Bayesian forecasting with a Small and Medium Scale Factor-Augmented Vector Autoregressive DSGE model

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Abstract

Advanced Bayesian methods are employed in estimating dynamic stochastic general equilibrium (DSGE) models. Although policymakers and practitioners are particularly interested in DSGE models, these are typically too stylized to be taken directly to the data and often yield weak prediction results. Hybrid models can deal with some of the DSGE model misspecifications. Major advances in Bayesian estimation methodology could allow these models to outperform well-known time series models and effectively deal with more complex real-world problems as richer sources of data become available. A comparative evaluation of the out-of-sample predictive performance of many different specifications of estimated DSGE models and various classes of VAR models is performed, using datasets from the US economy. Simple and hybrid DSGE models are implemented, such as DSGE-VAR and Factor Augmented DSGEs and tested against standard, Bayesian and Factor Augmented VARs. Moreover, small scale models including the real gross domestic product, the harmonized consumer price index and the nominal short-term federal funds interest rate, are comparatively assessed against medium scale models featuring additionally sticky nominal prices, wage contracts, habit formation, variable capital utilization and investment adjustment costs. The investigated period spans 1960:Q4 to 2010:Q4 and forecasts are produced for the out-of-sample testing period 1997:Q1-2010:Q4. This comparative validation can be useful to monetary policy analysis and macro-forecasting with the use of advanced Bayesian methods.

Keywords: Bayesian estimation, Forecasting, Metropolis-Hastings, Markov chain monte carlo, Marginal data density, Factor Augmented DSGE

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1 Introduction

The new micro-founded dynamic stochastic general equilibrium DSGE models appear to be particularly suited for evaluating the consequences of alternative macroeconomic policies, as shown in the works of Smets and Wouters (2003, 2004), Del Negro and Schorfheide (2004), Adolfson *et al.* (2008) and Christiano *et al.* (2005). However, the calibrated DSGE models face many important challenges such as the fragility of parameter estimates, statistical fit and the weak reliability of policy predictions as reported in Stock and Watson (2001), Ireland (2004) and Schorfheide (2010). In recent years Bayesian estimation of DSGE models has become popular for many reasons, mainly because it is a system-based estimation approach that offers the advantage of incorporating assumptions about the parameters based on economic theory. These assumptions can reduce weak identification issues.

Macroeconomists have extensively used Bayesian techniques during the last 20 years. One reason is that Bayesian methods afford researchers the chance to estimate and evaluate a wide variety of macro models that frequentist econometrics often find challenging. Bayesian methodology can be extremely useful in DSGE estimation and forecasting. The popularity of the Bayesian approach is also explained by the increasing computational power available to estimate and evaluate medium- to large-scale DSGE models using Markov Chain Monte Carlo (MCMC) simulators. These DSGE models can pose identification problems for frequentist estimation that no amount of data or computing power can overcome. New macro-research is also drawn to the estimation and evaluation framework of Bayesian statistics because DSGE models are often seen as abstractions of actual economies.

Increasing efforts have been undertaken to use DSGE models for forecasting. DSGE models were not considered as forecasting tools until the works of Smets and Wouters (2003, 2004) on the predictability of DSGE models compared to alternative non-structural models. In the macro-econometric literature, hybrid or mixture DSGE models have become popular in dealing with some of the model misspecifications as well as the trade-off between theoretical coherence and empirical fit (Schorfheide, 2010). They are categorized in additive hybrid models and hierarchical hybrid models. The hybrid models provide a complete analysis of the data law of motion and better capture the dynamic properties of the DSGE models. In the recent literature, different attempts of hybrid models have been introduced in solving, estimating and forecasting with DSGEs. Sargent (1989) and Altug (1989) proposed augmenting a DSGE model with measurement error terms that follow a first order autoregressive process, known as the DSGE-AR approach. Ireland (2004) proposed a method that is similar to the DSGE-AR, but imposing no restriction on the measurement errors, assuming that residuals follow a first-order vector autoregression (DSGE-AR à l'Ireland). A different approach called DSGE-VAR was proposed by Del Negro and Schorfheide (2004) and was based on the works of DeJong *et al.* (1996) and Ingram and Whiteman (1994). The problem of overfitting results in multicollinearity and loss of degrees of freedom, and leads to inefficient estimates and large out-of-sample forecasting errors. It is possible to overcome this problem by using the well-known "Minnesota" priors (Doan *et al.*, 1984). The use of "Minnesota" priors has been proposed to shrink the parameters space and thus overcome the curse of dimensionality. Following this idea in combining the DSGE model information and the VAR representation, two alternative econometric tools have been also introduced: the DSGE-FAVAR (Consolo *et al.*, 2009) and the Augmented VAR-DSGE model (Fernández-de-Córdoba and Torres, 2010). The main idea behind the Factor Augmented DSGE (DSGE-FAVAR) is the use of factors to improve the statistical identification in validating the models. Consequently, the VAR representation is replaced by a FAVAR model as the statistical benchmark.

In this study, we conduct an exhaustive empirical exercise that includes the comparison of the out-of-sample predictive performance of estimated DSGE models with that of standard VARs, Bayesian VARs and Factor Augmented VARs estimated on the same data set for the US economy. We focus on many different specifications of the DSGE models, i.e., the simple DSGE, the DSGE-VAR and specifically on the Factor Augmented DSGE (DSGE-FAVAR) model with emphasis on Bayesian estimation. The motivation comes from a group of recent papers that compares the forecasting performance of DSGE against VAR models, e.g., Smets and Wouters (2004), Ireland (2004), Del Negro and Schorfheide (2004), Del Negro *et al.* (2007), Adolfson *et al.* (2008), Christoffel *et al.* (2008), Rubaszek and Skrzypczynski (2008), Ghent (2009), Kolosa *et al.* (2009), Consolo *et al.* (2009) and Fernandez-de-Cordoba and Torres (2010) among others. We use comparatively a small scale model as in Del Negro and Schorfheide (2004) including the real GDP, the harmonized Consumer Price Index and the nominal short-term federal funds interest rate, as well as the medium scale model of Smets and Wouters (2007) which features sticky nominal price, wage contracts, habit formation, variable capital utilization and investment adjustment costs. The Smets and Wouters (2007) model is close in spirit to that of Christiano *et al.* (2005) to fit to US macroeconomic data. We use quarterly data of the US economy from 1960:Q4 to 2010:Q4 and we produce forecasts for the out-of-sample testing period 1997:Q1-2010:Q4. The remainder of this paper is organized as follows. Section 2 describes the standard and Bayesian VAR as well as the Factor Augmented VAR model. In section 3 the small and a medium scale DSGE models are analyzed and the hybrid DSGE-VAR and DSGE-FAVAR models are described in detail, both in a small and a medium scale specification. In section 4 the data are described and the empirical results of the comparative forecasting evaluation are illustrated and analyzed. Finally, section 5 concludes.

2 VAR Models

2.1 Classical VAR

As suggested by Sims (1980), the standard unrestricted VAR, has the following compact format

$$\mathbf{Y} = \mathbf{X\Phi} + \mathbf{U}, \tag{1}$$

where $\mathbf{Y}$ is a $(T \times n)$ matrix with rows Y_t', and $\mathbf{X}$ is a $(T \times k)$ matrix ($k = 1 + np, p =$number of lags) with rows $X_t' = [1, Y_{t-1}', ..., Y_{t-p}']$. $\mathbf{U}$ is a $(T \times n)$ matrix with rows u_t', $\mathbf{\Phi}$ is a $(k \times n) = [\Phi_{0,} \Phi_{1,} ..., \Phi_p]'$, while the one-step ahead forecast errors u_t have a multivariate $N(0, \Sigma_u)$ distribution conditional on past observations of Y.

2.2 Bayesian VAR

The Bayesian VAR, as described in Litterman (1981), Doan *et al.* (1984), Todd (1984), Litterman (1986) and Spencer (1993) has become a widely popular approach in dealing with overparameterization. One of main problems in using VAR models is that many parameters need to be estimated, although some of them may be insignificant. Instead of eliminating longer lags, the BVAR imposes restrictions on these coefficients by assuming that they are more likely to be near zero than the coefficients on shorter lags. Obviously, if there are strong effects from less important variables, the data can counter this assumption. Usually, the restrictions are imposed by specifying normal prior distributions with zero means and small standard deviations for all coefficients, with a decreasing standard deviation as the lags increase. The only exception is the coefficient on a variable's first lag that has a mean of unity. Litterman (1981) used a diffuse prior for the constant. The means of the prior are popularly called the "Minnesota Priors" due to the development of the idea at the University of Minnesota and the Federal Reserve Bank at Minneapolis. The basic principle behind the "Minnesota" prior is that all equations are centered around a random walk with drift. This idea has been modified by Kadiyala and Karlsson (1997) and Sims and Zha (1998). In Ingram and Whiteman (1994) a real business cycle model is used to generate a prior for a reduced form VAR, as a development of the "Minnesota" priors procedure. Also, a prior is placed on the parameters of a simple linearized DSGE, which is then compared with a Bayesian VAR in a forecasting exercise. Smets and Wouters (2003) extend this to medium scale New Keynesian models used in policy analysis. This approach has the advantage of providing information about which behavioural mechanisms produce forecast error or policy scenarios. However, it seems that it often fails to empirically fit compared to models with no behavioural structure. In Del Negro and Schorfheide (2004) and Del Negro *et al.* (2007) a DSGE prior is also developed for a VAR.

Formally speaking, these prior means can be written as follows

$$\Phi_i \sim N(1, \sigma^2_{\Phi_i}) \text{ and } \Phi_j \sim N(0, \sigma^2_{\Phi_j}), \tag{2}$$

where Φ_i denotes the coefficients associated with the lagged dependent variables in each equation of the VAR, while Φ_j represents any other coefficient. The prior variances $\sigma^2_{\Phi_i}$ and $\sigma^2_{\Phi_j}$ specify the uncertainty of the prior means, $\Phi_i = 1$ and $\Phi_j = 0$, respectively. In this study, we impose their prior mean on the first own lag for variables in growth rate, such as a white noise setting (Del Negro and Schorfheide 2004; Adolfson *et al.* 2007; Banbura *et al.* 2010). Instead, for level variables, we use the classical Minnesota prior (Del Negro and Schorfheide 2004). The specification of the standard deviation of the distribution of the prior imposed on variable j in equation i at lag m, for all i, j and m, denoted by $S(i, j, m)$, is specified as follows

$$S(i, j, m) = [w \times g(m) \times F(i, j)] \frac{\hat{\sigma}_i}{\hat{\sigma}_j}, \tag{3}$$

where

$$F(i, j) = \begin{cases} 1 \text{ if } i = j \\ k_{ij} \text{ otherwise, } 0 \leq k_{ij} \leq 1 \end{cases} \tag{4}$$

is the tightness of variable j in equation i relative to variable i and by increasing the interaction, i.e. it is possible for the value of k_{ij} to loosen the prior (Dua and Ray, 1995). The ratio $\frac{\hat{\sigma}_i}{\hat{\sigma}_j}$ consists of estimated standard errors of the univariate autoregression, for variables i and j. This ratio scales the variables to account for differences in the units of measurement, without taking into account the magnitudes of the variables. The term w measures the standard deviation on the first lag, and also indicates the overall tightness. A decrease in the value of w results in a tighter prior. The function $g(m) = m^{-d}$, $d > 0$ is the measurement of the tightness on lag m relative to lag 1, and is assumed to have a harmonic shape with a decay of d, which tightens the prior on increasing lags. Following the standard Minnesota prior settings, we choose the overall tightness (w) to be equal to 0.3, while the lag decay (d) is 1 and the interaction parameter (k_{ij}) is set equal to 0.5.

2.3 Factor Augmented VAR

A recent strand in the econometric literature mainly by Stock and Watson (2002), Forni and Reichlin (1996, 1998) and Forni *et al.* (1999, 2000) has shown that very large macroeconomic datasets can be properly modelled using dynamic factor models, where the factors can be considered as an "exhaustive summary of the information " in the data. The rationale underlying dynamic factor models is that the behavior of several variables is driven by few common forces, the factors, plus idiosyncratic shocks. Hence, the factors-approach can be useful in alleviating the omitted variable problem in empirical analysis using traditional small scale models. Bernanke and Boivin (2003) and Bernanke *et al.* (2005) utilized factors in the estimation of VAR to generate a more general specification. Chudik and Pesaran (2011) illustrated how a VAR augmented by factors could help in keeping the number of estimated parameters under control without loosing relevant information.

Let $\mathbf{X}_t$ denote an $N \times 1$ vector of economic time series and $\mathbf{Y}_t$ a vector of $M \times 1$ observable macroeconomic variables which are a subset of $\mathbf{X}_t$. In this context, most of the information contained in $\mathbf{X}_t$ is captured by $\mathbf{F}_t$, a $k \times 1$ vector of unobserved factors. The factors are interpreted as an addition to the observed variables, as common forces driving the dynamics of the economy. The relation between the "informational" time series $\mathbf{X}_t$, the observed variables $\mathbf{Y}_t$ and the factors $\mathbf{F}_t$ is represented by the following dynamic factor model

$$\mathbf{X}_t = \mathbf{\Lambda}^f \mathbf{F} + \mathbf{\Lambda}^y \mathbf{Y}_t + e_t, \tag{5}$$

where $\mathbf{\Lambda}^f$ is a $N \times k$ matrix of factor loadings, $\mathbf{\Lambda}^y$ is a $N \times M$ matrix of coefficients that bridge the observable $\mathbf{Y}_t$ and the macroeconomic dataset, and e_t is the vector of $N \times 1$ error terms. These terms are mean zero, normal distributed, and uncorrelated with a small cross-correlation. In fact, the estimator allows for some cross-correlation in e_t that must vanish as N goes to infinity. This representation nests also models where $\mathbf{X}_t$ depends on lagged values of the factors (Stock and Watson, 2002).

