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JØRGEN LANDSEM,
ERLEND NJØLSTAD,
KENNETH
SÆTERHAGEN
PAULSEN,
ØRJAN ROBSTAD AND
MAGNUS ÅSTEBØL

NORGES BANK

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Finding DORY*

Jørgen Landsem, Erlend Njølstad, Kenneth Sæterhagen Paulsen,

Ørjan Robstad and Magnus Åstebøl

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Abstract

This paper describes the semi-structural model DORY used by Norges Bank as a link between raw data, sector experts and the core policy model NEMO. While the primary objective in NEMO is to analyse business cycle fluctuations and monetary policy, DORY is used to identify the underlying trends in the main macro variables in Norway. DORY has been gradually developed over the last couple of years and has now been estimated using state of the art Bayesian estimation techniques.

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1 Introduction

This paper documents a semi-structural model for Norway named DORY. DORY is a flexible, unified framework used to identify underlying trends in the main macro variables in Norway. We specify a structure with cyclical interactions between the variables, common trends and variable-specific trends. The model includes information on a large set of observables and imposes simple economic relationships between the variables.

Norges Bank has always relied on a combination of models and expert judgment to produce forecasts. Our main policy model NEMO is used for analyzing and forecasting business cycle fluctuations.[1] Input data to, and forecasts from, NEMO are primarily in terms of deviation from a steady state (hereafter referred to as gaps). DORY complements NEMO by transforming raw data into gaps. This is done by decomposing data into gaps, trends, measurement errors and the steady state.

Norges Bank has used different methods to estimate gaps. For the output gap, a large set of indicators and suite of models has been developed, see Hagelund et al. (2018). For other gap estimates, Norges Bank has typically relied on univariate filters in combination with expert judgement. One issue with most univariate filters is that as real-time properties are quite poor, the historical estimates are revised substantially when new data becomes available. They also ignore potential common trends across macroeconomic variables. It is for example natural to assume that underlying trends in household credit and house prices are related. Using data on both house prices and credit could therefore be useful when estimating the respective trends. In fact, it has been shown that imposing some simple economic structure and adding information about developments in other variables can improve the precision of gap estimation (see e.g Stock and Watson (1989) and Basistha and Startz (2008)).

Another important drawback with univariate filters is that simple accounting relationships do not necessarily add up. For example, cyclical fluctuations in demand components should add up to the output gap at the same time as the underlying trend in demand components should add up to trend GDP. Estimating the gaps of the GDP components independently of the output gap could lead to trend and gap estimates that do not add up, especially if there is a large degree of fluctuation in inventories. These issues can easily be handled within a state space framework. It is also useful to have a unified

[1]See Kravik and Mimir (2019) for documentation of NEMO.

framework that allows experts to apply judgement in a consistent manner. For example, if the output gap is revised, this should have consequences for the gaps of the demand components.

DORY is a large, flexible system with many parameters and unobserved variables. We apply Bayesian techniques to estimate the model and use a version of the Kalman filter to find the most likely gap estimate of the variables given the raw data, model equations and parameter estimates. We also allow for judgement endogenously within the model.

The rest of the paper is organized as follows. Section 2 presents the data and transformations. Section 3 describes the structure of the model and Section 4 includes an overview of the model estimation. Some model properties and results are presented in Section 6.

2 Data and transformations

The dataset used in the estimation of DORY comprises annual data and runs from 1990 to 2019.[2] The observable domestic variables are mainland GDP and the associated demand components, house prices, credit, wages, disposable income, inflation, unemployment rate and the policy rate. Finally, we have the observed variables from the financial markets, which include the nominal exchange rate, nominal interest rate differential between Norway and its trading partners and the oil price in USD. The data sources are Statistics Norway, Thomson Reuters and Norges Bank.

The purpose of the model is to decompose the variation in the demeaned variables into stochastic, stationary trends and cyclical variation (gaps). The observable variables are transformed in the following way: First, variables that contain a trend with drift enter the model in growth rates. This includes the demand components, GDP, wages, consumer prices, house prices, unemployment rate, credit to households and enterprises and disposable income. The rest of these observables (exchange rate, interest rate, nominal interest rate differential between Norway and its trading partners and the oil price) are in levels. Second, all of these observable variables are demeaned.

In addition, we treat the output gap and the oil price gap as observable variables. For the output gap we use Norges Bank's estimate. Norges Bank already has a comprehensive

[2] Some of the data series start later. To deal with some missing values in the early part of the sample we apply the Kalman filter based on the model equation to estimate the missing values.

system in place for estimating the output gap, documented in Hagelund et al. (2018), which includes several models that are similar to DORY.

For the oil price gap we use a simple measure developed for running NEMO, as we do not expect there to be any information in DORY that can help us identify the trend in the oil price.[3]

For more information on the data, the variables and transformations, see Table 1 in the appendix.

3 The model

The state-space model we use is flexible and allows us to specify a structure with cyclical interactions between the variables, common trends and variable-specific trends. We organize the model into sections: a supply side, a demand side, prices and interest rates, and credit.

3.1 Supply

The stochastic process for changes in output (Δy_t) is given by equations (1)-(3).

$$\Delta y_t = \Delta \widehat{y}_t + \Delta y^*_t + e_{\tilde{y},t} \tag{1}$$

$$\widehat{y}_t = \lambda_{\widehat{y}} \cdot \widehat{y}_{t-1} - \alpha_{\widehat{r}} \cdot \widehat{r}_t + e_{\widehat{y},t} \tag{2}$$

$$\Delta y^*_t = \lambda_{y^*} \cdot \Delta y^*_{t-1} + e_{y^*,t} \tag{3}$$

Output growth (Δy) is decomposed into growth in the output gap ($\Delta\widehat{y}$), trend output growth (Δy^*) and a trend-level shock($e_{\tilde{y}}$). Equation (2) formulates the output gap as a function of the real interest rate gap ($\widehat{r}$), lagged output gap and shocks to the output gap ($e_{\widehat{y}}$).Trend output growth is modelled as an AR(1) process, subject to the shock e_{y^*}. This shock is meant to capture transitory changes in the path for trend output growth, such as changes in demographics and technological innovations. As in Garcia-Saltos et al.

[3]The oil price gap before 2015 is decomposed using an HP filter (lambda = 40 000), but is adjusted by sector expert judgement. After 2015 we have used a more mechanical method where the the oil price trend is a linear trend between 72 USD in 2015 Q3 and the endpoint of futures prices.

(2015), we also include a level-shock term ($e_{\tilde{y}}$) that is meant to capture one-time shifts in the level of the trend. Such shifts can capture real changes in production capacity, e.g. discovery and extraction of natural resources, but it can also capture noise in the data.

On the supply side, we also include the unemployment rate which is divided into changes in the unemployment gap ($\Delta\widehat{u}$) and changes in the trend unemployment rate (Δu^*).

$$\Delta u_t = \Delta\widehat{u}_t + \Delta u_t^* \tag{4}$$

$$\widehat{u}_t = \lambda_{\widehat{u}} \cdot \widehat{u}_{t-1} + \beta_{\widehat{y}} \cdot \widehat{y}_t + e_{\widehat{u},t} \tag{5}$$

$$\Delta u_t^* = \lambda_{u^*} \cdot \Delta u_{t-1}^* + e_{u^*,t} \tag{6}$$

The unemployment gap ($\widehat{u}$) is related to the output gap in equation (5). The change in the trend rate of unemployment (u^*) is subject to shocks (e_{u^*}), which permanently impact the trend rate of unemployment. Such shocks could for example be changes in labour market mismatch.

3.2 Demand

On the demand side, the model includes the same demand components as in NEMO, which sums up to our approximation of aggregate demand in mainland Norway (AD). The demand components are consumption (C), housing investment (I_H), corporate investment (I_C), oil investments (I_O), government expenditures (G), imports (IM), exports from the oil service sector (EX_O) and non-oil related exports (EX). Each demand component has the same structure in the model, denoted by X below.

$$\Delta X_t = \Delta\widehat{X}_t + \Delta X_t^* + e_{\tilde{X},t} \tag{7}$$

$$\widehat{X}_t = (\lambda_{\widehat{y}} + \lambda_{\widehat{X}}) \cdot \widehat{X}_{t-1} + e_{\widehat{X},t} \tag{8}$$

$$\Delta X_t^* = \Delta y_t^* + Z_{X^*,t} \tag{9}$$

$$Z_{X^*,t} = \lambda_{X^*} \cdot Z_{X^*,t-1} + e_{Z_{X^*},t} \tag{10}$$

As for output, the change in the demand component's is decomposed into cyclical variation, represented by the respective demand component's gap ($\Delta\widehat{X}_t$), trend growth (ΔX_t^*) and a trend-level shock ($e_{\tilde{X},t}$). For all demand components, the gaps are formulated as AR(1) processes denoted by equation (8). The persistence of the demand components gap is set equal to that of the output gap ($\lambda_{\widehat{y}}$) plus $\lambda_{\widehat{X}}$, where $\lambda_{\widehat{X}}$ is specific for each demand component. The trend growth rate of the demand components (ΔX^*) is given by the growth rate of trend output with temporary deviations, denoted by (Z_{X^*}). Equation (10) describes the law of motion for Z_{X^*}, which allows for the trend growth rate of a given demand component to deviate temporarily from the growth rate of trend output. Thus, trend output growth is imposed as a common trend for the demand components.

Further, two constraints are imposed on the demand components. First, equation (11) requires the weighted sum of the demand component gaps to be equal to the output gap, where $\delta_{X,t}$ represents the individual component's share of GDP. This constraint ensures that changes to the output gap are reflected in the demand components.

$$\widehat{y}_t = \sum_{X \in AD} \widehat{X}_t \cdot \delta_{X,t} \tag{11}$$

The second constraint is imposed such that the weighted sum of trend growth in the demand components is equal to trend growth in output. This is done by restricting the trend-level shocks to output growth to be equal to the trend-level shocks to the demand components and the contribution of inventories (Δl), see equation (12).

$$e_{\tilde{y},t} = \sum_{X \in AD} \delta_{X,t} \cdot e_{\tilde{X},t} + \Delta l_t \tag{12}$$

To see why, consider equation (13):

$$\Delta y_t = \sum_{X \in AD} \delta_{X,t} \cdot \Delta X_t + \Delta l_t \tag{13}$$

Inserting the decomposition of output from equation (1) and for the demand components from equation (7), we get:

$$\Delta\widehat{y}_t + \Delta y^*_t + e_{\tilde{y},t} = \sum_{X\in AD} \delta_{X,t} \cdot (\Delta\widehat{X}_t + \Delta X^*_t + e_{\tilde{X},t}) + \Delta l_t \tag{14}$$

Further, since the output gap and the weighted sum of the gaps of the demand components must be equal each period (see equation 11), this must also be true for the change in the output gap and the weighted sum of the change in each demand gap. Therefore, these terms cancel each other out, such that:

$$\Delta y^*_t + e_{\tilde{y},t} = \sum_{X\in AD} \delta_{X,t} \cdot (\Delta X^*_t + e_{\tilde{X},t}) + \Delta l_t \tag{15}$$

Finally, inserting for (12) yields that the trend in output is equal to the weighted sum of the trends in the demand components:

$$\Delta y^*_t = \sum_{X\in AD} \delta_{X,t} \cdot \Delta X^*_t \tag{16}$$

3.3 Prices

DORY also includes a set of price series: real wage growth, house price growth, the nominal exchange rate, an oil price gap and core inflation.

