in the model only after 1950. Depreciation of the real exchange rate have a positive effect on GDP in the short-run, though it is statistically insignificant at the 5% level. We also note evidence of persistence in the growth rate given that the lagged growth rate appears with a positive coefficient in the equation.

The equation for the real exchange rate, Δrex_t suggests that difference between

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[6]The use of impulse dummies helps us avoid the influence of probably unique events on the parameter estimates. Moreover, they contribute to bringing about symmetric/normal distributions of residuals.

growth rates of domestic and foreign public expenditures and between growth rates of domestic and foreign money affect the real exchange rate in the short-run. Specifically, relatively higher growth in domestic public consumption and money relative to abroad lead to a real appreciation of the exchange rate.

The latter two growth ratios are assumed to work through their effects on relative prices between home and abroad. Government expenditures are commonly believed to be biased towards purchases of non-tradables and thus tend to raise their prices relative to those of tradables. Therefore, an increase in government expenditures may increase the overall price level. If growth in public expenditure at home is higher than abroad, the overall domestic price level is likely to rise faster than the foreign price level, which in turn leads to a real appreciation of the exchange rate, *ceteris paribus.* Similarly, domestic money growth that is relatively higher than money growth abroad is likely to raise the domestic price level faster than the foreign price level and thereby lead to a real appreciation of the exchange rate. There is also some evidence that differences in money growth were particularly important to exchange rate movements during WWI and the interwar period, represented by the step dummy W2W.

There is also an indication of some persistence in changes in the real exchange rate, e.g., a depreciation tends to be followed by depreciation in the subsequent year. However, the equilibrium correction mechanism largely counteracts such persistence and ensures that movements in the real exchange are determined only by diverging growth paths between home and abroad.

Some impulse dummies are required to control for the relatively large exchange rate fluctuations around the end of WWI and during the 1920s. The remaining dummies may be associated with large changes in the nominal exchange rate and domestic prices since the late 1960s.[7]

The model for real money, Δrm_t, suggests that apart from reversion towards it long-run level, which is determined by GDP and the nominal interest rate, GDP growth tends to have a substantial effect on real money growth. The model also indicates a fairly small degree of persistence in real money growth.

It appears that we are able to obtain fairly stable parameters over time once we use the dummy variables. For example, allowance for separate income effects on real money growth before and after WWI does not suggest a change in the income effects; note the coefficient estimates of $\Delta y \times \mathrm{preW1}_t$ and $\Delta y \times \mathrm{postW1}_t$. The impulse dummies can be ascribed to episodes of excess money growth and relatively high inflation in 1916 and 1918, 16% deflation in 1926, 78% increase in money growth in 1941 and about 20% in 1947, which coincided with zero inflation.

However, system diagnostic tests suggest that the VECM could be misspecified.

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[7]Specifically, they may be associated with the devaluation of pound sterling in November 1967, the appreciation and the subsequent revaluation of the krone in 1973, the relatively high wage and price growth in Norway during the mid 1970s and the subsequent devaluations in 1977 and 1978. Moreover, the Norwegian government imposed wage and price control in the period 1978–1980, which may explain the real exchange rate depreciation in 1979–1980. Finally, the real exchange rate depreciation indicated by the impulse dummy for 1997 may be ascribed to the relatively strong appreciation of pound sterling against European currencies, about 14% against the krone, in this period.

We note that null hypothesis of normality can be rejected at the 5% level. In addition, the null hypothesis of no heteroscedasticity can be rejected even at the 1% level. The null hypothesis of no autocorrelation for the vector of the three residuals cannot be rejected at standard levels of significance.

A comparison of these system tests with tests based on single-equation models suggests that the apparent non-normality of equation errors and the absence of homoscedasticity can mainly be ascribed to the GDP equation, cf. Table 8. All three tests mentioned as well as other tests for model misspecification, i.e., ARCH and RESETs, suggest no misspecification of the equations for the real exchange rate and money growth, respectively, see Tables 9–10 in the next section.

5 Non-linear conditional models

Specification and estimation of non-linear multivariate models while conditioning on a number of variables can be undertaken more conveniently within the context of single-equation models rather than in a system. However, valid inference on key parameters such as those measuring the degree of equilibrium reversion in each period and those characterising the long-run relationships presupposes that variables in the system can be considered as weakly exogenous with respect to the parameters of interest.

In the following, we test whether our key variables (GDP, real exchange rate and real money) can be considered as weakly exogenous with respect to the long-run parameters and the associated adjustment coefficients. The outcome of these tests may also lend some support to our estimation of the long-run parameters within the static single-equation models. In addition, we examine possible simultaneity bias in the coefficient estimates, owing to endogenous right-hand-side variables, when their equations are estimated individually by OLS rather than as a system by the FIML method.

[Table 5 about here.]

Table 5 presents the outcome of the weak exogeneity tests. It appears that the real exchange rate and real money can be considered weakly exogenous with respect to the long-run parameters and the adjustment coefficient in the GDP equation and *vice versa*. Furthermore, the real exchange rate and real money seem to be weakly exogenous with respect to the long-run parameters and the adjustment coefficients in each others' equation. A joint test of weak exogeneity of all the three variables with respect to the parameters of interest does not reject the null hypothesis of weak exogeneity; the p-value is 28%. Hence, inference on these parameters may be valid within single-equation models of these variables.

In order to investigate possible simultaneity bias in the parameter estimates when moving from system to single-equation modelling, we estimated each of the equations in Table 4 by OLS and compared the coefficient estimates with their FIML

estimates in Table 4.

The OLS estimates of the linear ECMs were generally comparable to their corresponding FIML estimates, indicating negligible bias, especially in the ECMs of GDP and the real money (not reported). The OLS estimates of the (linear) real exchange rate model, however, differed somewhat from their FIML estimates. In particular, the estimated effects of differences in money growth ($\Delta(m0 - m0uk)_t$) and of the lagged real exchange rate (Δrex_{t-1}) became weaker when estimated by OLS, see Table 6.

[Table 6 about here.]

5.1 STR models of output, the real exchange rate and real money

This section develops non-linear single-equation equilibrium correction models (ECMs) of GDP, the real exchange rate and real money. We begin their development by formal tests of the adequacy of linear ECMs that are obtained by OLS estimation of each of the three equations in Table 4. To ease comparison with properties of the non-linear versions of the linear ECMs, the outcomes of a number of standard mis-specification tests for all of the linear models are reported in Tables 8–10.

[Table 7 about here.]

Table 7 presents tests for non-linear effects of STR form for different state variables in each of the three linear ECMs. The tests are based on the residuals from these models. We have limited the set of state variables mainly to the regressors in each of the three models and the time trend, t. In the latter case, the linearity test can be considered a test for smooth variation in the parameters of the linear ECM.

In the case of the ECM for GDP, Panel I shows that the null hypothesis of linearity can be rejected at the 1% level for $s_t = \Delta RR_t$; see the row for H_0.[8] The remaining test sequence shows the rejection of H_3 and H_2 at the 5% and 1% levels, respectively; see Section 2.1 for an explanation of the tests. We therefore assume that a logistic function of ΔRR_t is required to characterise the non-linear effects of the explanatory variables. We also note that linearity is nearly rejected, i.e., at the 10% level, for $s_t \in [t, \Delta co, \Delta rm_t]$, but we consider this evidence to be too weak against linearity to pursue non-linear modelling in these directions.

The evidence against linearity for the monetary policy and fiscal policy variables is largely consistent with a number of previous studies. A large number of studies

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[8]We have also tested the null hypothesis of linearity with contemporaneous and lagged levels of the real interest rates as transition variables, that is with $s = RR_t$ and RR_{t-1}. However, the null hypotheses was not rejected at the standard levels of significance as the p-values turned out to be 12% and 15%, respectively.

point out that contractionary monetary policy has a more pronounced effect on output than expansionary monetary policy; see *inter alia*, Cover (1992), Karras (1996) and Parker and Rothman (2004). Furthermore, effects of large monetary shocks may have a larger impact on output than small shocks if there are threshold effects in e.g. consumption and investment and if aggregate supply curves are highly convex and upward sloping, cf. the literature on multiple equilibria. The effects of monetary policy may also depend on the stage of the business cycle. For example Sensier et al. (2002) argue that monetary policy has stronger effects in expansions than in recessions. They also find evidence of non-linear effects of changes in the nominal interest rates on GDP growth in the UK. Moreover, the non-linear effects are of logistic form with annual changes in the nominal interest rate as the transition variable with a threshold value of 2.89 percentage points.

Asymmetric effects seem to be observed less often in connection with expansionary and contractionary fiscal policies than in connection with monetary policy. Nevertheless, a number of studies have reported evidence of smaller effects of fiscal expansions relative to those of contractions, and that fiscal policy has a stronger impact in recessions than in booms; see e.g. Kandil (2001) and the references therein.

In the case of the ECM of the real exchange rate, linearity is rejected at the 5% level for both $s_t = \widehat{u}_{rex,t-1}$ and $s_t = \Delta rex_{t-1}$. In addition, there seems to be strong evidence of smooth variation in parameters over time as linearity is rejected also for $s_t = t$. In this case, a permanent shift seems to occur in the parameters over time since a logistic function of t is favoured against an exponential function. For the other two transition variables, however, exponential functions turn out to be the preferred functions for characterising non-linear effects. We note that H_4 is rejected in the case of $s_t = t$, while H_3 is rejected for both $s_t = \Delta rex_{t-1}$ and $s_t = \widehat{u}_{rex,t-1}$, all at the 1% level.

Previously, Michael et al. (1997), Sarno (2000), Taylor et al. (2001) have developed STR models to characterise the behaviour of real exchange rates for a number of countries. These models suggest that the speed at which a real exchange rate moves towards its equilibrium level increases with the size of the deviation from its equilibrium, which is assumed to be a constant, as implied by the PPP hypothesis. More specifically, these studies reject the linearity of the real exchange rate process against the STR form of non-linearity for the lagged real exchange rate as the transition variable. The transition function is commonly specified as an exponential function of the lagged real exchange rate, though evidence of a logistic function is also found; see Michael et al. (1997).

Deviations from the equilibrium exchange rate define non-linear effects in our case as well. In contrast to the above studies, however, the equilibrium level of the real exchange rate is not constant but depends on the growth difference between home and abroad. In addition, our evidence of non-linearity and its form is based on a multivariate model where the equilibrium correction term is embedded in a model which controls for short-run effects of a number of presumably exogenous variables. In contrast, the evidence in previous studies is mainly based on autoregressive models of real exchange rates.

The last panel of Table 7 presents tests of the linearity of the ECM of real money.

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It appears that linearity can be rejected at the 5% level for three of the explanatory variables: $s_t \in [\Delta rm_{t-1}, \widehat{u}_{rm,t-1}, \Delta y_t]$. However, it is more strongly rejected in the case of $s_t = \Delta rm_{t-1}$ than in the other cases, that is, at the 1% level rather that at the 5% level. The sequence of tests conducted to determine the form of the non-linearity suggests logistic functions of both $s_t = \Delta rm_{t-1}$ and $s_t = \widehat{u}_{rm,t-1}$. In the case of $s_t = \Delta y_t$, however, none of the tests aimed at determining the form of non-linearity is rejected at the 5% level which undermines the evidence against linearity with Δy_t as the state variable.