For the estimation of the FAVAR model equation (5), we follow the two-step principal components approach proposed by Bernanke *et al.* (2005). In the first step factors are obtained from the observation equation by imposing the orthogonality restriction $\mathbf{F}'\mathbf{F}/T = \mathbf{I}$. This implies that $\hat{\mathbf{F}} = \sqrt{T}\hat{\mathbf{G}}$, where $\hat{\mathbf{G}}$ are the eigenvectors corresponding to the K largest eigenvalues of $\mathbf{X}\mathbf{X}^{'}$, sorted in descending order. Stock and Watson (2002) showed that the factors can be consistently estimated by the first r principal components of $\mathbf{X}$, even in the presence of moderate changes in the loading matrix $\mathbf{\Lambda}$. For this result to hold it is important that the estimated number of factors, k, is larger or equal than the true number r. Bai and Ng (2000) proposed a set of selection criteria to choose k that are generalizations of the BIC and AIC criteria. In the second step, we estimate the FAVAR equation replacing $\mathbf{F}_t$ by $\hat{\mathbf{F}}_t$. Following Bernanke *et al.* (2005), $\mathbf{Y}_t$ is removed from the space covered by the principal components. In a recent paper, Boivin *et al.* (2009) impose the constraint that $\mathbf{Y}_t$ is one of the common components in the first step, guaranteeing that the estimated latent factors $\hat{\mathbf{F}}_t$ recover the common dynamics which are not captured by $\mathbf{Y}_t$. The authors, comparing the two methodologies, concluded that the results are similar. As in Bernanke *et al.* (2005) we partition the matrix $\mathbf{X}_t$ in two categories of information variables: slow-moving and fast-moving. Slow-moving variables (e.g., real variables such as wages or spending) do not respond contemporaneously to unanticipated changes in monetary policy, while fast-moving (e.g., interest rates) respond contemporaneously to monetary shocks. We proceed to extracting two factors from slow variables and one factor from fast variables and we call them respectively "slow factors" and "fast factor". As suggested by Bai and Ng (2000) we use information criteria to determine the number of factors, extracting three factors (two slows and one fast) to strike a balance between the dimension of the panel data (112 series) and the parameters estimated in the VAR and FAVAR (number of endogenous variables and their lags). It is also worth noting that the factors are not uniquely identified, but this is not a problem in our context because we will not attempt a structural interpretation of the estimated factors. Finally, having determined the number of factors, we specify a Factor Augmented VAR by considering only one-lag of the factors according to BIC criterion. The potential identification of the macroeconomic shocks can be performed according to Bernanke *et al.* (2005) using the Cholesky decomposition.

3 DSGE Models

Only recently and after the seminal work of Smets and Wouters (2003, 2004) the DSGE models have been considered as forecasting tools in macroeconomic literature. Model validation, estimation and calibration are crucial issues in DSGE structure. The main problems are reported in Canova (1994). Calibrated DSGE models often yield fragile results when traditional econometric methods are used for estimation (Smets and Wouters 2003; Ireland 2004). Following this idea of combining the DSGE model information and the VAR representation, among other models that have been proposed in the literature, in this study we use the DSGE-VAR and DSGE-FAVAR hybrid models. Furthermore, we use comparatively the small scale model of Del Negro and Schorfheide (2004) including the real GDP, the harmonized Consumer Price Index and the nominal short-term federal funds interest rate, as well as the medium scale model of Smets and Wouters (2007) which features sticky nominal price, wage contracts, habit formation, variable capital utilization and investment adjustment costs. The Smets and Wouters (2007) model resembles that of Christiano *et al.* (2005) on US macroeconomic data.

3.1 Small Scale Model

Simple DSGE models with forward-looking features are usually referred to as a benchmarks in the literature. In a DSGE setup the economy is made up of four components. The first component is the representative household with habit persistent preferences. This household maximizes an additively separable utility function which is separable into consumption, real money balances and hours worked over an infinite lifetime. The household gains utility from consumption relative to the level of technology, real balances of money, and disutility from hours worked. The household earns interest from holding government bonds and earns real profits from the firms. Moreover, the representative household pays lump-sum taxes to the government. The second component is a perfectly competitive, representative final goods producer which is assumed to use a continuum of intermediate goods as inputs, and the prices for these inputs are given. The producers of these intermediate goods are monopolistic firms which use labour as the only input. The production technology is the same for all the monopolistic firms. Nominal rigidities are introduced in terms of price adjustment costs for the monopolistic firms. Each firm maximizes its profits over an infinite lifetime by choosing its labour input and its price. The third component is the government which spends in each period a fraction of the total output, which fluctuates exogenously. The government issues bonds and levies lump-sum taxes, which are the main part of its budget constraint. The last component is the monetary authority, which follows a Taylor rule regarding the inflation target and the output gap. There are three economic shocks: an exogenous monetary policy shock (in the monetary policy rule), and two autoregressive processes, AR(1), which model government spending and technology shocks. To solve the model, optimality conditions are derived for the maximization problems. After linearization around the steady-state, the economy is described by the following system of equations

$$\tilde{x}_t = E_t[\tilde{x}_{t+1}] - \frac{1}{\tau}(\tilde{R}_t - E_t[\tilde{\pi}_{t+1}]) + (1-\rho_g)\tilde{g}_t + \rho_Z \frac{1}{\tau}\tilde{z}_t \tag{6}$$

$$\tilde{\pi}_t = \beta E_t[\tilde{\pi}_{t+1}] + \kappa[\tilde{x}_t - \tilde{g}_t] \tag{7}$$

$$\tilde{R}_t = \rho_R \tilde{R}_{t-1} + (1-\rho_R)(\psi_1\tilde{\pi}_t + \psi_2\tilde{x}_t) + \epsilon_{R,t} \tag{8}$$

$$\tilde{g}_t = \rho_g \tilde{g}_{t-1} + \epsilon_{g,t} \tag{9}$$

$$\tilde{z}_t = \rho_z \tilde{z}_{t-1} + \epsilon_{z,t}, \tag{10}$$

where x is the detrended output (divided by the non-stationary technology process), π is the gross inflation rate, and R is the gross nominal interest rate. The tilde denotes percentage deviations from a steady state or, in the case of output, from a trend path (King, 2000; Woodford, 2003). The model can be solved by applying the algorithm proposed by Sims (2002). Define the vector of variables $\tilde{Z}_t = \left(\tilde{x}_t, \tilde{\pi}_t, \tilde{R}_t, \tilde{g}_t, \tilde{z}_t, E_t\tilde{x}_{t+1}, E_t\tilde{\pi}_{t+1}\right)$ and the vector of shocks as $\epsilon_t = (\epsilon_{R,t}, \epsilon_{g,t}, \epsilon_{z,t})$. Therefore the previous set of equations, (6) - (10), can be recasted into a set of matrices $(\Gamma_0, \Gamma_1, C, \Psi, \Pi)$ accordingly to the definition of the vectors $\tilde{Z}_t$ and ϵ_t

$$\Gamma_0 \tilde{Z}_t = C + \Gamma_1 \tilde{Z}_{t-1} + \Psi\epsilon_t + \Pi\eta_t, \tag{11}$$

where η_{t+1}, such that $E_t\eta_{t+1} \equiv E_t\left(y_{t+1} - E_t y_{t+1}\right) = 0$, is the expectations error.

As a solution to (11), we obtain the following transition equation as a policy function

$$\tilde{Z}_t = T(\theta)\tilde{Z}_{t-1} + R(\theta)\epsilon_t, \tag{12}$$

and in order to provide the mapping between the observable data and those computed as deviations from the steady state of the model we set the following measurement equations as in Del Negro and Schorfheide (2004)

$$\begin{aligned} \Delta \ln x_t &= \ln\gamma + \Delta\tilde{x}_t + \tilde{z}_t \\ \Delta \ln P_t &= \ln\pi^* + \tilde{\pi}_t \\ \ln R_t^a &= 4\left[(\ln r^* + \ln\pi^*) + \tilde{R}_t\right], \end{aligned} \tag{13}$$

which can be also casted into matrices as

$$Y_t = \Lambda_0(\theta) + \Lambda_1(\theta)\tilde{Z}_t + v_t, \tag{14}$$

where $Y_t = (\Delta\ln x_t, \Delta\ln P_t, \ln R_t^a)'$, $v_t = 0$ and Λ_0 and Λ_1 are defined accordingly. For completeness, we write the matrices T, R, Λ_0 and Λ_1 as a function of the structural parameters in the model, $\theta = \left(\ln\gamma, \ln\pi^*, \ln r^*, \kappa, \tau, \psi_1, \psi_2, \rho_R, \rho_g, \rho_Z, \sigma_R, \sigma_g, \sigma_Z\right)'$. Such a formulation derives from the rational expectations solution. The evolution of the variables of interest, Y_t, is therefore determined by (12) and (14) which impose a set of restrictions across the parameters on the moving average (MA) representation. Given that the MA representation can be very closely approximated by a finite order VAR representation, Del Negro and Schorfheide (2004) propose to evaluate the DSGE model by assessing the validity of the restrictions imposed by such a model with respect to an unrestricted VAR representation. The choice of the variables to be included in the VAR is however completely driven by those entering in the DSGE model regardless of the statistical goodness of the unrestricted VAR.

3.2 Medium Scale Model

The Smets and Wouters (2007) model is a medium scale model which features sticky nominal price and wage contracts, habit formation, variable capital utilization and investment adjustment costs. The demand side of the economy consists of consumption (c_t), investment (i_t), capital utilization (z_t) and government spending $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \sigma_g \eta_t^g + \rho_{ga} \eta_t^a$ that is assumed to be exogenous. The total output (y_t) is given by

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g, \tag{15}$$

where c_y is the steady-state share of consumption in output and equals $(1 - g_y - i_y)$, and g_y and i_y are respectively the steady-state exogenous spending-output ratio and investment-output. Also $z_y = R_*^k k_y$, where R_*^k, is the steady-state rental rate of capital and k_y is the steady-state capital-output ratio. The consumption Euler equation is provided by

$$\begin{aligned} c_t \;=\; & \frac{\lambda/\gamma}{1+\lambda/\gamma} c_{t-1} + \left(1 - \frac{\lambda/\gamma}{1+\lambda/\gamma}\right) E_t c_{t+1} + \frac{(\sigma_c - 1)\left(W_*^h L_* / C_*\right)}{\sigma_c (1+\lambda/\gamma)} (l_t - E_t l_{t+1}) - \\ & \frac{(1-\lambda/\gamma)}{\sigma_c (1+\lambda/\gamma)} (r_t - E_t \pi_{t+1} + \varepsilon_t^b), \end{aligned} \tag{16}$$

where l_t is the hours worked, r_t is the nominal interest rate and π_t is the rate of inflation. If the degree of habits is zero ($\lambda = 0$) and $\sigma_c = 1$, equation (16) reduces to the standard forward-looking consumption Euler equation. The disturbance is assumed to follow a first-order autoregressive process with an iid-Normal error term: $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$

The linearized investment equation is given by

$$i_t = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}} i_{t-1} + \left(1 - \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\right) E_t i_{t+1} + \frac{1}{(1+\beta\gamma^{(1-\sigma_c)})\gamma^2 \varphi} q_t + \varepsilon_t^i, \tag{17}$$

with i_t denoting the investment and q_t the real value of existing capital stock (Tobin's Q). φ is the steady-state elasticity of the capital adjustment cost function and β is the discount factor applied by households. The investment-specific technology process follows a first-order autoregressive process with an iid-Normal error term: $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$. The arbitrage equation for the value of capital is given by

$$q_t = \beta\gamma^{-\sigma}(1-\delta) E_t q_{t+1} + (1 - \beta\gamma^{-\sigma}(1-\delta)) E_t r_{t+1}^k - (r_t - E_t \pi_{t+1} + \varepsilon_t^b), \tag{18}$$

where $r_t^k = -(k_t - l_t) + w_t$ denotes the real rental rate of capital which is negatively related to the capital-labour ratio and positively to the real wage.

On the supply side of the economy, the aggregate production function is defined as

$$y_t = \phi_p (\alpha k_t^s + (1-\alpha) l_t + \varepsilon_t^a), \tag{19}$$

where ϕ_p and α are respectively one plus the share of fixed costs in production and the share of capital in production. The total factor productivity follows a first-order autoregressive process: $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$. The k_t^s represents capital services which is a linear function of lagged installed capital (k_{t-1}) and the degree of capital utilization, $k_t^s = k_{t-1} + z_t$. Capital utilization is proportional to the real rental rate of capital, $z_t = \frac{1-\Psi}{\Psi} r_t^k$, where Ψ is a positive function of the elasticity of the capital utilization adjustment cost function and normalized from zero (in equilibrium the rental rate on capital is constant) to one (the utilization of capital is constant). The accumulation process of installed capital is simply described as

$$k_t = \frac{1-\delta}{\gamma} k_{t-1} + \frac{\gamma - 1 + \delta}{\gamma} i_t + \left(1 - \frac{(1-\delta)}{\gamma}\left(1 + \beta\gamma^{(1-\sigma_c)}\gamma^2\varphi\right)\right) \varepsilon_t^i. \tag{20}$$

Monopolistic competition within the production sector and Calvo-pricing constraints gives the New-Keynesian

Phillips curve for inflation

$$\pi_t = \frac{\iota_p}{1+\beta\gamma^{(1-\sigma_c)}\iota_p}\pi_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}\iota_p}E_t\pi_{t+1} - \frac{1}{1+\beta\gamma^{(1-\sigma_c)}\iota_p}\frac{1-\beta\gamma^{(1-\sigma_c)}\xi_p(1-\xi_p)}{\big(\xi_p(\phi_p-1)\varepsilon_p+1\big)}\mu_t^p + \varepsilon_t^p, \quad (21)$$

where $\mu_t^p = \alpha(k_t^s - l_t) - w_t$ is the marginal cost of production and the price mark-up disturbance follows an ARMA(1,1) process $\varepsilon_t^p = \rho_p\varepsilon_{t-1}^p + \eta_t^p - \mu_p\eta_{t-1}^p$, where η_t^p is an iid-Normal price mark-up shock. The MA(1) term is included to capture the high-frequency fluctuations in inflation. If the degree of indexation to past inflation is zero, $\iota_p = 0$, the equation (21) becomes a standard forward-looking Phillips curve. The speed of adjustment depends on the degree of price stickiness (ξ_p), the curvature of the Kimball goods market aggregator (ε_p), and the steady-state mark-up which is related in equilibrium to ($\phi_p - 1$), the share of fixed costs in production. Monopolistic competition in the labour market also gives rise to a similar wage New-Keynesian Phillips curve

$$\begin{aligned} w_t \quad = \quad & \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}w_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}}\left(E_t w_{t+1} - E_t\pi_{t+1}\right) - \frac{1+\beta\gamma^{(1-\sigma_c)}\iota_w}{1+\beta\gamma^{(1-\sigma_c)}}\pi_t \\ & + \frac{\iota_w}{1+\beta\gamma^{(1-\sigma_c)}}\pi_{t-1} - \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\frac{1-\beta\gamma^{(1-\sigma_c)}\xi_w(1-\xi_w)}{(\xi_w(\phi_w-1)\varepsilon_w+1)}\mu_t^w + \varepsilon_t^w, \end{aligned} \quad (22)$$

where $\mu_t^w = w_t - \sigma_l l_t + \frac{1}{1-\lambda/\gamma}\left(c_t - \lambda/\gamma c_{t-1}\right)$ is the households' marginal benefit of supplying an extra unit of labour service and the wage mark-up shock is an ARMA(1,1) process, $\varepsilon_t^w = \rho_w\varepsilon_{t-1}^w + \eta_t^w - \mu_w\eta_{t-1}^w$,where η_t^w is an iid-Normal error term. Again, the MA(1) term captures the high-frequency fluctuations in wages. If the degree of indexation to past inflation is zero, $\iota_w = 0$, the equation (22) does not depend on lagged inflation. The speed of adjustment depends on the degree of wage stickiness (ξ_w), the curvature of the Kimball labour market aggregator (ε_w), and the steady-state labour market mark-up ($\phi_w - 1$).

