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Assessing estimates of the exchange rate pass-through

by

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Assessing estimates of the exchange rate pass-through

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Abstract

This paper uses Monte Carlo techniques to address the question: are structural VAR estimates of exchange rate pass-through a useful tool to evaluate macroeconomic models of open economies? The data generating process is a small open economy DSGE model with incomplete pass-through. The results suggest that (i) the pass-through estimates obtained from a first-differenced VAR exhibit a systematic downward bias; (ii) by contrast, estimates derived from a low order vector equilibrium correction model are fairly accurate; but (iii) standard cointegration tests have low power to detect the cointegration relations implied by the DSGE model.

JEL classification: C32, C52, F41
Keywords: Exchange rate pass-through, structural VAR, DSGE models, cointegration

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1 INTRODUCTION

A common approach to evaluate dynamic stochastic general equilibrium (DSGE) models is to compare impulse response functions from the DSGE model and impulse responses obtained from identified vector autoregressions (VARs). The VAR responses, which rely only on a minimum set of theoretical restrictions, are interpreted as 'stylised facts' that empirically relevant DSGE models should reproduce. Prominent examples are Rotemberg & Woodford (1997) and Christiano et al. (2005) who estimate the parameters of DSGE models by minimising a measure of the distance between the impulse responses to a monetary policy shock generated by an identified VAR and the responses to the monetary policy shock in the DSGE model. Choudhri et al. (2005) and Faruqee (2006) employ the same strategy to estimate 'new open economy macroeconomics' (NOEM) models with incomplete exchange rate pass-through, defining exchange rate pass-through as the impulse responses of a set of prices (import prices, export prices, producer prices, consumer prices) to a shock to the uncovered interest rate parity (UIP) condition.

Recently, several papers have examined the reliability of the structural VAR approach using Monte Carlo simulations. The basic idea in this literature is to generate artificial data from a DSGE model, construct impulse responses from a VAR estimated on the artificial data and ask whether the VAR recovers the DSGE model's responses. A maintained assumption is that the identification scheme used to identify the structural shocks in the VAR is consistent with the theoretical model. Chari et al. (2005), Erceg et al. (2005) and Christiano et al. (2006) assess the ability of a structural VAR to recover the impulse responses to a technology shock in a real business cycle (RBC) model. Their conclusions are not unanimous. Chari et al. (2005) conclude that a very large number of lags is needed for the VAR to well approximate their log-linearised RBC model. Erceg et al. (2005) find that, while the VAR responses have the same sign and shape as the true responses, quantitatively, the bias in the estimated responses could be considerable. Christiano et al. (2006) reach a more optimistic conclusion. They find that the VAR does a good job in recovering the responses from the RBC model, particularly if the technology shock is identified using short-run restrictions. Kapetanios et al. (2007) estimate a five variable VAR on data generated from a small open economy model and derive the impulse responses to shocks to productivity, monetary policy, foreign demand, fiscal policy and the risk premium. Their results suggest that the ability of the VAR to reproduce the theoretical shock responses varies across shocks. In particular, a high lag-order is required for the VAR to recover the responses to a risk premium shock and a domestic fiscal shock.

My paper extends this literature to assess the reliability of the structural VAR approach to estimating exchange rate pass-through. The motivating question is: are impulse responses of prices to a UIP shock a useful tool to evaluate and estimate DSGE models with incomplete exchange rate pass-through? To address this question I generate a large number of artificial datasets from a small open economy DSGE model, estimate a VAR on the artificial data and compare the responses of prices to a UIP shock in the VAR and the DSGE model. The DSGE model that serves as the data generating process incorporates many of the mechanisms for generating imperfect pass-through that have been proposed in the NOEM literature, including local currency price stickiness and distribution costs. In addition, the model incorporates mechanisms such as habit formation in consumption and structural inflation persistence that have been found to improve the empirical fit of monetary DSGE models.

The specification of the DSGE model implies that the nominal exchange rate and nominal prices are non-stationary unit root processes, but that relative prices and the real exchange rate are stationary. Given that exchange rate pass-through is usually defined in terms of the levels of prices and the nominal exchange rate, a conjecture is that the magnitude of the bias in the estimated VAR responses will depend on whether the correct cointegration rank has been imposed during estimation. To test this conjecture I compare the performance of two different VAR specifications: a pure first-differenced VAR and a VAR that includes the cointegration relations implied by the DSGE model. The first-differenced specification is by far the most common in the structural VAR literature on exchange rate pass-through.[1] As a second exercise, I investigate whether an econometrician would be able to infer the true cointegration rank and identify the cointegration relations using the maximum likelihood framework of Johansen (1988). My findings can be summarised as follows. The estimates of exchange rate pass-through obtained from a VAR estimated in first differences are biased downwards. This is true even when the VAR is specified with a large number of lags. The bias is attributable to the fact that the finite-order VAR in first differences is not a good approximation to the infinite order VAR implied by the DSGE model. By contrast, a low order vector equilibrium-correction model (VEqCM) that includes the cointegration relations implied by the DSGE model does a good job recovering the theoretical impulse responses. However, the results from the cointegration analysis raise doubts about whether, in practice, an econometrician would be able to infer the cointegration properties implied by the DSGE model.

The paper is organised as follows. Section 2 lays out the DSGE model that serves as the data generating process in the Monte Carlo exercise. Section 3 discusses the mapping from the DSGE model to a VAR, and the results of the simulation experiments are presented in section 4. Section 5 concludes the paper.

2 THE MODEL ECONOMY

This section presents the small open economy DSGE model that is used as the data generating process in the simulation experiments.

2.1 Firms

The production structure is the same as that considered by Choudhri et al. (2005). The home economy produces two goods: a non-tradable final consumption good and a tradable intermediate good. Firms in both sectors use domestic labour and a basket of domestic and imported intermediate goods as inputs. The assumption that imports do not enter directly in the consumption basket of households is consistent with the notion that all goods in the consumer price index contain a significant non-traded component. It follows that the direct effect of import prices on consumer prices will be muted, and this acts to limit the degree of exchange rate pass-through to consumer prices. The assumption that imported goods are used as inputs in the production of domestic goods implies a direct link between import prices and the production costs of domestic firms. The latter is potentially an important transmission channel for exchange rate changes in a small open economy (see e.g., McCallum & Nelson, 2000).

2.1.1 Final goods firms

Technology and factor demand There is a continuum of firms indexed by $c \in [0,1]$ that produces differentiated non-tradable final consumption goods. The market for final goods is characterised by monopolistic competition. The consumption good is produced using the following Cobb-Douglas technology

$$C_t(c) = Q_t(c)^{\gamma_c} H_t^c(c)^{1-\gamma_c}, \tag{1}$$

where $C_t(c)$ is the output of final good variety c at time t, $Q_t(c)$ and $H_t^c(c)$ are, respectively, the amounts of intermediate goods and labour used in the production of final good c and $\gamma_c \in [0,1]$. The aggregate labour index H_t is a constant elasticity of substitution (CES) aggregate of differentiated labour inputs indexed by $j \in [0,1]$

$$H_t \equiv \left[\int_0^1 H_t(j)^{\frac{\theta^h-1}{\theta^h}} dj \right]^{\frac{\theta^h}{\theta^h-1}}, \tag{2}$$

[1] See e.g., McCarthy (2000), Hahn (2003), Choudhri et al. (2005), and Faruqee (2006).

where $\theta^h > 1$ is the elasticity of substitution between labour types. Q_t is a composite of imported and domestically produced intermediate goods

$$Q_t \equiv \left[\alpha^{\frac{1}{\nu}} \left(Q_t^d\right)^{\frac{\nu-1}{\nu}} + (1-\alpha)^{\frac{1}{\nu}} \left(Q_t^m\right)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, \tag{3}$$

where $\alpha \in [0,1]$ and $\nu > 0$ denotes the elasticity of substitution between domestic and imported goods. Q_t^d and Q_t^m are quantity indices of differentiated domestic and foreign intermediate goods indexed by $i \in [0,1]$ and $m \in [0,1]$, respectively:[2]

$$Q_t^d \equiv \left[\int_0^1 Y_t^{dq}(i)^{\frac{\theta_t^y-1}{\theta_t^y}} di\right]^{\frac{\theta_t^y}{\theta_t^y-1}} \tag{4}$$

$$Q_t^m \equiv \left[\int_0^1 Y_t^{mq}(m)^{\frac{\theta_t^m-1}{\theta_t^m}} dm\right]^{\frac{\theta_t^m}{\theta_t^m-1}}, \tag{5}$$

where $Y_t^{dq}(i)$ and $Y_t^{mq}(m)$ denote the quantities of individual domestic and imported intermediate goods, respectively, used in the production of domestic final goods. The elasticities of substitution between varieties of domestic and imported intermediate goods in the domestic market are $\theta_t^y > 1$ and $\theta_t^m > 1$, respectively. Following e.g., Smets & Wouters (2003) and Adolfson et al. (2007), the substitution elasticities are assumed to be time-varying.

Final goods firms take the prices of intermediate goods and labour inputs as given. Cost minimisation implies that the demands for individual varieties of intermediate goods are

$$Y_t^{dq}(i) = \left(\frac{P_t^y(i)}{P_t^y}\right)^{-\theta_t^y} Q_t^d \tag{6}$$

$$Y_t^{mq}(m) = \left(\frac{P_t^m(m)}{P_t^m}\right)^{-\theta_t^m} Q_t^m. \tag{7}$$

The CES preference specification thus implies that the elasticities of substitution are equal to the elasticities of demand for individual goods. The price indices P_t^y and P_t^m are defined as

$$P_t^y \equiv \left[\int_0^1 P_t^y(i)^{1-\theta_t^y} di\right]^{\frac{1}{1-\theta_t^y}} \tag{8}$$

$$P_t^m \equiv \left[\int_0^1 P_t^m(m)^{1-\theta_t^m} dm\right]^{\frac{1}{1-\theta_t^m}}. \tag{9}$$

The price index for the composite intermediate good is

$$P_t^q \equiv \left[\alpha \left(P_t^y\right)^{1-\nu} + (1-\alpha)\left(P_t^m\right)^{1-\nu}\right]^{\frac{1}{1-\nu}}. \tag{10}$$

The demand for labour input j is

$$H_t^c(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\theta^h} H_t^c, \tag{11}$$

where $W_t(j)$ is the nominal wage paid to labour input j, and W_t is the aggregate wage index defined

[2] See table 1 for a schematic overview of the price and quantity indices in the DSGE model.

as

$$W_t \equiv \left[\int_0^1 W_t(j)^{1-\theta^h} dj\right]^{\frac{1}{1-\theta^h}}. \tag{12}$$

Aggregating over firms and using the fact that all firms are identical, the cost minimising choices of H_t^c and Q_t are characterised by

$$\begin{aligned} W_t &= \xi_t^c (1-\gamma_c)\frac{C_t}{H_t^c} && (13)\\ P_t^q &= \xi_t^c \gamma_c \frac{C_t}{Q_t}, && (14) \end{aligned}$$

where ξ_t^c denotes the nominal marginal costs of final goods firms. The marginal costs can be expressed as

$$\xi_t^c = \frac{W_t^{1-\gamma_c}(P_t^q)^{\gamma_c}}{(1-\gamma_c)^{1-\gamma_c}\gamma_c^{\gamma_c}}. \tag{15}$$

Finally, the final goods firms' demands for the composite imported and domestic intermediate goods are

$$\begin{aligned} Q_t^d &= \alpha\left(\frac{P_t^y}{P_t^q}\right)^{-\nu} Q_t && (16)\\ Q_t^m &= (1-\alpha)\left(\frac{P_t^m}{P_t^q}\right)^{-\nu} Q_t && (17) \end{aligned}$$

Price setting The aggregate consumption index is defined as

$$C_t \equiv \left[\int_0^1 C_t(c)^{\frac{\theta_t^c-1}{\theta_t^c}} dc\right]^{\frac{\theta_t^c}{\theta_t^c-1}}, \tag{18}$$

where $\theta_t^c > 1$ is the time-varying elasticity of substitution between individual consumption goods. The corresponding ideal price index is

$$P_t^c \equiv \left[\int_0^1 P_t^c(c)^{1-\theta_t^c} dc\right]^{\frac{1}{1-\theta_t^c}}, \tag{19}$$

and the demand for a single variety of the consumption good is

$$C_t(c) = \left(\frac{P_t^c(c)}{P_t^c}\right)^{-\theta_t^c} C_t. \tag{20}$$

Nominal price stickiness is modelled using the quadratic adjustment cost framework of Rotemberg (1982).[3] Following e.g., Price (1992), Ireland (2001) and Laxton & Pesenti (2003), I assume that there are costs associated with changing the inflation rate relative to past observed inflation. Specifically, adjustment costs are given by:

$$\Upsilon_{t+l}^c(c) \equiv \frac{\phi_c}{2}\left(\frac{P_{t+l}^c(c)/P_{t+l-1}^c(c)}{P_{t+l-1}^c/P_{t+l-2}^c} - 1\right)^2, \tag{21}$$

where $\phi_c >$ is an adjustment cost parameter.

Since all firms in the economy are owned by households, future profits are valued according to

[3]The list of NOEM papers which model price stickiness by assuming quadratic costs of price adjustment includes Bergin (2006), Corsetti et al. (2005), Laxton & Pesenti (2003), and Hunt & Rebucci (2005).

the households' stochastic discount factor $D_{t,t+l}$ (to be defined below). Firms set prices to maximise the expected discounted value of future profits subject to adjustment costs, that is, they maximise

$$E_t\left[\sum_{l=0}^{\infty} D_{t,t+l}\left(P^c_{t+l}(c)-\xi^c_{t+l}\right)\left(\frac{P^c_{t+l}(c)}{P^c_{t+l}}\right)^{-\theta^c_{t+l}} C_{t+l}\left(1-\Upsilon^c_{t+l}(c)\right)\right] \tag{22}$$

subject to (21). In a symmetric equilibrium, $P^c_t(c)=P^c_t$, and the optimal price satisfies

$$\begin{aligned} 0 \quad = \quad & -\left(P^c_t-\xi^c_t\right)\phi_c\left(\frac{\pi^c_t}{\pi^c_{t-1}}-1\right)\frac{\pi^c_t}{\pi^c_{t-1}} \\ & +\left((1-\theta^c_t)P^c_t+\theta^c_t\xi^c_t\right)\left(1-\frac{\phi_c}{2}\left(\frac{\pi^c_t}{\pi^c_{t-1}}-1\right)^2\right) \\ & +E_t\left[\ D_{t,t+1}\left(P^c_{t+1}-\xi^c_{t+1}\right)\tfrac{C_{t+1}}{C_t}\phi_c\left(\tfrac{\pi^c_{t+1}}{\pi^c_t}-1\right)\tfrac{\pi^c_{t+1}}{\pi^c_t}\ \right], \end{aligned} \tag{23}$$

where π^c_t is the gross inflation rate, $\pi^c_t \equiv P^c_t/P^c_{t-1}$. The price-setting rule is forward-looking and balances the costs of deviating from the optimal (frictionless) price and the costs associated with changing the inflation rate. The log-linearised inflation equation implied by this model (see equation A7 in appendix A) can be written as a forward-looking equation in the first difference of inflation. It is observationally equivalent to the inflation equation implied by the Calvo (1983) model when firms index non-optimised prices perfectly to last period's aggregate inflation rate (see e.g., Christiano et al., 2005).

If prices were flexible (i.e., $\phi_c=0$), firms would set prices according to the familiar mark-up rule:

$$P^c_t=\frac{\theta^c_t}{\theta^c_t-1}\xi^c_t. \tag{24}$$

2.1.2 Intermediate goods firms

Technology and factor demand There is a continuum of intermediate goods firms indexed by $i\in[0,1]$ operating in a monopolistically competitive market. The intermediate goods are produced with the following technology

$$Y_t(i)=Z_t(i)^{\gamma_y}H^y_t(i)^{1-\gamma_y}, \tag{25}$$

where $Y_t(i)$ denotes the output of intermediate good i, $\gamma_y\in[0,1]$ and $Z_t(i)$ and $H^y_t(i)$ are, respectively, units of the composite intermediate good and the composite labour index used in the production of variety i of the domestic intermediate good. The composite intermediate good is defined as

$$Z_t\equiv\left[\alpha^{\frac{1}{\nu}}\left(Z^d_t\right)^{\frac{\nu-1}{\nu}}+(1-\alpha)^{\frac{1}{\nu}}\left(Z^m_t\right)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, \tag{26}$$

where Z^d_t and Z^m_t are quantity indices of differentiated domestic and foreign intermediate goods, that is,

$$Z^d_t \equiv \left[\int_0^1 Y^{dz}_t(i)^{\frac{\theta^y_t-1}{\theta^y_t}}di\right]^{\frac{\theta^y_t}{\theta^y_t-1}} \tag{27}$$

$$Z^m_t \equiv \left[\int_0^1 Y^{mz}_t(m)^{\frac{\theta^m_t-1}{\theta^m_t}}dm\right]^{\frac{\theta^m_t}{\theta^m_t-1}}, \tag{28}$$

where $Y^{dz}_t(i)$ and $Y^{mz}_t(m)$ denote the quantities of individual domestic and imported intermediate goods, respectively, used in the production of domestic intermediate goods. The price index for the composite intermediate good, P_t^z, is defined as

$$P_t^z \equiv \left[\alpha \left(P_t^y\right)^{1-\nu} + (1-\alpha)\left(P_t^m\right)^{1-\nu}\right]^{\frac{1}{1-\nu}}. \tag{29}$$

Firms take the prices of the composite intermediate good and labour inputs as given. Cost minimisation with respect to H_t^y and Z_t implies (again using the fact that all intermediate goods firms are identical)

$$\begin{aligned} W_t &= \xi_t^y (1-\gamma_y) \frac{Y_t}{H_t^y} && (30) \\ P_t^z &= \xi_t^y \gamma_y \frac{Y_t}{Z_t}, && (31) \end{aligned}$$

where ξ_t^y denotes nominal marginal costs

$$\xi_t^y = \frac{W_t^{1-\gamma_y} (P_t^z)^{\gamma_y}}{(1-\gamma_y)^{1-\gamma_y} \gamma_y^{\gamma_y}}. \tag{32}$$

Demands for domestic and imported intermediate goods from domestic intermediate goods firms are

$$\begin{aligned} Z_t^d &= \alpha \left(\frac{P_t^y}{P_t^z}\right)^{-\nu} Z_t && (33) \\ Z_t^m &= (1-\alpha) \left(\frac{P_t^m}{P_t^z}\right)^{-\nu} Z_t && (34) \end{aligned}$$

Price setting As pointed out by Obstfeld & Rogoff (2000), there are more possibilities for modelling nominal rigidities in an open-economy setting than in a closed-economy setting. One issue is whether international goods markets should be characterised as being integrated or segmented. Another issue is that, with nominal price stickiness, the choice of price-setting currency will matter. In the following it is assumed that international goods markets are segmented, due to for example, transportation costs or formal or informal trade barriers. Intermediate goods firms thus have the option to set different prices in the domestic and foreign markets.

Domestic market The demand facing firm i in the domestic market is

$$Y_t^d(i) = \left(\frac{P_t^y(i)}{P_t^y}\right)^{-\theta_t^y} Y_t^d, \tag{35}$$

where $Y_t^d = Q_t^d + Z_t^d$ is the total demand for domestic intermediate goods from domestic firms. Firm i's price setting problem in the domestic market is

$$\max_{P_t^y(i)} E_t \left[\sum_{l=0}^{\infty} D_{t,t+l} \left(P_{t+l}^y(i) - \xi_{t+l}^y\right) \left(\frac{P_{t+l}^y(i)}{P_{t+l}^y}\right)^{-\theta_{t+l}^y} Y_{t+l}^d \left(1 - \Upsilon_{t+l}^y(i)\right)\right], \tag{36}$$

where the form of adjustment costs $\Upsilon_{t+l}^y(i)$ is

$$\Upsilon_{t+l}^y(i) \equiv \frac{\phi_y}{2} \left(\frac{P_{t+l}^y(i)/P_{t+l-1}^y(i)}{P_{t+l-1}^y/P_{t+l-2}^y} - 1\right)^2. \tag{37}$$

In equilibrium, $P_t^y(i) = P_t^y$, and the optimal price satisfies

$$\begin{aligned} 0 &= -\left(P_t^y - \xi_t^y\right)\phi_y\left(\frac{\pi_t^y}{\pi_{t-1}^y} - 1\right)\frac{\pi_t^y}{\pi_{t-1}^y} \\ &\quad + ((1-\theta_t^y)P_t^y + \theta_t^y\xi_t^y)\left(1 - \frac{\phi_y}{2}\left(\frac{\pi_t^y}{\pi_{t-1}^y} - 1\right)^2\right) \\ &\quad + E_t\left[\; D_{t,t+1}\left(P_{t+1}^y - \xi_{t+1}^y\right)\frac{Y_{t+1}^d}{Y_t^d}\phi_y\left(\frac{\pi_{t+1}^y}{\pi_t^y} - 1\right)\frac{\pi_{t+1}^y}{\pi_t^y}\;\right], \end{aligned} \tag{38}$$

where $\pi_t^y \equiv P_t^y / P_{t-1}^y$ is the gross inflation rate.

Foreign market In the Obstfeld & Rogoff (1995) Redux model, international goods markets are integrated, and the law of one price holds continuously. Moreover, because prices are set in the currency of the producer (so-called producer currency pricing, PCP), exchange rate pass-through to import prices is immediate and complete. Betts & Devereux (1996) extended the Redux model to allow for market segmentation and to allow a share of prices to be sticky in the currency of the buyer (so-called local currency pricing, LCP). Local currency pricing implies that import prices will respond only gradually to exchange rate changes, a feature consistent with the findings of a large empirical literature on exchange rate pass-through.[4] In this paper, following Choudhri et al. (2005) and Bergin (2006), I assume that a proportion ϖ of domestic intermediate goods firms engages in PCP, and a proportion $1-\varpi$ engages in LCP. Both PCP and LCP firms have the option to price discriminate between foreign and domestic markets.[5]

Corsetti & Dedola (2005) extended the basic NOEM framework to allow for distribution costs. In their model, the distribution of traded goods requires the input of local, non-traded goods and services. Here, following Choudhri et al. (2005), I assume that the distribution of one unit of the domestic traded good to foreign firms requires the input of δ_f units of foreign labour. The distribution sector is perfectly competitive. Let $\overline{P}_t^{xp}(i)$ and $\overline{P}_t^{xl}(i)$ be the ('wholesale') prices set by a representative PCP firm and LCP firm, respectively.[6] The Leontief production technology and the zero profit condition in the distribution sector imply that the ('retail') prices paid by foreign firms for a type i domestic good, $P_t^{xp}(i)$ and $P_t^{xl}(i)$, satisfy

$$\frac{P_t^{xp}(i)}{S_t} = \frac{\overline{P}_t^{xp}(i)}{S_t} + \delta_f W_t^f \tag{39}$$

$$P_t^{xl}(i) = \overline{P}_t^{xl}(i) + \delta_f W_t^f, \tag{40}$$

where S_t is the nominal exchange rate and W_t^f is the foreign wage level. The existence of a distribution sector thus implies that there will be a wedge between the wholesale and the retail price of imports in the foreign economy.

The aggregate export price index (in domestic currency) is

$$P_t^x \equiv \left[\varpi\left(P_t^{xp}\right)^{1-\theta_t^x} + (1-\varpi)(S_t P_t^{xl})^{1-\theta_t^x}\right]^{\frac{1}{1-\theta_t^x}}, \tag{41}$$

where P_t^{xp} and P_t^{xl} are the export price indices obtained by aggregating over PCP firms and LCP

[4] See Campa & Goldberg (2005) for a recent study.

[5] In this paper ϖ is treated as an exogenous parameter. Several recent papers have examined the optimal choice of invoicing currency in the context of NOEM models (e.g., Devereux et al., 2004; Bacchetta & van Wincoop, 2005; Goldberg & Tille, 2005). The choice is found to depend on several factors, including the exporting firm's market share in the foreign market, the degree of substitutability between foreign and domestic goods and relative monetary stability.