In addition to the right-hand-side variables appearing explicitly in the linear ECM for real money, we have also tested for possible non-linear effects with both the level and changes in real interest rates as transition variables. However, the hypothesis of linear effects was not rejected in either case, see the last two columns of Table 7.

The rejection of linearity for $s_t = \widehat{u}_{rm,t-1}$ is consistent with a number of studies of money demand. The suggested logistic form of the transition function is at variance with some of the well-known studies, though. Previously, the STR form of equilibrium correction models of money have been developed for e.g. the US, the UK, Italy and Germany; see Sarno et al. (2003), Teräsvirta and Eliasson (2001), Sarno (1999) and Lütkepohl et al. (1999). These studies specify the transition function as an exponential function of the lagged value of equilibrium correction terms. However, evidence for the UK and Germany also supports logistic transition functions of income growth and inflation, respectively.

5.2 The STR models

We specify the transition functions in light of the results in Table 7 and initially allow for non-linear effects of all explanatory variables in a model, except for the dummy variables. These general models are estimated by NLS and sequentially reduced to more parsimonious versions. In cases where several state variables were suggested, we make an effort to condition non-linear effects on each of these state variables, individually and jointly. Upon convergence of parameter estimates, we compare the performance of the different models of a variable in terms of explanatory power, interpretability and the extent to which they are able to represent the non-linear effects suggested by Table 7.

Tables 8–10 present the preferred models. To ease comparison with the linear models, statistically insignificant variables appearing in the linear models have not been left out to achieve more parsimonious models. The tables also report comprehensive evaluations of the models. Specifically, they lay out outcomes of a number of tests aimed at detecting possible violations of the standard assumptions about residuals and functional form misspecification. Moreover, these tables report the outcome of the corresponding tests for the linear models that were estimated by OLS, cf. Table 4. In Table 11, we examine to what extent the proposed models capture the state-dependent effects suggested by Table 7 through testing hypotheses of no remaining non-linear effects.

5.3 LSTR model of output

Table 8 presents the model of GDP with a logistic transition function of changes in real interest rates ΔRR_t. It appears that increases in real interest rates above 3.9 percentage points tend to substantially push up the speed of adjustment towards the long-run equilibrium for GDP, cf. Sensier et al. (2002). Specifically, the speed of adjustment increases up to –0.473 (= –0.069 –0.404) per annum compared with the typical speed of –0.069 when changes in real interest rates are relatively smaller. Moreover, the adjustment speed is more than four times higher than that implied by the linear model (–0.106), see Table 4.[9]

[Table 8 about here.]

Furthermore, the partial effect of the change in real interest rates becomes about ten times higher than suggested by the linear model of GDP, see Table 4. The LSTR model, however, indicates that only particularly large interest rate increases, i.e., above 3.9 percentage points, tend to have contractionary effects on GDP, see Figure 4. In particular, the estimated logistic transition function implies that cuts in real interest rates do not raise GDP growth.

[Figure 4 about here.]

Figure 2 shows that relatively large increases in the real interest rate occurred numerous times until about the early 1970s. Values of the transition function were mostly close to 1 during these occasions, owing to the step-form of the transition function, see Figure 4. Thus, the non-linear effects were quite active until the early 1970s. A closer examination of changes in the real interest rates suggests that large positive increases in the real interest rates mostly occurred during periods of large deflations until the late 1920s and due to sharp increases in nominal interest rates

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[9]It should be noted that in the case of LSTR models in Tables 8–10, estimates of the transition parameter γs are relatively large even when scaled by the sample standard deviations of the corresponding transition variables. Moreover, they are imprecisely estimated. In general, numerically large values of γ of a logistic transition function $F(\gamma, c; s_t)$ make it change rapidly at even small deviations between s_t and c, and its shape becomes consistent with a broad range of values of γ. The high standard deviations of γ are assumed to reflect this feature. In such cases, many observations in the neighbourhood of c are required to obtain precise estimates of γ, cf. Teräsvirta (1994). Given that threshold values c often represent non-typical values of s_t, imprecise estimates of γ are commonly encountered in the literature. This occurs particularly when $s_t = t$, as observations in the neighbourhood of c are few by the nature of t.

   In the case of ESTR models, however, relatively large standard errors, may indicate relatively poor fit of the model, relative to a linear version of the model. Moreover, particularly large values of the transition parameters γs in an estimated ESTR model may suggest that the model can be considered linear in practice. The transition function converges to a single value in such cases, and acts as an impulse dummy. This is, however, not the case for the real exchange rate model (M2) in Table 9.

in the period afterwards, see Figure 5.

[Figure 5 about here.]

The addition of the state-dependent effects leaves the coefficient estimates of the remaining variables largely unaltered. Numerically, the coefficient estimate of Δrex_t increases, while those of the impulse dummies $d1862_t$ and $d22_t$ fall.

The explanatory power of the LSTR model is 9% higher than that of the linear models, as measured by the ratio of the standard deviations. The diagnostics show that the standard assumption about the error term and the presumed adequacy of the functional form are not rejected at the 5% level. There is an indication of ARCH effects in the residual as the p-value of the test statistics is 4.6%. In contrast, the outcome of the corresponding tests of the linear model suggests that most of the tested assumptions are rejected at the 5% level, while the null hypotheses of no autocorrelation and the extended RESET (with both cubic and square terms) can be nearly rejected at the 10% level.

Finally, Table 11 shows that the null hypothesis of no remaining non-linearity of STR form with $s_t = \Delta RR_t$ is not rejected. Also, the null hypothesis of time variation in parameters is not rejected, which indicates absence of time variation in the model's parameters.

5.4   STR model of the real exchange rate

As noted above, comparison of the OLS estimates with the FIML estimates for the model of Δrex_t indicated some numerical differences. To separate the effect of non-linearisation on parameter estimates from that of potential simultaneity bias, we use the linear model with OLS estimates in Table 6 as the reference model.

[Table 9 about here.]

Panel I of Table 9 presents the NLS estimates of the LSTR model with $s = t$, that is with time variation in a subset of parameters. The logistic function implies a permanent shift in the coefficient of $\Delta(co-couk)_{t-1}$, the deviation between domestic and foreign growth rates of public consumption, quite early in the sample period: around 1845. Accordingly, the coefficient estimate falls from -0.484 to -0.044 ($= -0.484+0.440$), becoming virtually equal to that in the linear model. The remaining coefficient estimates remain comparable to those in the linear model. By allowing for such time variation, the explanatory power of the model increases by 3% relative to the linear model.

Panel II reports an extended model of the real exchange rate.[10]   This model

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[10]Initially, we allowed its parameters to change over time and with lagged deviations of the real exchange rate from its long-run relationship ($\widehat{u}_{rex,t-1}$). Owing to non-convergence of the parameter estimates, we had to condition on the logistic specification of time in Panel I and then

supports the shift in the coefficients of $\Delta(co-couk)_{t-1}$ around 1845. In this model the intercept term also becomes significant over time. However, the intercept term varies significantly with lagged deviations of the real exchange rate from its long-run relationship ($\widehat{u}_{rex,t-1}$). Due to the exponential transition function of $\widehat{u}_{rex,t-1}$ the intercept rises (at most) to 0.019 for particularly large positive or negative values of $\widehat{u}_{rex,t-1}$. Accordingly, the negative intercept of –0.0183, which induces a negative drift in the real exchange rate of 1.83% per annum, is virtually cancelled out whenever $\widehat{u}_{rex,t-1}$ is large (in absolute terms).

[Figure 6 about here.]

[Figure 7 about here.]

On the other hand, the influence of $\widehat{u}_{rex,t-1}$ on changes in the real exchange rate becomes larger whenever the negative intercept term is cancelled out. Thus the real exchange rate appreciation (or depreciation) can be larger than usual, i.e. when $\widehat{u}_{rex,t-1}$ is not particularly large and the negative intercept term is active. Figure 7 suggests that estimated values of the exponential transition function of $\widehat{u}_{rex,t-1}$ were often high and close to one when there was a downward trend in the real exchange rate, i.e. a tendency to appreciate, see Figure 1. One also gets the impression that relatively low values of the transition function often coincided with periods of relatively stable real exchange rate. Thus it seems that the downward trend in the real exchange rate does not vanish when the negative intercept term is outweighed. On the contrary, the downward trend over several periods becomes more pronounced on such occasions owing to the Balassa-Samuelson effect working through the $\widehat{u}_{rex,t-1}$ term.

The explanatory power of the extended model is 5% higher than that of the linear model and 2% higher than that of the non-linear model in Panel I. The diagnostics shows that the model satisfies the standard residual assumptions and that its functional form is adequate. We note that these tests are not rejected in the case of the linear model either. The tests for no remaining non-linearity in Table 11 indicate that time variation in the parameters have been adequately characterised though not fully satisfactorily. The results in the table also suggests that non-linearity with $s_t = \widehat{u}_{rex,t-1}$ is still a feature of the model. Moreover, there is also weak evidence of non-linearity with $s_t = \Delta rex_{t-1}$ in the extended model (M2), although not in the simpler model (M1). The p-values are 4% and 7%, respectively.

5.5   LSTR model of real money

The non-linear model of real money growth has been developed with its lagged value (Δrm_{t-1}) as the transition variable. The linearity hypothesis was also rejected with

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allow coefficients to vary with $\widehat{u}_{rex,t-1}$. Except for the intercept term, none of the other coefficients seemed to vary significantly with $\widehat{u}_{rex,t-1}$, hence they were excluded from the model for the sake of parsimony.

the deviation of money from its long-run level $(\widehat{u}_{rm,t-1})$, but less strongly than for $s_t = \Delta rm_{t-1}$.[11] The model with $s = \Delta rm_{t-1}$ in Table 10 improves only slightly on the linear model, but suggests a remarkably different dynamic behaviour of money growth than that implied by the linear model.

[Table 10 about here.]

[Figure 8 about here.]

It appears that adjustment towards the equilibrium level slows whenever real money growth is particularly high, i.e. above 5.8%. The estimated partial adjustment coefficient is –0.086 in general but may fall to –0.026 when real money growth becomes especially high. In addition, negative autocorrelation becomes an important feature of the growth process of real money. Note that the first autoregressive term becomes negative with a coefficient estimate of –0.20 ($\approx 0.172 - 0.371$) when Δrm_{t-1} exceeds 5.8%. Thus, real money growth seems to alternate between positive and negative growth rates. At levels below the threshold rate, the degree of persistence in the growth rate, as indicated by the sum of the two autoregressive terms, is almost absent. The relatively large estimate of the transition parameter in the logistic transition function of Δrm_{t-1} implies that the speed of adjustment falls abruptly whenever real money growth exceeds its estimated threshold value.

[Figure 9 about here.]

Interestingly, many periods of excess real money growth coincide with deflationary periods and wars. Figure 9 shows that shifts in the values of the transition function towards 1 occur mainly in deflationary periods and during the wars. Therefore, the low degree of equilibrium reversion during periods of excess real money growth seems to be consistent with the public's desire to have higher stocks of money than in equilibrium due to the relatively high return on money balances during deflationary periods. During war years, money stocks may also adjust more slowly towards their equilibrium levels due to relatively low liquidity in (real and financial) asset markets.

The table shows that the coefficient estimates of the short-run income effects have not been affected by allowance for non-linear effects.