The model is closed by the empirical monetary policy reaction function

$$r_t = \rho r_{t-1} + (1-\rho)\left[r_\pi\pi_t + r_Y(y_t - y_t^p)\right] + r_{\Delta y}\left[(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)\right] + \varepsilon_t^r, \quad (23)$$

where y_t^p is the flexible price level of output and $\varepsilon_t^r = \rho_r\varepsilon_{t-1}^r + \eta_t^r$ follows a first-order autoregressive process with an iid-Normal error term.

Equations (15) to (23) determine 14 endogenous variables: (y_t, c_t, i_t, q_t, k_t^s, k_t, z_t, r_t^k, μ_t^p, μ_t^w, π_t, w_t, l_t, r_t). The stochastic behaviour of the system of linear rational expectations equations is driven by 7 exogenous disturbances: total factor productivity (ε_t^a), investment-specific technology (ε_t^i), risk-premium (ε_t^b), exogenous spending (ε_t^g), price mark-up (ε_t^p), wage mark-up (ε_t^w), and monetary policy shock (ε_t^r). The model can be solved by applying the algorithm proposed by Sims (2002). As reported in Chib and Ramamurthy (2010), the vector of states is 53-dimensional, given the sticky price-wage and flexible price-wage settings (in asterisks): $\tilde{Z}_t = (y_t, k_t^s, l_t, r_t^k, w_t, \pi_t, \mu_t^p, c_t, r_t, z_t, q_t, i_t, k_t, \mu_t^w, E_t\pi_{t+1}, E_tc_{t+1}, E_tl_{t+1}, E_tq_{t+1}, E_tr_{t+1}^k, E_ti_{t+1}, E_tw_{t+1}, y_{t-1}, c_{t-1}, i_{t-1}, w_{t-1}, u_t^a, u_t^b, u_t^g, u_t^i, u_t^r, u_t^p, u_t^w, \varepsilon_t^p, \varepsilon_t^w, y_t^*, k_t^{s*}, l_t^*, r_t^{k*}, w_t^*, \pi_t^*, \mu_t^{p*}, c_t^*, r_t^*, z_t^*, q_t^*, i_t^*, k_t^*, \mu_t^{w*}, E_tc_{t+1}^*, E_tl_{t+1}^*, E_tq_{t+1}^*, E_tr_{t+1}^{k*}, E_ti_{t+1}^*, y_{t-1}^*)$.The vector of innovations is $\epsilon_t = \left(\varepsilon_t^a,\ \varepsilon_t^i,\ \varepsilon_t^b,\ \varepsilon_t^g,\ \varepsilon_t^p,\ \varepsilon_t^w,\ \varepsilon_t^r\right)$ and the vector of the endogenous rational expectations errors is $\eta_t = (\pi_t - E_{t-1}\pi_t, c_t - E_{t-1}c_t, l_t - E_{t-1}l_t, q_t - E_{t-1}q_t, r_t^k - E_{t-1}r_t^k, i_t - E_{t-1}i_t, w_t - E_{t-1}w_t, c_t^* - E_{t-1}c_t^*, l_t^* - E_{t-1}l_t^*, q_t^* - E_{t-1}q_t^*, r_t^{k*} - E_{t-1}r_t^{k*}, i_t^* - E_{t-1}i_t^*)$.

Therefore the previous set of equations, (15) - (23), can be recasted into a set of matrices $(\Gamma_0, \Gamma_1, C, \Psi, \Pi)$

accordingly to the definition of the vectors $\tilde{Z}_t$ and ϵ_t

$$\Gamma_0 \tilde{Z}_t = C + \Gamma_1 \tilde{Z}_{t-1} + \Psi \epsilon_t + \Pi \eta_t. \tag{24}$$

As a solution, we obtain the following transition equation as a policy function

$$\tilde{Z}_t = T(\theta)\, \tilde{Z}_{t-1} + R(\theta)\, \epsilon_t, \tag{25}$$

and in order to provide the mapping between the observable data and those computed as deviations from the steady state of the model we set the following measurement equations as

$$Y_t = \begin{bmatrix} \Delta \ln y_t \\ \Delta \ln c_t \\ \Delta \ln i_t \\ \Delta \ln w_t \\ \ln l_t \\ \Delta \ln P_t \\ \ln R_t^a \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \bar{l} \\ \overline{\pi} \\ \overline{r} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ l_t \\ \pi_t \\ r_t \end{bmatrix},$$

where ln denotes 100 times log and $\Delta \ln$ refers to the log difference. $\overline{\gamma} = 100(\gamma - 1)$ is the common quarterly trend growth rate to real GDP, consumption, investment and wages. Instead, $\overline{\pi} = 100(\Pi_* - 1)$ is quarterly steady-state inflation rate, $\overline{r} = 4 * 100(\beta^{-1}\gamma^{\sigma_c}\Pi_* - 1)$ is the steady-state nominal interest rate, and $\bar{l}$ is the steady-state hours worked, which is normalized to be equal to zero. We can write the following equation

$$Y_t = \Lambda_0(\theta) + \Lambda_1(\theta)\, \tilde{Z}_t + v_t, \tag{26}$$

where $Y_t = (\Delta \ln y_t, \Delta \ln c_t, \Delta \ln i_t, \Delta \ln w_t, \ln l_t, \Delta \ln P_t, \ln R_t^a)'$, $v_t = 0$ and Λ_0 and Λ_1 are defined accordingly. For completeness, we write the matrices T, R, Λ_0 and Λ_1 as a function of the structural parameters in the model.

3.3 Estimation of linearized DSGE Models

Several econometric procedures have been proposed to parameterize and evaluate DSGE models. Kydland and Prescott (1982) use calibration, Christiano and Eichenbaum (1992) consider the generalized method of moments (GMM) estimation of equilibrium relationships, while Rotemberg and Woodford (1997) and Christiano *et al.* (2005) use the minimum distance estimation based on the discrepancy among VAR and DSGE impulse response functions. Moreover the full-information likelihood-based estimation is considered by Altug (1989), McGrattan (1994), Leeper and Sims (1994) and Kim (2000). In recent years, Bayesian estimation became very popular. According to An and Schorfheide (2007) there are essentially three main characteristics. Firstly, the Bayesian estimation is system-based and fits the solved DSGE model to a vector of aggregate time series, as opposed to the GMM which is based on equilibrium relationships, such as the Euler equation for the consumption or the monetary policy rule. Secondly, it is based on the likelihood function generated by the DSGE model rather than the discrepancy between DSGE responses and VAR impulse responses. Thirdly, prior distributions can be used to incorporate additional information into the parameter estimation.

Priors distributions are important to estimate DSGE models. According to An and Schorfheide (2007)

Table 1: Prior Distributions for the small scale DSGE model parameters

Name	Density	Starting value	Mean	Standard deviation
$\ln\gamma$	Normal	0.500	0.500	0.250
$\ln\pi^*$	Normal	1.000	1.000	0.500
$\ln r^*$	Gamma	0.500	0.500	0.250
κ	Gamma	0.040	0.030	0.150
τ	Gamma	3.000	3.000	0.500
ψ_1	Gamma	1.500	1.500	0.250
ψ_2	Gamma	0.300	0.125	0.100
ρ_R	Beta	0.400	0.500	0.200
ρ_G	Beta	0.800	0.800	0.100
ρ_Z	Beta	0.200	0.200	0.100
σ_R	Inv.Gamma	0.100	0.100	0.139
σ_G	Inv.Gamma	0.300	0.350	0.323
σ_Z	Inv.Gamma	0.400	0.875	0.430

Note: The model parameters $\ln\gamma$, $ln\pi^*$, $\ln r^*$, σ_R, σ_g, and σ_z are scaled by 100 to convert them into percentages. The Inverse Gamma priors are of the form $p(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$, where v=4 and s equals 0.2, 0.5, and 0.7, respectively. Approximately 1.5% of the prior mass lies in the indeterminacy region of the parameter space. The prior is truncated to restrict it to the determinacy region of the DSGE model, to avoid multiple equilibria typical in rational expectations models .

priors might downweigh regions of the parameter space that are at odds with observations which are not contained in the estimation sample. Priors could add curvature to a likelihood function that is (nearly) flat for some parameters, given a strong influence to the shape of the posterior distribution. Table 1 lists the prior distributions for the structural parameters of the DSGE model which are adopted from Del Negro and Schorfheide (2004). Next, Table 2 reports the priors for the medium scale model following Smets and Wouters (2007).

In the Bayesian framework, the likelihood function is reweighted by a prior density as in An and Schorfheide (2007) and Fiorentini *et. al.* (2012). The prior is useful to add information which is contained in the estimation sample. Since priors are always subject to revisions, the shift from prior to posterior distribution can be considered as an indicator of the different sources of information. If the likelihood function peaks at a value that is at odds with the information that has been used to construct the prior distribution, then the marginal data density (MDD) of the DSGE model is defined as

$$p(Y) = \int L(\theta|Y)p(\theta)d\theta$$

The marginal data density is the integral of the likelihood ($L(\theta|Y)$) taken according to the prior distribution ($p(\theta)$), that is the weighted average of likelihood where the weights are given by priors. The MDD can be used to compare different models M_i, $p(Y|M_i)$.We can rewrite the log-marginal data density as

Table 2: Prior Distributions for the medium scale DSGE model parameters

Name	Density	Starting value	Mean	Standard deviation
φ	Normal	5.70	4	1.50
σ_c	Normal	1.35	1.50	0.37
h	Beta	0.70	0.70	0.10
ξ_w	Beta	0.70	0.50	0.10
σ_l	Normal	1.80	2.00	0.75
ξ_p	Beta	0.65	0.50	0.10
ι_w	Beta	0.44	0.50	0.15
ι_p	Beta	0.33	0.50	0.15
ψ	Beta	0.55	0.50	0.15
ϕ	Normal	1.55	1.25	0.12
r_π	Normal	1.80	1.50	0.25
ρ	Beta	0.88	0.75	0.10
r_y	Normal	0.08	0.12	0.05
$r_{\Delta y}$	Normal	0.22	0.12	0.05
$\overline{\pi}$	Gamma	0.70	0.62	0.10
$100(\beta^{-1}-1)$	Gamma	0.20	0.25	0.10
$\bar{l}$	Normal	1.30	0.00	2.00
$\overline{\gamma}$	Normal	0.40	0.40	0.10
α	Normal	0.24	0.30	0.05
σ_a	Inv. Gamma	0.46	0.10	2.00
σ_b	Inv. Gamma	0.18	0.10	2.00
σ_g	Inv. Gamma	0.61	0.10	2.00
σ_l	Inv. Gamma	0.46	0.10	2.00
σ_r	Inv. Gamma	0.24	0.10	2.00
σ_p	Inv. Gamma	0.15	0.10	2.00
σ_w	Inv. Gamma	0.21	0.10	2.00
ρ_a	Beta	0.97	0.50	0.20
ρ_b	Beta	0.27	0.50	0.20
ρ_g	Beta	0.99	0.50	0.20
ρ_l	Beta	0.57	0.50	0.20
ρ_r	Beta	0.30	0.50	0.20
ρ_p	Beta	0.87	0.50	0.20
ρ_w	Beta	0.95	0.50	0.20
μ_p	Beta	0.77	0.50	0.20
μ_w	Beta	0.89	0.50	0.20
ρ_{ga}	Beta	0.60	0.50	0.20

Notes: The following parameters are fixed in Smets and Wouters (2007): δ= 0.025, $\varepsilon_w = 10.0$, and ε_p= 10. The effective prior is truncated at the boundary of the determinacy region.

$$
\begin{aligned}
\ln(p(Y|M)) &= \sum_{t=1}^{T} \ln p(y_t|Y^{t-1}, M) \\
&= \sum_{t=1}^{T} \ln \left[\int p(y_t|Y^{t-1}, \theta, M) p(\theta|Y^{t-1}, M) d\theta \right],
\end{aligned}
$$

where $\ln(p(Y|M)$ can be interpreted as a predictive score (Good, 1952) and the model comparison based on posterior odds captures the relative one-step-ahead predictive performance. To compute the MDD, we consider the Geweke (1999) modified harmonic mean estimator. Harmonic mean estimators are based on the identity

$$
\frac{1}{p(Y)} = \int \frac{f(\theta)}{L(\theta|Y)p(\theta)} p(\theta|Y) d\theta,
$$

where $f(\theta)$ has the property that $\int f(\theta) d\theta = 1$ (Gelfand and Dey, 1994). Conditional on the choice of $f(\theta)$, an estimator is

$$
\widehat{p}_G(Y) = \left[\frac{1}{n_{sim}} \sum_{s=1}^{n_{sim}} \frac{f(\theta^{(s)})}{L(\theta^{(s)}|Y)p(\theta^{(s)})} \right]^{-1}, \tag{27}
$$

where $\theta^{(s)}$ is drawn from the posterior $p(\theta|Y)$. For a numerical approximation efficient, $f(\theta)$ should be chosen so that the summands are of equal magnitude. Geweke (1999) proposed to use the density of a truncated multivariate normal distribution

$$
\begin{aligned}
f(\theta) &= \tau^{-1}(2\pi)^{-\frac{d}{2}} |V_\theta|^{-\frac{1}{2}} \exp\left[-0.5(\theta - \overline{\theta})' V_\theta^{-1} (\theta - \overline{\theta})\right] \\
&\quad \times I \left\{ (\theta - \overline{\theta})' V_\theta^{-1} (\theta - \overline{\theta}) \leq F_{\chi_d^2}^{-1}(\tau) \right\}.
\end{aligned}
$$

As reported in An and Schorfheide (2007) rejoinder (excluding Lubik and Schorfheide, 2004), researchers tend to restrict the parameter space to the subspace in which the linearized DSGE model has a unique rational expectations solutions. We follow the adjustment proposed by Del Negro and Schorfheide (2004) and An and Schorfheide (2007b), with a small percentage (around 1.5%-2%) to the indeterminacy region. In the above $\overline{\theta}$ and V_θ are the posterior mean and covariance matrix computed from the output of the posterior simulator, d is the dimension of the parameter vector, $F_{\chi_d^2}$ is the cumulative density function of a χ^2 random variable with d degrees of freedom, and $\tau \in (0, 1)$. We set $\tau = 0.90$ which provides the most accurate computation as in Geweke (1999). We compute the log-marginal data density with $\tau = 0.95$ and $\tau = 0.99$, founding that the marginal likelihood is not sensitive to the value of τ. As shown in Schorfheide (2000), the estimated marginal likelihood does not so serially depend on τ, and the discrepancy of eq (27) across truncation levels τ and simulation runs is less than 0.60. If the posterior of θ is in fact normal then the summands in eq. (27) are approximately constant.

