An analysis of the Government Petroleum Fund's equity allocation

Norges Bank submitted the following letter to the Ministry of Finance on 15 March 2001

1. Introduction

In its letter of 20 December 2000, the Ministry of Finance announced that it would undertake a new evaluation of the Petroleum Fund's equity allocation. This was deemed necessary as a result of the increased size of the Fund and the longer investment horizon than assumed when the equity proportion was set at 40 per cent in the autumn of 1997. Norges Bank was requested to assess factors that are relevant to this evaluation. The Ministry was particularly interested in an assessment of the Fund's expected return and risk in connection with different equity allocations. Moreover, the Ministry wished to receive information concerning the equity allocation chosen by large international funds. The Ministry also requested Norges Bank to evaluate any operational consequences of a change in the equity allocation.

The choice of equity allocation is the one decision that will be of greatest importance to the Fund's return and risk. It is reasonable to assume that an increase in the equity proportion will increase both the Fund's return and risk. When choosing the equity allocation, a trade off must be made between return and risk. In this connection the owner's risk tolerance will be decisive. For this reason, Norges Bank will not provide specific advice concerning the equity allocations in this submission, but will instead discuss how changes in the equity proportion may influence the Fund's expected return and risk.

In Norges Bank's letter of 10 April 1997 to the Ministry of Finance, the Bank discussed what would be an appropriate investment strategy for the Petroleum Fund in the light of the upward adjustment of the Fund's investment horizon. In the letter, the Bank argued that part of the Fund's capital should be invested in equities because this would give the Fund a better risk-return trade-off. Norges Bank did not provide specific advice as to how large the equity allocation should be, but concluded that professional managers would not have any strong objections to an equity proportion of at least 30 per cent. It was also pointed out that other management institutions responsible for large portfolios with a long investment horizon often have a higher proportion than this. The analyses presented in the May 1997 letter are updated and expanded in this submission. In addition, a simulation model is used to examine the

properties of equity and fixed income portfolios over longer investment horizons. The submission also discusses the equity allocation chosen by other funds. The main conclusions of the analyses in this letter are summarised in section 8.

2. Choice of investment strategy

It is natural to evaluate the Petroleum Fund's investment strategy at regular intervals, partly because the Fund's size and investment horizon change over time. The greatest change in the Petroleum Fund's investment strategy occurred in 1998. Until then, the Fund's capital was only invested in government bonds in 8 countries. In the first half of 1998, the Petroleum Fund's investment universe was expanded to 21 countries and 40 per cent of the Fund's capital was invested in equities. In recent years, two new countries (Portugal and Greece) have been added to the country list for fixed income instruments, while equity investments in the same two countries as well as six emerging markets are now permitted.

The Petroleum Fund has already reached a size where even small changes in the Fund's investment strategy could lead to considerable transaction requirements. Changes in the strategy may also lay claim to resources in the operational management of the Fund. Both transaction costs and the capacity of operational management therefore make it important to prioritise and coordinate changes in the Petroleum Fund's investment strategy. For this reason, an overall evaluation of possible investment alternatives should preferably be made in order to arrive at an investment strategy that provides the desired risk-return trade-off. The choice of equity allocation is discussed in this letter, but other changes in the Fund's investment strategy may also be relevant. There are particularly three areas in which changes in the strategy are conceivable.

First, it is possible to expand the Petroleum Fund's investments to include new asset classes. Examples of such asset classes are corporate bonds, indexed (inflation-protected) bonds, bonds in emerging markets, private equities, commodities and real estate. These are asset classes that have properties which differ from those asset classes in which the Fund's capital is currently invested. A change in the Fund's investment strategy can consequently take place both by changing the distribution between existing asset classes and by including new asset classes. To the extent the inclusion of new asset classes in the Petroleum Fund's benchmark portfolio influences the equity allocation the Fund should have, an overall evaluation of possible changes in the Fund's strategy should be made.

Second, the exposure in existing asset classes (government bonds and listed shares) can be changed. This is perhaps most relevant with regard to equity exposure in various countries. For example, the Fund is currently invested in around 650 companies in the US stock market while the investment universe in this market consists of about 8000 companies. This means that there are many listed companies in which the Fund's capital is not invested. These are often companies that are not large enough to be included in the benchmark index used by the Fund for the US market (FTSE). As a result, the Fund has some excess equity exposure to large companies in the US market. The same applies to the Fund's exposure in stock markets in other countries.

A third factor that may be examined is how often the Fund should be rebalanced. Today, the Fund is rebalanced quarterly. This means that the equity allocation of the benchmark portfolio is set at 40 per cent at the beginning of each quarter, but that the proportion varies through the quarter in step with the relative price performance of equities and fixed income securities. Since the return on equities varies more than the return on fixed income instruments, a higher equity allocation will result in a greater need for rebalancing at the beginning of each quarter. A change in the Fund's equity proportion therefore raises the question of whether the rebalancing frequency should be changed.

Even though it would, in principle, be desirable to make an overall evaluation of possible changes in the investment strategy of the Petroleum Fund, Norges Bank considers it appropriate to make an evaluation of the Fund's equity allocation at this time. The rationale for this is that the choice of equity allocation will be the one decision that has the greatest impact on the Fund's return and risk. At the same time, this will be a decision that can be implemented swiftly in the operational management. However, a change in the Fund's equity allocation may have an effect on the appropriate rebalancing regime for the Fund. Norges Bank is therefore of the view that the Ministry of Finance should evaluate this question along with the choice of equity allocation. With regard to any new asset classes, it may be noted that Norges Bank in another letter to the Ministry of Finance provides advice about the inclusion of corporate bonds in the Fund's benchmark portfolio. In the longer term, it will probably also be necessary to evaluate whether the Petroleum Fund should be invested in emerging bond markets, private equities and real estate. It is important to consider carefully which of the possible changes in strategy can contribute to improving the Fund's risk-return trade-off. The

Ministry of Finance should also consider whether a coordinated plan for the work on assessing new changes in the strategy should be drawn up.

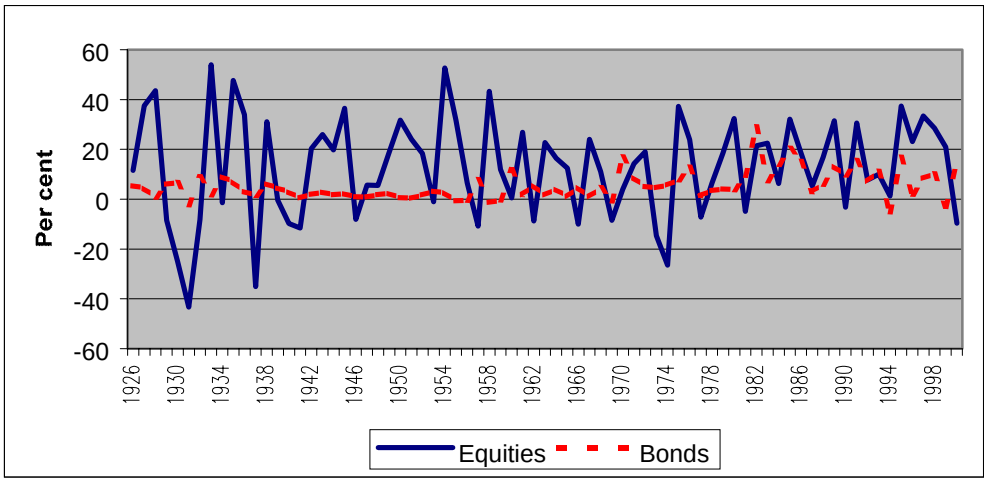
3. The return on equities and fixed income instruments

There appears to be broad agreement that equities are expected to provide a higher return than fixed income instruments, although there is greater uncertainty about the size of this excess return. There are two directions in the literature that discuss the difference in the return on equities and fixed income instruments. One studies historical data, while the other attempts to explain return differentials on the basis of different pricing models from financial theory. In the following, we look more closely at both these directions. At the end of the section we also examine possible implications of the sharp advances in equity prices the last few years.

Historical data

Most empirical studies of the stock market are based on the US market because this market provides long return series of high quality. Chart 1 shows the annual return on US equities and bonds in the period 1926-2000.[1] The chart illustrates that equities are considerably more risky than bonds in the short term. Whereas equities provided a negative return almost every third year, the return on bonds was negative approximately every tenth year.

Chart 1 Annual return on equities and bonds in the US in the period 1926-2000

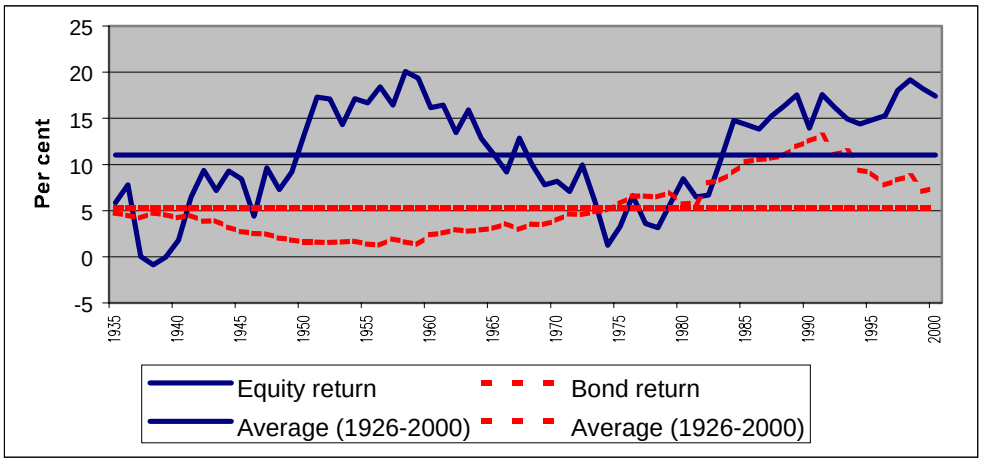


[1] The consulting firm Ibbotsen Associates has constructed annual data for returns on US equities and bonds for the period 1926 to 1998. With the help of Datastream, we have extended these time series to 2000. The return on equities refers to a broad market portfolio,

while the return on bonds refers to a portfolio with a duration of about 5. In this submission, we use nominal returns. The results would show little change if we had instead studied the real returns on equities and bonds.

In Chart 2, we have estimated the return in the US equity and bond market over rotating (fixed windows) ten-year periods. Since the Petroleum Fund has a long investment horizon, the properties of the return over ten-year periods will be more interesting than the properties of annual returns. We see that there are very few ten-year periods in which equities would have generated a lower return than bonds. There are, however, considerable variations with regard to the size of the excess return on equities. Moreover, we see that we have to go back to the 1930s to find a ten-year period in which equities provided a negative return (which is primarily related to the stock market crash in 1929). In the chart, we have also included the average return on equities and bonds in the period 1926-2000, which shows that equities have outperformed bonds by about 5.5 percentage points. The chart also shows that the 1990s was a decade when both the absolute return on equities and the outperformance of equities in relation to bonds were at a high level.