Finally, the diagnostic tests of the model do not indicate obvious misspecification of the model, though there is still some evidence of remaining non-linear effects dependent on Δrm_{t-1}, see Table 11. On the other hand, the null hypothesis of no remaining non-linearity with $\widehat{u}_{rm,t-1}$ as the transition variables can be accepted at a p-value of 5%. Also, there does not seem to be any evidence of parameter non-

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[11]Yet, we made an effort to develop a model with $s = \widehat{u}_{rm,t-1}$, but it did not seem to improve on the linear model in terms of explanatory power or in bringing forward new aspects of the behaviour of money growth.

constancy in this model.

[Table 11 about here.]

5.6 Dynamics of the linear vs the non-linear systems of equations

In the following, we compare the dynamic properties of the linear VECM with those of the corresponding system of non-linear equations. The dynamic properties of a system of equations can be summarised by calculating characteristic roots (eigenvalues) from the companion form representation of a system; see Lütkepohl (1991) and the appendix for more details. It appears that the dynamic behaviour of the system of equations can alter substantially when non-linearities are introduced.

To this end, we derived the reduced form of the linear ECMs in Table 4 and of the three non-linear equations (8–10) treating them as a system of equation. We then presented the reduced forms of both the linear and non-linear models in their companion form. In the case of the non-linear equations, we conditioned on different combinations of extreme values of the transition functions Fs, thus generalising the approach in Teräsvirta (1994) to a system of non-linear STR models.

[Table 12 about here.]

Table 12 presents the roots (eigenvalues) obtained from the companion form representation of the two systems of reduced form equations. The table shows the presence of both real and complex characteristic roots. Both the linear system and the non-linear system are stable since none of the roots has a modulus greater than one. In the following, we focus our attention only on the largest pair of complex pair roots for the sake of brevity. Complex roots imply cycles whose period length can be compared across different model specifications.

The first part of the table reports the roots for the linear system. We note that the characteristic polynomial contains a pair of complex roots, 0.82 +/- 0.44i, with a modulus of 0.93, which implies a cycle with a period of 12.8 years. This falls within the range implied by the system of non-linear equations, as discussed below.

The lower parts of Table 12 present roots based on the non-linear system for four different combinations of the transition functions. Here, $F_y = 0$ and $F_y = 1$ correspond to weak and strong responses, respectively, to ΔRR_t in the Δy_t equation; the equilibrium correction coefficient decreases from -0.069 to -0.473 as we change F_y from 0 to 1. $F_{rex} = 0$ and $F_{rex} = 1$ are associated with weak and strong responses, respectively, to $\Delta(co - couk)_{t-1}$ in the Δrex_t equation. Finally, $F_{rm} = 0$ and $F_{rm} = 1$ denote high and low degrees of equilibrium reversion, respectively, in the Δrm_t equation.

In the case with $F_y = 0, F_{rex} = 0, F_{rm} = 0$, the companion matrix contains a

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complex pair of roots, 0.84 +/- 0.39i, with a modulus of 0.93 and a period of 14.5 years. This is somewhat higher than the period implied by the linear system.

Strong response to ΔRR_t in the Δy_t equation, i.e. when $F_y = 1$, brings about a substantial reduction in the longest period associated with complex roots. For instance, when $F_y = 1, F_{rex} = 1, F_{rm} = 0$, the longest period associated with complex roots is reduced to 8.5 years. Note that the value of F_{rex} is irrelevant for these roots since the transition equation in the real exchange rate equation only affects the dynamic effects of exogenous variables or the constant term.

The degree of equilibrium reversion in the Δrm_t equation has a relatively small effect on the longest period associated with complex roots. Note that when we change F_{rm} from 1 to 0, *ceteris paribus*, the longest period increases from 8.5 years to 9.1 years only, see the case of $F_y = 1, F_{rex} = 1, F_{rm} = 1$. However, if we had weakened the interest rate response in the Δy_t equation by changing F_y from 1 to 0 while $F_{rex} = 1$ and $F_{rm} = 1$, the longest period associated with complex roots would have increased to 15.8 years.

In sum, shifts in the responsiveness to changes in real interest rates have the strongest impact on the dynamic properties of the system. Large increases in the real interest rate seem to stabilise the cycle not only in terms of increasing the direct responsiveness of output to the interest rate, but also through making output respond more strongly to deviations from its long-run equilibrium.

6   Concluding remarks

We have applied linear and non-linear models of STR form to characterise the behaviour of Norwegian GDP, real exchange rate and real money balances over a period of almost two centuries, 1830–2003. The employed data set for the Norwegian economy has just been compiled in its full length and is thus modelled for the very first time in this paper. It appears that non-linear behaviour is a pervasive property of these variables. Accordingly, models with non-linear dynamics and/or time variation in parameters have in general been found to have higher explanatory power than their linear counterparts.

In line with a number of previous studies, we find evidence of asymmetric effects of monetary policy on output. Specifically, large and contractionary monetary policy shocks tend to have significant effects on output, while small and/or expansionary monetary policy shocks tend to have negligible effects on output. We do not find evidence of asymmetric effects of fiscal policy, except that the role of fiscal policy in the Norwegian business cycles has increased substantially since the 1950s. Prior to that, the role of fiscal policy, as represented by public consumption, is rather passive.

The long-run (equilibrium) real exchange rate seems to depend on the ratio between domestic and foreign GDP (the UK), as implied by the Balassa-Samuelson hypothesis. Higher domestic growth than abroad over extended periods seems to largely explain the observed real appreciation of the krone against pound sterling over time. This occurs even though large deviations from the equilibrium path of the

real exchange rate also tend to modify the appreciation tendency of the Norwegian real exchange rate.

Adjustment of real money balances seems to depend substantially on lagged growth in real money balances. In particular, we observe that the speed of adjustment of the money stock to its long-run level tend to fall substantially in periods of excess growth in the real money stock. Historically, such periods of strong growth in real money are often associated with periods of deflation and/or wars. Accordingly, deviations from the equilibrium level become more persistent whenever the return on money stock is high, as in periods of deflation, and whenever asset markets are highly illiquid, as during periods of war.

This paper presents new empirical results for the Norwegian economy using an extended data set and sheds light on many aspects of output, the real exchange rate and real money balances over time. We have, however, only aimed at capturing the most apparent characteristics of the time series rather than at providing a precise description or explanation of their behaviour in different periods. In this sense, more research is warranted to obtain more detailed characterisations of the behaviour of these variables in specific periods. Also, it would be interesting to undertake the empirical analysis within the framework of other forms of non-linearity than the STR form which we selected. Our results should therefore be considered exploratory and intended to stimulate more research on Norwegian business cycles and especially their interaction with monetary and fiscal policies over time.

Appendix A: Data

Definitions and sources of the time series are provided in Section 3. Variable names in small letters denote the natural logs of the corresponding variables while Δ symbolises the first difference of the associated variable. In the following we specify the dummy variables.

- dyy: Denotes an impulse dummy that takes on a value of 1 in the year 19yy and zero elsewhere.

- d18yy: Is an impulse dummy that takes on a value of 1 in 18yy and zero elsewhere.

- M1820to13: Step dummy for Maddison's first growth regime. This is a step dummy that is 1 in the years 1830–1913 and zero afterwards. M1820to13 = PreW1.

- M14to49: Step dummy for Maddison's second growth regime. This is a step dummy that is 1 in the years 1914–1949 and zero afterwards.

- PreW1: Step dummy that takes on a value of 1 in the period 1830–1913 and zero elsewhere.

- PostW1: Step dummy for the inter-war and the post WWII period. Specifically, it takes on a value of 1 in the period 1929–1939 and in the period 1946–2003 and zero elsewhere.

- SD1950: Post World War II dummy. It has a value of 1 in the years 1950–2003 and zero elsewhere.

- W1: Step dummy for World War I. It has a value of 1 in the years 1914–1919 and zero elsewhere

- W2: Step dummy for World War II. It has a value of 1 in the years 1940–1944 and zero elsewhere

- W2W: Step dummy that takes on a value of 1 in the period 1915–1940 and zero elsewhere.

Appendix B: Dynamic properties of the nonlinear models

We draw on Lütkepohl (1991) who considers the case with n linear dynamic equations with n endogenous variables y_t and m exogenous variables x_t. The *structural form* of a model can be expressed as:

$$\mathbf{\Gamma}_0 \mathbf{y}_t = \sum_{i=1}^{q} \mathbf{\Gamma}_i \mathbf{y}_{t-i} + \sum_{i=0}^{q} \mathbf{D}_i \mathbf{x}_{t-i} + \boldsymbol{\varepsilon}_t \tag{8}$$

To investigate the dynamic properties of the model it is convenient to work with the *reduced form* of the model:

$$\mathbf{y}_t = \sum_{i=1}^{q} \mathbf{A}_i \mathbf{y}_{t-i} + \sum_{i=0}^{q} \mathbf{B}_i \mathbf{x}_{t-i} + \mathbf{u}_t \tag{9}$$

defining the $n \times n$ matrices $\mathbf{A}_i = \mathbf{\Gamma}_0^{-1} \mathbf{\Gamma}_i$, $i = 1, \ldots, q$, and the $n \times m$ matrices $\mathbf{B}_i = \mathbf{\Gamma}_0^{-1} \mathbf{D}_i$, $i = 0, \ldots, q$. The reduced form residuals are given by $\mathbf{u}_t = \mathbf{\Gamma}_0^{-1} \boldsymbol{\varepsilon}_t$.

The reduced form representation of the model can be expressed in its *companion form* as:

$$\mathbf{Z}_t = \mathbf{\Phi} \mathbf{Z}_{t-1} + \mathbf{\Psi} \mathbf{x}_t + \mathbf{U}_t \tag{10}$$

forming stacked $(n + m)q \times 1$ vectors with new variables

$$\mathbf{Z}_t = (\mathbf{y}'_t, \ldots, \mathbf{y}'_{t-q+1}, \mathbf{x}'_t, \ldots, \mathbf{x}'_{t-q+1})'$$

and

$$\mathbf{U}_t = (\mathbf{u}'_t, \mathbf{0}, \ldots, \mathbf{0})'$$

The matrices $\mathbf{\Phi}_{(n+m)q \times (n+m)q}$ and $\mathbf{\Psi}_{(n+m)q \times m}$ are formed by stacking the (reduced form) coefficient matrices A_i, B_i for $\forall i$ in the following way:

$$\mathbf{\Phi} = \left[\begin{array}{cccc|cccc} \mathbf{A}_1 & \cdots & \mathbf{A}_{q-1} & \mathbf{A}_q & \mathbf{B}_1 & \cdots & \mathbf{B}_{q-1} & \mathbf{B}_q \\ \mathbf{I}_n & & \mathbf{0}_n & \mathbf{0}_n & \mathbf{0}_n & & \mathbf{0}_n & \mathbf{0}_n \\ & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0}_n & \cdots & \mathbf{I}_n & \mathbf{0}_n & \mathbf{0}_n & \cdots & \mathbf{0}_n & \mathbf{0}_n \\ \hline & & & & \mathbf{0}_m & \cdots & \mathbf{0}_m & \mathbf{0}_m \\ & \mathbf{0}_{mq \times nq} & & & \mathbf{I}_m & & \mathbf{0}_m & \mathbf{0}_m \\ & & & & & \ddots & \vdots & \vdots \\ & & & & \mathbf{0}_m & \cdots & \mathbf{I}_m & \mathbf{0}_m \end{array}\right], \quad \mathbf{\Psi} = \left[\begin{array}{c} \mathbf{B}_0 \\ \mathbf{0}_n \\ \vdots \\ \mathbf{0}_n \\ \hline \mathbf{I}_m \\ \mathbf{0}_m \\ \vdots \\ \mathbf{0}_m \end{array}\right] \tag{11}$$

In our case, we have developed a system of nonlinear dynamic relationships of the type

$$\mathbf{Z}_t = [\Phi^1 + \Phi^2 \odot F(\Gamma, C, S)]\mathbf{Z}_{t-1} + [\Psi^1 + \Psi^2 \odot F(\Gamma, C, S)]\mathbf{x}_t + \mathbf{U}_t \tag{12}$$

where $\odot$ denotes element-wise multiplication of each element i, j in the matrices Φ^2, Ψ^2 with the corresponding element i, j in the matrix F with nonlinear transition functions $F_{ij}(\gamma_{ij}, c_{ij}, s_{t,ij})$ as its elements i, j or 0.