3.4 DSGE-VAR

Building on the work by Ingram and Whiteman (1994), the DSGE-VAR approach of Del Negro and Schorfheide (2004) was designed to improve forecasting and monetary policy analysis with VARs. Del Negro-Schorfheide's

(2004) approach is to use the DSGE model to build prior distributions for the VAR. This approach is employed both for the small and the medium scale DSGE model. Basically, the estimation initializes with an unrestricted VAR of order p

$$\mathbf{Y}_t = \mathbf{\Phi}_0 + \mathbf{\Phi}_1 \mathbf{Y}_{t-1} + ... + \mathbf{\Phi}_p \mathbf{Y}_{t-p} + \mathbf{u}_t. \tag{28}$$

In compact format

$$\mathbf{Y} = \mathbf{X\Phi} + \mathbf{U}, \tag{29}$$

$\mathbf{Y}$ is a $(T \times n)$ matrix with rows Y_t', $\mathbf{X}$ is a $(T \times k)$ matrix $(k = 1 + np, p =$number of lags) with rows $X_t' = [1, Y_{t-1}', ..., Y_{t-p}']$, $\mathbf{U}$ is a $(T \times n)$ matrix with rows u_t' and Φ is a $(k \times n) = [\Phi_{0,}\Phi_{1,}..., \Phi_p]'$. The one-step-ahead forecast errors u_t have a multivariate normal distribution $N(0, \Sigma_u)$ conditional on past observations of Y. The log-likelihood function of the data is a function of Φ and Σ_u

$$p(Y|\Phi, \Sigma_u) \propto |\Sigma_u|^{-\frac{T}{2}} \exp\left\{-\frac{1}{2}tr\left[\Sigma_u^{-1}\left(\mathbf{Y'Y} - \mathbf{\Phi'X'Y} - \mathbf{Y'X\Phi} + \mathbf{\Phi'X'X\Phi}\right)\right]\right\}. \tag{30}$$

The prior distribution for the VAR parameters proposed by Del Negro and Schorfheide (2004) is based on the statistical representation of the DSGE model given by a VAR approximation. Suppose that the actual observations are augmented with $T^* = \lambda T$ artificial observations (Y^*, X^*) generated from the DSGE model based on the parameter vector θ. The log-likelihood function for the combined sample of artificial and actual observations is obtained by premultiplying (30) with

$$p(\mathbf{Y}^*(\theta)|\Phi, \Sigma_u) \propto |\Sigma_u|^{-\lambda\frac{T}{2}} \exp\left\{-\frac{1}{2}tr\left[\Sigma_u^{-1}\left(\mathbf{Y}^{*\prime}\mathbf{Y}^* - \mathbf{\Phi}'\mathbf{X}^{*\prime}\mathbf{Y}^* - \mathbf{Y}^{*\prime}\mathbf{X}^*\mathbf{\Phi} + \mathbf{\Phi}'\mathbf{X}^{*\prime}\mathbf{X}^*\mathbf{\Phi}\right)\right]\right\}. \tag{31}$$

To remove the stochastic variation in the prior distribution from $p(Y^*(\theta)|\Phi, \Sigma_u)$, the nonstandardized sample moments $Y^{*\prime}Y$, $X^{*\prime}Y^*$, and $X^{*\prime}X^*$ are replaced by their expected values. Let Γ_{xx}^*, Γ_{yy}^*, Γ_{xy}^* and Γ_{yx}^* be the theoretical second-order moments of the variables Y and X implied by the DSGE model. Using the population moments, equation (31) is replaced with

$$p(\Phi, \Sigma_u|\theta) = c^{-1}(\theta)|\Sigma_u|^{-\frac{\lambda T+n+1}{2}} \exp\left\{-\frac{1}{2}tr\left[\lambda\Sigma_u^{-1}\left(\Gamma_{yy}^*(\theta) - \mathbf{\Phi}'\Gamma_{xy}^*(\theta) - \Gamma_{yx}^*(\theta)\mathbf{\Phi} + \mathbf{\Phi}'\Gamma_{xx}^*(\theta)\mathbf{\Phi}\right)\right]\right\}, \tag{32}$$

where an initial improper prior $p(\Phi, \Sigma_u) \propto |\Sigma_u|^{-\frac{(n+1)}{2}}$ is also added as shown in Del Negro and Schorfheide (2004). It is also true that we could have chosen a proper but "economics-free" prior on the VAR, as suggested by Sims (2007), e.g., some version of "Minnesota prior". However, in this study we followed the literature on DSGE-VARs with improper priors (e.g., as in Del Negro and Schorfheide, 2004). Provided that $\lambda T \geqslant k + n$ and $\Gamma_{xx}^*(\theta)$ is invertible, the prior density is proper and nondegenerate. Del Negro and Schorfheide (2004) show how the normalization factor $c(\theta)$ can be chosen to ensure that the density integrates to one. Hence, it can be defined as

$$\begin{aligned}\Phi^*(\theta) &= \Gamma_{xx}^{*-1}(\theta)\Gamma_{xy}^*(\theta) \\ \Sigma^*(\theta) &= \Gamma_{yy}^*(\theta) - \Gamma_{yx}^*(\theta)\Gamma_{xx}^{*-1}(\theta)\Gamma_{xy}^*(\theta).\end{aligned} \tag{33}$$

The moments are the dummy observation priors used in the mixture model. These vectors can be interpreted as the probability limits of the coefficients in a VAR estimated on the artificial observations generated by the DSGE model. Conditional on the vector of structural parameters in the DSGE model θ, the prior distributions for the VAR parameters $p(\Phi, \Sigma_u|\theta)$ are of the Inverted-Wishart (IW) and Normal forms

$$\begin{aligned} \Sigma_u \,|\theta &\sim IW\left((\lambda T\Sigma_u^* (\theta), \lambda T - k, n\right) \\ \mathbf{\Phi} \,|\Sigma_u, \theta &\sim N\left(\mathbf{\Phi}^* (\theta), \Sigma_u \otimes (\lambda T\Gamma_{XX} (\theta))^{-1}\right), \end{aligned} \tag{34}$$

where the parameter λ controls the degree of model misspecification with respect to the VAR: for small values of λ the discrepancy between the VAR and the DSGE-VAR is large and a sizeable distance is generated between the unrestricted VAR and DSGE estimators. Large values of λ correspond to small model misspecification and for $\lambda = \infty$ beliefs about DSGE misspecification degenerate to a point mass at zero. Bayesian estimation could be interpreted as estimation based on a sample in which data are augmented by a hypothetical sample in which observations are generated by the DSGE model, the so-called dummy prior observations (Theil and Goldberg, 1961; Ingram and Whiteman, 1994). Within this framework λ determines the length of the hypothetical sample. The posterior distributions of the VAR parameters are also of the Inverted-Wishart and Normal forms. Given the prior distribution, posterior distributions are derived by the Bayes theorem

$$\Sigma_u \,|\theta, \mathbf{Y} \sim IW\left((\lambda+1)\, T\hat{\Sigma}_{u,b} (\theta), (\lambda+1)\, T - k, n\right) \tag{35}$$

$$\mathbf{\Phi} \,|\Sigma_u, \theta, \mathbf{Y} \sim N\left(\hat{\mathbf{\Phi}}_b (\theta), \Sigma_u \otimes [\lambda T\Gamma_{XX} (\theta) + \mathbf{X}'\mathbf{X}]^{-1}\right) \tag{36}$$

$$\hat{\mathbf{\Phi}}_b (\theta) = (\lambda T\Gamma_{XX} (\theta) + \mathbf{X}'\mathbf{X})^{-1} (\lambda T\Gamma_{XY} (\theta) + \mathbf{X}'\mathbf{Y}) \tag{37}$$

$$\hat{\Sigma}_{u,b} (\theta) = \frac{1}{(\lambda+1)\, T} \left[(\lambda T\Gamma_{YY} (\theta) + \mathbf{Y}'\mathbf{Y}) - (\lambda T\Gamma_{XY} (\theta) + \mathbf{X}'\mathbf{Y})' \,\hat{\mathbf{\Phi}}_b (\theta)\right], \tag{38}$$

where the matrices $\hat{\mathbf{\Phi}}_b (\theta)$ and $\hat{\Sigma}_{u,b} (\theta)$ have the interpretation of maximum likelihood estimates of the VAR parameters based on the combined sample of actual observations and artificial observations generated by the DSGE. Equations (35) and (36) show that the smaller λ is, the closer the estimates are to the OLS estimates of an unrestricted VAR. Instead, the higher λ is, the closer the VAR estimates will be tilted towards the parameters in the VAR approximation of the DSGE model ($\hat{\mathbf{\Phi}}_b (\theta)$ and $\hat{\Sigma}_{u,b} (\theta)$). In order to obtain a non-degenerate prior density (34), which is a necessary condition for the existence of a well-defined Inverse-Wishart distribution and for computing meaningful marginal likelihoods, λ has to be greater than λ_{MIN}

$$\begin{aligned} \lambda_{MIN} &\geq \frac{n+k}{T}; k = 1 + p \times n \\ p &= \text{lags} \\ n &= \text{endogenous variables.} \end{aligned}$$

Hence, the optimal lambda must be greater than or equal to the minimum lambda $\left(\hat{\lambda} \geq \lambda_{MIN}\right)$.

Essentially, the DSGE-VAR tool allows the econometrician to draw posterior inferences about the DSGE model parameters θ. Del Negro and Schorfheide (2004) explain that the posterior estimate of θ has the interpretation of a minimum-distance estimator, where the discrepancy between the OLS estimates of the unrestricted VAR parameters and the VAR representation of the DSGE model is a sort of distance function. The estimated posterior of parameter vector θ depends on the hyperparameter λ. When $\lambda \to 0$, in the posterior the parameters are not informative, so the DSGE model is of no use in explaining the data. Unfortunately, the posteriors (36) and (35) do not have a closed form and we need a numerical method to solve the problem. The posterior simulator implemented by Del Negro and Schorfheide (2004) is the Markov Chain Monte Carlo Method and the algorithm used is the Random Walk - Metropolis Hastings (RW-MH) acceptance method. This procedure generates a Markov Chain from the posterior distribution of θ and this Markov Chain is used for Monte Carlo simulations. The optimal λ is given by maximizing the log of the marginal data density

$$\hat{\lambda} = \arg\max_{\lambda \geqslant \lambda_{MIN}} \ln p(\mathbf{Y}|\lambda).$$

According to the optimal lambda $\left(\hat{\lambda}\right)$, a corresponding optimal mixture model is chosen. This hybrid model is called DSGE-VAR$\left(\hat{\lambda}\right)$ and $\hat{\lambda}$ is the weight of the priors. It can also be interpreted as the restriction of the theoretical model on the actual data.

3.5 DSGE-FAVAR

According to Bernanke *et al.* (2005), a FAVAR benchmark for the evaluation of a DSGE model will include a vector of observable variables and a small vector of unobserved factors extracted from a large data-set of macroeconomic time series, that capture additional economic information relevant to model the dynamics of the observables. In this study we implement the DSGE-FAVAR model of Consolo *et al.* (2009). The statistical representation has the following specification

$$\begin{aligned}
\begin{pmatrix} \mathbf{Y}_t \\ \mathbf{F}_t \end{pmatrix} &= \begin{bmatrix} \mathbf{\Phi}_{11}(L) & \mathbf{\Phi}_{12}(L) \\ \mathbf{\Phi}_{21}(L) & \mathbf{\Phi}_{22}(L) \end{bmatrix} \begin{pmatrix} \mathbf{Y}_{t-1} \\ \mathbf{F}_{t-1} \end{pmatrix} + \begin{pmatrix} \mathbf{u}_t^Y \\ \mathbf{u}_t^F \end{pmatrix} \\
\mathbf{Y}_t &= (\Delta \ln x_t, \Delta \ln P_t, \ln R_t) \\
\mathbf{F}_t &= \left(F_{1t}^s, F_{2t}^s, F_{3t}^f\right),
\end{aligned} \tag{39}$$

whilst in case of the medium scale model

$$\begin{aligned}
\begin{pmatrix} \mathbf{Y}_t \\ \mathbf{F}_t \end{pmatrix} &= \begin{bmatrix} \mathbf{\Phi}_{11}(L) & \mathbf{\Phi}_{12}(L) \\ \mathbf{\Phi}_{21}(L) & \mathbf{\Phi}_{22}(L) \end{bmatrix} \begin{pmatrix} \mathbf{Y}_{t-1} \\ \mathbf{F}_{t-1} \end{pmatrix} + \begin{pmatrix} \mathbf{u}_t^Y \\ \mathbf{u}_t^F \end{pmatrix} \\
\mathbf{Y}_t &= (\Delta \ln y_t, \Delta \ln c_t, \Delta \ln i_t, \Delta \ln w_t, \ln l_t, \Delta \ln P_t, \ln R_t^a) \\
\mathbf{F}_t &= \left(F_{1t}^s, F_{2t}^s, F_{3t}^f\right),
\end{aligned} \tag{40}$$

where $\mathbf{Y}_t$ are the observable variables included in the simple DSGE model and $\mathbf{F}_t$ is a small vector of unobserved factors relevant to modelling the dynamics of $\mathbf{Y}_t$ (F_{1t}^s, F_{2t}^s are the two slow factors and F_{3t}^f is the fast factor). The system reduces to the standard VAR when $\Phi_{12}(L) = 0$. Importantly, and differently from Boivin and Giannoni (2006), this FAVAR is not interpreted as the reduced form of a DSGE model at hand. In fact, in this case the restrictions implied by the DSGE model on a general FAVAR are very difficult to trace and model evaluation becomes even more difficult to implement. A very tightly parameterized theory model can have a very highly parameterized reduced form if one is prepared to accept that the relevant theoretical concepts in the model are a combination of many macroeconomic and financial variables. The DSGE-FAVAR is implemented in the same way as the DSGE-VAR.