[6] The wholesale export prices correspond to the export prices 'at the docks'.

firms, respectively, that is

$$P_t^{xp} \equiv \left[\frac{1}{\varpi}\int_0^{\varpi} P_t^{xp}(i)^{1-\theta_t^x} di\right]^{\frac{1}{1-\theta_t^x}} \tag{42}$$

$$P_t^{xl} \equiv \left[\frac{1}{1-\varpi}\int_{\varpi}^{1} P_t^{xl}(i)^{1-\theta_t^x} di\right]^{\frac{1}{1-\theta_t^x}}, \tag{43}$$

and $\theta_t^x > 1$ denotes the elasticity of substitution between domestic intermediate goods in the foreign economy. The corresponding quantity indices are

$$Y_t^x \equiv \left[(\varpi)^{\frac{1}{\theta_t^x}} \left(Y_t^{xp}\right)^{\frac{\theta_t^x-1}{\theta_t^x}} + (1-\varpi)^{\frac{1}{\theta_t^x}} \left(Y_t^{xl}\right)^{\frac{\theta_t^x-1}{\theta_t^x}}\right]^{\frac{\theta_t^x}{\theta_t^x-1}} \tag{44}$$

and

$$Y_t^{xp} \equiv \left[\left(\frac{1}{\varpi}\right)^{\frac{1}{\theta_t^x}} \int_0^{\varpi} Y_t^{xp}(x)^{\frac{\theta_t^x-1}{\theta_t^x}} di\right]^{\frac{\theta_t^x}{\theta_t^x-1}} \tag{45}$$

$$Y_t^{xl} \equiv \left[\left(\frac{1}{1-\varpi}\right)^{\frac{1}{\theta_t^x}} \int_{\varpi}^{1} Y_t^{xl}(x)^{\frac{\theta_t^x-1}{\theta_t^x}} di\right]^{\frac{\theta_t^x}{\theta_t^x-1}} \tag{46}$$

A representative PCP firm sets $\overline{P}_t^{xp}(i)$ to maximise

$$E_t\left[\sum_{l=0}^{\infty} D_{t,t+l}\left(\overline{P}_{t+l}^{xp}(i) - \xi_{t+l}^{y}\right) Y_{t+l}^{xp}(i)\left(1-\Upsilon_{t+l}^{xp}(i)\right)\right] \tag{47}$$

subject to demand[7]

$$Y_{t+l}^{xp}(i) = \left(\frac{\overline{P}_{t+l}^{xp}(i)/S_{t+l} + \delta_f W_{t+l}^{f}}{P_{t+l}^{x}/S_{t+l}}\right)^{-\theta_{t+l}^{x}} Y_{t+l}^{x}, \tag{48}$$

and adjustment costs

$$\Upsilon_{t+l}^{xp}(i) \equiv \frac{\phi_x}{2}\left(\frac{\overline{P}_{t+l}^{xp}(i)/\overline{P}_{t+l-1}^{xp}(i)}{\overline{P}_{t+l-1}^{xp}/\overline{P}_{t+l-2}^{xp}} - 1\right)^2. \tag{49}$$

In equilibrium, $\overline{P}_t^{xp}(i) = \overline{P}_t^{xp}$, and the optimal price satisfies

$$\begin{aligned} 0 \;=\; & \left(\overline{P}_t^{xp} - \theta_t^x\left(\overline{P}_t^{xp} - \xi_t^y\right)\frac{\overline{P}_t^{xp}}{P_t^{xp}}\right)\left(1 - \frac{\phi_x}{2}\left(\frac{\overline{\pi}_t^{xp}}{\overline{\pi}_{t-1}^{xp}} - 1\right)^2\right) \\ & -\left(\overline{P}_t^{xp} - \xi_t^y\right)\phi_x\left(\frac{\overline{\pi}_t^{xp}}{\overline{\pi}_{t-1}^{xp}} - 1\right)\frac{\overline{\pi}_t^{xp}}{\overline{\pi}_{t-1}^{xp}} \\ & + E_t\left[\; D_{t,t+1}\left(P_{t+1}^{xp} - \xi_{t+1}^{y}\right)\frac{Y_{t+1}^{xp}}{Y_t^{xp}}\phi_x\left(\frac{\overline{\pi}_{t+1}^{xp}}{\overline{\pi}_t^{xp}} - 1\right)\frac{\overline{\pi}_{t+1}^{xp}}{\overline{\pi}_t^{xp}},\;\right] \end{aligned} \tag{50}$$

where $\overline{\pi}_t^{xp} \equiv \overline{P}_t^{xp}/\overline{P}_{t-1}^{xp}$.

The wedge between prices at the wholesale and retail levels implies that the price elasticity of

[7] This can be derived from

$$Y_{t+l}^{xp}(i) = \frac{1}{\varpi}\left(\frac{P_{t+l}^{xp}(i)}{P_{t+l}^{xp}}\right)^{-\theta_{t+l}^{x}} Y_{t+l}^{xp} = \left(\frac{P_{t+l}^{xp}(i)}{P_{t+l}^{x}}\right)^{-\theta_{t+l}^{x}} Y_{t+l}^{x}$$

demand as perceived by the exporter will be a function of the exchange rate. To see this, note that in the absence of costs of price adjustment (i.e., if $\phi_x = 0$), the optimal export price is

$$\overline{P}_t^{xp} = \frac{\theta_t^x}{\theta_t^x - 1}\xi_t^y + \frac{\delta_f}{\theta_t^x - 1} S_t W_t^f. \tag{51}$$

In the absence of distribution costs ($\delta_f = 0$), the export price in domestic currency is independent of the exchange rate, and the price-setting rule collapses to the standard mark-up rule. Moreover, if the elasticities of demand are the same across countries (i.e., $\theta_t^x = \theta_t^y$), the firm sets identical prices to the home and foreign markets. The existence of distribution costs thus creates a motive for price discrimination between markets. Moreover, distribution costs cause the optimal mark-up to vary positively with the level of the exchange rate. This can be seen more clearly by rewriting (51) as

$$\overline{P}_t^{xp} = \frac{\theta_t^x}{\theta_t^x - 1}\xi_t^y \left(1 + \frac{\delta_f}{\theta_t^x}\frac{S_t W_t^f}{\xi_t^y}\right). \tag{52}$$

In the face of an exchange rate depreciation, the exporter will find it optimal to absorb part of the exchange rate movement in her mark-up. From the point of view of the importing country, exchange rate pass-through to import prices at the docks is incomplete, even in the absence of nominal rigidities.

A representative LCP firm sets $\overline{P}_t^{xl}(i)$ to maximise

$$\max_{\overline{P}_t^{xl}(i)} E_t \left[\sum_{l=0}^{\infty} D_{t,t+l}\left(S_{t+l}\overline{P}_{t+l}^{xl}(i) - \xi_{t+l}^y\right) Y_{t+l}^{xl}(i)\left(1 - \Upsilon_{t+l}^{xl}(i)\right)\right] \tag{53}$$

subject to demand,[8]

$$Y_{t+l}^{xl}(i) = \left(\frac{\overline{P}_{t+l}^{xl}(i) + \delta_f W_{t+l}^f}{P_{t+l}^x / S_{t+l}}\right)^{-\theta_{t+l}^x} Y_{t+l}^x, \tag{54}$$

and adjustment costs

$$\Upsilon_{t+l}^{xl}(i) \equiv \frac{\phi_x}{2}\left(\frac{\overline{P}_{t+l}^{xl}(i)/\overline{P}_{t+l-1}^{xl}(i)}{\overline{P}_{t+l-1}^{xl}/\overline{P}_{t+l-2}^{xl}} - 1\right)^2. \tag{55}$$

The degree of price stickiness as measured by the parameter ϕ_x is thus assumed to be the same for PCP and LCP firms.[9] In equilibrium $\overline{P}_t^{xl}(i) = \overline{P}_t^{xl}$, and the first-order condition can be written

$$\begin{aligned} 0 \quad = \quad & \left(S_t\overline{P}_t^{xl} - \theta_t^x\left(S_t\overline{P}_t^{xl} - \xi_t^y\right)\frac{\overline{P}_t^{xl}}{P_t^{xl}}\right)\left(1 - \frac{\phi_x}{2}\left(\frac{\pi_t^{xl}}{\pi_{t-1}^{xl}} - 1\right)^2\right) \\ & - \left(S_t\overline{P}_t^{xl} - \xi_t^y\right)\phi_x\left(\frac{\pi_t^{xl}}{\pi_{t-1}^{xl}} - 1\right)\frac{\pi_t^{xl}}{\pi_{t-1}^{xl}} \\ & + E_t\left[\; D_{t,t+1}\left(S_{t+1}P_{t+1}^{xl} - \xi_{t+1}^y\right)\tfrac{Y_{t+1}^{xl}}{Y_t^{xl}}\phi_x\left(\tfrac{\pi_{t+1}^{xl}}{\pi_t^{xl}} - 1\right)\tfrac{\pi_{t+1}^{xl}}{\pi_t^{xl}} \;\right], \end{aligned} \tag{56}$$

[8]This follows from

$$Y_{t+l}^{xl}(i) = \frac{1}{1-\varpi}\left(\frac{P_{t+l}^{xl}(i)}{P_{t+l}^{xl}}\right)^{-\theta_{t+l}^x} Y_{t+l}^{xl} = \left(\frac{P_{t+l}^{xl}(i)}{P_{t+l}^x/S_{t+l}}\right)^{-\theta_{t+l}^x} Y_{t+l}^x$$

[9]This assumption has some empirical support. Using micro data for traded goods prices at the docks for the US, Gopinath & Rigobon (2006) find that the stickiness of prices invoiced in foreign currencies in terms of foreign currency is similar to the stickiness of prices invoiced in dollars in terms of dollars.

where $\overline{\pi}_t^{xl} \equiv \overline{P}_t^{xl}/\overline{P}_{t-1}^{xl}$. In the absence of adjustment costs (i.e., if $\phi_x = 0$), the optimal price is

$$\overline{P}_t^{xl} = \frac{\theta_t^x}{\theta_t^x - 1}\frac{\xi_t^y}{S_t} + \frac{\delta_f}{\theta_t^x - 1}W_t^f. \tag{57}$$

Thus, when prices are flexible, LCP and PCP firms set the same price. The choice of price-setting currency only matters in a situation where nominal prices are sticky.

Finally, aggregate export demand is assumed to be given by

$$Y_t^x = \alpha_f \left(\frac{P_t^x/S_t}{P_t^f}\right)^{-\nu_f} Y_t^f, \tag{58}$$

where α_f is (approximately) the share of home goods and ν_f the elasticity of substitution between home and foreign goods in the composite index of intermediate goods in the foreign economy, P_t^f is the foreign price level, and Y_t^f denotes aggregate demand for domestic intermediate goods in the foreign economy. Foreign output, prices and wages are assumed to exogenous to the small open economy.

2.1.3 Foreign firms

Foreign intermediate goods firms are treated symmetrically with domestic intermediate goods firms. A subset ϖ_f of firms engages in PCP, and a subset $1 - \varpi_f$ engages in LCP. The aggregate import quantity index is

$$Y_t^m \equiv \left[(\varpi_f)^{\frac{1}{\theta_t^m}} \left(Y_t^{mp}\right)^{\frac{\theta_t^m - 1}{\theta_t^m}} + (1 - \varpi_f)^{\frac{1}{\theta_t^m}} \left(Y_t^{ml}\right)^{\frac{\theta_t^m - 1}{\theta_t^m}}\right]^{\frac{\theta_t^m}{\theta_t^m - 1}}, \tag{59}$$

where Y_t^{mp} and Y_t^{ml} are, respectively, the production indices of PCP firms and LCP firms, defined as

$$Y_t^{mp} \equiv \left[\left(\frac{1}{\varpi_f}\right)^{\frac{1}{\theta_t^m}} \int_0^{\varpi_f} Y_t^{mp}(m)^{\frac{\theta_t^m - 1}{\theta_t^m}} dm\right]^{\frac{\theta_t^m}{\theta_t^m - 1}} \tag{60}$$

$$Y_t^{ml} \equiv \left[\left(\frac{1}{1 - \varpi_f}\right)^{\frac{1}{\theta_t^m}} \int_{\varpi_f}^{1} Y_t^{ml}(m)^{\frac{\theta_t^m - 1}{\theta_t^m}} dm\right]^{\frac{\theta_t^m}{\theta_t^m - 1}}. \tag{61}$$

The distribution of one unit of the imported good to domestic firms requires the input of δ units of domestic labour. The zero profit condition in the distribution sector implies that the prices paid by domestic firms for a type m imported good, $P_t^{mp}(m)$ and $P_t^{ml}(m)$, will be

$$S_t P_t^{mp}(m) = S_t \overline{P}_t^{mp}(m) + \delta W_t \tag{62}$$

$$P_t^{ml}(m) = \overline{P}_t^{ml}(m) + \delta W_t. \tag{63}$$

The aggregate import price index (in the importing country's currency) is

$$P_t^m \equiv \left[\varpi_f \left(S_t P_t^{mp}\right)^{1-\theta_t^m} + (1 - \varpi_f)(P_t^{ml})^{1-\theta_t^m}\right]^{\frac{1}{1-\theta_t^m}}, \tag{64}$$

where P_t^{mp} and P_t^{ml} are the price indices obtained by aggregating over PCP firms and LCP firms, respectively, that is

$$P_t^{mp} \equiv \left[\frac{1}{\varpi_f}\int_0^{\varpi_f} P_t^{mp}(m)^{1-\theta_t^m} dm\right]^{\frac{1}{1-\theta_t^m}} \quad (65)$$

$$P_t^{ml} \equiv \left[\frac{1}{1-\varpi_f}\int_{\varpi_f}^{1} P_t^{ml}(m)^{1-\theta_t^m} dm\right]^{\frac{1}{1-\theta_t^m}}. \quad (66)$$

Let $D_{t,t+l}^{f}$ denote the stochastic discount factor of foreign households and let ξ_t^f denote the marginal costs of foreign intermediate goods firms. A representative foreign LCP firm sets $\overline{P}_t^{ml}(m)$ to maximise

$$E_t\left[\sum_{l=0}^{\infty} D_{t,t+l}^{f}\left(\frac{\overline{P}_{t+l}^{ml}(m)}{S_{t+l}} - \xi_{t+l}^{f}\right) Y_{t+l}^{ml}(m)\left(1-\Upsilon_{t+l}^{ml}(m)\right)\right] \quad (67)$$

subject to demand[10]

$$Y_{t+l}^{ml}(m) = \left(\frac{\overline{P}_{t+l}^{ml}(m) + \delta W_{t+l}}{P_{t+l}^{m}}\right)^{-\theta_{t+l}^{m}} Y_{t+l}^{m}, \quad (68)$$

where $Y_t^m = Q_t^m + Z_t^m$ is the aggregate demand for imported intermediate goods from domestic firms. The specification of adjustment costs is

$$\Upsilon_{t+l}^{ml}(m) \equiv \frac{\phi_m}{2}\left(\frac{\overline{P}_{t+l}^{ml}(m)/\overline{P}_{t+l-1}^{ml}(m)}{\overline{P}_{t+l-1}^{ml}/\overline{P}_{t+l-2}^{ml}} - 1\right)^2. \quad (69)$$

In equilibrium, $\overline{P}_t^{ml} = \overline{P}_t^{ml}(m)$, and the optimal price satisfies

$$\begin{aligned} 0 = {} & \left(\frac{\overline{P}_t^{ml}}{S_t} - \theta_t^m\left(\frac{\overline{P}_t^{ml}}{S_t} - \xi_t^f\right)\frac{\overline{P}_t^{ml}}{P_t^{ml}}\right)\left(1 - \frac{\phi_m}{2}\left(\frac{\pi_t^{ml}}{\pi_{t-1}^{ml}} - 1\right)^2\right) \quad (70) \\ & - \left(\frac{\overline{P}_t^{ml}}{S_t} - \xi_t^f\right)\phi_m\left(\frac{\pi_t^{ml}}{\pi_{t-1}^{ml}} - 1\right)\frac{\pi_t^{ml}}{\pi_{t-1}^{ml}} \\ & + E_t\left[D_{t,t+1}^{f}\left(\frac{\overline{P}_{t+1}^{ml}}{S_{t+1}} - \xi_{t+1}^{f}\right)\frac{Y_{t+1}^{ml}}{Y_t^{ml}}\phi_m\left(\frac{\pi_{t+1}^{ml}}{\pi_t^{ml}} - 1\right)\frac{\pi_{t+1}^{ml}}{\pi_t^{ml}}\right], \end{aligned}$$

where $\pi_t^{ml} \equiv \overline{P}_t^{ml}/\overline{P}_{t-1}^{ml}$. If $\phi_m = 0$, the first-order condition simplifies to

$$\overline{P}_t^{ml} = \frac{\theta_t^m}{\theta_t^m - 1} S_t \xi_t^f + \frac{\delta}{\theta_t^m - 1} W_t \quad (71)$$

The foreign firm's optimal mark-up is a function of the exchange rate. Conditional on domestic wages and the marginal costs of foreign exporters, the exchange rate pass-through to domestic currency import prices at the wholesale level is incomplete, even if prices are perfectly flexible.

[10] This follows from

$$Y_{t+l}^{ml}(m) = \frac{1}{1-\varpi_f}\left(\frac{P_{t+l}^{ml}(m)}{P_{t+l}^{ml}}\right)^{-\theta_{t+l}^{m}} Y_{t+l}^{ml} = \left(\frac{P_{t+l}^{ml}(m)}{P_{t+l}^{m}}\right)^{-\theta_{t+l}^{m}} Y_{t+l}^{m}$$

Finally, a representative foreign PCP firm sets $\overline{P}_t^{mp}(m)$ to maximise

$$E_t\left[\sum_{l=0}^{\infty} D_{t,t+l}^{f}\left(\overline{P}_{t+l}^{mp}(m)-\xi_{t+l}^{f}\right)Y_{t+l}^{mp}(m)\left(1-\Upsilon_{t+l}^{mp}(m)\right)\right] \tag{72}$$

subject to demand[11]

$$Y_{t+l}^{mp}(m)=\left(\frac{S_{t+l}\overline{P}_{t+l}^{mp}(m)+\delta W_{t+l}}{P_{t+l}^{m}}\right)^{-\theta_{t+l}^{m}}Y_{t+l}^{m} \tag{73}$$

and adjustment costs

$$\Upsilon_{t+l}^{mp}(m)\equiv\frac{\phi_m}{2}\left(\frac{\overline{P}_{t+l}^{mp}(m)/\overline{P}_{t+l-1}^{mp}(m)}{\overline{P}_{t+l-1}^{mp}/\overline{P}_{t+l-2}^{mp}}-1\right)^2. \tag{74}$$

Imposing $\overline{P}_t^{mp}=\overline{P}_t^{mp}(m)$, the first-order condition can be written

$$\begin{aligned}0 \;=\; & \left(\overline{P}_t^{mp}-\theta_t^m\left(\overline{P}_t^{mp}-\xi_t^f\right)\frac{\overline{P}_t^{mp}}{P_t^{mp}}\right)\left(1-\frac{\phi_m}{2}\left(\frac{\overline{\pi}_t^{mp}}{\overline{\pi}_{t-1}^{mp}}-1\right)^2\right) \\ & -\left(\overline{P}_t^{mp}-\xi_t^f\right)\phi_m\left(\frac{\overline{\pi}_t^{mp}}{\overline{\pi}_{t-1}^{mp}}-1\right)\frac{\overline{\pi}_t^{mp}}{\overline{\pi}_{t-1}^{mp}} \\ & +E_t\left[\;D_{t,t+1}^{f}\left(\overline{P}_{t+1}^{mp}-\xi_{t+1}^{f}\right)\tfrac{Y_{t+1}^{mp}}{Y_t^{mp}}\phi_m\left(\tfrac{\overline{\pi}_{t+1}^{mp}}{\overline{\pi}_t^{mp}}-1\right)\tfrac{\overline{\pi}_{t+1}^{mp}}{\overline{\pi}_t^{mp}}\;\right],\end{aligned} \tag{75}$$

where $\overline{\pi}_t^{mp}\equiv\overline{P}_t^{mp}/\overline{P}_{t-1}^{mp}$.

2.2 *Households*

The economy is inhabited by a continuum of symmetric, infinitely lived households indexed by $j\in[0,1]$ that derive utility from leisure and consumption of the final good. Households get income from selling labour services, from holding one-period domestic and foreign bonds, and they receive the real profits from domestic firms. The adjustment costs incurred by domestic firms are also rebated to households. Each household is a monopoly supplier of a differentiated labour service and sets the wage rate subject to labour demand

$$H_t(j)=\left(\frac{W_t(j)}{W_t}\right)^{-\theta^h}H_t, \tag{76}$$

and quadratic costs of wage adjustment. The adjustment costs are measured in terms of the total wage bill. The specification of adjustment costs follows Laxton & Pesenti (2003) and is given by:

$$\Upsilon_t^w(j)\equiv\frac{\phi_w}{2}\left(\frac{W_t(j)/W_{t-1}(j)}{W_{t-1}/W_{t-2}}-1\right)^2. \tag{77}$$

The return on the foreign bond is given by $\kappa_t R_t^f$, where R_t^f is the gross nominal interest rate on foreign bonds and κ_t is a premium on foreign bond holdings. The premium is assumed to be a

[11] The demand function is derived from

$$Y_{t+l}^{mp}(m)=\frac{1}{\varpi_f}\left(\frac{P_{t+l}^{mp}(m)}{P_{t+l}^{mp}}\right)^{-\theta_{t+l}^{m}}Y_{t+l}^{mp}=\left(\frac{S_{t+l}P_{t+l}^{mp}(m)}{P_{t+l}^{m}}\right)^{-\theta_{t+l}^{m}}Y_{t+l}^{m},$$

function of the economy's real net foreign asset position

$$\kappa_t = \exp\left(-\psi\frac{S_t B_t^f}{P_t^c} + u_t\right), \tag{78}$$

where B_t^f is the aggregate holding of nominal foreign bonds in the economy, and u_t is a time-varying 'risk premium' shock.[12] The risk premium shock is assumed to follow a first-order autoregressive process

$$\ln u_t = \rho_u \ln u_{t-1} + \varepsilon_{u,t} \tag{79}$$

where $0 \leq \rho_u < 1$, and $\varepsilon_{u,t}$ is a white noise process. The specification of the risk premium implies that if the domestic economy is a net borrower ($B_t^f < 0$), it has to pay a premium on the foreign interest rate. This assumption ensures that net foreign assets are stationary.[13]

Household j's period $t+l$ budget constraint is

$$\begin{aligned} & P_{t+l}^c C_{t+l}(j) + \frac{B_{t+l}(j)}{R_{t+l}} + \frac{S_{t+l}B_{t+l}^f(j)}{\kappa_{t+l}R_{t+l}^f} \\ = \;& (1-\Upsilon_{t+l}^w(j))W_{t+l}(j)H_{t+l}(j) + B_{t+l-1}(j) + S_{t+l}B_{t+l-1}^f(j) + \Pi_{t+l}, \end{aligned} \tag{80}$$

where R_{t+l} is the (gross) nominal interest rate on domestic bonds, $B_{t+l}(j)$ and $B_{t+l}^f(j)$ are household j's holdings of nominal domestic and foreign bonds, and the variable Π_{t+l} includes all profits accruing to domestic households and the nominal adjustment costs that are rebated to households.