Real wage growth (Δw) is decomposed into changes in gap ($\Delta\widehat{w}$), trend growth rate (Δw^*) and a trend-level shock. Equation (18) links the real wage gap to the output gap, while equation (19) allows for shocks to trend wage growth with a decaying magnitude over time. Such shocks could be changes in the bargaining power of workers or terms of trade.

$$\Delta w_t = \Delta\widehat{w}_t + \Delta w^*_t + e_{\tilde{w},t} \tag{17}$$

$$\widehat{w}_t = \lambda_{\widehat{w}} \cdot \widehat{w}_{t-1} + \gamma \cdot \widehat{y}_t + e_{\widehat{w},t} \tag{18}$$

$$\Delta w^*_t = \lambda_{w^*} \cdot \Delta w^*_{t-1} + e_{w^*,t} \tag{19}$$

In addition to wages, we also include disposable income in the model. Disposable income is also divided into gap, trend and a trend-level shock.

$$\Delta w_{d,t}^{*} = \Delta C_{t}^{*} + e_{w_d,t} \tag{20}$$

$$\widehat{w}_{d,t} = \widehat{w}_{t} + \beta_{w_d} \cdot \widehat{y}_{t} - \alpha_{w_d} \cdot \widehat{r}_{t} + e_{w_d,t} \tag{21}$$

Trend growth in disposable income (Δw_{d}^{*}) is linked to trend growth in consumption (ΔC^{*}). Data on disposable income is introduced in order to better identify the consumption gap. Further, the disposable income gap ($\widehat{w}_{d}$) is linked to the wage gap ($\widehat{w}$), output gap ($\widehat{y}$) and the real interest rate gap ($\widehat{r}$). $\widehat{y}$ enters in equation (21) to capture changes in employment while $\widehat{r}$ captures net interest expense.

House prices are also included in the model and are decomposed into gap, trend and a trend-level shock as well. To help identify the gap for house prices, the housing investment gap and the real interest rate gap are introduced in equation (23). Cyclical changes in house prices and residential investment are expected to be correlated, while a negative relationship between the house price gap and the real interest rate gap is expected. Equation (24) allows for time-varying changes to trend growth in house prices.

$$\Delta P_{H,t} = \Delta \widehat{P_{H,t}} + \Delta P_{H,t}^{*} + e_{\tilde{P}_H,t} \tag{22}$$

$$\widehat{P}_{H,t} = \beta_{P_H,I_H} \cdot \widehat{I}_{H,t} - \gamma_{r} \cdot \widehat{r}_{t} + e_{\widehat{P}_H,t} \tag{23}$$

$$\Delta P_{H,t}^{*} = \lambda_{P_H^{*}} \cdot \Delta P_{H,t-1}^{*} + e_{P_H^{*},t} \tag{24}$$

An equation for the inflation gap ($\widehat{\pi}$) is included, defined as the difference between core inflation and the inflation target. The time-varying dynamics of the inflation gap are modelled as an AR(1) process.

$$\widehat{\pi}_{t} = \lambda_{\widehat{\pi}} \cdot \widehat{\pi}_{t-1} + e_{\widehat{\pi},t} \tag{25}$$

The exchange rate is decomposed into an exchange rate gap ($\widehat{s}$) and a trend (s^{*}). The gap is determined by a risk premium on the exchange rate (rp_{t}) and the interest rate differential between Norway and its trading partners (r_{d}). The risk premium is a function of its own lag, the oil price gap (op_t) and a shock, $e_{rp,t}$, that captures other factors that could affect the risk premium, such as international risk sentiment. The exchange rate trend is modeled as an AR(1) process.

$$s_t = \widehat{s}_t + s_t^* \tag{26}$$

$$\widehat{s}_t = rp_t - r_{d,t} \tag{27}$$

$$rp_t = \lambda_{rp} \cdot rp_{t-1} + \beta_{rp,op} \cdot \widehat{op}_t + e_{rp,t} \tag{28}$$

$$s_t^* = \lambda_s \cdot s_{t-1}^* + e_{s,t} \tag{29}$$

The oil price is divided into an oil price gap ($\widehat{op}$) and a trend level for the oil price (op^*). The oil price gap is linked to the gap for oil investment ($\widehat{I}_H$) and is subject to shocks ($e_{\widehat{op}}$). The trend oil price (op_t^*) is assumed to follow an AR(1) process.

$$op_t = \widehat{op}_t + op_t^* \tag{30}$$

$$\widehat{op}_t = \lambda_{\widehat{op}} \cdot op_{t-1} + \beta_{op,I_O} \cdot \widehat{I}_{O,t} + e_{\widehat{op},t} \tag{31}$$

$$op_t^* = \lambda_{op^*} \cdot op_{t-1}^* + e_{op^*,t} \tag{32}$$

3.4 Interest rate and credit

A measure of the real interest rate[4] and two credit variables, household and enterprise credit, are included in the model.

The real interest rate (r) is decomposed into the real interest rate gap ($\widehat{r}$) and the neutral real interest rate (r^*).

$$r_t = \widehat{r}_t + r_t^* \tag{33}$$

$$\widehat{r}_t = \lambda_{\widehat{r}} \cdot \widehat{r}_{t-1} + \alpha_{\widehat{y}} \cdot \widehat{y}_t + \alpha_{\widehat{\pi}} \cdot \widehat{\pi}_t + e_{\widehat{r},t} \tag{34}$$

$$r_t^* = \sigma \cdot \Delta y_t^* + z_{r^*,t} \tag{35}$$

$$z_{r^*,t} = \lambda_z \cdot z_{t-1} + e_{z_{r^*},t} \tag{36}$$

Equation (34) is a simple Taylor rule, linking the real interest rate gap to the output gap and the inflation gap ($\widehat{\pi}$). The neutral real interest rate in equation (35) is linked to trend output growth with temporary deviations described by the stochastic process z_{r*}.

Changes in household credit $\Delta b_{h,t}$ and enterprise credit $\Delta b_{e,t}$ are decomposed into gap, trend and a trend-level shock, similar to equation (7).

$$\Delta b_{h,t} = \Delta \widehat{b_{h,t}} + \Delta b_{h,t}^* + e_{\tilde{b}_h,t} \tag{37}$$

$$\Delta b_{e,t} = \Delta \widehat{b_{e,t}} + \Delta b_{e,t}^* + e_{\tilde{b}_e,t} \tag{38}$$

Equation (39) relates household credit gap ($\widehat{b}_h$) to the house price gap ($\widehat{P}$). The lag term ($\lambda_{\widehat{b}_h}$) enables the household credit gap to be more persistent than the house price gap. The enterprise credit gap ($\widehat{b}_e$) is conditioned on the corporate investment gap ($\widehat{I}_C$).

$$\widehat{b}_{h,t} = \lambda_{\widehat{b}_h} \cdot \widehat{b}_{h,t-1} + \alpha_{P_H} \cdot \widehat{P}_{H,t} + e_{\widehat{b}_h,t} \tag{39}$$

[4] The real interest rate is defined as the three-month nominal interest rate minus core inflation. See Table 1 for additional information.

$$\widehat{b}_{e,t} = \lambda_{\widehat{b}_e} \cdot \widehat{b}_{e,t-1} + \alpha_{I_C} \cdot \widehat{I}_{C,t} + e_{\widehat{b}_e,t} \tag{40}$$

Trend credit growth for both households ($\Delta b^*_{h,t}$) and enterprises (Δb^*_e) is driven by the shocks ($e_{b^*_h}$ and $e_{b^*_e}$).

$$\Delta b^*_{h,t} = \lambda_{b^*_h} \cdot \Delta b^*_{h,t-1} + e_{b^*_h,t} \tag{41}$$

$$\Delta b^*_{e,t} = \lambda_{b^*_e} \cdot \Delta b^*_{e,t-1} + e_{b^*_e,t} \tag{42}$$

4 Estimation

4.1 Estimation

The model presented in Section 3 is a large, flexible system with many parameters and unobserved variables. This makes it challenging to estimate the parameters of the model. We apply Bayesian techniques, which makes it possible to combine prior beliefs about the parameters and the moments of the model and data. We use the prior information/beliefs to shrink the possible parameter space. In the estimation, we first use the Kalman filter to evaluate the likelihood, then the likelihood is optimized by using the Artificial Bee Colony-algorithm by Karaboga and Basturk (2007). The methods are implemented using the *NB toolbox*.[5] The specific estimation methods are further explained in Appendix B.

For some parameters, we have rather strong prior beliefs about the parameter values and impose relatively tight priors. This applies to $\lambda_{\hat{y}}$, which determines the persistence of the output gap (see Section 3.1).The combination of a well-established model system and expert judgement makes us confident in the estimate of the output gap that we use in the Monetary Policy Report. Hence, we use a tight prior on $\lambda_{\hat{y}}$. We also impose relatively tight priors for $\lambda_{\widehat{X}}$. The demand components inherit $\lambda_{\hat{y}}$, where $\lambda_{\widehat{X}}$ is the persistence of the gap of demand component X relative to the output gap (see Section 3.2). We thereby assume that the persistence of the demand component gaps is close to that of the output gap.

[5]The NB Toolbox has been developed by Norges Bank and freely available for downloading at https://github.com/Coksp1/NBTOOLBOX.

For other parameters, such as β_{P_H,I_H}, which links the housing investment gap and the house price gap (see equation 23), there is little information outside the model. In that case, we are more agnostic as to what the parameter should be and use a uniform distribution. The uniform distribution makes any value (within a truncated range) equally likely.

For the variance parameters, we have either used the inverse gamma distribution $\mathcal{IG}(\alpha, \beta)$, with shape parameter α and scale parameter β, or used a uniform distribution. In cases where we have little information from outside the model or prior beliefs about the parameter values, we have kept our priors relatively loose in order to let the data inform us of the parameter values.

Table 2 in the appendix summarizes the priors imposed in the estimation. For all priors, we have truncated the distribution so that the parameters lie within a range that makes the model interpretable. No posterior mode of the parameters is on the edges of the truncated prior distributions.

To make the model more stable and easier to estimate, we do not use time-varying parameters in the estimation. But when running the Kalman filter, we allow the shares of the demand components to vary over time. For more on the Kalman filter, see Appendix B.

4.1.1 System priors

System priors are priors about the model's features and behaviour as a system, such as the moments of model variables, or the models' forecast error variance decomposition (FEVD). Often, it is easier to formulate priors on the system as a whole, rather than on individual parameters. We follow Andrle and Benes (2013), and use both priors on individual parameters and system priors in the estimation.

Specifically, we impose priors on the variance of some of the gaps in the model. In addition, we impose a system prior that limits how much of the forecast error variance decomposition (FEVD) that the trend-level shocks explain for some of the variables. This prior ensures that most of the variation in the observable variables are explained by the trend and gap shocks. See Table 2 in Appendix D for an overview of the system priors.

5 Judgement

Even though DORY uses a relatively large information set to estimate the gaps, we may want to impose judgement on the filtering in DORY. This judgement is typically based on alternative models and a larger information set than what is feasible to include in DORY. For example, we could have a model or qualitative information suggesting that house prices are overvalued, while the gap in DORY could be close to zero. In this case, we would like to impose judgement to push the house price gap in DORY in a more positive direction.