4 Empirical results

The dataset consists of quarterly data of the US economy from 1960:Q4 to 2010:Q4. The out-of sample period spans 1997:Q1 to 2010:Q4. To estimate the small scale models we use the log of the real output growth, the log of consumer price index, and the federal funds rate as short-term interest rate. In particular, the data for real output growth comes from the Bureau of Economic Analysis as Gross Domestic Product (GDP), while Consumer price index (CPI) data (seasonally adjusted, 1982-1984=100) are derived from the Bureau of Labor Statistics. Both series are taken in first difference logarithmic transformation. The interest rate series (FR rate) are constructed as in Clarida, Galì and Gertler (2000), namely for each quarter the interest rate is computed as the average federal funds rate during the first month of the quarter, including business days only. These three time series also represent the three equations of the small scale DSGE model. Also, the data is used to extract factors for FAVAR and DSGE-FAVAR models. Then, to estimate the medium scale DSGE model, we consider the log of the real output growth, the log difference of real consumption, log difference of real investment, the log difference of real wage, the log hours worked, the log difference of GDP deflator and the federal funds rate (Smets and Wouters, 2007; Del Negro and Schorfheide, 2012). Specifically, the real output growth (GDP) is given by the real Gross Domestic Product (GDPC), the log difference of the real consumption (CONS) by nominal personal consumption expenditures (PCEC), the log difference of real investment (INV) by nominal fixed private investment (FPI), and the log difference of GDP deflator (INFL) by the GDP price deflator (GDPDEF). The four series are constructed at a quarterly frequency by the Bureau of Economic Analysis (BEA). The real wage growth (WAGE) is given by the compensation per hour for the nonfarm business sector (PRS85006103) produced by the Bureau of Labor Statistics (BLS). The log hours worked (HOURS) are given by the average weekly hours of production (PRS85006023) and by the civilian employment (CE16OV) at monthly frequency provided by the BLS. The interest rate series (FRR) are constructed as in Clarida, Galì and Gertler (2000), i.e., for each quarter the interest rate is computed as the average federal funds rate during the first month of the quarter, including business days only. Finally, in this specification the complete dataset is used to extract factors for FAVAR and DSGE-FAVAR models.

In order to construct the FAVAR we extract factors from a balanced panel of 112 monthly and quarterly macroeconomic and financial time series, following the dataset built by Stock and Watson (2002). The dataset involves several measures of industrial production, interest rates, various price indices, employment and other important macroeconomic and also financial variables. In this set-up, the number of informational time series N is large (larger than time period T) and must be greater than the number of factors and observed variables in the FAVAR system ($k + M \ll N$). In the panel data used, there are some variables in monthly format, which are transformed into a quarterly data using end-of-period observations. All series have been transformed to induce stationarity. The series are taken as levels or transformed into logarithms, first or second difference (in level or logarithms) according to series characteristics. The Appendix contains a detailed description of all series and their corresponding transformations. Following Bernanke *et al.* (2005), we partition the data into two categories of information variables: slow and fast. Slow-moving variables (e.g., wages or spending) do not respond contemporaneously to unanticipated changes in monetary policy, while fast-moving variables (e.g., asset prices and interest rates) do respond contemporaneously to monetary shocks.

Then we extract two factors from the slow variables and one factor from the fast variables. The methodology implemented to extract the factors is principal components. Stock-Watson (1998) showed that factors can be consistently estimated by the first r principal components of a matrix X, even in the presence of moderate changes in the loading matrix Λ. For this result to hold it is important that the estimated number of factors k, is larger than or equal to the true number, r. Bai and Ng (2000) propose a set of selection criteria to choose k that are generalizations of the BIC and AIC criteria. As they suggest, we use information criteria to determine the number of factors but, as they are not so decisive, we limit the number of factors to three to strike a balance between the dimension of the panel data (112 series) and the parameters estimated in the VAR and FAVAR (number of endogenous variables and their lags). It is also worth noting that the factors are not uniquely identified, but this is not a problem in our context because we will not attempt a structural interpretation of the estimated factors.

We compare the out-of-sample forecasting performance of VAR models including BVAR and FAVAR and of the DSGE class including DSGE-VAR, DSGE-FAVAR, in terms of the Root Mean Squared Forecast Error (RMSE). According to Schwartz Bayesian information criterion (SIC), we implement the VAR based model from one to four lags. Hence, we compare the log of the marginal data densities (MDD) across lags. Based on the selection provided by MDD, a forecasting exercise is provided using a rolling procedure for h-steps-ahead. Most importantly, we compare the log of the marginal data densities (MDD). Based on the MDD a forecasting exercise is provided using a rolling procedure for h-steps-ahead. The variable forecasts for the small and medium scale models are estimated for the out-of-sample testing period 1997:Q1 - 2010:Q4. The forecasting investigation for the quarterly US data is performed over the one-, two-, three- and four-quarter-ahead horizon with a rolling estimation sample, based on the works of Marcellino (2004) and Brüggemann *et al.* (2008) for datasets of quarterly frequency. In particular, the models are re-estimated each quarter over the forecast horizon to update the estimate of the coefficients, before producing the quarter-ahead forecasts. Finally, in order to evaluate the models' forecast accuracy, we use the cross-model test statistic of Diebold and Mariano (1995).

Firstly, we report the estimation results for the log of Marginal Data Density (MDD). In particular, following Del Negro and Schorfheide (2006) we adopt the MDD as a measure of model fit, which arises naturally in the computation of posterior model odds. The MDD is calculated for the small and medium scale DSGE-VAR models using a different number of lags (from 1 up to 4). Each minimum λ ($\lambda_{\min}$) is produced based on the features of the model (number of observations, number of endogenous variables, number of lags), and the optimal lambda ($\widehat{\lambda}$) is calculated using the Markov Chain Monte Carlo with the Metropolis Hastings acceptance method (with 110,000 replications, we discard the first 10,000 ones and the following 100,000 replications are used in the estimation). Although we use the Random Walk - Metropolis Hastings (RW-MH) algorithm, we account for some problems it presents with the medium scale DSGE model of Smets-Wouters (2007) as shown in Chib and Ramamurthy (2010). In particular they propose replacing the commonly used single block RWM algorithm with a Metropolis-within-Gibbs algorithm that cycles over multiple, randomly selected blocks of parameters. Chib and Ramamurthy (2010) provide evidence that the RW-MH algorithm has a serial correlation problem at lags 2500 and it is proven difficult to tune up due to the dimensionality of the parameter space and the complexity of the posterior surface in case of many parameters, as with the Smets and Wouters (2007) model which estimates 36 parameters. To avoid the autocorrelation problem, Chib and Ramamurthy (2010) developed a Tailored randomized Block M-H (TaRB-MH) algorithm. Below in Tables 3 and 4 we show for each parameter the convergence diagnostic (CD) of Geweke (1992). The CD is a comparison between the first draws (10000) and the last draws (50,000), dropping out the middle

Table 3: Convergence Diagnostic (CD) of Geweke (1992) for the small scale model

Parameter	Convergence Diagnostic (CD)
$\ln \gamma$	0.670
$\ln \pi^*$	0.985
$\ln r^*$	-0.575
κ	0.850
τ	1.255
ψ_1	0.750
ψ_2	0.855
ρ_R	0.637
ρ_G	-1.255
ρ_Z	0.923
σ_R	0.855
σ_G	-0.348
σ_Z	-0.256

draws (more details are presented in Geweke, 1992 and Nakajima et al., 2011). The convergence diagnostic (CD) is computed by n_0 draws and the last n_1 draws, dropping out the middle draws. The CD statistics is computed by

$$CD = \frac{(\overline{x_0} - \overline{x_1})}{\sqrt{\frac{\widehat{\sigma_0}^2}{n_0} + \frac{\widehat{\sigma_1}^2}{n_1}}},$$

where $\overline{x_j} = \frac{1}{n_j} \sum_{i=m_j}^{m_j+n_j-1} x^{(i)}$, $x^{(i)}$ is the ith draw, and $\sqrt{\frac{\widehat{\sigma_j}^2}{n_j}}$ is the standard error of $\overline{x_j}$, respectively, for $j = 0, 1$. We set $m_0 = 1$, $n_0 = 10000$, $m_1 = 50001$, and $n_1 = 50000$. We compute $\widehat{\sigma_0}^2$ and $\widehat{\sigma_1}^2$ using a Parzen window with bandwidth of 1000 and 5000 respectively. If the MCMC algorithm has converged then Geweke's CD has a standard Normal distribution. It can be shown from the following tables, that all the diagnostics are less than 1.96 (in absolute value), indicating that the convergence has taken place.

The $\ln p(Y|M)$ is the log-MDD of the DSGE model specifications computed based on Geweke's (1999) modified harmonic mean estimator. The Bayes factor *(ratio of MDDs)*, as in An and Schorfheide (2007) helps us to understand the improvement of the log-MDD of a specific model. We compare different models against the benchmark model (M) maximizing the MDD. The prior distribution for the DSGE model parameters (θ) were already illustrated in Tables 1 and 2. This MDD measure has two dimensions: goodness of in-sample fit on the one hand and a penalty for model complexity or degrees of freedom on the other hand. The DSGE-VAR and the DSGE-FAVAR are estimated with a different number of lags on the sample 1960:Q4 -1996:Q4. From 1997:Q1, we start our forecasting evaluation as implemented in Herbst and Schorfheide (2012). The parameter λ is chosen from a grid which is unbounded from above. In our empirical exercise, the log of the MDD is computed over a discrete interval, $\ln p(Y|\lambda, M)$. The minimum value, $\lambda_{\min} = \frac{n+k}{T}$, is model dependent and is related to the existence of a well-defined Inverse-Wishart distribution. The $\lambda_{\min}$ refers to the VAR and FAVAR models nested in the DSGE-VAR and in the DSGE-FAVAR, respectively, since we cannot calculate the marginal likelihood in case of $\lambda = 0$. Therefore, we can show the log of MDD for any

Table 4: Convergence Diagnostic (CD) of Geweke (1992) for the medium scale model

Parameter	Convergence Diagnostic (CD)	Parameter	Convergence Diagnostic (CD)
φ	1.055	σ_a	0.784
σ_c	1.120	σ_b	0.965
h	-0.456	σ_g	-0.258
ξ_w	0.855	σ_l	-0.755
σ_l	-0.265	σ_r	-0.350
ξ_p	-0.955	σ_p	-0.470
ι_w	0.565	σ_w	0.695
ι_p	0.432	ρ_a	1.104
ψ	0.105	ρ_b	0.857
ϕ	0.958	ρ_g	-1.175
r_π	1.680	ρ_l	-1.025
ρ	-1.255	ρ_r	0.981
r_y	1.357	ρ_p	0.694
$r_{\Delta y}$	0.785	ρ_w	0.755
$\overline{\pi}$	0.995	μ_p	-0.575
$100(\beta^{-1}-1)$	0.258	μ_w	0.876
$\bar{l}$	-1.257	ρ_{ga}	0.759
$\overline{\gamma}$	0.755		
α	0.658		

value of λ larger than $\lambda_{\min}$. Importantly, $\lambda_{\min}$ depends on the degrees of freedom in the VAR or FAVAR and therefore, given estimation on the same number of available observations, $\lambda_{\min}$ for a DSGE-FAVAR will always be larger than $\lambda_{\min}$ for a DSGE-VAR. For the DSGE-VAR over the sample 1960:Q4-1996:Q4, the lambda grid is given by $\Lambda = \left\{ \begin{array}{c} 0,\ 0.05,\ 0.08,\ 0.10,\ 0.12,\ 0.15,\ 0.20,\ 0.25, \\ 0.30,\ 0.35,\ 0.40\ 0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9,\ 1,\ 10,\ 100 \end{array} \right\}$, while for the DSGE-FAVAR by$\Lambda = \left\{ \begin{array}{c} 0,\ 0.08,\ 0.1,\ 0.12,\ 0.14,0.15,\ 0.2,\ 0.25, \\ 0.3,\ 0,35,\ 0.4,\ 0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9,\ 1,\ 10,\ 100 \end{array} \right\}$. In both lambda intervals, we consider the λ_{MIN} across lags from 1 to 4.

Specifically, for the small scale model, Table 5 shows the main results related to the DSGE-VAR implemented using a different number of lags (from 1 up to 4). We compare different models against the benchmark

Table 5: Optimal lambda for the small scale DSGE-VAR and DGSE-FAVAR calculated with Markov Chain Monte Carlo and Metropolis Hastings method

	$\lambda_{\min}$	$\hat{\lambda}$	$\ln p(Y\vert\hat{\lambda}, M)$	Bayes Factor vs M_1
DSGE-VAR(1)	0.05	0.12	-563.440	$exp[37.26]$
DSGE-VAR(2)	0.08	0.15	-536.929	$exp[10.75]$
DSGE-VAR(3) (M_1)	0.1	0.2	-526.180	1
DSGE-VAR(4)	0.12	0.25	-530.421	$exp[4.24]$
	$\lambda_{\min}$	$\hat{\lambda}$	$\ln p(Y\vert\hat{\lambda}, M)$	Bayes Factor vs M_2
DSGE-FAVAR(1)	0.08	0.10	-546.742	$exp[30.22]$
DSGE-FAVAR(2)	0.1	0.15	-525.940	$exp[9.42]$
DSGE-FAVAR(3) (M_2)	0.12	0.20	-516.524	1
DSGE-FAVAR(4)	0.14	0.30	-522.191	$exp[5.67]$

Table 6: Optimal lambda for the medium scale DSGE-VAR and DGSE-FAVAR calculated with Markov Chain Monte Carlo and Metropolis Hastings method

	$\lambda_{\min}$	$\hat{\lambda}$	$\ln p(Y\|\hat{\lambda}, M)$	Bayes Factor vs M_1
DSGE-VAR(1)	0.11	0.4	-809.063	exp[0.965]
DSGE-VAR(2)	0.16	0.7	-808.197	exp[0.099]
DSGE-VAR(3) (M_1)	0.21	1	-808.098	1
DSGE-VAR(4)	0.26	1.25	-814.709	exp[6.611]
	$\lambda_{\min}$	$\hat{\lambda}$	$\ln p(Y\|\hat{\lambda}, M)$	Bayes Factor vs M_2
DSGE-FAVAR(1) (M_2)	0.13	0.4	-806.969	1
DSGE-FAVAR(2)	0.18	0.6	-807.740	exp[-0.36]
DSGE-FAVAR(3)	0.23	1.15	-808.100	exp[-1.131]
DSGE-FAVAR(4)	0.28	1.25	-812.525	exp[4.425]

model (M) maximizing the MDD. According to Table 5, we select the DSGE-VAR with 3 lags for the full sample 1960-1996. We repeat our exercise for the DSGE-FAVAR. We select one lag for the factors and we implement - as in case of the DSGE-VAR, - the DSGE-FAVAR with a different number lags from 1 to 4. As Table 5 shows, the DSGE-FAVAR with 3 lags is chosen. Then, for the Smets-Wouters model (2007), according to Table 6, we select the DSGE-VAR with 3 lags for the full sample 1960-1996. We repeat our exercise for the DSGE-FAVAR. We select one lag for the factors and we implement - as in case of the DSGE-VAR, - the DSGE-FAVAR with a different number lags from 1 to 4. As Table 6 shows, the DSGE-FAVAR with 1 lag is chosen.