A representative household chooses a sequence $\left\{C_{t+l}(j), B_{t+l}(j), B_{t+l}^f(j), W_{t+l}(j)\right\}_{l=0}^{\infty}$ to maximise

$$[E_t[\sum_{l=0}^{\infty}\beta^l\left(\ln\left(\frac{C_{t+l}(j)-\zeta C_{t+l-1}}{1-\zeta}\right) - \eta\frac{H_{t+l}^{1+\chi}}{1+\chi}\right)]] \tag{81}$$

subject to the budget constraint (80). The parameter $\zeta \in [0,1)$ reflects the assumption of (external) habit formation in consumption, and $\chi \in (0,\infty)$ is the inverse of the Frisch elasticity of labour supply (i.e., the elasticity of labour supply with respect to real wages for a constant marginal utility of wealth). The parameter $\eta > 0$ is a scale parameter and $\beta \in (0,1]$ is the subjective discount factor. The stochastic discount factor $D_{t,t+l}$ is defined as

$$D_{t,t+l} = \beta^l \frac{C_t - \zeta C_{t-1}}{C_{t+l} - \zeta C_{t+l-1}} \frac{P_t^c}{P_{t+l}^c} \tag{82}$$

Making use of the fact that all households are identical, the first-order conditions with respect to consumption and bond holdings can be combined to give

$$\frac{1}{R_t} = E_t D_{t,t+1} \tag{83}$$

$$\frac{1}{\kappa_t R_t^f} = E_t\left[D_{t,t+1}\frac{S_{t+1}}{S_t}\right]. \tag{84}$$

The first equation is the consumption Euler equation reflecting the households' desire to smooth

[12] As discussed by Bergin (2006), the mean-zero disturbance term u_t can be interpreted as a proxy for a time-varying risk premium omitted by linearisation, or as capturing the stochastic bias in exchange rate expectations in a noise trader model.

[13] See Schmitt-Grohe & Uribe (2003) for a discussion of alternative ways to ensure stationary net foreign assets in a small open economy. In the standard small open economy model with incomplete international asset markets, equilibrium dynamics have a random walk component. That is, transitory shocks have permanent effects on wealth and consumption.

consumption over time. With habit formation, consumption dated $t-1$ enters the Euler equation. The second equation is the UIP condition which characterises the optimal portfolio allocation of foreign and domestic bonds. The first-order condition with respect to wages can be written

$$
\begin{aligned}
0 \quad = \quad & \frac{P_t^c\left(C_t-\zeta C_{t-1}\right)}{1-\zeta}\eta\frac{\theta^h H_t^{\chi}}{W_t} \\
& -(\theta^h-1)\left(1-\frac{\phi_w}{2}\left(\frac{\pi_t^w}{\pi_{t-1}^w}-1\right)^2\right) \\
& -\phi_w\left(\frac{\pi_t^w}{\pi_{t-1}^w}-1\right)\frac{\pi_t^w}{\pi_{t-1}^w} \\
& +E_t\left[\ D_{t,t+1}\pi_{t+1}^w\frac{H_{t+1}}{H_t}\phi_w\left(\frac{\pi_{t+1}^w}{\pi_t^w}-1\right)\frac{\pi_{t+1}^w}{\pi_t^w}\ \right],
\end{aligned}
\tag{85}
$$

where $\pi_t^w \equiv W_t/W_{t-1}$. In the absence of adjustment costs ($\phi_w=0$), the optimal real wage is a mark-up over the marginal rate of substitution between leisure and consumption

$$
\frac{W_t}{P_t^c}=\frac{\theta^h}{\theta^h-1}\eta H_t^{\chi}\frac{\left(C_t-\zeta C_{t-1}\right)}{1-\zeta}. \tag{86}
$$

2.3 *Monetary authorities*

The central bank sets short-term interest rates according to the following simple feedback rule

$$
R_t=\rho_R R_{t-1}+\left(1-\rho_R\right)\left(R+\rho_\pi\left(\pi_t^c-\pi^c\right)\right), \tag{87}
$$

where R is the steady-state level of the nominal interest rate, π^c is the inflation target and $\rho_\pi>0$. The parameter $0<\rho_R<1$ measures the degree of interest rate smoothing.

2.4 *Market clearing*

The market clearing conditions for the domestic labour market and the intermediate goods market are

$$
H_t=H_t^y+H_t^c+H_t^m \tag{88}
$$

$$
Y_t=Y_t^d+Y_t^x, \tag{89}
$$

where $H_t^m=\delta Y_t^m$. Only foreign bonds are assumed to be traded internationally, hence the domestic bond is in zero net supply at the domestic level (i.e., $B_t=0$). Net foreign assets evolve according to

$$
\frac{S_t B_t^f}{\kappa_t R_t^f}=S_t B_{t-1}^f+\overline{P}_t^x Y_t^x-\overline{P}_t^m Y_t^m \tag{90}
$$

2.5 *Mark-up shocks*

The model derived above only has one shock; the risk premium or UIP shock. If the purpose is to estimate the DSGE model by matching impulse responses to a UIP shock, there is no need to introduce additional shocks. In fact, one of the advantages of the impulse response matching approach is that it allows the researcher to leave most of the exogenous shocks unspecified. However, if the dimension of the VAR is greater than the number of shocks, a VAR fitted to data generated from the DSGE model will have a singular variance-covariance matrix. This is the stochastic singularity problem discussed by e.g., Ingram et al. (1994). One strategy for dealing with this problem is to add shocks until the number of shocks is at least as great as the number of variables in the VAR. This is the approach taken in this paper. More precisely, I introduce four mark-ups shocks. The elasticities of substitution between varieties of goods are characterised by the following processes

$$\ln\theta_t^c = (1-\rho_c)\ln\theta^c + \rho_c \ln\theta_{t-1}^c + \varepsilon_{c,t} \quad (91)$$
$$\ln\theta_t^y = (1-\rho_y)\ln\theta^y + \rho_y \ln\theta_{t-1}^y + \varepsilon_{y,t} \quad (92)$$
$$\ln\theta_t^x = (1-\rho_x)\ln\theta^x + \rho_x \ln\theta_{t-1}^x + \varepsilon_{x,t} \quad (93)$$
$$\ln\theta_t^m = (1-\rho_m)\ln\theta^m + \rho_m \ln\theta_{t-1}^m + \varepsilon_{m,t} \quad (94)$$

where $0 \leq \rho_i < 1$ and the $\varepsilon_{i,t}$ are independent white noise processes, $i = \{c, y, x, m\}$. Variables without time-subscripts denote steady-state values. The motivation for adding this particular set of shocks is that the mark-up shocks have a direct effect on the price setting equations in the structural model and hence, on the variables included in the VAR. This turned out to be important to avoid a (near) singular variance-covariance matrix. However, I do not attach a strong structural interpretation to the mark-up shocks. An alternative would be to add serially correlated errors to the observation equations in the state space representation. Such 'measurement errors' could be interpreted as capturing the effects of structural shocks that are omitted from the model or other forms of misspecification of the DSGE model.

2.6 Calibration

In the calibration one period is taken to be one quarter. The calibration is guided by the following principles: first, the parameters should be within the range suggested by the literature and second, the model should loosely match the standard deviations and first-order autocorrelations of UK prices and exchange rates over the period 1980–2003.

Table 2 lists the values of the parameters in the baseline calibration of the model. The subjective discount factor is set to $1.03^{-0.25}$ to yield a steady-state annualised real interest rate of 3%. The habit persistence parameter (ζ) is set to 0.85, which is close to the value chosen by Kapetanios et al. (2007) for the UK. There appears to be little consensus in the literature about the appropriate value for the inverse of the Frisch elasticity of labour demand (χ). Choudhri et al. (2005) choose an initial value of 0.5 for this parameter, but later allow it to vary between zero and infinity. In the baseline calibration in this paper, the inverse Frisch elasticity is set to 3, which is the same value used in Hunt & Rebucci (2005) in a version of the IMF's Global Economy Model. The weight on leisure in the utility function (η) is chosen to yield a steady-state level of labour supply equal to unity ($H = 1$).

Based on the data for revenue shares of intermediate goods reported in Choudhri et al. (2005), the Cobb-Douglas shares of intermediate goods in the production functions for final goods and intermediate goods (γ_c, γ_y) are set to 0.42 and 0.77, respectively. The share of domestic intermediate goods in the aggregate intermediate good (α) is set to 0.85, and the elasticity of substitution between domestic and foreign intermediate goods (ν) is 1.5. The range considered by the literature for the latter is quite large. Groen & Matsumoto (2004) use the value 1.5 in their calibrated model of the UK economy. The distribution cost parameters (δ, δ_f) are set to 0.4, slightly higher than the 0.3 used by Hunt & Rebucci (2005).

The steady-state values of the elasticities of substitution between varieties of goods sold in the domestic market (i.e., θ^c, θ^y and θ^m) are set to 6. This implies a steady-state mark-up of 20% for final goods and domestic intermediate goods. Again, these numbers are comparable to what has been used in models of the UK economy. Benigno & Thoenissen (2003) assume that the substitution elasticity between traded goods is 6.5, and Kapetanios et al. (2007) set the elasticity of substitution between varieties of domestic goods sold in the domestic market to 5. The elasticity of substitution between types of labour services is also set to 6, in line with the values in Hunt & Rebucci (2005) and Benigno & Thoenissen (2003). Finally, the elasticity of substitution between varieties of domestic goods sold in foreign markets is set to 15. This is based on the argument in Kapetanios et al. (2007) that domestic firms face more competitive demand conditions in foreign markets.

The annual domestic inflation target is 2%. The parameters in the monetary policy rule are taken from Kapetanios et al. (2007). The weight on interest rate smoothing in the monetary policy rule (ρ_R) is 0.65, and the weight on inflation (ρ_π) is 1.8.

The adjustment costs parameters associated with changing the rates of change in prices and wages ($\phi_c, \phi_y, \phi_m, \phi_x, \phi_w$) are set to 400.

The share of PCP firms in the foreign economy (ω_f) is set to 0.4, while the share of PCP firms in exports (ω) is 0.6. Data on invoicing currency in UK trade from the years 1999 to 2002 show that the share of UK imports and exports that are invoiced in sterling is around 40% and 50% respectively.[14] To get short-run pass-through to import prices more in line with the empirical estimates I had to use a somewhat higher value for the share of LCP firms in the foreign economy than what is suggested by the data on invoicing currency. Admittedly, this is not entirely satisfying.

The steady-state levels of foreign output y_f and real wages w_f are normalised to unity. The implicit inflation target in the foreign economy (π_f) is identical to the domestic inflation target. This implies that the rate of exchange rate depreciation is zero in the steady-state. Moreover, assuming that domestic and foreign households have the same subjective discount rates, the steady-state interest rates will be the same. This is consistent with a zero risk premium ($\kappa = 1$) and zero net foreign assets ($B_f = 0$) in the steady-state. The elasticity of substitution between foreign and domestic goods in the foreign economy (ν_f) is set to 1.5, the same as in the domestic economy.

The sensitivity of the risk premium to net foreign assets is set to 0.02. During the calibration process I found that setting this parameter too low caused the model to become non-invertible (see section 3). The parameters in the processes for the risk premium and the demand elasticities were chosen to make the standard deviation and autocorrelation of the inflation rates and exchange rate depreciation roughly match those in the data. Table 3 reports the standard deviations and the first-order autocorrelations in the model and in the UK data 1980Q1–2003Q4.

2.7 Model solution and properties

To solve the model, I first compute a first-order approximation (in logs) of the equilibrium conditions around a non-stochastic steady state. Several solution algorithms are available for linearised rational expectations models (e.g., Blanchard & Kahn, 1980; Anderson & Moore, 1985; Klein, 2000; Sims, 2002). Depending on the eigenvalues of the system there are three possibilities: there are no stable rational expectations solutions, there exists a unique stable solution, or there are multiple stable solutions. According to Blanchard & Kahn (1980, prop. 1), there exists a unique stable solution if the number of eigenvalues outside the unit circle equals the number of non-predetermined ('forward-looking') variables. In this paper, the log-linearised model is solved using the procedures implemented in Dynare, which is a collection of Matlab routines for solving rational expectations models (see Juillard, 2005). The equations in the log-linearised model are listed in appendix A.

2.8 Is the model empirically relevant?

As a check on the calibration I examined whether the DSGE model is empirically relevant in the following sense: the estimation of a VAR on artificial data generated from the DSGE model should yield similar estimates of exchange rate pass-through to those obtained when estimating a VAR on actual UK data when using the same sample size, the same set of variables and the same identification scheme.

The estimated fourth-order VAR includes the following variables: UK import prices of manufactures ($\overline{P}_t^m$), export prices of manufactures ($\overline{P}_t^x$), producer prices of manufactures (P_t^y), consumer

[14] These numbers can be found on `http://customs.hmrc.gov.uk/`

prices (P_t^c), and a nominal effective exchange rate (S_t).[15] An increase in the exchange rate S_t corresponds to a depreciation of sterling. The data are quarterly, covering the period 1980Q1–2003Q4, and all the price series are seasonally adjusted and measured in domestic currency. Variable definitions and sources are provided in appendix B.

In line with common practice in the literature, the variables are differenced prior to estimation. Also in line with common practice, the exchange rate shock is identified by placing the exchange rate first in a recursive ordering of the variables. Under this identification scheme, exchange rate shocks have a contemporaneous effect on the price indices, but shocks to the price equations affect the exchange rate with at least a one-period lag. This assumption could be justified by the existence of time lags in the publication of official statistics (see Choudhri et al., 2005). Note that, if interest is only in the exchange rate shock, the ordering of the variables placed before or after the exchange rate is irrelevant.

Figure 1 plots the accumulated impulse responses of import prices, export prices, producer prices and consumer prices to a one standard deviation shock to the exchange rate. The responses are normalised by the accumulated response of the exchange rate. The normalised impulse responses can be interpreted as a measure of exchange rate pass-through.[16] Exchange rate pass-through to import prices is 39% within the first quarter, increasing to 55% within one year and to about 70% in the longer run. The immediate response of export prices is somewhat lower; pass-through is 16% after one quarter, 47% after one year and increasing to 60% in the long run. The response of producer prices is smaller and more gradual; pass-through is 15% within one year and increases to 27% after five years. The pass-through to consumer prices is close to zero at all horizons. The long-run pass-through is approximately 7%. These estimates are broadly in line with the estimates reported for the UK in other structural VAR studies such as McCarthy (2000) and Faruqee (2006).

As a next step, I conducted the following simulation experiment: using the log-linearised solution to the DSGE model as the data generating process, I simulated 5000 synthetic datasets of length $T = 100$ for $y_t' = \{\Delta\ln S_t, \Delta\ln\overline{P}_t^m, \Delta\ln\overline{P}_t^x, \Delta\ln P_t^y, \Delta\ln P_t^c\}$. For each synthetic dataset I estimated a VAR(4) and computed the impulse responses to an exchange rate shock using the same recursive identification scheme as above.[17] Figure 2 plots the pointwise mean of the normalised responses to an exchange rate shock. Exchange rate pass-through to import prices is 45% in the first quarter and stabilises at 75% after about 12 quarters. The pass-through to export prices is lower; 32% in the first quarter and close to 40% in the long-run. The short-run pass-through to producer and consumer prices is close to zero. After twenty periods the pass-through is 25% and 10%, respectively. Evidently, the overall pattern of the pass-through estimates is broadly similar to the estimates obtained using actual UK data.

3 MAPPING FROM THE DSGE MODEL TO A VAR

Adopting the notation in Fernández-Villaverde et al. (2007), the log-linear transition equations describing the model solution can be expressed in state space form as

$$\begin{aligned} x_{t+1} &= Ax_t + Bw_t \\ y_t &= Cx_t + Dw_t, \end{aligned} \tag{95}$$

[15]This is the same set of variables as considered by Faruqee (2006), with the exception that he also includes wages in the VAR. I have confirmed that the pass-through estimates reported in this section are robust to the inclusion of wages in the model.

[16]The normalisation facilitates a comparison with single-equation estimates of pass-through defined as the dynamic responses of prices to a one per cent permanent exchange rate change.

[17]This identification scheme is not consistent with the DSGE model presented above. However, the point of this exercise is to show that, if I use a similar sample size and the same identification scheme, I get results that are not too dissimilar from what was found using actual UK data. In the Monte Carlo experiments in section 4 I use an identification scheme which is compatible with the DSGE model.

where w_t is an $m \times 1$ vector of structural shocks satisfying $E[w_t] = 0, E[w_t w_t'] = I$ and $E[w_t w_{t-j}] = 0$ for $j \neq 0$, x_t is an $n \times 1$ vector of state variables, and y_t is a $k \times 1$ vector of variables observed by the econometrician. The eigenvalues of A are all strictly less than one in modulus, hence the model is stationary. In what follows I will focus on the case where D is square (i.e., $m = k$) and D^{-1} exists. The impulse responses from the structural shocks w_t to y_t are given by the moving average (MA) representation

$$y_t = d(L)w_t = \sum_{j=0}^{\infty} d_j L^j w_t, \tag{96}$$

where L is the lag operator ($L^j y_t \equiv y_{t-j}$), $d_0 = D$ and $d_j = CA^{j-1}B$ for $j \geq 1$.

3.1 Invertibility

An infinite order VAR is defined by

$$y_t = \sum_{j=1}^{\infty} A_j y_{t-j} + G\nu_t, \tag{97}$$

where $E[\nu_t] = 0$, $E[\nu_t \nu_t'] = I$, $E[\nu_t \nu_{t-j}] = 0$ for $j \neq 0$. The orthogonalisation of the VAR innovations implicit in G is void of economic content and does not impose any restrictions on the model. The covariance matrix of the VAR innovations $u_t = G\nu_t$ is $E[G\nu_t \nu_t' G'] = GG' = \Sigma_u$. The MA representation of (97) is

$$y_t = c(L)\nu_t \tag{98}$$

where $c(L) = \sum_{j=0}^{\infty} c_j L^j = (I - \sum_{j=1}^{\infty} A_j L^j)^{-1} G$.

A potential source of discrepancies between the VAR impulse responses and the responses from the log-linearised solution to the DSGE model is that the MA representation (96) is non-invertible. By construction, the MA representation associated with the infinite order VAR (98) is fundamental in the sense that the innovations ν_t can be expressed as a linear combination of current and past observations of y_t. However, there exists an infinite number of other, non-fundamental, MA representations that are observationally equivalent to (98), but which cannot be recovered from the infinite order VAR. These MA representations are non-invertible, meaning that they cannot be inverted to yield an infinite order VAR. In general, we cannot rule out the possibility that a DSGE model has a non-invertible MA representation for a given set of observables.[18] That is, we cannot rule out the possibility that some of the roots of the characteristic equation associated with (96) are inside the unit circle. If this is the case, the impulse responses derived from an infinite order VAR will be misleading, as the structural shocks cannot be recovered from the innovations to the VAR. Whether the MA components of a model are invertible or non-invertible will in general depend on which variables are included in the VAR.

Fernández-Villaverde et al. (2007) show that when D is square and D^{-1} exists, a necessary and sufficient condition for invertibility is that the eigenvalues of $A - BD^{-1}C$ are strictly less than one in modulus. If this condition is satisfied, y_t has an infinite order VAR representation given by

$$y_t = \sum_{j=1}^{\infty} C(A - BD^{-1}C)^{j-1} BD^{-1} y_{t-j} + Dw_t. \tag{99}$$

The rate at which the autoregressive coefficients converge to zero is determined by the largest eigenvalue of $A - BD^{-1}C$. If this eigenvalue is close to unity, a low order VAR is likely to be a poor approximation to the infinite order VAR. Two special cases are worth noting. First, as can be seen from (95), if all the variables in x_t are observed by the econometrician (implying that $A = C$

[18]Lippi & Reichlin (1994) and Fernández-Villaverde et al. (2007) provide examples of economic models with non-invertible MA components.

and $B = D$), the process for y_t will be a VAR(1). Second, if all the endogenous state variables are observable and included in y_t, and the exogenous state variables follow a VAR(1), then y_t has a VAR(2) representation (see e.g., Kapetanios et al., 2007; Ravenna, 2007).

If one or more of the eigenvalues of $A - BD^{-1}C$ are exactly equal to one in modulus, y_t does not have a VAR representation; the autoregressive coefficients do not converge to zero as the number of lags tends to infinity. Fernández-Villaverde et al. (2007) refer to this as a 'benign borderline case'. Often, roots on the unit circle indicate that the variables in the VAR have been overdifferenced (see Watson, 1994).

3.2 Identification

If the model is invertible, the impulse responses from the infinite order VAR (97) with $G\nu_t = Dw_t$ correspond to the impulse responses to the structural shocks in the DSGE model (96). In practice, however, D is unknown, and the econometrician is faced with an identification problem. A prerequisite for estimating DSGE models by matching impulse responses, is that the identification restrictions imposed on the VAR are compatible with the theoretical model. As discussed above, the pass-through literature has typically achieved exact identification by setting $G = \Gamma_{tr}$, where Γ_{tr} is the lower triangular Choleski factor of the estimated variance-covariance matrix of the VAR residuals, $\widehat{\Sigma}_u$. However, this identification scheme is not consistent with the DSGE model set out in section 2. Hence, $G = \Gamma_{tr}$ will yield biased estimates of the model's impulse responses.[19]

In the simulation experiments in this paper I employ an identification scheme suggested by Del Negro & Schorfheide (2004). Using a QR decomposition of D, the impact responses of y_t to the structural shocks w_t can be expressed as

$$\left(\frac{\partial y_t}{\partial w_t}\right)_{DSGE} = D = \Gamma^*_{tr}\Omega^*, \tag{100}$$

where Γ^*_{tr} is lower triangular and Ω^* satisfies $(\Omega^*)'\Omega^* = I$. The VAR is identified by setting $G = \Gamma_{tr}\Omega^*$. With this identification scheme, the impact responses computed from the VAR will differ from D only to the extent that Γ_{tr} differs from Γ^*_{tr} (that is, only to the extent that the estimated variance-covariance matrix $\widehat{\Sigma}_u$ differs from DD'). Thus, in the absence of misspecification of the VAR, the identification scheme succeeds in recovering the true impact responses.

4 SIMULATION EXPERIMENTS

This section presents the results of the simulation experiments. I consider two different VARs: a VAR in first differences of nominal prices and the exchange rate, and a VAR in relative prices and the first difference of consumer prices. The latter is equivalent to a VEqCM that includes the cointegration relations implied by the DSGE model as regressors. As a second exercise, I examine whether an econometrician who uses standard techniques for determining cointegration rank and for testing restrictions on the cointegration relations will be able to infer the cointegration properties of the DSGE model.

4.1 Monte Carlo design

I generate $M = 5000$ datasets of lengths $T = 1100$ and $T = 1200$ using the state space representation of the log-linearised DSGE model as the data generating process.[20] Each sample is initialised

[19] Canova & Pina (2005) show that when the DSGE model does not imply a recursive ordering of the variables, the VAR responses to a monetary policy shock identified with a recursive identification scheme can be very misleading.

[20] To examine the sensitivity of the results to the number of Monte Carlo replications I conducted preliminary experiments using $M = \{1000, 2000, \ldots, 10000\}$ and found that the pointwise mean and standard deviations of the impulse responses obtained with $M = 5000$ and $M = 10000$ are essentially indistinguishable.

using the steady-state values of the variables. To limit the influence of the initial conditions, I discard the first 1000 observations in each replication and leave $T = 100$ and $T = 200$ observations for estimation of the VAR. The simulations are performed in Matlab, and the built-in function `randn.m` is used to generate the pseudo-random normal errors. I use the same random numbers in all experiments. This is achieved by fixing the seed for the random number generator.

For each dataset I estimate a VAR and compute the accumulated responses of prices to a UIP shock. The UIP shock is identified using the Del Negro & Schorfheide (2004) identification scheme discussed in the previous section.

The selection of lag-order is an important preliminary step in VAR analyses. I conduct experiments for four different methods of lag-order selection: the Akaike information criterion (AIC), the Hannan-Quinn criterion (HQ), the Schwarz criterion (SC), and the sequential likelihood-ratio test (LR) (see Lütkepohl, 1991 for a discussion). The LR test is implemented using the small-sample correction suggested in Sims (1980) and using a 5% significance level for the individual tests. I also report results for a fixed lag-length ($L = 2$ and $L = 4$ for the VAR in first differences, $L = 3$ and $L = 5$ for the VEqCM and the VAR in levels).

