Central Bank Collateral Framework as an Unconventional Policy Tool*

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Abstract

This paper investigates the macroeconomic effects of easing collateral standards in open market operations (OMOs), an unconventional policy tool that central banks in advanced economies (AE) implemented during the Great Recession to attenuate the effects of financial market disruptions on the real economy. Using a New-Keynesian general equilibrium model with an explicit banking sector and central bank collateralized lending, we study the central bank policy of broadening the range of assets that are accepted as collateral in OMOs, e.g. accepting a wider range of government bonds or taking corporate loans as eligible. Taking the analysis to the experience of central banks in AE during the Great Recession, when the policy rate endogenously hits the zero lower bound (ZLB) due to sudden rise in funding stress, active use of collateral policies helps mitigate the sharp drop in asset prices, credit, investment, and output. Our analysis also reveals that under severe financial market conditions, central banks do not need to wait until the policy rate hits the ZLB: active collateral policies reduce the need for looser conventional policy response and can help central banks avoid the ZLB.

Keywords: Central bank collateralized lending; Unconventional monetary policy; Zero lower bound.
JEL Codes: E52; E58; E61.

*The views expressed in this paper are those of the authors and do not necessarily reflect the official views or the policies of Norges Bank and the Norwegian Ministry of Finance.

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1 Introduction

The Global Financial Crisis significantly re-shaped our understanding of macro-financial linkages and posed questions beyond the scope of conventional monetary policy making. As the federal funds rate had been reduced at an unprecedented pace and eventually hit the effective zero lower bound (ZLB), the US Federal Reserve (Fed) resorted to various unconventional policy measures. Figure 1 shows a particular unconventional policy response: easing collateral standards in open market operations. Often called 'quantitative easing', such policies (including other measures such as large-scale asset purchases and target-specific liquidity facilities), have generally been regarded particularly helpful to ease the credit market disruptions by mitigating the rise in liquidity premia and help attenuate the spillovers from the financial sector to the real economy. In this paper, we study a widely-used unconventional policy, easing collateral constraints on financial intermediaries in receiving funds from the central bank in open market operations. In particular, building a New-Keynesian general equilibrium model with an explicit banking sector and central bank collateralized lending, we trace how precisely such unconventional policies transmit to financial aggregates and real economic activity including the case of ZLB, and shed light on what would be the real economic outcome in the absence of such policies.

The collateral policy of central banks, the type or the fraction of assets that central banks take as eligible when lending to commercial banks, is generally absent in standard New Keynesian DSGE literature. The central bank is typically assumed to set the policy rate in response to fluctuations in key macroeconomic aggregates, such as inflation and output gap, where money supply is passively adjusted by the central bank to satiate money demand. This standard framework is regarded as well-suited for monetary policy analysis during normal times, given that policy rates hover sufficiently above the effective ZLB and central banks do not actively re-calibrate their collateral standards. Nonetheless, severe disruptions in credit markets coupled with policy rates fast reaching the ZLB at the onset of the crisis have left the use of unconventional policies as the only venue to follow and proved limitations of standard approach to monetary policy making. Continuing to follow accommodative policies at the ZLB, the Fed and other major central banks have significantly eased the collateral standards, among other measures.[1]

[1]For example, the Fed launched new lending programmes such as the Term Auction Facility, the Commercial

Despite overwhelming evidence on easing the collateral standards as a crisis management tool, the precise transmission of such policy is yet to be understood. How would an easing in the collateral policy, e.g., (i) by increasing the fraction of safe assets (e.g. Treasury securities or government bonds) that are pledgable for borrowing from the central bank, which we call as *safe asset collateral policy*; or (ii) by widening the eligible pool of assets by including risky securities (including commercial loans and corporate securities as well), which we call as *risky asset collateral policy*, affect bank balance sheets, their lending behavior, credit and bond spreads, and eventually, real aggregates such as investment and output?

A model suitable to address these questions requires at least three departures from a standard New-Keynesian model. First, the existence of collateralized lending market between the central bank and commercial banks, by definition, requires a model with an explicit banking sector (which optimally chooses how much to demand monetary injection from the central bank in addition to loan supply and deposit demand decisions) and an explicit role for the central bank (e.g. central bank setting the collateral standards as a policy tool). Second, in equilibrium, for different assets to effectively serve as collateral (e.g. Treasury notes and commercial loans), asset portfolio of commercial banks should be determinate and endogenous. Moreover, for active collateral policy and hence changes in the composition of central bank's balance sheet to matter for the real economy, different assets should have different returns, leading to multiple interest rates and liquidity premia. Third, the commercial banks' problem of choosing the demand for injection should be non-trivial: banks would otherwise demand an indefinitely high level of injection at a given policy rate to expand their balance sheets and earn unbounded profits. Therefore, a proper approach should incorporate an endogenous limit to central bank liquidity injection due to potentially risky banking activity.

Paper Funding Facility and the Treasury Securities Lending Facility in which it extended short-term credit to depository institutions, purchased three-month commercial paper and exchanged Treasury securities for mortgage-backed securities and commercial paper, respectively. The Fed is certainly not the only example regarding unconventional policies. The Bank of England (BoE) bought commercial paper, corporate bonds and government bonds under the programme called the Asset Purchase Facility. The European Central Bank (ECB) started to intervene directly in securities markets under the Securities Markets Programme followed by a purchase programme for bank-issued covered bonds and the Outright Monetary Transactions (OMT) programme. We discuss briefly the unconventional policy responses in Section 2. On the effect of quantitative easing policies on financial and real aggregates, see, among others, Christensen et al. (2014), McAndrews et al. (2015), Taylor and Williams (2009), Wu (2008), Krishnamurthy and Vissing-Jorgensen (2011), Gagnon et al. (2011), Campbell et al. (2011), Brave and Genay (2011), Walentin (2014), Gambacorta et al. (2014), Lutz (2015), and Greenwood et al. (2015) for particular emphasis on QE announcements. For the effect of various QE policies at a global scale, see Bowman et al. (2015), Ahmed and Zlate (2014), Fratzscher et al. (2015), and Neely (2015).

Along these lines, we build a New-Keynesian model with an explicit banking sector following Gertler and Karadi (2011), Gertler and Kiyotaki (2010) and Mimir (2016), where we further consider the requirement of holding collateral to receive injection from the central bank in the spirit of Schabert (2015).

In the model, banks are subject to two constraints. First, they are constrained in their ability to raise funds from households due to a moral hazard problem. In particular, banks' incentive to divert assets for their own benefit (or likelihood of their lending in excessively risky projects) creates a moral hazard problem between the funders and the bank, leading to a *funding* constraint for the bankers. Second, they are further constrained in receiving funds from the central bank via a *collateral* constraint. Following the common practice, we assume that the central bank takes only a certain fraction of government bonds or corporate loans as pledgable. In equilibrium, therefore, banks should strike a balance between dynamic costs and benefits of holding pledgable assets, e.g. higher corporate lending may tighten the funding constraint but at the same time may ease the collateral constraint.

The model exhibits a double acceleration due to funding and collateral constraints. The first is due to the conventional financial amplification mechanism as in Gertler and Karadi (2011). Consider, for instance, an unfavorable productivity shock that leads to a decline in output, investment, credit and asset prices. In response, banks' balance sheets deteriorate, funding constraint gets tighter, and therefore investment demand and asset prices decline. In turn, banks' balance sheet conditions deteriorate, and credit and output decline even further. The second acceleration is similar in notion to Kiyotaki and Moore (1997) and Iacoviello (2005), and is due to collateral requirement for receiving monetary injection from the central bank. If corporate loans are deemed eligible for receiving injection, then an exogenous decline in asset prices reduces the value of corporate loans pledgable to the central bank, and therefore induces a decline in monetary injection from the central bank. In turn, banks face a lower funding base, leading to further decline in credit, investment demand, asset prices, and output. In sum, an unfavorable shock is endogenously propagated through the economy not only due to a tighter funding constraint (as in Gertler and Karadi, 2011) but also due to a tighter collateral constraint.

The central bank, however, is endowed with a rich set of tools to attenuate such a propagation:

Besides the conventional policy rate, the central bank can implement either safe asset collateral policy, or risky asset collateral policy, or both policies at the same time. In formulating these policy tools, we assume that the central bank follows simple and implementable policy rules. It sets the fraction of eligible safe assets (e.g. a κ_t^b fraction of Treasury bills) in response to fluctuations with bond spreads: a policy rule of higher κ_t^b in response to higher bond spreads (along with persistence in the policy rule). Alternatively, the central bank can widen the eligible pool of assets by accepting corporate loans in addition to Treasury Bills (setting both $\kappa_t^b > 0$ and $\kappa_t > 0$). Similarly, we assume that the central bank sets a higher κ_t in response to higher corporate loan spreads (corporate lending rate net of risk-free rate). By following such policy rules, or "active collateral policies" as we label throughout the text, the central bank can mitigate the effects of unfavorable financial conditions on the real economy.

Our results suggest the following: First, we analytically show that both safe and risky asset collateral policies help mitigate the rise in bond and credit spreads, respectively. In particular, both types of collateral policies lower liquidity risk premium charged on holding those assets. Second, in a partial equilibrium setup, we show that active collateral policies, if accompanied by looser conventional policy, have stronger effect on the risk premia. Specifically, the extent to which a higher eligibility for safe and risky assets reduces the bond and credit spreads is decreasing in the policy rate. Hence, hitting the ZLB and thus being unable to reduce the policy rate further limits the effectiveness of active collateral policies. These analytical results also suggest that under unfavorable financial conditions, the central banks do not need to wait until the policy rate hits the ZLB in order to implement active collateral policies. Our numerical results below also confirm these findings. Third, our quantitative analysis also reveals that during bad times due to a sudden rise in funding stress on banks, loosening in central bank collateral requirements attenuate the effect of financial shocks on the real economy. In particular, it shows that by reducing either bond or credit spreads, safe and risky asset collateral policies help banks receive a higher funding base and provide credit to non-financial sector at more favorable terms. This mitigates the fall in credit, investment and output and increases inflation. We also show that this policy is more effective if accompanied with a looser conventional policy response. Hence, our results underline the importance of coordination between conventional and collateral policies. If the policy rate is at the ZLB, however, an active collateral policy is the only viable policy alternative.

To this end, we take our analysis to the Fed's recent experience, starting active collateral policies when the federal funds rate hit the ZLB. In particular, we consider a sudden rise in funding stress for banks that leads to an unprecedented rise in corporate loan spreads (by about 600 basis points, see e.g. Gilchrist and Zakrajek, 2012) and a sharp decline in investment (by about 4% in cyclical terms). In response to the equilibrium decline in inflation and output, the central bank lowers the policy rate aggressively, and eventually hits the ZLB (in six quarters as observed in the U.S. data). We postulate that the central bank then starts implementing active collateral policies (i.e. higher κ_t^b in response to higher bond spreads, or higher κ_t in response to higher corporate spreads), and continues following these rules as long as the policy rate is at the ZLB. The results suggest that, similarly to our analysis before, active collateral policies help mitigate the abrupt fluctuations in real and financial aggregates. Our results suggest that, in the absence of active collateral policies, investment, for instance, would drop by about 4%, and would be persistently below its long-run level. In sum, our take is that Fed's collateral practice at the time has served as a powerful tool to mitigate the effect of financial crisis on the real economy, and can serve as a strong viable policy option to use for central banks in general.

Our paper is related to a burgeoning literature on the effect of quantitative easing policies in theoretical models. Cúrdia and Woodford (2010), for instance, show that targeted asset purchases (rather than quantitative easing in the strict sense) may be effective by reducing credit spreads when financial markets are severely disrupted. Similarly and with a particular emphasis on banking sector balance sheet effects, Gertler and Karadi (2013) show that large scale asset purchases (QE1) has partially offset the disruption in financial intermediation. Although both studies are important cornerstones in the literature in terms of incorporating financial sector in DSGE models and analyzing QE-type policies, they lack a significant ingredient of how most central banks lend to financial institutions in real life, i.e. collateralized lending. On the other hand, Schabert (2015) investigates central bank collateralized lending in a standard New Keynesian model without a banking sector, which is not suitable to investigate the effects of collateralized lending on bank balance sheets. In this regard, the contribution of our paper is to provide a unified framework of an explicit banking sector and central bank collateralized lending, enriching the set of policy tools often considered much narrowly in standard New Keynesian models. Using this unified framework, we are able to study (i) the macroeconomic effects of a sudden disruption in financial sector (as had been observed at the outset of the crisis) and its reflection on bank balance sheets, and (ii) the performance of central bank collateralized lending policy in mitigating the adverse effects of such disruption (reflecting the Fed's practice at the time) as well as the effectiveness of such policies during when the policy rate endogenously hits the ZLB.

The paper proceeds as follows: Section 2 presents an overview of advanced economy central banks' collateral framework, and their use during the crisis. Section 3 presents the model economy. Section 4 presents the results and the model dynamics. Section 5 provides the analytical insights of the model. Section 6 gives the results for the case of occasionally binding ZLB, and finally Section 7 concludes.

2 Central Bank Collateral Frameworks

The collateral frameworks and the terms of use across central banks varies in different aspects and in different market situations; and can also be discretionary depending on the market-wide stress.[2] According to the BIS reports, as of July 2012, the main classification of collateral frameworks is built on three basic styles: uniform vs. differentiated, narrow vs. wide, and earmarked vs. pooled. As a first classification, central banks including the Bank of Japan (BoJ) and the ECB have applied 'uniform' collateral eligibility, while others including the Fed and the BoE have had differentiated eligibility. In those differentiated frameworks, non-routine liquidity operations allow for less liquid collateral. Second, in terms of eligible issuer type, there is also a variation among the different jurisdictions. Central banks of advanced countries, mostly, applied 'wide' (low restriction) collateral eligibility criteria where the obligations of private financial and non-financial entities are also accepted as collateral for liquidity operations. Bank of Canada (BoC), the Fed and the Bank of Korea (BoK), on the other hand, kept the criteria narrow (high restriction) and accepted only public sector debt for central bank open market operations (OMOs), but have used wider criteria for their standing facilities and specified loan programs. Third, central bank collateral frameworks were differentiated also in terms of the allowance of collateral pooling. While the ECB and the

[2] See BIS Markets Committee Report (2013) and BIS CGFS Papers No. 53 (2015).

BoJ mostly accepted pooled collateral, central banks including the Fed, BoC, Swiss National Bank (SNB) and BoK have accepted only earmarked collateral which are related to specific loans for OMOs, though these latter countries allowed pooled collateral for their standing facilities.

Beyond these general differences in basic styles of collateral frameworks, central banks also differ, more specifically, in collateral types, minimum rating requirements, haircuts and risk control measures.

Prior to 2007, the Fed bought and sold only treasuries in its OMOs and accepted treasuries, direct agency debt, and agency mortgage-backed securities (MBS) as collateral for its temporary OMOs, repurchase and reverse repurchase agreement (Repos) transactions. During the same period more than half of the collateral pledged by banks to the ECB were liquid government bonds. With the arrival of financial crisis central banks extended their eligible assets to dampen the market wide stress on financial system and to mitigate the negative repercussions on the real economy. For instance, the BoC, the BoJ and the ECB included foreign-currency denominated assets into their eligible collateral basket. The ECB also extended the eligible collateral set through fixed term deposits and additional types of credit claims. Moreover, the BoJ extended the set far enough to include obligations of real estate investment corporations. Figure 1 shows how the Fed significantly broadened the range of collateral eligible to obtain credit whereas it only buys and sells treasuries to selected counterparties in normal times. The figure includes total value of collateral pledged to the Fed in the Term Auction Facility (TAF), Primary Dealer Credit Facility (PDCF) and the Term Securities Lending Facility (TSLF) operations.

A number of central banks responded the financial crisis by reducing their minimum rating requirements, such as the reduction of requirements for securities of deposit-taking intermediaries by the Reserve Bank of Australia and for marketable securities and credit claims by the ECB. Another adjustment applied on collateral frameworks with the arrival of the crisis is the change in haircuts. Lower size of haircuts in Japan and the US are prominent examples.

All of the measures taken by central banks through collateral frameworks as a response to the crisis, some of which are retained currently and even broadened recently, can be summarized as the loosening of collateral standards. In our paper, the level of tightness for collateral standards are represented by two critical policy parameters, κ_t^b and κ_t, which are fractions of eligible Treasury securities and corporate securities, respectively. In a collateral framework setting where only Treasury securities are eligible for central bank lending, the standards can be eased by increasing κ_t^b or widening the eligible pool by accepting corporate securities as collateral as well ($\kappa_t > 0$). With these two policy parameters we refrain from the dispersion of collateral standards while capturing the main transmission mechanism of these frameworks.

3 The Model

The model economy is composed of households, banks, intermediate goods producers, retailers, final goods producers, capital goods producers, government and central bank. Households supply labor to intermediate goods producers and put their deposits at the banks. Banks, in turn, use these deposits, their own equity capital, as well as monetary injection from the central bank to finance their lending operations and holding of government bonds. The modeling of the financial sector follows Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), except that here we explicitly consider central bank collateralized lending in open market operations. In particular, banks are required to pledge collateral to receive injection from the central bank. Following the common practice, we assume that the central bank takes a certain fraction of government bonds as pledgable. In further analyses, we also allow for corporate loans as pledgable assets in light of the Fed's practice at the time (e.g. the TAF).

The non-financial firms are standard as in typical New-Keynesian models. Intermediate goods producers use capital (that they purchase from capital producers by borrowing from banks via issuing equity) and labor supplied by the households to produce intermediate goods. These goods are then bought by monopolistically competitive retailers, which are then aggregated by perfectly competitive final goods producing firms. Below is a detailed description of the model economy.