In Tables 7 (small scale) and 8 (medium scale) we compare the logarithm of the MDD of the hybrid models, DSGE-VAR and DSGE-FAVAR against the DSGE, the Bayesian VAR, the VAR and the Factor Augmented VAR. The DSGE-FAVAR shows in both models (small and medium scale) the maximum MDD. For the VAR and FAVAR, the reported MDD is function of the $\lambda_{\min}$ lambda. Numerical Standard Errors are calculated following Chib (1995). The MDD comparison is useful to select the lag length for the models in each category (VAR, FAVAR, BVAR, DSGE-VAR, DSGE-FAVAR). As reported above, VAR and FAVAR models are nested in the two hybrid models, the DSGE-VAR and the DSGE-FAVAR. The marginal data density for the VAR and FAVAR is calculated using Bayesian methods, similarly to the DSGE-VAR and DSGE-FAVAR, as a function of the $\lambda_{\min}$. The minimum value, is model dependent and is related to the existence of a well-defined Inverse-Wishart distribution.

Table 9 reports the RMSE for all variables of the small scale model. An exhaustive exercise was conducted with one to four lags based on the Schwartz Bayesian information criterion (SIC). The results provide evidence that in general three lags is the optimal number for all models. Only in case of BVAR the optimal number of lags was two, but the SIC score was very close to the one corresponding to three lags. Overall, the results from RMSE are in accordance with those from the MDD estimation. For the VAR and FAVAR, the forecasting evaluation is produced with a $\lambda = 0$. As reported in Del Negro and Schorfheide (2004), when $\lambda = 0$ the estimation of the posterior mean of Φ conditional on θ equals the OLS estimate. In particular, for the GDP series the DSGE-FAVAR model provides the lowest RMSE for the first two forecasting horizons (i.e., one- and two-steps ahead) while the simple DSGE and the FAVAR outperform the other models for three- and

Table 7: Log of the Marginal Data Density for the sample 1960:Q4-1996:Q4 of the small scale model

	$lnp(Y\|M)$	Numerical Standard Error
DSGE	-588.435	0.0035
DSGE-VAR(3)	-526.180	0.0050
DSGE-FAVAR(3)	**-516.524**	0.0025
BVAR(1)	-570.244	0.0066
BVAR(2)	-534.305	0.0060
BVAR(3)	-524.858	0.0038
BVAR(4)	-535.723	0.0055
VAR(1)	-568.056	0.0030
VAR(2)	-541.083	0.0060
VAR(3)	-534.063	0.0063
VAR(4)	-534.063	0.0065
FAVAR(1)	-547.398	0.0040
FAVAR(2)	-528.824	0.0055
FAVAR(3)	-522.426	0.0058
FAVAR(4)	-532.383	0.0060

Table 8: Log of the Marginal Data Density for the sample 1960:Q4-1996:Q4 of the medium scale

	$lnp(Y\|M)$	Numerical Standard Error
DSGE	-848.446	0.0075
DSGE-VAR(3)	-808.098	0.0090
DSGE-FAVAR(1)	-806.969	0.0067
BVAR(1)	-851.690	0.0088
BVAR(2)	-829.570	0.0078
BVAR(3)	-843.311	0.0085
BVAR(4)	-857.530	0.0079
VAR(1)	-825.827	0.0082
VAR(2)	-848.846	0.0083
VAR(3)	-870.105	0.0095
VAR(4)	-872.710	0.0075
FAVAR(1)	-814.320	0.0081
FAVAR(2)	-845.356	0.0078
FAVAR(3)	-847.550	0.0080
FAVAR(4)	-848.746	0.0084

Table 9: Root Mean Square Forecast Error (RMSE) for GDP, CPI and FF rate based on small scale modeling

	VAR	BVAR	FAVAR	DSGE	DSGE-VAR	DSGE-FAVAR
GDP						
1	0.721	0.724	0.715	0.690	0.720	0.679
2	0.734	0.731	0.706	0.733	0.722	0.692
3	0.742	0.737	0.707	0.685	0.728	0.701
4	0.748	0.737	0.683	0.688	0.732	0.698
CPI						
1	0.953	0.940	0.995	0.841	0.955	0.792
2	0.929	0.927	1.046	0.849	0.920	0.781
3	0.897	0.906	1.061	0.790	0.890	0.778
4	0.890	0.908	1.050	0.954	0.886	0.779
FFR						
1	4.176	4.122	2.067	3.397	4.135	1.926
2	4.096	4.115	2.549	3.570	3.985	1.979
3	4.055	4.155	2.890	3.108	3.951	2.037
4	4.150	4.268	2.607	4.314	4.043	2.007

four-quarters-ahead. The VAR, BVAR and DSGE-VAR models present similar predictive performance and on average they generate the highest forecast errors. Next, in case of the CPI variable, the DSGE-FAVAR model clearly outperforms all other models for all steps-ahead. The simple DGSE and the DSGE-VAR outrank with a few exceptions the other model classes. The VAR model seems slightly better than BVAR, whilst the FAVAR provides with relatively high scores for the RMSE especially for two-, three- and four-quarters-ahead. The results for the FFR series provide further evidence of the DSGE-FAVAR superiority. Specifically, when comparing the RMSE scores of all model classes, DSGE-FAVAR is consistently the best performer in each forecasting horizon. The next lowest error is produced by the FAVAR model for all quarters-ahead. The DSGE model is better than DSGE-VAR for one- two- and three-step-ahead forecasts, while the DGSE-VAR provides with relatively lowest error in the fourth-quarter. Overall, the BVAR and the VAR models produce similar scores and they both underperform relatively to the other models.

Next, Table 10 reports the RMSE for all models and variables in the medium scale scenario. Starting with CONS variable, the DSGE-FAVAR and the DSGE-VAR outperform the other models in one- and three-quarters-ahead for the former and two- and four-quarters-ahead for the latter model, respectively. VAR and BVAR models produce a low RMSE, yet their predictive performance is significantly lower than that of DSGE-FAVAR and the DSGE-VAR. For the INV variable, the medium scale DSGE model outperforms all other models consistently for all steps-ahead. Only FAVAR shows a relatively good forecastability, yet the superiority of the FAVAR approach is clear. In case of GDP, the picture that emerges from the results is undisputed: when comparing the RMSE scores of all model classes, the medium scale DSGE is consistently the best performer in each forecasting horizon. Furthermore, for the HOURS variable the VAR model seems better than all the other models, whilst the BVAR provides with a relatively lower score for the RMSE only for the four-quarter-ahead. The other models in this case clearly underperform relatively to VAR or BVAR. Regarding INFL variable, the medium scale DSGE model clearly outperforms all other models in all forecasting horizons. VAR, BVAR, FAVAR models produce high scores for the RMSE whilst DSGE-VAR and

DSGE-FAVAR show a similar performance with the DSGE, yet evidently inferior in terms of predictability. For the WAGE series, the results indicate that VAR models are better for the first two quarters-ahead and BVAR models for three- and four-steps ahead. It resembles the case of HOURS variable with some variations. Again all other models present a worse predictive performance and on average they generate the highest forecast errors. Finally, the results for the FRR series provide further evidence of the medium scale DSGE superiority. In all steps-ahead the DSGE model outranks the other models while especially VAR, BVAR and FAVAR models seem to fall rather short in terms of RMSE.

Table 10: Root Mean Square Forecast Error (RMSE) for CONS, INV, GDP, HOURS, INFL, WAGE and FFR based on medium scale modeling

	VAR	BVAR	FAVAR	DSGE	DSGE-VAR	DSGE-FAVAR
CONS						
1	0.755	0.755	0.761	0.787	0.762	0.748
2	0.741	0.747	0.759	0.776	0.741	0.743
3	0.758	0.754	0.752	0.770	0.744	0.739
4	0.746	0.747	0.750	0.755	0.738	0.740
INV						
1	2.731	2.735	2.728	2.698	2.763	2.866
2	2.713	2.724	2.726	2.682	2.740	2.880
3	2.745	2.736	2.701	2.671	2.744	2.865
4	2.724	2.731	2.701	2.690	2.735	2.875
GDP						
1	0.791	0.791	0.805	0.776	0.784	0.781
2	0.779	0.780	0.797	0.763	0.769	0.770
3	0.793	0.789	0.793	0.758	0.771	0.765
4	0.794	0.796	0.804	0.765	0.777	0.775
HOURS						
1	3.094	3.098	3.432	3.266	3.981	4.155
2	3.236	3.241	3.575	3.361	4.093	4.397
3	3.356	3.371	3.751	3.528	4.378	4.628
4	3.549	3.543	3.942	3.689	4.560	4.879
INFL						
1	0.568	0.563	0.432	0.264	0.298	0.311
2	0.500	0.499	0.434	0.265	0.277	0.307
3	0.554	0.538	0.437	0.268	0.290	0.306
4	0.543	0.546	0.429	0.265	0.284	0.305
WAGE						
1	0.793	0.796	0.794	0.985	0.871	0.815
2	0.794	0.796	0.798	0.990	0.885	0.822
3	0.801	0.800	0.807	1.014	0.893	0.836
4	0.803	0.801	0.817	0.998	0.887	0.842
FFR						
1	1.037	1.032	0.798	0.442	0.558	0.670
2	1.019	1.015	0.823	0.451	0.583	0.701
3	1.100	1.079	0.856	0.463	0.595	0.714
4	1.098	1.093	0.873	0.474	0.601	0.738

The Diebold-Mariano (DM) pairwise test is employed in order to evaluate the comparative forecast accuracy. The results are reported in Tables 11-13 for the small scale specification and in Tables 14-20 for the medium scale model. The Diebold-Mariano test is based on the squared prediction errors. The DM test has been conducted on the best performer of each category, namely VAR, BVAR, FAVAR, DSGE, DSGE-VAR, DSGE-FAVAR model and for each examined macro-variable, based on the Log of Marginal Data Density. For example, for the medium scale model, the DM test has been implemented pairwise on the VAR (1), BVAR (2), FAVAR (1), DSGE, DSGE-VAR (3), DSGE-FAVAR (1) model specifications. Firstly, we examine the DM results of differential predictability for the small scale formulation. The DM test statistics for GDP indicate that none of the models consistently outperforms any of the other for all quarter-ahead forecasts, namely their pairwise forecast comparison shows no statistically significant difference at the 5% and 1% level. Only in case of VAR vs. DSGE-VAR and BVAR vs. DSGE-VAR, for three-steps ahead, differential predictability is significant at the 5% level as well as for the VAR-BVAR pair for four-quarters-ahead. On the contrary for the CPI series, the DSGE-FAVAR model in any pair shows a distinctively significant predictability at 1% in all step-ahead forecasts. In fact, most models for all forecast horizons appear to have a significant pairwise predictability at the 1% level. Some exceptions include the pairs VAR-BVAR, VAR-DSGE-VAR and BVAR-DSGE for two- and four-steps-ahead. In accordance with the MDD and RMSE results, it is evident that in case of CPI the DSGE-FAVAR set-up outperforms the other models. Finally, in case of FFR the DM results lead to a more diverse and variant assessment of differential predictability, albeit the majority of cases produce a statistically significant DM score. While it appears that no particular model consistently and comparatively outperforms any of the other, yet the DSGE-FAVAR presents significant scores at the 1% level in almost all pairwise comparisons. Specifically, the DSGE-FAVAR is superior when examined with the VAR, BVAR, simple DSGE and DSGE-VAR models and only when compared to FAVAR especially in one- and four-steps-ahead, it shows weak or no differential predictability. Overall, in all other cases except DSGE-FAVAR, many test statistics are not significant for four-quarters-ahead and the combined investigation of the MDD, RMSE and DM results is not indicative of a consistent outranking classification among the other investigated models for all forecasting horizons.

Next, we conduct a comparative predictability analysis based on the DM results for the medium scale specification. The results for the CONS variable indicate that none of the models consistently outperforms any of the other for all quarter-ahead forecasts, namely their pairwise forecast comparison shows no statistically significant difference at the 5% and 1% level. However, only in case of DSGE-VAR vs. FAVAR and DSGE-FAVAR vs. FAVAR, for different steps-ahead, differential predictability is significant at the 5% level. This accords with the RMSE results that DSGE-VAR and DSGE-FAVAR models provided with the best forecasting ability. Similarly, for the INV series the majority of cases produce statistically insignificant DM scores, thus no particular model consistently and comparatively outperforms any of the other. The only exception is the DSGE medium scale model compared against the DSGE-VAR and DSGE-FAVAR where mostly for the first-quarter-ahead and marginally for the second-step, the differential predictability is significant at 1% or 5% level. In case of GDP the DM results lead to a diverse assessment of comparative predictability. It appears

Table 11: Pairwise forecast comparison for the GDP with the Diebold-Mariano test: small scale model

GDP	PERIODS			
	1	2	3	4
VAR vs BVAR	1.247	1.365	1.317	2.017
VAR vs FAVAR	0.928	0.524	0.320	0.628
VAR vs DSGE	1.134	0.105	1.544	1.457
VAR vs DSGE-VAR	0.525	1.891	2.163	2.067
VAR vs DSGE-FAVAR	0.720	1.389	1.635	1.742
BVAR vs FAVAR	0.963	0.492	0.271	0.533
BVAR vs DSGE	1.181	0.218	1.469	1.322
BVAR vs DSGE-VAR	1.209	1.846	2.339	1.397
BVAR vs DSGE-FAVAR	0.943	1.353	1.518	1.580
FAVAR vs DSGE	0.568	0.555	0.282	0.068
FAVAR vs DSGE-VAR	0.929	0.396	0.193	0.500
FAVAR vs DSGE-FAVAR	0.945	0.225	0.078	0.184
DSGE vs DSGE-VAR	1.153	1.225	1.352	1.296
DSGE vs DSGE-FAVAR	1.267	1.839	1.304	0.711
DSGE-VAR vs DSGE-FAVAR	0.716	1.112	1.348	1.592

Notes: The Diebold-Mariano (1995) test is based on squared prediction errors.