Lütkepohl (1990) shows that, as long as the lag-order goes to infinity with the sample size, the orthogonalised impulse response functions computed from a finite order VAR estimated by OLS are consistent and asymptotically normal, even if the true order of the process is infinite. In this sense, any discrepancies between the impulse responses from the VAR and the log-linearised DSGE model can be attributed to a small-sample bias. It is nevertheless instructive to decompose the overall difference between the DSGE model's impulse responses and the VAR impulse responses into (i) bias arising from approximating an infinite order VAR with a finite order VAR, and (ii) small-sample estimation bias for a given lag-order. The first source of bias, which Chari et al. (2005) label the 'specification error', is given by the difference between the DSGE model's responses and those obtained from the population version of the finite order VAR for a given lag-order. The coefficients in the population version of a finite order VAR can be interpreted as the probability limits of the OLS estimators or, what the OLS estimates would converge to if the number of observations went to infinity while keeping the lag-order fixed (Christiano et al., 2006). Fernández-Villaverde et al. (2007) provide formulas for these coefficients as functions of the matrices A, B, C and D in the state space representation (95). Hence, the magnitude of the specification error can be assessed without resorting to simulation exercises.[21] For a given lag-order, the bias arising from the specification error persists even in large samples. Regarding the small-sample estimation bias; VAR impulse responses are non-linear functions of the autoregressive coefficients and the covariance matrix of the VAR residuals. It is well known that OLS estimates of the autoregressive coefficients in VARs are biased downward in small samples.

4.2 VAR in first differences

The first model I consider is a VAR in first differences of nominal prices and the exchange rate:

$$\Delta y_t = A_1 \Delta y_{t-1} + A_2 \Delta y_{t-2} + \ldots + A_p \Delta y_{t-p} + \varepsilon_t \tag{101}$$

where

$$\Delta y_t' = \{\Delta \ln \overline{P}_t^m, \Delta \ln \overline{P}_t^x, \Delta \ln P_t^y, \Delta \ln P_t^c, \Delta \ln S_t\}.$$

With this vector of observables, the matrix $A - BD^{-1}C$ has four roots equal to one, while the remaining roots are all smaller than one in modulus. This implies that, technically, the model does not have a VAR representation. The reason why a VAR representation fails to exist in this case, is that the variables included in the VAR are overdifferenced.

[21] I am grateful to Jesús Fernández-Villaverde for sharing the Matlab program `ssvar.m` which calculates the coefficients of the population version VAR.

Table 4 reports the distribution of the lag-orders chosen by the different lag-order selection criteria for sample sizes $T = 100$ and $T = 200$. The maximum lag-length is set to five. As expected, the SC is the most conservative and selects the lowest average lag-order. For sample size $T = 100$ the SC chooses a lag-length of one in 71.5% of the replications. By contrast, the AIC and the HQ select a lag-order of two in approximately 90% of the replications. With a sample size of $T = 200$, the average lag-order increases for all the criteria: the SC picks a lag-order of two in 98% of the datasets, the AIC and the HQ select a lag-order of two in 89% and 100% of the replications respectively. For both sample sizes, the LR test selects a somewhat higher lag-order than the information criteria.

Figure 3 plots the outcome of the simulation experiment with $T = 100$ and a fixed lag-length $L = 2$. The solid lines represent the pointwise mean of the accumulated impulse responses, and the shaded areas correspond to the pointwise mean plus/minus 1.96 times the pointwise standard deviations. The lines with points correspond to a 95% interval for the pointwise responses, calculated by reading off the 2.5 and 97.5 percentiles of the ordered responses at each horizon. Finally, the lines with circles depict the impulse responses from the DSGE model. Figure 4 plots the accumulated responses normalised on the exchange rate response.

Looking at the normalised responses, we see that the VAR estimates of exchange rate pass-through are biased downwards. Whereas in the DSGE model the exchange rate pass-through is nearly complete after twenty quarters, the mean of the VAR estimates of long-run pass-through is 72% for import prices, 35% for export prices, 15% for consumer prices and 20% for producer prices.[22] From the bottom panel of figure 3 it is evident that the downward bias to some extent reflects that the exchange rate behaves almost like a random walk in the VAR, whereas there is significant (but not complete) reversion in the exchange rate towards the original level following a UIP shock in the DSGE model. The bias in the nominal exchange rate response is transmitted to import prices. By contrast, the estimated VAR responses of consumer and producer prices are smaller than the true responses. This suggests that the downward bias in the VAR estimates of pass-through to these prices would remain even if the VAR had accurately captured the exchange rate response. Figures 5 and 6 plot the outcome of an experiment with $L = 2$ and $T = 200$. The biases in the impulse responses remain in the larger sample, the main effect of adding observations is to lower the standard deviations of the simulated responses.

Figures 7 and 8 decompose the overall bias into small-sample bias and bias arising from approximating an infinite order VAR with a VAR(2). The latter is measured as the difference between the true impulse responses (lines with circles) and the responses from the population version of a VAR(2) (solid lines). It is evident that the dominant source of bias is the specification error. For a given lag-order, this bias persists in large samples. The small-sample bias is measured as the difference between the responses from the population VAR(2) and the mean responses from the Monte Carlo experiments for $T = 100$ (dotted lines) and $T = 200$ (lines with points). The impulse responses of the exchange rate and import prices are biased downward in small samples. For these variables, the small-sample bias and the specification error bias are of opposite signs. Hence, the effect of adding more observations is to increase the overall bias in the impulse responses. For consumer and producer prices, the opposite is true. For these variables the small-sample bias reinforces the downward bias induced by the specification error.

Next, I examine how many lags are needed for the VAR to be able to recover the true impulse responses. Figures 9 and 10 show the impulse responses from the DSGE model (circled lines), together with the responses from the population version of the VAR for lag-orders $L = \{2, 4, 10, 20\}$. As expected, increasing the number of lags reduces the biases. However, even with as many as twenty lags, the VAR does not accurately capture the responses of prices to a UIP shock.

As we have seen, standard lag-order selection criteria do not detect the need for longer lags.

[22]Extending the horizon beyond twenty quarters, the exchange rate pass-through to all prices is 100% in the DSGE model.

This raises the question whether the misspecification of the lag order is picked up by standard misspecification tests. Table 5 reports the rejection frequencies at the 5% significance level for single-equation and vector tests for residual autocorrelation up to order four. The test is the $F-$approximation to the Lagrange Multiplier (LM) test for autocorrelation described in Doornik (1996). In the small sample ($T = 100$), the tests reject only slightly more often than expected when using a 5% significance level when $L = 4$ or the lag-order is determined by the sequential LR tests. However, when a conservative criterion such as the SC is used, the autocorrelation tests are rejected in a large number of the replications. The autocorrelation tests thus help detect misspecification when the lag-order is very low. When the sample size is increased to $T = 200$, the rejection frequencies increase for all the criteria and for both the fixed lag-lengths.

Erceg et al. (2005) suggest measuring the bias in the impulse responses by the average absolute per cent difference between the mean response and the theoretical response for each variable, that is

$$bias_i^H = \frac{1}{H}\sum_{j=1}^{H}\left|\frac{r_{i,j}^{VAR} - r_{i,j}^{DSGE}}{r_{i,j}^{DSGE}}\right|, \tag{102}$$

where $r_{i,j}^{DSGE}$ and $r_{i,j}^{VAR}$ are the DSGE model's responses and the mean across datasets of the VAR responses of variable i to a UIP shock at horizon j respectively. Tables 6 and 7 report the biases for $H = 10$ and $H = 20$ for different lag-order criteria and sample sizes $T = 100$ and $T = 200$. The results confirm that adding observations increases the bias in the responses of exchange rates and import prices, but reduces the biases in the estimated responses of consumer prices and producer prices. At both horizons and for both sample sizes the average bias is minimised for $L = 4$. The average bias is largest when the lag-order is chosen to minimise the SC.

As a final point, note that a reduction in bias from estimating a higher order VAR may come at the cost of higher variance. Using VARs estimated by leading practitioners as data generating processes, Ivanov & Kilian (2005) find that underestimation of the true lag-order is beneficial in very small samples because the bias induced by choosing a low lag-order is more than offset by a reduction in variance. If the primary purpose of the VAR analysis is to construct accurate impulse responses, the authors recommend using the SC for sample sizes up to 120 quarters and the HQ for larger sample sizes. However, Ivanov & Kilian (2005) do not explore the case where the data generating process is an infinite order VAR, in which case the trade-offs between bias and variance are likely to be different.

4.3 VEqCM

The fact that the monetary policy rule is specified in terms of inflation and not the price level induces a common stochastic trend in the nominal variables in the log-linearised DSGE model.[23] Hence, while nominal prices and the exchange rate contain a unit root, the real exchange rate and relative prices are stationary. Estimating a VAR in first differences implies a loss of information, and in this sense it is not surprising that a VAR that omits the cointegration relations does a poor job in recovering the responses of the levels of prices and the exchange rate. Here I examine whether I obtain a better approximation of the DSGE model by estimating a VEqCM that includes the cointegration relations implied by the theoretical model. That is, I consider the system

$$\Delta y_t = \alpha\beta' y_{t-1} + A_1^*\Delta y_{t-1} + A_2^*\Delta y_{t-2} + \ldots + A_p^*\Delta y_{t-p} + \varepsilon_t \tag{103}$$

[23] The foreign price level is stationary around a deterministic trend.

with

$$\beta' y_{t-1} = \left\{ \begin{array}{c} \ln \overline{P}^m_{t-1} - \ln P^c_{t-1} \\ \ln \overline{P}^x_{t-1} - \ln P^c_{t-1} \\ \ln P^y_{t-1} - \ln P^c_{t-1} \\ \ln S_{t-1} + \ln P^f_{t-1} - \ln P^c_{t-1} \end{array} \right\}$$

Estimating (103) is (almost) the same as estimating a VAR in the real exchange rate, relative prices and consumer price inflation[24] , that is,

$$y^\dagger_t = A^\dagger_1 y^\dagger_{t-1} + A^\dagger_2 y^\dagger_{t-2} + \ldots + A^\dagger_{p+1} y^\dagger_{t-(p+1)} + \varepsilon^\dagger_t \tag{104}$$

where

$$(y^\dagger_t)' = \left\{ \Delta \ln P^c_t, \ln\left(\overline{P}^m_t / P^c_t\right), \ln\left(\overline{P}^x_t / P^c_t\right), \ln\left(P^y_t / P^c_t\right), \ln\left(S_t P^f_t / P^c_t\right) \right\}$$

When the observation vector is $y^\dagger_t$, all the roots of the matrix $A - BD^{-1}C$ are smaller than one in modulus. Hence, the model is invertible, and $y^\dagger_t$ has a VAR representation. Including the cointegration relations thus removes the unit roots in the MA components that appear in the VARMA representation for the first differences. This is a common finding in the literature (see e.g., Del Negro et al., 2005).

Table 8 reports the distribution of the lag-orders chosen by different selection criteria for $T = 100$ and $T = 200$. The maximum lag-length is six. The SC selects a lag-length of two for both sample sizes. On average, the AIC chooses a higher lag-order: when the sample size is $T = 100$ the AIC chooses $L = 2$ in 47.4% of the datasets and $L = 3$ in 46.2% of the datasets. Figures 11 and 12 plot the outcome of the simulation experiment with $T = 100$. and a fixed lag-length $L = 3$. The VAR approximation to the DSGE model is good even with a moderate number of lags. This is confirmed in figures 13 and 14 which plot the responses computed from the population version of the VAR for lag-orders $L = \{2, 3, 20\}$. There is some bias in the impulse responses for $L = 2$, but for $L = 3$ the estimated responses are close to the true responses. Note that this holds even if the VAR does not include all the state-variables in the DSGE model (e.g., the VAR does not include consumption or the nominal interest rate).

Figures 15 and 16 plot the impulse responses from the population version of the VEqCM(3) together with the true responses and the mean responses from a VEqCM(3) estimated on sample sizes $T = 100$ and $T = 200$. In this case, the small-sample estimation bias is the dominant source of bias in the responses. For all prices except import prices, the estimate of exchange rate pass-through is biased upwards, implying that for a given lag-order, adding observations does not reduce the bias. This is confirmed in tables 9 and 10 which report the average biases over the first ten and twenty quarters respectively, for different lag-order criteria and sample sizes $T = 100$ and $T = 200$. Notice that, in contrast to what was the case for the first-differenced model, the average bias is minimised when the lag-order is based on a conservative lag-order criterion such as the SC.

To summarise, provided the cointegration relations implied by the model are included as additional regressors, the state space representation of the log-linearised DSGE models can be approximated with a low order VAR. This raises the question of whether, in practice, the econometrician would be able to infer the cointegration rank and identify the cointegration relations using standard techniques.

4.4 Cointegration analysis

This section asks the question: will an econometrician armed with standard techniques be able to infer the correct cointegration rank and identify the cointegration relations implied by the theory?

[24]The only difference is that an extra lag of $\ln P^c_t$ is included in the latter from the inclusion of $\Delta \ln P^c_{t-(p+1)} \equiv \ln P^c_{t-(p+1)} - \ln P^c_{t-(p+2)}$.

The experiment is constructed as follows. I generate 5000 artificial datasets of lengths $T = 100$ and $T = 200$ from the DSGE model. Dataseries for the levels of the variables are obtained by cumulating the series for the first differences.[25] For a given synthetic dataset I estimate an unrestricted VAR in levels of the variables and determine the cointegration rank using the sequential testing procedure based on the trace test-statistic proposed by Johansen (1988).[26] Next, I test the restrictions on the cointegration space implied by the DSGE model using the standard LR test for known cointegration vectors (see Johansen, 1995, chap. 7).

The VAR is fitted with an unrestricted constant term and a restricted drift term. The specification of the deterministic terms is consistent with the data generating process. To see this, note that the monetary policy rule and the positive inflation target imply that nominal prices will have both a deterministic trend and a stochastic trend. Both trends are cancelled in the cointegrating relations, implying that relative prices are stationary around a constant mean. That is, $\ln \overline{P}_t^m - \ln P_t^c \sim I(0), \ln \overline{P}_t^x - \ln P_t^c \sim I(0)$, and $\ln P_t^y - \ln P_t^c \sim I(0)$. Since the inflation target in the foreign economy is assumed to be the same as the domestic inflation target, the process for the foreign price level contains the same deterministic trend as the domestic price level, and there is no linear trend in the nominal exchange rate. However, since the foreign price level is not included in the VAR, the fourth cointegration relation will be stationary around a deterministic trend. That is, $\ln S_t - \ln P_t^c + 0.005t \sim I(0)$.

Table 11 reports the distribution of lag-orders chosen by the different selection criteria when the maximum lag-length is set to six. The average lag-order selected is two or three for both sample sizes, with SC being the most conservative criterion.

The trace test is derived under the assumption that the errors are serially uncorrelated and normally distributed with mean zero. Good practice dictates that these assumptions be checked before testing for cointegration. Table 12 reports the rejection frequencies across 5000 datasets for the single-equation and vector tests for non-normality in the residuals described in Doornik & Hansen (1994). The rejection frequencies are close to the nominal 5% level for both sample sizes and across different lag-order criteria. Table 13 reports the rejection frequencies for tests of no autocorrelation up to order five in the residuals. For sample size $T = 100$ and lag-length $L = 3$, the rejection frequencies for the single-equation tests are around 10%. The vector test rejects the null hypothesis in 23% of the datasets. Similar rejection frequencies are obtained when the lag-length is determined using the AIC or sequential LR tests. However, when a conservative criterion like the SC or HQ is used, the rejection frequencies are much higher. When the lag-order is chosen to minimise the SC, the vector test rejects the null of no autocorrelation in 59.1% of the datasets. For all criteria except the SC, the rejection frequencies are lower in the larger sample $T = 200$. Below I report the outcome of the cointegration tests for all the lag-order selection criteria. In practice, however, researchers often supplement the information criteria with tests for residual autocorrelation, and when there is a contradiction, overrule the lag-order selected by the former. This suggests that less weight should be placed on the results of the cointegration analysis obtained when the lag-order is selected using the SC or the HQ criterion.

Table 14 shows the frequencies of preferred cointegration rank for different sample sizes and for different methods of lag-order selection. The non-standard 5% critical values for the trace test are taken from MacKinnon et al. (1999). The numbers in parentheses correspond to the frequencies of preferred rank when the test statistic is adjusted using the small-sample correction suggested by Reinsel & Ahn (1988). When $T = 100$ and $L = 3$, the correct cointegration rank is selected in only 2.7% of the datasets. In 17.5% of the datasets the trace test suggests that the rank is zero, in which case a model in first differences is appropriate. Using the small-sample adjusted test statistics, the trace test chooses the correct rank in only 0.3% of the datasets. In 61.2% of the datasets the

[25]The initial values of the (log) levels of the variables are set to zero. Since the levels series are unit root processes and thus have infinite memory, dropping observations at the beginning of the sample does not eliminate the dependence on the initial values.

[26]See chapter 2 of this thesis for details on the cointegration tests.

trace test would lead us to conclude that the variables are not cointegrated. The results are more encouraging when a sample size of $T = 200$ is used. However, for $L = 3$ the trace test still picks the true cointegration rank in only 31% of the replications.

When the lag-order is endogenous, the correct rank is chosen most frequently when the lag-order is determined using the SC. For $T = 100$ the correct rank is chosen in 20% of the datasets. With a sample size of $T = 200$ the corresponding number is 53%. For the purpose of choosing the correct cointegration rank, a low lag-order appears to be beneficial.

As a second exercise, I examine how often the restrictions on the cointegration vector implied by the DSGE model are rejected when using the standard LR test for known cointegration vectors. Table 15 reports the rejection frequencies for the individual and joint tests of the following hypotheses

$$\ln \overline{P}_t^m - \ln P_t^c \sim I(0), \ln \overline{P}_t^x - \ln P_t^c \sim I(0), \ln P_t^y - \ln P_t^c \sim I(0) \text{ and } \ln S_t - \ln P_t^c + 0.005t \sim I(0)$$

The tests are conditional on the maintained hypothesis that the cointegration rank is 4 ($r = 4$). For $T = 100$ and $L = 3$, the rejection frequencies for the individual hypotheses are 20% when using a nominal test size of 5%. The rejection frequency for the joint hypothesis is 88%. These results raise doubts about whether, in practice, the econometrician will be able to identify the cointegration relations implied by the DSGE model.

Again it is instructive to see whether the results are driven by the specification error or by small-sample estimation bias. In particular, it is of interest to see whether the frequent rejections of the autocorrelation tests are due to the omission of MA terms or due to the fact that the autocorrelation tests are oversized in small samples. To address this issue I redo the above Monte Carlo experiments, this time using the population version of a VEqCM(5) and a VEqCM(3) as the data generating processes. Table 16 reports the distribution of chosen lag-lengths and table 17 reports the outcome of the trace test in this case. The results are similar to the results obtained when the log-linearised solution to the DSGE model is used as the data generating process. This finding suggests that the poor performance of the sequential testing procedure is not due to approximating an infinite order VAR with a low order VAR, but is due to small-sample problems. Interestingly, the same seems to hold for the autocorrelation test. When the data generating process is a VEqCM(3) and the estimated model is a VAR(3) in levels of the data, the rejection frequencies of the autocorrelation tests are 10% for the single-equation tests and 23% for the vector test (see table 18). This suggests that the autocorrelation test is oversized in small samples. This is consistent with the Monte Carlo evidence presented in Brüggemann et al. (2006). Table 19 illustrates a well known result in the literature (see e.g. Gredenhoff & Jacobson, 2001), namely that the LR tests for restrictions on the cointegration space are oversized in small samples.

Most of the existing literature that has examined the finite-sample performance of cointegration tests assumes that the data generating process is a (reduced form) VAR(MA), in which case it is possible to study the effects of marginal changes in the reduced form parameters (e.g., in the elements in the adjustment coefficients to the cointegration relations).[27] Here, the underlying data generating process is a restricted VAR where the parameters are explicit functions of the parameters in the DSGE model. Changing a structural parameter in the DSGE model will typically affect all the dynamic coefficients in the population VAR as well as the variance-covariance matrix of the error terms. A natural extension of the analysis in this paper would be to identify which feature(s) of the DSGE model is responsible for the low power of the cointegration test.

[27] A notable exception is Söderlind & Vredin (1996) who examine the properties of cointegration tests when the data are generated by a monetary equilibrium business cycle model.

5 CONCLUDING REMARKS

This paper has examined the ability of a structural VAR to recover the dynamic responses of a set of prices to a risk premium shock. The main results can be summarised as follows. The estimates of exchange rate pass-through obtained from a first-differenced VAR are systematically biased downwards. The bias in the estimated responses can largely be attributed to the fact that a low order first-differenced VAR is not a good approximation to the VARMA model implied by the DSGE model. Moreover, small-sample estimation bias sometimes acts to offset the bias arising from the approximation error. When the cointegration relations implied by the DSGE model are included in the VAR, even a VAR with a modest number of lags is able to recover the true impulse responses. However, an econometrician using standard tests for cointegration rank and for testing restrictions on the cointegration space would in general not be able to infer the correct rank or identify the true cointegration relations. Interesting extensions of the analysis in this paper are therefore to examine the properties of structural VAR estimates of exchange rate pass-through when the model is estimated in levels, or when the cointegration rank is chosen on the basis of data-based cointegration tests.