3.1 Households

There is an infinitely-lived representative household with a [0,1]-continuum of members. Within the household, there are $1 - f$ "workers" and f "bankers". Each banker operates a financial intermediary (that can be called as a "bank") that facilitates flow of funds from the households to the firms. Workers supply labor h, receive w as real wage per labor hour, and deposit their savings at the banks owned by the banker members of other households. There is perfect consumption insurance within the household, i.e., workers return their wage and bankers their dividends back to the household.[3]

The representative household derives utility from an aggregate consumption good c_t and leisure $l_t = 1 - h_t$. She enters the period t with the risk-free gross nominal rate of return R_t on their real deposits d_{t-1} and real money balances m_{t-1}. Further receiving lump-sum transfers from the government and divident payments from the firms owned, Ξ_t, the household decides how much to consume, save and supply labor. Formally, the representative household solves

$$\max_{\{c_t,h_t,d_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t - h_c c_{t-1}, h_t\right) \tag{1}$$

subject to the flow budget constraints

$$c_t + d_t + m_t = w_t h_t + \frac{R_t d_{t-1}}{\pi_t} + \frac{m_{t-1}}{\pi_t} + \Xi_t \tag{2}$$

and the following cash-in-advance constraint

$$c_t \leq \frac{R_t d_{t-1}}{\pi_t} + \frac{m_{t-1}}{\pi_t} - d_t. \tag{3}$$

where $u(t)$ is the period-t utility function, satisfying standard assumptions $u_c > 0$, $u_l > 0$, $u_{cc} < 0$ and $u_{ll} < 0$. $\beta \in (0,1)$ is the subjective discount factor, and E_t is the expectation operator conditional on information set available at the beginning of t. $h_c \in [0,1)$ governs the degree of internal habit formation in consumption.[4]

Equation (2) represents the household's period budget constraint. The household carries over d_{t-1} and m_{t-1}, the balance of real deposits held at the banks and the real money holdings at the end of $t-1$, to the current period t. These terms are deflated by the gross inflation rate π_t realized from $t-1$ to t. By carrying over deposits to the current period, household earns R_t.

[3] The assumption of perfect consumption insurance within the household makes the agency problem that we introduce in Section 3.2 more tractable.

[4] Habit formation in consumption, a now-standard feature in medium-scale New-Keynesian models, help match the observed hump-shaped response of consumption to disturbances driving the economy (see, e.g., Christiano and Eichenbaum, 2005)

Households need to hold money to finance consumption expenditures. In particular, following Lucas (1982) and Cooley and Hansen (1989), we assume that asset markets open first, and goods market opens thereafter. Therefore, consumption cannot exceed real return earned in the asset market (beginning of period-t real money balances and real return from holding deposits). As long as $R_t > 1$, the household would hold the amount of cash just sufficient to finance her desired level of consumption, i.e., the equation (3) binds.

Household preferences over consumption and labor is governed by a CRRA-type utility function given by

$$u(c_t, h_t) = \frac{(c_t - h_c c_{t-1})^{1-\sigma} - 1}{1-\sigma} - \psi \frac{h_t^{1+\upsilon}}{1+\upsilon} \tag{4}$$

where $\sigma > 0$ is the degree of relative risk aversion, $\upsilon > 0$ represents the (inverse) Frisch elasticity of labor supply, and $\psi > 0$ is the relative disutility weight on labor. Accordingly, the solution to the households' problem yields the following optimality conditions:

$$\zeta_t = (c_t - h_c c_{t-1})^{-\sigma} - \beta h_c (c_{t+1} - h_c c_t)^{-\sigma} \tag{5}$$

$$\zeta_t = \beta E_t \left[\frac{R_{t+1} \zeta_{t+1}}{\pi_{t+1}} \right] \tag{6}$$

$$\frac{\psi h_t^{\upsilon}}{\zeta_t} = \frac{w_t}{R_{t+1}} \tag{7}$$

where ζ_t is the marginal utility of consuming an additional unit of income today. Equation (6) is the standard consumption-savings optimality condition, which equates the marginal cost of foregoing consumption today to the expected discounted benefit of savings, taking into account the household's preferences in habit formation. Equation (7) is the consumption-leisure optimality condition, which, due to the existence of cash-in-advance constraint, is inter-temporal and reflects the trade-off between consumption and labor across periods.

Finally, the stochastic discount factor, which is taken as given by the sectors owned by the household, is given by $\Lambda_{t+s|t} = \beta E_t \left[\frac{\partial u(t+s)/\partial c_{t+s}}{\partial u(t)/\partial c_t} \Pi_{k=1}^{s} \frac{1}{\pi_{t+k}} \right]$. A no-Ponzi condition on households,

$\lim_{T\to\infty} E_t \beta^T \Lambda_{t+T|t} D_{t+T} \geq 0$, as well as $m_t \geq 0$ and $d_t \geq 0$ for all $t > 0$ completes the household's problem.

3.2 Banks

Banks finance their lending activity by using their own net worth, household deposits as well as monetary injection from the central bank. The injection is provided against eligible assets, similar to central banking practice in open market operations (see Section 2). We take government bonds and corporate loans as eligible assets to receive injection from the central bank. To this end, we provide formal representation of the banking sector.

Let n_{jt} denote the bank j's net worth (the amount of wealth that the banker j has) at period t. Banks use these internal funds, deposits from the households, d_{jt}, as well as monetary injection from the central bank in open market operations, in_{jt}, to finance their lending ($q_t s_{jt}$) and the purchase of government bonds (b_{jt}). Thus, period-t balance sheet of a bank j is given by

$$q_t s_{jt} + b_{jt} = d_{jt} + in_{jt} + n_{jt}. \tag{8}$$

where loans serve as state-contingent claims s_{jt} toward the ownership of firms' physical capital which are traded at the market price q_t. The balance sheet of the bank j is presented in Table 1 below:

Table 1: Bank j's Balance Sheet

Assets	Liabilities
Loans ($q_t s_{jt}$)	Household deposits (d_{jt})
Government bonds (b_{jt})	Central bank injection (in_{jt})
	Net Worth (n_{jt})

Next period's net worth, n_{jt+1}, is determined by earnings on assets and the outlays due to liabilities. In particular,

$$n_{jt+1}\pi_{t+1} = R_{kt+1} q_t s_{jt} + R_{bt+1} b_{jt} - R_{t+1} d_{jt} - R_{mt+1} in_{jt} \tag{9}$$

where R_{kt+1} is the gross nominal return on the purchased firm equity, R_{bt+1} is the gross nominal return on government bond holdings, R_{t+1} is the gross nominal cost of deposit borrowed from worker $i \neq j$, and R_{mt+1} is the central bank's money injection rate (the conventional policy rate). Combining equations (8) and (9) yields a convenient expression for how the net worth evolves over time:

$$n_{jt+1}\pi_{t+1} = (R_{kt+1} - R_{t+1})q_t s_{jt} + (R_{bt+1} - R_{t+1})b_{jt} + (R_{t+1} - R_{mt+1})in_{jt} + R_{t+1}n_{jt} \tag{10}$$

Equation (10) suggests that bank j accumulates net worth to the extent that the return on lending exceeds the risk-free rate ($R_{kt+1} - R_{t+1} > 0$), the return on holding government bonds is above the risk-free rate ($R_{bt+1} - R_{t+1} > 0$), and the risk-free rate is above the cost of injection ($R_{t+1} - R_{mt+1} > 0$). Furthermore, bank j can accumulate net worth by using internal funds instead of taking deposits from households, earning R_{t+1}.

Central bank injection, in_{jt}, is provided only against eligible assets. In particular, the injection that bank j receives is constrained by a certain fraction of its government bond and firm equity holdings at the price R_{mt+1}, given by

$$in_{jt} \leq \kappa_t^b \frac{b_{jt}}{R_{mt+1}} + \kappa_t \frac{q_t s_{jt}}{R_{mt+1}} \tag{11}$$

where κ_t^b and κ_t are the fractions of bank bond and firm equity holdings accepted as collateral, respectively, which are chosen by the central bank.[5]

The fractions, κ_t^b and κ_t, and the injection rate, R_{mt+1}, are the central bank policy instruments. As we are going to study below, in response to financial market developments, the central bank can set κ_t^b and κ_t to adjust the amount of collateral eligible for banks to receive central bank injection. Moreover, the central bank can set R_{mt+1}, the price of money, in response to fluctuations in macroeconomic aggregates such as inflation and output gap as in conventional New-Keynesian

[5] Here we take these assets as homogenous classes of assets. In particular, government bonds are assumed to be homogenous, having similar risk structures in terms of liquidity, maturity and default, and that corporate papers –or corporate loans in our framework– are homogenous in that they have the same maturity (one quarter), default risk and liquidity profile. While these assumptions are restrictive (see, e.g., Section 2), we maintain them for the tractability of the model.

models.

Bankers are assumed to have a finite life and survive to the next period with probability $0 < \theta < 1$ to ensure that they need external financing on top of internal financing.[6] At the end of each period, $1-\theta$ measure of new bankers are born and are remitted $\frac{\epsilon^b}{1-\theta}$ fraction of the net worth owned by exiting bankers. Given this framework, the bank j's objective is to maximize the expected present discounted value of the terminal net worth of the financial firm, V_{jt}, by choosing the amount of claims toward the ownership of non-financial firms' physical capital, s_{jt}, government bond holdings, b_{jt}, and injection from the central bank, in_{jt}. That is,

$$V_{jt} = \max_{s_{jt}, b_{jt}, in_{jt}} E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t,t+1+i} \Big\{ (R_{kt+1+i} - R_{t+1+i})(q_{t+i} s_{jt+i}) + (R_{bt+1+i} - R_{t+1+i}) b_{jt+i}$$

$$+ (R_{t+1+i} - R_{mt+1+i}) in_{jt+i} + R_{t+1+i} n_{jt+i} \Big\}, \tag{12}$$

where $\Lambda_{t+1+i|t}$ is the $1+i$ periods ahead stochastic discount factor of households. Note that as long as $E_t \beta^{i+1} \Lambda_{t,t+1+i} R_{kt+1+i} - R_{t+1+i} > 0$, $E_t \beta^{i+1} \Lambda_{t,t+1+i} R_{bt+1+i} - R_{t+1+i} > 0$, and $E_t \beta^{i+1} \Lambda_{t,t+1+i} R_{t+1+i} - R_{mt+1+i} > 0$ holds, the problem above implies an unbounded cash flow for the operating banks. Therefore, we introduce a moral hazard problem between banks and households to ensure a bounded maximization problem.

The key feature of the financial sector unfolds around a moral hazard problem between banks and households. Households believe that banks might divert a certain fraction (λ) of their assets for their own benefit.[7] In this case, the depositors shall initiate a bank run that leads to the liquidation of the bank altogether. In this regard, λ can be interpreted as reflecting the degree of funding stress on banks. Therefore, to prevent liquidation by bank runs, the bank j's optimal plan regarding the choice of s_{jt}, b_{jt} and in_{jt} at any date t should satisfy an incentive compatibility constraint

[6] This assumption ensures that bankers never accumulate enough net worth to finance all their asset purchases via internal funds.

[7] One can interpret possibility of diverting funds as bankers' inability to manage funds fully in the interest of depositors or that bankers might invest in too risky projects that commands an excessively high return for bankers but a low return for depositors.

$$V_{jt} \geq \lambda_t(q_t s_{jt} + \omega b_{jt}) \tag{13}$$

where $\omega > 0$ is a constant, showing the riskiness of government bonds (if any). This constraint suggests that, for depositors to put deposits at the banks, what the bankers would lose if they were to divert funds should be greater than or equal to what they would gain by diverting assets. In particular, the liquidation cost to bank j of diverting funds, V_{jt}, should be greater than or equal to the diverted portion of assets, $\lambda_t(q_t s_{jt} + \omega b_{jt})$.[8] $\omega > 0$ is a constant that enables existence of a non-zero return differential between government bonds and corporate loans, and in turn, ensures a well-determinate asset portfolio of banks.

The key financial disturbance that we consider is the innovations to the level of funding stress on banks. In particular, we let λ follow a stochastic AR(1) process: $\lambda_t = (1 - \rho_\lambda)\overline{\lambda} + \rho_\lambda \lambda_{t-1} + \epsilon_{\lambda_t}$, where ρ_λ is the persistence parameter, $\overline{\lambda}$ is the long-run deterministic value of λ, and ϵ_{λ_t} is the funding stress shock.

Bank j's Maximization Problem. *Bank j chooses the amount of claims toward the ownership of nonfinancial firms' physical capital, s_{jt}, government bond holdings, b_{jt}, and injection from the central bank, in_{jt}, to maximize the expected present discounted value of terminal net worth, V_{jt}, given by (12), subject to the balance sheet given by (8), the collateral constraint for central bank injection given by (11), and the incentive compatibility constraint given by (13).*$\diamondsuit$

Proposition 1. *One can show that V_{jt} is linear in $(s_{jt}, b_{jt}, in_{jt}, n_{jt})$, and optimal V^*_{jt} satisfies*

$$V^*_{jt} = \nu^s_t q_t s^*_{jt} + \nu^b_t b^*_{jt} + \nu^n_t n^*_{jt} + \nu^{in}_t in^*_{jt} \tag{14}$$

where ν^s_t, the expected discounted marginal value of extending credit, ν^b_t, the expected discounted marginal value of holding government bonds, ν^n_t, the expected discounted marginal value of accumulating net worth, and ν^{in}_t, the expected discounted marginal value of borrowing from the central

[8]Due to bankers' such tendency to divert funds, depositors restrict the amount they deposit at the banks, which then renders bank balance sheet matter for business cycle fluctuations. The incentive compatibility constraint of the form given in equation (13) is similar to Gertler and Kiyotaki (2010) except that here we have government bond holdings.

bank in open market operations, can be expressed recursively as given by:

$$\nu_t^s = E_t\left\{(1-\theta)\beta\Lambda_{t,t+1}(R_{kt+1} - R_{t+1}) + \theta\beta\Lambda_{t,t+1}\chi_t^s\nu_{t+1}^s\right\} \tag{15}$$

$$\nu_t^b = E_t\left\{(1-\theta)\beta\Lambda_{t,t+1}(R_{bt+1} - R_{t+1}) + \theta\beta\Lambda_{t,t+1}\chi_t^b\nu_{t+1}^b\right\} \tag{16}$$

$$\nu_t^{in} = E_t\left\{(1-\theta)\beta\Lambda_{t,t+1}(R_{t+1} - R_{mt+1}) + \theta\beta\Lambda_{t,t+1}\chi_t^{in}\nu_{t+1}^{in}\right\} \tag{17}$$

$$\nu_t^n = E_t\left\{(1-\theta)\beta\Lambda_{t,t+1}R_{t+1} + \theta\beta\Lambda_{t,t+1}\chi_t^n\nu_{t+1}^n\right\} \tag{18}$$

where $\chi_{t,t+1}^s = \frac{q_{t+1}s_{jt+1}}{q_t s_{jt}}$, $\chi_{t,t+1}^b = \frac{b_{jt+1}}{b_{jt}}$, $\chi_{t,t+1}^{in} = \frac{in_{jt+1}}{in_{jt}}$ *and* $\chi_{t,t+1}^n = \frac{n_{jt+1}}{n_{jt}}$ *represent growth rates of loans, bond holdings, injection from the central bank and the net worth, respectively.* $\diamondsuit$

Proof 1. *See Appendix A.*

As evident from Proposition 1, the higher the lending spread $(R_{kt+1} - R_{t+1})$, the higher the expected discounted marginal value of extending loans (ν_t^s). In particular, ν_t^s is a weighted average of lending spread that bank j earns conditional on failing to survive at the end of t (the first term in equation (15)) and the continuation value conditional on the survival (the second term). Similarly, the higher the spread between return on government bond holding and risk-free return $(R_{bt+1} - R_{t+1})$, the higher the expected discounted marginal value of holding government bonds (ν_t^b). Moreover, the higher the cost of borrowing from the central bank, R_{mt+1}, the lower the expected discounted marginal value of central bank funding (ν_t^{in}). Last, the higher the risk-free deposit rate –which is the opportunity cost of raising funds by borrowing from households–, the higher the expected discounted marginal benefit of accumulating net worth (ν_t^n).

Our methodological approach is to linearly approximate the stochastic equilibrium around the deterministic steady state. The optimal behavior of a banker is to increase the value of the bank by raising the amount of loans (assets), government bond holdings, injection from the central bank and the net worth to the point where the amount of assets that they can divert is equal to the liquidation cost. Therefore, the incentive compatibility condition (equation (13)) is always binding.[9] Moreover, we assume that the collateral constraint (equation (11)) is always binding, i.e., the central bank ensures that the constraint always binds to effectively steer money market conditions through change in collateral policies (e.g. change in κ_t^b or κ_t).

Proposition 2 [Bank j's Asset Portfolio]. *The expected discounted marginal value of extending credit over and above the expected discounted marginal value of holding government bonds depends on (1) the relative degree of bank's ability to divert government bonds (ω) compared to corporate loans; (2) the fraction of government bonds that are accepted by the central bank as collateral relative to the fraction of corporate loans that are deemed eligible (κ_t^b compared to κ_t). In particular, the following holds:*

$$\nu_t^s - \frac{\nu_t^b}{\omega} = \nu_t^{in}\left[\frac{1}{\omega}\frac{\kappa_t^b}{R_{mt+1}} - \frac{\kappa_t}{R_{mt+1}}\right] \tag{19}$$

This condition then determines the bank j's asset portfolio between credit and government bond holdings. $\diamondsuit$

Proof 2. *See Appendix D.* $\diamondsuit$

To isolate the effect of central bank injection policy on bank asset portfolio structure, first let $\omega = 1$ without loss of generality. Then (i) if the central bank sets $\kappa_t^b = \kappa_t$, the bank's asset portfolio would be indeterminate; (ii) if the central bank sets κ_t^b greater than κ_t (that is, the central bank accepts a higher fraction of government bonds as collateral than corporate loans), ceteris paribus, the expected discounted marginal value of extending credit compared to the expected discounted marginal value of government bond holding rises. For $0 < \omega < 1$, the asset portfolio would still be determinate even if $\kappa_t^b = \kappa_t$ (even if the central bank is indifferent in terms of eligibility of the assets for receiving the injection).