Table 12: Pairwise forecast comparison for the CPI with the Diebold-Mariano test: small scale model

CPI	PERIODS			
	1	2	3	4
VAR vs BVAR	6.107	0.678	3.748	4.898
VAR vs FAVAR	2.243	3.900	4.404	4.215
VAR vs DSGE	5.142	5.532	3.298	2.071
VAR vs DSGE-VAR	2.039	2.513	2.963	0.995
VAR vs DSGE-FAVAR	3.878	3.174	2.304	2.382
BVAR vs FAVAR	2.835	3.856	4.211	3.749
BVAR vs DSGE	4.879	5.630	3.510	1.537
BVAR vs DSGE-VAR	6.821	2.944	4.316	4.607
BVAR vs DSGE-FAVAR	3.671	3.083	2.454	2.684
FAVAR vs DSGE	4.082	4.740	4.084	2.728
FAVAR vs DSGE-VAR	2.206	4.153	4.407	4.113
FAVAR vs DSGE-FAVAR	3.529	3.622	3.445	3.433
DSGE vs DSGE-VAR	5.127	5.458	3.198	2.318
DSGE vs DSGE-FAVAR	2.311	1.741	0.584	2.442
DSGE-VAR vs DSGE-FAVAR	3.881	2.914	2.210	2.323

Notes: As in Table 11

Table 13: Pairwise forecast comparison for the FFR with the Diebold-Mariano test: small scale model

FF rate	PERIODS			
	1	2	3	4
VAR vs BVAR	8.367	1.492	3.097	2.927
VAR vs FAVAR	5.133	1.914	1.107	1.317
VAR vs DSGE	9.078	4.832	3.601	1.607
VAR vs DSGE-VAR	8.407	4.480	4.138	3.505
VAR vs DSGE-FAVAR	7.110	3.416	2.429	2.509
BVAR vs FAVAR	5.061	1.943	1.198	1.410
BVAR vs DSGE	9.102	4.878	3.728	0.416
BVAR vs DSGE-VAR	5.023	4.451	3.845	3.585
BVAR vs DSGE-FAVAR	7.063	3.456	2.521	2.601
FAVAR vs DSGE	3.813	1.329	0.236	1.386
FAVAR vs DSGE-VAR	5.072	1.796	1.018	1.236
FAVAR vs DSGE-FAVAR	0.653	1.483	1.731	0.974
DSGE vs DSGE-VAR	9.047	4.877	3.516	2.377
DSGE vs DSGE-FAVAR	5.969	2.877	1.705	2.482
DSGE-VAR vs DSGE-FAVAR	7.060	3.313	2.350	2.435

Notes: As in Table 11

Table 14: Pairwise forecast comparison for the CONS with the Diebold-Mariano test: medium scale model

CONS	PERIODS			
	1	2	3	4
VAR vs BVAR	0.244	1.257	1.378	0.443
VAR vs FAVAR	1.195	2.797	0.371	0.378
VAR vs DSGE	0.720	0.500	0.170	0.126
VAR vs DSGE-VAR	0.999	0.162	0.786	0.844
VAR vs DSGE-FAVAR	0.801	0.228	0.797	0.283
BVAR vs FAVAR	1.172	1.891	0.101	0.345
BVAR vs DSGE	0.718	0.415	0.228	0.123
BVAR vs DSGE-VAR	0.992	0.768	0.646	0.852
BVAR vs DSGE-FAVAR	0.812	0.438	0.702	0.292
FAVAR vs DSGE	0.520	0.225	0.220	0.066
FAVAR vs DSGE-VAR	0.406	2.611	2.123	1.949
FAVAR vs DSGE-FAVAR	2.624	2.235	1.394	0.795
DSGE vs DSGE-VAR	0.479	0.479	0.321	0.234
DSGE vs DSGE-FAVAR	0.762	0.436	0.371	0.179
DSGE-VAR vs DSGE-FAVAR	3.462	0.197	0.792	0.143

Notes: As in Table 11

Table 15: Pairwise forecast comparison for the INV with the Diebold-Mariano test: medium scale model

INV	PERIODS			
	1	2	3	4
VAR vs BVAR	0.924	1.335	0.754	0.965
VAR vs FAVAR	0.302	0.723	0.951	0.650
VAR vs DSGE	1.048	0.685	0.854	0.409
VAR vs DSGE-VAR	0.756	0.447	0.002	0.118
VAR vs DSGE-FAVAR	1.784	1.325	0.710	0.867
BVAR vs FAVAR	0.615	0.119	0.931	0.726
BVAR vs DSGE	1.112	0.835	0.837	0.461
BVAR vs DSGE-VAR	0.632	0.257	0.083	0.047
BVAR vs DSGE-FAVAR	1.687	1.207	0.784	0.807
FAVAR vs DSGE	1.342	1.337	0.712	0.222
FAVAR vs DSGE-VAR	1.018	0.316	0.659	0.512
FAVAR vs DSGE-FAVAR	2.026	1.393	1.198	1.165
DSGE vs DSGE-VAR	2.793	1.655	1.472	1.189
DSGE vs DSGE-FAVAR	2.979	1.944	1.595	1.515
DSGE-VAR vs DSGE-FAVAR	2.994	2.017	1.630	1.608

Notes: As in Table 11

Table 16: Pairwise forecast comparison for the GDP with the Diebold-Mariano test: medium scale model

GDP	PERIODS			
	1	2	3	4
VAR vs BVAR	0.027	0.405	1.334	0.462
VAR vs FAVAR	3.028	2.764	0.011	1.421
VAR vs DSGE	0.645	0.462	0.952	0.890
VAR vs DSGE-VAR	1.209	1.303	1.622	1.736
VAR vs DSGE-FAVAR	1.690	1.065	1.898	1.738
BVAR vs FAVAR	2.529	2.395	0.293	0.780
BVAR vs DSGE	0.675	0.535	0.875	1.009
BVAR vs DSGE-VAR	1.114	2.025	1.497	1.666
BVAR vs DSGE-FAVAR	1.785	1.401	1.846	1.800
FAVAR vs DSGE	1.095	0.897	0.768	0.995
FAVAR vs DSGE-VAR	3.030	2.326	1.960	1.950
FAVAR vs DSGE-FAVAR	2.910	2.020	1.825	1.840
DSGE vs DSGE-VAR	0.313	0.223	0.357	0.424
DSGE vs DSGE-FAVAR	0.235	0.250	0.212	0.399
DSGE-VAR vs DSGE-FAVAR	0.343	0.245	1.350	0.569

Notes: As in Table 11

that the FAVAR model produces significant DM results in some pairwise comparisons (e.g., against BVAR, DSGE-VAR and DSGE-FAVAR) in all steps ahead. Nevertheless, this is not corroborated by the RMSE results where the DSGE model was the superior model. On the contrary for the HOURS variable, the VAR and BVAR models in any pair show a distinctively significant predictability at 1% in all step-ahead forecasts. In fact, most models for all forecast horizons appear to have a significant pairwise predictability at the 1% level. Some exceptions include the pairs FAVAR-DSGE, DSGE-DSGE-VAR and VAR-BVAR mostly for the longest four-steps-ahead horizon. In accordance with the MDD and RMSE results, it is evident that in case of HOURS the VAR set-up outperforms the other models. Moreover, the DM results lead to the same conclusion as far as the INFL series is concerned. Based on the MDD and RMSE results the medium scale DSGE model in any pair shows a distinctively significant predictability at 1% in all step-ahead forecasts. In fact most pairwise comparisons for all models indicate a statistically significant differential predictability. Regarding the WAGE variable a more variant assessment emerges, albeit all pairwise comparisons excluding VAR and BVAR models produce a high DM score and reveal a strong differential predictability in all forecasting horizons. This conclusion weakens the MDD and RMSE superiority of the VAR and BVAR models in terms of distinctive predictability. Finally, in case of FFR the DM results are in full accordance with the RMSE results, hence the highly statistically significant DM scores verify that the medium scale DSGE comparatively outperforms any other model at the 1% level. While the majority of pairwise investigation shows a distinctive differential predictability in terms of the DM test for almost all models, the results for the DSGE model are indicative of a consistent outranking classification among the other models for all forecasting horizons.

Overall, we tried to further investigate whether the better performance of FAVAR and DSGE-FAVAR under a small scale modeling might be originated from possible model misspecification which eventually results in omitted variable bias. Also, the performance comparison among non-factor models i.e., VAR, BVAR, DSGE and DSGE-VAR could also depend on the specification of the baseline DSGE model. Hence, a larger DSGE model might reduce the performance gap between models with and without augmented factors. Consequently, it made sense to examine if the results for the small scale model would hold for a medium scale DSGE model. The results corroborated in part this argument, yet the DSGE-FAVAR still outperformed some models in the medium scale implementation as well. The complexity of the medium scale model could have resulted in forecasting the three key variables i.e., growth, inflation, and short-term interest rate more accurately. The small scale DSGE model can be considered as a special case of the medium scale model, by removing some of its features such as capital accumulation and wage stickiness. Indeed, the CPI forecast of the medium scale model is more precise than the one from the small scale model, a fact that could be attributed to a more sophisticated Phillips curve relationship and the presence of wage stickiness. For the same reason, the medium scale FFR forecasts are slightly more accurate than in the case of a small scale model. On the contrary, the richness of the medium scale specification does not seem to improve the predictability of the GDP. Hence, the accuracy of the GDP forecasts from the medium scale model is similar to the one generated by the small scale model. This result is in full accordance with the recent literature by Del Negro and Schorfheide (2012a, 2012b). Moreover, for the short-term prediction horizons, better accuracy is reported in case of the small scale DSGE, whilst instead for the long-run similar values for the two models emerge. In case of CPI and FFR, the scale-dimension of the DSGE model has an impact on the hybrid models too.

Table 17: Pairwise forecast comparison for the HOURS with the Diebold-Mariano test: medium scale model

HOURS	PERIODS			
	1	2	3	4
VAR vs BVAR	1.787	0.306	2.722	0.651
VAR vs FAVAR	3.296	2.900	2.254	1.932
VAR vs DSGE	4.967	1.295	2.459	2.005
VAR vs DSGE-VAR	3.777	2.767	2.248	2.043
VAR vs DSGE-FAVAR	3.980	2.865	2.285	2.074
BVAR vs FAVAR	3.250	2.769	2.230	1.900
BVAR vs DSGE	4.966	1.477	2.304	2.158
BVAR vs DSGE-VAR	3.754	2.663	2.239	2.024
BVAR vs DSGE-FAVAR	3.961	2.789	2.278	2.060
FAVAR vs DSGE	2.150	1.547	1.434	1.410
FAVAR vs DSGE-VAR	3.994	2.362	2.126	2.014
FAVAR vs DSGE-FAVAR	4.299	2.668	2.241	2.093
DSGE vs DSGE-VAR	3.382	2.076	1.912	1.814
DSGE vs DSGE-FAVAR	3.683	2.355	2.036	1.919
DSGE-VAR vs DSGE-FAVAR	4.663	3.068	2.287	2.084

Notes: As in Table 11

Table 18: Pairwise forecast comparison for the INFL with the Diebold-Mariano test: medium scale model

INFL	PERIODS			
	1	2	3	4
VAR vs BVAR	2.316	0.476	2.562	0.764
VAR vs FAVAR	10.661	2.494	5.385	4.908
VAR vs DSGE	7.926	4.127	4.659	3.903
VAR vs DSGE-VAR	10.030	4.523	5.609	4.857
VAR vs DSGE-FAVAR	9.746	4.530	5.552	4.858
BVAR vs FAVAR	9.521	2.341	4.015	4.530
BVAR vs DSGE	7.733	4.054	4.302	3.889
BVAR vs DSGE-VAR	9.719	4.420	5.114	4.796
BVAR vs DSGE-FAVAR	9.372	4.400	4.974	4.767
FAVAR vs DSGE	5.656	4.078	3.547	2.913
FAVAR vs DSGE-VAR	8.044	5.001	4.716	4.061
FAVAR vs DSGE-FAVAR	7.825	5.554	4.746	4.164
DSGE vs DSGE-VAR	2.171	1.002	1.120	0.765
DSGE vs DSGE-FAVAR	2.872	1.958	1.620	1.314
DSGE-VAR vs DSGE-FAVAR	2.968	3.129	2.317	2.450

Notes: As in Table 11

Table 19: Pairwise forecast comparison for the WAGE with the Diebold-Mariano test: medium scale model

WAGE	PERIODS			
	1	2	3	4
VAR vs BVAR	1.909	0.870	0.761	0.911
VAR vs FAVAR	0.060	0.417	0.664	1.656
VAR vs DSGE	3.102	3.703	4.431	3.931
VAR vs DSGE-VAR	1.975	2.681	3.626	3.295
VAR vs DSGE-FAVAR	1.030	1.709	2.680	2.533
BVAR vs FAVAR	0.144	0.222	0.786	1.510
BVAR vs DSGE	3.022	3.605	4.407	3.922
BVAR vs DSGE-VAR	1.863	2.548	3.597	3.267
BVAR vs DSGE-FAVAR	0.855	1.494	2.564	2.363
FAVAR vs DSGE	3.697	4.179	4.916	4.080
FAVAR vs DSGE-VAR	2.729	3.346	4.612	3.493
FAVAR vs DSGE-FAVAR	2.279	2.932	4.232	3.346
DSGE vs DSGE-VAR	4.658	5.037	5.170	4.482
DSGE vs DSGE-FAVAR	3.948	4.373	4.849	4.089
DSGE-VAR vs DSGE-FAVAR	2.913	3.450	4.122	3.140

Notes: As in Table 11

5 Conclusions

In this paper we employed advanced Bayesian methods for estimating dynamic stochastic general equilibrium (DSGE) models. These models appear to be particularly suited for conducting policy evaluation, as shown in the works of Smets and Wouters (2003, 2004), Del Negro and Schorfheide (2004), Adolfson *et al.* (2008) and Christiano *et al.* (2005). However, calibrated DSGE models face many important challenges such as the fragility of parameter estimates, statistical fit and the weak reliability of policy forecasts as reported in Stock and Watson (2001), Ireland (2004) and Schorfheide (2010). In recent years Bayesian estimation has become popular mainly because it provides a system-based estimation approach that offers the advantage of employing prior assumptions about the parameters based on economic theory. The popularity of the Bayesian approach in DSGE modeling and forecasting is also explained by the increasing computational power available to estimate large-scale DSGE models using Markov Chain Monte Carlo simulations. DSGE models can pose identification problems for frequentist estimation that no amount of data or computing power can overcome. New macroeconomic research is drawn to the application of Bayesian statistics because DSGE models are often seen as abstractions of actual economies.