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A LOG-LINEARISED MODEL

Letting variables with a hat denote percentage deviations from the deterministic steady state (i.e., $\widehat{X}_t = \ln X_t - \ln X$), the log-linearised equilibrium conditions can be written

$$\widehat{C}_t = \gamma_c \widehat{Q}_t + (1-\gamma_c)\widehat{H}_t^c \tag{A1}$$

$$\widehat{Q}_t^d = -\nu\left(\widehat{p}_t^y - \widehat{p}_t^q\right) + \widehat{Q}_t \tag{A2}$$

$$\widehat{Q}_t^m = -\nu\left(\widehat{p}_t^m - \widehat{p}_t^q\right) + \widehat{Q}_t \tag{A3}$$

$$\widehat{w}_t = \widehat{\vartheta}_t^c + \widehat{C}_t - \widehat{H}_t^c \tag{A4}$$

$$\widehat{p}_t^q = \widehat{\vartheta}_t^c + \widehat{C}_t - \widehat{Q}_t \tag{A5}$$

$$\widehat{p}_t^q = \alpha\left(\frac{p^y}{p^q}\right)^{1-\nu}\widehat{p}_t^y + (1-\alpha)\left(\frac{p^m}{p^q}\right)^{1-\nu}\widehat{p}_t^m \tag{A6}$$

$$\widehat{\pi}_t^c = \frac{\beta}{1+\beta}E_t\widehat{\pi}_{t+1}^c + \frac{1}{1+\beta}\widehat{\pi}_{t-1}^c + \frac{\theta^c(\theta^c-1)}{(1+\beta)\phi_c}\widehat{\vartheta}_t^c - \frac{\theta^c}{(1+\beta)\phi_c}\widehat{\theta}_t^c \tag{A7}$$

$$\widehat{Y}_t = \gamma_y \widehat{Z}_t + (1-\gamma_y)\widehat{H}_t^y \tag{A8}$$

$$\widehat{Z}_t^d = -\nu\left(\widehat{p}_t^y - \widehat{p}_t^z\right) + \widehat{Z}_t \tag{A9}$$

$$\widehat{Z}_t^m = -\nu\left(\widehat{p}_t^m - \widehat{p}_t^z\right) + \widehat{Z}_t \tag{A10}$$

$$\widehat{w}_t = \widehat{\vartheta}_t^y + \widehat{Y}_t - \widehat{H}_t^y \tag{A11}$$

$$\widehat{p}_t^z = \widehat{\vartheta}_t^y + \widehat{Y}_t - \widehat{Z}_t \tag{A12}$$

$$\widehat{p}_t^z = \alpha\left(\frac{p^y}{p^z}\right)^{1-\nu}\widehat{p}_t^y + (1-\alpha)\left(\frac{p^m}{p^z}\right)^{1-\nu}\widehat{p}_t^m \tag{A13}$$

$$\widehat{Y}_t^d = \left(\frac{Q^d}{Y^d}\right)\widehat{Q}_t^d + \left(\frac{Z^d}{Y^d}\right)\widehat{Z}_t^d \tag{A14}$$

$$\widehat{Y}_t^m = \left(\frac{Q^m}{Y^m}\right)\widehat{Q}_t^m + \left(\frac{Z^m}{Y^m}\right)\widehat{Z}_t^m \tag{A15}$$

$$\widehat{Y}_t = \left(\frac{Y^d}{Y}\right)\widehat{Y}_t^d + \left(\frac{Y^x}{Y}\right)\widehat{Y}_t^x \tag{A16}$$

$$\widehat{\pi}_t^y = \frac{\beta}{1+\beta}E_t\widehat{\pi}_{t+1}^y + \frac{1}{1+\beta}\widehat{\pi}_{t-1}^y + \frac{\theta^y(\theta^y-1)}{(1+\beta)\phi_y}\left(\widehat{\vartheta}_t^y - \widehat{p}_t^y\right) - \frac{\theta^y}{(1+\beta)\phi_y}\widehat{\theta}_t^y \tag{A17}$$

$$\widehat{\overline{p}}_t^x = \varpi\widehat{\overline{p}}_t^{xp} + (1-\varpi)\widehat{\overline{p}}_t^{xl} \tag{A18}$$

$$\widehat{p}_t^x = \left(\frac{\overline{p}^x}{p^x}\right)\widehat{\overline{p}}_t^x + \delta_f\left(\frac{sw^f}{p^x}\right)\left(\widehat{w}_t^f + \widehat{s}_t\right) \tag{A19}$$

$$\widehat{\overline{\pi}}_t^{xl} = \frac{\beta}{1+\beta}E_t\widehat{\overline{\pi}}_{t+1}^{xl} + \frac{1}{1+\beta}\widehat{\overline{\pi}}_{t-1}^{xl} - \frac{\theta^x-1}{(1+\beta)\phi_x}\frac{\overline{p}^x}{\overline{p}^x-\vartheta^y}\frac{\overline{p}^x}{p^x}\left(\widehat{\overline{p}}_t^{xl} - \left(\widehat{\overline{p}}_t^x\right)^{opt}\right) \tag{A20}$$

$$\widehat{\overline{\pi}}_t^{xp} = \frac{\beta}{1+\beta}E_t\widehat{\overline{\pi}}_{t+1}^{xp} + \frac{1}{1+\beta}\widehat{\overline{\pi}}_{t-1}^{xp} - \frac{\theta^x-1}{(1+\beta)\phi_x}\frac{\overline{p}^x}{\overline{p}^x-\vartheta^y}\frac{\overline{p}^x}{p^x}\left(\widehat{\overline{p}}_t^{xp} - \left(\widehat{\overline{p}}_t^x\right)^{opt}\right) \tag{A21}$$

$$\overline{p}^x\left(\widehat{\overline{p}}_t^x\right)^{opt} = \frac{\theta^x}{\theta^x-1}\vartheta^y\left(\widehat{\vartheta}_t^y - \frac{1}{\theta^x-1}\widehat{\theta}_t^x\right) + \frac{\delta_f}{\theta^x-1}sw^f\left(\widehat{s}_t + \widehat{w}_t^f - \frac{\theta^x}{\theta^x-1}\widehat{\theta}_t^x\right) \tag{A22}$$

$$\widehat{Y}_t^x = -\nu_f\left(\widehat{p}_t^x - \widehat{s}_t\right) + \widehat{Y}_t^f \tag{A23}$$

$$\widehat{H}_t^m = \widehat{Y}_t^m \tag{A24}$$

$$\widehat{p}_t^m = \left(\frac{\overline{p}^m}{p^m}\right)\widehat{\overline{p}}_t^m + \delta\left(\frac{w}{p^m}\right)\widehat{w}_t \tag{A25}$$

$$\widehat{\overline{p}}_t^m = \varpi_f \widehat{\overline{p}}_t^{mp} + (1-\varpi_f)\widehat{\overline{p}}_t^{ml} \tag{A26}$$

$$\widehat{\overline{\pi}}_t^{ml} = \frac{\beta}{1+\beta} E_t \widehat{\overline{\pi}}_{t+1}^{ml} + \frac{1}{1+\beta}\widehat{\overline{\pi}}_{t-1}^{ml} - \frac{\theta^m - 1}{(1+\beta)\phi_m} \frac{\overline{p}^m}{\overline{p}^m - s\vartheta^f} \frac{\overline{p}^m}{p^m} \left(\widehat{\overline{p}}_t^{ml} - \left(\widehat{\overline{p}}_t^m\right)^{opt} \right) \tag{A27}$$

$$\widehat{\overline{\pi}}_t^{mp} = \frac{\beta}{1+\beta} E_t \widehat{\overline{\pi}}_{t+1}^{mp} + \frac{1}{1+\beta}\widehat{\overline{\pi}}_{t-1}^{mp} - \frac{\theta^m - 1}{(1+\beta)\phi_m} \frac{\overline{p}^m}{\overline{p}^m - s\vartheta^f} \frac{\overline{p}^m}{p^m} \left(\widehat{\overline{p}}_t^{mp} - \left(\widehat{\overline{p}}_t^m\right)^{opt} \right) \tag{A28}$$

$$\overline{p}^m \left(\widehat{\overline{p}}_t^m\right)^{opt} = \frac{\theta^m}{\theta^m - 1} s\vartheta^f \left(\widehat{s}_t + \widehat{\vartheta}_t^f - \frac{1}{\theta^m - 1}\widehat{\theta}_t^m \right) + \frac{\delta}{\theta^m - 1} w \left(\widehat{w}_t - \frac{\theta^m}{\theta^m - 1}\widehat{\theta}_t^m \right) \tag{A29}$$

$$\widehat{C}_t = \frac{1}{1+\zeta} E_t \widehat{C}_{t+1} + \frac{\zeta}{1+\zeta}\widehat{C}_{t-1} - \frac{1-\zeta}{1+\zeta}\left(\widehat{R}_t - E_t \widehat{\pi}_{t+1}^c \right) \tag{A30}$$

$$\widehat{R}_t = \widehat{R}_t^f + E_t \widehat{\Delta S}_{t+1} + \widehat{\kappa}_t \tag{A31}$$

$$\widehat{\pi}_t^w = \frac{\beta}{1+\beta} E_t \widehat{\pi}_{t+1}^w + \frac{1}{1+\beta}\widehat{\pi}_{t-1}^w - \frac{\theta^h - 1}{(1+\beta)\phi_w} \left(\widehat{w}_t - \chi \widehat{H}_t - \frac{\widehat{C}_t - \zeta \widehat{C}_{t-1}}{1-\zeta} \right) \tag{A32}$$

$$\widehat{\kappa}_t = -\psi s \widehat{b}_t^f + \widehat{u}_t \tag{A33}$$

$$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + \frac{(1-\rho_R)\rho_\pi \pi_c}{R}\widehat{\pi}_t^c \tag{A34}$$

$$\widehat{H}_t = \left(\frac{H^c}{H}\right)\widehat{H}_t^c + \left(\frac{H^y}{H}\right)\widehat{H}_t^y + \left(\frac{H^m}{H}\right)\widehat{H}_t^m \tag{A35}$$

$$\frac{s\beta \widehat{b}_t^f}{\pi^f} = \frac{s\widehat{b}_{t-1}^f}{\pi^f} + \overline{p}^x Y^x \left(\widehat{\overline{p}}_t^x + \widehat{Y}_t^x \right) - \overline{p}^m Y^m \left(\widehat{\overline{p}}_t^m + \widehat{Y}_t^m \right) \tag{A36}$$

$$\widehat{\theta}_t^c = \rho_c \widehat{\theta}_{t-1}^c + \varepsilon_{c,t} \tag{A37}$$

$$\widehat{\theta}_t^y = \rho_y \widehat{\theta}_{t-1}^y + \varepsilon_{y,t} \tag{A38}$$

$$\widehat{\theta}_t^x = \rho_x \widehat{\theta}_{t-1}^x + \varepsilon_{x,t} \tag{A39}$$

$$\widehat{\theta}_t^m = \rho_m \widehat{\theta}_{t-1}^m + \varepsilon_{m,t} \tag{A40}$$

$$\widehat{u}_t = \rho_u \widehat{u}_{t-1} + \widehat{\varepsilon}_{u,t} \tag{A41}$$

$$\widehat{\pi}_t^y = \widehat{p}_t^y - \widehat{p}_{t-1}^y + \widehat{\pi}_t^c \tag{A42}$$

$$\widehat{\overline{\pi}}_t^m = \widehat{\overline{p}}_t^m - \widehat{\overline{p}}_{t-1}^m + \widehat{\pi}_t^c \tag{A43}$$

$$\widehat{\pi}_t^m = \widehat{p}_t^m - \widehat{p}_{t-1}^m + \widehat{\pi}_t^c \tag{A44}$$

$$\widehat{\overline{\pi}}_t^{mp} = \widehat{\overline{p}}_t^{mp} - \widehat{\overline{p}}_{t-1}^{mp} + \widehat{\pi}_t^c - \widehat{\Delta S}_t \tag{A45}$$

$$\widehat{\overline{\pi}}_t^{ml} = \widehat{\overline{p}}_t^{ml} - \widehat{\overline{p}}_{t-1}^{ml} + \widehat{\pi}_t^c \tag{A46}$$

$$\widehat{\Delta S}_t = \widehat{s}_t - \widehat{s}_{t-1} + \widehat{\pi}_t^c - \widehat{\pi}_t^f \tag{A47}$$

$$\widehat{\pi}_t^w = \widehat{w}_t - \widehat{w}_{t-1} + \widehat{\pi}_t^c \tag{A48}$$

$$\widehat{\overline{\pi}}_t^x = \widehat{\overline{p}}_t^x - \widehat{\overline{p}}_{t-1}^x + \widehat{\pi}_t^c \tag{A49}$$

$$\widehat{\overline{\pi}}_t^{xl} = \widehat{\overline{p}}_t^{xl} - \widehat{\overline{p}}_{t-1}^{xl} + \widehat{\pi}_t^c - \widehat{\Delta S}_t \tag{A50}$$

$$\widehat{\overline{\pi}}_t^{xp} = \widehat{\overline{p}}_t^{xp} - \widehat{\overline{p}}_{t-1}^{xp} + \widehat{\pi}_t^c \tag{A51}$$

- $\overline{P}^m$: Import price of manufactured goods, local currency (source: OECD International Trade and Competitiveness Indicators).[28]
- S: Nominal effective exchange rate (source: OECD Economic Outlook [Q.GBR.EXCHEB]).
- P^c: RPIX, retail price index excl. mortgage interest payments (source: UK National Statistics [CHMK]/Bank of England).[29]
- $\overline{P}^x$: Export price of manufactured goods, local currency (source: OECD International Trade and Competitiveness Indicators).
- P^y: Producer price index all manufacturing excl. duty (source: UK National Statistics [PVNQ]).

[28] All nominal variables are converted to a common baseyear 2000=100.

[29] As no official seasonally adjusted RPIX exists this series was seasonally adjusted using the X12 method as implemented in EViews.

Table 1: The price and quantity indices in the DSGE model

Quantity index	Price index	Demand functions
$Q_t = \left[\alpha^{\frac{1}{\nu}}\left(Q_t^d\right)^{\frac{\nu-1}{\nu}} + (1-\alpha)^{\frac{1}{\nu}}\left(Q_t^m\right)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}$	$P_t^q = \left[\alpha\left(P_t^y\right)^{1-\nu} + (1-\alpha)\left(P_t^m\right)^{1-\nu}\right]^{\frac{1}{1-\nu}}$	$Q_t^d = \alpha\left(\frac{P_t^y}{P_t^q}\right)^{-\nu} Q_t,\ Q_t^m = (1-\alpha)\left(\frac{P_t^m}{P_t^q}\right)^{-\nu} Q_t$
$Q_t^d = \left[\int_0^1 Y_t^{dq}(i)^{\frac{\theta_t^y-1}{\theta_t^y}} di\right]^{\frac{\theta_t^y}{\theta_t^y-1}}$	$P_t^y = \left[\int_0^1 P_t^y(i)^{1-\theta_t^y} di\right]^{\frac{1}{1-\theta_t^y}}$	$Y_t^{dq}(i) = \left(\frac{P_t^y(i)}{P_t^y}\right)^{-\theta_t^y} Q_t^d$
$Q_t^m = \left[\int_0^1 Y_t^{mq}(m)^{\frac{\theta_t^m-1}{\theta_t^m}} dm\right]^{\frac{\theta_t^m}{\theta_t^m-1}}$	$P_t^m = \left[\int_0^1 P_t^m(m)^{1-\theta_t^m} dm\right]^{\frac{1}{1-\theta_t^m}}$	$Y_t^{mq}(m) = \left(\frac{P_t^m(m)}{P_t^m}\right)^{-\theta_t^m} Q_t^m$
$Z_t = \left[\alpha^{\frac{1}{\nu}}\left(Z_t^d\right)^{\frac{\nu-1}{\nu}} + (1-\alpha)^{\frac{1}{\nu}}\left(Z_t^m\right)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}$	$P_t^z = \left[\alpha\left(P_t^y\right)^{1-\nu} + (1-\alpha)\left(P_t^m\right)^{1-\nu}\right]^{\frac{1}{1-\nu}}$	$Z_t^d = \alpha\left(\frac{P_t^y}{P_t^z}\right)^{-\nu} Z_t,\ Z_t^m = (1-\alpha)\left(\frac{P_t^m}{P_t^z}\right)^{-\nu} Z_t$
$Z_t^d = \left[\int_0^1 Y_t^{dz}(i)^{\frac{\theta_t^y-1}{\theta_t^y}} di\right]^{\frac{\theta_t^y}{\theta_t^y-1}}$	$P_t^y = \left[\int_0^1 P_t^y(i)^{1-\theta_t^y} di\right]^{\frac{1}{1-\theta_t^y}}$	$Y_t^{dz}(i) = \left(\frac{P_t^y(i)}{P_t^y}\right)^{-\theta_t^y} Z_t^d$
$Z_t^m = \left[\int_0^1 Y_t^{mz}(m)^{\frac{\theta_t^m-1}{\theta_t^m}} dm\right]^{\frac{\theta_t^m}{\theta_t^m-1}}$	$P_t^m = \left[\int_0^1 P_t^m(m)^{1-\theta_t^m} dm\right]^{\frac{1}{1-\theta_t^m}}$	$Y_t^{mz}(m) = \left(\frac{P_t^m(m)}{P_t^m}\right)^{-\theta_t^m} Z_t^m$
$Y_t^x = \left[\varpi^{\frac{1}{\theta_t^x}}\left(Y_t^{xp}\right)^{\frac{\theta_t^x-1}{\theta_t^x}} + (1-\varpi)^{\frac{1}{\theta_t^x}}\left(Y_t^{xl}\right)^{\frac{\theta_t^x-1}{\theta_t^x}}\right]^{\frac{\theta_t^x}{\theta_t^x-1}}$	$P_t^x = \left[\varpi\left(P_t^{xp}\right)^{1-\theta_t^x} + (1-\varpi)(S_t P_t^{xl})^{1-\theta_t^x}\right]^{\frac{1}{1-\theta_t^x}}$	$Y_t^{xp} = \varpi\left(\frac{P_t^{xp}}{P_t^x}\right)^{-\theta_t^x} Y_t^x,$ $Y_t^{xl} = (1-\varpi)\left(\frac{S_t P_t^{xl}}{P_t^x}\right)^{-\theta_t^x} Y_t^x$
$Y_t^{xp} = \left[\left(\frac{1}{\varpi}\right)^{\frac{1}{\theta_t^x}} \int_0^{\varpi} Y_t^{xp}(i)^{\frac{\theta_t^x-1}{\theta_t^x}} di\right]^{\frac{\theta_t^x}{\theta_t^x-1}}$	$P_t^{xp} = \left[\frac{1}{\varpi}\int_0^{\varpi} P_t^{xp}(i)^{1-\theta_t^x} di\right]^{\frac{1}{1-\theta_t^x}}$	$Y_t^{xp}(i) = \frac{1}{\varpi}\left(\frac{P_t^{xp}(i)}{P_t^{xp}}\right)^{-\theta_{t+l}^x} Y_t^{xp}$
$Y_t^{xl} = \left[\left(\frac{1}{1-\varpi}\right)^{\frac{1}{\theta_t^x}} \int_{\varpi}^1 Y_t^{xl}(i)^{\frac{\theta_t^x-1}{\theta_t^x}} di\right]^{\frac{\theta_t^x}{\theta_t^x-1}}$	$P_t^{xl} = \left[\frac{1}{1-\varpi}\int_{\varpi}^1 P_t^{xl}(i)^{1-\theta_t^x} di\right]^{\frac{1}{1-\theta_t^x}}$	$Y_t^{xl}(i) = \frac{1}{1-\varpi}\left(\frac{P_t^{xl}(i)}{P_t^{xl}}\right)^{-\theta_t^x} Y_t^{xl}$
$Y_t^m = \left[(\varpi_f)^{\frac{1}{\theta_t^m}}\left(Y_t^{mp}\right)^{\frac{\theta_t^m-1}{\theta_t^m}} + (1-\varpi_f)^{\frac{1}{\theta_t^m}}\left(Y_t^{ml}\right)^{\frac{\theta_t^m-1}{\theta_t^m}}\right]^{\frac{\theta_t^m}{\theta_t^m-1}}$	$P_t^m = \left[\varpi_f\left(S_t P_t^{mp}\right)^{1-\theta_t^m} + (1-\varpi_f)(P_t^{ml})^{1-\theta_t^m}\right]^{\frac{1}{1-\theta_t^m}}$	$Y_t^{mp} = \varpi_f\left(\frac{S_t P_t^{mp}}{P_t^m}\right)^{-\theta_t^m} Y_t^m,$ $Y_t^{ml} = (1-\varpi_f)\left(\frac{P_t^{ml}}{P_t^m}\right)^{-\theta_t^m} Y_t^m$

Continued on next page

Quantity index	Price index	Demand functions
$Y_t^{mp}=\left[\left(\frac{1}{\varpi_f}\right)^{\frac{1}{\theta_t^m}}\int_0^{\varpi_f} Y_t^{mp}(m)^{\frac{\theta_t^m-1}{\theta_t^m}}dm\right]^{\frac{\theta_t^m}{\theta_t^m-1}}$	$P_t^{mp}=\left[\frac{1}{\varpi_f}\int_0^{\varpi_f} P_t^{mp}(m)^{1-\theta_t^m}dm\right]^{\frac{1}{1-\theta_t^m}}$	$Y_t^{mp}(m)=\frac{1}{\varpi_f}\left(\frac{P_t^{mp}(m)}{P_t^{mp}}\right)^{-\theta_t^m}Y_t^{mp}$
$Y_t^{ml}=\left[\left(\frac{1}{1-\varpi_f}\right)^{\frac{1}{\theta_t^m}}\int_{\varpi_f}^{1} Y_t^{ml}(m)^{\frac{\theta_t^m-1}{\theta_t^m}}dm\right]^{\frac{\theta_t^m}{\theta_t^m-1}}$	$P_t^{ml}=\left[\frac{1}{1-\varpi_f}\int_{\varpi_f}^{1} P_t^{ml}(m)^{1-\theta_t^m}dm\right]^{\frac{1}{1-\theta_t^m}}$	$Y_t^{ml}(m)=\frac{1}{1-\varpi_f}\left(\frac{P_t^{ml}(m)}{P_t^{ml}}\right)^{-\theta_t^m}Y_t^{ml}$
$C_t=\left[\int_0^1 C_t(c)^{\frac{\theta_t^c-1}{\theta_t^c}}dc\right]^{\frac{\theta_t^c}{\theta_t^c-1}}$	$P_t^c=\left[\int_0^1 P_t^c(c)^{1-\theta_t^c}dc\right]^{\frac{1}{1-\theta_t^c}}$	$C_t(c)=\left(\frac{P_t^c(c)}{P_t^c}\right)^{-\theta_t^c}C_t$
$H_t=\left[\int_0^1 H_t(j)^{\frac{\theta^h-1}{\theta^h}}dj\right]^{\frac{\theta^h}{\theta^h-1}}$	$W_t=\left[\int_0^1 W_t(j)^{1-\theta^h}dj\right]^{\frac{1}{1-\theta^h}}$	$H_t(j)=\left(\frac{W_t(j)}{W_t}\right)^{-\theta^h}H_t$

Table 2: Baseline calibration

Parameter	Value
Share of intermediate goods production of final goods γ_c	0.42
Share of intermediate goods production of intermediate goods γ_y	0.77
Elasticity of substitution varieties of domestic intermediate goods domestic market θ^y	6
Elasticity of substitution varieties of domestic intermediate goods foreign market θ^x	15
Elasticity of substitution varieties of domestic final goods θ^c	6
Elasticity of substitution varieties of imported intermediate goods θ^m	6
Elasticity of substitution differentiated labour services θ^h	6
Share of domestic intermediate goods production of domestic goods α	0.85
Share of domestic intermediate goods production of goods in foreign economy α_f	0.064
Elasticity of substitution domestic and foreign goods domestic economy ν	1.5
Elasticity of substitution between domestic and foreign goods foreign economy ν_f	1.5
Habit persistence parameter ζ	0.85
Inverse of Frisch elasticity of labour supply χ	3
Weight on labour in utility function η	0.570
Discount factor β	$1.03^{-0.25}$
Inflation target π^c	1.005
Units of labour required to distribute one unit of imported intermediate good δ	0.4
Units of labour required to distribute one unit of imported intermediate good foreign economy δ_f	0.4
Adjustment cost parameter domestic final goods prices ϕ_c	400
Adjustment cost parameter domestic intermediate goods prices ϕ_y	400
Adjustment cost parameter export prices ϕ_x	400
Adjustment cost parameter import prices ϕ_m	400
Adjustment cost parameter wages ϕ_w	400
Proportion of PCP firms domestic economy ϖ	0.6
Proportion of PCP firms foreign economy ϖ_f	0.4
Sensitivity of premium on foreign bond holdings w.r.t. net foreign assets ψ	0.02
Coefficient on lagged interest rates in interest rate rule ρ_R	0.65
Coefficient on inflation in interest rate rule ρ_π	1.8
AR coefficient in process for θ_t^y, ρ_y	0.3
AR coefficient in process for θ_t^x, ρ_x	0.75
AR coefficient in process for θ_t^c, ρ_c	0.5
AR coefficient in process for θ_t^m, ρ_m	0.5
AR coefficient in process for risk premium shock ρ_u	0.9
Standard deviation shock to $\theta_t^y, \varepsilon_{y,t}$	0.2
Standard deviation shock to $\theta_t^x, \varepsilon_{x,t}$	0.35
Standard deviation shock to $\theta_t^c, \varepsilon_{c,t}$	0.2
Standard deviation shock to $\theta_t^m, \varepsilon_{m,t}$	0.35
Standard deviation risk premium shock $\varepsilon_{u,t}$	0.005
Foreign inflation target π_f	1.005

Table 3: *Second order moments: DSGE model and UK data 1980Q1–2003Q4.*

Standard deviation	Data	Model
$\Delta \ln S_t$	0.031	0.035
$\Delta \ln \overline{P}_t^m$	0.017	0.019
$\Delta \ln \overline{P}_t^x$	0.013	0.018
$\Delta \ln P_t^y$	0.004	0.006
$\Delta \ln P_t^c$	0.003	0.006

First-order autocorrelation	Data	Model
$\Delta \ln S_t$	0.21	-0.07
$\Delta \ln \overline{P}_t^m$	0.36	0.29
$\Delta \ln \overline{P}_t^x$	0.29	0.30
$\Delta \ln P_t^y$	0.76	0.86
$\Delta \ln P_t^c$	0.79	0.81

Table 4: Distribution of chosen lag-length for different lag-order selection criteria. VAR in first differences. In per cent.