If we index by $F^\dagger()$ a given choice of values for the transition function elements $F^\dagger_{ij}$, we can write the corresponding coefficient matrices of the companion form representation of the model as

$$\Phi^\dagger = [\Phi^1 + \Phi^2 \odot F^\dagger(\Gamma, C, S)]$$

$$\Psi^\dagger = [\Psi^1 + \Psi^2 \odot F^\dagger(\Gamma, C, S)]$$

The corresponding companion form is given by

$$\mathbf{Z}_t = \Phi^\dagger \mathbf{Z}_{t-1} + \Psi^\dagger \mathbf{x}_t + \mathbf{U}_t \tag{13}$$

The eigenvalues (characteristic roots) of the system matrix $\Phi^\dagger$ are useful to summarise the characteristics of the dynamic behaviour of the system, and by varying $F^\dagger()$ we can explore the dynamic properties of the system when the transition function takes on different values. Admittedly, this is a rather crude approximation to the dynamic properties of the system, but it gives a rough indication about the dynamics of particular regime combinations stemming from the matrix of transition functions $F^\dagger()$.

To calculate the roots of the system matrix $\Phi^\dagger$, we have used the Gauss function `EIG`, which calculates the eigenvalues of a general matrix.

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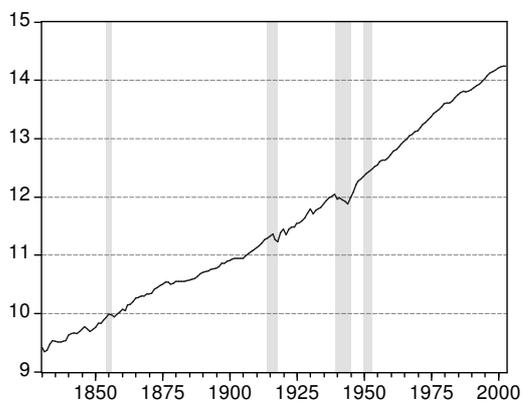
(a) Real GDP, $\log(Y)$
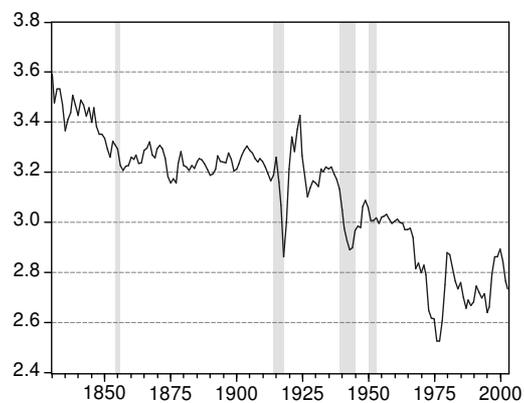
(b) Real exchange rate, $\log(REX)$
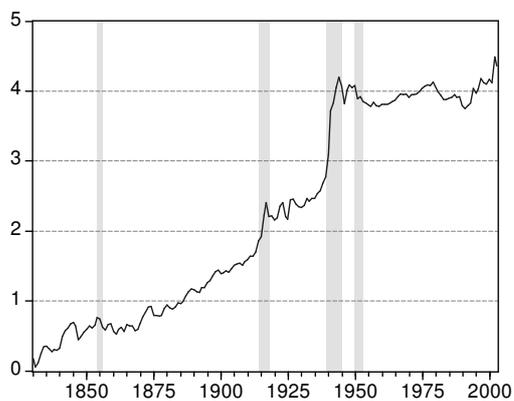
(c) Real narrow money, $\log(M0/CPI)$
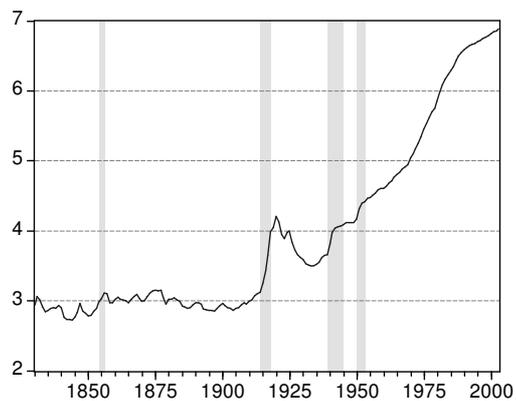
(d) Consumer price index, $\log(CPI)$
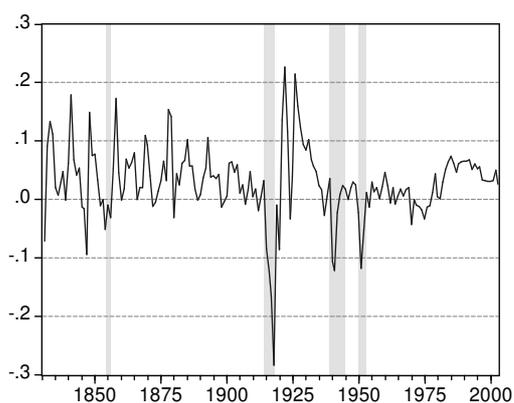
(e) Real interest rate, RR
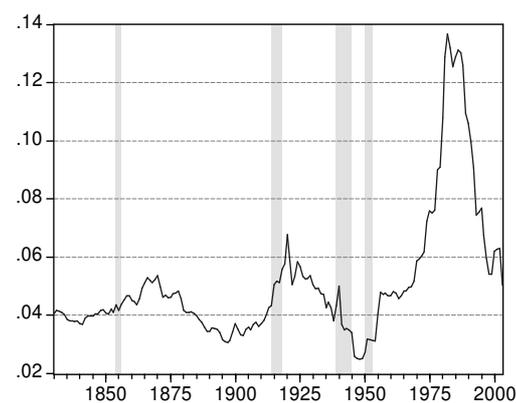
(f) Annual bond yield, R

Figure 1: Historical data 1831–2003 (levels). Here and elsewhere in this paper, the shaded areas designate the Crimean war, WW I & II and the Korean war.

(a) $\Delta \log(Y)$
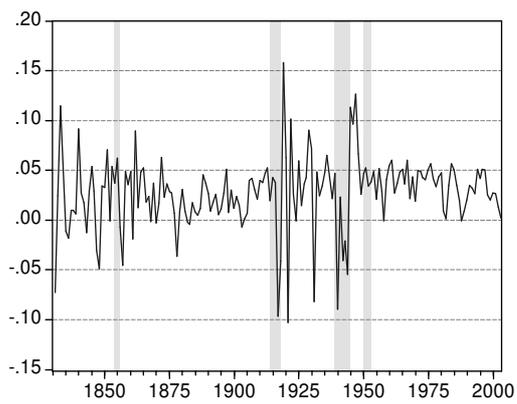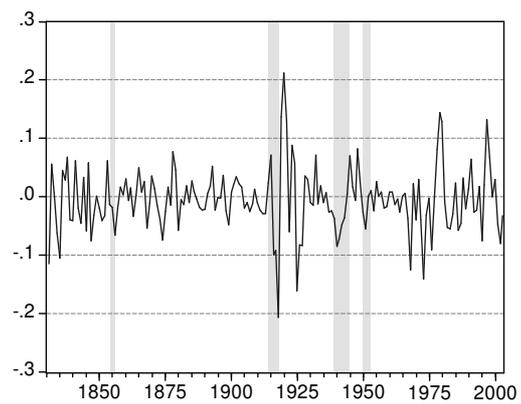
(b) $\Delta \log(REX)$
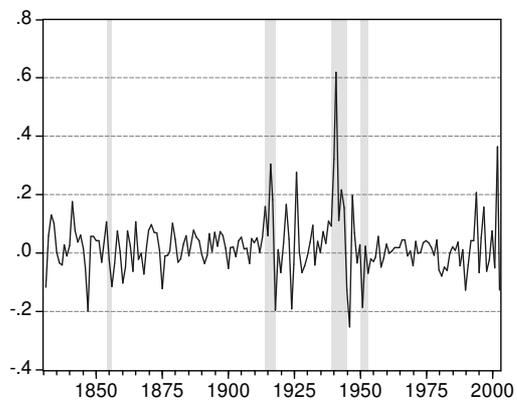
(c) $\Delta \log(M0/CPI)$
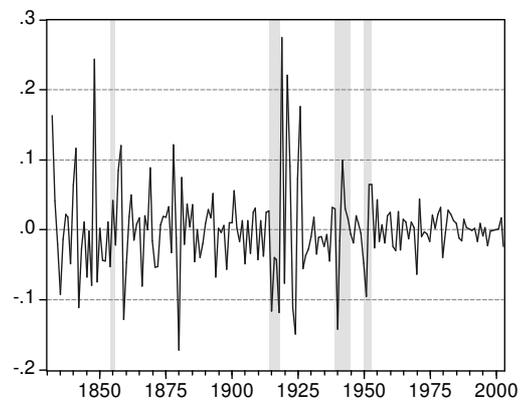
(d) ΔRR