Chart 2 Return on equities and bonds in the US over rotating 10-year periods



In the above, we have looked at the return on US equities and bonds in the period 1926 to 2000. Even though this is a relatively long period that includes financial crises, recessions and wars, there are many who are of the view that the US stock market in this period provides too positive a picture of the return on and risk associated with equity investments. The US stock market has experienced a long period without such serious crises as, for example, the markets in Japan, Germany and Italy. In the literature, this problem is known as survivorship bias. A

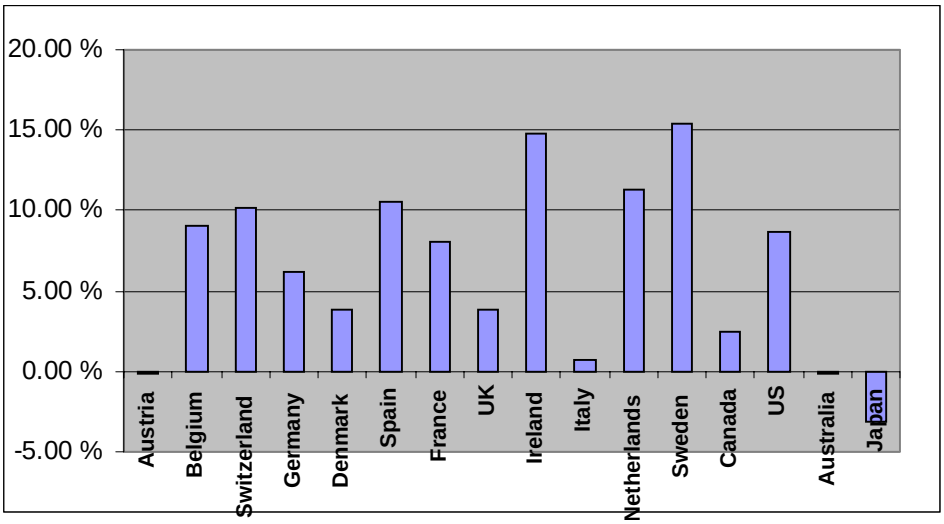
continuous and complete time series with rates of return for the US market exists precisely because the country has avoided major crises. It may therefore be the case that historical data for the US for the period 1926 to 2000 do not provide a representative picture of what the stock market will be like in future periods. As a result, there has been considerable interest in studying both longer time periods in the US and equity and bond markets in other countries. We refer to some of these studies in the following.

Siegel (1998)[2] studies US equity and bond data for the period 1802 to 1997. In this period, the average return on equities was 8.4 per cent, while the return on bonds was 4.8 per cent. The excess return for equities of 3.6 percentage points has thus been lower than was the case in the period 1926-2000 (5.5 percentage points).

[2] J.J. Siegel (1998): "Stocks for the long run", Irwin Inc.

Chart 3 shows the average return in equity and bond markets in the period 1987-2000 for most of the developed countries included in the Petroleum Fund's benchmark portfolio. The chart shows that equities have generated a substantial excess return in many of the countries in this period. Japan, Austria and Australia are the only countries where the return in equity markets has been less favourable than in bond markets. The average (equally weighted) excess return on equities in the 16 countries was 6.4 percentage points during the period.

Chart 3 Difference in returns on equities and bonds in selected countries in the period 1987 to 2000



One possible objection to the results presented in Chart 3 is that the period is relatively short. It would therefore be desirable to study whether differences in the return on equities and bonds in these countries also hold true for longer periods. However, it is difficult to find satisfactory data on returns further back in time. Jorion and Goetzman (1999)[3] have attempted to construct these return series for equities. They study the return on equities in 39 markets from the 1920s to 1996. One disadvantage of these series is that in most markets there are only figures on equity prices and not dividend payments. The real return on equities is therefore measured only by real price changes.[4] Another disadvantage of Jorion and Goetzmann's work is that they do not have access to corresponding time series for the return on bonds. Jorion and Goetzmann find that in the period they study the real return has been highest in the US stock market. In this market, the real return was 4.3 per cent while the median for all 39 markets was 0.8 per cent. It is interesting to note that only in 11 of the 39 markets has there been no break in the time series for the return on equities. These results may indicate that the high observed return on US equities may to some extent reflect a long period without serious crises in this market (survivorship bias).

[3] P. Jorio and W.N. Goetzmann (1999): "Global stock markets in the twentieth century", Journal of Finance.

[4] This may imply that the results for European markets are less favourable in relative terms since it has been more common to pay dividends in these markets.

Other methods for estimating the return on equities

In the above, we have studied the historical return on equities and bonds. We have good data for the US market in the period 1926-2000 and for 16 different countries in the period 1987-2000. These data show that the return on equities has historically been in the order of 5-6 percentage points higher than the return on bonds. There are, however, several reasons why these figures cannot necessarily be used as estimates for future return differentials between equities and bonds. First, empirical studies show that the return differentials vary considerably over time and between markets. Chart 2 shows, for example, that in the period 1926-2000 the average excess return on equities over ten-year periods varied from –5 percentage points to nearly +20 percentage points. As a result, there is considerable uncertainty associated with the estimates of historical return differentials. Second, it is difficult to explain the empirical results with the help of theoretical financial models. Equilibrium models that relate the expected return on equities to key economic variables, such as risk aversion, consumption

variation and time preferences, have not been able to explain to any extent the high excess return on equities. The term equity premium puzzle is used to describe the lack of accord between empirical results and theoretical calculations. As a result of both the uncertainty associated with historical rates of return and the equity premium puzzle, it has become common to use several different methods to estimate the return in the stock market. Two methods that are widely used in the literature are to evaluate the return in the stock market on the basis of the dividend-price ratio or the earnings-price ratio. These methods presuppose that the stock market is rationally priced without a precise formulation of the underlying equilibrium model. In the following we look more closely at the two methods.

Dividend-price ratio

In the dividend model, the value of the stock market is determined by future dividends and investors' required return. If one assumes that investors are rational and markets are efficient, the following expression for the expected return on equities (E[R]) may be derived under certain assumptions:

$$(1) \qquad \mathrm{E}[\mathrm{R}] = \frac{E(D_1)}{P_0} + g$$

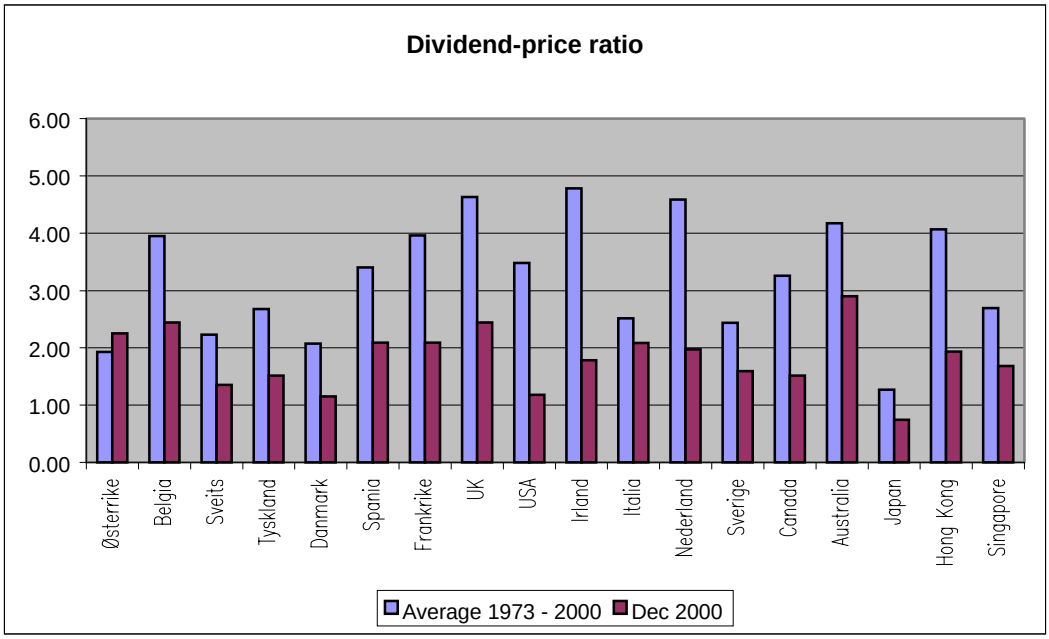
The first term on the right side is the ratio of expected dividends next year to the current price (D/P), while the second term is an estimate for the expected annual rise in companies' dividends (g). In (1), it is assumed, among other things, that the required return does not vary over time and that future dividends increase at a constant rate forever. It follows from (1) that we need an estimate for the increase in dividends and the dividend-price ratio in order to comment on the expected return on equities.

One possible starting point for estimating the rise in dividends may be to consider a reasonable estimate for long-term growth in the countries in question. Real growth is determined by growth in employment and labour productivity growth. Adherents of the "new economy" estimate high productivity growth and annual growth rates in the range 3 to 4 per cent in the long term. Others are more sceptical about the "new economy" and estimate long-term growth rates of 2 to 2.5 per cent. If, as an illustration, we assume that the long-term growth potential is about 2.5 per cent annually and that the global inflation rate is 2.5 per cent,

nominal growth may be estimated at about 5 per cent a year. Under certain assumptions, this may also be an estimate for the rise in dividends.

Chart 4 shows the average dividend-price ratio for the period 1973-2000 for stock markets in a number of developed countries. For Italy, Spain and Sweden, the time series start in the 1980s, while for the US and the UK they start in the 1960s. The companies' dividend-price ratio varies considerably across countries. The average for all countries was a little less than 2 per cent at the end of 2000. If we assume nominal growth (and rise in dividends) of 5 per cent, it follows from (1) that the expected return on equities will be about 7 per cent.

Chart 4 Dividend-price ratio in the period 1973-2000 in selected countries



Earnings-price ratio

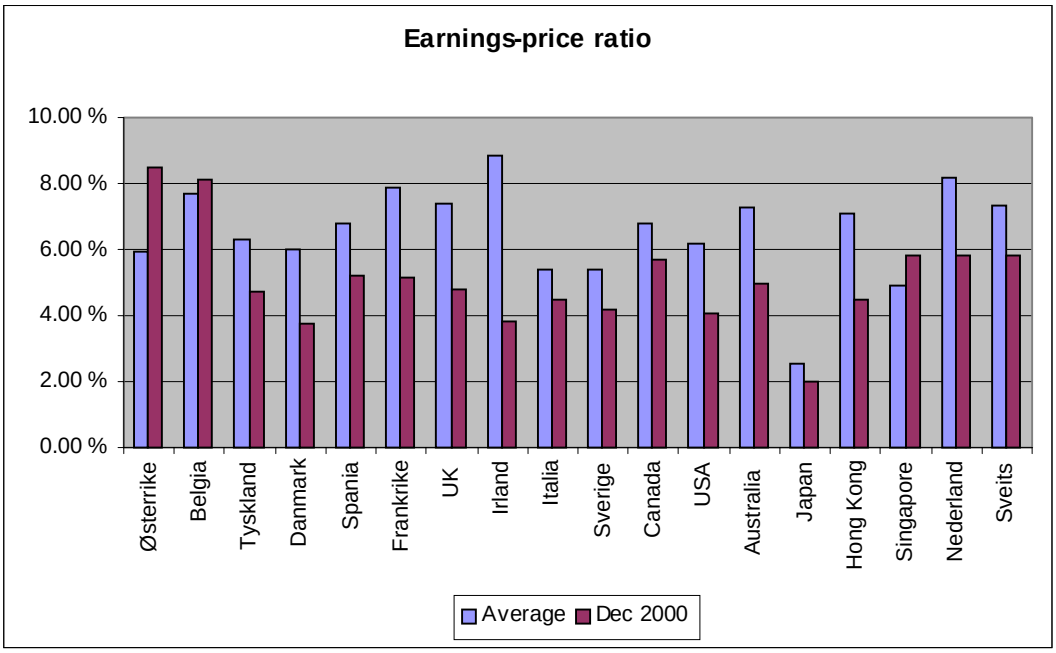
Siegel (1998) has argued that the ratio of companies' earnings to share price can be used to determine the long-term real return on equities. In the US, the average real return on equities was 7.7 per cent in the period 1926-1997, while the median for the earnings-price ratio was 7.2 per cent. The accord between the two figures holds true even when the period is extended back to 1881.