In DORY, there are two ways of imposing judgement. The first is to condition on a specific value for the gap. This is done by making the relevant gap observable for the period in which we impose judgement, while the gap will be estimated for the other periods.

The other way of imposing judgement is by ”pushing” the gaps in a certain direction. We do this in two steps. First, we run the filter to find the gap without judgement. Then we change some values based on judgement. In the second step, we run the filter again conditioning on the new value of the gap for the given period. This way of imposing judgement is useful when we have a view on the value of the gap at a certain point in time, but still would like DORY to update the gaps in a consistent way in light of new information.

Both ways of imposing judgement ensure that the imposed judgement will influence the filtering of the other variables, based on the economic relationships in the model. During the forecasting process, some rounds of iterations are usually needed before the process converges and the gaps are used as observables in NEMO.

6 Results

In this section, we highlight some results and key properties of the model. Additional results are provided in Section C of the appendix. The results are model-driven (without any additional judgement).

Figure 1 shows Norges Banks output gap[6] decomposed into the different demand components' gaps. By assumption, the output gap will always be fully explained by the demand gaps. Note that Figure 1 shows the reduced form contributions, i.e. not the contributions from the shocks. Thus, the figure shows the propagation of shocks through demand and not the underlying shocks driving the cycle. The figure shows that the different investment components seem to be important drivers of the business cycle in Norway.

Figure 1: Decomposition of the output gap. Percent

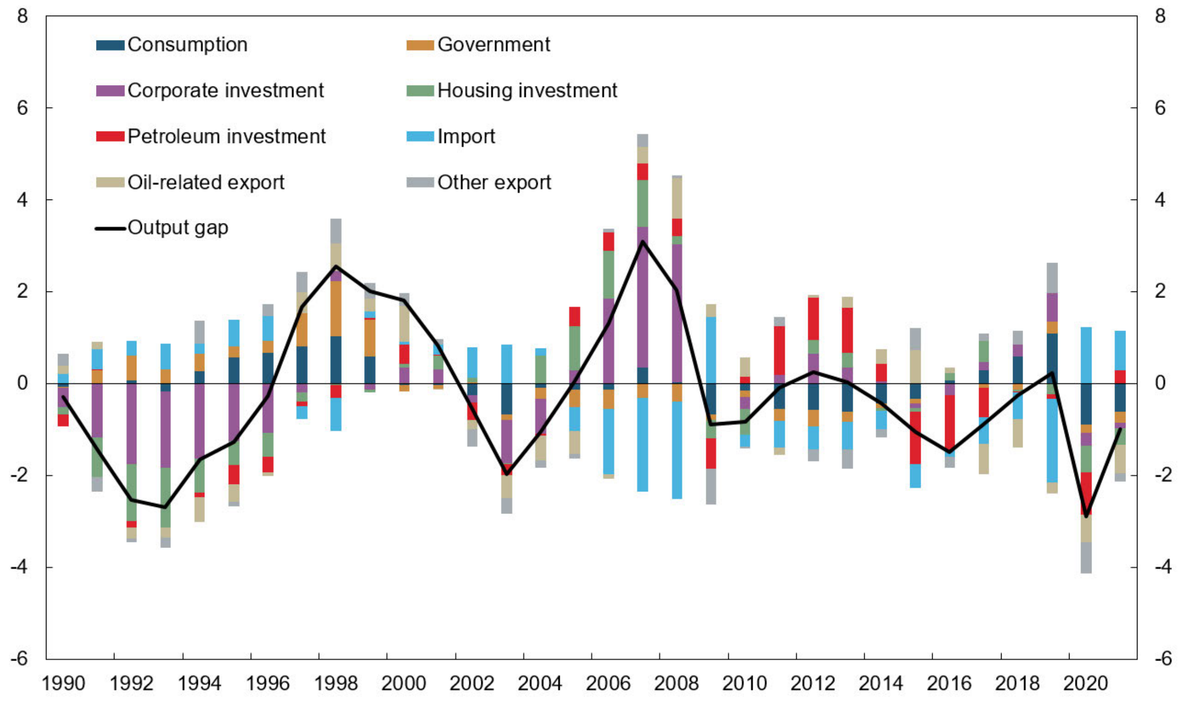
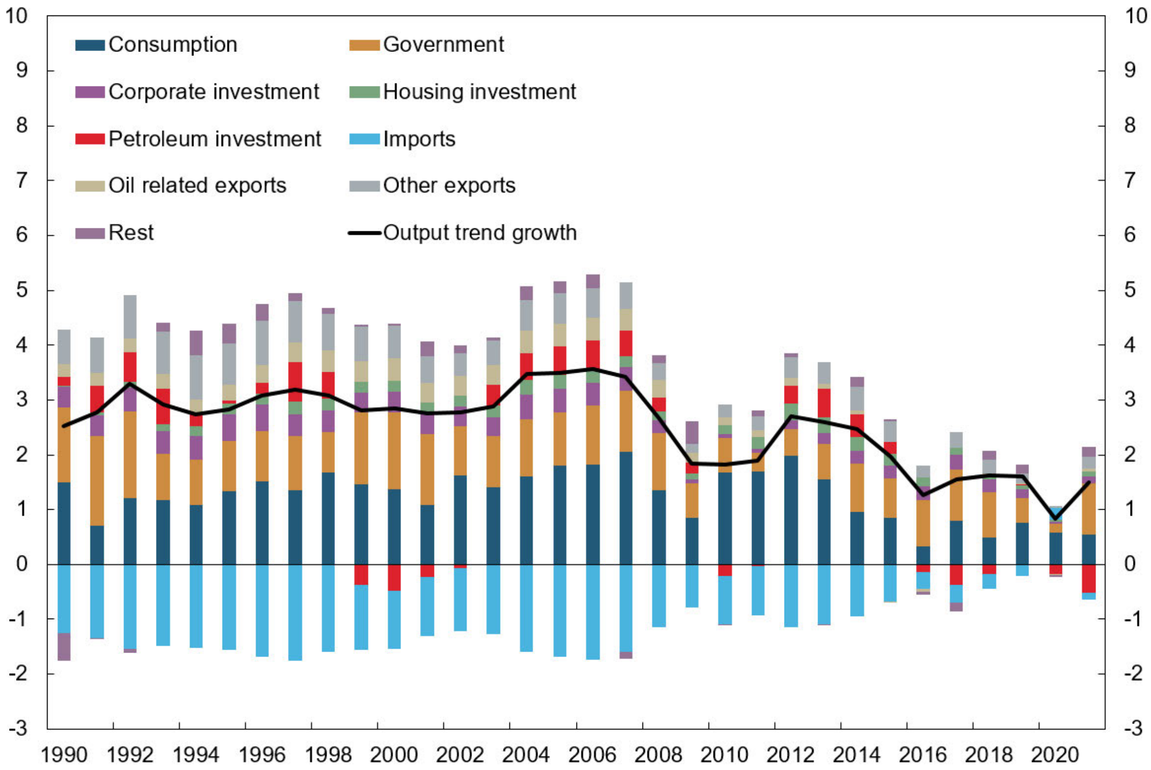


Figure 2 shows the contributions from the demand components' trends to trend output growth. As in Figure 1, this is reduced form contributions. By assumption, the trends of the demand components sum to trend output. The figure shows that the trend growth rates of consumption and government expenditure have historically accounted for a large

[6]The model treats the output gap as an observable variable, and we condition on Norges Banks official output gap. The methods used to estimate Norges Banks output gap are further described in Hagelund et al. (2018).

part of trend output growth.

Figure 2: Decomposition of trend output growth. Percent

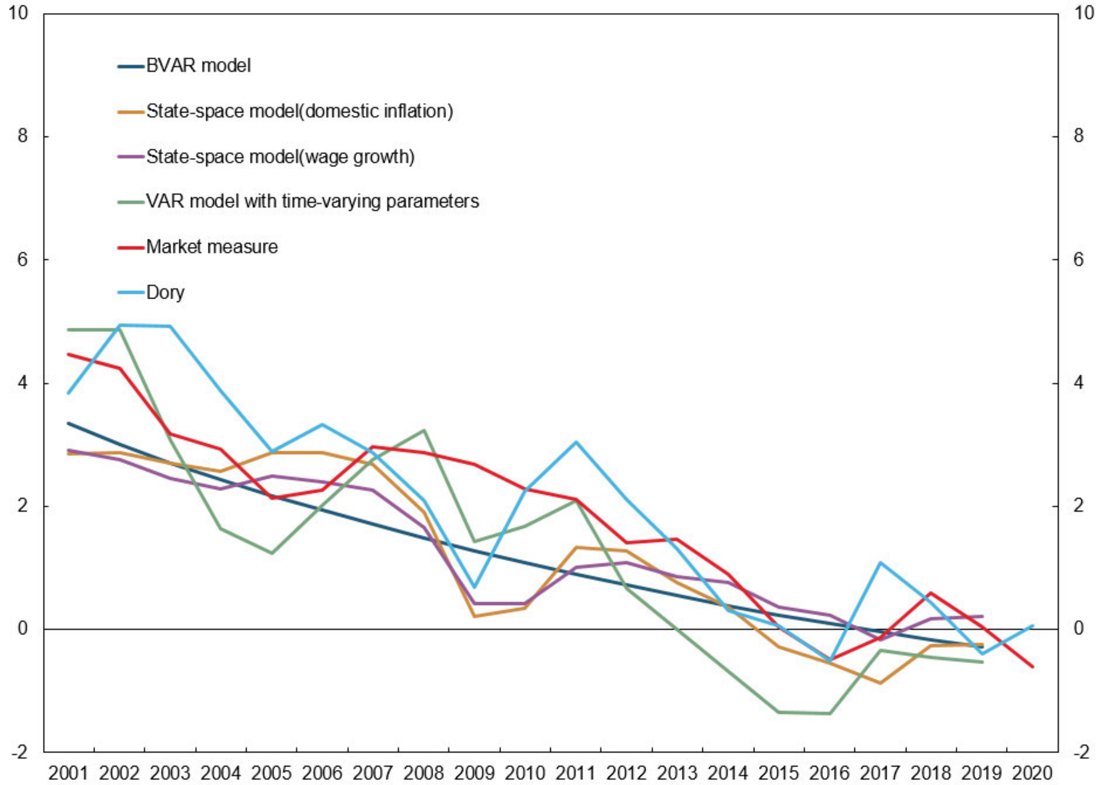


Next, in Figure 6 we decompose growth in mainland GDP. More specifically, the figure shows the estimated contributions from gap, trend-level and trend-growth shocks to variation in output growth around its mean. Among other things, it is evident that the estimated contributions from trend shocks to output growth have changed over the sample period. Before the financial crisis of 2008, the contribution from trend-growth shocks is consistently positive, while it is estimated to have been negative ever since the crisis. This is consistent with the observed fall in productivity growth over the same period. In more recent years, the gap component pulls in the opposite direction to that of the trend component. See Appendix C for decompositions of other observable variables.

Further, we also compare the estimated gaps from DORY to the gaps in Kravik and Mimir (2019). The gaps in Kravik and Mimir (2019) are estimated using different methods. Overall, the estimates are quite similar, see Figures 4 and 5 in Appendix C.