Proposition 3 [Bank Leverage]. *The costly enforcement problem described above, i.e. the incentive compatibility constraint given by (13) and the collateral constraint for central bank injec-*

[9] At the optimum, one can re-express the incentive compatibility constraint using our verified conjecture for V_{jt}^*. In particular, by combining equations (13) and (14), the incentive compatibility constraint becomes $\nu_t^s q_t s_{jt} + \nu_t^b b_{jt} + \nu_t^n n_{jt} + \nu_t^{in} in_{jt} \geq \lambda_t(q_t s_{jt} + \omega b_{jt})$.

tion given by (11), limits bank's leverage to the point where its incentive to divert funds is exactly offset by its loss from the diversion. Therefore, there exists an endogenous borrowing constraint on the bank's ability to acquire assets. In particular, the bank leverage is given by:

$$lev_{jt} = \frac{\nu_t^n}{\lambda_t - \nu_t^s - \frac{\nu_t^{in}\kappa_t}{R_{mt+1}}} \tag{20}$$

where lev_{jt} is the bank j's leverage.[10] ♢

Proof 3. *See Appendix D.* ♢

The bank's leverage increases in the expected discounted marginal benefit of extending credit (ν_t^s), the expected discounted marginal benefit of accumulating net worth (ν_t^n), and the expected discounted marginal benefit of injection (ν_t^{in}). Intuitively, a rise in ν_t^n, ν_t^s or ν_t^{in} implies that financial intermediation is expected to be more profitable in the future, which makes it less attractive to divert funds today, and thus, makes depositors more willing to trust and put deposits at the bank. Similarly, the bank's leverage is decreasing with the fraction of funds (λ) divertible. Finally, central bank accepting a higher fraction of corporate loans as collateral (i.e. a rise in κ_t) relaxes the collateral requirements on banks, which, similarly, makes financial intermediation more viable and loosens the endogenous limit on bank's leverage.[11]

Since none of the components of lev_{jt} depend on bank-specific factors, lev_{jt} is equal to lev_t. Therefore, we can aggregate equation (20) over j and obtain the following aggregate relationship:

$$q_t s_t + \omega b_t = lev_t n_t, \tag{21}$$

where $q_t s_t$ is the outstanding loans to intermediate goods sector, b_t is the amount of government securities held by the banking sector and n_t represents the aggregate level of net worth.[12] Equation (21) shows that aggregate banking sector assets in this economy can only be up to an endogenous multiple of aggregate bank capital. Note that endogenous fluctuations in asset prices feeding into

[10] We define leverage as the ratio of the total divertable assets to net worth.

[11] As shown in Appendix D, since $\left[\lambda_t - \nu_t^s + \frac{\nu_t^{in}\kappa_t}{R_{mt+1}}\right] = \left[\lambda_t - \nu_t^b - \frac{\nu_t^{in}\kappa_t^b}{R_{mt+1}}\right]$, one can also show that $lev_{jt} = \frac{\nu_t^n}{\lambda_t - \nu_t^b - \frac{\nu_t^{in}\kappa_t^b}{R_{mt+1}}}$. Therefore, a rise in κ_t^b also induces a looser constraint on bank's raising up its leverage.

[12] We present the evolution of aggregate net worth shortly below.

bank's equity capital will render bank balance sheet conditions affecting the model dynamics. This will be the main source of the financial accelerator mechanism in the model.

Aggregate net worth of banks is the sum of surviving bankers' net worth (n_{et+1}) and the start-up funds of the new entrants ($n_{n,t+1}$): $n_{t+1} = n_{e,t+1} + n_{n,t+1}$. The start-up funds for new entrants are equal to $\frac{\epsilon^b}{1-\theta}$ fraction of exiting banks' net worth, $(1-\theta)n_t$. Therefore,

$$n_{n,t+1}\pi_{t+1} = \epsilon^b n_t \tag{22}$$

Since θ fraction of bankers survive to the next period, the net worth evolution for surviving bankers can be expressed as:

$$n_{e,t+1}\pi_{t+1} = \theta \varrho_{t,t+1} n_t \tag{23}$$

where $\varrho_{t,t+1} = \frac{n_{jt+1}\pi_{t+1}}{n_{jt}}$ is the existing bank j's growth rate of net worth. Therefore, the evolution of aggregate net worth for the entire banking system is given by

$$n_{t+1}\pi_{t+1} = \theta \varrho_{t,t+1} n_t + \epsilon^b n_t. \tag{24}$$

where $\varrho_{t,t+1} = \left[(R_{kt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t}{R_{mt+1}}\right] lev_{jt} + R_{t+1}$.[13] Hence, equation (10) can be re-written at an aggregate level as

$$n_{jt+1}\pi_{t+1} = \theta \left\{ \left[(R_{kt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t}{R_{mt+1}}\right] lev_{jt} + R_{t+1} \right\} n_{jt} + \epsilon^b n_{jt} \tag{25}$$

Thus, the growth of aggregate net worth depends positively on loan-deposit spreads, the difference between cost of borrowing from households and from the central bank, endogenous bank leverage, risk-free deposit rate, survival probability, and the fraction of start-up funds.

[13]For the derivation, see Appendix D.

3.3 Firms

We now turn to the non-financial side of the economy. The productive units in the economy are the intermediate goods producing firms. They use physical capital and labor to produce intermediate goods. These goods are then bought by monopolistically competitive retail-goods-producing firms, and then bundled to a final aggregate consumption good. At the end of t, competitive capital goods producers buy depreciated capital from the intermediate-goods-producers and investment goods from the final-goods-producers to produce new capital goods to be used in the next period.

3.3.1 Intermediate Goods Producers

Intermediate goods producers use physical capital and labor to produce the intermediate good, y_t. They acquire capital from the financial intermediaries by issuing equity claims s_t, which is equal to the level of capital acquired, k_{t+1}, at a price q_t. Therefore, $q_t s_t = q_t k_{t+1}$ for all t. The financing of capital expenditures is thus frictionless, i.e. the firm is able to issue a perfectly state-contingent security to obtain capital.

Moreover, firms can choose how much to utilize capital, u_t.[14] In line with the related literature, we assume that the higher the utilization, the higher the depreciation rate. In particular, the effective depreciation rate, δ_u, satisfies $\delta_u = \delta + \frac{\varphi_u}{1+\varrho} u_t^{1+\varrho}$, where δ is the long-run depreciation rate, $\varphi_u > 0$ is a scale parameter, and $\varrho > 0$ is the elasticity of marginal depreciation with respect to the utilization rate. The other factor input, labor (h_t), is supplied by worker members of the household.

Firms use a constant-return-to-scale production technology given by

$$y_t \equiv exp(z_t)F(k_t, h_t) = exp(z_t)\left(u_t k_t\right)^{\alpha} h^{1-\alpha} \tag{26}$$

where z_t is the total factor productivity, and is assumed to be governed by a stationary AR(1) process: $z_t = \rho_z z_{t-1} + \epsilon_{z_t}$ with $\epsilon_{z_t} \in i.i.d.N(0, \sigma^{\epsilon_z})$. At each t, the firm's problem of choosing the utilization rate of capital and labor demand yields

[14]Empirical evidence suggests that the utilization rate of capital varies along the business cycles (higher during booms and lower during recessions). Moreover, higher utilization leads to a higher depreciation rate. Introducing this feature helps match the observed inflation and output dynamics in response to monetary policy shocks (Christiano and Eichenbaum, 2005).

$$\alpha mc_t \frac{y_t}{u_t} = \varphi_u(u_t^{\varrho})k_t \tag{27}$$

$$w_t = mc_t exp(z_t)F_h(k_t, h_t) \tag{28}$$

where mc_t is the real marginal cost. The banks' claim against the ownership of the firm pays out its dividend via the marginal product of capital in the next period. Hence, ex-post nominal return to capital to the intermediary should satisfy

$$R_{kt} = \frac{mc_t exp(z_t)F_k(k_t, h_t) + q_t(1-\delta)}{q_{t-1}}\pi_t \tag{29}$$

3.3.2 Capital Producers

Capital producers purchase capital goods from the intermediate goods producing firms and investment goods from final goods producers to produce new capital goods to be used in the next period. They are subject to adjustment costs, $\Phi(\frac{i_t}{k_t})$ where $\Phi'(.) \geq 0$ and $\Phi''(.) \leq 0$. The capital goods are then sold to intermediate-goods-producing firms at a price q_t. Their problem of choosing i_t to maximize their profits, $q_t k_{t+1} - q_t(1-\delta(u_t))k_t - i_t$, subject to the aggregate law of motion for capital, $k_{t+1} = (1-\delta(u_t))k_t + \Phi(\frac{i_t}{k_t})k_t$ yields a standard q-relation for the price of capital:

$$q_t = \left[\Phi'\left(\frac{i_t}{k_t}\right)\right]^{-1} \tag{30}$$

It is easy to verify that net investment, $i_t - \delta(u_t)k_t$ is positive if and only if $q_t \geq 1$, and that q_t is a sufficient statistic for the level of investment. We assume a conventional functional form for $\Phi(.)$: $\Phi(.) = \frac{i_t}{k_t} - \frac{\psi_k}{2}\left(\frac{i_t}{k_t} - \delta(u_t)\right)^2$, where ψ_k then governs the sensitivity of price of capital to investment-to-capital ratio.

3.3.3 Retailers and Final Good Bundlers

There is a unit measure of monopolistically competitive retailers indexed by i. Each retailer i buys the intermediate good in a competitive market at a common price P_{it}^{int}, differentiates it at no cost into y_{it}, and sells it at P_{it} to perfectly competitive final goods producers. Final goods are then a constant-elasticity-of-substitution (CES) aggregate of retail goods

$$y_t = \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{31}$$

where $\varepsilon > 1$ is the elasticity of substitution across the varieties. The final goods producers' demand for each retail good i, y_{it}, satisfies

$$y_{it} = \left[\frac{P_{it}}{P_t}\right]^{-\varepsilon} y_t \tag{32}$$

where the aggregate price of final goods, P_t, is

$$P_t = \left[\int_0^1 P_{it}^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \tag{33}$$

Retailers face standard Calvo-Yun type price stickiness, i.e. they may not be able to change their prices with a constant probability ϕ in future periods. Those that can change their prices set the price optimally.

Retailers' maximization of expected discounted real profits given the iso-elastic demands for each retail good (equation (32)) yields a standard optimality condition: a retailer who is able to change its price at t sets the price such that the expected discounted difference between the real marginal cost ($\frac{P_t^{int}}{P_t}$) and real marginal revenue ($\frac{P_t^*}{P_t}$) is zero, given the environment that the firm could reset its price only with a certain probability in the future. Formally, retailer i, that is allowed to set its price, solves

$$\max_{P_t^*} \sum_{k=0}^{\infty} \phi^k E_t \left[\Lambda_{t+k|t} \left[\frac{P_t^* - P_{t+k}^{int}}{P_{t+k}}\right] y_{i,t+k}\right] \tag{34}$$

subject to (32), and where P_t^* is the optimal price chosen by the retailer i.[15] The optimal price then satisfies:

$$\sum_{k=0}^{\infty} E_t \left[\Lambda_{t+k|t} \phi^k y_{i,t+k} \left[\frac{P_t^*}{P_{t+k}} - \frac{\epsilon}{\epsilon-1} \frac{P_{t+k}^{int}}{P_{t+k}}\right]\right] = 0 \tag{35}$$

[15]We focus on the symmetric equilibrium such that optimizing retailers at a given time choose the same price.

The conventional approach in most New-Keynesian literature is to log-linearize this equation around a non-inflationary steady state, and proceed to the standard New-Keynesian Phillips curve. However, since we assume a non-zero inflation at the deterministic steady state, we represent equation (35) in a recursive format. In particular, equation (35) can be re-expressed as

$$z_{1,t} = \frac{\epsilon}{\epsilon - 1} z_{2,t} \tag{36}$$

where

$$z_{1,t} = \widetilde{p_t}^{1-\epsilon} y_t + \beta \frac{\zeta_{t+1}}{\zeta_t} \phi \left(\frac{\widetilde{p_t}}{\widetilde{p_{t+1}}} \right)^{1-\epsilon} \pi_{t+1}^{\epsilon-1} z_{1,t+1} \tag{37}$$

$$z_{2,t} = \widetilde{p_t}^{-\epsilon} y_t mc_t + \beta \frac{\zeta_{t+1}}{\zeta_t} \phi \left(\frac{\widetilde{p_t}}{\widetilde{p_{t+1}}} \right)^{-\epsilon} \pi_{t+1}^{\epsilon} z_{2,t+1} \tag{38}$$

and $\widetilde{p_t} = \frac{P_t^*}{P_t}$ is the relative price dispersion.

Finally, given that the exact distribution of prices across the varieties does not matter, the evolution of aggregate price simply satisfies $P_t^{1-\epsilon} = \phi(P_{t-1})^{1-\epsilon} + (1-\phi)(P_t^*)^{1-\epsilon}$. Dividing this expression by $P_t^{1-\epsilon}$ yields

$$1 = \phi(\pi_t)^{\epsilon-1} + (1-\phi)\widetilde{p_t}^{1-\epsilon} \tag{39}$$

3.4 Government

We keep the fiscal side simple. Government issues new bonds, pay the interest on previous period's bonds that come due, and receive central bank earnings. The government then transfers the net revenue back to households in a lump-sum fashion. The government's budget constraint reads as

$$b_t^t - (b_t^c - \frac{b_{t-1}^c}{\pi_t}) + \tau_t^m = \frac{R_{bt} b_{t-1}^t}{\pi_t} + \tau_t - R_{bt} \left[\frac{b_{t-1}^c}{\pi_t} - \frac{b_{t-2}^c}{\pi_t \pi_{t-1}} \right] \tag{40}$$

where τ_t^m is the net revenue of the central bank remitted to the government, and τ_t is the net lump-sum transfer to the households. b_t^t the total supply of government bonds that are held by households (b_t) and the central bank (b^c_t), satisfying $b^t_t = b^c_t + b_t$.[16]

Moreover, b^t_t grows with a constant rate $\Gamma > \beta$

$$b^t_t = \Gamma \frac{b^t_{t-1}}{\pi_t} \tag{41}$$

where $b^t_{-1} > 0$.[17]

3.5 Central Bank

The central bank requires eligible collateral to lend to bank via open market operations. For the benchmark case, we take government bonds as the only eligible asset as in central banks' conventional collateral framework. In later analyses, we also consider corporate loans as eligible assets (as observed during financially turbulent times).

Central bank budget constraint implies that change in money supply is equal to the sum of change in central bank bond holdings and change in central bank injection. Formally, the budget constraint of the central bank can be expressed as

$$b^c_t + \tau^m_t + repo_t = \frac{R_{bt} b^c_{t-1}}{\pi_t} + \frac{R_{mt} in_{t-1}}{\pi_t} + m_t - \frac{m_{t-1}}{\pi_t} - R_{bt}\left[\frac{b^c_{t-1}}{\pi_t} - \frac{b^c_{t-2}}{\pi_t \pi_{t-1}}\right] - \Psi_t \tag{42}$$

where b^c_t corresponds to central bank bond holdings at t, Ψ_t is the cost of injection that the central bank incurs due to risky assets on its balance sheet, and τ^m_t is the central bank net revenue due to earnings from holding assets and open market operations and is transferred to the government.[18]

The central bank can provide liquidity by outright money purchases:

$$\kappa^b_t \frac{b_t}{R_{mt+1}} = b^c_t - \frac{b^c_{t-1}}{\pi_t} + m^r_t \tag{43}$$

[16] $(b^c_t - \frac{b^c_{t-1}}{\pi_t})$ is the amount of bond that central bank facilitates for its open market operations with the financial sector. Note that total injection to the banking sector is the sum of repo and this bond change (direct bond purchase of central bank), $in_t = repo_t + b^c_t - \frac{b^c_{t-1}}{\pi_t}$, and for these liquidity operations central bank charges the policy rate, R_m to the financial sector. To eliminate the problem of double counting in central bank bond holding revenues we assume that government is not supposed to pay R_b to the central bank for the amount of $(b^c_t - \frac{b^c_{t-1}}{\pi_t})$, which is used for liquidity operations.

[17] The condition $\Gamma > \beta$ ensures that the gross nominal interest rate is greater than 1 at the deterministic steady state.

[18] Central bank's transfer of its net revenues to the government is a common central banking practice. Moreover, we set Ψ_t, the cost of injection that the central bank incurs, at a very small number so that it does not affect model dynamics.

Moreover, $repo_t$ is given by

$$repo_t = m_t^r + \kappa_t \frac{q_t s_t}{R_{mt+1}} \tag{44}$$

Equations (43) and (44) then imply that

$$in_t = repo_t + b_t^c - \frac{b_{t-1}^c}{\pi_t} \tag{45}$$

Therefore, the central bank balance sheet can be read as in Table 2:

TABLE 2: CENTRAL BANK BALANCE SHEET

Assets	Liabilities
Government bonds balances	Money holdings
Central bank operations (repo)	Net Worth
	(remittances to the government)

where the explicit formula for the central bank revenue is $\tau_t^m = (R_{bt} - 1)\frac{b_{t-2}^c}{\pi_t \pi_{t-1}} + (R_{mt} - 1)\frac{in_{t-1}}{\pi_t}$. Combining this expression with equation (42) then yields

$$\frac{b_{t-1}^c}{\pi_t} - \frac{b_{t-2}^c}{\pi_t \pi_{t-1}} + in_t - \frac{in_{t-1}}{\pi_t} = m_t - \frac{m_{t-1}}{\pi_t} \tag{46}$$

That is, change in money supply should be equal to the change in central bank bond holdings plus the change in central bank injection.

The consolidated government budget constraint is given by

$$in_t + \tau_t + \frac{R_{bt} b_{t-1}}{\pi_t} = b_t + m_t - \frac{m_{t-1}}{\pi_t} + - \frac{R_{mt} in_{t-1}}{\pi_t} \tag{47}$$

The central bank has three policy instruments: First, central bank sets the conventional policy rate, R_{mt}. We assume that the central bank responds to inflation and output gaps, as in conventional New-Keynesian models, and further, exhibits some degree of policy persistence. Formally,

$$R_{mt} = R_{mt-1}^{\rho_r} \left[\overline{R_m} \left(\frac{\pi_t}{\overline{\pi}} \right)^{\varphi_\pi} \left(\frac{y_t}{\overline{y}} \right)^{\varphi_y} \right]^{1-\rho_r} \tag{48}$$

where $\overline{R_m}, \overline{\pi}$ and $\overline{y}$ are the long-run deterministic levels of policy rate, inflation, and output, respectively. ρ_r is the policy persistence, φ_π and φ_y capture the degree of central bank's reaction to inflation and output gaps.