Figure 2: Growth rates in real GDP, the real exchange rate, narrow real money balances and changes in real interest rates 1831–2003.
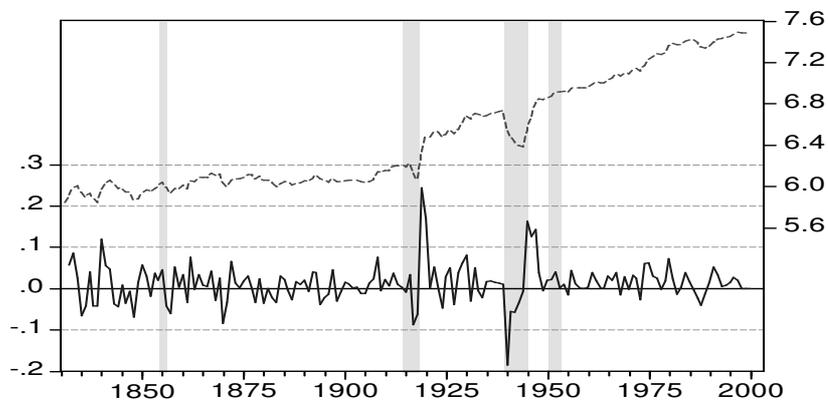
(a) Relative GDP, $\log(Y/YUK)$
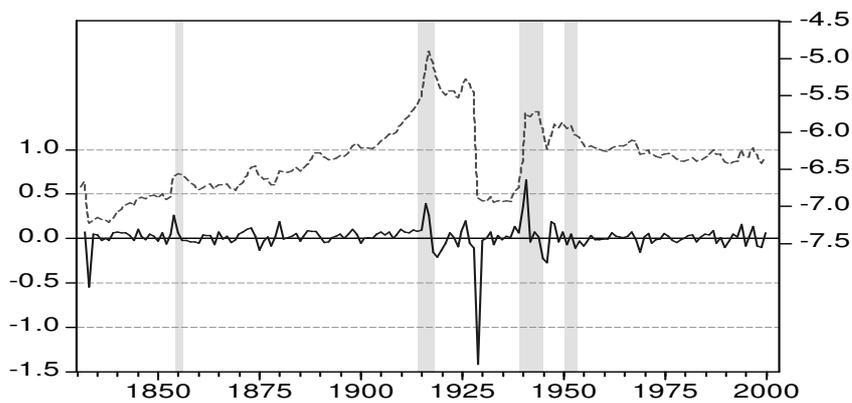
(b) Relative narrow money, $\log(M0/M0UK)$
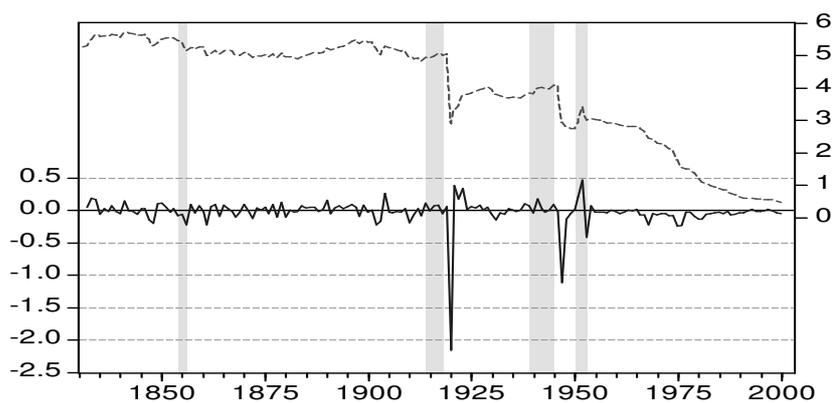
(c) Relative public consumption, $\log(CO/COUK)$

Figure 3: Growth rates (left scale) and ratios (right scale) of Norwegian GDP, narrow money and public consumption relative to the UK.
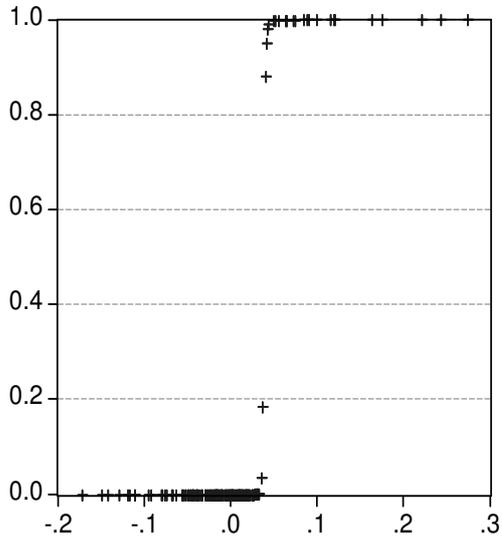
(a) Transition function $F_y \times s$
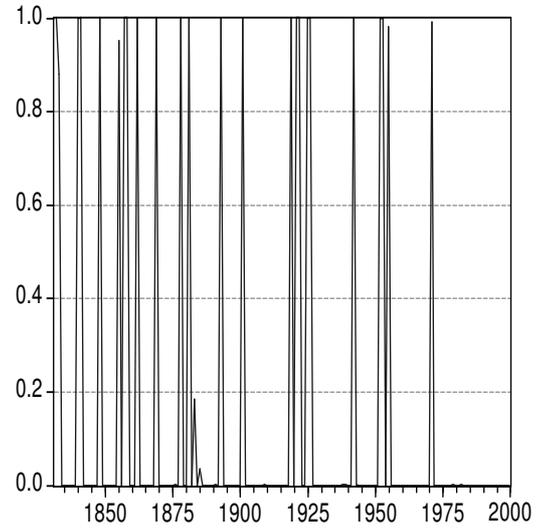
(b) Transition function over time

Figure 4: Logistic transition F_y function for the model of GDP, 1831–2000; $s_t = \Delta RR_t$.
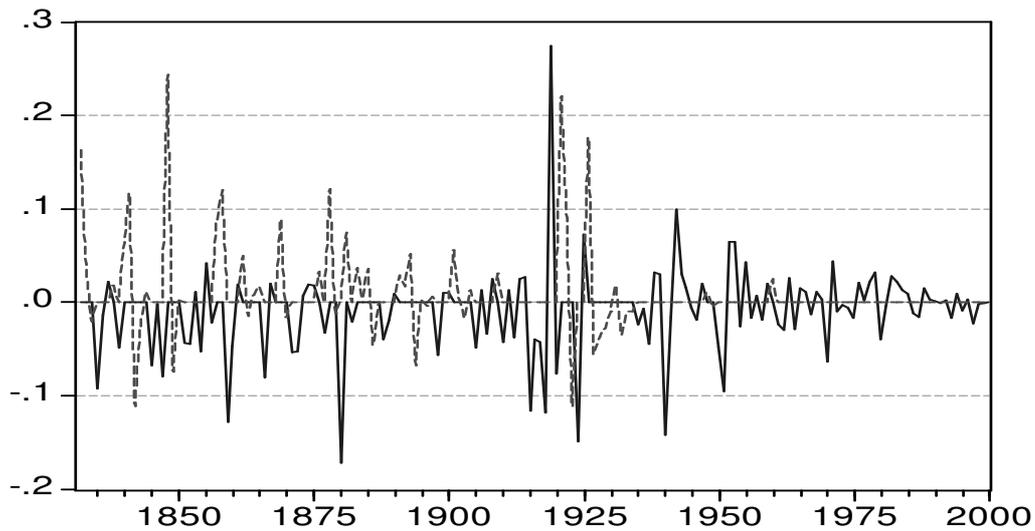
Figure 5: Changes in real interest rates in periods with inflation (solid line) and deflation (dotted line).
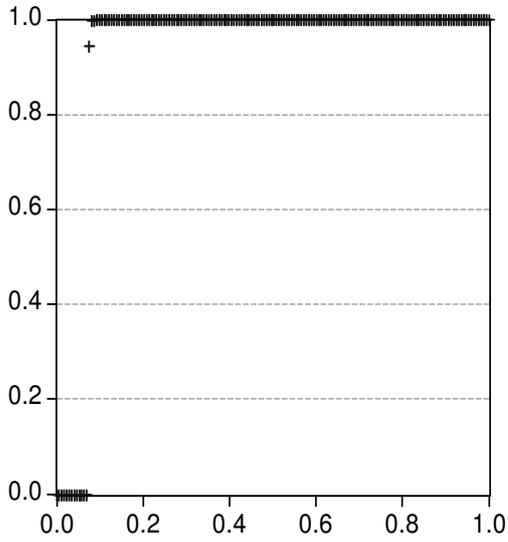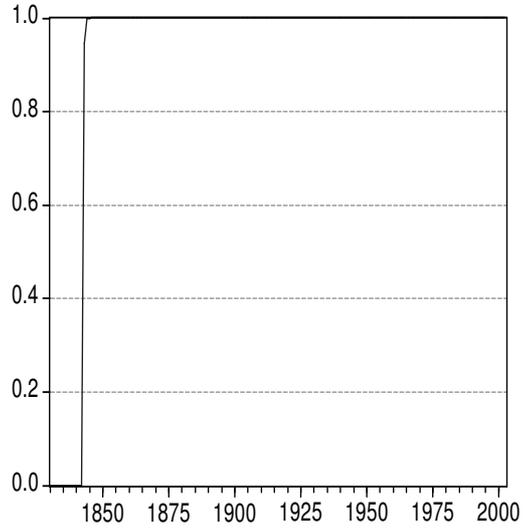
(a) Transition function $F_{rex} \times s$ (b) Transition function over time

Figure 6: Logistic transition function for the real exchange rate model M1, 1831–2000; $s_t = t$.
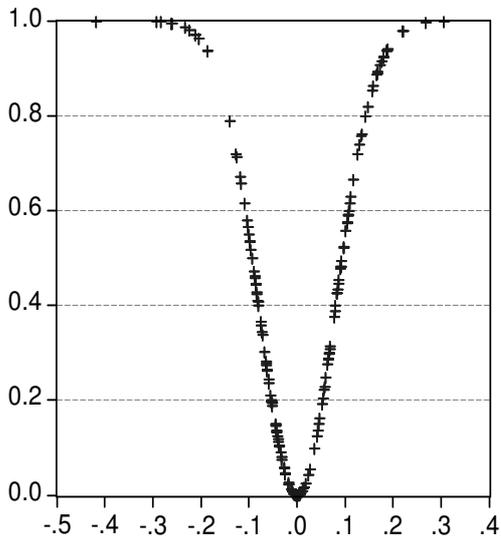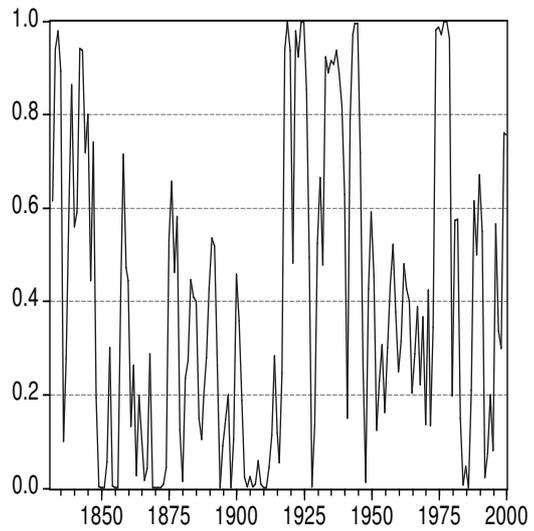
(a) Transition function $F_{rex} \times s$ (b) Transition function over time

Figure 7: Exponential transition function F_{rex} for the real exchange rate model M2, 1831–2000; $s_t = \widehat{u}_{rex,t-1}$.
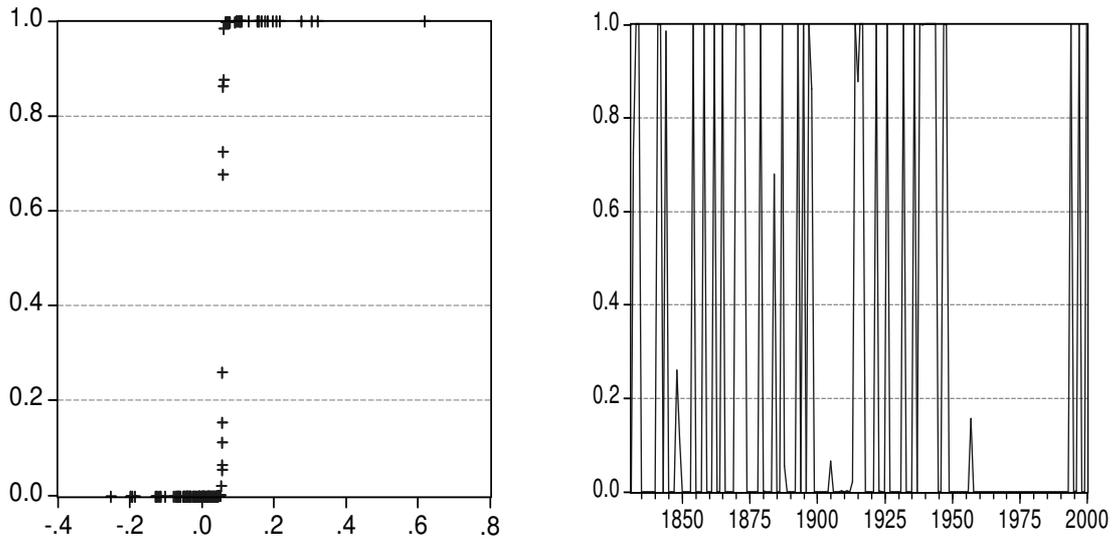
(a) Transition function $F_{rm} \times s$ (b) Transition function over time