A long-term relationship between the earnings-price ratio and the real return on equities may be because companies' earnings, either paid out in dividends or reinvested, are the real source

of value for shareholders. A reduction in dividends is therefore not necessarily bad news for shareholders. This is because the reinvestment of part of the companies' cash flows can result in higher earnings for companies, forming the basis for growth in future dividend payments. In the long term, the earnings-price ratio may therefore be a reasonable estimate of the proportion of long-term cash flows to shareholders in relation to the capital shareholders' invest in the same companies.

Chart 5 shows the average for the period 1973-2000 for the earnings-price ratio in a number of markets. The time series for Italy, Spain and Sweden start in the 1980s, while the series for the US and the UK start in the 1960s.

Chart 5 Earnings-price ratio in the period 1973-2000 in selected countries



There are considerable variations between countries, with a maximum of 8.8 per cent in Ireland and a minimum of 2.6 per cent in Japan. The average for all countries is 6.6 per cent. If we start with the average for countries and assume that the global inflation rate in the period ahead will be about 2.5 per cent, the expected nominal return on equities can be estimated at 9 per cent.

It should be emphasised that the estimates for the expected return on equities on the basis of the dividend-price ratio and the earnings-price ratio are based on a number of simplifying assumptions. There is thus considerable uncertainty associated with this type of estimate. The

estimates should therefore be looked upon as an illustration of how estimates for the return on equities can be made on the basis of methods other than looking at historical rates of return.

Developments in the stock market in recent years

In the period 1995-2000, the average (equally weighted) return on equities in developed markets was about 20 per cent. This is about 11 percentage points higher than the return on fixed income instruments. The sharp advance in equity prices has resulted in a discussion concerning the "correct" equity price level. There are several indicators that are used to assess whether the level of the stock market is reasonable:

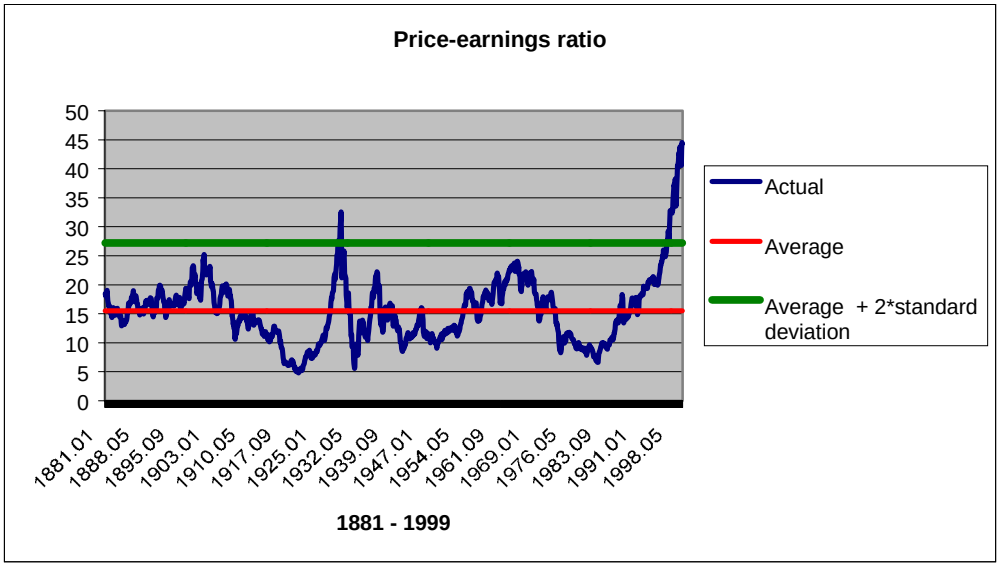
- The value of a share is basically linked to the company's future earnings potential and dividend capacity. In the long term, share values are determined by future dividends and shareholders' required return. One possible indicator is the dividend-price ratio. A low ratio means that the current payment to investors is low compared with the amount they must pay to obtain this current payment. This may be an indication that the market is priced higher than the level implied by underlying earnings in the companies.
- Another indicator is the ratio of the companies' total earnings to the companies' total market capitalisation. In this type of study, it is common to look at the price-earnings ratio (and not the earnings-price ratio as in Chart 5). A high ratio indicates that investors must pay more for each krone earned by the companies. Shiller (2000)[5] shows that in the US stock market there has historically been a negative correlation between the price-earnings ratio in one year and the real return on equities in the following ten-year period.
- A third indicator is Tobin's Q, which is defined as the ratio of the value in the stock market to the replacement cost of the companies' fixed assets. If the value in the stock market exceeds the replacement cost, it is assumed that companies will increase fixed investment. If the value in the market is lower than the replacement cost, it is assumed that fixed investment will be postponed and that the number of corporate acquisitions will increase. Tobin's Q should therefore be around 1 in long-term equilibrium.

[5] R.J. Shiller (2000): "Irrational Exuberance", Princeton University Press.

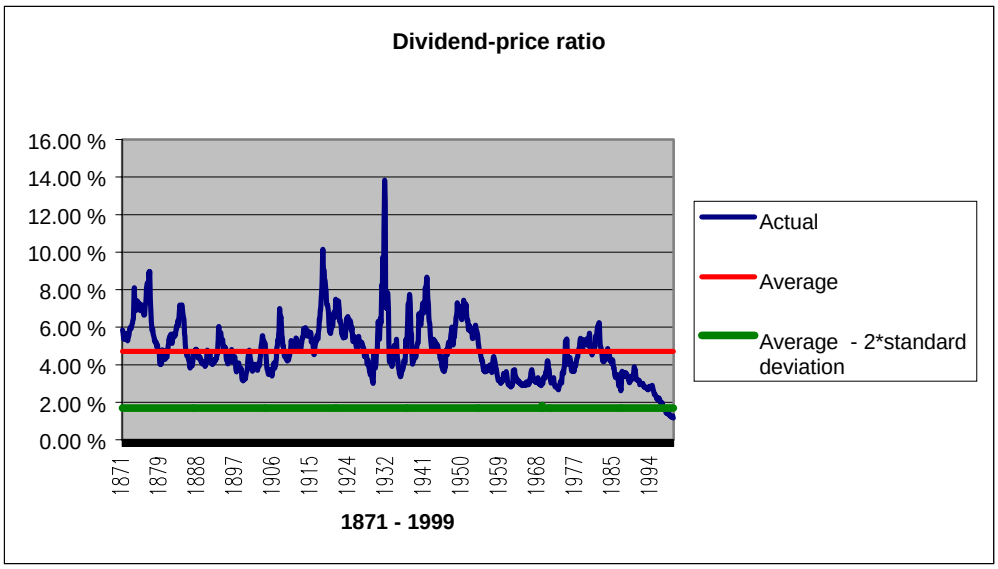
It should be emphasised that for several of these indicators it is better to compare developments over time in the same market than to compare different markets. This is because differences between countries can be influenced by differences in tax rules, accounting standards, the interest rate level, the structure of capital markets and growth rates for the economy.

We have access to long time series for the price-earnings ratio and dividend-price ratio in the US stock market. These time series, which are obtained from Shiller (2000), start in the last century and continue to 1999. Chart 6 shows that at the end of 1999 both indicators were at a level not observed earlier. The indicators were also far from the average over the period. In addition, we see from Charts 4 and 5 that the dividend-price ratio in most countries at the end of 2000 was lower than the average for the period 1973-2000, while the price-earnings ratio was higher than the average. This reflects the sharp rise in share prices in the last half of the 1990s.

Chart 6 Dividend-price ratio and price-earnings ratio for the US equity market in the period 1871 to 1999



Equity prices have fallen in many markets the past year, with technology shares recording the steepest decline. For example, the US technology exchange Nasdaq fell by about 60 per cent from the peak on 10 March 2000 to mid-March 2001. In many countries, however, the recent decline in equity prices has been small compared with the strong advances in stock markets in the last half of the 1990s. When analysing developments in stock markets in the period ahead, it may therefore be useful to first examine the possible reasons for this rise in equity prices. On a general basis, it may be said that there are three possible explanations for the sharp rise in equity prices.



One explanation may be that the risk premium on equities has been reduced. The risk premium is the extra return investors require to hold shares, and represents compensation for the uncertainty associated with the return on equities. The size of the risk premium depends on both variations in the return on equities and investors' attitude towards risk. A decline in the risk premium may thus be due to reduced volatility in the stock market or a reduction in investors' risk aversion. Studies of stock market volatility provide no clear indications that the short-term variations in the return on equities have been reduced in recent years. It is conceivable, however, that increased access to information has prompted investors to evaluate to a greater extent the risk of equities on the basis of more long-term considerations. Similarly, it cannot be ruled out that the sharp rise in equity prices in many countries in the 1990s has reduced investors' subjective estimates of the risk of investing in equities. A decline in the risk premium may also be related to a change in share ownership. Previously,

equities were primarily owned by a small group of the population. As living standards increased, additional groups have become active in the stock market. It is now easier to buy shares, and financial instruments have been developed which make it easier to manage the risk of equity investments. Furthermore, changes in the authorities' regulations meant that institutional investors, such as pension funds and life insurance companies, were given an opportunity to invest in equities. This has increased the demand for equities and reduced the risk premium. Equities have shifted from being a "narrowly held risk" to being a "broadly held risk", ie more people are sharing risk in the stock market.

Another possible explanation for the rise in share prices is that investors expect higher growth in corporate earnings and dividend payment capacity. It is not uncommon to use growth in the economy as an estimate for long-term growth in corporate earnings. This can best be estimated as potential real GDP growth since growth in corporate earnings must be linked to growth in the production potential of the economy. Real GDP growth depends on growth in both employment and labour productivity. However, for many of the countries in which the Petroleum Fund's capital is invested growth in the labour force will be limited since the share of the working-age population is declining due to demographic changes. This means that higher economic growth must primarily reflect higher productivity gains.