Second, the central bank can determine the type and the fraction of assets accepted as collateral in open market operations (κ_t^b and κ_t). For the benchmark case, we take $\kappa_t^b > 0$ and $\kappa_t = 0$ as exogenously fixed. In later sections where we study "active collateral policies", we assume that central bank sets κ_t^b and κ_t in response to financial market developments (see Section 5). Finally, central bank sets the ratio of treasury repos to the total outright purchases of bonds, $\Omega > 0$: $m_t^r = \Omega m_t^h$.

3.6 Competitive Equilibrium

We solve the model locally around a deterministic steady state. A competitive equilibrium of this model economy is defined by sequences of allocations $\{c_t, k_{t+1}, i_t, l_t, h_t, y_t, d_t, b_t, n_t, n_{et}, n_{nt}, in_{t+1}, m_t, m_t^r, s_t, b_t^c, b_t^t, \Lambda_{t,t+1}, lev_t, \nu_t^n, \nu_t^s, \nu_t^b, \nu_t^i, \varrho_{t,t+1}, \chi_{t,t+1}, \chi_{t,t+1}^s, \chi_{t,t+1}^b, \chi_{t,t+1}^{in}, \widetilde{Z}_{t+1}, Z_t^1, Z_t^2, s_t^p, \tau_t, \tau_t^m, A_t\}$, prices $\{q_t, R_{mt+1}, R_{kt+1}, R_{bt+1}, R_{t+1}, \pi_t, w_t, mc_t\}$, the government policy parameters $\{\varphi_\pi, \varphi_y, \rho_r, \kappa, \kappa^b, \Omega\}$ and exogenous processes $\{z_t, \lambda_t\}_{t=0}^\infty$ such that the optimality conditions of utility maximizer workers, net worth maximizer bankers, profit maximizer of intermediate good, final good and capital goods producers are satisfied, and goods, labor, bonds and money markets clear. A complete set of these conditions are given in Appendix C.

4 Quantitative Results

In the benchmark economy, we study the model dynamics under productivity, funding stress and monetary policy shocks, where the central bank takes government bonds as the only eligible asset in open market operations.

4.1 Calibration

Table 3 presents the structural parameters for the baseline model. For most parameters, we use conventional values used in the literature. For parameters related to the financial sector, we conduct a joint calibration strategy.

We set the quarterly discount factor, β as 0.9935 to match the 2.62% average annualized real deposit rate (for the period that we have data for the US commercial and industrial loan rates before the Great Recession, 1986Q3-2007Q4). The relative utility weight of labour ψ is set to 17.58 to fix hours worked to one third at the deterministic steady state. The (inverse) Frisch elasticity of labor supply is set at 0.276, following Gertler and Karadi (2011). The share of capital in the production function is set at 0.36 to match the average labour share of income. The steady-state utilization rate of capital is normalized at 1. The elasticity of marginal depreciation with respect to the utilization rate is set at 3, within the range studied in the literature, e.g. Kimball and Kimball (1997) and Gertler and Karadi (2011). The quarterly depreciation rate of capital (after adjusted for utilization) is set to 2.2% following Bachmann et al. (2013). We set the capital adjustment cost parameter equal to 26 to match the long-run elasticity of the price of capital with respect to the investment-to-capital ratio of 0.65, following Christensen and Dib (2008). Finally, the parameters related to the New-Keynesian features, ϕ, the probability that firms are not allowed to change their prices, and ϵ, the elasticity of substitution across the varieties, are set at conventional values. In particular, we set ϕ equal to 0.66 following Klenow and Malin (2010), that implies an average frequency of price changes of approximately 3 quarters.[19] Moreover, we set ϵ equal to 11, implying a price mark-up of 10% over the marginal cost at the deterministic steady state.

Regarding the financial sector parameters, we set θ equal to 0.94, within the range studied in the related literature, e.g. Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). For the remaining parameters, $\overline{\lambda}$, the long-run level of diversion parameter, and ϵ^b, the fraction of net worth transferred to new entrants, we target two financial variables: leverage ratio of 9.89, the US commercial banking sector's aggregate leverage ratio for 1986Q3-2007Q4, and credit spread of 180 annualized basis points.[20] The resulting values for $\overline{\lambda}$ and ϵ are 0.3748 and 0.0096, respectively.

The monetary policy rule parameters are set following Smets and Wouters (2007). In particular,

[19] Klenow and Malin (2010) document that the mean (non-sale) price durations of non-durable and services goods are 8.3 and 9.6 months, respectively. Weighting these price durations by their shares in the CPI yields 2.93 quarters. This level of duration in turn implies ϕ=0.66. This value is well in the range used in the New-Keynesian literature (for a review, see Schmitt-Grohe and Uribe, 2010).

[20] Since there are numerous definitions for credit spread in the data, depending on firm and loan characteristics, and there is no direct empirical counterpart for the model-based spread, we take an average value (180 basis points) following the related literature.

we set the policy persistence, ρ_r, equal to 0.81, the degree of long-run reaction to deviation of inflation from its long-run deterministic value, φ_π, equal to 2.03, and the degree of long-run reaction to output gap, φ_y, equal to 0.125. We set Γ, the growth rate of government bonds, equal to 1.0059 to match the observed long-run average annual inflation rate of 2.4%. Moreover, we set Ω equal to 0.032 to match the observed repo over M1 ratio of 0.032. [21]

Regarding the shock processes, we set the persistence of TFP process, ρ_z, and the standard deviation of innovations to the TFP, σ_z, for the period 1986Q3-2007Q4. The resulting values are 0.9315 and 0.007, respectively, and are in line with the conventional estimates. We set the persistence of funding stress shock, ρ_λ, at a lower value (compared to the persistence in the productivity) as it is best to think of financial shocks as rare events. In particular, we set ρ_λ equal to 0.66 following Gertler and Karadi (2011) and Dedola et al. (2013). For the standard deviation of innovations to the funding stress, σ_λ, we take 0.03 that yields an on-impact equilibrium increase in the credit spread of around 100 basis points in annualized terms, in line with the empirical evidence laid out by Gilchrist and Zakrajsek (2011).

4.2 Model Dynamics

Figure 2 shows the equilibrium responses of key model variables to a one-standard-deviation unexpected decrease in total factor productivity (TFP). Since a lower TFP reduces the marginal product of capital and make credit less attractive, non-financial firms demand less capital. In turn, bank credit and asset prices decline (by about 0.4%). The exogenous decline in aggregate supply of intermediate goods drives the nominal marginal costs up that the retailers face. Hence, the retailers that are allowed to change their prices set the optimizing price above the average price level. Average price level, P_t, then rises, but due to stickiness in price adjustment, not as much as the increase in nominal marginal costs. As a result, average mark-up in the economy falls, and inflation goes above its long-run value (by about .7% points). Given the degree of monetary policy reaction to inflation and output gaps, the policy rate increases (by about 40 basis points), raising the price of injection to financial intermediaries. In turn, total injection to banks as well as the money supply decreases, depressing credit even further. Due to sharp drop in credit, the credit

[21] Due to data limitations on repurchase agreements of the Fed, the ratio is calibrated to the period 2003-2007.

spread rises only marginally by about 2 basis points in equilibrium.

Funding stress shock, on the other hand, has a first-order effect on credit spreads as shown by Figure 3. In response to a sudden rise in funding stress, banks find it harder to attract funds and experience tightening in their endogenous leverage constraint. Hence, banks start deleveraging, extend less credit to firms (credit falls by about .15% points on impact) and credit spread rises by about 100 annualized basis points (as set by the calibration strategy). Since non-financial firms finance their capital expenditures by bank credit, they reduce their capital demand, leading to .3% decline in investment and .15% reduction in asset prices. Moreover, due to lower demand for intermediate goods, nominal marginal costs that the retailers face go down. In turn, average mark-up in the economy declines, and inflation goes below its long-run value (by about .3% points). Due to the decline in inflation and output gap, the policy rate goes down (by about 20 basis points), leading to a mild increase in central bank injection. The increase in the injection, though, appears not sufficient to offset the equilibrium decline in real and financial variables.

These equilibrium responses are by and large in line with the data. Gilchrist and Zakrajsek (2011), for instance, document that in response to a one-standard-deviation shock to financial sector bond premium (which closely resembles the funding stress shock that we study), output declines by .4% and investment by 1% point at the peak. Moreover, they exhibit hump-shaped responses and reaching their peak responses after about 6-7 quarters. Moreover, inflation goes down by .2% point, the federal funds rate is reduced by about 30 basis points, and financial bond premium rises by about 100 basis points (in annualized terms). Our model performs well in capturing the actual responses of premium, the policy rate, and inflation, as in the US data. Regarding the responses of real variables (such as investment and output), the model exhibits similar dynamics, yet imply weaker responses (e.g. investment declines by about half of the data). Our take is that the model can either be enriched to strengthen the amplification (for instance, by introducing non-financial firms' balance sheet conditions mattering for the business cycles), or that standard deviation of innovations to the funding stress can be set higher to match the real responses (yet, in expense of matching the credit spread's response). For the former, we would like to emphasize that we deliberately keep the model as simple and tractable as possible to focus on key features of central bank collateralized lending. For the latter, the model dynamics and the results we discuss below are robust to considering a higher standard deviation for the funding stress shock.

Next, we provide further insights on the model dynamics due to funding stress shock. First, we let κ_t, the fraction of corporate loans accepted as eligible, take a positive value. In particular, for illustrative purposes, we first set κ_t equal to 0.025. For the sake of experiment, we then consider $\kappa_t = 0.050$. As evident from Figure 4, the economy with $\kappa_t > 0$ exhibits a second acceleration in addition to the conventional financial accelerator. Due to the decline in credit, the collateral that banks can pledge declines. In response, banks receive lower injection from the central bank, and thus, the decline in credit, investment, asset prices, and output is amplified. Moreover, Figure 4 shows that the greater the κ_t, the stronger the amplification.

Before we proceed to active collateral policies where the central bank sets κ_t^b and κ_t in response to financial market developments, we would like to present model dynamics in response exogenous changes in κ_t^b and κ_t. First, we consider a one-time increase in κ_t^b, the fraction of Treasury bonds accepted as collateral (Figure 5, dashed lines). For the sake of analysis, we follow an illustrative calibration: we let κ_t^b follow a stochastic AR(1) process: $\kappa_t^b = (\kappa_{t-1}^b)^{\rho_{\kappa^b}} \left[\overline{\kappa^b}\right]^{1-\rho_{\kappa^b}} exp(\epsilon_{\kappa^b t})$, where ρ_{κ^b} is set to 0.81 (the same persistence that we consider for the policy rate rule), and standard deviation of shock to κ_t^b is set equal to 0.0044 that yields an on-impact 5% points increase in total injection.

As higher κ_t^b implies looser collateral constraint on banks and leads to higher central bank injection, banks extend more credit to non-financial firms. In turn, demand for investment goods rises which leads to a rise in asset prices. As banks balance sheets improve (due to rise in asset prices), banks extend even higher credit. As a result, investment and output rises, while credit spread declines. Moreover, looser credit market conditions lead to higher demand for intermediate goods, pushing aggregate price and inflation up. As evident, the κ_t^b shock behaves like a demand shock, output and inflation moving in the same direction.

Similar results are obtained for the κ_t shock (Figure 5, solid lines). In particular, we consider a one-time increase in κ that yields the same on-impact increase in the total injection (of 5% points).[22] While an increase in κ_t yields similar responses in central bank injection, credit, investment, asset

[22] Similarly, we set the persistence in κ_t equal to 0.81, and the standard deviation of shock to κ_t that yields the same on-impact increase in the total injection of 5% points as we had in the previous analysis. The resulting standard deviation is 1.40610^{-4}.

prices, and output, the magnitudes of dynamics differ particularly for inflation (and hence the policy rate). Our take is that a rise in κ_t triggers both the demand and the supply of intermediate goods. For instance, it makes lending operations more profitable for banks, which then leads to higher supply of intermediate goods. And similar to the κ_t^b shock, it leads to an increase in the demand for intermediate goods. In turn, inflation moves only marginally in response.

As a benchmark, we also report the model dynamics due to an unexpected 25-basis-points increase in the conventional policy rate (Figure 6). Since the central bank financing becomes more expensive, repo and total injection decrease. As a result, banks find it harder to extend credit, leading to an equilibrium decrease in credit and investment, and an increase in credit spread. Since aggregate demand (consumption and investment) falls more than output, inflation goes down as well.

5 The Collateral Framework as a Policy Tool

Besides the conventional policy rate, central bank is endowed with two other tools: The central bank can change collateral standards in response to financial market developments by changing the fraction of safe assets (κ_t^b) or the fraction of corporate loans (κ_t) that are accepted as collateral in open market operations. In Section 5.1, we provide analytical insights on how changes in κ_t^b or κ_t, the collateral policy parameters, affect model dynamics. While the analytical derivations are based on a partial equilibrium setup, it sets the stage for further analyses where we consider active collateral policies. In particular, we let the central bank follow a simple policy rule for setting the collateral standards. The analytical insights shed light on how to appropriately design (in particular, which financial variables to target) the collateral policy rules.

5.1 Analytical Insights: Transmission Channel of Central Bank Collateral Policy

In this section, we provide analytical insights on which financial variables to target in simple collateral policy rules.

Proposition 4 [Safe Asset Collateral Policy]. *A wider range for the eligibility of safe as-*

sets, i.e. higher κ_t^b, lowers liquidity risk premium charged on holding those assets, i.e. bond spreads. A tighter conventional policy attenuates this effect and the ZLB may reduce the effectiveness of this collateral policy.$\diamondsuit$

Proof 4. *Expected discounted marginal benefit of increasing the holdings of safe assets is given by equation (16). This expected marginal benefit is an increasing function of bond spreads, $(R_{bt+1} - R_{t+1})$. Moreover, the first order condition with respect to b_t from the banks' profit maximization problem is given by*

$$\nu_t^b = \frac{\lambda \mu_t \omega}{(1+\mu_t)} - \gamma_t \frac{\kappa_t^b}{R_{mt+1}(1+\mu_t)}$$

Taking the partial derivative of this first order condition with respect to κ_t^b gives

$$\frac{\partial \nu_t^b}{\partial \kappa_t^b} = -\frac{\gamma_t}{R_{mt+1}(1+\mu_t)} < 0$$

showing that increasing the fraction of safe assets accepted as collateral, a higher κ_t^b, reduces the expected discounted marginal benefit of increasing the holdings of safe assets, ν_t^b, which is an increasing function of bond spreads. Therefore, a higher κ_t^b leads to a lower $(R_{bt+1} - R_{t+1})$. Taking the second partial derivative of the above condition with respect to R_{mt}, central bank policy rate, gives

$$\frac{\partial^2 \nu_t^b}{\partial \kappa_t^b \partial R_{mt+1}} = \frac{\gamma_t}{R_{mt+1}^2(1+\mu_t)} > 0$$

showing that the extent to which a higher κ_t^b reduces $(R_{bt+1} - R_{t+1})$ is decreasing in central bank policy rate, R_{mt+1}. Hence, being unable to reduce the policy rate further and thus hitting the ZLB reduces the effectiveness of this collateral policy.$\diamondsuit$

Proposition 5 [Risky Asset Collateral Policy]. *A wider range for the eligibility of risky assets, i.e. higher κ_t, lowers liquidity risk premium charged on holding those assets, i.e. credit spreads. A tighter conventional policy attenuates this effect and the ZLB may reduce the effectiveness of this collateral policy.*$\diamondsuit$

Proof 5. *Expected discounted marginal benefit of increasing the holdings of risky assets is given by equation (15). This expected marginal benefit is an increasing function of credit spreads,* $(R_{kt+1} - R_{t+1})$*. Moreover, the first order condition with respect to* s_t *from the banks' profit maximization problem is given by*

$$\nu_t^s = \frac{\lambda \mu_t}{(1+\mu_t)} - \gamma_t \frac{\kappa_t}{R_{mt+1}(1+\mu_t)}$$

Taking the partial derivative of this first order condition with respect to κ_t *gives*

$$\frac{\partial \nu_t^s}{\partial \kappa_t} = -\frac{\gamma_t}{R_{mt+1}(1+\mu_t)} < 0$$

showing that increasing the fraction of risky assets accepted as collateral, a higher κ_t*, reduces the expected discounted marginal benefit of increasing the holdings of risky assets,* ν_t^s*, which is an increasing function of credit spreads. Therefore, a higher* κ_t *leads to a lower* $(R_{kt+1} - R_{t+1})$*. Taking the second partial derivative of the above condition with respect to* R_{mt+1}*, central bank policy rate, gives*

$$\frac{\partial^2 \nu_t^s}{\partial \kappa_t \partial R_{mt+1}} = \frac{\gamma_t}{R_{mt+1}^2(1+\mu_t)} > 0$$

showing that the extent to which a higher κ_t *reduces* $(R_{kt+1} - R_{t+1})$ *is decreasing in central bank policy rate,* R_{mt+1}*. Hence, being unable to reduce the policy rate further and thus hitting the ZLB reduces the effectiveness of this collateral policy.*$\diamondsuit$

In light on Propositions 4 and 5, we consider the following simple collateral policy rules:

$$\kappa_t^b = (\kappa_{t-1}^b)^{\rho_{\kappa^b}} \left[\overline{\kappa^b} \left(\frac{BS_t}{\overline{BS}} \right)^{\varphi_{\kappa^b}} \right]^{1-\rho_{\kappa^b}} \tag{49}$$

$$\kappa_t = (\kappa_{t-1})^{\rho_\kappa} \left[\overline{\kappa} \left(\frac{CS_t}{\overline{CS}} \right)^{\varphi_\kappa} \right]^{1-\rho_\kappa} \tag{50}$$

where ρ_{κ^b} and ρ_κ are the policy persistence parameters, φ's are the policy reaction parameters, and $\overline{\kappa^b}$ and $\overline{\kappa}$ are the values of κ_t^b and κ_t at their long-run deterministic steady states, respectively. Last, BS and CS denote bond spread and credit spread, respectively.

