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by

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Inflation Targeting Rules: History-Dependent or Forward-Looking?*

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Abstract

This paper discusses the optimal use of inflation forecasts in an inflation targeting setting with reaction and implementation lags. It distinguishes between the optimality of history-dependence versus forward-lookingness in monetary policy-making. It is shown that monetary policy strategy is inverted relative to private sector pricing behaviour: if private sector price setting is backward-looking, policy should be forward-looking, and vice versa.

Keywords: Monetary policy, targeting rule, optimal horizon, history dependence.

JEL classification codes: E52,E61.

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1. Introduction

The conduct of monetary policy from both a prescriptive and a descriptive perspective has been an increasingly central topic in macroeconomics in the past decade. An important issue has been what role forecasts of future economic conditions play in policymaking. Since it takes time for monetary policy to affect the economy, a central argument has been that policy should be forward-looking and inflation forecasts therefore should be an integral part of the inflation-targeting strategy. This paper shows that this argument depends on the degree to which private sector price setters exert forward-looking behaviour in addition to the length of their reaction and implementation lags.

Svensson (1997) defines inflation targeting as a policy that minimizes expected loss where inflation deviations from some target is the important argument. This optimizing approach has been denoted a targeting rule. In his important early contribution to this targeting-rule approach, Svensson considers an inflation-targeting central bank with a mandate to minimize expected loss given by

$$\min E_{t_0} \sum_{t=t_0}^{\infty} L_t,$$

where the period loss (L_t) is given by a quadratic function, taking the inflation gap ($\pi - \pi^*$) and output gap ($y - y^*$) as arguments, i.e.

$$L_t = \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda\,(y_t - y_t^*)^2\right],$$

where λ is the relative weight on output-gap stabilization. Svensson notes that the optimal inflation targeting horizon, i.e., the time it is expected to take for inflation to return to its target level depends positively on λ. A central bank that places a higher relative weight on the output gap should conduct an interest-rate setting policy that brings forecasts of future inflation into line with the inflation target over a longer period of time. The Svensson model exhibits private sector agents that are backward-looking and react with a lag to available information. Monetary policymaking will then only have a lagged and gradual effect on the macroeconomy. These assumptions introduce a role for forecasts as indicators of monetary policy.

An important element of recent macroeconomic research is the recognition of the forward-looking nature of private-sector decision making. Forward-looking behaviour implies that private agents make forecasts about the future and base decisions on these forecasts. Such behaviour can be motivated by, e.g., consumption smoothing or adjustment costs. Hence, by being able to commit to future policies, the central bank is able to influence private sector forecasts which lead private agents to behave in a way that is

more consistent with policy objectives. Woodford (1999b) points out that the optimal commitment policy is therefore history-dependent, that is, must honor the promises made in earlier periods about future policies. In the New Keynesian macroeconomic tradition,[1] history-dependence implies interest rate (policy) inertia, policy should only gradually respond to the state variables. In contrast to the situation where private sector behaviour is backward-looking, forecasts do not play an important role in policymaking.

This illustrates that an important problem with the targeting rule approach is the model dependency of the policy strategy; the optimal strategy is based on and intertwined with the particular model considered. In other words, differentiating between the more *general* policy prescriptions and the highly model-specific prescriptions is difficult. The lack of consensus on how to model the macroeconomy is therefore critical for the implementation of the targeting rule approach in policymaking.

A step forward in making targeting rules more operational is to discuss the properties of the targeting rule strategy under different assumptions regarding private sector behaviour. The policymaker may then pick the strategy according to his belief about the macroeconomy. An important and controversial issue is the degree to which private sector decisions are forward-looking. This paper shows that the assumption made about how forward-looking private sector pricing decisions is crucial for whether the central bank should follow a forward-looking or a history-dependent strategy, or a combination of both. It is shown that the policy strategy is inverted relative to private sector decision making: if the private sector is forward-looking, the monetary policy strategy should be history-dependent, and vice versa. Moreover, the paper also discusses how reaction lags (behaviour based on a lagged information set) and implementation lags[2] (behaviour based on lagged expectations of contemporaneous variables) influence the targeting rule approach.