$T = 100$					
	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$
LR	0.00	67.34	11.86	11.50	9.30
AIC	0.16	90.56	6.72	1.78	0.78
HQ	8.82	91.16	0.02	0.00	0.00
SC	71.52	28.48	0.00	0.00	0.00

$T = 200$					
	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$
LR	0.00	43.08	19.34	20.28	17.30
AIC	0.00	89.42	8.32	1.92	0.34
HQ	0.00	100.00	0.00	0.00	0.00
SC	2.44	97.56	0.00	0.00	0.00

Table 5: Rejection frequencies for single-equation and vector tests for residual autocorrelation up to order four. First-differenced VAR. 5% significance level.

$T = 100$						
	$\Delta \ln \overline{P}_t^m$	$\Delta \ln \overline{P}_t^x$	$\Delta \ln P_t^c$	$\Delta \ln P_t^y$	$\Delta \ln S_t$	Vector test
LR	4.9	7.1	7.4	7.4	6.4	6.7
AIC	5.7	8.3	8.5	8.1	7.4	12.4
HQ	6.4	11.2	15.1	11.2	8.4	22.5
SC	11.3	37.1	66.4	39.5	11.0	75.9
$L = 2$	6.2	9.3	9.4	8.7	8.2	16.4
$L = 4$	5.6	6.5	6.7	8.8	5.9	10.7

$T = 200$						
	$\Delta \ln \overline{P}_t^m$	$\Delta \ln \overline{P}_t^x$	$\Delta \ln P_t^c$	$\Delta \ln P_t^y$	$\Delta \ln S_t$	Vector test
LR						
AIC	6.4	13.8	14.3	15.9	9.6	36.7
HQ	7.3	15.6	16.6	17.3	11.2	42.8
SC	7.5	17.0	18.4	18.8	11.1	44.1
$L = 2$	7.3	15.6	16.6	17.3	11.2	42.8
$L = 4$	6.8	9.9	10.1	15.8	7.4	23.6

Table 6: Absolute value of per cent difference between pointwise mean of estimated accumulated responses and DSGE model's responses over first ten quarters.

$T = 100$

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$
LR	5.6	8.8	25.6	24.1	30.6
AIC	6.0	10.7	29.7	24.8	34.6
HQ	6.6	11.2	30.4	25.1	36.8
SC	11.0	11.9	33.3	24.8	42.9
$L = 2$	6.2	11.5	30.3	25.2	36.2
$L = 4$	6.0	5.4	25.5	22.2	21.7

$T = 200$

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$
LR	8.9	10.2	24.3	20.9	34.8
AIC	10.3	13.4	27.1	23.5	42.4
HQ	10.5	14.1	27.7	24.1	43.8
SC	10.7	14.0	27.7	24.1	43.9
$L = 2$	10.5	14.1	27.7	24.1	43.8
$L = 4$	8.0	8.4	22.0	18.7	29.4

Table 7: Absolute value of per cent difference between pointwise mean of estimated responses and DSGE model's responses over first twenty quarters.

$T = 100$

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$
LR	39.4	6.6	38.7	29.4	81.1
AIC	42.6	7.7	40.0	30.5	88.6
HQ	44.5	8.0	40.2	30.5	92.2
SC	54.3	9.1	38.6	26.7	100.6
$L = 2$	43.6	8.1	40.5	31.0	91.3
$L = 4$	33.0	5.1	35.4	26.9	63.8

$T = 200$

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$
LR	45.9	7.3	34.7	25.4	88.9
AIC	51.0	9.5	38.1	28.5	101.4
HQ	51.7	9.9	38.7	29.1	103.6
SC	52.1	9.9	38.6	29.0	103.8
$L = 2$	51.7	9.9	38.7	29.1	103.6
$L = 4$	42.5	6.3	33.2	23.0	80.0

Table 8: Distribution of chosen lag-length for different lag-order selection criteria. VEqCM. In per cent.

			$T=100$			
	$L=1$	$L=2$	$L=3$	$L=4$	$L=5$	$L=6$
LR	0.0	27.1	49.9	7.3	7.1	8.5
AIC	0.0	47.4	46.2	3.7	1.4	1.2
HQ	0.0	95.3	4.7	0.0	0.0	0.0
SC	0.0	100.0	0.0	0.0	0.0	0.0
			$T=200$			
	$L=1$	$L=2$	$L=3$	$L=4$	$L=5$	$L=6$
LR	0.0	0.4	80.7	7.0	5.5	6.4
AIC	0.0	3.4	95.0	1.6	0.1	0.0
HQ	0.0	62.1	37.9	0.0	0.0	0.0
SC	0.0	99.7	0.3	0.0	0.0	0.0

Table 9: Absolute value of per cent difference between pointwise mean of estimated accumulated responses from VEqCM and DSGE model's responses over first ten quarters.

			$T=100$		
	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$
LR	21.7	26.3	27.7	22.0	25.0
AIC	19.2	24.2	25.2	19.3	22.7
HQ	15.9	22.4	22.4	16.2	19.9
SC	15.5	22.2	22.0	15.9	19.6
$L=3$	20.7	24.9	26.2	20.6	23.8
$L=5$	27.9	31.9	33.2	27.8	30.8
			$T=200$		
	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$
LR	7.7	12.6	16.1	11.4	8.7
AIC	7.0	12.0	15.5	10.8	8.1
HQ	4.8	12.3	14.3	9.1	7.0
SC	3.0	12.7	13.4	7.8	6.1
$L=3$	7.0	11.9	15.5	10.8	8.1
$L=5$	9.8	14.5	17.6	13.3	10.8

Table 10: Absolute value of per cent difference between pointwise mean of estimated accumulated responses from VEqCM and DSGE model's responses over first twenty quarters.

$T = 100$

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$
LR	20.0	25.6	27.3	23.8	20.2
AIC	17.7	22.8	24.5	21.0	17.8
HQ	15.3	18.9	20.9	17.4	14.9
SC	15.1	18.5	20.5	17.0	14.7
$L = 3$	18.4	24.6	25.9	22.5	19.3
$L = 5$	26.7	32.5	33.9	30.5	27.4

$T = 200$

	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$
LR	8.4	10.7	14.2	10.4	9.2
AIC	8.3	9.8	13.2	9.5	9.1
HQ	6.2	7.3	11.1	7.3	9.3
SC	4.9	7.5	9.6	5.7	9.7
$L = 3$	8.4	9.8	13.2	9.5	9.1
$L = 5$	8.5	13.5	17.0	13.3	9.3

Table 11: Distribution of chosen lag-length for different lag-order selection criteria. 5% significance level in individual LR tests. Variables in (log) levels. In per cent.

$T = 100$

	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$	$L = 6$
LR	0.0	13.8	55.4	8.9	9.4	12.4
AIC	0.0	27.7	59.3	6.3	3.1	3.6
HQ	0.0	86.5	13.4	0.0	0.0	0.0
SC	0.0	99.9	0.1	0.0	0.0	0.0

$T = 200$

	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$	$L = 6$
LR	0.0	0.0	78.1	8.2	6.4	7.2
AIC	0.0	0.1	97.6	2.2	0.1	0.0
HQ	0.0	14.2	85.8	0.0	0.0	0.0
SC	0.0	90.5	9.5	0.0	0.0	0.0

Table 12: Rejection frequencies for single-equation and vector tests for non-normality. 5% significance level.

	$T = 100$					
	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$	Vector test
LR	5.0	5.1	4.8	4.8	5.0	5.2
AIC	5.2	5.0	4.9	4.8	5.2	5.2
HQ	4.7	4.9	5.5	4.8	4.9	5.4
SC	4.8	5.0	5.1	4.8	5.1	5.4
$L = 3$	5.1	4.9	4.9	4.6	4.6	5.0
$L = 5$	5.4	4.8	5.1	5.5	5.4	5.2

	$T = 200$					
	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$	Vector test
LR	5.2	4.9	5.3	4.9	5.3	5.6
AIC	5.0	4.9	5.4	5.1	5.2	5.5
HQ	5.4	4.9	5.5	5.0	5.3	5.5
SC	5.0	5.1	4.9	5.4	5.3	5.7
$L = 3$	5.0	5.0	5.5	5.2	5.2	5.5
$L = 5$	5.1	4.9	5.1	5.3	5.4	6.0

Table 13: Rejection frequencies for single-equation and vector tests for residual autocorrelation up to order 5. 5% significance level.

	$T = 100$					
	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$	Vector test
LR	10.2	9.5	13.0	11.6	9.4	19.5
AIC	9.5	9.0	16.6	11.7	8.9	22.5
HQ	10.5	14.5	46.5	21.0	8.8	50.0
SC	11.5	16.9	55.5	24.1	9.1	59.1
$L = 3$	11.0	9.6	10.5	11.4	10.3	23.2
$L = 5$	12.8	12.0	11.8	13.7	12.4	32.6

	$T = 200$					
	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$	Vector test
LR	5.9	4.8	7.1	6.2	5.5	5.5
AIC	7.1	5.6	8.4	7.1	6.6	9.7
HQ	7.4	8.3	20.5	12.3	6.7	22.1
SC	11.1	33.7	89.5	53.2	7.6	89.6
$L = 3$	7.4	5.8	8.6	7.3	6.8	10.7
$L = 5$	6.1	5.3	7.1	6.9	6.1	8.6

Table 14: Frequencies of chosen cointegration rank using Johansen's trace test. Numbers in parentheses denote the preferred rank when using a small-sample correction to the trace test.

$T = 100$						
	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
LR	10.7 (51.7)	31.4 (28.6)	33.6 (11.8)	17.6 (6.1)	5.5 (1.5)	1.2 (0.4)
AIC	10.8 (42.2)	28.4 (26.1)	30.2 (16.9)	21.1 (11.3)	8.2 (3.2)	1.3 (0.5)
HQ	2.5 (8.1)	8.4 (14.5)	28.6 (36.5)	39.7 (30.9)	17.6 (8.7)	3.3 (1.3)
SC	0.0 (0.6)	3.8 (12.2)	29.0 (41.2)	43.8 (34.8)	19.8 (9.8)	3.6 (1.5)
$L = 3$	17.5 (61.2)	41.3 (29.7)	28.0 (7.2)	10.0 (1.6)	2.7 (0.3)	0.5 (0.0)
$L = 5$	13.7 (84.4)	40.1 (13.9)	32.2 (1.7)	10.5 (0.2)	2.5 (0.0)	0.5 (0.0)

$T = 200$						
	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
LR	0.3 (2.1)	3.9 (11.0)	19.0 (29.5)	43.5 (38.4)	28.1 (16.3)	5.2 (2.6)
AIC	0.1 (0.5)	2.1 (6.9)	16.2 (28.7)	45.5 (41.8)	30.5 (19.1)	5.7 (3.0)
HQ	0.0 (0.5)	1.9 (6.3)	14.4 (25.4)	42.4 (39.4)	33.6 (23.4)	7.6 (5.0)
SC	0.0 (0.1)	0.3 (1.0)	2.4 (4.2)	25.6 (31.0)	53.2 (48.7)	18.4 (15.1)
$L = 3$	0.1 (0.5)	2.0 (6.8)	16.1 (28.5)	45.5 (42.0)	30.6 (19.2)	5.7 (3.0)
$L = 5$	1.5 (12.3)	11.9 (32.9)	32.0 (32.7)	37.2 (16.9)	14.7 (4.3)	2.6 (0.1)

Table 15: Rejection frequencies for LR tests of restrictions on cointegration space conditional on $r = 4$. 5% significance level. Numbers in parentheses are rejection frequencies based only on the datasets in which the correct cointegration rank is chosen.

$T = 100$					
	$\ln(\overline{P}_t^m/P_t^c) \sim I(0)$	$\ln(\overline{P}_t^x/P_t^c) \sim I(0)$	$\ln(P_t^y/P_t^c) \sim I(0)$	$\ln(S_t/P_t^c) + 0.005t \sim I(0)$	Joint
LR	27.9	24.8	24.4	26.5	81.6 (94.6)
AIC	29.1	24.2	23.9	27.3	79.2 (91.7)
HQ	38.8	29.4	28.4	35.8	83.3 (92.1)
SC	41.2	30.3	29.6	38.1	84.2 (91.0)
$L = 3$	22.4	20.2	19.7	21.1	73.4 (81.5)
$L = 5$	26.8	25.9	25.7	26.3	88.1 (94.4)

$T = 200$					
	$\ln(\overline{P}_t^m/P_t^c) \sim I(0)$	$\ln(\overline{P}_t^x/P_t^c) \sim I(0)$	$\ln(P_t^y/P_t^c) \sim I(0)$	$\ln(S_t/P_t^c) + 0.005t \sim I(0)$	Joint
LR	19.4	13.5	15.4	15.4	40.6 (45.7)
AIC	18.2	13.4	14.7	14.5	37.2 (43.2)
HQ	20.0	15.1	16.4	16.6	40.3 (49.0)
SC	29.3	25.5	25.0	26.0	60.5 (64.5)
$L = 3$	18.2	13.2	14.5	14.3	36.7 (42.8)
$L = 5$	22.0	14.3	16.7	17.9	48.0 (57.9)

Table 16: *Distribution of chosen lag-length for different lag-order selection criteria. Data generated from VEqCM(5) [VEqCM(3)]. 5% significance level in individual LR tests. Variables in (log) levels. In per cent.*

$T = 100$						
	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$	$L = 6$
LR	0.0 [0.0]	14.1 [36.3]	54.4 [32.8]	9.0 [8.6]	10.7 [10.0]	11.8 [12.3]
AIC	0.0 [0.0]	27.5 [61.8]	59.5 [29.5]	6.6 [3.9]	3.0 [2.0]	3.3 [2.8]
HQ	0.0 [0.0]	87.8 [98.4]	12.2 [1.5]	0.0 [0.0]	0.0 [0.0]	0.0 [0.0]
SC	0.0 [0.0]	99.9 [100.0]	0.1 [0.0]	0.0 [0.0]	0.0 [0.0]	0.0 [0.0]
$T = 200$						
	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$	$L = 6$
LR	0.0 [0.0]	0.0 [6.9]	79.2 [75.3]	7.1 [5.8]	7.0 [5.3]	6.7 [6.7]
AIC	0.0 [0.0]	0.1 [24.5]	97.7 [74.3]	2.1 [1.1]	0.1 [0.0]	0.0 [0.0]
HQ	0.0 [0.0]	16.1 [93.0]	83.9 [7.0]	0.0 [0.0]	0.0 [0.0]	0.0 [0.0]
SC	0.0 [0.0]	92.0 [100.0]	8.0 [0.0]	0.0 [0.0]	0.0 [0.0]	0.0 [0.0]

Table 17: *Frequencies of chosen cointegration rank using Johansen's trace test. 5% significance level. Data generated from VEqCM(5) [VEqCM(3)].*

$T = 100$						
	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
LR	11.5 [0.6]	33.2 [14.5]	31.6 [39.7]	17.5 [33.6]	5.3 [9.9]	0.8 [1.6]
AIC	11.5 [0.4]	30.6 [10.7]	28.6 [38.0]	20.5 [36.1]	7.6 [12.8]	1.2 [2.0]
HQ	2.1 [0.3]	8.7 [6.8]	29.0 [35.0]	39.8 [40.2]	18.1 [15.4]	2.5 [2.3]
SC	0.0 [0.3]	3.5 [6.7]	29.6 [34.9]	44.0 [40.3]	19.9 [15.5]	2.9 [2.3]
$L = 3$	18.9 [2.5]	42.1 [23.1]	27.2 [41.9]	9.5 [25.7]	1.9 [5.9]	0.3 [0.1]
$L = 5$	14.6 [5.2]	41.6 [30.1]	31.8 [40.3]	9.7 [19.4]	2.0 [4.4]	0.2 [0.6]
$T = 200$						
	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
LR	0.2 [0.0]	5.1 [0.3]	25.1 [6.8]	44.8 [52.3]	21.8 [37.1]	3.0 [3.5]
AIC	0.0 [0.0]	3.1 [0.0]	22.6 [3.3]	46.6 [48.5]	24.5 [43.5]	3.2 [4.7]
HQ	0.0 [0.0]	2.9 [0.0]	20.0 [1.4]	42.8 [37.2]	29.9 [54.5]	4.3 [6.9]
SC	0.0 [0.0]	0.3 [0.0]	2.5 [0.3]	27.3 [27.6]	57.5 [62.8]	12.4 [9.2]
$L = 3$	0.0 [0.0]	3.1 [0.0]	22.5 [4.1]	46.7 [54.0]	24.5 [38.6]	3.1 [3.3]
$L = 5$	1.5 [0.0]	16.2 [1.7]	37.1 [22.6]	33.2 [52.0]	10.2 [21.5]	1.7 [2.3]

Table 18: Rejection frequencies for single-equation and vector tests for residual autocorrelation up to order five. Data generated by VEqCM(5) [VEqCM(3)]. 5% significance level.

$T = 100$						
	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$	Vector test
LR	10.1 [9.2]	9.8 [9.5]	13.1 [25.6]	11.6 [12.3]	9.7 [9.7]	18.5 [18.6]
AIC	9.3 [8.1]	8.9 [8.3]	16.5 [39.0]	12.5 [12.8]	8.6 [8.2]	21.7 [22.8]
HQ	9.7 [9.1]	14.4 [9.1]	48.2 [66.2]	22.3 [14.5]	7.8 [8.5]	50.2 [38.3]
SC	10.2 [9.1]	16.6 [9.1]	56.9 [67.6]	25.5 [14.4]	8.1 [8.4]	58.4 [39.1]
$L = 3$	10.5 [11.0]	8.7 [9.5]	9.8 [10.0]	11.3 [13.3]	10.1 [10.9]	23.3 [23.6]
$L = 5$	12.3 [13.0]	12.1 [13.2]	12.4 [13.2]	12.7 [14.4]	12.0 [12.5]	33.0 [32.8]

$T = 200$						
	$\ln \overline{P}_t^m$	$\ln \overline{P}_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln S_t$	Vector test
LR	5.4 [4.9]	4.7 [4.8]	6.7 [11.0]	6.3 [6.2]	5.3 [5.1]	5.1 [4.6]
AIC	5.9 [5.2]	4.8 [4.9]	8.5 [27.7]	7.1 [6.6]	5.9 [5.6]	8.8 [11.7]
HQ	6.3 [5.8]	8.5 [5.4]	22.6 [92.2]	13.2 [7.9]	5.8 [7.1]	22.1 [51.2]
SC	10.9 [5.6]	36.3 [5.6]	90.9 [98.9]	55.8 [8.3]	6.8 [7.2]	90.2 [58.2]
$L = 3$	6.1 [5.7]	5.0 [5.7]	8.7 [6.1]	7.2 [6.8]	6.1 [5.9]	9.7 [7.9]
$L = 5$	5.5 [6.0]	5.7 [5.6]	5.1 [5.4]	6.2 [6.6]	6.0 [6.1]	7.3 [7.4]

Table 19: Rejection frequencies for LR tests of restrictions on cointegration space conditional on $r = 4$. 5% significance level. Data generated by VEqCM(5) [VEqCM(3)].

$T = 100$					
	$\ln(\overline{P}_t^m/P_t^c) \sim I(0)$	$\ln(\overline{P}_t^x/P_t^c) \sim I(0)$	$\ln(P_t^y/P_t^c) \sim I(0)$	$\ln(S_t/P_t^c) + 0.005t \sim I(0)$	Joint
LR	29.0 [34.4]	23.3 [22.3]	23.2 [25.3]	26.9 [35.5]	81.3 [74.6]
AIC	29.7 [33.7]	23.0 [20.5]	22.4 [23.8]	27.4 [8.2]	79.1 [69.7]
HQ	38.9 [33.8]	28.0 [18.9]	27.0 [23.5]	35.9 [35.5]	83.2 [67.1]
SC	17.1 [33.9]	30.0 [18.9]	8.4 [23.4]	26.5 [35.6]	58.4 [66.8]
$L = 3$	23.1 [29.3]	18.7 [18.3]	18.7 [20.1]	21.9 [29.9]	73.7 [65.9]
$L = 5$	28.1 [34.9]	24.9 [25.2]	25.1 [26.1]	27.6 [35.9]	87.8 [85.1]

$T = 200$					
	$\ln(\overline{P}_t^m/P_t^c) \sim I(0)$	$\ln(\overline{P}_t^x/P_t^c) \sim I(0)$	$\ln(P_t^y/P_t^c) \sim I(0)$	$\ln(S_t/P_t^c) + 0.005t \sim I(0)$	Joint
LR	14.5 [16.4]	12.2 [11.8]	12.6 [12.6]	14.2 [16.8]	35.1 [28.6]
AIC	14.3 [18.0]	11.7 [12.6]	12.3 [14.2]	13.8 [18.0]	31.8 [28.1]
HQ	17.5 [24.0]	14.0 [16.2]	14.6 [20.9]	16.4 [24.3]	36.1 [34.0]
SC	31.4 [24.5]	25.8 [16.5]	26.4 [21.8]	27.7 [24.9]	60.7 [34.6]
$L = 3$	14.4 [15.3]	11.5 [10.9]	12.2 [11.4]	13.7 [15.3]	31.4 [25.2]
$L = 5$	15.4 [17.5]	12.4 [13.0]	13.3 [13.3]	15.0 [18.1]	40.9 [35.3]

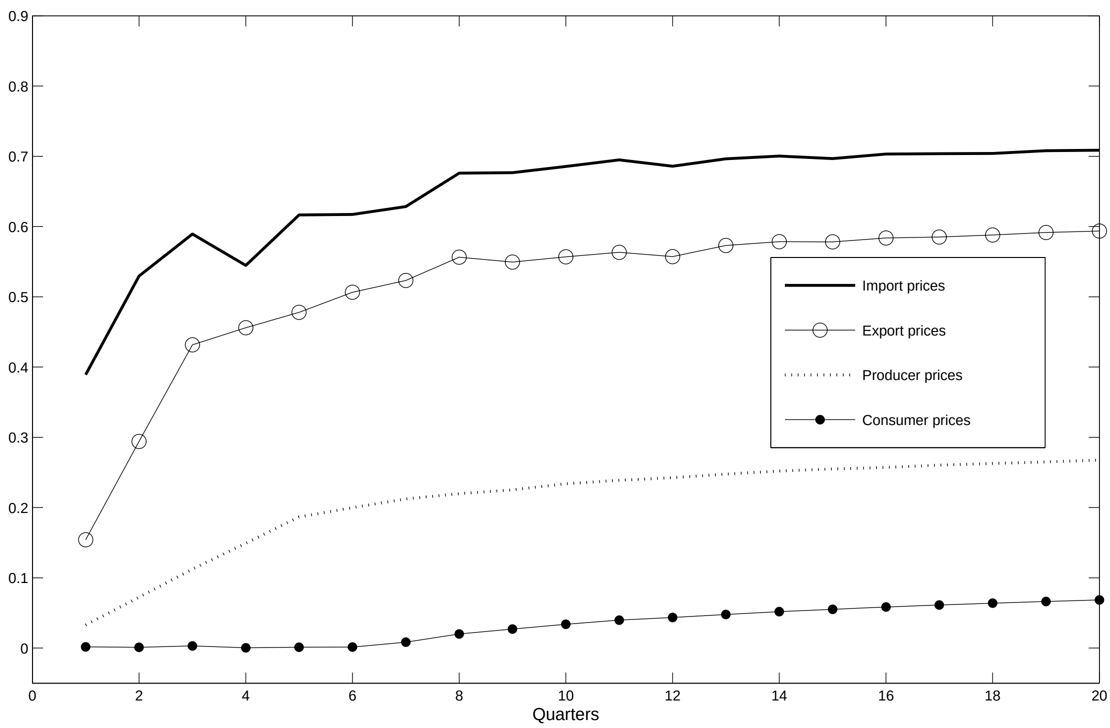

Figure 1: Normalised impulse responses to exchange rate shock. UK data. First-differenced VAR(4). Exchange rate first in recursive ordering.