A third possible explanation for the sharp rise in equity prices is that the market is not rationally priced and that we are facing a financial bubble. There are several historical examples showing that equity prices in periods have not been based on underlying corporate values. For example, it appeared that the sharp growth in the Japanese stock market at the end of the 1980s could not be explained by economic fundamentals in Japan. The sharp fall in technology shares over the past year has also shown that the rise in these equity prices at the end of the 1990s could not be justified on the basis of reasonable estimates for earnings growth in these companies.

It is difficult to know which of the three explanations is the most relevant to the rise in equity prices at the end of the 1990s. It is also conceivable that it is a combination of these factors that explain the sharp advances in many countries' stock markets in these years. However, the three explanations have different implications for the expected return in stock markets in the years ahead. If the explanation for the rise in equity prices is lower risk premia, the return on equities will be noticeably lower in the years ahead than at the end of the 1990s. This is due to

two factors. First, the risk premium has been reduced. The second is that the actual decline in the risk premium provides an extraordinarily high return on equities in the period the premium is reduced because equity prices rise in order to adjust to the lower risk premium. If the rise in the stock market had been related to investors' expectations of higher growth in corporate earnings, the excess return on equities in the years ahead might be of the same order as we have seen from a longer historical perspective. But if earnings growth alone is to explain the rise in equity prices, this may mean that in some markets earnings growth must be higher than growth in the economy. It is conceivable that this can occur for limited periods, but this situation is less probable in the long term.

If the fundamental preconditions for a rise in equity prices were to change or if a rise in equity prices is not related to underlying fundamentals, a correction of equity prices could occur. Some are of the view that the decline in equity prices the past year represents such a correction. Others point out that this decline has been small compared with the sharp advances in equity prices at the end of the 1990s and that there is therefore a risk that equity prices will fall further.

4. Risk associated with equities and fixed income instruments

The risk associated with equities and fixed income instruments can be measured in several ways. The most common measure is the standard deviation of the return. This is a measure of dispersion which indicates the magnitude of variations in the return. Another measure that is widely used is the probability that the return will fall below a certain limit (shortfall risk). In the case of equities, the focus is sometimes on the probability of a negative return, while other times the focus is on the probability that the return on equities will be lower than the return on money market investments or bonds. In this section, we will use both risk measures.

It is interesting to see how the risk of equities and fixed income instruments changes over time and whether there are considerable differences between countries. We will shed light on the first by using long time series for the US, while the second is examined with the help of shorter time series for many countries. Chart 7 shows the standard deviation of the annual return on US equities and bonds in the period 1926-2000. The chart shows that the standard deviation of equities is nearly four times as high as the standard deviation of bonds for the entire period. This is mostly due to the very high risk for equities in the 1930s. In the last 50 years, the standard deviation of equities has been about 15 per cent, while the standard

deviation of bonds has been about 5 per cent. The chart also shows that there are wide variations in the standard deviation over time when this is calculated over rotating ten-year periods.

Chart 7 Standard deviation of US equities and bonds over rotating 10-year periods

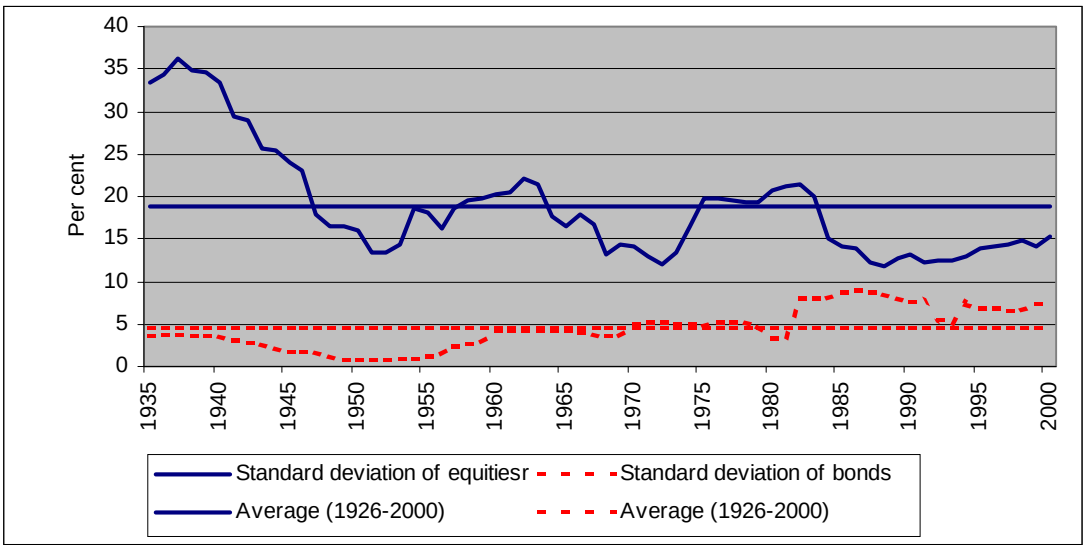
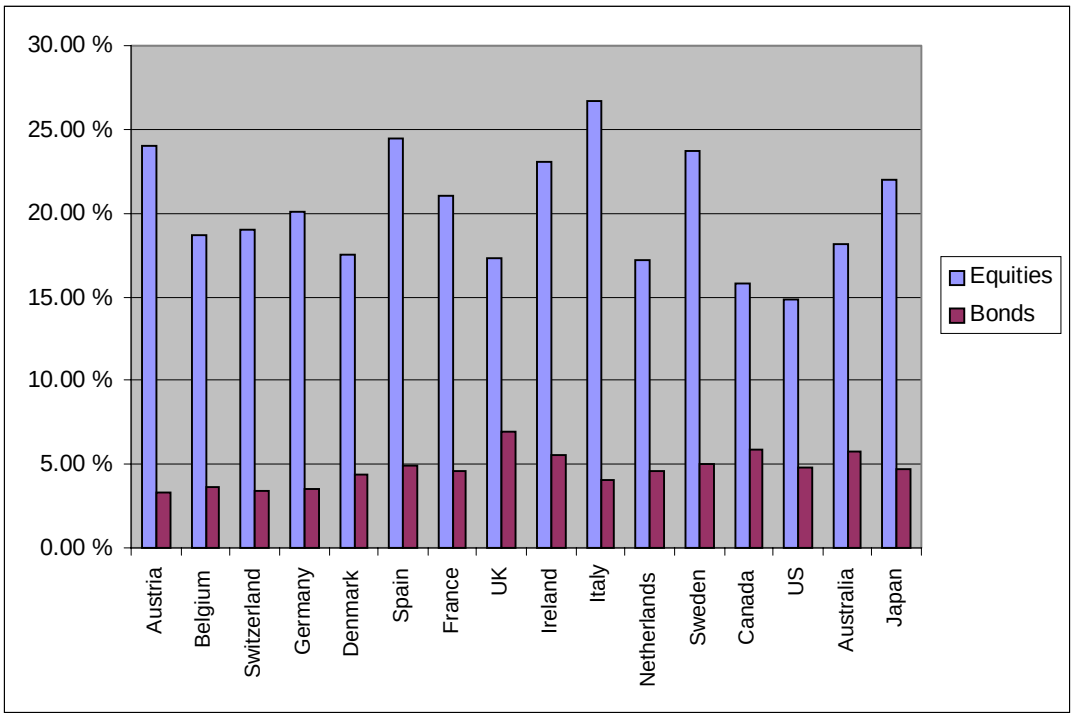


Chart 8 shows historical estimates for the standard deviation of equities and bonds in 16 different countries. Even thought there are considerable differences between countries, the chart nevertheless presents more or less the same picture as the results from the US market; the standard deviation of equities is between 3 and 4 times higher than the standard deviation of bonds.

Chart 8 Standard deviation of equities and bonds in the period 1987 to 2000 in selected countries



In Chart 7 and 8 we have looked at the volatility of equities and bonds in some countries. However, the Petroleum Fund's investments are spread among many countries. In isolation, this means that the volatility of the Fund's equity and fixed income portfolio is slightly lower than the volatility of investments in individual markets. This is because there is no perfect correlation between the return in individual markets, with diversification gains arising by spreading investments in many countries. When measuring the risk of a portfolio that is invested in a number of countries, however, it is also necessary to decide how to treat variations in exchange rates. Since the purpose of the management of the Petroleum Fund is to ensure the Fund's international purchasing power, it may be appropriate to disregard fluctuations in the krone exchange rate against other currencies when evaluating the Fund's risk. If we disregard these exchange rate fluctuations, the calculations for the period 1987-2000 show that the standard deviation of the Fund's fixed income portfolio is slightly lower than 5 per cent, while the standard deviation of the Fund's equity portfolio is a little less than 15 per cent.

In the analysis above, we have focused on the annual standard deviation of equities and fixed income instruments. These figures can provide an indication of how much the return on equities and fixed income assets will fluctuate from one year to the next. The results show that from this perspective equities are considerably more risky than fixed income instruments. There are, however, two important factors when evaluating the risk of equities and fixed income instruments that are not captured by this type of study. The first is that we are primarily interested in how the distribution between equities and fixed income assets influences portfolio risk. This risk depends not only on the standard deviation of equities and fixed income instruments but also on the correlation between them. This means that even though equities have a higher risk than fixed income instruments, it is not certain that the portfolio risk rises when equities are included. The diversification gains achieved by the imperfect correlation between the return on equities and fixed income instruments may in fact offset the effects on portfolio risk associated with the higher volatility of equities compared with fixed income instruments. The second factor that is important to take into account is that the Petroleum Fund has a long investment horizon, and that we are therefore particularly interested in long-term portfolio risk. This will depend on the properties of time series for the return. We often distinguish between three different types of time series:

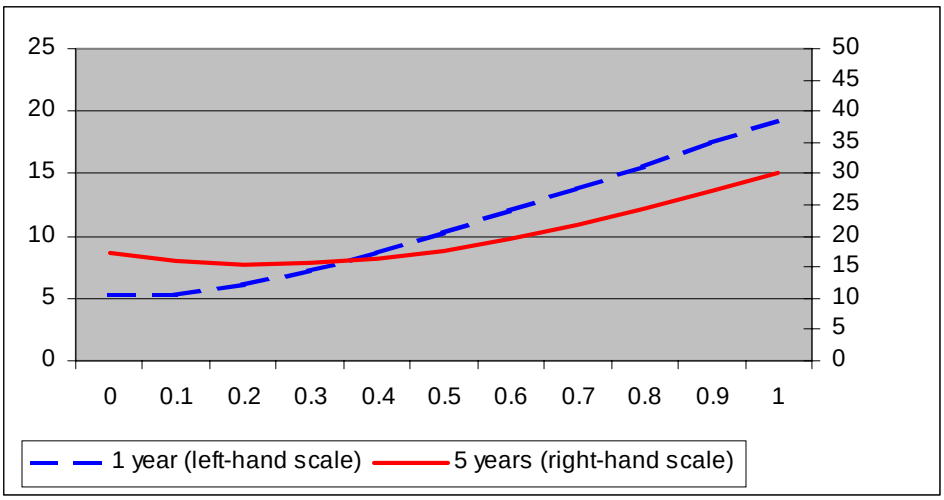
i) Mean reversion implies that a high return in some periods will be followed by a lower return in subsequent periods. For equities, mean reversion could be due to a change in the risk premium over time. Another possible explanation is that in periods equity prices deviate from the underlying fundamental values, but that such deviations are corrected over time.

ii) A random walk implies that the return in the following periods is independent of the return in earlier periods.

iii) Mean aversion means that a high return will be followed by a high return and vice versa. Return series cannot have such properties over long time periods, but for limited periods we can, for example, imagine a situation whereby trends in the nominal interest rate level result in mean aversion properties for the return on fixed income instruments.