Intuitively, both of these policies increase the market price and liquidity of assets that are newly accepted as collateral by the central bank, and reduce liquidity risk premium charged on those assets since the demand for those assets rises in financial markets. This increase in demand together with the positive impact of collateral policy on their market prices and liquidity shows the extent to which those newly eligible assets become more appealing for market participants due to their liquidity buffer status. Moreover, the increased attractiveness of assets that are newly accepted as collateral alter banks' balance sheets, making banks acquire more of those assets and reduce the other. Through a partial equilibrium analysis, Propositions 4 and 5 show that how this basic mechanism works in the model and how hitting the ZLB limits the effectiveness of central bank collateral policy.

5.2 Quantitative Insights: Active Collateral Policy

As the underlying intuition is provided in Section 5.1, central bank follows a simple policy rule of higher κ_t^b in response to higher bond spreads and higher κ_t in response to higher credit spreads in the active collateral policy framework. Along these lines, we consider below the model dynamics due to funding stress shock for four different cases: First, we report the dynamics under the passive injection policy where κ_t^b and κ_t are set fixed at 0.075 and 0.025, respectively. Second, we consider an active κ_t policy where the central bank sets a higher κ_t in response to higher credit spread, with the degree of reaction, φ_κ, equal to 0.02, and the policy persistence, ρ_κ, equal to 0.81). Third, we consider an active safe asset collateral policy where the central bank sets a higher κ_t^b in response to higher bond spread, with the degree of reaction, φ_{κ^b}, equal to 7, and the policy persistence, ρ_{κ^b}, equal to 0.81. The policy response coefficients, φ_κ and φ_{κ^b}, are set so that total injection rises only marginally on impact. In this regard, our results should be read as a lower bound on the effectiveness of active collateral policies. The collateral policy persistence parameters, ρ_κ and ρ_{κ^b} are set in line with the persistence in conventional policy rate rule.

The results are summarized in Figure 7. Under the passive injection policy, a rise in funding stress leads to a drop in credit, investment, asset prices and output. Moreover, inflation falls below its long-run value, and in response, the policy rate falls. Endowed with a single policy tool, the central bank can only mildly mitigate the effect of rise in funding stress on real and financial variables. Active injection policies, on the other hand, appear a strong viable option to follow. Increasing the fraction of corporate loans eligible for collateral (active risky asset collateral policy) or widening the fraction of safe assets as collateral (active safe asset collateral policy) both help attenuate the effect of financial market disruption on real and financial variables. When implemented in coordination, the effectiveness of collateral policy is even stronger: the drop in investment, credit and output are reduced by more than half (compared to the passive policy). Moreover, the results suggest that active use of collateral policies help reduce the need for looser conventional policy response: the policy rate declines only marginally when both collateral policies are actively used.

Moreover, active collateral policies reduce fluctuations in credit spread, the key variable that reflects the degree of financial market imperfections. In this regard, such policies appear helpful in smoothing households' intertemporal wedge, and in this regard, may hint improvement in aggregate welfare. Similarly, active policies help reduce fluctuations in bond spreads, the spread that would be nil in the absence of liquidity premia on these assets. Since smoothing fluctuations in bond spread reduces fluctuations in asset prices, and thus in banks' balance sheet conditions, and in turn, in financial and real variables, such active collateral policy renders better aggregate outcomes.[23]

6 Fed's Recent Experience and Active Collateral Policy: The Case of ZLB

Due to severe financial market disruptions at the onset of Great Recession, the federal funds rate has been reduced aggressively and eventually hit the effective ZLB. As we discuss extensively throughout the text, the Fed has then implemented easier collateral policies (i.e. accepting a wider range of Treasury assets or accepting corporate loans as well) to mitigate the spillover from financial market disruptions on the real economy. In this section, we study a very similar scenario. We consider a series of unfavorable funding stress shocks that induces the policy rate decline from its steady state level and hit the ZLB in about 5 quarters, similar to the path the federal funds rate has actually exhibited at the time. When the policy rate hit the ZLB endogenously, the central

[23] We leave a strict welfare analysis, e.g. the optimal degree of reaction to credit spreads, to future research.

bank then starts implementing active collateral policies (as we studied in detail -and justified on theoretical grounds- in Section 5).[24] In particular, the central bank follows the collateral policy rules (equations (49) and (50)) at the ZLB.

Formulating the collateral policy rules is challenging since there is no direct empirical counterpart for the model-based credit or bond spreads, nor do we have long enough time series data or even enough time variation in collateral fractions (κ_t^b or κ_t) to properly estimate collateral policy rules. Therefore, our analysis should be taken as a well-disciplined illustration of how or why active collateral policies can serve as a strong viable policy option to use during severe financial market disruptions (and especially when the conventional policy tool is at its limits). To formulate the collateral policy rules, we conduct a simple and intuitive calibration strategy. First, we set ρ_κ, the degree of inertia in risky asset collateral policy, at 0.50, i.e. at a low value that allows the central bank to quickly re-adjust the collateral standard when the policy rate hit the ZLB. We set φ_κ, the strength of reaction to credit spread at 0.02 (as we set in the previous section). Given these policy parameters, we set ρ_κ and φ_κ such that the two policies (safe asset or risky asset collateral policies) yield similar first-order impact on the path of injection. In particular, we set ρ_κ^b and φ_{κ^b} at 0.30 and 50, respectively, so that the paths of injection during when the policy rate is at the ZLB are roughly the same under the two policy rules. Finally, we study the effect of active collateral policy when both policy rules are followed at the ZLB.

The results are presented in Figure 8. First, both collateral policies (individually or in combination) appear helpful in mitigating the effect of funding stress on the real economy. For a given path of injection, though, the safe asset collateral policy appears more effective: the recovery in credit, investment and output is stronger under the calibrated safe asset collateral policy. Second, the safe asset collateral policy helps reduce the likelihood of a deflation by stimulating aggregate demand sufficiently strongly. For the same path of injection, however, the risky asset collateral policy cannot help avoid the deflation (if not push the economy into). As we have discussed in Section 4.2, a looser risky asset collateral policy may trigger both aggregate demand and supply, and in this experiment, the aggregate supply disproportionately more, and in turn, a risky asset collateral policy cannot have an inflationary effect and help avoid the deflation. Last, active use of

[24] In order to solve the model with occasionally binding ZLB, we use Occbin algorithm developed by Guerrieri and Iacoviello (2015).

collateral policies helps the economy 'normalize', i.e. policy rate moving above the ZLB, earlier by about a few quarters than what would have been in the absence of these policies.

We would like to note that active collateral policies in our model imply a significantly stronger recovery in real and financial aggregates compared to the actual US experience at the time. Enriching the theoretical model with other salient features of the Great Recession, e.g. deterioration in non-financial firms balance sheet conditions, heightened macroeconomic and policy uncertainty, banks unwilling to extend credit due to gloomy economic prospects, etc., would help the model account better for both the deepness and the duration of the recession as well as the following slower recovery. Moreover, the recovery in aggregate supply conditions due to risky asset collateral policy would then be milder, which in turn would yield a much weaker decline (if not increase) in inflation, and help bring the model closer to the data. Yet, our aim is not to fully account for the macroeconomic dynamics during the Great Recession, which would necessarily be too ambitious for a theoretical model, but rather put the effect of collateral policies under the spot light in a well-disciplined experiment.

7 Conclusion

This paper studies the macroeconomic and financial effects of a widely-used unconventional policy tool, the Fed's policy of broadening the eligible pool of safe and risky assets acceptable as collateral in open market operations. We investigate this question in a New-Keynesian general equilibrium model with an explicit banking sector, taking into account the possibility of conventional policy rate endogenously hitting the ZLB.

Our results are three fold: First, we analytically show that following an active policy rule of higher fraction of safe assets in response to higher bond liquidity premium (safe asset collateral policy as we have labeled) or an active policy rule of higher fraction of risky assets in response to higher credit spread (risky asset collateral policy) helps mitigate fluctuations in these premia, the key sources of financial market imperfections in the model. Justifying analytically the use of such policies, we then quantitatively show that, in response to a sudden rise in funding stress, active use of collateral policies helps mitigate the sharp drop in investment, credit, and asset prices.

Moreover, active collateral policies reduce the need for looser conventional policy response and can help central banks avoid the ZLB. We further show that safe asset collateral policy helps avoid a deflationary regime by providing strong demand stimulus. Last, given our illustrative calibration for the collateral policy rules, both safe asset and risky asset collateral policies help the economy 'normalize', i.e. the policy rate moving away from the ZLB and starts increasing towards its long-run level, a few quarters earlier than what would have been in the absence of such policies.

Our model provides a rich framework to study various features of central bank open market operations such as optimal policy, direct liquidity injection, emerging market central banks accepting foreign-currency assets as collateral and firm balance sheet channel. These points are left to future work.

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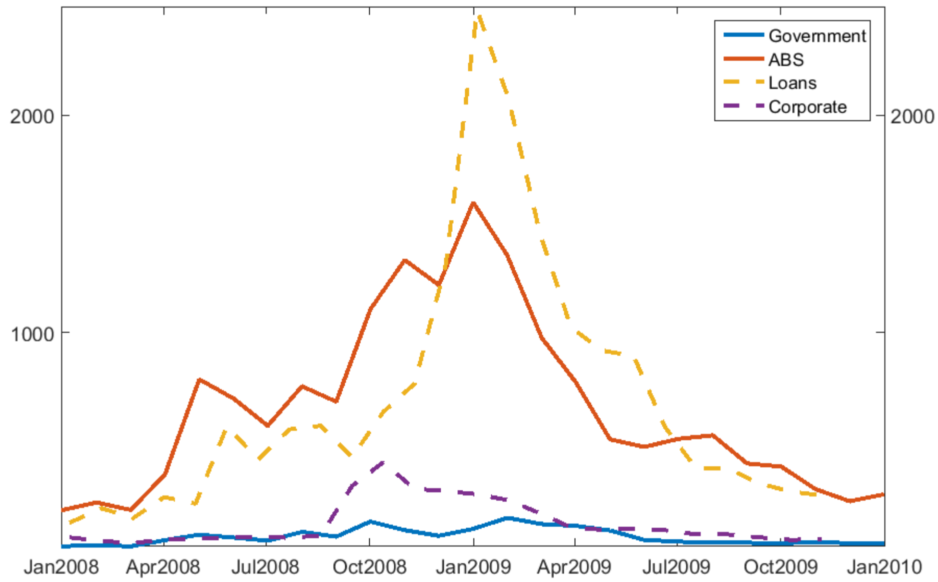
Figure 1: Collateral Pledged to the Fed ($ Billions)



Notes. This figure includes the collateral for the TAF, PDCF and TSLF. Government refers to the US Treasuries, agency debt or municipal bonds. ABS is asset-backed securities including agency-guaranteed mortgage backed securities. Loans are commercial and consumer loans including commercial real estate loans and residential mortgages, respectively. Corporate stands for corporate market instruments.

Figure 2: Impulse responses to an unfavorable productivity shock

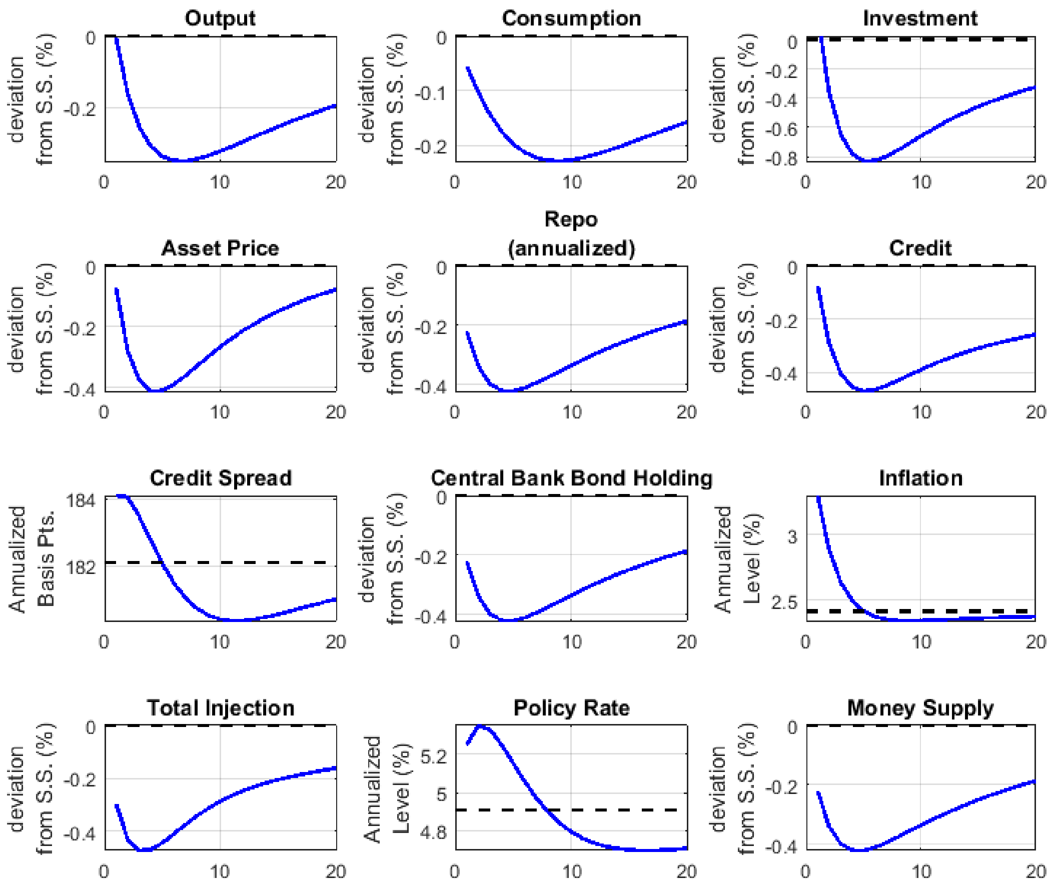


Figure 3: Impulse responses to an unfavorable financial shock (an increase in λ).

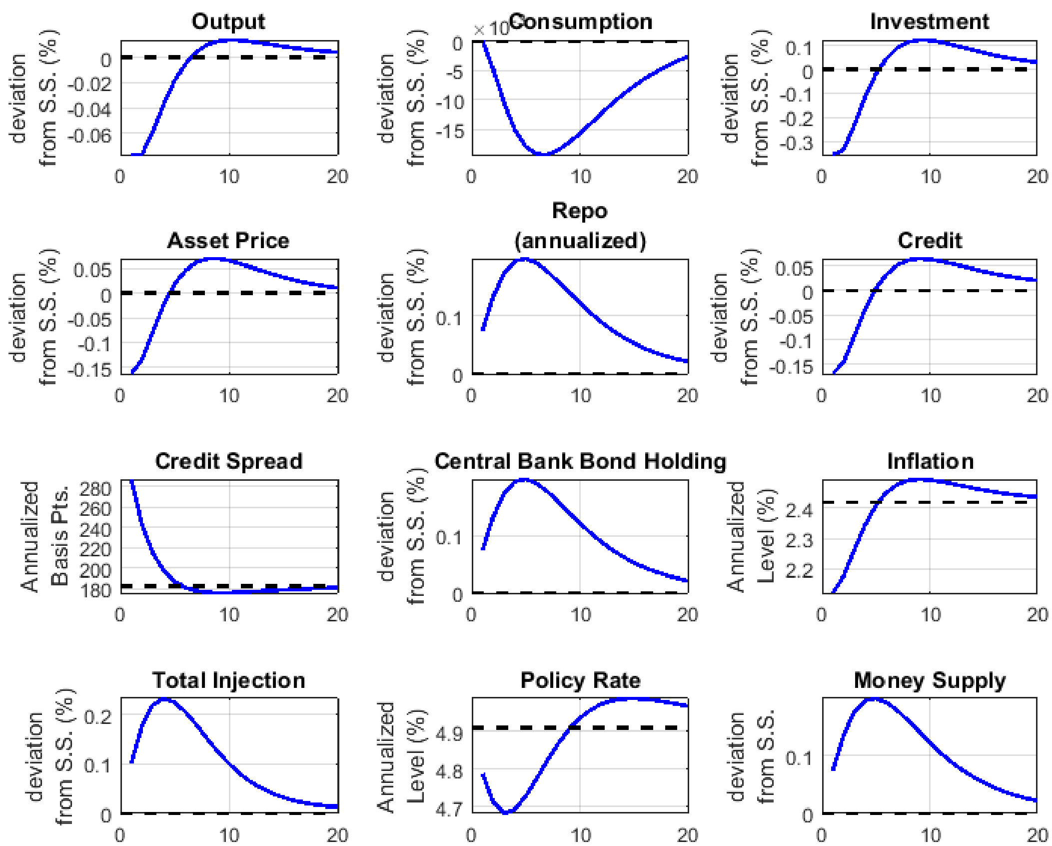


Figure 4: An Example on Double Acceleration in the Model: accepting corporate loans as collateral ($\kappa_t > 0$)