Lastly, Figure 3 shows the estimate of the neutral real money market interest rate in DORY. The neutral real interest rate shows a downward trend over time. The figure also shows other estimates of the neutral real interest rate for Norway from Brubakk et al. (2018), and we see that the estimate from DORY indicates a fall in the neutral real interest rate in line with previous estimates.

Figure 3: Estimates of the neutral real money market interest rate. Percent



7 Conclusion

This paper presents the DORY model used by Norges Bank as a link between raw data, sector expert judgement and the core policy model NEMO. DORY is used to identify the underlying trends in the main macro variables in Norway. In the model, we specify a structure with cyclical interactions between the variables, common trends and variable-specific trends. As a multivariate filter, the model incorporates information on several variables and economic relationships between the variables. This should help identify underlying trends in the variables of interest. Further, the multivariate setting/setup allows us to incorporate simple accounting relationships; for example, that the cyclical fluctuations in the demand components should add up to the output gap. The model is also a useful framework to allow for adding/applying judgement in a consistent manner. DORY has been gradually developed over the last couple of years and has now been estimated using Bayesian estimation techniques.

A Data

Table 1: Data

Observable variable	Description	Transformation
Consumption (c_t)	Private consumption. Fixed prices. Source: Statistics Norway	Demeaned growth rate.
Housing Investment (jh_t)	Gross investment in housing. Fixed prices. Source: Statistics Norway.	Demeaned growth rate.
Business investment (jc_t)	Firms gross investment. Fixed prices. Source: Statistics Norway.	Demeaned growth rate.
Government expenditures (g_t)	Public sector consumption and gross investment. Fixed prices. Source: Statistics Norway.	Demeaned growth rate.
Petroleum investment (jos_t)	Gross investment in oil activities and sea transport. Fixed prices. Source: Statistics Norway.	Demeaned growth rate.
Imports (im_t)	Imports for mainland Norway. Fixed prices. Source: Statistics Norway.	Demeaned growth rate.
Exports (e_t)	Exports from mainland Norway. Fixed prices. Source: Statistics Norway.	Demeaned growth rate.
Oil exports (exo_t)	Exports from oil services. Fixed prices. Source: Statistics Norway and Norges Bank.	Demeaned growth rate.
Inventories (l_t)	Inventories. Source: Statistics Norway	Demeaned growth rate.
Wage growth (w_t)	Real wage growth. Source: Statistics Norway.	Nominal wage growth minus the growth rate of CPI-ATE and demeaned.
Unemployment rate (u_t)	Registered unemployed as a share of the labour force. Source: Norwegian Labour and Welfare Administration (NAV) .	First differences of the unemployment rate.
Inflation (π_t)	CPI-ATE (CPI adjusted for taxes and excluding energy prices). Sources: Statistics Norway.	Growth rate of CPI-ATE minus the inflation target (2.5 percent until 2017 and 2 percent since).
House prices (ph_t)	Nominal house prices deflated by CPI-ATE. Sources: Eiendom Norge, Eiendomsverdi. Finn.no, Norges Bank and Statistics Norway .	Growth rate minus growth rate of CPI-ATE and demeaned.
Disposable income ($w_{d,t}$)	Real disposable income for households. Source: Statistics Norway.	Growth rate minus growth rate of CPI-ATE and demeaned.
Interest rate (r_t)	Real money market interest rate. Source: Norges Bank	3-month nominal interest rate, Norwegian Interbank Offered Rate (Nibor) minus the growth rate of CPI-ATE.
Exchange rate (s_t)	Import-weighted exchange rate measured against the currencies of 44 countries (I-44). Source: Norges Bank.	In logs and demeaned.
Interest rate differential Norway and trading partners ((r_d,t)	Difference between Norwegian money market interest rate and trading partners. Trade-weighted. Source: Refinitiv Datastream and Norges Bank.	Demeaned.
Credit to households ($b_{h,t}$)	Credit indicator for households (C2). Source: Statistics Norway.	Taken in logs, divided by CPI adjusted for taxes and excluding energy prices (CPI-ATE) and demeaned.
Credit to enterprises ($b_{e,t}$)	Credit indicator for non-financial enterprises (C3). Source: Statistics Norway.	Taken in logs, divided by CPI adjusted for taxes and excluding energy prices (CPI-ATE) and demeaned.
Output gap ($\widehat{y}_t$)	Norges Banks official estimate of the output gap. Documented in Hagelund et al. (2018)	
Oil price gap (op_t)	Brent blend USD per barrel. Sources: Norges Bank and Statistics Norway.	The oil price gap before 2015 is decomposed using an HP filter (lambda = 40 000), but is adjusted by sector expert judgement. After 2015 we have used a more mechanical method where the oil price trend is a linear trend between 72 USD in 2015 Q3 and the endpoint of futures prices.

B Algorithm

The DORY model can be put on the form

$$D(\theta_t)X_t = G(\theta_t)X_{t-1} + C(\theta_t)U_t, \tag{43}$$

$$\theta_t = F(\theta_{t-1}, X_{t-1}), \tag{44}$$

where X_t are the endogenous variables. U_t are the exogenous variables, with size N. Let the number of equations be given by M, which must also be the number of endogenous variables. $D(\theta_t)$, $G(\theta_t)$ and $C(\theta_t)$ are all matrices which are a function of potentially time-varying parameters θ_t. $D(\theta_t)$ and $G(\theta_t)$ has size MxM, while $C(\theta_t)$ has size MxN. θ_t is the parameter vector of size Q of the model. θ_t may change over time following the process $F(\theta_{t-1}, X_{t-1}) : \mathbb{R}^{\mathbb{M}} \rightarrow \mathbb{R}^{\mathbb{Q}}$.

The solution to the linear system in equation (43) given θ_t is

$$X_t = D(\theta_t)^{-1}G(\theta_t)X_{t-1} + D(\theta_t)^{-1}C(\theta_t)U_t, \tag{45}$$

or in a more compact representation

$$X_t = A(\theta_t)X_{t-1} + B(\theta_t)U_t. \tag{46}$$

B.1 Filtering and smoothing

We can rewrite the model in equation (46) into a state-space representation. The measurement equation can be posted as

$$Y_t = HX_t. \tag{47}$$

Y_t are the observable variables with size $O \times 1$, H is the observation matrix with size $O \times M$. The state equation linking the current state of the state variables with its own lags and some exogenous disturbances is given by (46). The disturbances (U_t) are assumed to be normally distributed with covariance matrix I,[7] i.e

[7] It is assumed that the exogenous disturbances are uncorrelated across time.

$$U_t \sim N(0, I). \tag{48}$$

Please see Hamilton (1994) Section 13.1, for a more thorough description of the state-space representation.

B.1.1 Kalman filter

The piecewise linear Kalman filter can be used to get estimates of state variables, or the unobservable variables, given the observable variables and the initial parameter values of the model.

Let us start out with some definitions. $X_{t|t-1} = E_{t-1}[X_t]$ is the expectation of X_t given information on the observed variables up until time $t-1$, $\theta_{t|t} = E_t[\theta_t]$ is the expectation of θ_t given information on the observed variables up until time t [8], while $X_{t|t} = E_t[X_t]$ is the expectation of X_t given information on the observed variables up until time t. The associated variance of the observation equation is then

$$\begin{aligned} F_t &= E[(Y_t - Y_{t|t-1})(Y_t - Y_{t|t-1})'] \\ &= HP_{t|t-1}H'. \end{aligned} \tag{49}$$

where $P_{t|t-1} = E[(X_t - X_{t|t-1})(X_t - X_{t|t-1})']$ is the error when forecasting X_t given information on the observed variables up until time $t-1$. We need F_t as we want to update the projection of $X_{t|t-1}$ given the new information on Y_t

$$X_{t|t} = X_{t|t-1} + P_{t|t-1}H'F_t^{-1}(Y_t - HX_{t|t-1}) \tag{50}$$

The associated variance is given by

$$\begin{aligned} P_{t|t} &= E[(X_t - X_{t|t})(X_t - X_{t|t})'] \\ &= P_{t|t-1} - P_{t|t-1}H'F_t^{-1}HP_{t|t-1} \end{aligned} \tag{51}$$

Before starting the same filtering step for $t+1$ we must produce the one-step ahead forecast of the different measures. This is done by first updating $\theta_{t|t}$ according to $F(\theta_{t|t-1}, X_{t|t})$,

[8] Be aware that we do a simplification and ignore the uncertainty in the estimate of θ_t.

and resolve the model at these parameter values, and applying the following

$$X_{t+1|t} = A(\theta_{t|t})X_{t|t}, \tag{52}$$

$$P_{t+1|t} = A(\theta_{t|t})P_{t|t}A(\theta_{t|t})' + B(\theta_{t|t})B(\theta_{t|t})', \tag{53}$$

To initialize the steps of the filter we use $X_{1|0} = 0$ and the solution to the fixed point problem

$$P_{1|0} = A(\theta_0)P_{1|0}A(\theta_0)' + B(\theta_0)B(\theta_0)', \tag{54}$$

The full likelihood for the model $\mathbb{M}$ over T periods given the initial values of the parameters θ_0 can be calculated as

$$\mathcal{L}(Y|\theta_0, \mathbb{M}) = \frac{TOlog(2\pi)}{2} + \frac{\sum_{t=1}^{T} \ell_t}{2}, \tag{55}$$

where

$$\ell_t = (2\pi)^{O/2}|F_t|^{-1/2}e^{-\frac{1}{2}(Y_t - HX_{t|t-1})'F_t^{-1}(Y_t - HX_{t|t-1})}, \tag{56}$$

and where Y is constructed by stacking Y_t over all time periods.