It can be demonstrated that the variation in the return will increase proportionately with time if the return series follows a random walk. This means that the standard deviation will increase by the square root of time. With mean reversion, the risk will increase less than this, while it will increase more if the return follows a mean aversion process. It is therefore interesting to know the time series properties of the return on equities and fixed income instruments. We need long time series to study this, and they only exist for the US market. Chart 9 shows the risk of a portfolio with US equities and fixed income instruments with investment horizons of 1 and 5 years respectively. Since we only have 15 independent observations of the return over 5-year periods, there is considerable uncertainty associated with the estimate for the standard deviation for this horizon.

Chart 9 Relationship between equity allocation and standard deviation of a portfolio with investment horizon of 1 year and 5 years respectively

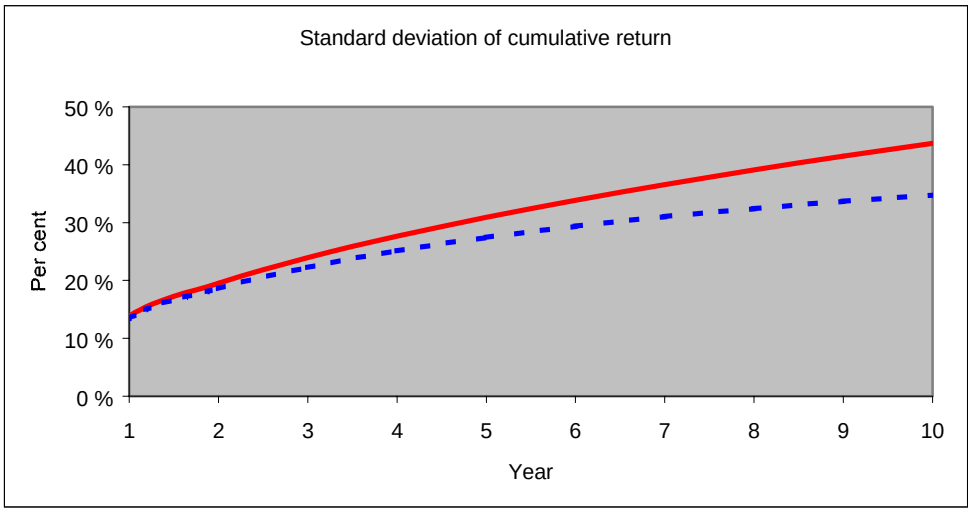


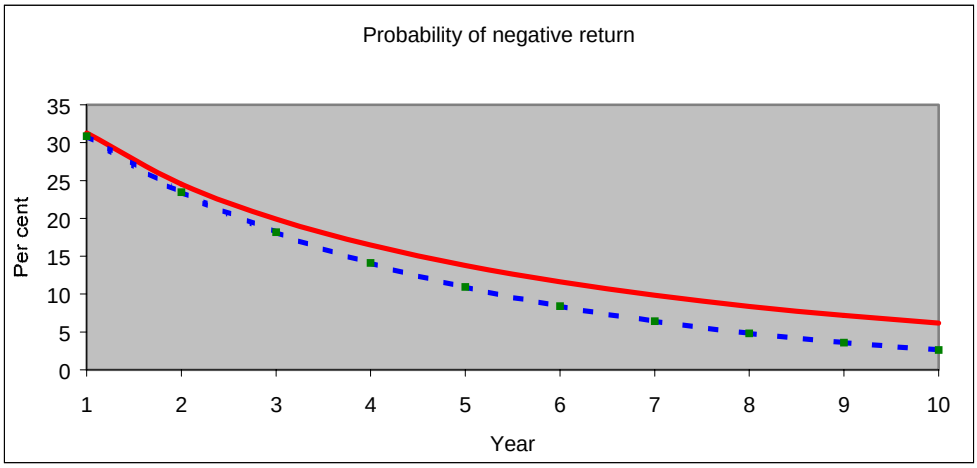
The chart provides a basis for several interesting observations of the properties of the portfolio risk. First, we see that if the investment horizon is 1 year, even a limited equity allocation will result in an increase in portfolio risk. This is not the case when the investment horizon is 5 years. The risk will then be reduced when equities are included in the portfolio. The chart shows that the portfolio with the lowest risk over a five-year horizon includes 80 per cent fixed income instruments and 20 per cent equities. The chart also shows that a portfolio with 50 per cent fixed income instruments and 50 per cent equities has about the same risk as a portfolio that has only invested in fixed income instruments. Second, we see that the risk of a fixed income portfolio is more than trebled when the horizon increases from 1 to 5 years (from 5 per cent to 17 per cent). By way of comparison, the risk of an equity portfolio is less than doubled with this increase in the investment horizon (from 19 per cent to 30 per cent). In the above, we showed that under the assumption of a random walk, the risk will rise by the square root of the number of periods (i.e. with a root of 5 in our case). This means that US data for the period 1926-2000 show that the risk of a fixed income portfolio increases more than what we would expect if there was a random walk, while the risk of an equity portfolio increases less than would be expected. It may indicate that there is mean aversion in fixed income instruments and mean reversion in equities. It is important to emphasise, however, that we have so few observations of the return over five-year periods that it is not possible to draw definite conclusions.

The time series are not sufficiently long to make a meaningful analysis of how the risk of an equity portfolio evolves over horizons that are longer than 5 years. We have therefore instead attempted to examine this question with the help of a simulation model. Simulations have been carried out under two different assumptions concerning the time series properties of the return on equities. We have assumed that equity prices either follow a random walk or a mean reversion process.[6] Chart 10 shows the change in the risk of a portfolio with a return of 8 per cent and a standard deviation of 15 per cent when the horizon increases. The chart shows that if the return follows a mean reversion process, the risk over long horizons will be lower than if the return follows a random walk process. The chart also shows how the probability of a negative cumulative return is influenced by the length of the investment horizon and the time series properties of the return. When risk is measured in this way, we find that risk declines with the investment horizon. This is true irrespective of the time series properties of the equity return.

[6] Equity prices are assumed to follow an Ornstein-Uhlenbeck mean reversion process. This process is calibrated so that it has properties resembling the properties of historical equity prices in the US.

Chart 10 Simulation and relationship between risk and investment horizon for a portfolio with a return of 8 per cent and a standard deviation of 15 per cent. The solid curve shows a random walk and the dashed curve shows mean reversion





Over a longer time period, it is not sufficient to focus on the standard deviation or the probability of a negative return. Account should then also be taken of seldom events that can have a substantial influence on changes in the value of equities and bonds (event risk). For example, wars, natural disasters or hyperinflation may result in a considerable fall in the value of both equities and fixed income instruments. Jorion and Goetzmann (1999) illustrate how

these events have influenced the value of equities. Of the 24 stock markets for which Jorion and Goetzmann had observations in 1931, as many as 17 markets were closed for shorter or longer periods in the period to 1996. Jorion and Goetzmann show that equity prices fell markedly in a number of the markets that were closed. For example, the real value of Japanese equities dropped by as much as 95 per cent in the period 1944-1949, while German equities fell by 84 per cent in the period 1944-1950. It is important to note that this type of risk is not captured by a risk measure that shows variations in the return on equities the last 10 or 20 years.

5. Some portfolio calculations

There are two methods that are often used to illustrate how the properties of a portfolio depend on the equity allocation. One is a portfolio model that shows combinations of equities and fixed income instruments that provide the best risk-return trade-off. The second is a simulation model that studies how the properties of a portfolio change when the investment horizon increases. We will use both methods in this section.

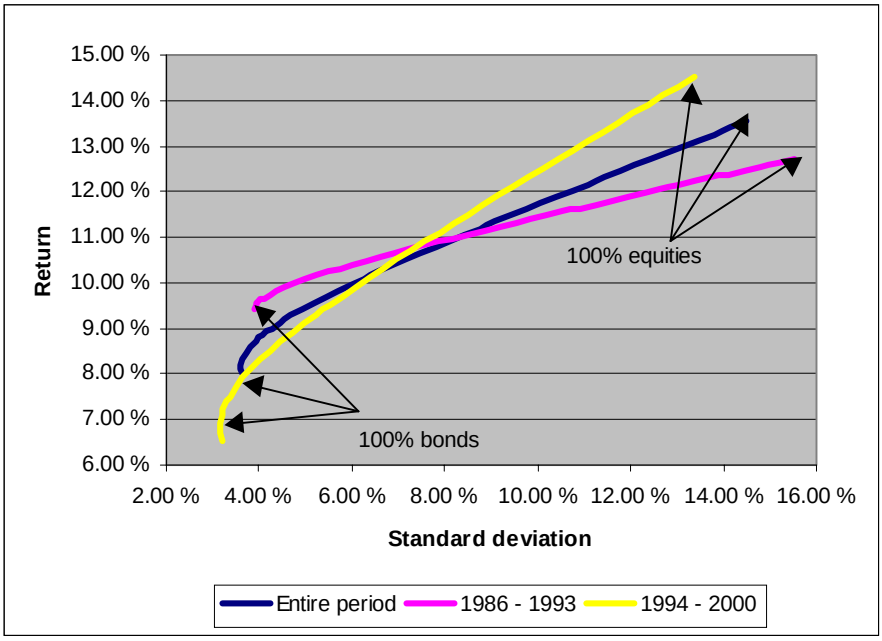
Portfolio theory starts with the assumption that an investor chooses between different asset classes on the basis of the investments' expected return and the standard deviation of the return. A high expected return is positive, while a high standard deviation (risk) is negative. If this optimisation function is to be used, either the properties of the return on financial assets must be fully described by the expected return and the standard deviation or investors' utility functions must be quadratic.[7] Even though this is often perceived as fairly restrictive assumptions, it is common both in financial theory and in practice to assume that investors choose portfolios on the basis of the expected return and standard deviation. Investors' risk tolerance is decisive for weighing the return against the standard deviation.

[7] A normal distribution of the return means that the entire outcome for the return on an asset class is described by the expected return and the standard deviation of the return. A quadratic utility function means that the investor has increasing absolute risk aversion, i.e. the proportion of the risky asset class in the portfolio declines as wealth increases.