This study included an exhaustive comparative evaluation of the out-of-sample predictive performance of many different specifications of small and medium scale estimated DSGE models and various classes of VAR models, using datasets from the US economy. Simple and hybrid DSGE models were implemented, such as DSGE-VARs and Factor Augmented DGSEs (DSGE-FAVAR), and tested against standard VARs, Bayesian VARs and Factor Augmented VARs (FAVAR).We used comparatively a small scale model as in Del Negro and Schorfheide (2004) including the real gross domestic product, the harmonized Consumer Price Index and the nominal short-term federal funds interest rate, as well as the medium scale model of Smets and Wouters (2007) which additionally features sticky nominal price, wage contracts, habit formation, variable capital utilization and investment adjustment costs. The Smets and Wouters (2007) model is close in spirit

Table 20: Pairwise forecast comparison for the FFR with the Diebold-Mariano test: medium scale model

FFR	PERIODS			
	1	2	3	4
VAR vs BVAR	2.941	1.381	3.225	1.287
VAR vs FAVAR	8.823	4.960	4.643	3.823
VAR vs DSGE	6.714	3.832	3.418	2.914
VAR vs DSGE-VAR	7.574	4.304	3.830	3.240
VAR vs DSGE-FAVAR	8.117	4.608	4.157	3.458
BVAR vs FAVAR	8.671	4.838	4.373	3.712
BVAR vs DSGE	6.634	3.786	3.284	2.867
BVAR vs DSGE-VAR	7.467	4.237	3.654	3.177
BVAR vs DSGE-FAVAR	7.987	4.521	3.943	3.382
FAVAR vs DSGE	5.232	3.195	2.643	2.347
FAVAR vs DSGE-VAR	6.102	3.692	2.976	2.637
FAVAR vs DSGE-FAVAR	6.514	3.965	3.192	2.686
DSGE vs DSGE-VAR	3.633	2.330	1.970	1.593
DSGE vs DSGE-FAVAR	4.512	2.809	2.314	1.944
DSGE-VAR vs DSGE-FAVAR	5.577	3.382	2.695	2.336

Notes: As in Table 11

to that of Christiano *et al.* (2005) to fit to US macroeconomic data. We used quarterly time series data of the US economy from 1960:Q4 to 2010:Q4 and we produced forecasts for the out-of-sample testing period 1997:Q1-2010:Q4. The results were evaluated with the use of Bayesian method of the marginal data density (MDD) as well as the root mean squared forecast error. The Diebold-Mariano (1995) pairwise test was also employed to measure comparatively the differential forecastability.

For the small scale DSGE implementation the best forecasting performance for the CPI and FFR macroeconomic variables was consistently produced by the DSGE-FAVAR model, with few exceptions regarding the forecast horizons. For the GDP, different models provided with the most accurate forecasts depending on the forecast horizon and the statistical measure of predictability used. In particular, apart from the DSGE-FAVAR, the FAVAR and the simple DSGE models were the best performers, whilst BVAR and DSGE-VAR specifications provided with less satisfying forecasting results. Moreover, the medium scale simple DSGE specification of Smets and Wouters (2007) produced consistently the best forecasts for the INV, GDP, INFL and FFR series, while the DSGE-FAVAR was a also good performer but slightly worse than the medium scale DSGE. In case of HOURS and WAGE variables the VAR and BVAR models consistently outperformed any of the other models for all quarter-ahead forecasts. Finally, the DSGE-FAVAR outranked the other models for the CONS macroeconomic variable in the medium scale modeling of Smets and Wouters (2007).

The present comparative model validation can be useful to monetary policy analysis and macro-forecasting with the use of advanced Bayesian methods. Although policymakers and practitioners are particularly interested in DSGE models, these are typically too stylized to be taken directly to the data and often yield weak prediction results. Very recently, hybrid models have become popular for dealing with some of the DSGE model misspecifications. Major advances in Bayesian estimation methodology as shown in this study, will allow large scale DSGE models to compete and outperform well-known time-series models (e.g., VARs) and effectively deal with more complex real-world problems as richer sources of data become available.

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A Appendix

The source of the data is the Federal Reserve Economic Data - Federal Reserve Bank of Saint Louis (http://research.stlouisfed.org/fred2/). In order to construct the FAVAR we extract factors from a balanced panel of 112 monthly and quarterly macroeconomic and financial time series, following the dataset built by Stock and Watson (2002). The dataset involves several measures of industrial production, interest rates, various price indices, employment and other important macroeconomic and also financial variables. In the following Table, the first column has the series number, the second the series acronym, the third the series description, the fourth the transformation codes and the fifth column denotes a slow-moving variable with 1 and a fast-moving one with 0. The transformed series are tested using the Box-Jenkins procedure and the Dickey-Fuller test. Following Bernanke *et al.* (2005), the transformation codes are as follows: 1 - no transformation; 2 - first difference; 4 - logarithm; 5 - first difference of logarithm; 6 - second difference; 7 - second difference of logarithm.

Date	Long Description	Tcode	SlowCode
PAYEMS	Total Nonfarm Payrolls: All Employees	5	1
DSPIC96	Real Disposable Personal Income	5	1
NAPM	ISM Manufacturing: PMI Composite Index	1	1
UNRATE	Civilian Unemployment Rate	1	1
INDPRO	Industrial Production Index (Index 2007=100)	5	1
PCEPI	Personal Consumption Expenditures: Chain-type Price Index (Index 2005=100)	5	1
PPIACO	Producer Price Index: All Commodities (Index 1982=100)	5	1
FEDFUNDS	Effective Federal Funds Rate	1	0
IPDCONGD	Industrial Production: Durable Consumer Goods (Index 2007=100)	5	1
IPBUSEQ	Industrial Production: Business Equipment (Index 2007=100)	5	1
IPMAT	Industrial Production: Materials (Index 2007=100)	5	1
IPCONGD	Industrial Production: Consumer Goods (Index 2007=100)	5	1
IPNCONGD	Industrial Production: Nondurable Consumer Goods (Index 2007=100)	5	1
IPFINAL	Industrial Production: Final Products (Market Group) (Index 2007=100)	5	1
UNEMPLOY	Unemployed	5	1
EMRATIO	Civilian Employment-Population Ratio (%)	1	1
CE16OV	Civilian Employment	5	1
CLF16OV	Civilian Labor Force	5	1
CIVPART	Civilian Participation Rate (%)	1	1
UEMP27OV	Civilians Unemployed for 27 Weeks and Over	5	1
UEMPLT5	Civilians Unemployed - Less Than 5 Weeks	5	1
UEMP15OV	Civilians Unemployed - 15 Weeks & Over	5	1
UEMP15T26	Civilians Unemployed for 15-26 Weeks	5	1
UEMP5TO14	Civilians Unemployed for 5-14 Weeks	5	1
MANEMP	Employees on Nonfarm Payrolls: Manufacturing	5	1
USPRIV	All Employees: Total Private Industries	5	1
USCONS	All Employees: Construction	5	1
USFIRE	All Employees: Financial Activities	5	1
USTRADE	All Employees: Retail Trade	5	1
DMANEMP	All Employees: Durable Goods Manufacturing	5	1
USGOOD	All Employees: Goods-Producing Industries	5	1
USEHS	All Employees: Education & Health Services	5	1
USLAH	All Employees: Leisure & Hospitality	5	1
SRVPRD	All Employees: Service-Providing Industries	5	1
USINFO	All Employees: Information Services	5	1
USPBS	All Employees: Professional & Business Services	5	1
USTPU	All Employees: Trade, Transportation & Utilities	5	1
NDMANEMP	All Employees: Nondurable Goods Manufacturing	5	1
USMINE	All Employees: Natural Resources & Mining	5	1
USWTRADE	All Employees: Wholesale Trade	5	1
USSERV	All Employees: Other Services	5	1
AHEMAN	Average Hourly Earnings: Manufacturing	5	1
AHECONS	Average Hourly Earnings: Construction (NSA)	5	1
PPIIDC	Producer Price Index: Industrial Commodities (NSA)	5	1

Date	Long Description	Tcode	SlowCode
PPIFGS	Producer Price Index: Finished Goods (Index 1982=100)	5	1
PPICPE	Producer Price Index: Finished Goods: Capital Equipment (Index 1982=100)	5	1
PPICRM	Producer Price Index: Crude Materials for Further Processing (Index 1982=100)	5	1
PPIITM	Producer Price Index: Intermediate Materials: Supplies & Components (Index 1982=100)	5	1
PPIENG	Producer Price Index: Fuels & Related Products & Power (Index 1982=100)	5	1
PPIFCG	Producer Price Index: Finished Consumer Goods (Index 1982=100)	5	1
PFCGEF	Producer Price Index: Finished Consumer Goods Excluding Foods (Index 1982=100)	5	1
CPIAUCSL	Consumer Price Index for All Urban Consumers: All Items (Index 1982=100)	5	1
CPIAUCNS	Consumer Price Index for All Urban Consumers: All Items (Index 1982-84=100)	5	1
CPILFESL	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy (Index 1982-84=100)	5	1
CPILFENS	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy (NSA Index 1982=100)	5	1
CPIUFDNS	Consumer Price Index for All Urban Consumers: Food (NSA Index 1982=100)	5	1
CPIENGNS	Consumer Price Index for All Urban Consumers: Energy (NSA Index 1982=100)	5	1
CPIENGSL	Consumer Price Index for All Urban Consumers: Energy (Index 1982-1984=100)	5	1
CPILEGSL	Consumer Price Index for All Urban Consumers: All Items Less Energy (Index 1982-1984=100)	5	1
CPIMEDSL	Consumer Price Index for All Urban Consumers: Medical Care (Index 1982-1984=100)	5	1
PPIFCF	Producer Price Index: Finished Consumer Foods (Index 1982=100)	5	1
AAA	Moody's Seasoned Aaa Corporate Bond Yield	1	0
BAA	Moody's Seasoned Baa Corporate Bond Yield	1	0
M2SL	M2 Money Stock	6	0
M2NS	M2 Money Stock (NSA)	6	0
M1NS	M1 Money Stock (NSA)	6	0
M3SL	M3 Money Stock (DISCONTINUED SERIES)	6	0
GS5	5-Year Treasury Constant Maturity Rate	1	0
GS10	10-Year Treasury Constant Maturity Rate	1	0
GS1	1-Year Treasury Constant Maturity Rate	1	0
GS3	3-Year Treasury Constant Maturity Rate	1	0
TB3MS	3-Month Treasury Bill: Secondary Market Rate	1	0
TB6MS	6-Month Treasury Bill: Secondary Market Rate	1	0
HOUST	Housing Starts: Total: New Privately Owned Housing Units Started	5	0
PERMIT	New Private Housing Units Authorized by Building Permits	5	0
HOUSTMW	Housing Starts in Midwest Census Region	5	0
HOUSTW	Housing Starts in West Census Region	5	0
HOUSTNE	Housing Starts in Northeast Census Region	5	0
HOUSTS	Housing Starts in South Census Region	5	0
PERMITS	New Private Housing Units Authorized by Building Permits - South	5	0
PERMITMW	New Private Housing Units Authorized by Building Permits - Midwest	5	0
PERMITW	New Private Housing Units Authorized by Building Permits - West	5	0
PERMITNE	New Private Housing Units Authorized by Building Permits - Northeast	5	0
PDI	Personal Dividend Income	5	0
SPREAD1	3mo-FYFF	1	0
SPREAD2	6mo-FYFF	1	0
SPREAD3	1yr-FYFF	1	0
SPREAD4	2yr-FYFF	1	0
SPREAD5	3yr-FYFF	1	0
SPREAD6	5yr-FYFF	1	0
SPREAD7	7yr-FYFF	1	0
SPREAD8	10yr-FYFF	1	0
PCEC96	Real Personal Consumption Expenditures (Billions of Chained 2005 Dollars)	5	1
UNLPNBS	Nonfarm Business Sector: Unit Nonlabor Payments (Index 2005=100)	5	1
IPDNBS	Nonfarm Business Sector: Implicit Price Deflator (Index 2005=100)	5	1
OUTNFB	Nonfarm Business Sector: Output (Index 2005=100)	5	1
HOANBS	Nonfarm Business Sector: Hours of All Persons (Index 2005=100)	5	1
COMPNFB	Nonfarm Business Sector: Compensation Per Hour (Index 2005=100)	5	1
ULCNFB	Nonfarm Business Sector: Unit Labor Cost (Index 2005=100)	5	1
COMPRNFB	Nonfarm Business Sector: Real Compensation Per Hour (Index 2005=100)	5	1
OPHNFB	Nonfarm Business Sector: Output Per Hour of All Persons (Index 2005=100)	5	1
OPHPBS	Business Sector: Output Per Hour of All Persons (Index 2005=100)	5	1
ULCBS	Business Sector: Unit Labor Cost (Index 2005=100)	5	1
RCPHBS	Business Sector: Real Compensation Per Hour (Index 2005=100)	5	1
HCOMPBS	Business Sector: Compensation Per Hour (Index 2005=100)	5	1
OUTBS	Business Sector: Output (Ineex 2005=100)	5	1
HOABS	Business Sector: Hours of All Persons (Index 2005=100)	5	1
IPDBS	Business Sector: Implicit Price Deflator (Index 2005=100)	5	1
CP	Corporate Profits After Tax	5	0
GDPDEF	Gross Domestic Product: Implicit Price Deflator	5	0
PRFI	Private Residential Fixed Investment	5	0
SP500	S&P500 Index	5	0