B.1.2 Kalman smoother

In contrast to the piecewise linear Kalman filter, the piecewise linear Kalman smoother uses all the information in the observable variables to estimate the unobservable variables, i.e. $X_{t|T} = E_T[X_t]$. The first part of the smoother is to run through the filter. Let us initialize $R_{T+1} = 0$, then by a backward recursion, starting with $t = T$, on the following equations you will get the smoothed estimates

$$R_t = A(\theta_{t|t})'R_{t+1}, \tag{57}$$

$$R_{t-1}^i = R_t^i + F_t^{-1}(Y_t - HX_{t|t-1}) - K_t'R_t, \tag{58}$$

where R_t^i refers to the elements of r_t that are restricted to the observed variables only and $K_t = A(\theta_{t+1})P_{t|t-1}H'F_t^{-1}$. Then finally, given R_t we can get the smoothed estimate of X_t from

$$X_{t|T} = X_{t|t-1} + P_{t|t-1}R_t. \tag{59}$$

Smoothed estimate of U_t can be found from

$$u_{t|T} = B(\theta_{t|t})^{-1}(X_{t|T} - A(\theta_{t|t})X_{t-1|T}) \tag{60}$$

for $t > 1$, while for $t = 1$ we get

$$u_{1|T} = B(\theta_{1|1})^{-1}(X_{1|T} - X_{1|0}). \tag{61}$$

B.2 Estimation

We follow Andrle and Benes (2013)) and estimate DORY with system priors. In the model we allow for time-varying parameters, which means that we are interested in estimating the initial value of the process θ_t, i.e θ_0. As normal in this literature we first formulate a set of marginal independent priors

$$p(\theta_0|\mathbb{M}) = p(\theta_0^1) \times \cdots \times p(\theta_0^Q), \tag{62}$$

where $\mathbb{M}$ indicates that the prior is set under the condition of knowing the model. In additional we want to apply priors to some properties of the model. These properties will be a function of the model and the parameters of the model, i.e. $\mathbb{Z} = H(\mathbb{M}, \theta_0)$. These properties will themselves form a probabilistic model $\mathbb{Z} \sim h(\mathbb{S})$, where h is a distribution function with parameters $\mathbb{S}$. This will result in the system prior on the form $p(\mathbb{S}|\theta_0, \mathbb{M})$, and the joint prior is then given by

$$p(\theta_0|\mathbb{M}, \mathbb{S}) = p(\mathbb{S}|\theta_0, \mathbb{M}) \times p(\theta_0|\mathbb{M}). \tag{63}$$

A Bayesian approach constitutes of estimating the parameters θ_0 by maximization of the posterior distribution given by

$$p(\theta_0|Y, \mathbb{M}, \mathbb{S}) = \frac{\mathcal{L}(Y|\theta_0, \mathbb{M}) \times p(\theta_0|\mathbb{M}, \mathbb{S})}{p(Y|\mathbb{M})} \propto \mathcal{L}(Y|\theta_0, \mathbb{M}) \times p(\theta_0|\mathbb{M}, \mathbb{S}). \tag{64}$$

C Additional results

Figure 4: Estimated gaps. DORY and Kravik and Mimir (2019). Percent

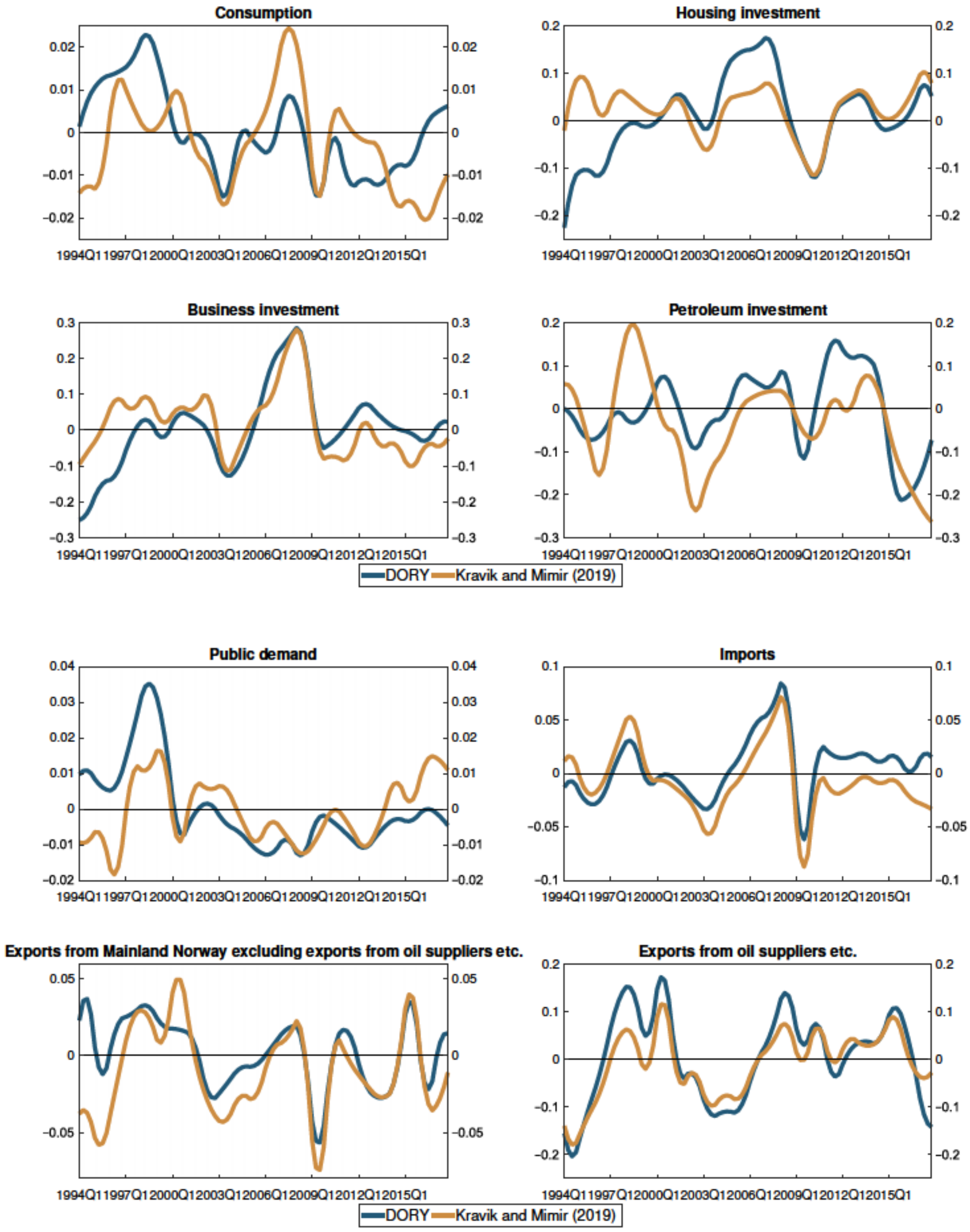


Figure 5: Estimated gaps. DORY and Kravik and Mimir (2019). Percent

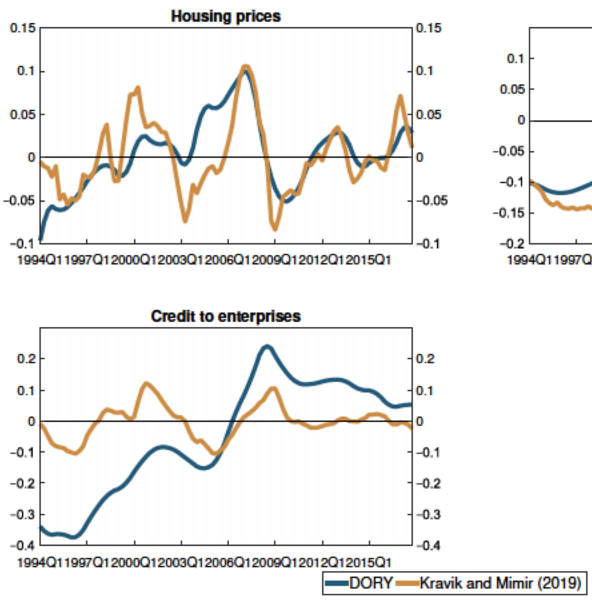
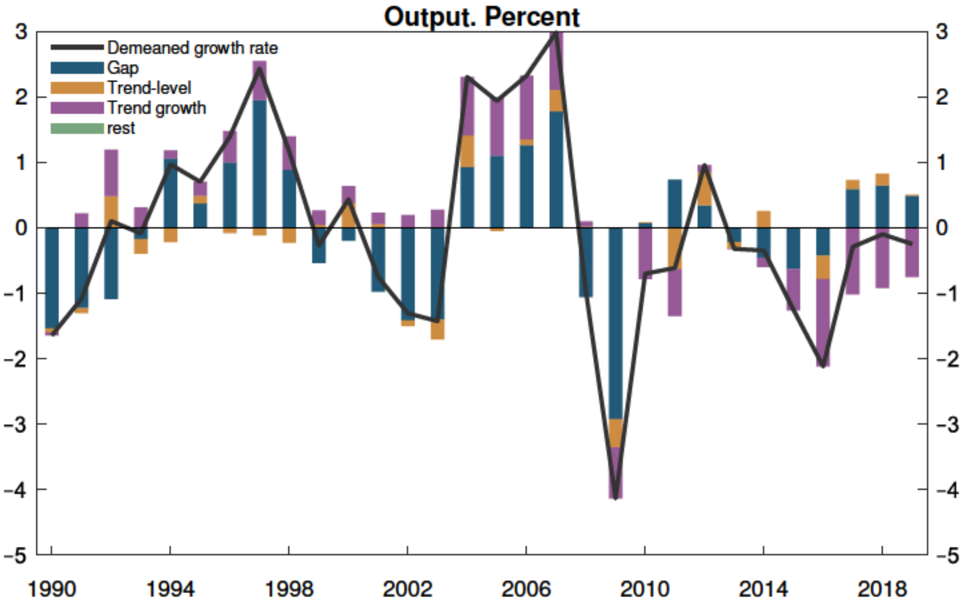


Figure 6: Decompositions of observables



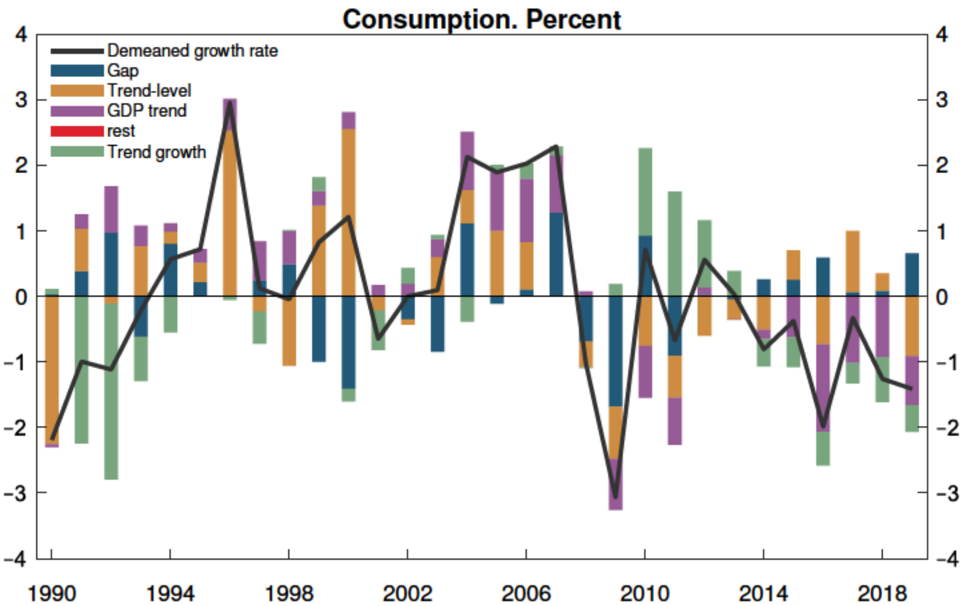
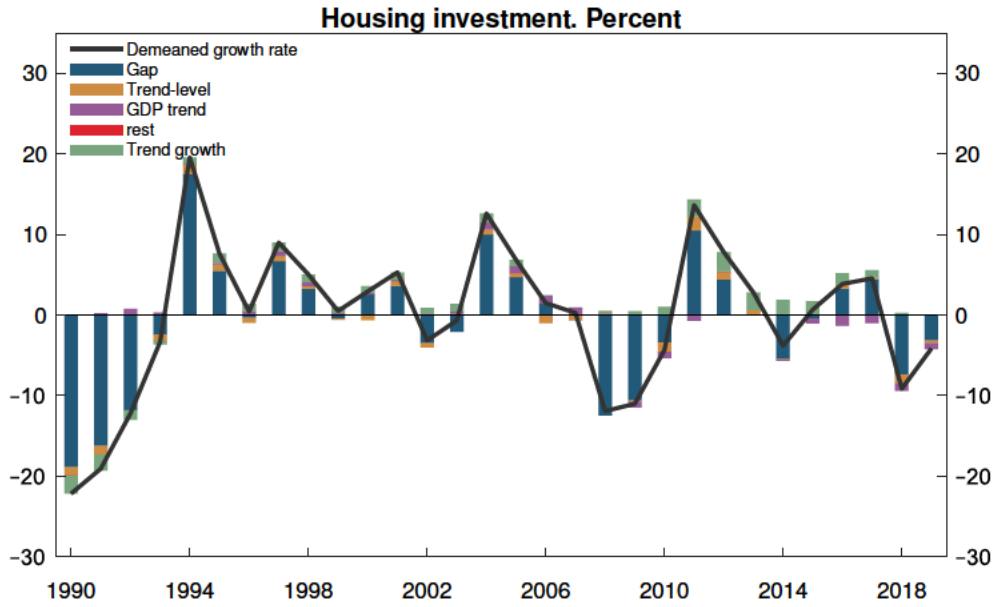