We have constructed a portfolio model in which the return and standard deviation of equities and fixed income instruments are estimated by weighting indices for the US, Europe and Japan using the Petroleum Fund's regional weights. Equity and bond indices provide a basis

for estimating the correlation between the return on the two asset classes. We have used return data for the period 1986-2000. In the calculations, we have measured the return in local currency, which means that we disregard changes in the krone exchange rate and variations in this. The results are shown in Chart 11. The chart shows that if we started with a portfolio that was exclusively invested in fixed income instruments, the portfolio risk could have been reduced somewhat by investing a small share in equities. After having achieved this diversification gain, a continued increase in the equity allocation would have almost resulted in a linear relationship between return and risk. If the entire portfolio was invested in equities, the standard deviation would have increased from about 4 to 14 per cent, while the return would have increased from 8 per cent to 13.5 per cent. The relationship between return and risk as a function of the equity allocation has varied over time. When the time series are split up into sub-periods, it is seen that the trade-off between equities and fixed income instruments is more favourable in some periods than in others.

Chart 11 Efficient frontier for an equity and fixed income portfolio with the same regional weights as the Petroleum Fund

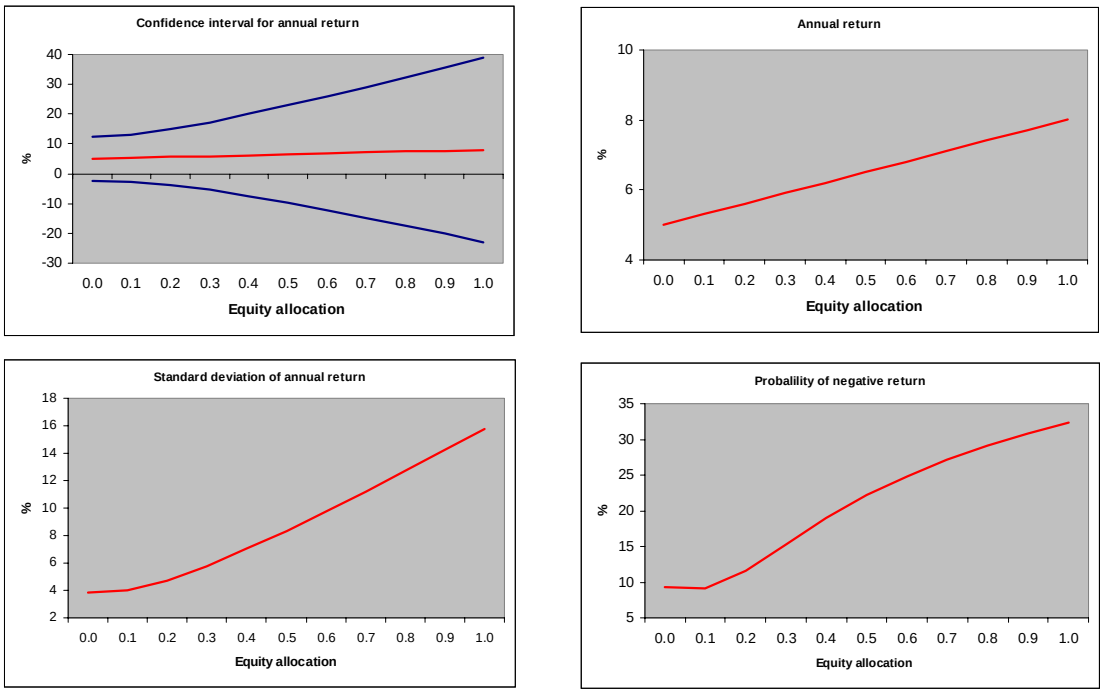


How the properties of a portfolio are influenced by a change in the equity allocation depends on the assumptions made concerning the return and risk for equities and fixed income instruments and the correlation between them. It is difficult to know which assumptions will be correct for the years ahead. In the period 1986-2000, the average return on fixed income instruments was about 8 per cent. In the light of the current level of bond yields, it may be

unreasonable to assume this return in the portfolio calculations. A more neutral estimate for the return on bonds may be the yield, which at the moment is about 5 per cent in many countries. Our estimate for the return on equities is based on this return on bonds and a slightly lower estimate for the risk premium than implied by historical data. In the calculations, we have set the return on equities at 8 per cent. The estimate for the standard deviation of equities and bonds and the correlation between them are based on historical estimates. We have disregarded variations in the krone exchange rate when calculating the standard deviation of the return.[8] Chart 12 shows how changes in the equity allocation influence the portfolio's properties.

[8] It should be emphasised that the estimates for the return and risk are associated with considerable uncertainty. In the annex, we look more closely at the portfolio properties under other assumptions concerning the return on equities and fixed income instruments.

Chart 12 Relationship between equity allocation and properties of a global portfolio with the same regional weights as the Petroleum Fund



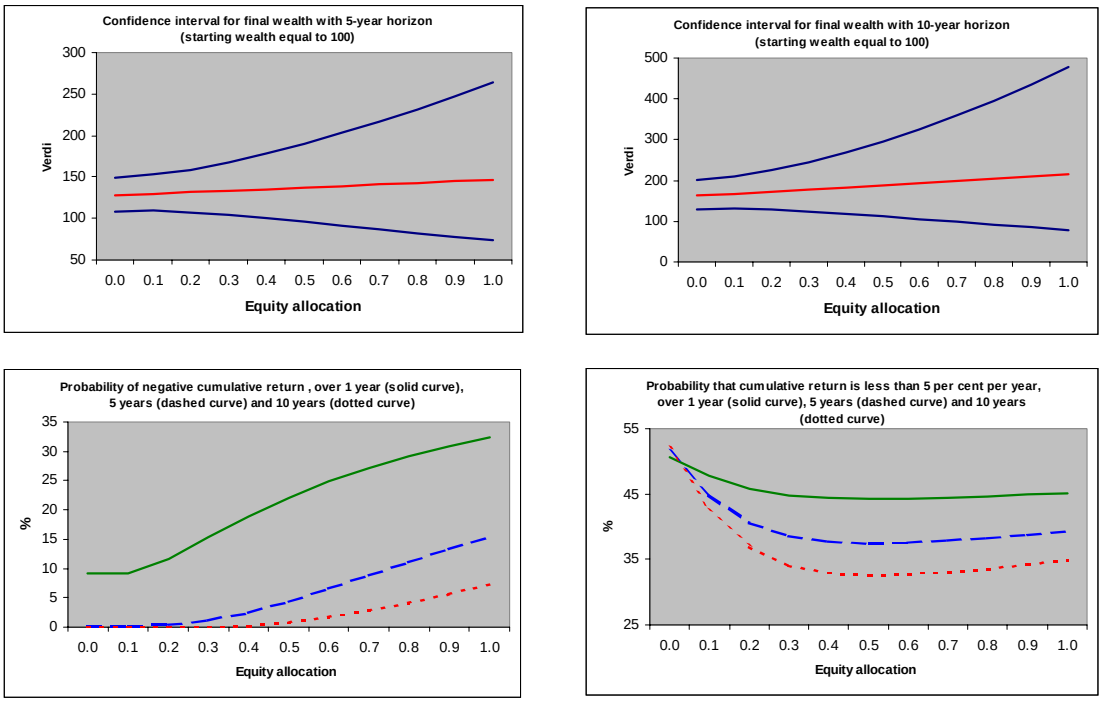
The chart focuses on how a change in the equity proportion influences the portfolio's return and risk measured over one-year horizons. The chart depicting confidence intervals shows the most probable variations in the annual return.[9] The chart shows that the confidence interval for the return on a fixed income portfolio is from –2.5 per cent to +12.5 per cent, while the

corresponding interval for an equity portfolio is from –23 per cent to +39 per cent. The chart depicting the standard deviation shows how the risk of a portfolio changes with the equity allocation. We see that the risk rises almost proportionately with the equity allocation and that the standard deviation of an equity portfolio is approximately three times higher than a fixed income portfolio. If the risk is measured as the probability of a negative return, the picture is about the same. The chart shows that a pure fixed income portfolio will have a negative return almost every fifth year, while a pure equity portfolio will have a negative return almost every third year. A portfolio with 40 per cent equities and 60 per cent fixed income instruments (as the Petroleum Fund) will, according to the calculations above, have a negative return approximately every sixth year.

[9] In the analyses in this section, we present confidence intervals that are calculated by adding to and subtracting from two standard deviations from the expectations value of the return. If the return on equities and fixed income instruments is normally distributed, this will represent a 95 per cent confidence interval. This means that the return in 19 out of 20 years will be within this interval. Empirical studies show, however, that the return on equities and fixed income instruments more often assume extreme values than the normal distribution would imply. Nor is the return symmetrical around the expectations value as is the case with the normal distribution. This means that our estimates for the probability of a negative return and the confidence interval for the return may underestimate the real risk of an equity and fixed income portfolio. The error size depends on the magnitude of the deviation between the normal distribution and the actual distribution of the return. These deviations will depend on the time horizon being considered and how diversified the portfolios are.

In the analyses above, we have focused on how a change in the equity proportion influences the return and risk over one-year horizons. As noted earlier, however, it is perhaps more interesting to study the long-term development of a portfolio's properties. In order to shed light on this, we use a simulation model. The model includes two asset classes (equities and fixed income instruments) and it is assumed that the return on asset classes follows a random walk process. The assumptions concerning the return, the standard deviation and correlation are the same as for the calculations presented in Chart 12. Chart 13 shows the confidence interval for final wealth, the probability of a negative cumulative return and the probability that the cumulative return is lower than 5 per cent a year with investment horizons of 1, 5 and 10 years.

Chart 13 Confidence interval for final wealth, probability of negative cumulative return and probability that the cumulative return is lower than 5 per cent per year with investment horizons of 1, 5 and 10 years



The chart shows that the outcome scope for final wealth increases the higher the equity proportion is and the longer the investment horizon. On this basis, it is natural to conclude that the risk increases both with the equity allocation and the investment horizon. However, a higher equity proportion and longer horizon will also increase the return on the portfolio. A key question will then be how the relationship between return and risk is influenced by changes in the equity proportion and the investment horizon. One way to illustrate this is to look at the probability of a negative cumulative return over different investment horizons. The chart shows how this probability is influenced by the equity proportion. The relationships are now no longer as clear-cut. In general, we see that the probability of a negative cumulative return falls with the investment horizon. We have also calculated how the equity proportion influences the probability that the cumulative return on the portfolio will be lower than a fixed return of 5 per cent a year. Since the expected return on fixed income instruments is 5 per cent a year, the probability that the fixed income portfolio will generate a return that is lower than 5 per cent a year is naturally 50 per cent. The probability that an equity portfolio will provide a cumulative return lower than 5 per cent a year is lower than this.

Since the Petroleum Fund today has an equity allocation of 40 per cent, it will be particularly interesting to see how changes in the equity proportion will influence the Fund's return and risk. Annex 1 presents tables showing the results of analyses in this section for equity allocations of between 30 and 70 per cent. These tables show that, under our assumptions, an increase of, for example, 10 percentage points in the equity allocation will increase the expected return on the Fund's capital by 0.3 percentage point, which over a ten-year period will probably result in an increase in the return of more than 5 percentage points. At the same time, this increase in the equity allocation will increase the probability of a negative annual return somewhat, while the probability of a negative return over a ten-year period will not be affected to any extent.