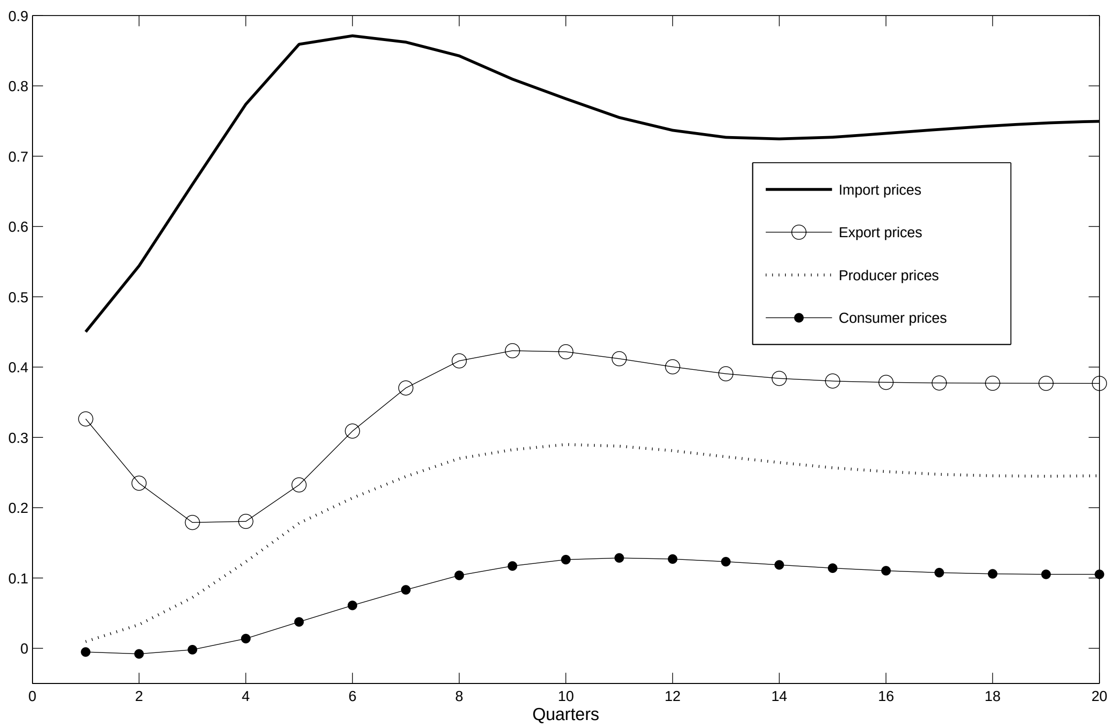

Figure 2: Normalised responses to exchange rate shock. Mean of 5000 datasets from DSGE model. $T = 100$. *First-differenced VAR(4). Exchange rate first in recursive ordering.*

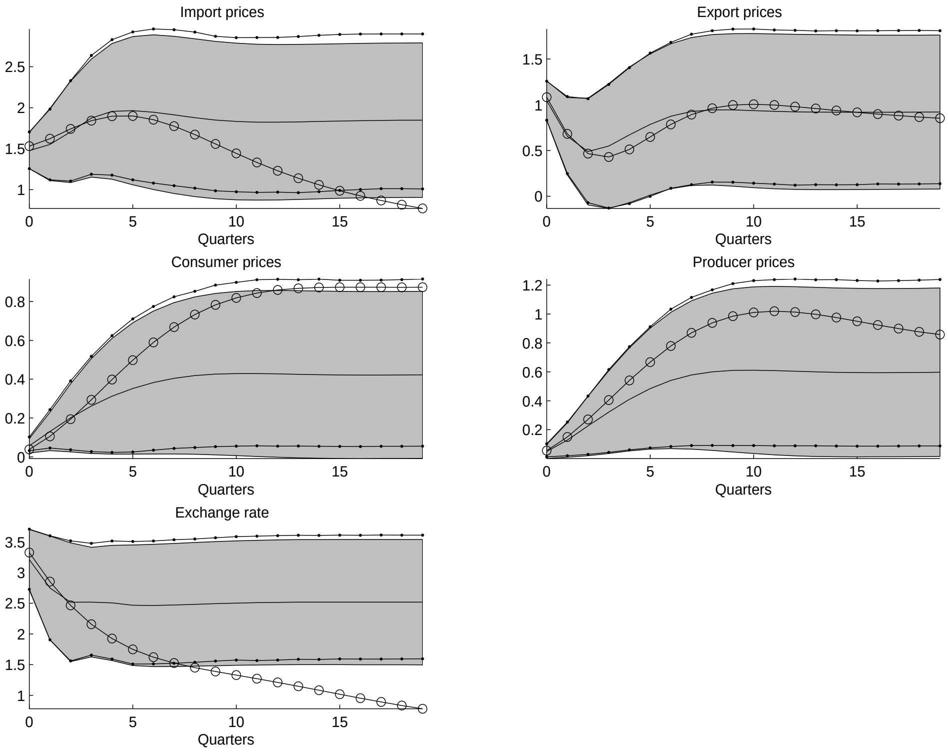

Figure 3: Responses to a one standard deviation UIP shock. In per cent. $T = 100$*,* $L = 2$*. Line with circles: DSGE model responses. Solid line: Pointwise mean of simulated responses. Shaded area: pointwise mean plus/minus* 1.96 *times the standard deviations of the simulated responses. Line with points: 95% percentile interval for simulated responses.*

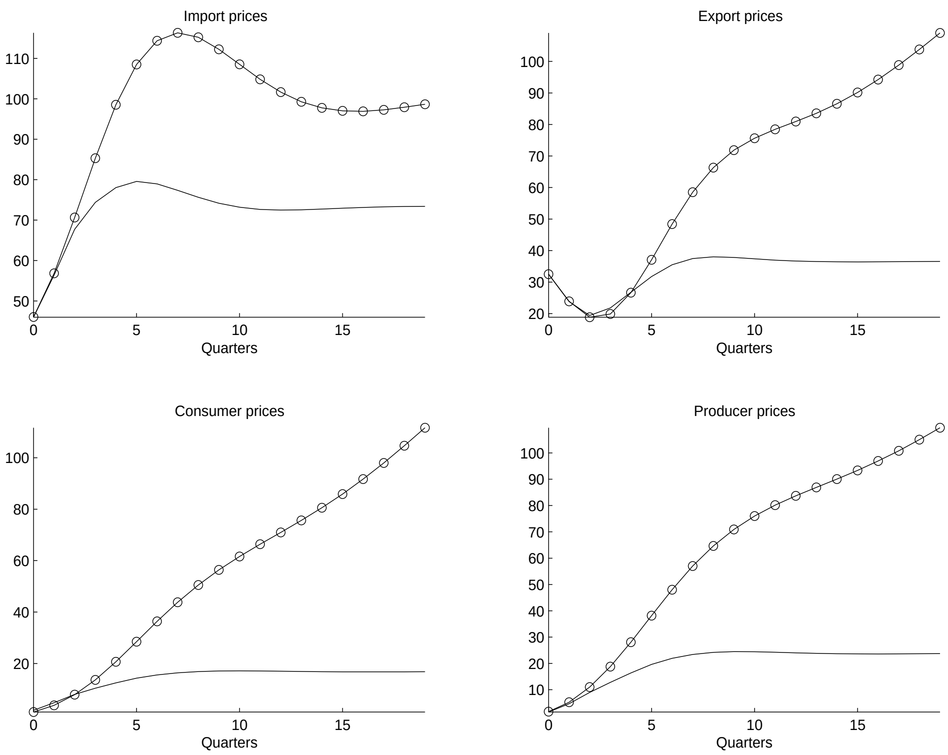

Figure 4: Responses to UIP shock normalised on exchange rate response. In per cent. $T = 100$*,* $L = 2$*. Line with circles: DSGE model responses. Solid line: pointwise mean of simulated responses.*

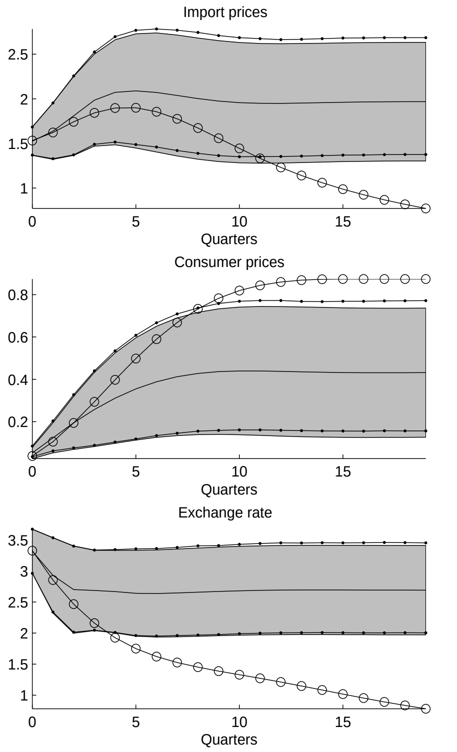

Figure 5: Accumulated responses to one standard deviation UIP shock. In per cent. $T = 200$, $L = 2$. *Line with circles: DSGE model responses. Solid line: Pointwise mean of simulated responses. Shaded area: pointwise mean plus/minus* 1.96 *times the standard deviations of the simulated responses. Line with points: 95% percentile interval for simulated responses.*

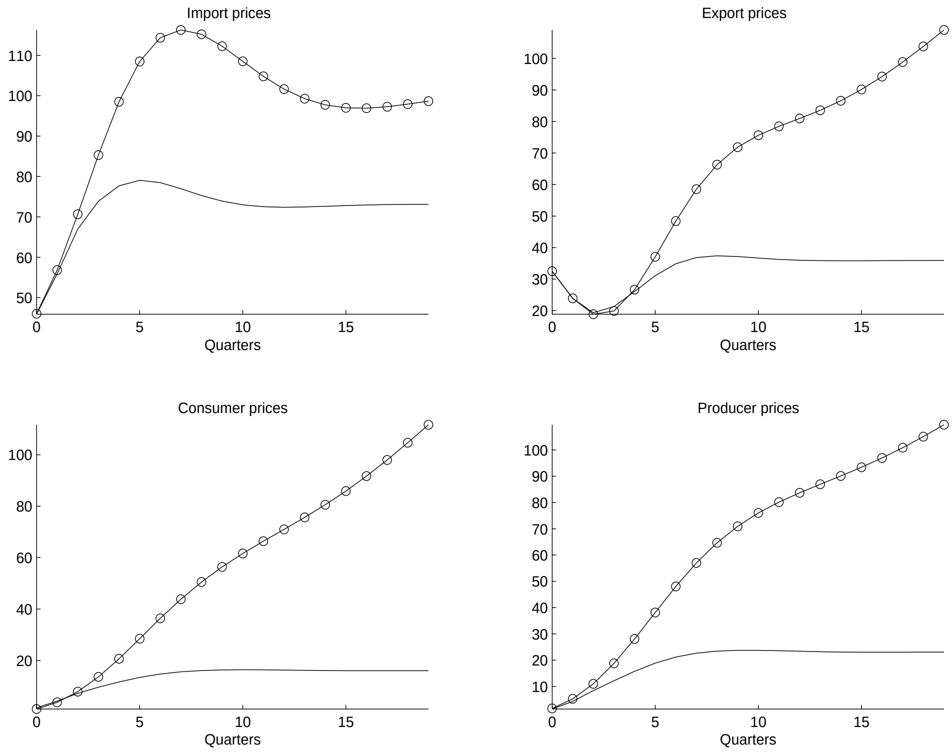

Figure 6: Responses to UIP shock normalised on exchange rate response. In per cent. $T = 200$, $L = 2$. *Line with circles: DSGE model responses. Solid line: pointwise mean of simulated responses.*

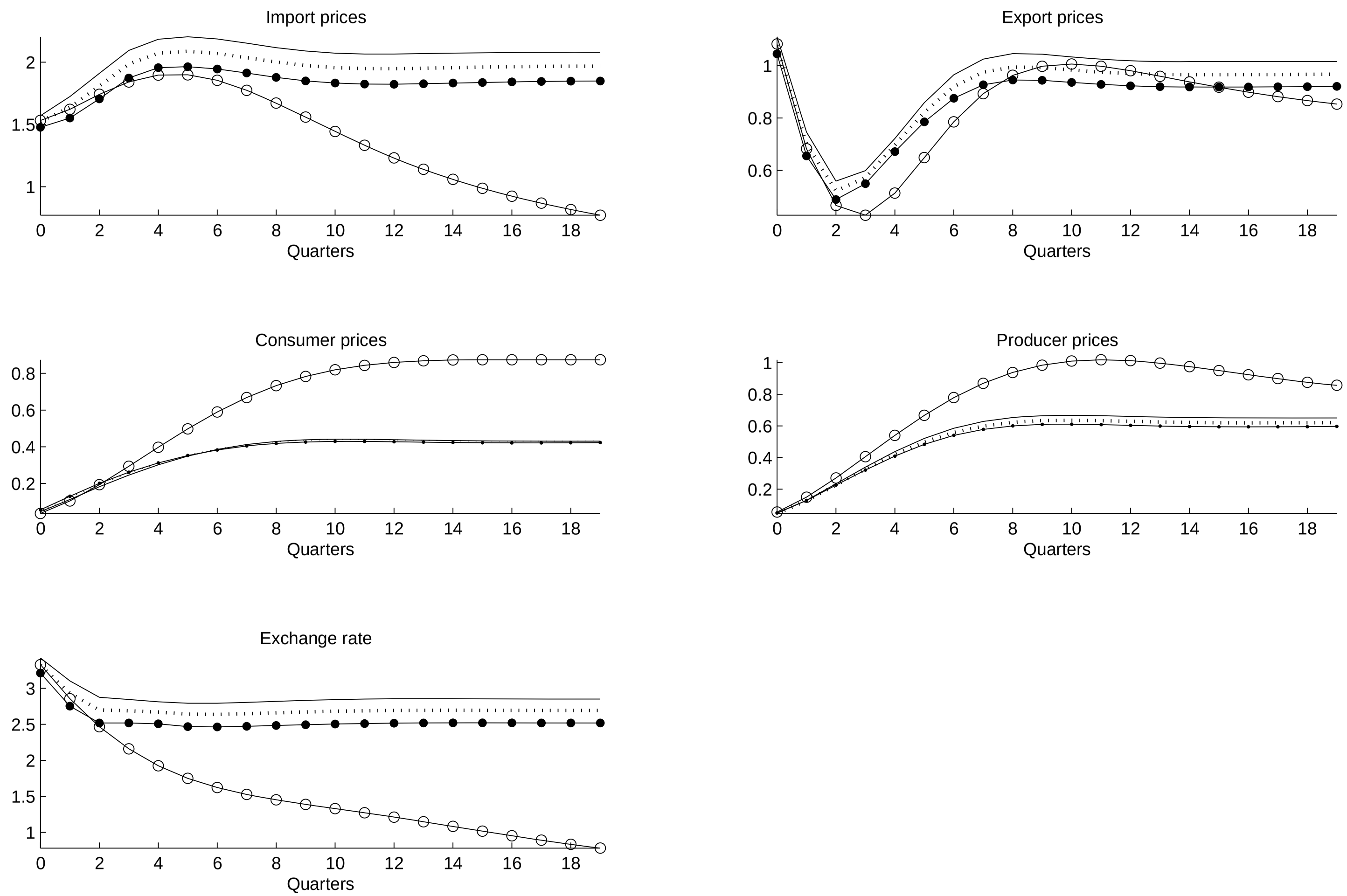

Figure 7: Accumulated responses to one standard deviation UIP shock. In per cent. $L = 2$. Line with circles: DSGE model responses. Solid line: Responses from population VAR. Line with points: Responses from VAR when $T = 200$. Dashed line: Responses from VAR when $T = 100$.

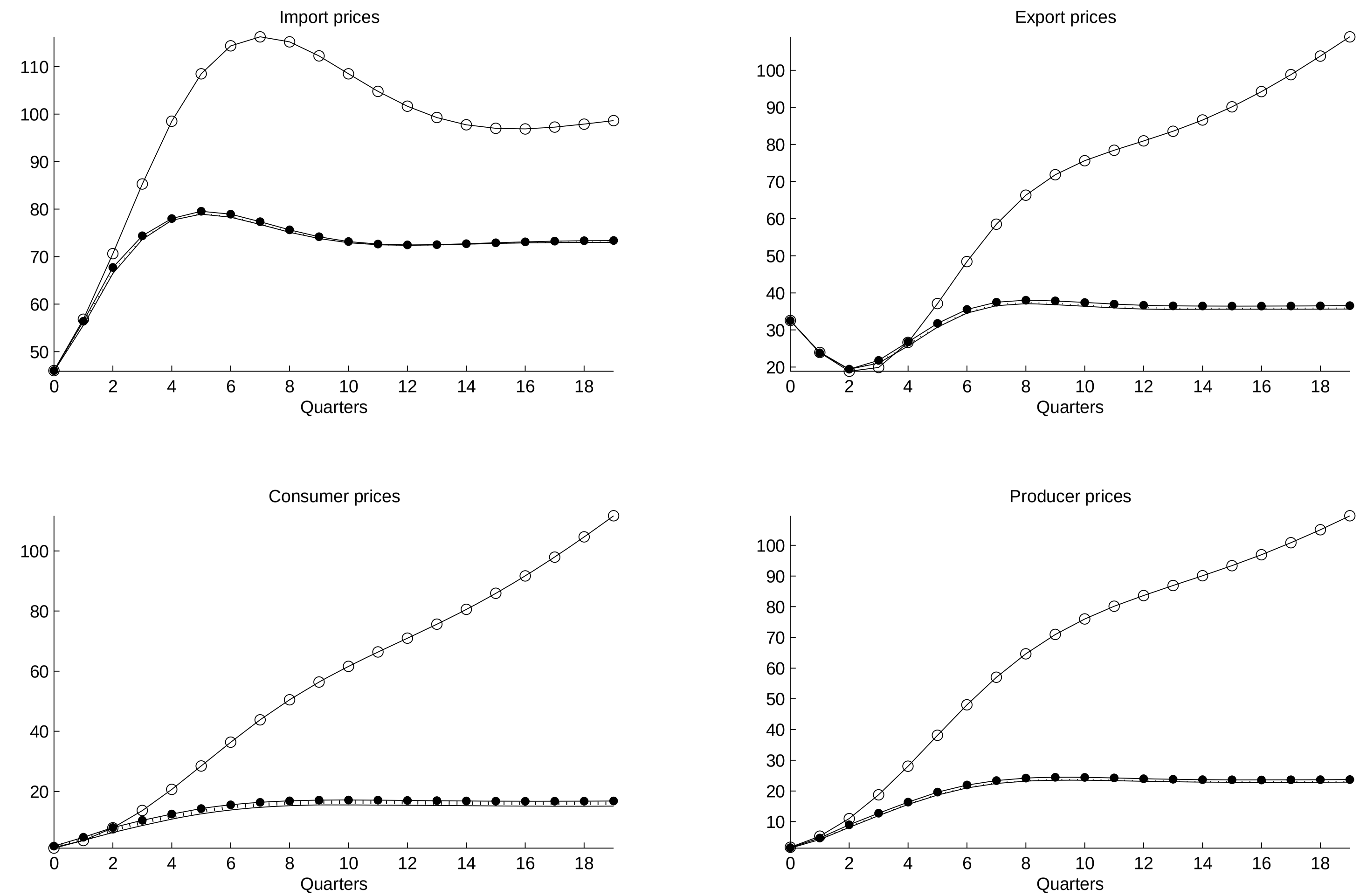

Figure 8: Normalised responses to one standard deviation UIP shock. In per cent. $L = 2$. Line with circles: DSGE model responses. Solid line: Responses from population VAR. Line with points: Responses from VAR when $T = 200$. Dashed line: Responses from VAR when $T = 100$.

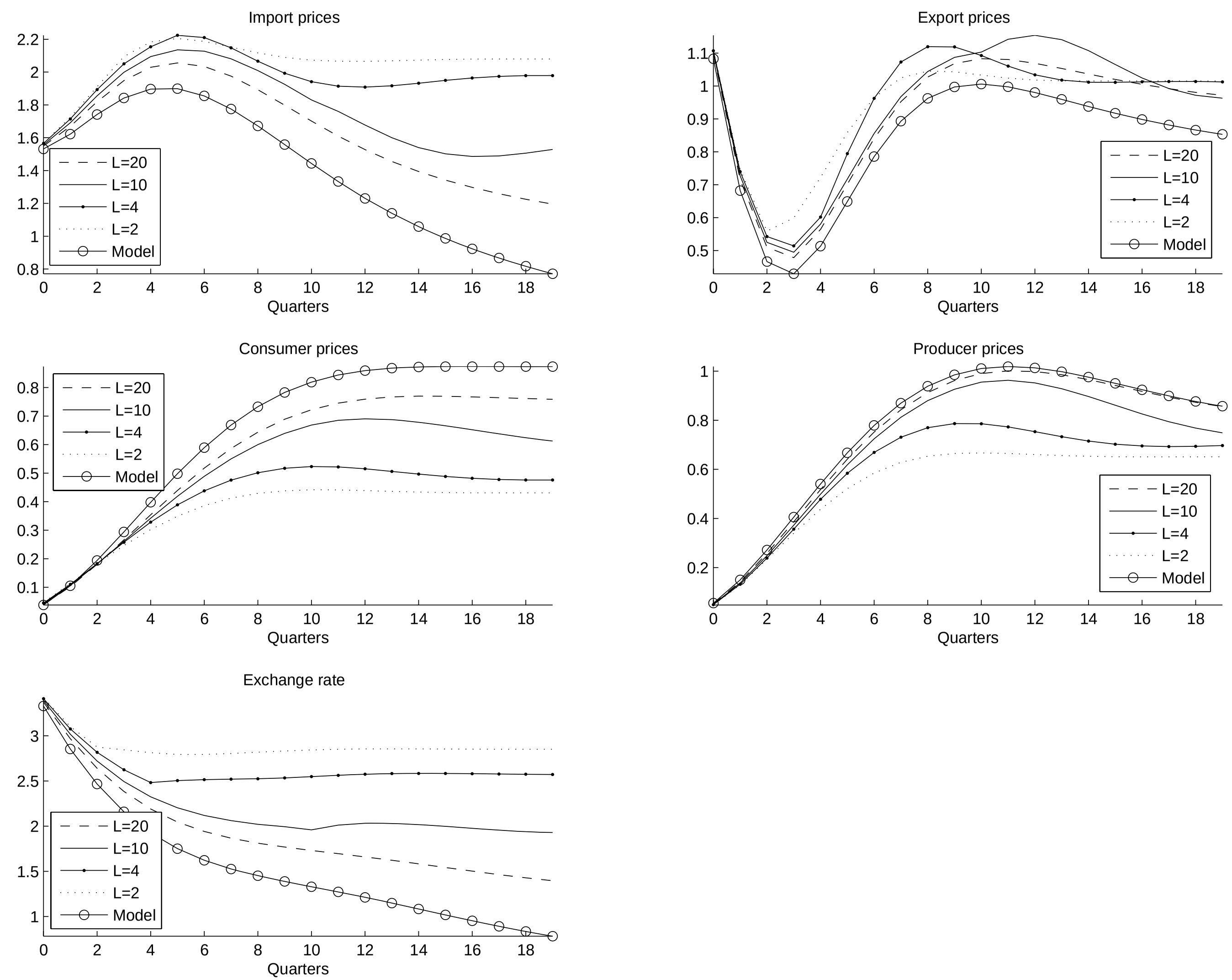

Figure 9: Accumulated responses to one standard deviation UIP shock in population version of VAR for different lag-orders. In per cent.