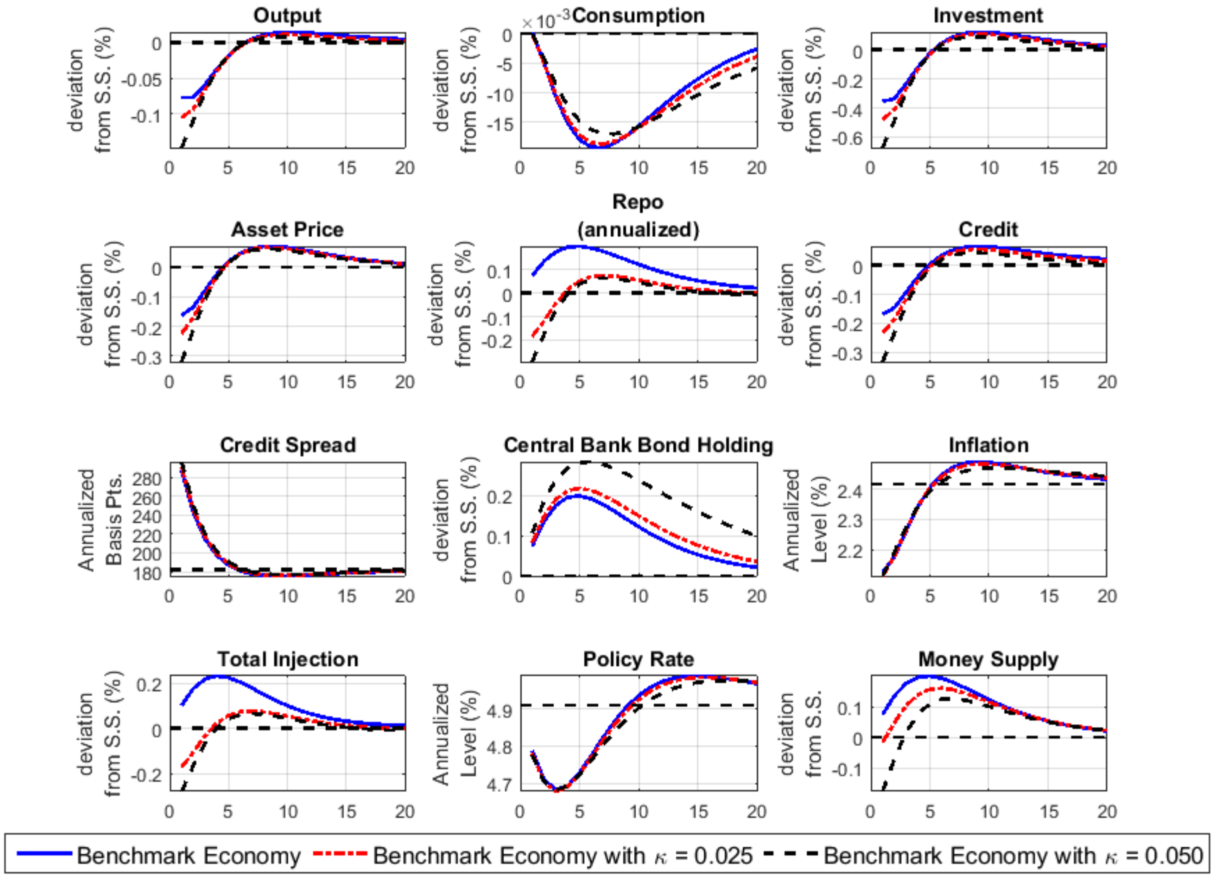


Figure 5: Impulse responses to an easing in collateral policy (an increase in κ_t^b or κ_t)

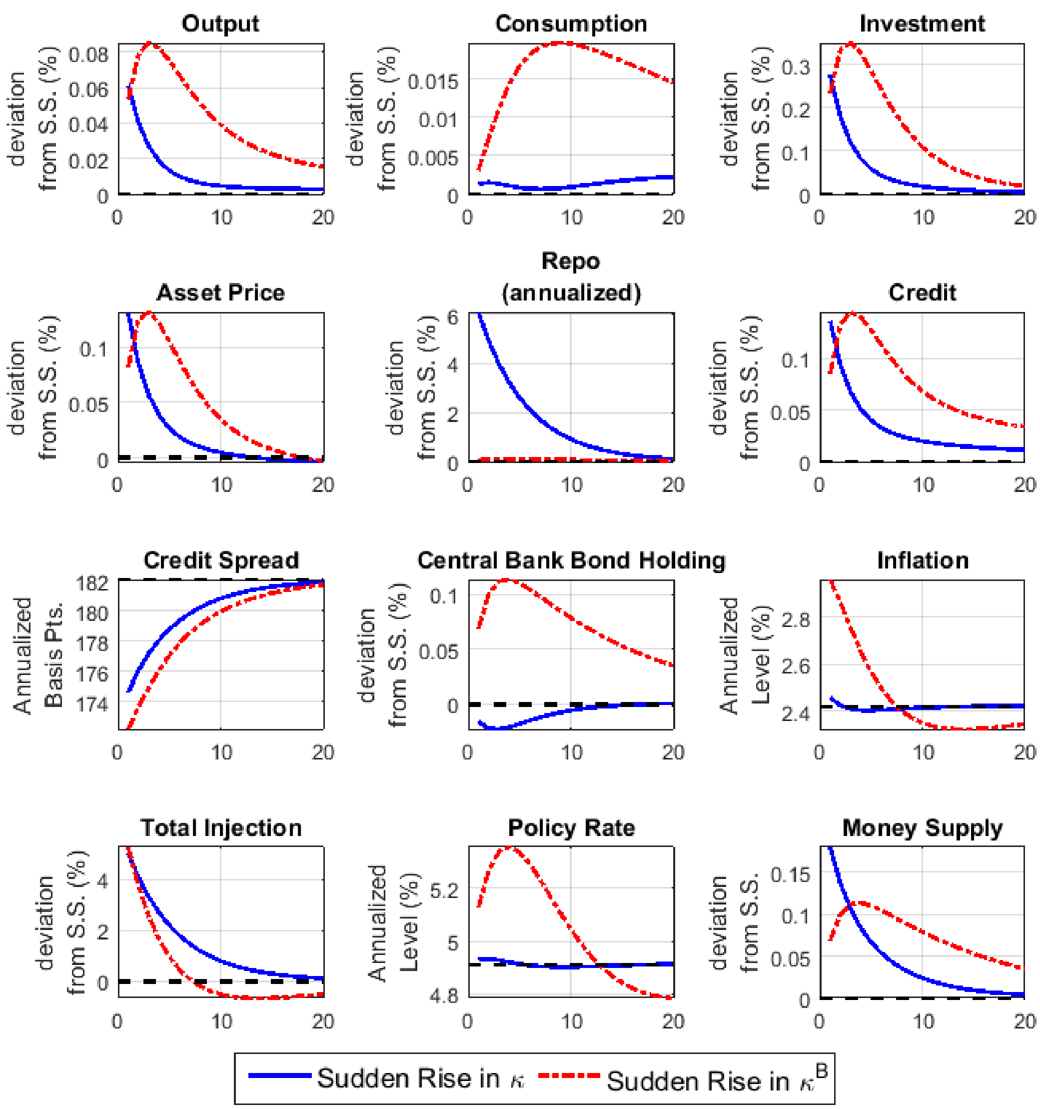


Figure 6: Impulse responses to a 25-bpts tightening (in quarterly terms) in the conventional policy rate

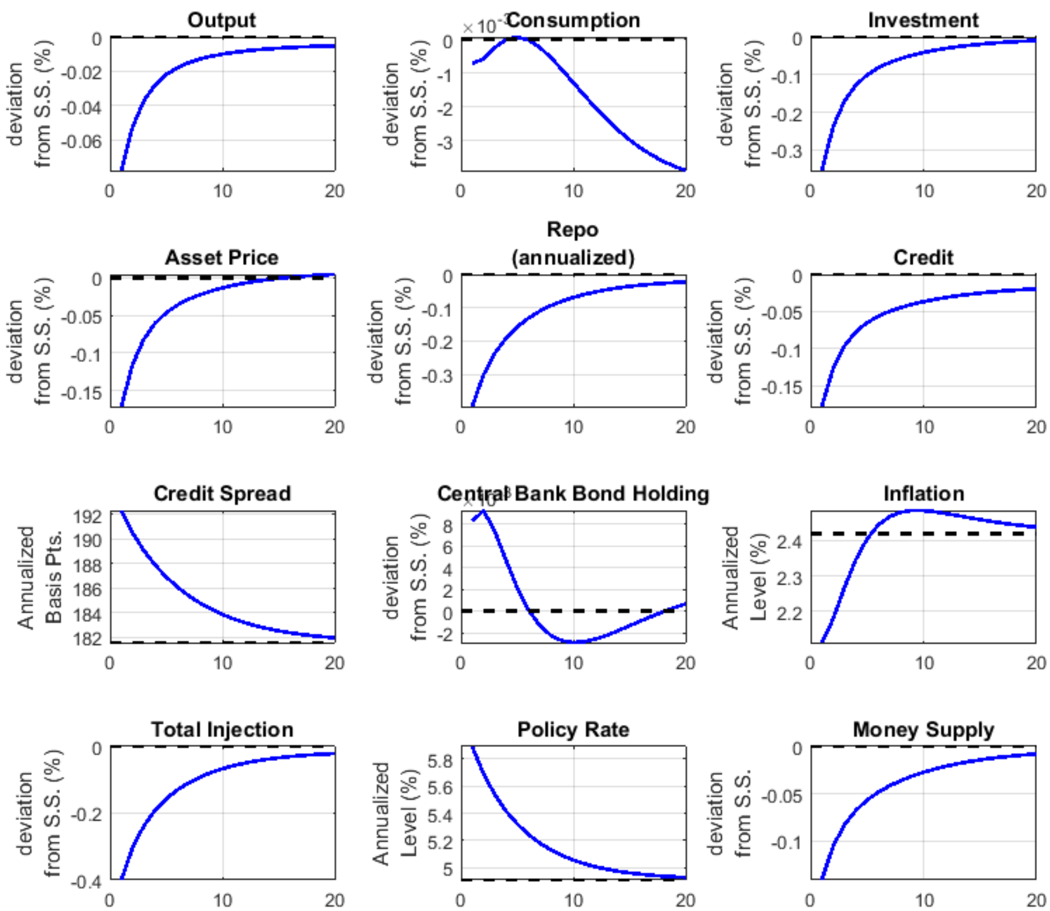


Figure 7: Impulse responses to an unfavorable financial shock

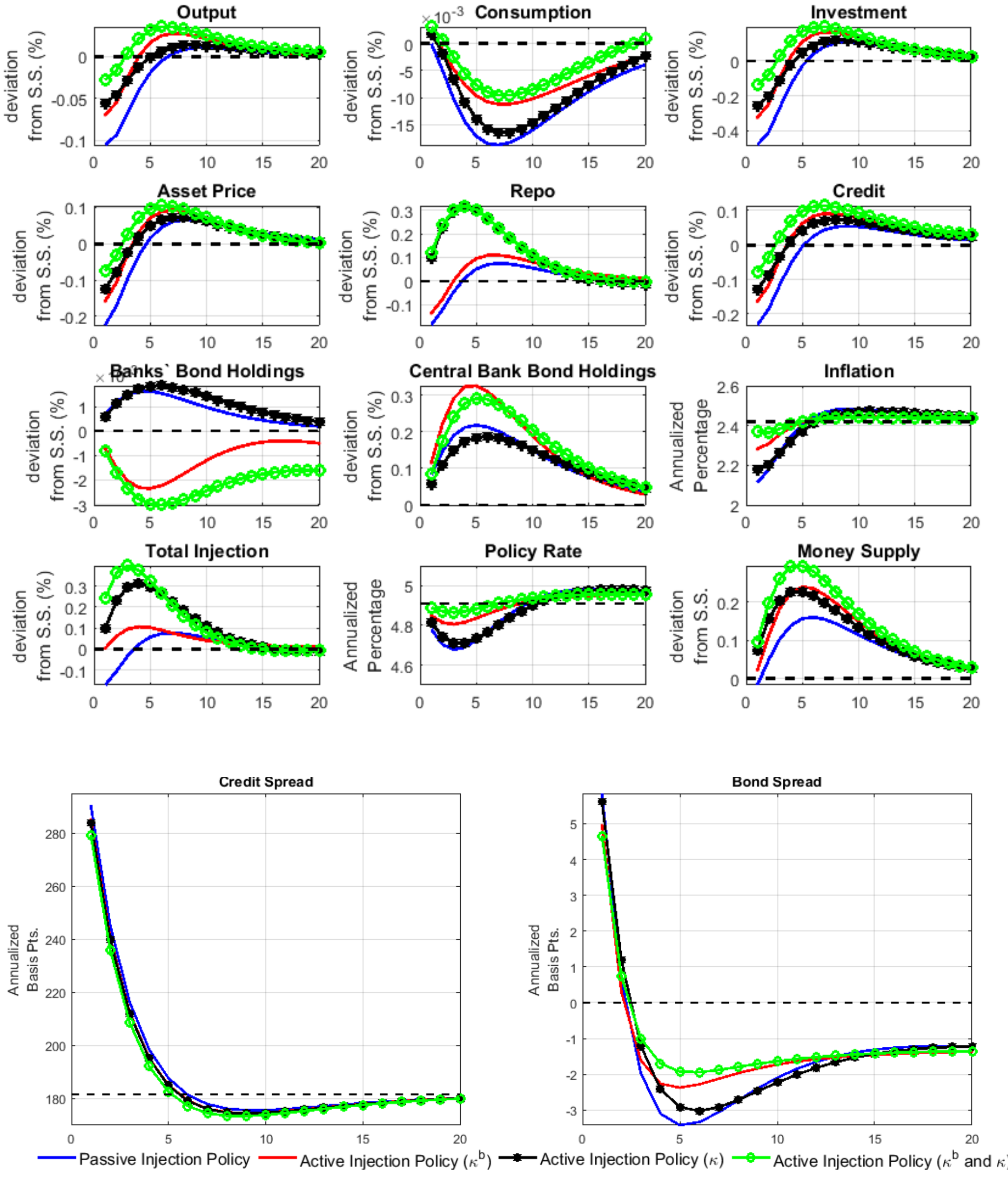


Figure 8: Using Active Collateral Policies when the policy rate endogenously hits the ZLB (due to increase in funding stress, λ)

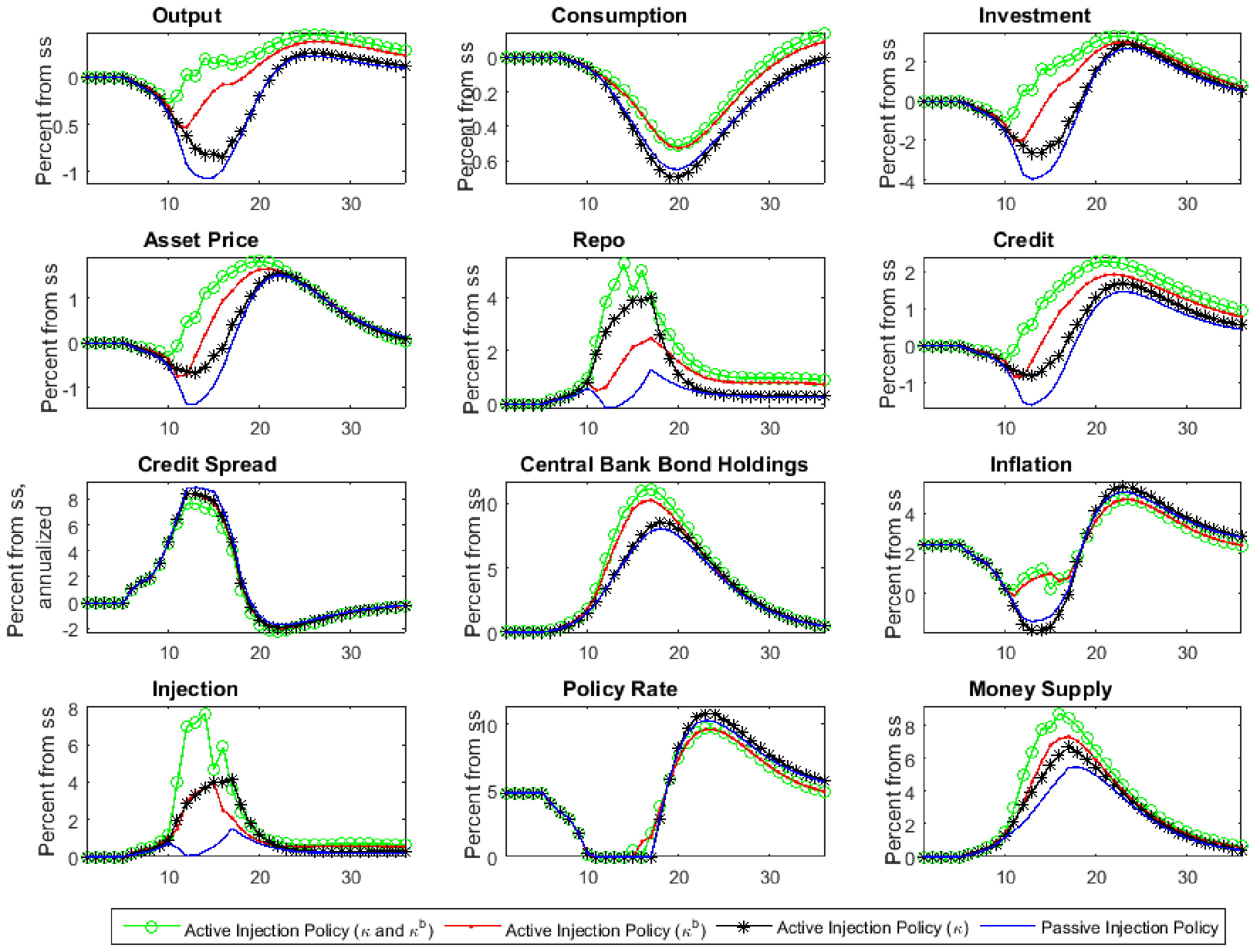


Table 3: Structural Parameters

Description	Value	Target
Preferences		
Quarterly discount factor (β)	0.9935	Data, annualized US real deposit rate (2.16%)
Relative risk aversion (σ)	2	RBC
Relative utility weight of leisure (ψ)	17.58	Hours worked (0.33)
Labor supply elasticity (υ)	0.276	Gertler and Karadi (2011), Frisch elasticity of ~ 4
Habit persistence (h_c)	0.815	Gertler and Karadi (2011)
Non-financial firms		
Share of capital in output (α)	0.36	Labor share of output (0.64)
Capital adjustment cost parameter (φ)	26	Christensen and Dib (2008)
Depreciation rate of capital (δ)	0.022	Bachmann et al. (2013)
Elast. of marginal depreciation w.r.t. utilization (ϱ)	3	Related literature
Depreciation rate, scale parameter (φ_u)	0.033	Related literature
Capacity utilization parameter (u)	1	Related literature
Elasticity of substitution across varieties	11	Gross mark-up of 10%
Calvo price stickiness parameter	0.66	Klenow and Malin (2010)
Financial Intermediaries		
Fraction of diverted loans (λ)	0.3748	Data, credit spread (180 basis points)
Prop. transfer to the entering bankers (ϵ^b)	0.0096	Data, leverage ratio of 9.89 for US commercial banks
Survival probability of the bankers (θ)	0.938	Gertler and Karadi (2011), Gertler and Kiyotaki (2010)
Fraction of divertable government bonds (ω)	0.00001	A small value (without loss of generality).
Policy Parameters		
Policy persistence (ρ_r)	0.81	Smets and Wouters (2007) posterior mode
L.R. reaction to inflation (φ_π)	2.03	Smets and Wouters (2007) posterior mode
L.R. reaction to output gap (φ_y)	0.125	Related literature, in line with Smets and Wouters (2007)
Fraction of government bonds eligible for CB injection (κ^b)	0.075	Injection-to-output ratio of 0.7%
Fraction of corporate loans eligible for CB injection (κ)	0 or 0.025	Injection-to-output ratio of 7%
Shock Processes		
Persistence of TFP process (ρ_z)	0.9315	Estimated AR(1) persistence from detrended log(TFP)
Std. deviation of productivity shocks (σ_z)	0.007	Estimated standard deviation for the TFP
Persistence of funding stress shock (ρ_λ)	0.66	Gertler and Karadi (2011), Dedola et al. (2013)
Std. deviation of funding stress shock (σ_λ)	0.003	On-impact increase in the credit spread of about 1% points
Other Parameters		
Government bond growth rate (Γ)	1.00599	L.R. inflation rate of 2.4% (annualized)
Ratio of treasury repo to total outright purchases of bonds	0.032	Repo over broad money (0.032)
Injection cost parameter	0.0001	Real cost of central bank injection