Section 2 presents the model. Section 3 discusses the solutions under different institutional arrangements. Section 4 provides some concluding remarks.

2. The demand management problem

Assume that the central bank faces the Phillips curve given by

$$\pi_{t+j+n} = \theta\pi_{t+j+n+1|t} + (1-\theta)\pi_{t+j+n-1|t} + \gamma\hat{y}_{t+j|t} + \varepsilon_{t+j+n}, \tag{1}$$

where π_t is the rate of inflation, $\hat{y}_t \equiv y_t - y_t^*$ is the output gap, ε_t is a white-noise cost-push shock, $n \geq 0$ and $j \geq 0$ are respectively the reaction and implementation lags of

[1] See, e.g., Clarida et al. (1999) and Woodford (2002).

[2] The effects of implementation lags on optimal monetary policy are also discussed in Svensson and Woodford (1999a).

the private sector, and $\theta \in [0,1]$ measures the degree to which price-setters are forward-looking. For notational convenience, the rational expectation of a variable x at time t_1 given the information available at time t_0 is denoted by $x_{t_1|t_0}$. All variables are measured as deviation from their steady-state values.

The hybrid Phillips curve in (1) is similar to the New Keynesian Phillips curve studied by, among others, Clarida et al. (1999), Woodford (2002) and Svensson and Woodford (1999b). It allows, however, for a backward-looking component, which can be rationalized by adaptive expectation formation (see Roberts, 1995, 1997), or rule-of-thumb pricing behaviour (Gali and Gertler, 1999). For simplicity, we assume that the central bank uses the output gap as its instrument. We are thus implicitly making the assumption that the path for the expected, optimal output gap is attainable through an appropriate setting of the central-bank instrument (the short-term interest rate or base money).

The problem of the central bank is now to minimize the loss function subject to the Phillips curve by choosing an optimal path for the output gap. If the inflation target is normalized to zero, the Lagrangian for the policy problem is given by

$$\mathcal{L} = E_{t_0} \sum_{t=t_0}^{\infty} \left(\frac{1}{2} \left(\pi_t^2 + \lambda \hat{y}_t^2 \right) + \mu_t \left(\pi_{t+j+n} - \theta \pi_{t+1+j+n} - (1-\theta) \pi_{t-1+j+n} - \gamma \hat{y}_{t+j} - \varepsilon_{t+j+n} \right) \right),$$

where μ is the Lagrange multiplier associated with the constraint (1).

3. Solutions

The solution to the above problem depends on the institutional arrangement. If the central bank is able to commit to a strategy which is time-inconsistent, the relevant solution would be the commitment solution, in which the central bank utilizes private sector expectations to minimize its loss function. If the central bank is unable to commit, private sector expectations are taken as given and the relevant solution would be the discretionary solution.[3]

3.1. The timeless commitment solution

We first analyze the situation where the central bank credibly commits to its strategy and takes into account that this influences private sector expectations. The first-order

[3]McCallum and Nelson (2000) consider the solution to a similar problem where price setters' degree of forward lookingness is allowed to vary. They focus, however, on measuring the difference between the discretionary and timeless commitment solutions, that is, the cost of not being able to commit. Svensson (2001) discusses optimality conditions when pricing decisions are either fully forward-looking or fully backward-looking.

conditions for this problem are given by

$$\frac{\partial \mathcal{L}}{\partial \pi_{t+j+n}} = \pi_{t+j+n|t} + \mu_t - \theta\mu_{t-1} = 0, \quad \text{for } t = t_0, \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial \pi_{t+j+n}} = \pi_{t+j+n|t} + \mu_t - \theta\mu_{t-1} - (1-\theta)\mu_{t+1|t} = 0, \quad \text{for } t > t_0, \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial \hat{y}_{t+j}} = \lambda\hat{y}_{t+j|t} - \mu_t\gamma = 0, \quad \text{for } t \geq t_0. \tag{4}$$