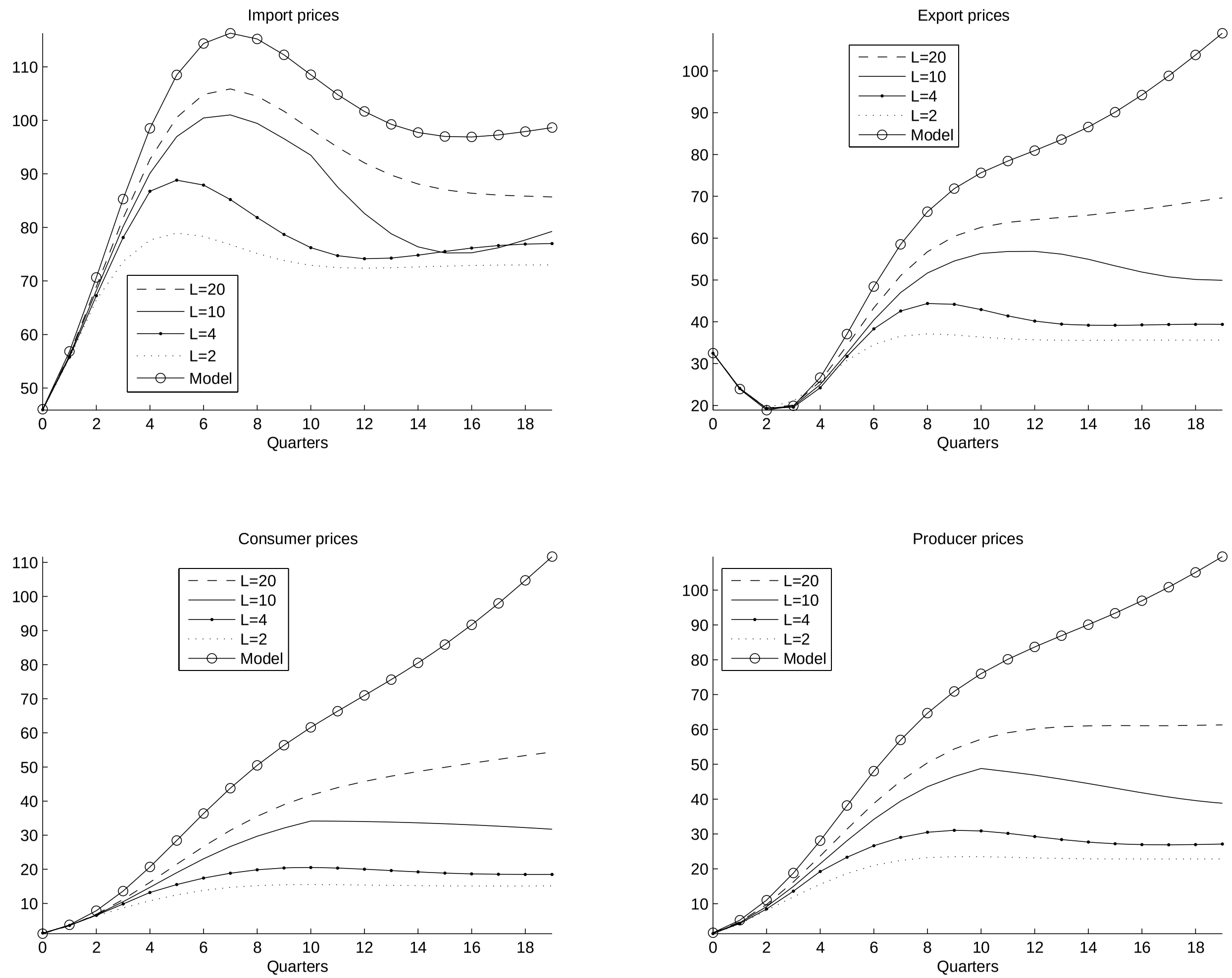

Figure 10: Normalised impulse responses to one standard deviation UIP shock in population version of VAR for different lag-orders. In per cent.

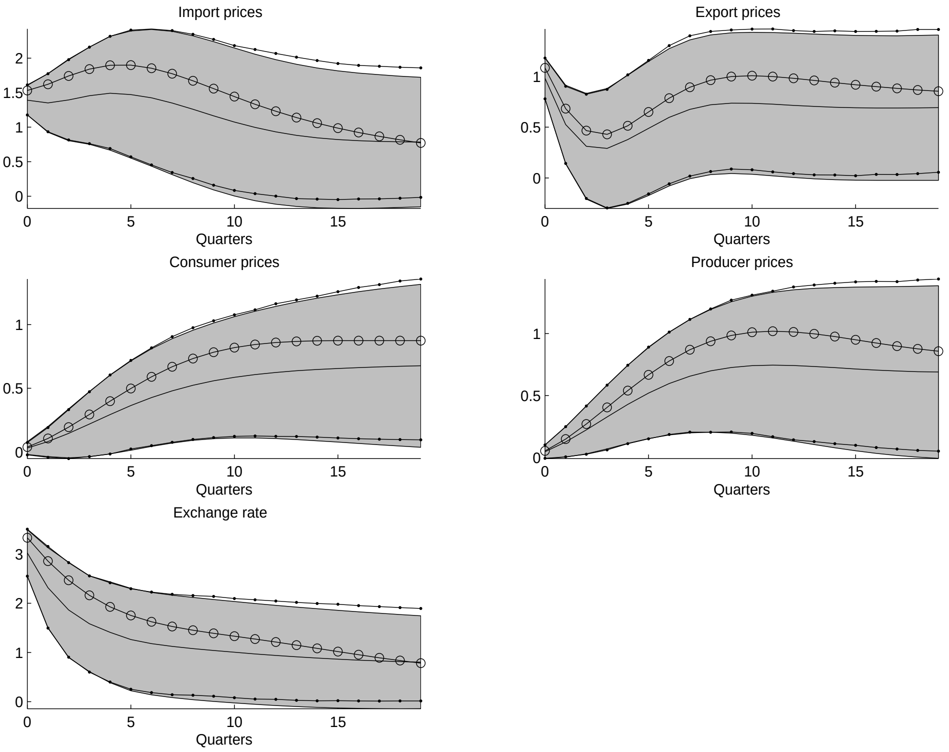

Figure 11: Accumulated responses to one standard deviation UIP shock. In per cent. VEqCM. $T = 100$*,* $L = 3$*. Line with circles: DSGE model responses. Solid line: Pointwise mean of simulated responses. Shaded area: pointwise mean plus/minus* 1.96 *times the standard deviations of the simulated responses. Line with points: 95% percentile interval for simulated responses.*

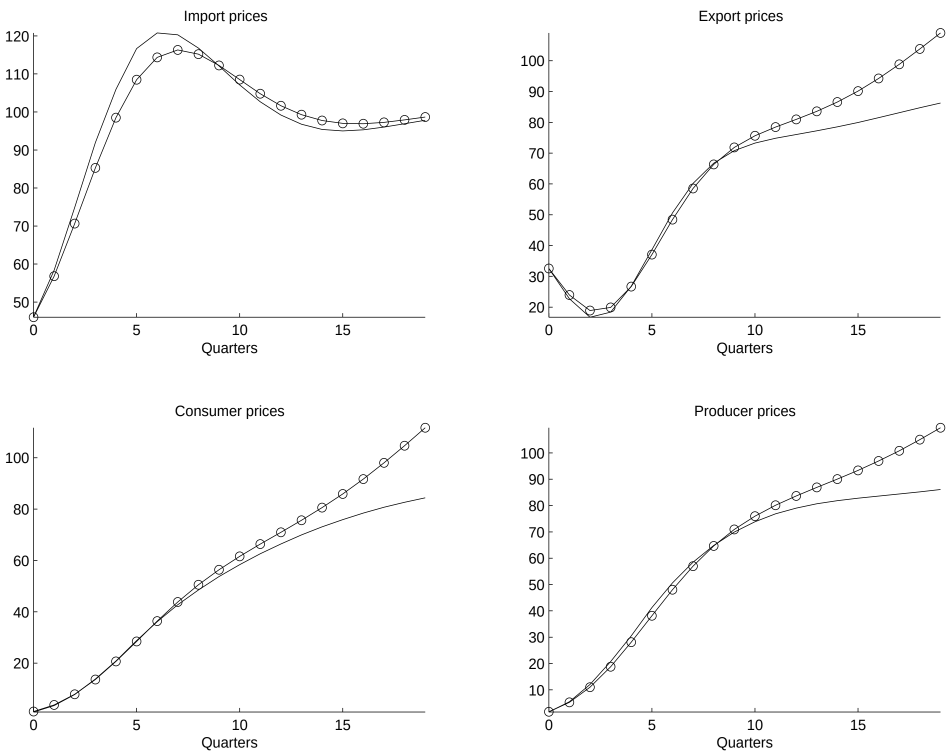

Figure 12: Normalised responses to one standard deviation UIP shock. In per cent. VEqCM. $T = 100$*,* $L = 3$*. Line with circles: DSGE model responses. Solid line: pointwise mean of simulated responses.*

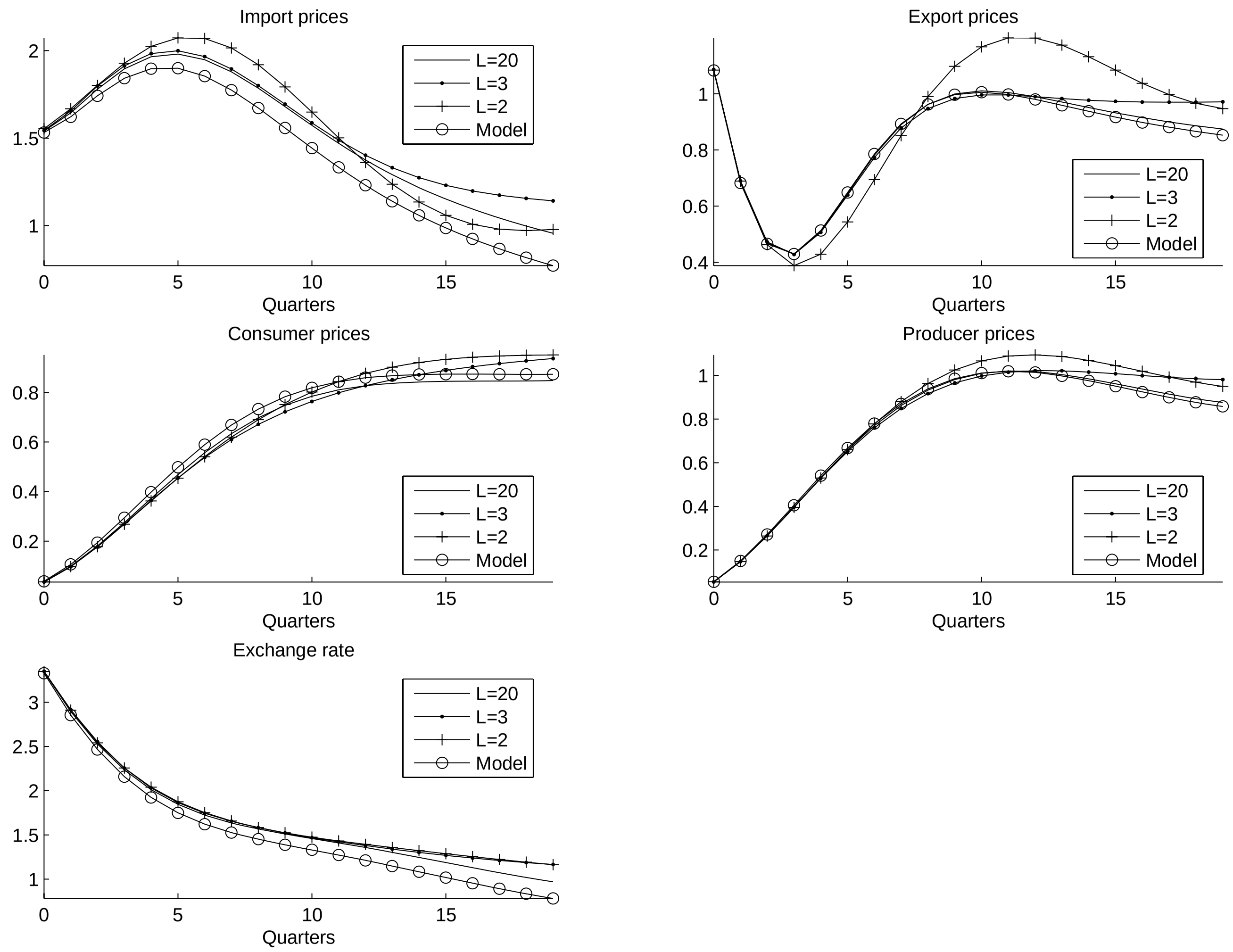

Figure 13: Accumulated responses to one standard deviation UIP shock from population version of VEqCM for different lag-orders. In per cent.

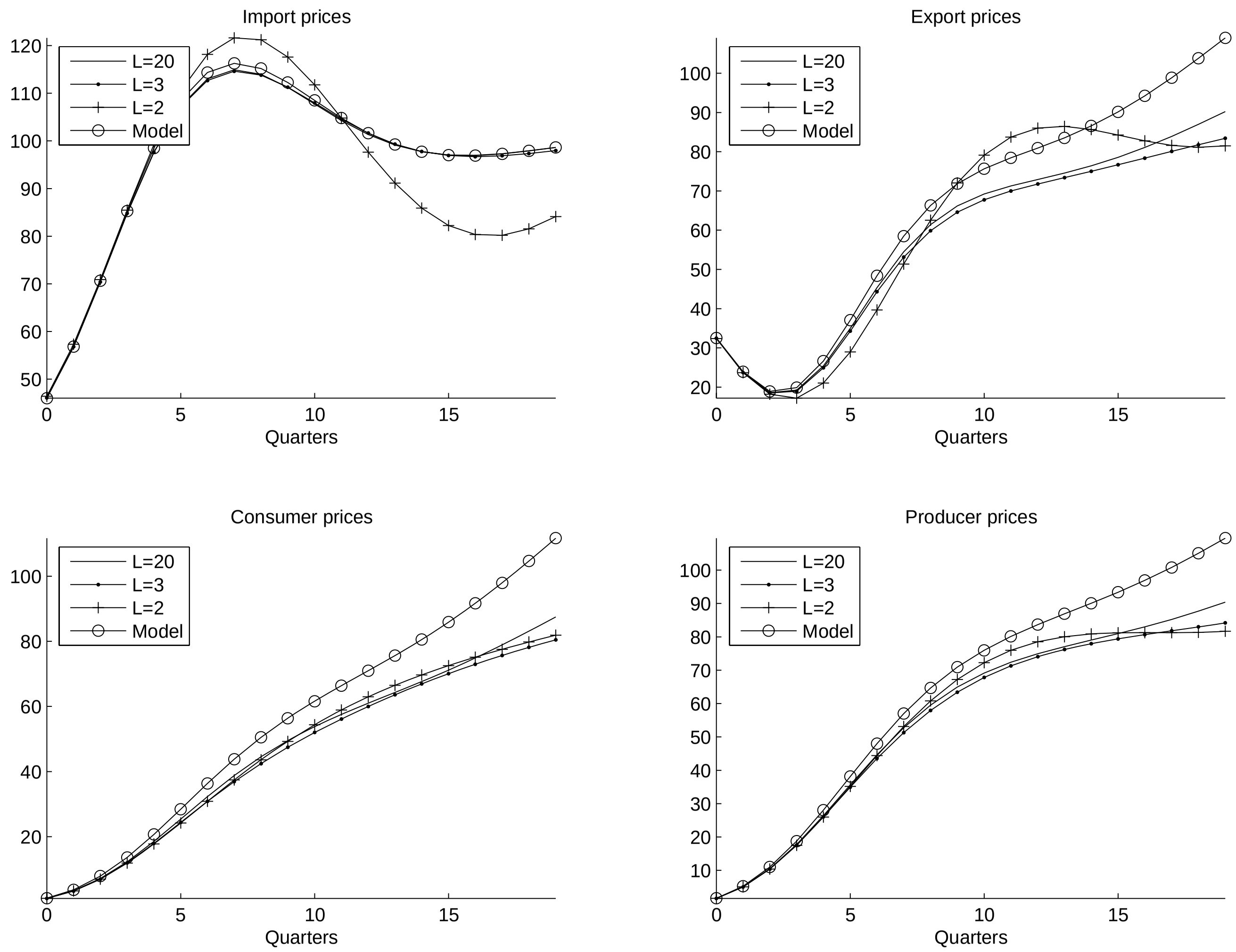

Figure 14: Normalised responses to one standard deviation UIP shock from population version of VEqCM for different lag-orders. In per cent.

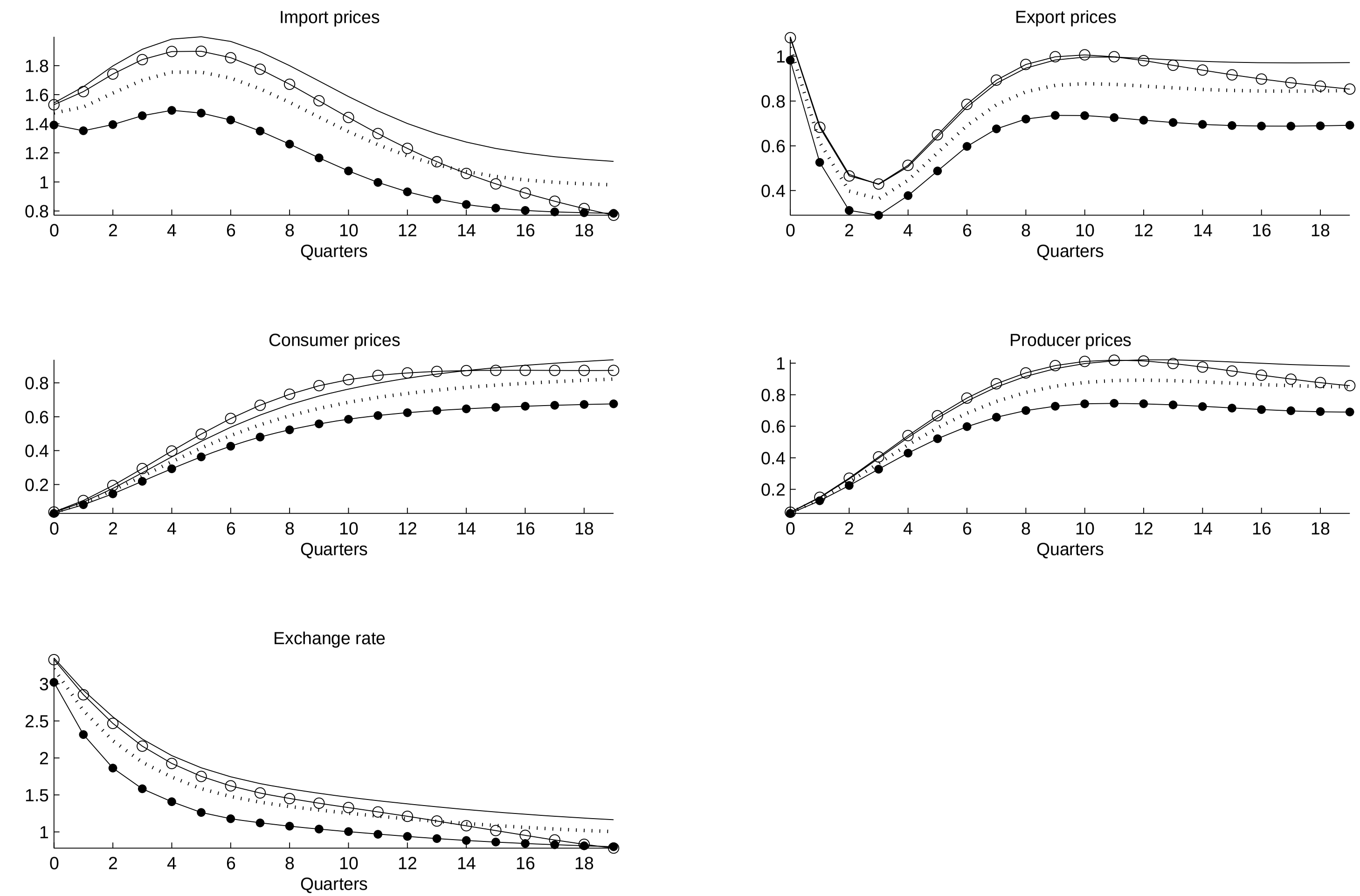

Figure 15: Accumulated responses to one standard deviation UIP shock from VEqCM. In per cent. $L = 3$. Line with circles: DSGE model responses. Solid line: Responses from population VAR. Line with points: Responses from VAR when $T = 200$. Dashed line: Responses from VAR when $T = 100$.

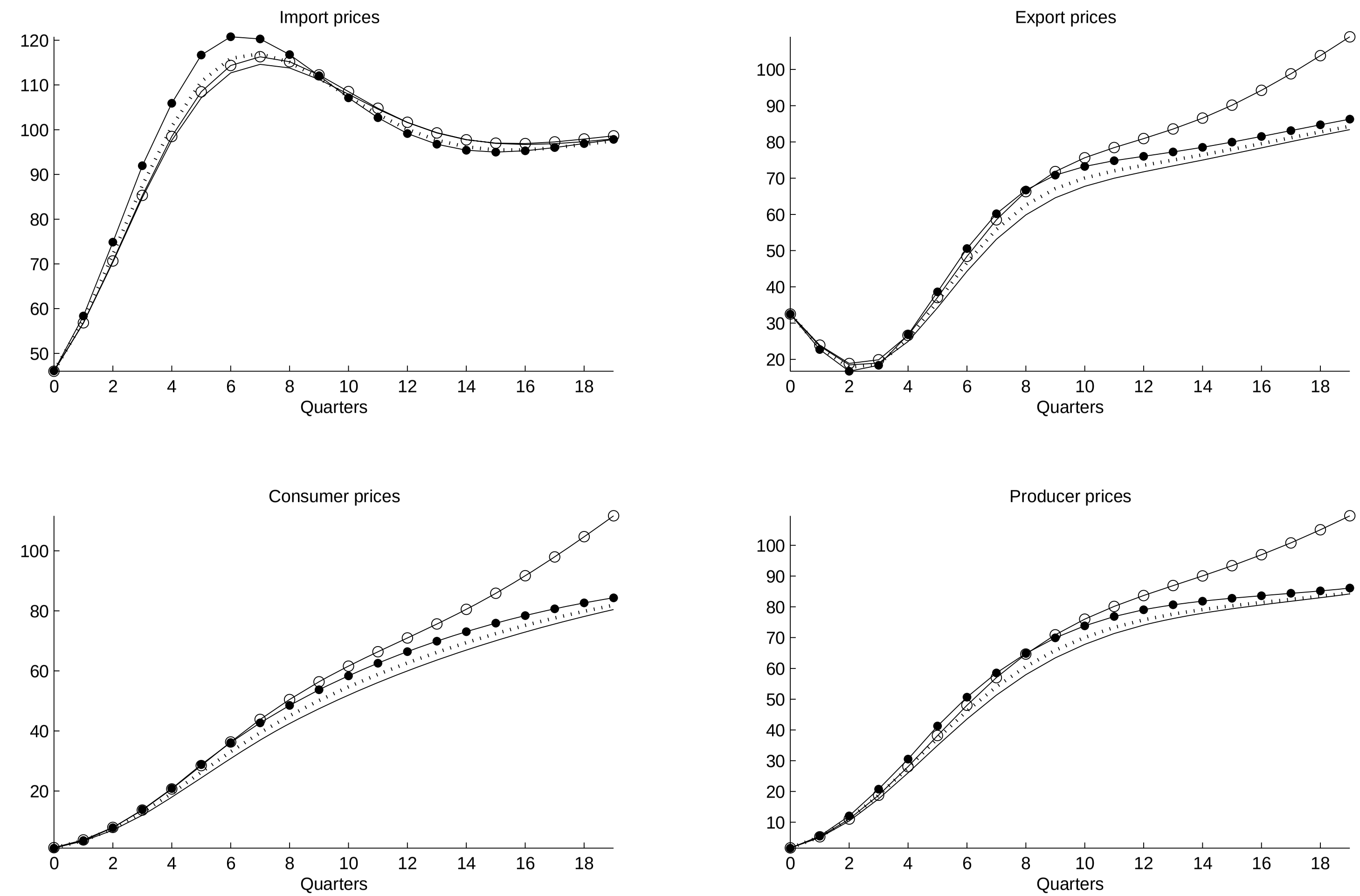

Figure 16: Normalised responses to one standard deviation UIP shock from VEqCM. In per cent. $L = 3$. Line with circles: DSGE model responses. Solid line: Responses from population VAR. Line with points: Responses from VAR when $T = 200$. Dashed line: Responses from VAR when $T = 100$.

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