Figure 7: Decompositions of observables



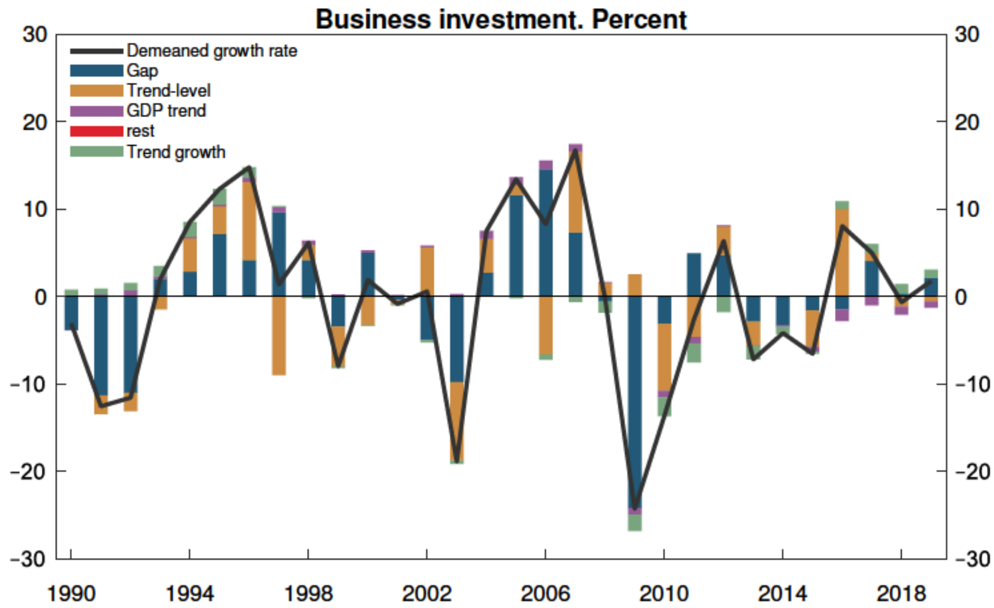
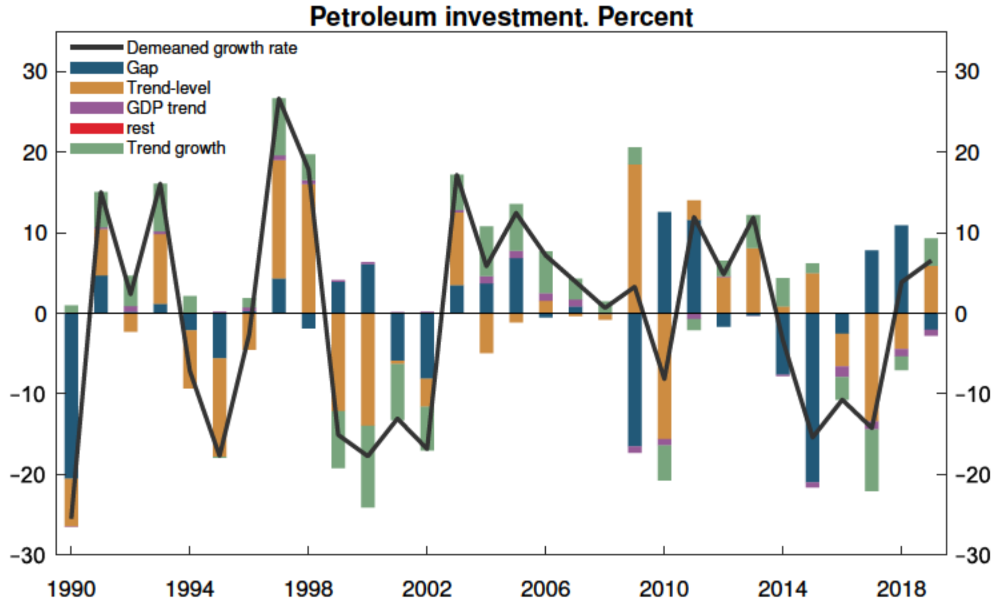


Figure 8: Decompositions of observables



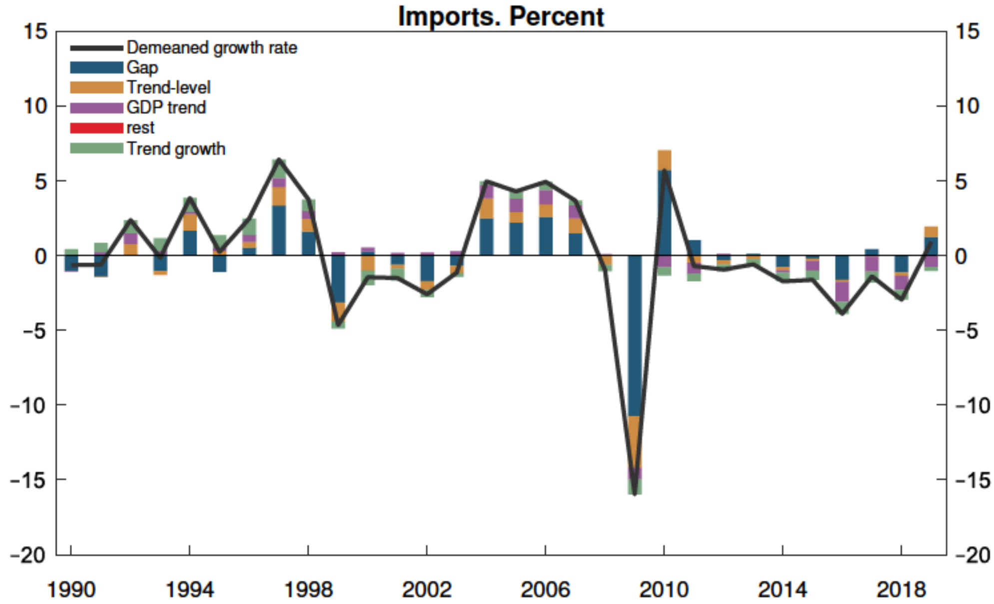
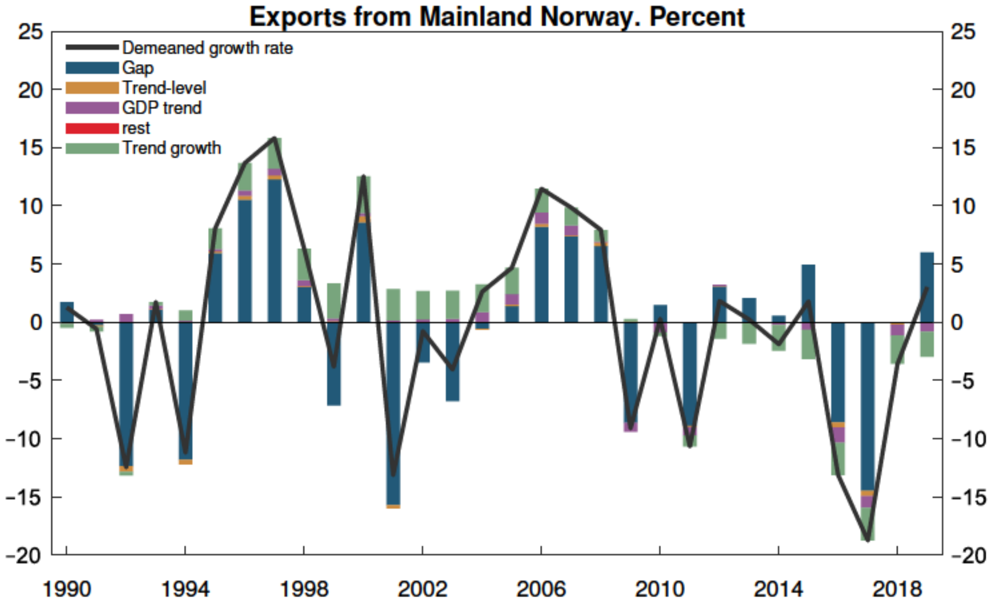