Appendix A: Banks' Profit Maximization Problem

Let us conjecture that the bank's franchise value is given by

$$V_{jt} = \nu_t^s q_t s_{jt} + \nu_t^b b_{jt} + \nu_t^n n_{jt} + \nu_t^{in} in_{jt}. \tag{A.1}$$

Comparing the conjectured solution for V_{jt} to the expected discounted terminal net worth yields the following expressions:

$$\nu_t^s q_t s_{jt} = E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t,t+1+i} \left[R_{kt+1+i} - R_{t+1+i}\right] q_{t+i} s_{jt+i} \tag{A.2}$$

$$\nu_t^b b_{jt} = E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t,t+1+i} \left[R_{bt+1+i} - R_{t+1+i}\right] b_{jt+i} \tag{A.3}$$

$$\nu_t^{in} in_{jt} = E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t,t+1+i} \left[R_{t+1+i} - R_{mt+1+i}\right] in_{jt+i} \tag{A.4}$$

$$\nu_t^n n_{jt} = E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t,t+1+i} R_{t+1+i} n_{jt+i}. \tag{A.5}$$

We write ν_t^s, ν_t^b, ν_t^{in} and ν_t^n recursively using the above expressions. Let us begin with ν_t^s. To ease the notation, let us drop expectations for now:

$$\nu_t^s = \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t,t+1+i} \left[R_{kt+1+i} - R_{t+1+i}\right] \chi_{t,t+i}^s, \tag{A.6}$$

where $\chi_{t,t+i}^s = \frac{q_{t+i} s_{jt+i}}{q_t s_{jt}}$. Let us separate (A.6) into two parts:

$$\nu_t^s = (1-\theta)\beta\Lambda_{t,t+1}\left[R_{kt+1} - R_{t+1}\right] + \sum_{i=1}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t,t+1+i} \left[R_{kt+1+i} - R_{t+1+i}\right] \chi_{t,t+i}^s. \tag{A.7}$$

Rearrange the second term on the right-hand side of expression (A.7):

$$\nu_t^s = (1-\theta)\beta\Lambda_{t,t+1}\left[R_{kt+1} - R_{t+1}\right] + \beta\Lambda_{t,t+1}\theta\chi_{t,t+1}^s \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t+1,t+2+i} \left[R_{kt+2+i} - R_{t+2+i}\right] \chi_{t+1,t+1+i}^s. \tag{A.8}$$

The infinite sum on the right-hand side of equation (A.8) is the one-period updated version of equation (A.6), given by

$$\nu_{t+1}^s = \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t+1,t+2+i} \left[R_{kt+2+i} - R_{t+2+i}\right] \chi_{t+1,t+1+i}^s, \tag{A.9}$$

where $\chi_{t+1,t+1+i}^s = \frac{q_{t+1+i} s_{jt+1+i}}{q_{t+1} s_{jt+1}}$.
Hence, we can rewrite (A.8) with expectations as follows:

$$\nu_t^s = E_t[(1-\theta)\beta\Lambda_{t,t+1}\left[R_{kt+1} - R_{t+1}\right] + \beta\Lambda_{t,t+1}\theta\chi_{t,t+1}^s \nu_{t+1}^s]. \tag{A.10}$$

Following the same steps we have the following expressions for ν_t^s, ν_t^s, ν_t^s, respectively as in equations (16),(17),(18):

$$\nu_t^b = E_t\left\{(1-\theta)\beta\Lambda_{t,t+1}(R_{bt+1} - R_{t+1}) + \theta\beta\Lambda_{t,t+1}\chi_{t,t+1}^b \nu_{t+1}^b\right\}, \tag{A.11}$$

$$\nu_t^{in} = E_t\left\{(1-\theta)\beta\Lambda_{t,t+1}(R_{t+1} - R_{mt+1}) + \theta\beta\Lambda_{t,t+1}\chi_{t,t+1}^{in} \nu_{t+1}^{in}\right\}, \tag{A.12}$$

and

$$\nu_t^n = E_t\left\{(1-\theta)\beta\Lambda_{t,t+1}R_{t+1} + \theta\beta\Lambda_{t,t+1}\chi_{t,t+1}^n \nu_{t+1}^n\right\} \tag{A.13}$$

where $\chi_{t,t+1}^b = \frac{b_{jt+1}}{b_{jt}}$, $\chi_{t,t+1}^{in} = \frac{in_{jt+1}}{in_{jt}}$ and $\chi_{t,t+1}^n = \frac{n_{jt+1}}{n_{jt}}$.

Appendix B: Aggregate Resource Constraint

Households' budget constraint is given by

$$c_t + d_t + m_t = w_t h_t + \frac{R_t d_{t-1}}{\pi_t} + \frac{m_{t-1}}{\pi_t} + \tau_t^f + \tau_t^c + \tau_t \tag{B.1}$$

where $\tau_t^f, \tau_t^c, \tau_t$ represent lump-sum transfers from the final-goods producing firms, capital-goods producing firms, and the government, respectively. Combining equation (B.1) with the banks' aggregate balance sheet:

$$q_t k_{t+1} + b_t = d_t + n_t + in_t \tag{B.2}$$

where we make use of the relation that $s_t = k_{t+1}$ (that firms make use of equity issuance to fully finance their capital expenditures). Therefore, combining (B.1) and (B.2) yields:

$$c_t + [q_t k_{t+1} + b_t - n_t - in_t] + m_t = w_t h_t + \frac{R_t}{\pi_t}[q_{t-1}k_t + b_{t-1} - n_{t-1} - in_{t-1}] + \frac{m_{t-1}}{\pi_t} + \tau_t^f + \tau_t^c + \tau_t \tag{B.3}$$

Combining the terms with n_t then yields:

$$c_t + [q_t k_{t+1} + b_t - in_t] + m_t = w_t h_t + \frac{R_t}{\pi_t}[q_{t-1}k_t + b_{t-1} - in_{t-1}] + \left[n_t - \frac{R_t}{\pi_t} n_{t-1}\right] + \frac{m_{t-1}}{\pi_t} + \tau_t^f + \tau_t^c + \tau_t \tag{B.4}$$

Note that the evolution of net worth of banks is given by:

$$n_{t+1}\pi_{t+1} = (R_{k,t+1} - R_{t+1})q_t k_{t+1} + (R_{b,t+1} - R_{t+1})b_t + (R_{t+1} - R_{m,t+1})in_t + R_{t+1}n_t \tag{B.5}$$

Taking one-period lag of equation (B.5), and plugging into the equation (B.4) yields:

$$c_t + [q_t k_{t+1} + b_t - in_t] + m_t = w_t h_t + \frac{R_{kt}}{\pi_t} q_{t-1}k_t + \frac{R_{bt}}{\pi_t} b_{t-1} - \frac{R_{mt}}{\pi_t} in_{t-1} + \frac{m_{t-1}}{\pi_t} + \tau_t^f + \tau_t^c + \tau_t \tag{B.6}$$

Now combine the profit of final goods producing firms, τ_t^f, that satisfy $\tau_t^f = Y_t + q_t(1-\delta)k_t - w_t h_t - \frac{R_{kt}}{\pi_t} q_{t-1} k_t$, with the equation (B.6) yields:

$$c_t + [q_t k_{t+1} + b_t - in_t] + m_t = \frac{R_{bt}}{\pi_t} b_{t-1} - \frac{R_{mt}}{\pi_t} in_{t-1} + \frac{m_{t-1}}{\pi_t} + [Y_t + q_t(1-\delta)] + \tau_t^c + \tau_t \tag{B.7}$$

Next combine the profit of capital goods producing firms, τ_t^c, that satisfy $\tau_t^c = q_t k_{t+1} - q_t(1-\delta)k_t - i_t$, with the equation (B.7) yields:

$$c_t + i_t = Y_t - \left[b_t - \frac{R_{bt}}{\pi_t} b_{t-1}\right] + \left[in_t - \frac{R_{mt}}{\pi_t} in_{t-1}\right] - \left[m_t - \frac{m_{t-1}}{\pi_t}\right] + \tau_t \tag{B.8}$$

Note that the balance sheet of the central bank is given by:

$$b_t^c + \tau_t^m + repo_t = \frac{R_{bt} b_{t-1}^c}{\pi_t} + \frac{R_{mt} in_{t-1}}{\pi_t} + m_t - \frac{m_{t-1}}{\pi_t} - R_{bt}\left[\frac{b_{t-1}^c}{\pi_t} - \frac{b_{t-2}^c}{\pi_t \pi_{t-1}}\right] - \Psi_t \tag{B.9}$$

where Ψ_t is the real cost incurred by the central bank due to injection. Furthermore, note that the government's budget constraint is:

$$b_t^t - \left(b_t^c - \frac{b_{t-1}^c}{\pi_t}\right) + \tau_t^m = \frac{R_{bt} b_{t-1}^t}{\pi_t} + \tau_t - R_{bt}\left[\frac{b_{t-1}^c}{\pi_t} - \frac{b_{t-2}^c}{\pi_t \pi_{t-1}}\right] \tag{B.10}$$

Combining equations (B.9), (B.10) and the bonds market equilibrium condition $b_t^t = b_t^c + b_t$, we obtain the lump-sum transfers by the government to the households, which is given by:

$$\tau_t = b_t - \frac{R_{bt} b_{t-1}}{\pi_t} + \frac{R_{mt} in_{t-1}}{\pi_t} - in_t + m_t - \frac{m_{t-1}}{\pi_t} - \Psi_t \tag{B.11}$$

Finally, combining equation (B.11) with equation (B.8) yields the aggregate resource constraint:

$$c_t + i_t + \Psi_t = Y_t \tag{B.12}$$

Appendix C: Competitive Equilibrium Conditions

The following are the optimality and market clearing conditions that are satisfied in a competitive equilibrium as defined in section 3.6:

$$U_c(t) = (c_t - h_c c_{t-1})^{-\sigma} - \beta(c_{t+1} - h_c c_t)^{-\sigma} \tag{EC-1}$$

$$\Lambda_{t,t+1} = \frac{U_c(t+1)}{U_c(t)} \frac{1}{\pi_{t+1}} \tag{EC-2}$$

$$1 = \beta E_t R_{t+1} \Lambda_{t,t+1} \tag{EC-3}$$

$$c_t = \frac{m_{t-1}}{\pi_t} + \frac{R_t d_{t-1}}{\pi_t} - d_t \tag{EC-4}$$

$$\frac{\psi h_t^{\upsilon}}{U_c(t)} = \frac{w_t}{R_{t+1}} \tag{EC-5}$$

$$lev_t = \frac{\nu_t^n}{\lambda - \nu_t^s - \nu_t^i \frac{\kappa_t}{R_{mt+1}}} \tag{EC-6}$$

$$q_t s_t + \omega b_t = lev_t n_t \tag{EC-7}$$

$$q_t s_t + b_t = d_t + n_t + in_t \tag{EC-8}$$

$$\varrho_{t,t+1} = \Big[(R_{kt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t}{R_{mt+1}}\Big] lev_t + R_{t+1} \tag{EC-9}$$

$$\chi_{t,t+1} = \varrho_{t,t+1} \frac{lev_{t+1}}{lev_t} \tag{EC-10}$$

$$n_{e,t+1}\pi_{t+1} = \theta \varrho_{t,t+1} n_t \tag{EC-11}$$

$$n_{n,t+1}\pi_{t+1} = \epsilon^b n_t \tag{EC-12}$$

$$n_{t+1} = n_{e,t+1} + n_{n,t+1} \tag{EC-13}$$

$$\nu_t^s = E_t \left\{(1-\theta)\beta\Lambda_{t,t+1} (R_{kt+1} - R_{t+1}) + \beta\Lambda_{t,t+1}\theta\chi_{t,t+1}^s \nu_{t+1}^s\right\} \tag{EC-14}$$

$$\nu_t^b = E_t \left\{(1-\theta)\beta\Lambda_{t,t+1} (R_{bt+1} - R_{t+1}) + \beta\Lambda_{t,t+1}\theta\chi_{t,t+1}^b \nu_{t+1}^b\right\} \tag{EC-15}$$

$$\nu_t^{in} = E_t \left\{(1-\theta)\beta\Lambda_{t,t+1} (R_{t+1} - R_{mt+1}) + \beta\Lambda_{t,t+1}\theta\chi_{t,t+1}^{in} \nu_{t+1}^{in}\right\} \tag{EC-16}$$

$$\nu_t^n = E_t \left\{(1-\theta)\beta\Lambda_{t,t+1} R_{t+1} + \beta\Lambda_{t,t+1}\theta\varrho_{t,t+1} \nu_{t+1}^n\right\} \tag{EC-17}$$

$$w_t = mc_t exp(z_t)(1-\alpha)k_t^{\alpha} h_t^{-\alpha} \tag{EC-18}$$

$$R_{kt} = \frac{mc_t exp(z_t)\alpha k_t^{\alpha-1} h_t^{1-\alpha} + q_t(1-\delta)}{q_{t-1}} \pi_t \tag{EC-19}$$

$$k_{t+1} = (1-\delta)k_t + \Big(\frac{i_t}{k_t} - \frac{\phi_k}{2}(\frac{i_t}{k_t} - \delta)^2\Big) k_t \tag{EC-20}$$

$$q_t = \frac{1}{1 - \phi_k(\frac{i_t}{k_t} - \delta)} \tag{EC-21}$$

$$\frac{y_t}{sp_t} = c_t + i_t + \Psi_t \tag{EC-22}$$

$$s_t = k_{t+1} \tag{EC-23}$$

$$1 = l_t + h_t \tag{EC-24}$$

$$Z_{1,t} = \frac{\epsilon^p}{\epsilon^p - 1} Z_{2,t} \tag{EC-25}$$

$$Z_{1,t} = \tilde{p_t}^{(1-\epsilon^p)} \frac{y_t}{sp_t} + \phi\beta E_t \frac{U_c(t+1)}{U_c(t)} \frac{1}{\pi_{t+1}} \pi_{t+1}^{(\epsilon^p - 1)} \Big(\frac{\tilde{p_t}}{\tilde{p}_{t+1}}\Big)^{(1-\epsilon^p)} Z_{1,t+1} \tag{EC-26}$$

$$Z_{2,t} = \tilde{p_t}^{(-\epsilon^p)} \frac{y_t}{sp_t} mc_t + \phi\beta E_t \frac{U_c(t+1)}{U_c(t)} \frac{1}{\pi_{t+1}} \pi_{t+1}^{\epsilon^p} \Big(\frac{\tilde{p_t}}{\tilde{p}_{t+1}}\Big)^{(-\epsilon^p)} Z_{2,t+1} \tag{EC-27}$$

$$1 = (1-\phi)(\tilde{z_t})^{1-\epsilon^p} + \phi \pi_t^{\epsilon^p - 1} \tag{EC-28}$$

$$sp_t = (1-\phi)(\tilde{z_t})^{-\epsilon^p} + \phi sp_{t-1} \pi_t^{\epsilon^p} \tag{EC-29}$$

$$b_t^t = b_t^c + b_t \tag{EC-30}$$

$$b_t^t + \tau_t^m = \frac{R_{bt} b_{t-1}^t}{\pi_t} + \tau_t \tag{EC-31}$$

$$in_t = \kappa_t^b \frac{b_t}{R_{mt+1}} + \kappa_t \frac{q_t s_t}{R_{mt+1}} \tag{EC-32}$$

$$R_{bt} = \omega(R_{kt} - R_t) + \big(\frac{\kappa_t \omega}{R_{mt}}\big)(R_t - R_{mt}) - \big(\frac{\kappa_t^b}{R_{mt}}\big)(R_t - R_{mt}) + R_t \tag{EC-33}$$

$$b_t^t = \Gamma \frac{b_{t-1}^t}{\pi_t} \tag{EC-34}$$

$$b_t^c + \tau_t^m + repo_t = \frac{R_{bt} b_{t-1}^c}{\pi_t} + \frac{R_{mt} in_{t-1}}{\pi_t} + m_t - \frac{m_{t-1}}{\pi_t} - R_{bt}\left[\frac{b_{t-1}^c}{\pi_t} - \frac{b_{t-2}^c}{\pi_t \pi_{t-1}}\right] - \Psi_t \tag{EC-35}$$

$$\tau_t^m = \frac{b_{t-2}^c}{\pi_t \pi_{t-1}}(R_{bt} - 1) + \frac{R_{mt} in_{t-1}}{\pi_t} - \frac{in_{t-1}}{\pi_t} \tag{EC-36}$$

$$A_t = \frac{\omega b_t}{\omega b_t + q_t s_t} \tag{EC-37}$$

$$\chi_t^s = \Big(\frac{1 - A_{t+1}}{1 - A_t}\Big) \varrho_{t,t+1} \frac{lev_{t+1}}{lev_t} \tag{EC-38}$$

$$\chi_t^b = \frac{A_{t+1}}{A_t} \varrho_{t,t+1} \frac{lev_{t+1}}{lev_t} \tag{EC-39}$$

$$\chi_t^{in} = \Big[\frac{R_t^m}{R_{t+1}^m} \frac{\kappa_t(1 - A_{t+1}) + \kappa_t^b A_{t+1}}{\kappa_t(1 - A_t) + \kappa_t^b A_t}\Big] \varrho_{t,t+1} \frac{lev_{t+1}}{lev_t} \tag{EC-40}$$

$$\kappa_t^b \frac{b_t}{R_{mt+1}} = b_t^c - \frac{b_{t-1}^c}{\pi_t} + m_t^r \tag{EC-41}$$

$$m_t^r = \Omega_t m_t \tag{EC-42}$$

$$y_t = exp(z_t) k_t^\alpha h_t^{1-\alpha} \tag{EC-43}$$

$$z_{t+1} = \rho_z z_t + \epsilon_{z,t+1} \tag{EC-44}$$

Appendix D: Derivations

EC-1:

Marginal utility of consumption.