If the policy commitment is assumed to have been made sufficiently long time ago ($t_0 = -\infty$), the first-order condition for this period, (2), can be disregarded,[4] and equations (3) and (4) describe the timeless commitment solution, see Woodford (1999a). By substituting out the Lagrange multipliers, the timeless commitment solution is given by

$$\hat{y}_{t+j|t} = \theta\hat{y}_{t+j-1|t-1} + (1-\theta)\hat{y}_{t+j+1|t} - \frac{\gamma}{\lambda}\pi_{t+j+n|t}. \tag{5}$$

The solution implies that the central bank at time t sets the j-period-ahead output gap reflecting a weighted average of the expected $j-1$-period-ahead output gap, as set in the previous period, and the expected $(j+1)$-period-ahead output gap, as well as inflation-forecast deviations from target $j+n$ periods ahead.[5] It gives rise to the following propositions.

Proposition 3.1. ***Direction of Strategy in the Timeless Perspective.***
The optimal timeless inflation-targeting strategy is inverted relative to the private sector pricing strategy - the policy strategy is as forward (backward)-looking as the private sector is backward (forward)-looking.

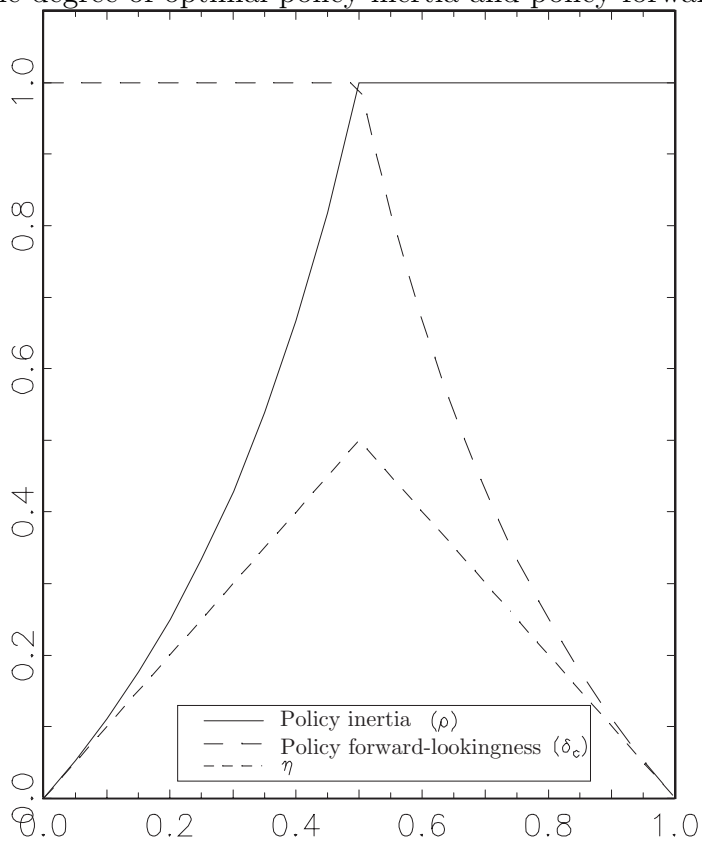
Proposition 3.2. ***Effect of implementation lags.***
The effect of a j-period private sector implementation lag is that the policymaker sets the expected j-period-ahead output gap, and disregards information on inflation forecast deviations from target from the time of the policy decision and $j-1$ periods ahead.

Proposition 3.3. ***Effect of reaction lags.***
The effect of a n-period private sector reaction lag is that the policymaker disregards information on inflation forecast deviations from target from the time of policy decision and $n-1$ periods ahead.

[4] As pointed out by Jensen and McCallum (2002), however, the timeless perspective policy rule will not be the optimal rule given that the discount factor for the representative agent is less than unity.

[5] It should be noted that expectations of future variables (forecasts) can always be expressed as a linear function of current state variables in a linear model. It is, however, more informative to represent the first-order conditions by expectations terms, since they clearly show that it is the information that affect expectations that matters for policy.

Figure 3.1: The degree of optimal policy inertia and policy forward-lookingness.