Figure 8: Logistic transition function F_{rm} for the model of real money M1, 1831–2000; $s_t = \Delta rm_{t-1}$.
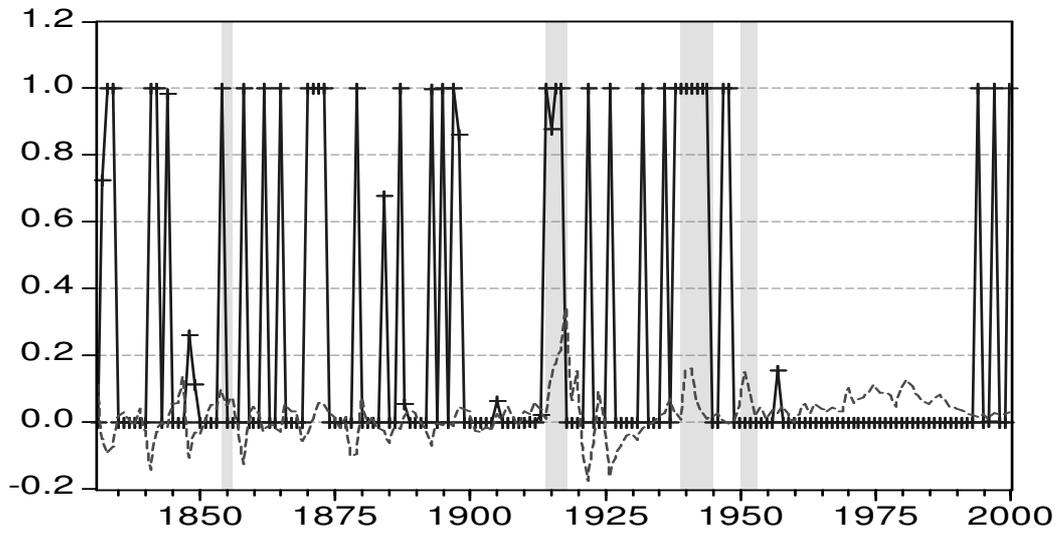
Figure 9: Inflation and values of the transition function F_{rm} over time.

Table 1: Summary statistics of the data

	Period	Variable	Mean	Stdev	Min	Max
Total period	1831 − 2003	Δy_t	2.9	3.7	-9.7	17.1
		Δrex_t	3.0	11.2	-22.4	85.7
		Δrm_t	-0.3	5.5	-18.6	23.6
		RR_t	2.6	6.5	-34.9	21.2
		R	5.1	2.3	2.5	13.7
		Δcpi	2.5	6.8	-16.2	40.5
Subperiods	1831 − 1870	Δy_t	2.4	4.0	-7.0	12.2
		Δrex_t	1.5	7.5	-18.0	19.3
		Δrm_t	-0.6	4.6	-10.8	7.1
		RR_t	4.0	6.1	-10.4	16.9
		R_t	4.3	0.5	3.7	5.4
		Δcpi_t	0.3	6.1	-13.2	14.5
	1870 − 1914	Δy_t	2.2	1.9	-3.5	6.5
		Δrex_t	3.0	4.9	-11.5	17.2
		Δrm_t	-0.4	3.0	-7.1	8.1
		RR_t	3.6	3.9	-3.3	14.8
		R_t	3.9	0.6	3.1	5.4
		Δcpi_t	0.3	3.8	-10.0	7.5
	1914 − 1945	Δy_t	2.5	6.3	-9.7	17.1
		Δrex_t	9.2	19.5	-17.9	85.7
		Δrm_t	-0.3	8.7	-18.6	23.6
		RR_t	1.1	11.7	-34.9	21.2
		R_t	4.8	0.8	3.4	6.8
		Δcpi_t	3.7	11.7	-16.2	40.5
	1945 − 1970	Δy_t	5.0	3.0	0.0	13.6
		Δrex_t	-0.8	8.2	-22.4	22.0
		Δrm_t	-0.3	3.9	-11.8	8.6
		RR_t	0.2	3.5	-12.9	4.7
		R_t	4.1	1.0	2.5	5.9
		Δcpi_t	3.9	3.6	-0.5	16.1
	1970 − 1990	Δy_t	3.5	1.9	0.0	5.9
		Δrex_t	-0.7	4.6	-11.9	4.6
		Δrm_t	-0.5	6.8	-13.1	15.6
		RR_t	2.8	3.4	-4.9	7.2
		R_t	8.7	2.8	5.0	13.7
		Δcpi_t	8.2	2.5	4.1	13.6
	1990 − 2003	Δy_t	3.1	1.5	0.3	5.3
		Δrex_t	4.1	15.3	-11.9	44.0
		Δrm_t	0.6	6.0	-7.6	14.1
		RR_t	4.7	1.5	2.6	6.8
		R_t	7.1	1.7	5.0	10.6
		Δcpi_t	2.5	0.8	1.2	4.1

Note: Summary statistics for GDP growth (Δy_t), real exchange rate depreciation (Δrex_t), real money growth (Δrm_t), real and nominal interest rate levels (RR_t and R_t), and annual inflation (Δcpi_t). Mean, Stdev, Min, Max are all measured in per cent.

Table 2: Time series properties of Norwegian variables

I. ADF tests with:	Intercept and trend		Intercept	
	$t-$ADF	$\widehat{\varrho}$	$t-$ADF	$\widehat{\varrho}$
y	-0.995	0.985	2.186	1.005
rex	-3.394	0.860	-1.509	0.973
$m0$	-2.412	0.974	0.487	1.001
y-yuk	-2.225	0.954	0.293	1.002
co	-1.936	0.973	1.937	1.005
cpi	-1.005	0.992	1.474	1.006
R	-2.433	0.959	-1.997	0.972
RR	-5.320	0.496	-5.269	0.509
II.				
Δy			-6.276	0.030
Δrex			-7.426	-0.079
$\Delta m0$			-5.493	0.500
$\Delta(y$-$yuk)$			-6.727	0.079
Δco			-6.069	0.016
Δcpi			-4.944	0.558
ΔR			-4.216	0.529
DF crit. v. 5%	-3.44		-2.88	
DF crit. v. 1%	-4.02		-3.47	

Note: All results are based on data for the period 1836–2000. Panel I. Column 2 reports t-ADF values while column 3 reports estimates of the associated ϱ, which is the sum of the autoregressive coefficients in the ADF model. Columns 4 and 5 report the t-ADF values and the estimates of ϱ in the case of ADF models with intercepts. The ADF tests are based on ADF models with 3 lags of the difference terms. Panel II. Here we report the outcome of ADF tests conducted on the first differences of the above variables, exclusive the real interest rate (RR). The last two rows report asymptotic Dickey-Fuller critical values at the 5% and 1% level, respectively.

Table 3: Long-run models of GDP, the real exchange rate and money demand

I. GDP:

$$y_t = 0.399\ yuk_t + 0.014\ t + 0.317\ co\times SD50_t + 5.109\ SD50_t$$

$$+ 7.990\ M1830to13_t - 0.233\ W1_t + 8.248\ M14to49_t - 0.116\ W2_t + \widehat{u}_{y,t}$$

$$\Delta\widehat{u}_{y,t} = -\ \underset{(0.059)}{0.278}\ \widehat{u}_{y,t-1} + \underset{(0.081)}{0.177}\ \Delta\widehat{u}_{y,t-1} - \underset{(0.076)}{0.078}\ \Delta\widehat{u}_{y,t-2}$$

$$+ \underset{(0.075)}{0.139}\ \Delta\widehat{u}_{y,t-3}$$

DF 5% : –3.50; MacKinnon 5% = –4.23

II. Real exchange rate:

$$rex_t = 5.640 - 0.389\ (y - yuk)_t + \widehat{u}_{rex,t}$$

$$\Delta\widehat{u}_{rex,t} = -\ \underset{(0.040)}{0.216}\ \widehat{u}_{rex,t-1} + \underset{(0.072)}{0.376}\ \Delta\widehat{u}_{rex,t-1}$$

DF 5%: –2.90; MacKinnon 5% = –3.40

III. Money:

$$\widehat{m0}_t = -\ 9.915 + 0.982\ cpi_t + 1.111\ y_t - 0.120\ R_t$$

$$(m0 - cpi - y)_t = -\ 8.883 - 0.086\ R_t + \widehat{u}_{rm,t}$$

$$\Delta\widehat{u}_{rm,t} = -\ \underset{(0.020)}{0.047}\ \widehat{u}_{rm,t-1} + \underset{(0.077)}{0.432}\ \Delta\widehat{u}_{rm,t-1} - \underset{(0.083)}{0.130}\ \Delta\widehat{u}_{rm,t-2}$$

$$+ \underset{(0.079)}{0.141}\ \Delta\widehat{u}_{rm,t-3}$$

Note: This table employs the two-step procedure proposed by Engle and Granger (1987) to estimate and evaluate the long run relationships for GDP, the real exchange rate and money demand. A variable name in small letters indicates the natural log of the variable. The long-run OLS estimates are based on annual data for the period 1831–2000. These are followed by ADF tests using the residuals from the estimated long-run relationships. The parentheses contain estimated standard errors of the associated coefficients.