EC-2:

Notation for the ratio of utilities of intertemporal consumption.

EC-3:

Standard euler equation.

EC-4:

Cash in advance constraint.

EC-5:

Using equation (6) and (7) gives EC-5.

EC-6:

Recall the lagrangian of the bank's problem:

$$\mathcal{L} = \nu_t^s q_t s_{jt} + \nu_t^b b_{jt} + \nu_t^n n_{jt} + \nu_t^{in} in_{jt} + \mu_t[V_t - \lambda(q_t s_t + \omega b_t)] + \gamma_t[\kappa_t^b \frac{b_{jt}}{R_{mt+1}} + \kappa_t \frac{q_t s_{jt}}{R_{mt+1}} - in_{jt}]$$

and take the derivations with respect to s_t, b_t and in_t
$s_t : q_t(1+\mu_t)\nu_t^s - \lambda\mu_t q_t + \gamma_t \frac{q_t \kappa_t}{R_{mt+1}} = 0,$
$b_t : (1+\mu_t)\nu_t^b - \lambda\mu_t\omega + \gamma_t \frac{\kappa_t^b}{R_{mt+1}} = 0,$
$in_t : (1+\mu_t)\nu_t^{in} - \gamma_t = 0.$
Combining first two derivations we get

$$(1+\mu_t)\nu_t^s + \gamma_t \frac{\kappa_t}{R_{mt+1}} = \frac{(1+\mu_t)}{\omega}\nu_t^b + \frac{\gamma_t}{\omega}\frac{\kappa_t^b}{R_{mt+1}},$$

and from the third derivation we have $\gamma_t = (1+\mu_t)\nu_t^{in}$. Combining these two equations we obtain

$$(1+\mu_t)\nu_t^s + (1+\mu_t)\nu_t^{in}\frac{\kappa_t}{R_{mt+1}} = (1+\mu_t)\frac{\nu_t^b}{\omega} + (1+\mu_t)\frac{\nu_t^{in}}{\omega}\frac{\kappa_t^b}{R_{mt+1}},$$

and finally eliminating $(1+\mu_t)$ we have $\left[\lambda - \nu_t^s - \frac{\nu_t^{in}\kappa_t}{R_{mt+1}}\right] = \left[\lambda - \frac{\nu_t^b}{\omega} - \frac{\nu_t^{in}\kappa_t^b}{R_{mt+1}\omega}\right]$.
Rewrite the injection constraint,

$$in_{jt} \leq \kappa_t^b \frac{b_j t}{R_{mt+1}} + \kappa_t \frac{q_t s_j t}{R_{mt+1}}, \tag{D.1}$$

which binds for each bank as long as the marginal gain of increasing the injection is positive. We are interested in where constraint binds because the policy parameters κ_t^b and κ_t can only be used as effective monetary policy tools in that case. Substituting injection in the incentive compatibility constraint in equation (A.1) we get

$$\left[\nu_t^s + \frac{\nu_t^{in}\kappa_t}{R_{mt+1}}\right] q_t s_{jt} + \left[\nu_t^b + \frac{\nu_t^{in}\kappa_t^b}{R_{mt+1}}\right] b_{jt} + \nu_t^n n_{jt} \geq \lambda(q_t s_{jt} + \omega b_{jt}), \tag{D.2}$$

and rearranging it and taking into account the constraint when it binds we obtain

$$\nu_t^n n_{jt} = \left[\lambda - \nu_t^s - \frac{\nu_t^{in}\kappa_t}{R_{mt+1}}\right] q_t s_{jt} + \left[\lambda - \frac{\nu_t^b}{\omega} - \frac{\nu_t^{in}\kappa_t^b}{R_{mt+1}\omega}\right]\omega b_{jt}. \tag{D.3}$$

Since $\left[\lambda - \nu_t^s + \frac{\nu_t^{in}\kappa_t}{R_{mt+1}}\right] = \left[\lambda - \frac{\nu_t^b}{\omega} - \frac{\nu_t^{in}\kappa_t^b}{R_{mt+1}}\right]$ we can rewrite the equation above as

$$q_t s_{jt} + \omega b_{jt} = \frac{\nu_t^n}{\left[\lambda - \nu_t^s - \frac{\nu_t^{in}\kappa_t}{R_{mt+1}}\right]} n_{jt} = \frac{\nu_t^n}{\left[\lambda - \frac{\nu_t^b}{\omega} - \frac{\nu_t^{in}\kappa_t^b}{R_{mt+1}}\right]} \tag{D.4}$$

We define the ratio of total assets over net worth as the leverage $lev_t = \frac{\nu_t^n}{\left[\lambda - \nu_t^s - \frac{\nu_t^{in}\kappa_t}{R_{mt+1}}\right]}$.

EC-7:

We define lev_t above.

EC-8:

This is the balance sheet of the bank by definition.

EC-9:

Recall the net worth evolution equation (10). We first substitute for in_{jt} using (11) with equality.

$$n_{t+1}\pi_{t+1} = \Big[(R_{kt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t}{R_{mt+1}}\Big]q_t s_t \\ + \Big[(R_{bt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t^b}{R_{mt+1}}\Big]b_t + R_{t+1}n_t \quad \text{(D.5)}$$

We then use (20) to substitute for b_t.

$$n_{t+1}\pi_{t+1} = \underbrace{\Big[(R_{kt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t}{R_{mt+1}}\Big]}_{\mathcal{X}_1} q_t s_t \\ - \underbrace{\Big[(R_{bt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t^b}{R_{mt+1}}\Big]\frac{1}{\omega}}_{\mathcal{X}_2} q_t s_t \\ + \Big[(R_{bt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t^b}{R_{mt+1}}\Big]\frac{lev_t n_t}{\omega} + R_{t+1}n_t \quad \text{(D.6)}$$

Next we need to show that $\mathcal{X}_1 = \mathcal{X}_2$. We rewrite the bank's problem differently:

$$\max_{s_t, b_t} E_t \sum_{i=0}^{\infty}(1-\theta)\theta^i\beta^i\Lambda_{t,t+1+i}n_{t+1}\pi_{t+1} = E_t \sum_{i=0}^{\infty}(1-\theta)\theta^i\beta^i\Lambda_{t,t+1+i}\Big\{\Big[(R_{kt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t}{R_{mt+1}}\Big]q_t s_t \\ + \Big[(R_{bt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t^b}{R_{mt+1}}\Big]b_t + R_{t+1}n_t\Big\} \quad \text{(D.7)}$$

subject to

$$V_t = \Big[\nu_t^s + \frac{\nu_t^{in}\kappa_t}{R_{mt+1}}\Big]q_t s_{jt} + \Big[\nu_t^b + \frac{\nu_t^{in}\kappa_t^b}{R_{mt+1}}\Big]b_{jt} + \nu_t^n n_{jt} \geq \lambda(q_t s_{jt} + \omega b_{jt}) \quad \text{(D.8)}$$

The first order conditions with respect to b_t and s_t imply:

$$(1-\theta)\theta^i\beta^i\Lambda_{t,t+1+i}\Big[(R_{kt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t}{R_{mt+1}}\Big] + \mu_t(\nu_t^s + \nu_t^{in}\frac{\kappa_t}{R_{mt+1}} - \lambda) = 0 \quad \text{(D.9)}$$

$$(1-\theta)\theta^i\beta^i\Lambda_{t,t+1+i}\Big[(R_{bt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t^b}{R_{mt+1}}\Big]\frac{1}{\omega} + \mu_t(\frac{\nu_t^b}{\omega} + \frac{\nu_t^{in}}{\omega}\frac{\kappa_t^b}{R_{mt+1}} - \lambda) = 0 \quad \text{(D.10)}$$

Since $\nu_t^s + \nu_t^{in}\frac{\kappa_t}{R_{mt+1}} = \frac{\nu_t^b}{\omega} + \frac{\nu_t^{in}}{\omega}\frac{\kappa_t^b}{R_{mt+1}}$ from the derivation (EC-32), $\mathcal{X}_1 = \mathcal{X}_2$. Then (D.6) can be written as:

$$n_{t+1}\pi_{t+1} = \Big[(R_{bt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t^b}{R_{mt+1}}\Big]\frac{lev_t n_t}{\omega} + R_{t+1}n_t \quad \text{(D.11)}$$

$$= \Big\{\Big[(R_{bt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t^b}{R_{mt+1}}\Big]\frac{lev_t}{\omega} + R_{t+1}\Big\}n_t \quad \text{(D.12)}$$

Hence,

$$\frac{n_{t+1}\pi_{t+1}}{n_t} = \varrho_{t,t+1} = \Big[(R_{bt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t^b}{R_{mt+1}}\Big]\frac{lev_t}{\omega} + R_{t+1}, \quad \text{(D.13)}$$

or similarly,

$$\varrho_{t,t+1} = \Big[(R_{kt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t}{R_{mt+1}}\Big]lev_t + R_{t+1}. \quad \text{(D.14)}$$

EC-10:

We name the growth rate of total assets as $\chi_{t,t+1}$ and by definition in equation EC-7, it is the product of growth rates of net worth and leverage.

EC-11:

This is the condition for surviving bankers in equation (23).

EC-12:

This is the condition for remittance of new entrant bankers in equation (22).

EC-13:

This is the condition for net worth of the entire banking system as the sum of existing bankers' and new entrants' net worth. See equation (24).

EC-14:

The derivation is in Appendix A.

EC-15:

The derivation is in Appendix A.

EC-16:

The derivation is in Appendix A.

EC-17:

The derivation is in Appendix A.

EC-18:

The intermediate goods producer solves the following problem

$$\max_{h_t,k_t} \quad P_t^{int} y_t - W_t h_t - R_{kt} Q_{t-1} k_t + Q_t(1-\delta)k_t \tag{D.15}$$

subject to

$$y_t = exp(z_t) k_t^{\alpha} h_t^{1-\alpha} \tag{D.16}$$

Dividing by P_t and substituting the constraint in and making use of $mc_t = P_t^{int}/P_t$, we get

$$\max_{h_t,k_t} \quad mc_t exp(z_t) k_t^{\alpha} h_t^{1-\alpha} - w_t h_t - \frac{R_{kt} q_{t-1} k_t}{\pi_t} + q_t(1-\delta)k_t \tag{D.17}$$

The first order conditions yield the following equations:

$$w_t = mc_t exp(z_t)(1-\alpha) k_t^{\alpha} h_t^{-\alpha} \tag{D.18}$$

$$R_{kt} = \left[\frac{mc_t exp(z_t) \alpha k_t^{\alpha-1} h_t^{1-\alpha} + q_t(1-\delta)}{q_{t-1}} \right] \pi_t \tag{D.19}$$

EC-19:

See above.

EC-20:

This is the capital accumulation condition.

EC-21:

Recall the problem of capital producers

$$\max_{i_t} q_t k_{t+1} - q_t(1-\delta)k_t - i_t \tag{D.20}$$

subject to the capital accumulation technology,

$$k_{t+1} = (1-\delta)k_t + \Phi\Big(\frac{i_t}{k_t}\Big)k_t. \tag{D.21}$$

We can write the Lagrangian as

$$\mathcal{L} = q_t k_{t+1} - q_t(1-\delta)k_t - i_t + \lambda_t \Big[(1-\delta)k_t + \Phi\Big(\frac{i_t}{k_t}\Big)k_t\Big] \tag{D.22}$$

Taking derivations with respect i_t and k_{t+1} we get

$i_t : (-1) + \lambda_t - \Phi^{'}\left(\frac{i_t}{k_t}\right)k_t$ where $\Phi^{'}\left(\frac{i_t}{k_t}\right)k_t = 1 - \phi_k\left(\frac{i_t}{k_t} - \delta\right)$

and

$k_{t+1} : q_t + \lambda_t = 0.$

Using the last condition of λ_t and substituting in the previous condition we have EC-21.

EC-22:

See Appendix B.

EC-23:

The amount of capital acquired for the next period is equal to claims that issued to banks.

EC-24:

Time is allocated to leisure and labor.

EC-25:

The derivation is in the text.

EC-26:

The derivation is in the text.

EC-27:

The derivation is in the text.

EC-28:

Given by (39).

EC-29:

Let sp_t is a measure of price dispersion, $sp_t = \int_0^1 (P_t(i)/P_t)^{-\epsilon^p} di$. Since each period $1-\phi$ fraction of firms reset their prices, sp_t can be written as

$$sp_t = (1-\phi)(\tilde{p}_t)^{-\epsilon^p} + (1-\phi)\phi(\tilde{p}_{t-1})^{-\epsilon^p} + (1-\phi)\phi^2(\tilde{p}_{t-1})^{-\epsilon^p} + ...$$

which can be rearranged as the following with the assumption of no initial dispersion:

$$sp_t = (1-\phi)(\tilde{p}_t)^{-\epsilon^p} + \phi sp_{t-1}\pi_t^{\epsilon^p}$$

EC-30:

Government bonds that can be used for open market operations are the sum of holdings of central bank and the banks.

EC-31:

Budget constraint of the government.

EC-32:

The amount of injection when the constraint binds.

EC-33:

From the derivation of EC-9 we have:

$$(R_{kt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t}{R_{mt+1}} = \Big[(R_{bt+1} - R_{t+1}) + (R_{t+1} - R_{mt+1})\frac{\kappa_t^b}{R_{mt+1}}\Big]\frac{1}{\omega}$$

From here it follows that:

$$R_{bt} = \omega(R_{kt} - R_t) + (\frac{\kappa_t \omega}{R_{mt}})(R_t - R_{mt}) - (\frac{\kappa_t^b}{R_{mt}})(R_t - R_{mt}) + R_t$$

EC-34:

The evolution of government bonds.

EC-35:

Budget constraint of the central bank.

EC-36:

Earnings of central bank.

EC-37:

Ratio of government bond holdings of banking sector to total assets.

EC-38:

Recall the growth rate of total loans $\chi_{t,t+1} = \frac{q_{t+1}s_{t+1}+\omega b_{t+1}}{q_t s_t + \omega b_t} = \varrho_{t,t+1}\frac{lev_{t+1}}{lev_t}$. Using the ratio of government bond holdings of banking sector to total assets, $A_t = \frac{\omega b_t}{\omega b_t + q_t s_t}$, we can rewrite the growth rate of total loans as:

$$\frac{q_{t+1}s_{t+1}+\frac{A_{t+1}}{1-A_{t+1}}q_{t+1}s_{t+1}}{q_ts_t+\frac{A_t}{1-A_t}q_{t+1}s_{t+1}} = \frac{\left(1+\frac{A_{t+1}}{1-A_{t+1}}\right)q_{t+1}s_{t+1}}{\left(1+\frac{A_t}{1-A_t}\right)q_ts_t} = \left(\frac{1-A_t}{1-A_{t+1}}\right)\nu^s_{t,t+1} = \varrho_{t,t+1}\frac{lev_{t+1}}{lev_t}.$$

By rearranging it we have $\nu^s_{t,t+1} = \left(\frac{1-A_{t+1}}{1-A_t}\right)\varrho_{t,t+1}\frac{lev_{t+1}}{lev_t}$.

EC-39:

Similar to algebra as in the derivation of EC-38, we write $\chi_{t,t+1} = \frac{b_{t+1}+\frac{1-A_{t+1}}{A_{t+1}}b_{t+1}}{\frac{1-A_t}{A_t}b_t+b_t} = \frac{b_{t+1}}{b_t}\frac{1+\frac{1-A_{t+1}}{A_{t+1}}}{1+\frac{1-A_t}{A_t}}$. Then we have $\nu^b_{t,t+1} = \frac{A_{t+1}}{A_t}\varrho_{t,t+1}\frac{lev_{t+1}}{lev_t}$.

EC-40:

The growth rate of injection is $\nu^{in}_{t,t+1} = \frac{\kappa^b_{t+1}\frac{b_{t+1}}{R_{mt+1}}+\kappa_{t+1}\frac{q_{t+1}s_{t+1}}{R_{mt+1}}}{\kappa^b_t\frac{b_t}{R_{mt}}+\kappa_t\frac{q_ts_t}{R_{mt}}}$ and let denote loan as $L_t = q_ts_t+\omega b_t$. Then we have

$$\nu^{in}_{t,t+1} = \frac{\kappa^b_{t+1}\frac{A_{t+1}L_{t+1}}{R_{mt+1}}+\kappa_{t+1}\frac{(1-A_{t+1})L_{t+1}}{R_{mt+1}}}{\kappa^b_t\frac{A_tL_t}{R_m}+\kappa_t\frac{(1-A)L_t}{R_m}}.$$

which can be rewritten as

$$\nu^{in}_{t,t+1} = \frac{\kappa^b_{t+1}A_{t+1}+\kappa_{t+1}(1-A_{t+1})}{\kappa^b_tA_t+\kappa_t(1-A_t)}\frac{R_{mt}}{R_{mt+1}}\frac{L_{t+1}}{L_t}$$

and finally substituting the growth rate of L_t we get EC-40.

EC-41:

The total amount of money market operations with government bonds (outright purchase plus repo operations with bonds).

EC-42:

Central bank sets the ratio of treasury repos to money stock which is actually net outright purchase of central bank up to time t if the initial bond holdings of central bank.

EC-43:

Cobb-Douglas production function.

EC-44:

This is the productivity shock process.