The closed-form solution of (5) is given by[6]

$$\hat{y}_{t+j|t} = \rho \hat{y}_{t+j-1|t-1} - \frac{\gamma}{\lambda (1-\eta)} \sum_{i=0}^{\infty} \delta_c^i \pi_{t+j+n+i|t} \tag{6}$$

where $\rho = \frac{\theta}{(1-\eta)} \in [0,1]$ denotes the degree of optimal policy inertia, $\delta_c = \frac{(1-\theta)}{(1-\eta)} \in [0,1]$ denotes the degree to which the policymaker should optimally discount the inflation forecasts, where $\eta = \frac{1-\sqrt{1-4\theta(1-\theta)}}{2} \in [0, 0.5]$. Equation (6) shows that optimal inflation targeting is equivalent to responding to the *discounted* and *forward-accumulated* inflation-forecast deviations from target. The greater the effect of output on inflation, the stronger the response should be. The greater weight placed on output in the loss function, the weaker the response should be.

The degree of optimal policy inertia (ρ) and policy forward-lookingness (δ_c) are both complicated functions of only one argument: the degree to which price-setters are forward-looking (θ). ρ, δ_c and η are plotted in Figure 3.1. δ_c is unity for $\theta \in [0, 0.5]$ (full policy forward-lookingness) and then declines monotonically if price-setters become

[6]For the solutions to forward-looking models with lagged dependent variables, see Røisland (2000) and Pesaran (1987).

more forward-looking.[7] Policy is completely backward-looking if if price setters are fully forward-looking. Moreover, policy inertia (ρ) increases in θ, from a value of zero in the case when price setters are fully backward-looking, and reaches unity (full policy inertia) at $\theta \in [0.5, 1]$. This implies that the monetary-policy response to future discounted inflation deviations is strongest at $\theta = 0.5$

The two extreme cases are worth noting: when price setters are assumed to be completely backward-looking ($\theta = 0$) and when price setters are assumed to be completely forward-looking ($\theta = 1$). If the price setters are entirely backward-looking (as in Svensson, 1997), the closed-form solution is given by

$$\hat{y}_{t+j|t} = -\frac{\gamma}{\lambda} \sum_{i=0}^{\infty} \pi_{t+j+n+i|t}, \text{for } t > t_0. \tag{7}$$

In this case, the central bank should only adjust the output gap inversely to the undiscounted sum of expected future inflation deviations from target. There is no role for policy inertia as the private sector does not respond to information about the future.

If private sector price setting is entirely forward-looking (as in Clarida et al., 1999), the closed-form solution is given by

$$\hat{y}_{t+j|t} = \hat{y}_{t+j-1|t-1} - \frac{\gamma}{\lambda} \pi_{t+j+n|t}, \text{for } t > t_0. \tag{8}$$

In this case, forecasts play only a role as long as there are implementation and reaction lags. Moreover, policy should be fully inertial, with the growth in the expected output gap set in negative proportion to the $(j + n)$-period-ahead inflation forecast deviations from target.

3.2. Discretion

The previous section assumed that the central bank was able to commit to a timeless strategy. Such commitment may not be feasible and the central bank may be expected to renege on any promises given in the past that are not consistent with optimal policy from that time on, thus leaving the timeless perspective. The appropriate equilibrium would then be that of discretion where the central bank takes private-sector expectations as given when optimizing. The discretionary solution is described by the following first-

[7] Giannoni and Woodford (2001) show that also in the context of optimal monetary policy, where the policymaker maximizes the welfare of the consumers, inflation inertia creates a role for forward-looking policymaking.

order conditions:

$$\frac{\partial \mathcal{L}}{\partial \pi_{t+j+n}} = \pi_{t+j+n|t} + \mu_t - (1-\theta)\mu_{t+1|t} = 0, \quad \forall t, \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial \hat{y}_{t+j}} = \lambda y_{t+j|t} - \mu_t \gamma = 0, \quad \forall t. \tag{10}$$

The discretionary solution to the demand management problem is given by

$$\hat{y}_{t+j|t} = (1-\theta)\hat{y}_{t+j+1|t} - \frac{\gamma}{\lambda}\pi_{t+j+n|t}. \tag{11}$$

The solution implies that the expected j-period-ahead future output gap is set as a function of the expected future $j+1$-period-ahead output gap and the inflation forecast $n+j$-periods ahead. It gives rise to the following proposition.