Figure 9: Decompositions of observables



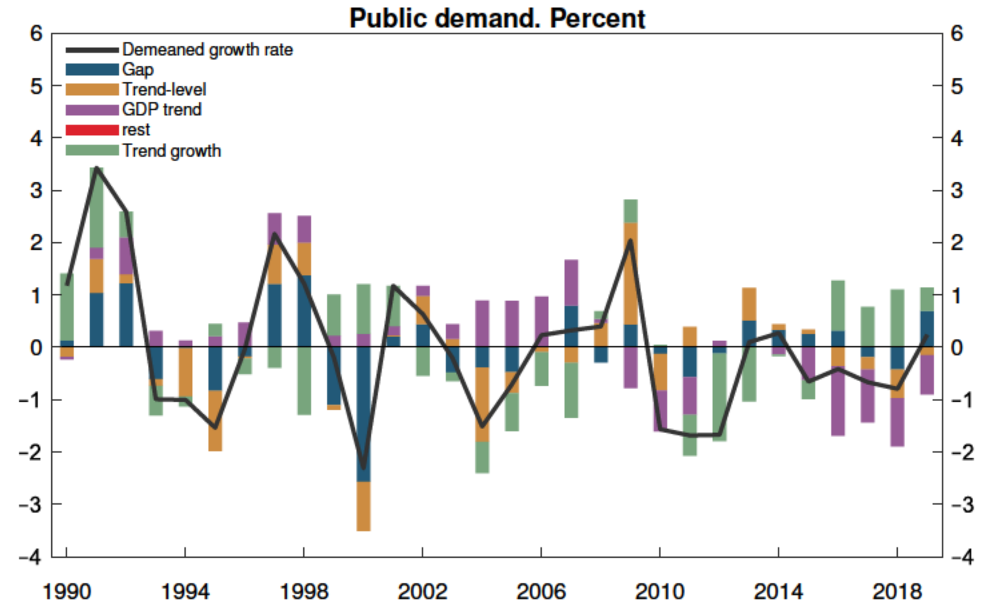


Figure 10: Decompositions of observables

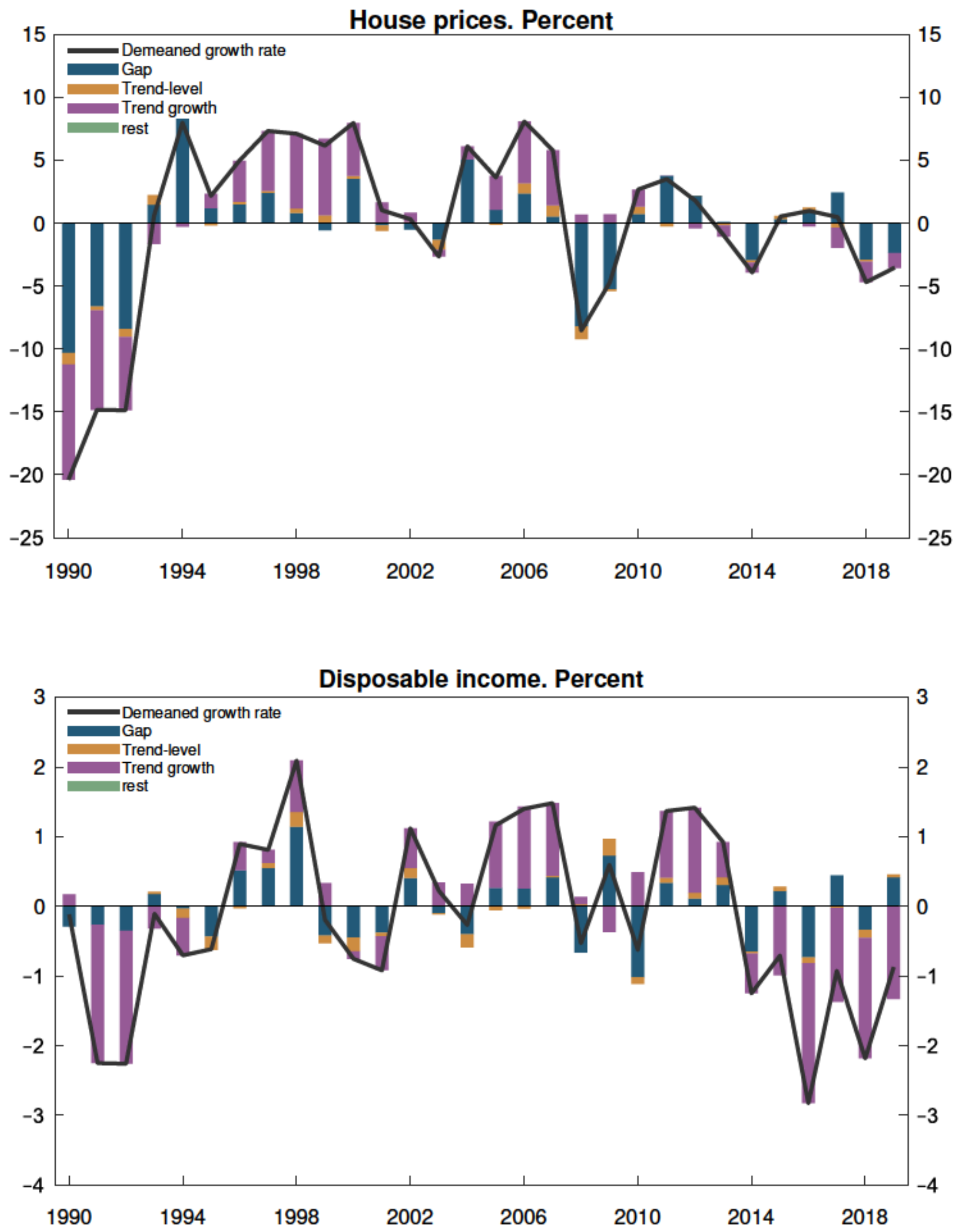
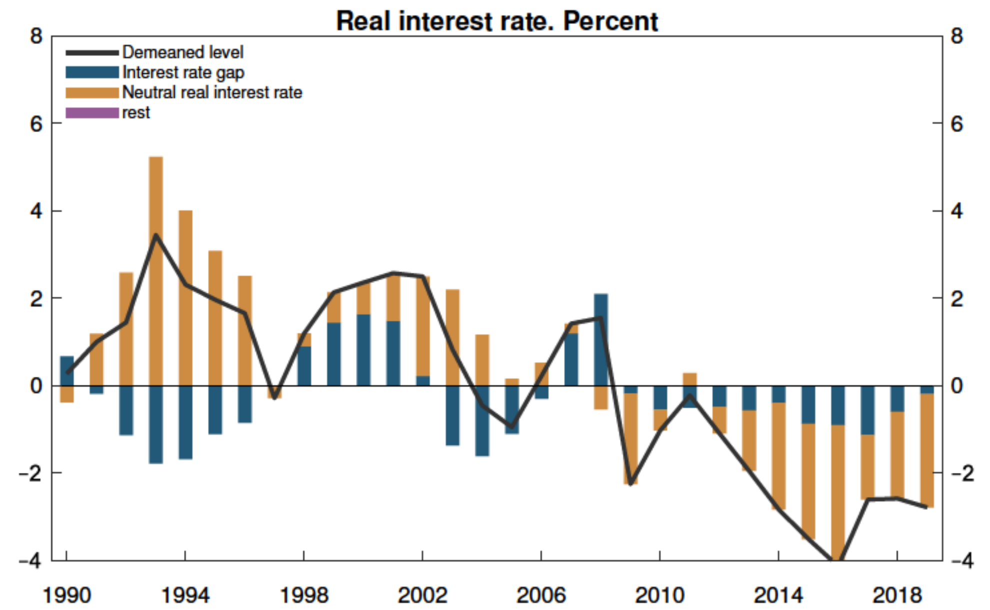


Figure 11: Decompositions of observables



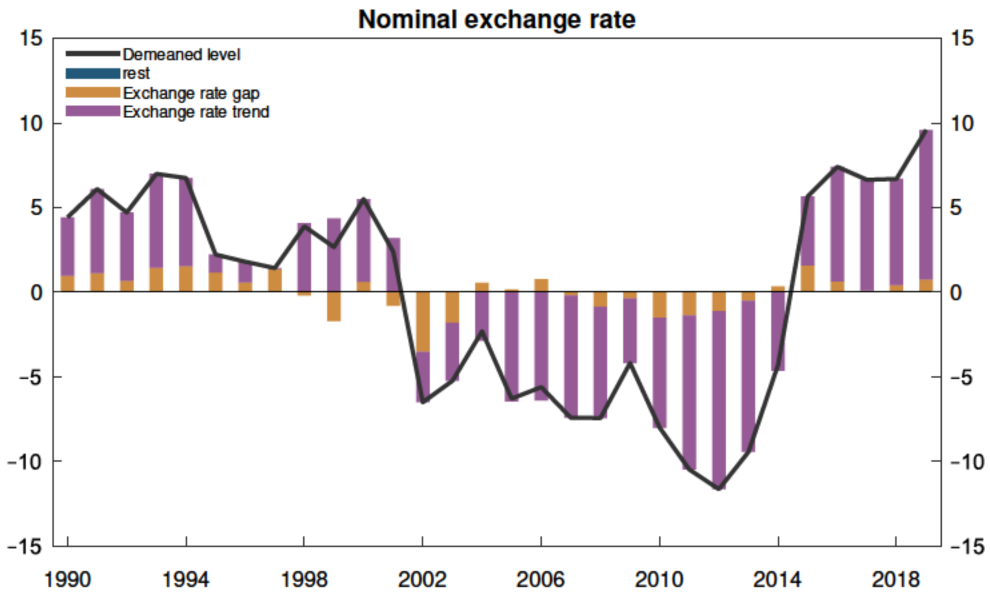
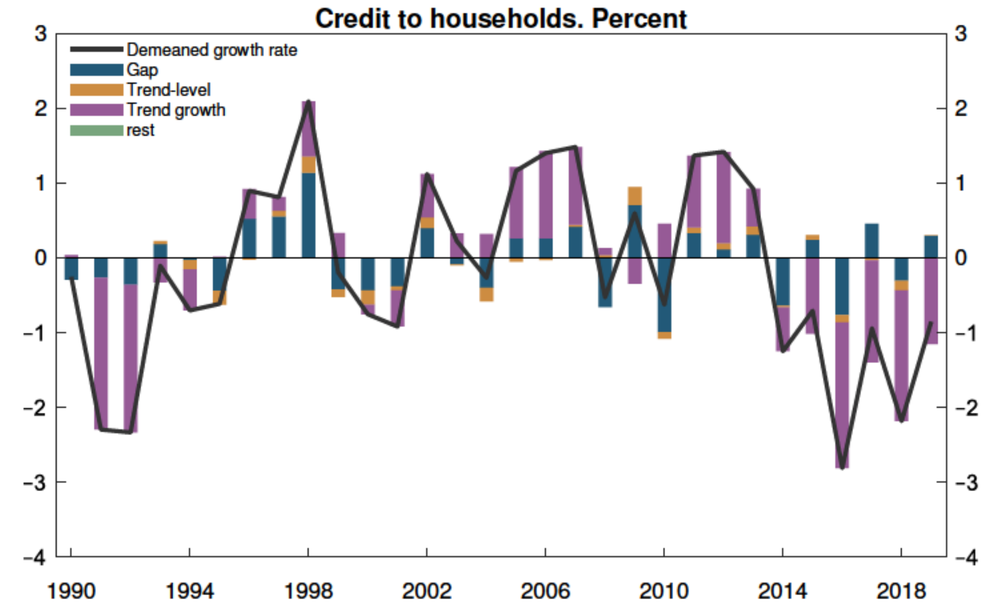