In the simulations above, we have assumed that the return on both equities and fixed income instruments follows a random walk. In section 4 we showed that this was not necessarily the case. The results of our studies of US data could indicate that there was mean reversion in equities, so that the risk of equities increased less over time than that implied by the assumption of a random walk. The uncertainty associated with the results from the US market and the unavailability of results from other markets means that in the simulation model we have not assumed mean reversion in equities. It is nevertheless important to emphasise that if equities follow a mean reversion process, the uncertainty concerning the cumulative return on equities will be less than indicated by the simulations above.

A common argument for increasing the equity proportion when the investment horizon is longer is the reduced probability that equities perform less favourably than alternative investments (shortfall risk falls). However, it is not sufficient to focus on the probability of a lower return on equities. It is also important to consider how much lower the return on equities may be. Even though the probability of a lower return is reduced when the horizon is extended, the size of any lower return will increase.

The relationship between the equity proportion and the investment horizon has been the subject of considerable discussion in the literature. It can be shown that if investors have constant relative risk aversion[10] and the return on equities follows a random walk process, the equity proportion will be independent of the investment horizon. If, on the other hand, the return on equities follows a mean reversion process and investors have special preferences for avoiding a negative return on the portfolio, it can be argued that the equity proportion should

increase with the length of the investment horizon. The relationship between the equity proportion and the investment horizon will then be a question of the time series properties of the return on equities and fixed income instruments and investors' attitudes towards risk.

[10] Relative risk aversion (RRA) is a measure of an investor's attitude towards a proportional loss of wealth. An investor with increasing (decreasing) RRA will invest more (less) in equities when wealth increases.

In the analyses above, we have used the standard deviation, confidence interval and the probability of a negative return as a measure of the portfolio's risk. The Ministry of Finance, when requesting the Bank to assess the equity allocation, has emphasised that it appears that the Fund's capital will be higher than envisaged when the equity proportion was established in 1997. Higher capital in the Petroleum Fund will mean that variations in the krone value of the Fund will be greater even with an unchanged investment strategy. To what extent this risk is relevant to the question concerning the equity proportion will depend on the owner's risk tolerance.

6. Equity proportions in other fund[11]

[11] The Petroleum Fund's annual report for 2000 contains a feature article that provides a thorough review of the investment strategy of other funds.

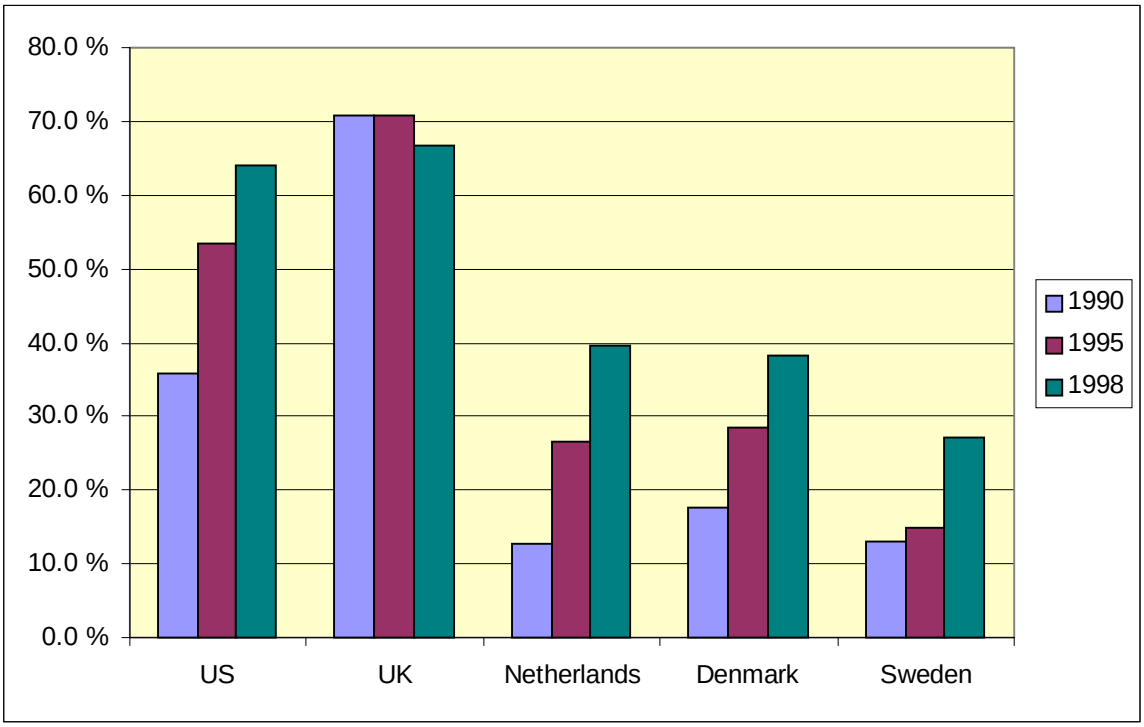
The investment strategy a fund should have depends on the purpose of the management of the fund. The purpose depends on both what the fund is to be used for and when it shall be used (investment horizon). It is natural that funds with various purposes also choose different investment strategies. It will therefore be most relevant to compare the Petroleum Fund's investment strategy with the investment strategy of funds with a similar purpose. Few funds of this type exist. The most obvious ones are perhaps the Alaska Permanent Fund, the Alberta Heritage Fund and the oil funds of Kuwait and Oman. It may also be relevant to compare the Petroleum Fund with institutions that manage their own countries' accumulated surpluses, such as the Government of Singapore Investment Corporation.

The problem with some of these funds is that little information exists concerning the investment strategy they have chosen. It is easiest to obtain information about the strategy of the Alaska Permanent Fund and the Alberta Heritage Fund. The Alaska Permanent Fund has a

portfolio of about USD 27 billion, with an equity proportion of 49 per cent. The Alberta Heritage Fund has a portfolio of CAD 12.4 billion, and is divided into a long-term portfolio of CAD 7.0 billion and a more short-term portfolio of CAD 5.4 billion. The equity proportion of the first portfolio is 55 per cent, while the other portfolio does not include equities. However, the Alberta Heritage Fund has adopted a plan to transfer the short-term portfolio to the long-term portfolio by 2005. This means that the total equity proportion will gradually increase to 55 per cent. In addition to investments in equities and fixed income instruments, both the Alaska Permanent Fund and the Alberta Heritage Fund invest in real estate.

In the case of pension funds, information about the investment strategy in general and the equity proportion in particular is readily accessible. Chart 14 shows the average equity proportion for pension funds in selected countries.

Chart 14 Average equity allocation in pension funds in selected countries. Figures for Sweden refer only to private pension funds and are from 1997
Source: OECD Institutional Investor Yearbook 2000



The chart shows that the average equity proportion in pension funds varies considerably across countries. In the US and the UK, the equity proportion in pension funds has traditionally been high, while it has been substantially lower in continental Europe. We also

see that there is a clear trend towards higher equity proportions in all countries with the exception of the UK.

In continental Europe, there has been a clear tendency in recent years for the equity proportion to increase at the expense of fixed income instruments and for investments to be shifted from domestic investments to international equity and bond markets. According to market surveys, the average equity proportion of European pension funds in 2000 was higher than in earlier years, partly because of the relaxation of rules for pension funds' investment strategies in some countries. In a number of European countries, however, restrictions on the size of the equity exposure of pension funds still exist. The EU Commission has proposed a directive which, in the longer term, will give pension funds increased flexibility for choosing the asset distribution.[12]

[12] The EU Commission proposal to the European Parliament and the Council Directive on the activities of institutions for occupational retirement provision, 11 October 2000.

In Sweden, only private pension funds are included in Chart 14. AP funds are a key component of the public pension system. With effect from January 2001, the management structure of these funds was changed as a result of the Swedish pension reform. Four independent and competing funds have been established with the same investment rules and with each managing about SEK 125 billion. These funds serve as a buffer between payments of social security premiums of 16 per cent of pensionable income and payments of pensions. At the moment, it is not known what type of investment strategy these funds will choose. According to the general rules that apply to all four funds, at least 30 per cent shall be invested in fixed income securities (i.e. an implicit maximim 70 per cent in equities). In the Swedish pension system, 2.5 per cent of pensionable income is to be paid into a defined-contribution pension scheme with individual investment and manager choices. Those who have not chosen special managers will have their capital invested in a separate fund, AP-Fund 7. This fund is smaller than each of the four buffer funds, but is expected to increase considerably in coming years. The fund has an equity proportion of 85 per cent.

Europe's largest pension fund, ABP, is located in the Netherlands. The Dutch authorities have not stipulated corresponding limitations on asset allocations that apply in many other European countries. The Dutch pension fund therefore has on average a higher equity

proportion than the average in continental Europe (45 per cent against 36 per cent[13]). The equity portion of ABP also includes smaller investments in private equities (see Table 1).

[13] Source: Greenwich Associates: "Investment Management, Continental Europe", October 2000.

Table 1 ABP's asset distribution (October 2000)

Asset class	**Percentage share**
Fixed income securities	40
Listed equities	48
Private equities	4
Real estate	8

The world's largest manager of pension funds is the US Teachers Insurance and Annuity Association – College Retirement Equity Fund (TIAA-CREF). In many respects this company is considered to be one of many offering management services to others, on a par with the external managers used by the Petroleum Fund. They do not, however, offer their management services in the ordinary market. TIAA-CREF manages pension funds of teachers, professors, etc. in colleges and universities throughout the US. The company is therefore more comparable to a pension fund. TIAA-CREF manages a little more than USD 290 billion, with more than half invested in listed equities in the US and the rest of the world.

The review of other funds' investment strategy shows that many funds have a higher equity proportion than the Petroleum Fund. Another difference is that some funds invest part of their capital in asset classes which at the moment the Petroleum Fund does not invest in (such as unlisted equities and real estate)

7. Operational considerations

Norges Bank has an organisation that can deal with any transitional phase with an increasing equity allocation and a higher volume of equity management. A higher equity allocation will not entail anything qualitatively new for the implementation of management, but it will still have some operational consequences that should be noted.

Current management costs

Equity management is more expensive than management of fixed-income securities so that an increase in the equity allocation will increase management costs. In 2000, average management costs for the equity portfolio came to 20.8 basis points, while they were 4.5 basis points for the fixed income portfolio. The size of the differential in 2000, however, also reflected the fact that a considerably higher portion of the equity portfolio than the fixed income portfolio was managed externally, and that active management in the equity portfolio was substantially higher. The distribution between internal and external management is now being changed, with a more equal distribution between equity and fixed income portfolios, thereby narrowing the cost differential.

A Canadian consulting firm, Cost Effectiveness Measurement Inc. (CEM) has cost data for several hundred US and European pension funds and, after being commissioned by Norges Bank, has produced an analysis of the costs of managing the Petroleum Fund. On the basis of the material we have received from CEM, a normal cost differential between equity and fixed income management will be 4-5 basis points if the management is handled internally. If active management is carried out externally, a normal cost differential will be about 15 basis points. The data indicate that in the case of internal index management there will be little difference in costs for equities and fixed income instruments, while for external index management it is about 1 basis point more costly for equities than for fixed income instruments.