Table 4: Linear system of GDP, the real exchange rate and real money

$\Delta \widehat{y}_t = \begin{array}{c} 0.017 \\ (0.003) \end{array} + \begin{array}{c} 0.254 \ \Delta y_{t-1} \\ (0.058) \end{array} - \begin{array}{c} 0.106 \ \widehat{u}_{y,t-1} \\ (0.036) \end{array} + \begin{array}{c} 0.262 \ \Delta co \times \text{SD50}_t \\ (0.070) \end{array}$

$\begin{array}{cccc} - & 0.068 \ \Delta RR_t + & 0.047 \ \Delta rex_t + & 0.065 \ \Delta rm_t + & 0.072 \ \text{d1862} \\ & (0.033) & (0.044) & (0.027) & (0.022) \end{array}$

$\begin{array}{ccccc} - & 0.128 \ \text{d17} + & 0.151 \ \text{d19} - & 0.139 \ \text{d21} + & 0.091 \ \text{d22} - & 0.111 \ \text{d31} \\ & (0.022) & (0.025) & (0.024) & (0.024) & (0.022) \end{array}$

$\begin{array}{ccccc} - & 0.133 \ \text{d40} - & 0.078 \ \text{d44} + & 0.106 \ \text{d45} + & 0.060 \ \text{d46} + & 0.075 \ \text{d47} \\ & (0.024) & (0.023) & (0.023) & (0.023) & (0.023) \end{array}$

$$\widehat{\sigma}_y = 0.022$$

$\Delta \widehat{rex}_t = \begin{array}{c} 0.156 \ \Delta rex_{t-1} \\ (0.058) \end{array} - \begin{array}{c} 0.132 \ \widehat{u}_{rex,t-1} \\ (0.028) \end{array} - \begin{array}{c} 0.033 \ \Delta(co - couk)_{t-1} \\ (0.013) \end{array}$

$\begin{array}{cc} - & 0.160 \ \Delta(m0 - m0uk)_t - & 0.134 \ \Delta(m0 - m0uk) \times \text{W2W}_t \\ & (0.034) & (0.061) \end{array}$

$\begin{array}{ccccc} - & 0.261 \ \text{d18} + & 0.144 \ \text{d20} + & 0.118 \ \text{d23} - & 0.376 \ \text{d29} - & 0.118 \ \text{d68} \\ & (0.038) & (0.037) & (0.035) & (0.081) & (0.035) \end{array}$

$\begin{array}{ccccc} - & 0.150 \ \text{d73} - & 0.131 \ \text{d76} + & 0.089 \ \text{d79} + & 0.110 \ \text{d80} + & 0.119 \ \text{d97} \\ & (0.034) & (0.035) & (0.035) & (0.036) & (0.035) \end{array}$

$$\widehat{\sigma}_{rex} = 0.037$$

$\Delta \widehat{rm}_t = \begin{array}{c} 0.097 \ \Delta rm_{t-1} \\ (0.056) \end{array} - \begin{array}{c} 0.135 \ \Delta rm_{t-2} \\ (0.056) \end{array} - \begin{array}{c} 0.068 \ \widehat{u}_{rm,t-1} \\ (0.013) \end{array}$

$\begin{array}{cccc} + & 0.432 \ \Delta y \times \text{preW1}_t + & 0.406 \ \Delta y \times \text{postW1}_t + & 0.083 \ \text{W1}_t + & 0.287 \ \text{W2}_t \\ & (0.194) & (0.150) & (0.029) & (0.037) \end{array}$

$\begin{array}{ccccc} + & 0.230 \ \text{d16} - & 0.221 \ \text{d18} + & 0.239 \ \text{d26} + & 0.322 \ \text{d41} + & 0.205 \ \text{d47} \\ & (0.064) & (0.068) & (0.058) & (0.065) & (0.062) \end{array}$

$$\widehat{\sigma}_{rm} = 0.061$$

System diagnostics

Vector Normality	$\chi^2(6)$	=	$15.02[.02]^*$
Vector AR$_{1-2}$	$F_{18,407}$	=	$0.68[.84]$
Vector Heterosced.	$F_{324,558}$	=	$2.04[.00]^{**}$

Note: This simultaneous vector error correction model (VECM) has been estimated by FIML and the diagnostic tests are the standard tests for systems of linear equations as implemented in PcGive (v. 10.0); see Doornik and Hendry (2001). Sample: 1834–2000. Here and elsewhere in this paper a * denotes rejection of the corresponding null hypothesis at the 5% level while ** indicate rejection at the 1% level.

Table 5: Testing validity of single-equation models

	Weak exogeneity tests		
	Equation Δy_t	Equation Δrex_t	Equation Δrm_t
$\widehat{u}_{rm,t-1}$ and $\widehat{u}_{rex,t-1}$	$\chi^2(2) = 3.18[.20]$		
$\widehat{u}_{y,t-1}$ and $\widehat{u}_{rm,t-1}$		$\chi^2(2) = 1.27[.53]$	
$\widehat{u}_{y,t-1}$ and $\widehat{u}_{rex,t-1}$			$\chi^2(2) = 3.08[.21]$
Joint test:	$\chi^2(6) = 7.44\ [.28]$		

Note: The tests of weak exogeneity with respect to parameters of the long-run relationships and the associated adjustment coefficients have been conducted by including the three deviation terms in each of the equation in the VECM of Table 4. After reestimation by FIML, zero restrictions on the indicated pair of deviations terms are tested within the indicated equations. Thereafter, a joint test is conducted where the three pairs of restrictions are imposed jointly and tested. The square brackets contain the p-values of the chi-square test statistics under the null hypotheses.

Table 6: ECM of the real exchange rate

$$\Delta \widehat{rex}_t = \underset{(0.060)}{0.094}\ \Delta rex_{t-1} - \underset{(0.029)}{0.111}\ \widehat{u}_{rex,t-1} - \underset{(0.014)}{0.045}\ \Delta(co - couk)_{t-1}$$

$$- \underset{(0.031)}{0.082}\ \Delta(m0 - m0uk)_t - \underset{(0.061)}{0.177}\ \Delta(m0 - m0uk)\mathrm{W2W}_t$$

$$- \underset{(0.038)}{0.258}\ \mathrm{d}18 + \underset{(0.039)}{0.144}\ \mathrm{d}20 + \underset{(0.037)}{0.122}\ \mathrm{d}23 - \underset{(0.084)}{0.338}\ \mathrm{d}29 - \underset{(0.037)}{0.121}\ \mathrm{d}68$$

$$- \underset{(0.037)}{0.149}\ \mathrm{d}73 - \underset{(0.037)}{0.124}\ \mathrm{d}76 + \underset{(0.038)}{0.111}\ \mathrm{d}79 + \underset{(0.038)}{0.105}\ \mathrm{d}80 + \underset{(0.037)}{0.132}\ \mathrm{d}97$$

Diagnostics

$\widehat{\sigma}_{rex}$		$=$	0.037	
Normal	$\chi^2(2)$	$=$	3.369	$[.190]$
AR_{1-3}	$\mathrm{F}_{3,148}$	$=$	1.834	$[.143]$
Het_{Xi^2}	$\mathrm{F}_{20,148}$	$=$	1.230	$[.238]$
ARCH_{1-3}	$\mathrm{F}_{3,162}$	$=$	0.704	$[.551]$
RESET (sq.)	$\mathrm{F}_{1,153}$	$=$	0.476	$[.491]$
RESET (sq.& cub.)	$\mathrm{F}_{2,152}$	$=$	0.356	$[.701]$

Note: The model has been estimated by OLS using data for the period 1832–2000. p-values are shown in square brackets. The tests are the standard misspecification tests for linear models; cf. Table 8.

Table 7: Testing for non-linearity and for its form

I. ECM of Δy_t

$s =$	t	Δy_{t-1}	$\widehat{u}_{y,t-1}$	Δco_t	Δrex_t	ΔRR_t	Δrm_t
H_0: $F_{18,133}$	1.60[.07]	1.28[.21]	1.41[.13]	1.62[.06]	1.02[.44]	2.18[.01]**	1.55[.08]
H_{04}: $F_{6,133}$						0.25[.96]	
H_{03}: $F_{6,139}$						2.72[.02]*	
H_{02}: $F_{6,145}$						3.53[.00]**	

II. ECM of Δrex_t

$s =$	t	Δrex_{t-1}	$\widehat{u}_{rex,t-1}$	$\Delta(m0- m0uk)_t$	$\Delta(co-couk)_{t-1}$
H_0: $F_{15,139}$	2.56[.00]**	1.88[.03]*	1.73[.05]*	1.04[.42]	1.67[.06]
H_{04}: $F_{5,139}$	5.67[.00]**	1.56[.18]	1.03[.40]		
H_{03}: $F_{5,144}$	0.57[.72]	3.48[.01]**	3.47[.01]**		
H_{02}: $F_{5,149}$	1.16[.33]	0.48[.79]	0.63[.68]		

III. ECM of Δrm_t

$s =$	t	Δrm_{t-1}	Δrm_{t-2}	$\widehat{u}_{rm,t-1}$	Δy_t	RR_t	ΔRR_t
H_0: $F_{15,141}$	0.96[.51]	2.46[.00]**	1.38[.17]	1.96[.02]*	1.77[.04]*	0.44[0.96]	0.88[.59]
H_{04}: $F_{5,141}$		3.05[.01]**		3.29[.01]**	1.55[.18]		
H_{03}: $F_{5,146}$		1.65[.15]		0.81[.54]	1.97[.09]		
H_{02}: $F_{5,151}$		2.34[.05]*		1.60[.16]	1.67[.15]		

Note: The F-tests associated with H_0 test the null hypotheses of linear effects from a variable against the alternative hypotheses of non-linear effects of STR form. The other F-tests are aimed at determining the form of non-linearity.

Table 8: Non-linear ECM of Norwegian GDP

M1: LSTR ECM of Δy_t with $s = \Delta RR_t$

$$\Delta \widehat{y}_t = \underset{(0.003)}{0.019} + \underset{(0.053)}{0.225} \, \Delta y_{t-1} - \underset{(0.038)}{0.069} \, \widehat{u}_{y,t-1} + \underset{(0.067)}{0.238} \, \Delta co \times \text{SD50}_t$$

$$+ \underset{(0.049)}{0.015} \, \Delta RR_t + \underset{(0.033)}{0.072} \, \Delta rex_t + \underset{(0.020)}{0.073} \, \Delta rm_t + \underset{(0.022)}{0.048} \, \text{d1862}$$

$$- \underset{(0.021)}{0.125} \, \text{d17} + \underset{(0.029)}{0.173} \, \text{d19} - \underset{(0.026)}{0.119} \, \text{d21} + \underset{(0.025)}{0.032} \, \text{d22} - \underset{(0.021)}{0.114} \, \text{d31}$$

$$- \underset{(0.023)}{0.126} \, \text{d40} - \underset{(0.021)}{0.083} \, \text{d44} + \underset{(0.022)}{0.105} \, \text{d45} + \underset{(0.022)}{0.064} \, \text{d46} + \underset{(0.022)}{0.072} \, \text{d47}$$

$$+ \left(\underset{(0.010)}{0.033} - \underset{(0.106)}{0.464} \, \Delta RR_t - \underset{(0.095)}{0.404} \, \widehat{u}_{y,t-1} \right) \times$$

$$\left(1 + \exp(\underset{(17655)}{-231.196} \times \tfrac{1}{0.0612} \times (\Delta RR_t - \underset{(0.127)}{0.039})) \right)^{-1}$$

$\widehat{\sigma}_{\Delta y, M1} = 0.0208; \ \widehat{\sigma}_y = 0.023; \ \widehat{\sigma}_{\Delta y, M1} / \widehat{\sigma}_{\Delta y} = 0.91$

Diagnostics	LSTR ECM (M1)			Linear model		
Normality	$\chi^2(2)$	$=$	2.804 [.246]	$\chi^2(2)$	$=$	9.20 [.010]**
AR_{1-3}	$F_{3,136}$	$=$	0.694 [.557]	$F_{3,145}$	$=$	2.132 [.099]
Het_{Xi^2}	$F_{32,134}$	$=$	1.131 [.307]	$F_{24,145}$	$=$	4.070 [.000]**
$ARCH_{1-3}$	$F_{3,158}$	$=$	2.735 [.046]*	$F_{3,162}$	$=$	2.897 [.037]*
RESET (sq.)	$F_{1,144}$	$=$	0.028 [.868]	$F_{1,150}$	$=$	4.003 [.047]*
RESET (sq. and cub.)	$F_{2,143}$	$=$	0.405 [.668]	$F_{2,149}$	$=$	2.266 [.107]

Note: M1 is our preferred LSTR model with $s_t = \Delta RR_t$. The transition parameter has been scaled by the empirical std. deviation of ΔRR_t. The panel of diagnostics lays out observed test-statistics and the associate p-values in square bracket for a number of standard tests for model misspecification. Specifically, we test the following null hypotheses: the null hypothesis of normally distributed errors, tested by Jarque-Bera chi-square test; No residual autocorrelation up to order 3; No residual heteroscedasticity, which has been tested by including the regressors and their squares; No ARCH effects up to order 3; And finally, the null hypothesis of correct model specification, through two RESETs. The outcome of the first RESET refers to the case when the significance of the square of the fitted value is tested in the model while the second one refers to the case when the joint significance of the second and third power of the fitted value is tested. Sample 1832–2000; Method: NLS.