Figure 12: Decompositions of observables



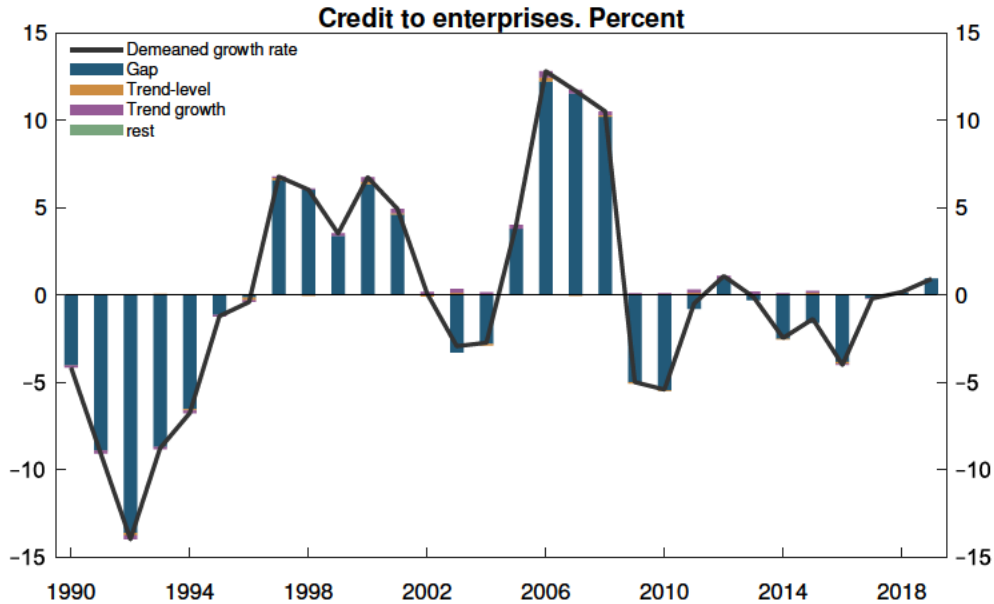
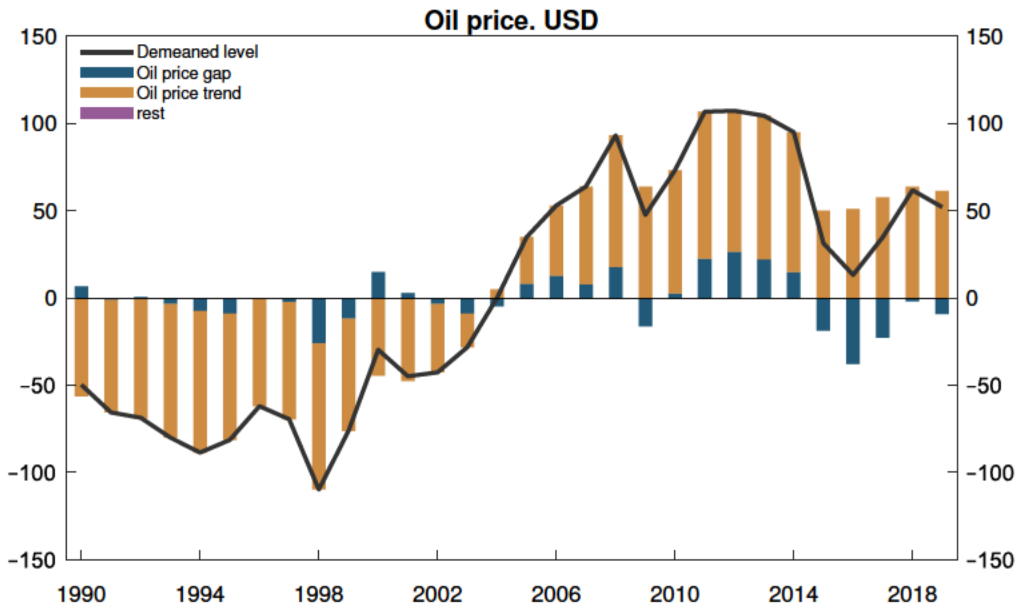


Figure 13: Decompositions of observables



D Priors

Table 2: Marginal priors

Parameter	Mean	Standard deviation	Distribution	Lower bound	Upper bound	Posterior Mode
std of $s*$	3	5	inverse gamma	0	5	3.95
std of z_{r*}	3	5	inverse gamma	0	5	2.06
std of $\tilde{e}_y$	1	10	inverse gamma	0	5	0.37
std of e_{y^*}	1	10	inverse gamma	0	5	0.45
std of $e_{\widehat{y}}$	2	10	inverse gamma	0	5	1.89
std of $\tilde{e}_w$	3	5	inverse gamma	0	5	0.73
std of e_{w^*}	2	10	inverse gamma	0	5	0.6
std of $e_{\widehat{w}}$	1	10	inverse gamma	0	5	0.25
std of e_{u^*}	2	10	inverse gamma	0	5	1.91
std of $e_{\widehat{u}}$	1	10	inverse gamma	0	5	0.32
std of $\tilde{e}_{rp}$	3	5	inverse gamma	0	5	2.11
std of $e_{\widehat{r}}$	3	5	inverse gamma	0	5	0.92
std of $\tilde{e}_{P_H}$	3	5	inverse gamma	0	10	1.11
std of $e_{P_H^*}$	1	10	inverse gamma	0	10	1.92
std of $e_{\widehat{P_H}}$	5	10	inverse gamma	0	10	1.19
std of e_{op^*}	3	5	inverse gamma	0	5	4.85
std of $e_{\widehat{op}}$	3	5	inverse gamma	0	5	4.94
std of $e_{\widehat{\pi}}$	3	5	inverse gamma	0	5	0.55
std of $\tilde{e}_C$	4	10	inverse gamma	0	8	3.3
std of e_{C^*}	4	10	inverse gamma	0	12	4.58
std of $e_{\widehat{C}}$	4	10	inverse gamma	0	12	1.07
std of $\tilde{e}_{I_O}$	4	10	inverse gamma	0	8	7.18
std of $e_{I_O^*}$	4	10	inverse gamma	0	12	6.19
std of $e_{\widehat{I_O}}$	4	10	inverse gamma	0	12	10.17
std of $\tilde{e}_{I_H}$	4	10	inverse gamma	0	8	1.77
std of $e_{I_H^*}$	4	10	inverse gamma	0	12	1.24
std of $e_{\widehat{I_H}}$	4	10	inverse gamma	0	12	5.2
std of $\tilde{e}_{I_C}$	4	10	inverse gamma	0	8	5.95
std of $e_{I_C^*}$	4	10	inverse gamma	0	12	1.42
std of $e_{\widehat{I_C}}$	4	10	inverse gamma	0	12	6.71
std of $\tilde{e}_{I_M}$	Calibrated					1.2
std of $e_{I_M^*}$	Calibrated					0.7
std of $e_{\widehat{I_M}}$	Calibrated					2.1
std of $\tilde{e}_G$	4	10	inverse gamma	0	8	1.38
std of e_{G^*}	4	10	inverse gamma	0	12	2.03
std of $e_{\widehat{g}}$	4	10	inverse gamma	0	12	1.24
std of $\tilde{e}_{EX_O}$	4	10	inverse gamma	0	8	1.41
std of $e_{EX_O^*}$	4	10	inverse gamma	0	12	1.33
std of $e_{\widehat{EX_O}}$	4	10	inverse gamma	0	12	8.19
std of $\tilde{e}_{EX}$	4	10	inverse gamma	0	8	1.55
std of e_{EX^*}	4	10	inverse gamma	0	12	0.83
std of $e_{\widehat{EX}}$	4	10	inverse gamma	0	12	2.14
std of $e_{w_d^*}$	1	2	inverse gamma	0	3	0.42
std of $\tilde{e}_{w_d}$	1	2	inverse gamma	0	3	0.42
std of $e_{\widehat{w_d}}$	1	2	inverse gamma	0	3	0.51
std of $\tilde{e}_{b_H}$	1	10	inverse gamma	0	5	0.29
std of $e_{b_H^*}$	0.50	10	inverse gamma	0	5	0.77
std of $e_{\widehat{b_h}}$	1	10	inverse gamma	0	5	0.28
std of $\tilde{e}_{b_e}$	1	10	inverse gamma	0	5	0.39
std of $e_{b_e^*}$	1	10	inverse gamma	0	5	0.39
std of $e_{\widehat{b_e}}$	1	10	inverse gamma	0	5	1

Parameter	Mean	Standard deviation	Distribution	Lower bound	Upper bound	Posterior Mode
σ	1	2	normal	0.90	1.10	0.97
λ_z	0.50	0.20	beta	0	1	0.52
λ_{y^*}	0.90	0.10	gamma	0	1	0.83
$\lambda_{\widehat{y}}$	Calibrated					0.66
λ_{w^*}	0.50	0.20	beta	0	1	0.45
$\lambda_{\widehat{w}}$	0.50	0.20	beta	0	1	0.56
λ_{u^*}	0.50	0.20	beta	0	1	0.51
$\lambda_{\widehat{u}}$	0.50	0.20	beta	0	1	0.57
λ_{s^*}	0.50	0.20	beta	0	1	0.77
λ_{rp}	0.50	0.20	beta	0	1	0.66
$\lambda_{\widehat{r}}$	0.50	0.20	beta	0	1	0.31
$\lambda_{P_H^*}$	0.90	0.10	gamma	0	1	0.82
λ_{op^*}	0.50	0.20	beta	0	1	0.97
$\lambda_{\widehat{op}}$	0.50	0.20	beta	0	1	0.35
$\lambda_{\widehat{\pi}}$	0.50	0.20	beta	0	1	0.71
λ_{C^*}	0.90	0.20	gamma	0	1	0.81
$\lambda_{\widehat{C}}$	0	0.10	normal	-1	0.25	0.01
$\lambda_{I_O^*}$	0.90	0.10	gamma	0	1	0.83
$\lambda_{\widehat{I_O}}$	0	0.10	normal	-1	0.25	-0.94
λ_{jh^*}	0.90	0.05	gamma	0	1	0.87
$\lambda_{\widehat{I_H}}$	0	0.10	normal	-1	0.25	0.09
$\lambda_{I_{C^*}}$	0.90	0.05	gamma	0	1	0.89
$\lambda_{\widehat{I_C}}$	0	0.10	normal	-1	0.25	0
λ_{IM^*}	0.90	0.20	gamma	0	1	0.74
$\lambda_{\widehat{IM}}$	0	0.20	normal	-1	0.25	-0.09
λ_{G^*}	0.90	0.10	gamma	0	1	0.53
$\lambda_{\widehat{G}}$	0	0.10	normal	-1	0.25	0.03
$\lambda_{EX_O^*}$	0.90	0.10	gamma	0	1	0.86
$\lambda_{\widehat{EX_O}}$	0	0.20	normal	-1	0.25	0.08
λ_{EX^*}	0.90	0.10	gamma	0	1	0.82
$\lambda_{\widehat{EX}}$	0	0.20	normal	-1	0.25	-0.15
$\lambda_{b_{h^*}}$	0.80	0.10	gamma	0	1	0.85
$\lambda_{\widehat{b_h}}$	0.85	0.05	gamma	0	1	0.87
λ_{b_*}	0.80	0.10	gamma	0	1	0.68
$\lambda_{\widehat{b_e}}$	0.80	0.10	gamma	0	1	0.81
γ_r	0	5	uniform	0	5	0.15
γ	Calibrated					0.29
$\beta_{rp,op}$	0	1	uniform	0	1	0.04
β_{P_H,I_H}	0	0.30	normal	-1	2	0.48
β_{op,I_O}	0	1	uniform	0	1	0.9
β_{w_d}	0.10	0.20	gamma	0	1	0.1
$\alpha_{\widehat{y}}$	1	0.30	normal	0	2	0.83
$\alpha_{\widehat{r}}$	0.15	0.50	normal	0	2	0.93
α_{P_H}	0.50	0.20	beta	0	1	0.38
$\alpha_{\widehat{P}}$	0	2	uniform	0	2	0.36
α_{I_C}	0.50	0.20	beta	0	1	0.5
α_{W_d}	0	1	uniform	0	1	0.41

Table 3: System priors

Moment prior on the standard deviation of variable:	Prior distribution	Prior mean	Prior std	Posterior mean
$\widehat{I_H}$	Normal	6	0.7	6.7
$\widehat{b_h}$	Normal	10	1	8.9
$\widehat{b_e}$	Normal	10	1	12.1
$\widehat{P_h}$	Normal	7	2	4.5

Moment prior on FEVD[a]:	Prior distribution	Lower bound	Upper bound
	Uniform	0	0.5

[a]Restricting the share that noise can explain of the forward error variance decomposition (FEVD) for the following variables: $\Delta y, \Delta w, \Delta c, \Delta I_H, \Delta I_C, \Delta I_O, \Delta G, \Delta EX_O, \Delta EX, \Delta IM, \Delta b_h, \Delta b_e, \Delta P_H, \Delta w_d$.

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