In the Petroleum Fund, active management accounts for 38 per cent of the equity portfolio, distributed as 26 percentage points externally and 12 percentage points internally. In the fixed income portfolio, 7.5 per cent is under external management. With this distribution, and on the basis of the cost figures from CEM, the average cost differential between equity and fixed income management will be about 8 basis points.

Cost of new allocations

In addition to current management costs, one-off costs are incurred when new capital is to be invested in securities markets. In the annual report for 2000, the cost of new investments in stock markets was estimated at 25 basis points. For new investments in the fixed income market, costs can be estimated at less than 5 basis points. This means that the establishment cost for new equity portfolios is about 20 basis points higher than for new fixed income portfolios.

Cost of rebalancing

There is considerably greater price volatility in stock markets than in fixed income markets. As noted earlier, with a higher equity allocation, the absolute deviations from fixed asset and regional weights will as a rule be greater, and in the current rebalancing regime the need for transactions in connection with rebalancing will increase to a corresponding extent. With other rebalancing regimes the effect on the transaction volume may be reduced. However, the cost differential between equity and fixed income transactions will be approximately the same as for new investments.

Implementation of a higher equity allocation

A decision concerning immediate implementation involves a risk of high transaction costs. The most cost-effective approach would be to allow the equity allocation to drift up to the new target zone by channelling new capital to the equity portfolio and, at the same time, adjusting for any higher return in stock markets compared with fixed income markets. The benchmark portfolio must then be changed in accordance with this.

This gradual transition means that we cannot know in advance how long the implementation will take. This will depend on the size of new capital inflows and on market developments. According to the National Budget for 2001, the size of the Petroleum Fund is expected to reach NOK 590 billion at the end of 2001 and about NOK 750 billion at the end of 2002. With a quarterly addition of NOK 35-40 billion, a 10 percentage point increase in the equity allocation could most likely be accomplished in six months without the sale of fixed income securities.

In general, the expected excess return with a swift increase in the equity allocation should be weighed against the higher risk this may entail. In any event, however, it would be advantageous to avoid increasing the equity allocation at a faster pace than the level implied by transfers of new capital and market developments.

8. Summary

In choosing the equity allocation of the Petroleum Fund it is necessary to weigh return against risk. In this submission, we have discussed how changes in the equity allocation can influence the Fund’s return and risk.

Several methods can be used to estimate the size of the return on equities and fixed income instruments. A natural point of departure is historical return rates. These show that the average return on equities has been between 5 and 6 percentage points higher than the return on fixed income instruments. The estimates for historical return differentials, however, are associated with considerable uncertainty. It is also difficult to explain such large return differentials on the basis of theoretical models for the pricing of financial assets.

An alternative method for estimating the return on equities is to use the dividend-price ratio or the earnings-price ratio. We have used this method to make some illustrative calculations. They show that the return on equities in the period ahead may be lower than the historical average. The calculations are, however, very sensitive to underlying assumptions.

There has also been a discussion of the implications of the sharp rise in equity prices in the latter part of the 1990s for the future return on equities. Usually, a rise in equity prices can be explained by a lower risk premium or expectations of higher corporate earnings. If the rise in prices reflects reduced share premia, the return on equities will be lower in the years ahead than the historical performance. If the rise in equity prices is related to investor expectations of higher corporate earnings, the excess return on equities in the years ahead may be of the same magnitude as we have seen in a longer historical context.

If the fundamental preconditions for an increase in equity prices should change or if the rise in prices is not related to underlying fundamentals, equity prices may fall. Some are of the view that the decline in equity prices over the past year represents such a correction. Others point out that this decline is small compared with the substantial price advances at the end of the 1990s and that there is thus a risk that equity prices can fall further.

The risk of equities is often measured by the standard deviation of the return or by the probability that the return on equities will either be negative or lower than the return on other asset classes. We have used both these methods to illustrate the risk of equities. On the basis of data on historical returns, we find that the standard deviation of annual returns is about three times higher for equities than for fixed income instruments. However, the Petroleum Fund has a long investment horizon, a factor that makes it important to focus on risk measured over longer time periods. How risk changes with the length of the horizon depends

on the time series properties of the return on equities and fixed income instruments. We do not have access to sufficiently long time periods for most markets to examine time series properties. Our studies of US data for the period 1926-2000 may indicate, however, that the standard deviation of equities increases more slowly than the standard deviation of fixed income securities when the length of the investment horizon is increased. If this applies more generally, this would in isolation be an argument for increasing the equity allocation when the investment horizon is longer.

When deciding on the equity allocation for a fund, it may be natural to look at what other funds have chosen. In the US and the UK, there are many funds that have a higher equity allocation than the Petroleum Fund. In continental Europe, pension funds have on average a slightly lower equity proportion than the Petroleum Fund. However, the equity proportion in these countries is increasing, partly because the authorities in many of them have relaxed restrictions on investments by pension funds. An important difference between the investment strategy of the Petroleum Fund and other funds is that many funds invest part of their capital in asset classes which the Petroleum Fund does not invest in at the moment (such as private equities and real estate).

A change in the Petroleum Fund's equity allocation will not entail anything quantitatively new in the operational management. Norges Bank has an organisation for handling any change in the Fund's equity allocation. The most cost-effective way to change the equity allocation is to use new capital in the Fund to implement a gradual change in the Fund's composition.

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Annex 1

In section 5, we showed how a change in the equity allocation would influence the risk and return on a portfolio. In the analyses in that section, we assumed that the return on fixed income securities was 5 per cent and the return on equities was 8 per cent. The standard deviation of equities and fixed income instruments and the correlation between them were calculated on the basis of historical data. The effect of a change in the equity allocation on a portfolio's properties will, however, depend on the assumptions made concerning returns, standard deviation and correlation. In this annex we will illustrate this by applying different assumptions concerning the return on equities and fixed income securities. We assume throughout that the return on fixed income securities is 5 per cent, but allow the risk premium on equities to be 2 and 4 per cent respectively. We also include relevant results from section 5 in order to simplify comparisons. Below, the results are shown in tables and only for equity allocations between 30 per cent and 70 per cent. This has been done in order to demonstrate more precisely how different return differentials influence the portfolio's properties.

In the tables, we estimate the probability of a negative return, confidence intervals for annual returns and confidence intervals for final wealth with different investment horizons. We have assumed in all the calculations that the return on equities and fixed income securities is normally distributed. Confidence intervals are estimated in such a way that if the assumption of normal distribution holds true, 95 per cent of the observations can be expected to be within this interval.

It is important to emphasise that the results presented in this annex are sensitive to the assumptions made concerning the properties of equities and fixed income securities with regard to returns, standard deviation, correlation and the statistical distribution. If, for example, returns have a broader statistical distribution than the normal distribution, the results below may underestimate the portfolio risk. The estimate for the probability of a negative return will in part be sensitive to our assumptions concerning the level of the return on equities and fixed income securities. The results below must therefore be seen in the light of the assumptions made.

Table 1 Confidence interval for annual return as a function of equity allocation. The table shows the lower limit in the interval, expectations value and upper limit in the interval

Equity	Equities 8%, bonds 5%			Equities 7%, bonds 5%			Equities 9%, bonds 5%		
allocation	Lower	Expectations	Upper	Lower	Expectations	Upper	Lower	Expectations	Upper
0.3	-5.4 %	5.9 %	17.2 %	-5.7 %	5.6 %	16.9 %	-5.2 %	6.2 %	17.6 %
0.4	-7.5 %	6.2 %	19.9 %	-7.9 %	5.8 %	19.5 %	-7.2 %	6.6 %	20.4 %
0.5	-9.9 %	6.5 %	22.9 %	-10.3 %	6.0 %	22.3 %	-9.5 %	7.0 %	23.5 %
0.6	-12.3 %	6.8 %	25.9 %	-12.8 %	6.2 %	25.2 %	-11.8 %	7.4 %	26.6 %
0.7	-14.9 %	7.1 %	29.1 %	-15.4 %	6.4 %	28.2 %	-14.3 %	7.8 %	29.9 %

Table 2 Probability of negative cumulative return as a function of equity allocation and investment horizon

Equity	Equities 8%, bonds 5%			Equities 7%, bonds 5%			Equities 9%, bonds 5%		
allocation	1 year	5 years	10 years	1 year	5 years	10 years	1 year	5 years	10 years
0.3	15.3 %	1.1 %	0.1 %	16.5 %	1.5 %	0.1 %	14.1 %	0.8 %	0.0 %
0.4	18.9 %	2.4 %	0.3 %	20.5 %	3.3 %	0.5 %	17.5 %	1.8 %	0.2 %
0.5	22.2 %	4.3 %	0.8 %	24.0 %	5.7 %	1.3 %	20.5 %	3.3 %	0.5 %
0.6	24.9 %	6.5 %	1.6 %	26.9 %	8.4 %	2.6 %	23.0 %	5.0 %	1.0 %
0.7	27.2 %	8.8 %	2.8 %	29.3 %	11.2 %	4.3 %	25.2 %	6.8 %	1.7 %

Table 3 Confidence interval for final wealth as a function of equity allocation and investment horizon. Starting wealth is equal to 100. The table shows the lower limit in the interval, expectations value and upper limit in the interval

Equities 8%, bonds 5%

Equity	1 year			5 years			10 years		
allocation	Lower	Expectations	Upper	Lower	Expectations	Upper	Lower	Expectations	Upper
0.3	94.6	105.9	117.2	104.1	133.2	168.0	124.7	177.5	245.2
0.4	92.5	106.2	119.9	100.1	135.1	178.5	118.7	182.6	268.9
0.5	90.1	106.5	122.9	95.8	137.1	190.2	112.2	187.9	295.9
0.6	87.7	106.8	125.9	91.3	139.0	203.0	105.4	193.3	326.2
0.7	85.1	107.1	129.1	86.8	141.0	216.8	98.6	198.8	359.6

Equities 7%, bonds 5%

Equity	1 year			5 years			10 years		
allocation	Lower	Expectations	Upper	Lower	Expectations	Upper	Lower	Expectations	Upper
0.3	94.3	105.6	116.9	102.7	131.4	165.6	121.3	172.6	238.4
0.4	92.1	105.8	119.5	98.3	132.6	175.1	114.4	175.9	258.9
0.5	89.7	106.0	122.3	93.6	133.9	185.8	107.1	179.3	282.3
0.6	87.2	106.2	125.2	88.8	135.2	197.4	99.7	182.7	308.3
0.7	84.6	106.4	128.2	84.1	136.5	209.8	92.4	186.2	336.7

Equities 9%, bonds 5%

Equity	1 year			5 years			10 years		
allocation	Lower	Expectations	Upper	Lower	Expectations	Upper	Lower	Expectations	Upper
0.3	94.8	106.2	117.6	105.6	135.1	170.4	128.2	182.5	252.2
0.4	92.8	106.6	120.4	102.0	137.7	181.8	123.2	189.5	279.1
0.5	90.5	107.0	123.5	98.0	140.3	194.7	117.4	196.8	310.1
0.6	88.2	107.4	126.6	93.8	142.9	208.8	111.3	204.3	344.9
0.7	85.7	107.8	129.9	89.7	145.6	224.0	105.1	212.1	383.7