Proposition 3.4. ***Direction of Discretionary Strategy.***
The optimal discretionary inflation-targeting strategy is as forward-looking as the private sector pricing strategy is backward-looking.

The closed-form solution of (11) is given by

$$\hat{y}_{t+j|t} = -\frac{\gamma}{\lambda}\sum_{i=0}^{\infty} \delta_d^i \pi_{t+j+n+i|t}, \tag{12}$$

where $\delta_d = (1-\theta)$. The discount factor is negatively related to the degree to which private sector price-setters are forward-looking. There is no role for policy inertia. If price-setters are entirely backward-looking, the discretionary solution is identical to the commitment solution in equation (7). In the forward-looking case, the central bank sets the output gap according to the rule

$$\hat{y}_{t+j|t} = -\frac{\gamma}{\lambda}\pi_{t+j+n|t},$$

and the central bank should only consider forecasts of inflation to the extent that there are implementation or reaction lags. Note that Propositions 3.2 and 3.3 also hold for the discretionary optimizing case, irrespective of how forward-looking private sector price setters are.

4. Concluding remarks

This paper shows that private sector price setting determines the degree to which an inflation-targeting central bank should have a forward-looking or a history-dependent policy. It is shown that optimal monetary policy is equivalent to responding to the

discounted and forward accumulated inflation forecast deviations from target. Moreover, policy should be history-dependent, except if pricing decisions are completely backward-looking. Implementation and reaction lags introduce a separate role for using inflation forecasts in policymaking, since policy then only will have an effect in the future.

References

Clarida, Richard, Jordi Galí, and Mark Gertler (1999). "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature*, 37:4:1661–1707.

Gali, Jordi and Mark Gertler (1999). "Inflation Dynamics: A Structural Econometric Analysis." *Journal of Monetary Economics*, 44:195–222.

Giannoni, Marc P. and Michael Woodford (2001). "Optimal Interest-Rate Rules." Manuscript Princeton University.

Jensen, Christian and Bennett T. McCallum (2002). "The Non-Optimality of Proposed Monetary Policy Rules Under Timeless-Perspective Commitment." NBER Working Paper No. w8882.

McCallum, Bennett T. and Edward Nelson (2000). "Timeless Perspective vs Discretionary Monetary Policy in Forward-Looking Models." NBER Working Paper 7915.

Pesaran, M. Hashem (1987). *The Limits to Rational Expectations.* Basil Blackwell, New York, USA.

Roberts, John M. (1995). "New Keynesian Economics and the Phillips Curve." *Journal of Money, Credit and Banking*, 27:4, Part 1:975–84.

— (1997). "Is Inflation Sticky?" *Journal of Monetary Economics*, 39(2):173–96.

Røisland, Øistein (2000). "Rules and Institutional Arrangements for Monetary Policy." Doctoral dissertation. Department of Economics, University of Oslo.

Svensson, Lars E.O. (1997). "Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets." *European Economic Review*, 41:1111–1146.

— (2001). "What is Wrong with Taylor Rules? Using Judgment in Monetary Policy Through Targeting Rules." Manuscript Princeton University.

Svensson, Lars E.O. and Michael Woodford (1999a). "Implementing Optimal Policy Through Inflation-Forecast Targeting." Manuscript, Institute for International Economic Studies, Stockholm University.

— (1999b). "Implementing Optimal Policy Through Inflation-Forecast Targeting." Working paper, Department of Economics, Princeton University.

Woodford, Michael (1999a). "Commentary: How Should Monetary Policy Be Conducted in an Era of Price Stability?" In "New Challenges for Monetary Policy," Federal Reserve Bank of Kansas City, Jackson Hole, Wyoming.

— (1999b). "Optimal Monetary Policy Inertia." Working Paper, Princeton University.

— (2002). *Interest and Prices: Foundations of a Theory of Monetary Policy.* In preparation for Princeton University Press. Copyright Princeton University Press, 2002, Princeton.

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