Table 9: Non-linear ECM of the real exchange rate

M1: LSTR ECM of Δrex_t with $s = t$

$$\Delta \widehat{rex}_t = \underset{(0.059)}{0.095} \ \Delta rex_{t-1} - \underset{(0.028)}{0.099} \ \widehat{u}_{rex,t-1} - \underset{(0.132)}{0.484} \ \Delta(co - couk)_{t-1}$$

$$- \underset{(0.030)}{0.083} \ \Delta(m0 - m0uk)_t - \underset{(0.060)}{0.172} \ \Delta(m0 - m0uk)\text{W2W}_t + IDs$$

$$+ \left(- \underset{(0.003)}{0.004} + \underset{(0.133)}{0.440} \ \Delta(co - couk)_{t-1} \right) \times \left(1 + \exp(\underset{(NC)}{-11604} \times (t/T - \underset{(\text{large})}{0.074})) \right)^{-1}$$

$$\widehat{\sigma}_{rex,M1} = 0.0356, \ \widehat{\sigma}_{rex,M1}/\widehat{\sigma}_{rex} = 0.97$$

M2: STR ECM of Δrex_t with $s_1 = t$ and $s_2 = \widehat{u}_{rex,t-1}$

$$\Delta \widehat{rex}_t = \underset{(0.057)}{0.129} \ \Delta rex_{t-1} - \underset{(0.026)}{0.117} \ \widehat{u}_{rex,t-1} - \underset{(0.127)}{0.533} \ \Delta(co - couk)_{t-1}$$

$$- \underset{(0.029)}{0.060} \ \Delta(m0 - m0uk)_t - \underset{(0.060)}{0.192} \ \Delta(m0 - m0uk)\text{W2W}_t$$

$$- \underset{(0.036)}{0.255} \ \text{d18} + \underset{(0.037)}{0.139} \ \text{d20} + \underset{(0.036)}{0.124} \ \text{d23} - \underset{(0.072)}{0.324} \ \text{d29} - \underset{(0.034)}{0.117} \ \text{d68}$$

$$- \underset{(0.035)}{0.148} \ \text{d73} - \underset{(0.036)}{0.126} \ \text{d76} + \underset{(0.036)}{0.107} \ \text{d79} + \underset{(0.036)}{0.102} \ \text{d80} + \underset{(0.035)}{0.128} \ \text{d97}$$

$$+ \left(- \underset{(0.005)}{0.0183} + \underset{(0.127)}{0.491} \ \Delta(co - couk)_{t-1} \right) \times \left(1 + \exp(- \ 11603.97 \ \times (t/T - \ 0.074)) \right)^{-1}$$

$$+ \underset{(0.005)}{0.019} \ \times \left(1 - \exp(- \ \underset{(50.892)}{79.245} \ \times (\widehat{u}_{rex,t-1} - \underset{(0.00)}{0.00})^2) \right)$$

$$\widehat{\sigma}_{rex,M2} = 0.0349, \ \widehat{\sigma}_{rex} = 0.0367; \ \widehat{\sigma}_{rex,M2}/\widehat{\sigma}_{rex} = 0.95$$

Diagnostics	STR ECM (M2)			Linear ECM		
Normality	$\chi^2(2)$	= 4.153	[.125]	$\chi^2(2)$	= 3.369	[.190]
AR_{1-3}	$F_{3,141}$	= 2.072	[.107]	$F_{3,148}$	= 1.834	[.143]
Het_{Xi^2}	$F_{29,137}$	= 1.226	[.218]	$F_{20,148}$	= 1.230	[.238]
$ARCH_{1-3}$	$F_{3,162}$	= 0.213	[.888]	$F_{3,162}$	= 0.704	[.551]
RESET (sq.)	$F_{1,146}$	= 2.324	[.130]	$F_{1,153}$	= 0.476	[.491]
RESET (sq. and cub.)	$F_{2,145}$	= 1.313	[.272]	$F_{2,152}$	= 0.356	[.701]

Note: M1 is our preferred LSTR model with $s = t$, which has been scaled by the total number of observations T (= 169). The dummies have been suppressed to save space. Their effects are represented by the term "IDs" and are almost identical to those presented in M2. *NC* means Not Computed. M2 is our preferred model with both LSTR and ESTR type of effects triggered by the two transition variables t and $\widehat{u}_{rex,t-1}$, respectively. M2 has been estimated by conditioning on the estimate of γ and c from M1. The transition parameter in the ESTR term has been scaled by the empirical std. deviation of $\widehat{u}_{rex,t-1}$. We have also computed the ratios between the estimated standard deviation of the residuals from M1 and M2 relative to that from the linear ECM of rex in Table 4. The tests are those proposed by Eitrheim and Teräsvirta (1996). The square brackets contain p-values. The sample period is 1832–2000; Method: NLS.

Table 10: Non-linear ECM of money demand

M1: LSTR ECM of Δrm_t with $s = \Delta rm_{t-1}$

$$\Delta \widehat{rm}_t = \underset{(0.100)}{0.172} \Delta rm_{t-1} - \underset{(0.061)}{0.189} \Delta rm_{t-2} - \underset{(0.017)}{0.086} \widehat{u}_{rm,t-1}$$

$$+ \underset{(0.206)}{0.442} \Delta y \times \text{preW1}_t + \underset{(0.157)}{0.421} \Delta y \times \text{postW1}_t + \underset{(0.031)}{0.129} \text{W1}_t + \underset{(0.041)}{0.293} \text{W2}_t$$

$$+ \underset{(0.068)}{0.180} \text{d16} - \underset{(0.068)}{0.257} \text{d18} + \underset{(0.060)}{0.240} \text{d26} + \underset{(0.068)}{0.383} \text{d41} + \underset{(0.067)}{0.237} \text{d47}$$

$$+ \left(\underset{(0.018)}{0.036} - \underset{(0.154)}{0.371} \Delta rm_{t-1} + \underset{(0.029)}{0.060} \widehat{u}_{rm,t-1} \right) \times$$

$$\left(1 + \exp(- \underset{(2980.639)}{216.322} \times \tfrac{1}{0.0949}(\Delta rm_{t-1} - \underset{(0.014)}{0.058})) \right)^{-1}$$

$$\widehat{\sigma}_{rm,M1} = 0.0588, \ \widehat{\sigma}_{rm} = 0.0596; \ \widehat{\sigma}_{rm,M1}/\widehat{\sigma}_{rm} = 0.99$$

Diagnostics	LSTR ECM (M1)				Linear model			
Normal	$\chi^2(2)$	=	4.84	[.089]	$\chi^2(2)$	=	8.282	[.016]*
AR_{1-3}	$F_{3,145}$	=	1.26	[.292]	$F_{3,150}$	=	1.384	[.250]
Het_{Xi^2}	$F_{27,140}$	=	1.21	[.231]	$F_{17,150}$	=	1.159	[.305]
$ARCH_{1-3}$	$F_{3,161}$	=	0.75	[.522]	$F_{3,161}$	=	0.231	[.875]
RESET (sq.)	$F_{1,150}$	=	0.21	[.645]	$F_{1,155}$	=	0.224	[.637]
RESET (sq. and cub.)	$F_{2,149}$	=	0.73	[.482]	$F_{2,154}$	=	0.323	[.724]

Note: M1 is our preferred LSTR model with $s_t = \Delta rm_{t-1}$. The transition parameter has been scaled by the empirical std. deviation of Δrm_{t-1}. The sample period is 1834–2000; Method: NLS.

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Table 11: Testing for no remaining non-linearity

STR ECM of:		Δy_t	Δrex_t	Δrm_t
$s =$		ΔRR_t	Δrex_{t-1}	Δrm_{t-1}
	M1:	$F_{18,127}$: 1.10[.36]	$F_{15,135}$: 1.63[.07]	$F_{15,\,136}$: 2.15[.01]**
	M2:		$F_{15,132}$: 1.80[.04]*	
$s =$			$\widehat{u}_{rex,t-1}$	$\widehat{u}_{rm,t-1}$
	M1:		$F_{15,135}$: 2.14[.01]**	$F_{15,136}$: 1.73[.05]*
	M2:		$F_{15,132}$: 2.08[.02]*	
$s =$		t	t	t
	M1:	$F_{21,124}$: 1.39[.14]	$F_{18,132}$: 1.70[.05]*	$F_{15,136}$: 1.13[.33]
	M2:		$F_{18,129}$: 1.49[.11]	

Note: The first row indicates the non-linear ECM of a given variable while the rows headed by s present the transition variable (s) defining the non-linear model, see Tables 8–10. The $F_{df1,df2}$-tests test whether there is any remaining non-linearity of STR type for a given s in the non-linear ECMs. We also test whether there is any remaining non-linearity when s = t, the time trend, cf. Table 7.

Table 12: Dynamic properties of the linear and non-linear systems

Properties of the complete system		
Roots	Modulus	Period
Linear VECM (Table 4)		
0.82 +/- 0.44i	0.93	12.8
0.83	0.83	
0.06 +/- 0.38i	0.38	4.5
0.28	0.28	
System of the STR models (Tables 8–10)		
$F_y = 0, F_{rex} = 0, F_{rm} = 0$		
0.84 +/- 0.39i	0.93	14.5
0.86	0.86	
0.10 +/- 0.45i	0.46	4.7
0.24	0.24	
0.15	0.15	
$F_y = 1, F_{rex} = 1, F_{rm} = 1$		
0.86	0.86	
0.74	0.74	
0.41 +/- 0.34i	0.53	9.1
-0.10 +/- 0.45i	0.46	3.5
0.15	0.15	
$F_y = 1, F_{rex} = 1, F_{rm} = 0$		
0.85	0.85	
0.65	0.65	
0.42 +/- 0.38i	0.57	8.5
0.12 +/- 0.44i	0.46	4.8
0.15	0.15	
$F_y = 0, F_{rex} = 1, F_{rm} = 1$		
0.86 +/- 0.36i	0.94	15.8
0.87	0.87	
-0.10 +/- 0.45i	0.45	
0.24	0.24	3.5
0.15	0.15	

Note: We have used the Gauss function `EIG` to calculate the roots. $F_y = 0$: Weak response to ΔRR_t. $F_y = 1$: Strong response to ΔRR_t. $F_{rex} = 0$: Weak response to $\Delta(co-couk)_{t-1}$, $F_{rex} = 1$: Strong response to $\Delta(co - couk)_{t-1}$. $F_{rm} = 0$: Strong equilibrium reversion. $F_{rm} = 1$: Weak equilibrium reversion.

KEYWORDS: