A complete documentation of Norges Bank's policy model NEMO[1]

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Abstract

This paper documents the theoretical structure and all derivations of the current version of Norges Bank's policy model *Norwegian Economy Model* (NEMO). The model consists of households, firms, an explicit treatment of the oil sector, a credit market (including a separate banking sector), a housing sector and a foreign sector. Monetary policy works through a standard policy rule or by minimizing a loss function. We set up all maximation problems, derive the first order conditions and show how the variables are made stationary. We list all shocks to the model, derive the steady state, and lastly, we provide the full parametrization of the model.

Contents

1 Introduction

This paper documents the theoretical structure and all derivations of the current version of Norges Bank's DSGE policy model *NEMO*.[1] The model includes households, firms, an explicit treatment of the credit market, including a separate banking sector and a role for housing services and house prices. In 2017 an oil sector was incorporated in NEMO and the block-exogenous foreign sector was improved. Monetary policy in the model works either via a standard policy rule or through optimal monetary policy (i.e. minimizing an operational loss function). The DSGE literature is large and NEMO builds on numerous different sources. Key references are listed at the end.

1.1 Syntax and notation

Throughout this document, P_t^X denotes the nominal price of real variable X in period t. The consumption good is the numeraire and has the price P_t. $W_{X,t}$ is the nominal wage rate in sector X. Moreover, $R_t^X \equiv 1 + r_t^X$ is the "gross interest rate" associated with sector or variable X, where r_t^X is the net interest rate. All other variables are expressed in real terms unless otherwise stated.

There is exogenous labor augmenting technological growth in the intermediate sector which causes the economy to grow at rate $\pi_t^z \equiv Z_t/Z_{t-1}$. We let $\widetilde{X}_t$ indicate the stationary form of X_t. Hence, for consumption, $\widetilde{C}_t \equiv \frac{C_t}{Z_t}$. The housing sector is assumed to have a weaker technology growth rate of π_t^z/π_t^h, where $\pi_t^h \equiv Z_t^h/Z_{t-1}^h$, hence $\widetilde{H}_t \equiv H_t \frac{Z_t^h}{Z_t}$.[2] We use the notation X_{ss} for variable X in the steady state.

2 Households

Each household supplies a differentiated labor input to intermediate firms and the oil supply sector. Wages are set by the households under the assumption of monopolistic competition. Households obtain utility from consumption, leisure, housing services and deposits. Direct utility from deposits ensures that households are both gross lenders and gross borrowers (alternatively, we could have modeled two different types of households: savers and spenders). Preferences are additively separable. We have also separated the households problem into two maximization problems: that of the households and that of the entrepreneurs. We do this to simplify the maximization problem and to clarify the decision-making by the households in the model. The entrepreneurs' part of the problem is covered in section 5.

2.1 Maximization problem

Lifetime expected utility of household j at time s can be represented as

$$U_s(j) = E_s \sum_{t=s}^{\infty} \beta^{t-s} \left[u(C_t(j)) + d(D_t(j)) + w(H_t(j)) - v(L_t(j)) \right], \tag{1}$$

where β is the discount factor, C_t denotes consumption, D_t is deposits, H_t is housing stock and L_t is supply labor. The in-period utility functions are defined as:

$$\begin{aligned} u(C_t(j)) &= z_t^u \left(1 - b^c/\pi_{ss}^z\right) \ln\left[\frac{C_t(j) - b^c C_{t-1}}{1 - b^c/\pi_{ss}^z}\right], \\ d(D_t(j)) &= z_t^d \left(1 - b^d/\pi_{ss}^z\right) \ln\left[\frac{D_t(j) - b^d D_{t-1}}{1 - b^d/\pi_{ss}^z}\right], \\ v(L_t(j)) &= \frac{1-b^l}{1+\zeta}\left[\frac{L_t(j) - b^l L_{t-1}}{1-b^l}\right]^{1+\zeta}, \\ w(H_t(j)) &= z_t^h \left(1 - b^h \pi_{ss}^h/\pi_{ss}^z\right) \ln\left[\frac{H_t(j) - b_h H_{t-1}}{1 - b^h \pi_{ss}^h/\pi_{ss}^z}\right], \end{aligned} \tag{2}$$

where the small letter z's are random preference shocks that follow AR processes (a list of all shocks can be found in chapter 16). The b parameters govern habit persistence and the π_{ss}^z denotes the exogenous steady-state (labor augmenting) technology growth rate (i.e. $\pi_t^z = Z_t/Z_{t-1}$). The weaker technology growth rate of the housing sector, π_t^z/π_t^h, is equal to π_{ss}^z/π_{ss}^h in the steady state. The degree of disutility of supplying labor is captured by the parameter $\zeta > 0$. The log in-period utility functions for consumption, deposits and housing imply an intertemporal elasticity of substitution equal to 1, which secures a balanced growth path.

[1] Please note that this document is continuously revised, and changes to the model will be documented in updated versions.

[2] The *relative* growth rate of the housing sector to the economy's growth rate is $\frac{\pi_t^z}{\pi_t^h}/\pi_t^z = \frac{1}{\pi_t^h}$. The growth rate of real house prices to consumer prices is π_t^h.

The household's budget constraint is:

$$\begin{aligned}
&P_tC_t(j) + P_tD_t(j) + P_t^H H_t(j) + \left(r_{t-1}^F + \delta_t^B(j)\right) P_{t-1}B_{h,t-1}(j) \\
&= W_t(j)L_t(j)\left[1-\gamma_t(j)\right] + P_tI_{B,t}(j) + R_{t-1}^d P_{t-1}D_{t-1}(j) \\
&\quad + (1-\delta_H)P_t^H H_{t-1}(j) + DIV_t(j) - TAX_t(j),
\end{aligned} \tag{3}$$

where P_t is the price level of final goods, P_t^H is the price level of the housing stock, r_t^F is the nominal net mortgage interest rate faced by households, R_t^d is the gross interest on household's deposits, $\delta_t^B(j)$ denotes household j's amortization rate (mortgage repayment share), $B_{h,t}(j)$ is real household's borrowing, $W_t(j)$ is the nominal wage rate (in both the intermediate sector and the oil sector) set by household j, $\gamma_t(j)$ is wage adjustment costs (defined below in (8)), $L_t(j)$ is the total amount of hours worked (in both the intermediate sector and the oil sector), $I_{B,t}(j)$ indicates new real loans by household j, δ_H denotes depreciation of the housing stock and $DIV_t(j)$ and $TAX_t(j)$ are dividends (in nominal terms) paid out to the household and taxes paid by the household, respectively.

Household's borrowing process:

$$B_{h,t}(j) = \left(1-\delta_t^B(j)\right)\frac{P_{t-1}}{P_t}B_{h,t-1}(j) + I_{B,t}(j). \tag{4}$$

Loan installments (repayments) are assumed to follow from an (approximated) annuity loan repayment formula[3]:

$$\delta_{t+1}^B(j) = \left(1-\frac{I_{B,t}(j)}{B_{h,t}(j)}\right)\left(\delta_t^B(j)\right)^{\alpha^h} + \frac{I_{B,t}(j)}{B_{h,t}(j)}\left(1-\alpha^h\right)^{\kappa^h}, \tag{5}$$

where α^h and κ^h are exogenous parameters. If α^h is 0, we have that $\delta_{t+1}^B(j) = 1$ for all t, i.e. $B_{h,t}(j) = I_{B,t}(j)$, but if $\alpha^h > 0$, the above repayment formula captures that the amortization rate is low during the first years of a mortgage, when interest payments are high, and thereafter increasing.

Household j's new loans are constrained by a relationship between the expected value of the household's housing stock in the next period and household j's mortgage (assumed to hold with equality):

$$I_{B,t}(j) = \phi_t E_t\left[\frac{P_{t+1}^H}{P_{t+1}}\frac{P_{t+1}}{P_t}H_t(j) - B_{h,t}(j)\right], \tag{6}$$

where ϕ_t governs the constraint on new loans and is a shock and assumed to follow an AR process. See (477) for the relationship with the loan-to-value ratio. Household j faces the following labor demand curve (see derivation in the section on the intermediate goods sector, (61)):

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\psi_t} L_t, \tag{7}$$

where W_t is the wage rate and ψ_t is the elasticity of substitution between differentiated labor and the wage markup shock, which is assumed to follow an AR process. We further assume that there is sluggish wage adjustment due to resource costs that are measured in terms of the total wage bill. Wage adjustments costs are specified as

$$\gamma_t(j) \equiv \frac{\phi^W}{2}\left[\frac{W_t(j)/W_{t-1}(j)}{W_{t-1}/W_{t-2}} - 1\right]^2. \tag{8}$$

As can be seen from (8), costs are related to changes in wage inflation relative to the past observed rate for the whole economy. The parameter $\phi^W > 0$ determines how costly it is to change the wage inflation rate. In NEMO, the adjustment costs of wages and prices are fully indexed.

To obtain an easier expression to maximize, some algebra is required. Combining (4) and (6) yields:

$$B_{h,t}(j) = \frac{\left(1-\delta_t^B(j)\right)}{1+\phi_t}\frac{P_{t-1}}{P_t}B_{h,t-1}(j) + \frac{\phi_t}{1+\phi_t}E_t\left[\frac{P_{t+1}^H}{P_{t+1}}\frac{P_{t+1}}{P_t}H_t(j)\right]. \tag{9}$$

Similarly, combining (5) and (4) gives

[3] See reference #[13], Gelain et.al (2014b)

$$\delta_{t+1}^{B}(j)=\left(1-\frac{B_{h,t}(j)-\left(1-\delta_t^B(j)\right)\frac{P_{t-1}}{P_t}B_{h,t-1}(j)}{B_{h,t}(j)}\right)\left(\delta_t^B(j)\right)^{\alpha^h}$$
$$+\frac{B_{h,t}(j)-\left(1-\delta_t^B(j)\right)\frac{P_{t-1}}{P_t}B_{h,t-1}(j)}{B_{h,t}(j)}\left(1-\alpha^h\right)^{\kappa^h}$$
$$\Leftrightarrow$$
$$\delta_{t+1}^{B}(j)=\left(1-\delta_t^B(j)\right)\frac{P_{t-1}}{P_t}\frac{B_{h,t-1}(j)}{B_{h,t}(j)}\left[\left(\delta_t^B(j)\right)^{\alpha^h}-\left(1-\alpha^h\right)^{\kappa^h}\right]+\left(1-\alpha^h\right)^{\kappa^h}. \tag{10}$$

Maximizing (1) by substituting in for C_t from the budget constraint and subject to the constraints in (9) and (10) (and also substituting in for $I_t^B(j)$ from (4) into the budget constraint gives rise to the following Lagrangian:

$$\mathcal{L}(W_t, D_t, B_{h,t}, H_t, \delta_t^B, \omega_t, \mu_t) =$$
$$E_s\sum_{t=s}^{\infty}\beta^{t-s}\left[\begin{array}{c} u\left(\begin{array}{c}\frac{W_t(j)}{P_t}L_t(j)\left[1-\gamma_t(j)\right]\\ +B_{h,t}(j)+R_{t-1}^{d}\frac{P_{t-1}}{P_t}D_{t-1}(j)\\ +\left(1-\delta_H\right)\frac{P_t^H}{P_t}H_{t-1}(j)\\ +\frac{DIV_t(j)}{P_t}-\frac{TAX_t(j)}{P_t}-D_t(j)\\ -\frac{P_t^H}{P_t}H_t(j)-R_{t-1}^{F}\frac{P_{t-1}}{P_t}B_{h,t-1}(j)\end{array}\right)\\ +d(D_t(j))+w(H_t(j))-v(L_t(j))\\ -\omega_t\left[B_{h,t}(j)-\frac{P_{t-1}}{P_t}\frac{\left(1-\delta_t^B(j)\right)}{1+\phi_t}B_{h,t-1}(j)-\frac{\phi_t}{1+\phi_t}E_t\left[\frac{P_{t+1}^H}{P_{t+1}}\frac{P_{t+1}}{P_t}H_t(j)\right]\right]\\ -\mu_t\left[\delta_{t+1}^B(j)-\left(1-\delta_t^B(j)\right)\frac{P_{t-1}}{P_t}\frac{B_{h,t-1}(j)}{B_{h,t}(j)}\left[\left(\delta_t^B(j)\right)^{\alpha^h}-\left(1-\alpha^h\right)^{\kappa^h}\right]-\left(1-\alpha^h\right)^{\kappa^h}\right]\end{array}\right], \tag{11}$$

where ω_t is the Lagrangian multiplier associated with the borrowing constraint (9) and μ_t is the Lagrangian multiplier associated with the repayment constraint (10). FOC (suppressing index j)

w.r.t. $B_{h,t}$

$$u'(C_t)-E_t\left[\beta u'(C_{t+1})R_t^F\frac{P_t}{P_{t+1}}\right]-\omega_t$$
$$+E_t\left[\omega_{t+1}\beta\frac{P_t}{P_{t+1}}\frac{(1-\delta_{t+1}^B)}{1+\phi_{t+1}}\right]-\mu_t\frac{B_{h,t-1}}{B_{h,t}^2}\frac{P_{t-1}}{P_t}(1-\delta_t^B)\left[\left(\delta_t^B\right)^{\alpha^h}-(1-\alpha^h)^{\kappa^h}\right]$$
$$+E_t\left[\mu_{t+1}\frac{P_t}{P_{t+1}}\frac{\beta}{B_{h,t+1}}(1-\delta_{t+1}^B)\left[\left(\delta_{t+1}^B\right)^{\alpha^h}-(1-\alpha^h)^{\kappa^h}\right]\right]=0 \tag{12}$$

$$\Leftrightarrow \mid *\frac{B_{h,t}}{u'(C_t)} \tag{13}$$

$$B_{h,t}-B_{h,t}E_t\left[\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{P_t}{P_{t+1}}\right]R_t^F-\frac{\omega_t}{u'(C_t)}B_{h,t}$$
$$+E_t\left[\frac{\omega_{t+1}}{u'(C_{t+1})}\frac{u'(C_{t+1})}{u'(C_t)}\beta\frac{P_t}{P_{t+1}}\frac{(1-\delta_{t+1}^B)}{1+\phi_{t+1}}B_{h,t}\right]-\frac{\mu_t}{u'(C_t)}\frac{B_{h,t-1}}{B_{h,t}}\frac{P_{t-1}}{P_t}(1-\delta_t^B)\left[\left(\delta_t^B\right)^{\alpha^h}-(1-\alpha^h)^{\kappa^h}\right] \tag{14}$$
$$+E_t\left[\beta\frac{\mu_{t+1}}{u'(C_{t+1})}\frac{u'(C_{t+1})}{u'(C_t)}\frac{P_t}{P_{t+1}}\frac{B_{h,t}}{B_{h,t+1}}(1-\delta_{t+1}^B)\left[\left(\delta_{t+1}^B\right)^{\alpha^h}-(1-\alpha^h)^{\kappa^h}\right]\right]=0,$$

w.r.t. D_t

$$-u'(C_t) + E_t\left[\beta u'(C_{t+1})\frac{P_t}{P_{t+1}}\right]R_t^d + d'(D_t) = 0$$
$$\Leftrightarrow$$
$$E_t\left[\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{P_t}{P_{t+1}}\right]R_t^d = 1 - \frac{d'(D_t)}{u'(C_t)} \tag{15}$$

w.r.t. H_t:

$$-u'(C_t)\frac{P_t^H}{P_t} + E_t\left[\beta(1-\delta_H)u'(C_{t+1})\frac{P_{t+1}^H}{P_{t+1}}\right] + w'(H_t) + E_t\left[\omega_t\frac{\phi_t}{1+\phi_t}\frac{P_{t+1}^H}{P_{t+1}}\frac{P_{t+1}}{P_t}\right] = 0$$
$$\Leftrightarrow$$
$$\frac{w'(H_t)}{u'(C_t)} = \frac{P_t^H}{P_t} - (1-\delta_H)E_t\left[\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{P_{t+1}^H}{P_{t+1}}\right] - \frac{\omega_t}{u'(C_t)}\frac{\phi_t}{1+\phi_t}E_t\left[\frac{P_{t+1}^H}{P_{t+1}}\frac{P_{t+1}}{P_t}\right], \tag{16}$$

w.r.t. δ_t^B

$$-\mu_{t-1} + \mu_t\beta\frac{B_{h,t-1}}{B_{h,t}}\frac{P_{t-1}}{P_t}\left[\alpha^h\left(\delta_t^B\right)^{\alpha^h-1}(1-\delta_t^B) - \left(\delta_t^B\right)^{\alpha^h} + (1-\alpha^h)^{\kappa^h}\right] - \omega_t\beta\left[\frac{B_{h,t-1}}{1+\phi_t}\frac{P_{t-1}}{P_t}\right] = 0, \tag{17}$$

w.r.t. W_t (recall (8))

$$-v'(L_t)\frac{\delta L_t}{\delta W_t} + u'(C_t)\left[\left(\frac{L_t}{P_t} + \frac{W_t}{P_t}\frac{\delta L_t}{\delta W_t}\right)(1-\gamma_t) - \frac{W_t}{P_t}L_t\frac{\delta\gamma_t}{\delta W_t}\right]$$
$$-E_t\left[\beta u'(C_{t+1})\frac{W_{t+1}}{P_{t+1}}L_{t+1}\frac{\delta\gamma_{t+1}}{\delta W_t}\right] = 0$$
$$\Leftrightarrow$$
$$v'(L_t)\psi_t\frac{L_t}{W_t} + u'(C_t)\left[\frac{L_t}{P_t}(1-\psi_t)(1-\gamma_t) - \frac{W_t}{P_t}L_t\phi^W\left(\frac{W_t/W_{t-1}}{W_{t-1}/W_{t-2}} - 1\right)\frac{1/W_{t-1}}{W_{t-1}/W_{t-2}}\right]$$
$$+E_t\left[\beta u'(C_{t+1})\frac{W_{t+1}}{P_{t+1}}L_{t+1}\phi^W\left(\frac{W_{t+1}/W_t}{W_t/W_{t-1}} - 1\right)\frac{W_{t+1}}{W_t/W_{t-1}}\left(\frac{1}{W_t}\right)^2\right] = 0$$
$$\Leftrightarrow$$
$$\frac{v'(L_t)}{u'(C_t)}\psi_t\frac{P_t}{W_t} = \left[(\psi_t - 1)(1-\gamma_t) + \phi^W\left(\frac{W_t/W_{t-1}}{W_{t-1}/W_{t-2}} - 1\right)\frac{W_t/W_{t-1}}{W_{t-1}/W_{t-2}}\right]$$
$$-E_t\left[\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{P_t}{P_{t+1}}\frac{L_{t+1}}{L_t}\phi^W\left(\frac{W_{t+1}/W_t}{W_t/W_{t-1}} - 1\right)\frac{(W_{t+1}/W_t)^2}{W_t/W_{t-1}}\right], \tag{18}$$

where we have used that in symmetric equilibrium all households will set the same wage rate, hence: $W_t(j) = \overline{W_t}$ for all j, so $W_t = \left[\int_0^1 W_t(j)^{1-\psi_t}dj\right]^{\frac{1}{1-\psi_t}} = \left[\int_0^1 \overline{W_t}^{1-\psi_t}dj\right]^{\frac{1}{1-\psi_t}} = \overline{W_t}\int_0^1 1dj = \overline{W_t} = W_t(j)$ and

$$\frac{\delta L_t(j)}{\delta W_t(j)} = -\psi_t\left(\frac{W_t(j)}{W_t}\right)^{-(\psi_t+1)}\frac{L_t}{W_t} \underset{\text{in symmetric eq. } (W(j)=W)}{\Rightarrow} \frac{\delta L_t(j)}{\delta W_t(j)} = -\psi_t\frac{L_t}{W_t}.$$

We have also used that

$$\frac{\delta\gamma_t(j)}{\delta W_t(j)} = \phi^W\left[\frac{W_t(j)/W_{t-1}(j)}{W_{t-1}/W_{t-2}} - 1\right]\frac{1/W_{t-1}(j)}{W_{t-1}/W_{t-2}},$$

$$\frac{\delta\gamma_{t+1}(j)}{\delta W_t(j)} = -\phi^W\left[\frac{W_{t+1}(j)/W_t(j)}{W_t/W_{t-1}} - 1\right]\frac{W_{t+1}(j)}{W_t/W_{t-1}}\left(\frac{1}{W_t(j)}\right)^2.$$

2.2 Making the equations stationary

2.2.1 The in-period utility functions

From (2) we have: $u(C_t) = z_t^u (1 - b^c/\pi^z) \ln \left[\frac{C_t - b^c C_{t-1}}{1 - b^c/\pi^z}\right] \Rightarrow u'(C_t) = z_t^u \left[\frac{C_t - b^c C_{t-1}}{1 - b^c/\pi^z}\right]^{-1} = z_t^u \left[\frac{\widetilde{C}_t - b^c \widetilde{C}_{t-1} \frac{1}{\pi_t^z}}{1 - b^c/\pi^z}\right]^{-1} \frac{1}{Z_t} \Rightarrow$

$$Z_t u'(C_t) = \widetilde{u}'\left(\widetilde{C}_t\right) = z_t^u \left[\frac{\widetilde{C}_t - b^c \widetilde{C}_{t-1} \frac{1}{\pi_t^z}}{1 - b^c/\pi_{ss}^z}\right]^{-1}, \tag{19}$$

where $\widetilde{u}'\left(\widetilde{C}_t\right)$ is stationary. Similarly,

$$\widetilde{d}'\left(\widetilde{D}_t\right) = z_t^d \left[\frac{\widetilde{D}_t - b^d \widetilde{D}_{t-1} \frac{1}{\pi_t^z}}{1 - b^d/\pi_{ss}^z}\right]^{-1} (= Z_t d'(D_t)), \tag{20}$$

$$v'(L_t) = \left[\frac{L_t - b^l L_{t-1}}{1 - b^l}\right]^{\zeta}, \tag{21}$$

$$\widetilde{w}'(\widetilde{H}_t) = z_t^h \left[\frac{\widetilde{H}_t - b_h \widetilde{H}_{t-1} \frac{\pi_t^h}{\pi_t^z}}{1 - b^h \pi_{ss}^h/\pi_{ss}^z}\right]^{-1} \left(= \frac{Z_t}{Z_t^h} w'(H_t)\right). \tag{22}$$

2.2.2 First-order conditions

First, define $\widetilde{\omega}_t = \frac{\omega_t}{u'(C_t)}$ and $\widetilde{\mu}_t = \frac{\mu_t}{Z_t u'(C_t)}$, which are both stationary. The stochastic discount factor is defined as: $\Delta_{t,t+i} \equiv \beta^i \frac{u'(C_{t+i})}{u'(C_t)} \frac{1}{\pi_{t+i}}$,where $\pi_{t+i} \equiv \frac{P_{t+i}}{P_t}$. Note that

$$\Delta_{t,t+i} = \Lambda_{t,t+i} \equiv \beta^i \frac{\widetilde{u}'\left(\widetilde{C}_{t+i}\right)}{\widetilde{u}'\left(\widetilde{C}_t\right)} \frac{1}{\pi_{t+i}} \frac{1}{\pi_{t+i}^z}, \tag{23}$$

where $\Lambda_{t,t+i}$ is the stochastic discount factor in stationary terms. We also define the stochastic discount factor for one period ahead as

$$\Delta = \Lambda \equiv \beta \frac{\widetilde{u}'\left(\widetilde{C}_{t+1}\right)}{\widetilde{u}'\left(\widetilde{C}_t\right)} \frac{1}{\pi_{t+1}} \frac{1}{\pi_{t+1}^z}. \tag{24}$$

Wage adjustment costs are already stationary. By using $W_t(j) = W_t$ and defining $\pi_t^W \equiv \frac{W_t}{W_{t-1}}$ equation (8) can be written:

$$\gamma_t \equiv \frac{\phi^W}{2} \left[\frac{\pi_t^W}{\pi_{t-1}^W} - 1\right]^2. \tag{25}$$

FOCs are given by:
w.r.t. $B_{h,t}$ (recall (14))

$$\begin{aligned} &\widetilde{B}_{h,t} - \widetilde{B}_{h,t} R_t^F E_t[\Lambda] - \widetilde{\omega}_t \widetilde{B}_{h,t} \\ &+ E_t\left[\widetilde{\omega}_{t+1} \Lambda \frac{(1 - \delta_{t+1}^B)}{1 + \phi_{t+1}}\right] \widetilde{B}_{h,t} - \widetilde{\mu}_t (1 - \delta_t^B) \frac{\widetilde{B}_{h,t-1}}{\widetilde{B}_{h,t}} \frac{1}{\pi_t \pi_t^z} \left[\left(\delta_t^B\right)^{\alpha^h} - (1 - \alpha^h)^{\kappa^h}\right] \\ &+ E_t\left[\widetilde{\mu}_{t+1} \Lambda (1 - \delta_{t+1}^B) \frac{\widetilde{B}_{h,t}}{\widetilde{B}_{h,t+1}} \left[\left(\delta_{t+1}^B\right)^{\alpha^h} - (1 - \alpha^h)^{\kappa^h}\right]\right] = 0, \end{aligned} \tag{26}$$

w.r.t. D_t (recall (15))

$$E_t[\Lambda] R_t^d = \left[1 - \frac{\widetilde{d}'\left(\widetilde{D}_t\right)}{\widetilde{u}'\left(\widetilde{C}_t\right)}\right], \tag{27}$$

w.r.t. H_t (recall (16) and note that $\widetilde{P}_t^H \equiv \frac{P_t^H}{P_t} \frac{1}{Z_t^h}$, which is stationary)

$$\frac{\widetilde{w}'(\widetilde{H}_t)}{\widetilde{u}'\left(\widetilde{C}_t\right)} = \widetilde{P}^H_t - (1-\delta_H)\, E_t\left[\Lambda \widetilde{P}^H_{t+1}\pi^h_{t+1}\pi_{t+1}\right] - \widetilde{\omega}_t \frac{\phi_t}{1+\phi_t} E_t\left[\widetilde{P}^H_{t+1}\pi^h_{t+1}\pi_{t+1}\right].$$

It is assumed that households *do not* have complete rational expectations w.r.t. house prices. Instead, we assume that a share b^{sa} of households expects house prices to follow a moving average process, whereas a share $(1-b^{sa})$ has rational expectations. To model this assumption, the first-order conditions w.r.t. house prices need to be log-linearized. Let Y_{ss} indicate the steady-state value of Y_t and let $\hat{Y}$ indicate the gap (log-deviation) from the steady-state value, $\hat{Y}_t = \log(Y_t/Y_{ss})$. Then, house price expectations are (in gap form) given by

$$E_t\left[\widehat{\widetilde{P}^H_{t+1}}\right] = b^{sa}\widehat{X^H_t} + (1-b^{sa})\widehat{\widetilde{P}^H_{t+1}}, \tag{28}$$

where

$$\widehat{X^H_t} = \lambda^{sa}\widehat{\widetilde{P}^H_{t-1}} + (1-\lambda^{sa})\widehat{X^H_{t-1}} \tag{29}$$

is the moving average process. The log-linearized FOC

w.r.t. H_t:

$$\begin{aligned}\frac{\widetilde{w}'(\widetilde{H}_t)}{\widetilde{u}'\left(\widetilde{C}_t\right)} \approx\ & \widetilde{P}^H_t - (1-\delta_H)\,\Lambda_{ss}\pi_{ss}\widetilde{P}^H_{ss}\pi^h_{ss}\left[1+\widehat{\Lambda}+\widehat{\pi_{t+1}}+\widehat{\pi^h_{t+1}}+b^{sa}\widehat{X^H_t}+(1-b^{sa})\widehat{\widetilde{P}^H_{t+1}}\right] \\ & -\frac{\phi_t}{1+\phi_t}\widetilde{\omega}_{ss}\pi_{ss}\widetilde{P}^H_{ss}\pi^h_{ss}\left[1+\widehat{\widetilde{\omega}_t}+\widehat{\pi_{t+1}}+\widehat{\pi^h_{t+1}}+b^{sa}\widehat{X^H_t}+(1-b^{sa})\widehat{\widetilde{P}^H_{t+1}}\right],\end{aligned} \tag{30}$$

Other FOCs are given by

w.r.t. δ^B_t (recall (17), *leaded*):

$$-\widetilde{\mu}_t + E_t\left[\widetilde{\mu}_{t+1}\Lambda\frac{\widetilde{B}_{h,t}}{\widetilde{B}_{h,t+1}}\left[\alpha^h\left(\delta^B_{t+1}\right)^{\alpha^h-1}(1-\delta^B_{t+1}) - \left(\delta^B_{t+1}\right)^{\alpha^h} + (1-\alpha^h)^{\kappa^h}\right] - \widetilde{\omega}_{t+1}\Lambda\frac{\widetilde{B}_{h,t}}{1+\phi_{t+1}}\right] = 0, \tag{31}$$

w.r.t. W_t (recall (18), and define $\pi^W_t \equiv \frac{W_t}{W_{t-1}}$):

$$\frac{W_t}{P_t} = \psi_t Z_t MRS(L_t,\widetilde{C}_t)\left[\begin{array}{c}[\psi_t-1]\,[1-\gamma_t] \\ +\phi^W\frac{\pi^W_t}{\pi^W_{t-1}}\left(\frac{\pi^W_t}{\pi^W_{t-1}}-1\right) \\ -E\left[\Delta\phi^W\frac{P_t}{P_{t+1}}\frac{L_{t+1}}{L_t}\frac{(\pi^W_{t+1})^2}{\pi^W_t}\left(\frac{\pi^W_{t+1}}{\pi^W_t}-1\right)\right]\end{array}\right]^{-1}$$

$\Longrightarrow$

$$\widetilde{W}_t = \psi_t MRS(L_t,\widetilde{C}_t)\left[\begin{array}{c}[\psi_t-1]\,[1-\gamma_t] \\ +\phi^W\frac{\pi^W_t}{\pi^W_{t-1}}\left(\frac{\pi^W_t}{\pi^W_{t-1}}-1\right) \\ -E\left[\Lambda\phi^W\frac{L_{t+1}}{L_t}\frac{\pi^W_{t+1}}{\pi^W_t}\left(\frac{\pi^W_{t+1}}{\pi^W_t}-1\right)\right]\end{array}\right]^{-1}, \tag{32}$$

where we have used that

$$MRS(L_t,C_t) = \frac{v'(L_t)}{u'(C_t)} = \frac{Z_t v'(L_t)}{Z_t u'(C_t)} = \frac{Z_t v'(L_t)}{\widetilde{u}'\left(\widetilde{C}_t\right)} = Z_t MRS(L_t,\widetilde{C}_t). \tag{33}$$

2.2.3 The constraints

Lastly, the constraints need to be stationary. From (9) we have: $\widetilde{B}_{h,t} = \frac{(1-\delta^B_t)}{1+\phi_t}\frac{1}{\pi_t\pi^z_t}\widetilde{B}_{h,t-1} + \frac{\phi_t}{1+\phi_t}E_t\left[\widetilde{P}^H_{t+1}\pi^h_{t+1}\pi_{t+1}\widetilde{H}_t\right]$. By log-linearizing the second term and again using the assumption that households do not have complete rational expectations w.r.t. housing prices, (28), we get

$$\widetilde{B}_{h,t} = \frac{(1-\delta^B_t)}{1+\phi_t}\frac{1}{\pi_t\pi^z_t}\widetilde{B}_{h,t-1} + \frac{\phi_t}{1+\phi_t}\widetilde{H}_{ss}\pi_{ss}\widetilde{P}^H_{ss}\pi^h_{ss}\left[1+\widehat{\widetilde{H}_t}+\widehat{\pi_{t+1}}+\widehat{\pi^h_{t+1}}+b^{sa}\widehat{X^H_t}+(1-b^{sa})\widehat{\widetilde{P}^H_{t+1}}\right]. \tag{34}$$

From (10) and *lagged* we have that

$$\delta^B_t = (1-\delta^B_{t-1})\frac{1}{\pi_{t-1}\pi^z_{t-1}}\frac{\widetilde{B}_{h,t-2}}{\widetilde{B}_{h,t-1}}\left[\left(\delta^B_{t-1}\right)^{\alpha^h} - (1-\alpha^h)^{\kappa^h}\right] + \left(1-\alpha^h\right)^{\kappa^h}. \tag{35}$$

2.2.4 Equations included in the model

The following equations are included in the model file: (29), (27), (26), (24), (20), (19), (34), (35), (31), (30), (22), (32), (33), (21) and (25).

2.3 Steady-state equations

2.3.1 The in-period utility functions

$$\widetilde{u}'\left(\widetilde{C}_{ss}\right) = \frac{z^u_{ss}}{\widetilde{C}_{ss}}, \tag{36}$$

$$\widetilde{d}'\left(\widetilde{D}_{ss}\right) = \frac{z^d_{ss}}{\widetilde{D}_{ss}}, \tag{37}$$

$$v'\left(L_t\right) = L^{\zeta}_{ss}, \tag{38}$$

$$\widetilde{w}'(\widetilde{H}_{ss}) = \frac{z^h_{ss}}{\widetilde{H}_{ss}}. \tag{39}$$

2.3.2 First-order conditions

Discount factor

$$\Lambda_{ss} \equiv \frac{\beta}{\pi_{ss}\pi^z_{ss}}. \tag{40}$$

FOC

w.r.t. $B_{h,t}$:

$$\widetilde{B}_{h,ss} - \widetilde{B}_{h,ss}R^F_{ss}\Lambda_{ss} - \widetilde{\omega}_{ss}\widetilde{B}_{h,ss} + \widetilde{\omega}_{ss}\Lambda_{ss}\frac{(1-\delta^B_{ss})}{1+\phi_{ss}}\widetilde{B}_{h,ss}$$
$$-\widetilde{\mu}_{ss}(1-\delta^B_{ss})\frac{1}{\pi_{ss}\pi^z_{ss}}\left[\left(\delta^B_{ss}\right)^{\alpha^h} - (1-\alpha^h)^{\kappa^h}\right] + \widetilde{\mu}_{ss}\Lambda_{ss}(1-\delta^B_{ss})\left[\left(\delta^B_{ss}\right)^{\alpha^h} - (1-\alpha^h)^{\kappa^h}\right] = 0. \tag{41}$$

w.r.t. D_t:

$$\Lambda_{ss}R^d_{ss} = \left[1 - \frac{\widetilde{d}'\left(\widetilde{D}_{ss}\right)}{\widetilde{u}'\left(\widetilde{C}_{ss}\right)}\right]. \tag{42}$$

w.r.t. H_t:

$$\frac{\widetilde{w}'(\widetilde{H}_{ss})}{\widetilde{u}'\left(\widetilde{C}_{ss}\right)} = \widetilde{P}^H_{ss} - (1-\delta_H)\Lambda_{ss}\pi_{ss}\widetilde{P}^H_{ss}\pi^h_{ss} - \frac{\phi_{ss}}{1+\phi_{ss}}\widetilde{\omega}_{ss}\pi_{ss}\widetilde{P}^H_{ss}\pi^h_{ss}. \tag{43}$$

w.r.t. δ^B_t:

$$-\widetilde{\mu}_{ss} + \widetilde{\mu}_{ss}\Lambda_{ss}\left[\alpha^h\left(\delta^B_{ss}\right)^{\alpha^h-1}(1-\delta^B_{ss}) - \left(\delta^B_{ss}\right)^{\alpha^h} + (1-\alpha^h)^{\kappa^h}\right] - \widetilde{\omega}_{ss}\Lambda_{ss}\frac{\widetilde{B}_{h,ss}}{1+\phi_{ss}} = 0. \tag{44}$$

w.r.t. W_t:

$$\gamma_{ss} = 0, \tag{45}$$

$$\gamma'_{ss} = 0, \tag{46}$$

$$\widetilde{W}_{ss} = \frac{\psi_{ss}}{\psi_{ss}-1}MRS(L_{ss},\widetilde{C}_{ss}). \tag{47}$$

Definition of MRS:

$$MRS(L_{ss},\widetilde{C}_{ss}) = \frac{v'\left(L_{ss}\right)}{\widetilde{u}'\left(\widetilde{C}_{ss}\right)}. \tag{48}$$

2.3.3 The constraints

$$\widetilde{B}_{h,ss} = \left[1 - \frac{(1-\delta^B_{ss})}{1+\phi_{ss}} \frac{1}{\pi_{ss}\pi^z_{ss}}\right]^{-1} \frac{\phi_{ss}}{1+\phi_{ss}} \widetilde{H}_{ss}\pi_{ss}\widetilde{P}^H_{ss}\pi^h_{ss}, \tag{49}$$

$$\delta^B_{ss} = \frac{1-\delta^B_{ss}}{\pi_{ss}\pi^z_{ss}} \left[\left(\delta^B_{ss}\right)^{\alpha^h} - (1-\alpha^h)^{\kappa^h}\right] + \left(1-\alpha^h\right)^{\kappa^h}. \tag{50}$$

3 Intermediate goods sector

A continuum of firms in the intermediate goods sector use capital and labor to produce a differentiated intermediate good which is sold under monopolistic competition to the final goods producers at home and abroad as exports.

3.1 Maximization problem

The intermediate firm n sells good $Q_t(n)$ to the final good sector and exports good $M^*_t(n)$. It has the following production function (where $T_t(n) = Q_t(n) + M^*_t(n)$):

$$T_t(n) = \left[(1-\alpha)^{\frac{1}{\xi}} (Z_t z^L_t L_{I,t}(n))^{1-\frac{1}{\xi}} + \alpha^{\frac{1}{\xi}} \overline{K}_{I,t}(n)^{1-\frac{1}{\xi}}\right]^{\frac{\xi}{\xi-1}}, \tag{51}$$

where $\alpha \in [0,1]$ determines the capital share and ξ denotes the elasticity of substitution between labor and capital. The variables $L_{I,t}(n)$ and $\overline{K}_{I,t}(n)$ denote, respectively, hours and effective capital used by firm n in period t. There are two exogenous shocks to productivity in the model: Z_t refers to an exogenous permanent (level) technology process, which grows at the gross rate π^z_t, whereas z^L_t denotes a temporary (stationary) shock to productivity (or labor utilization), which follows an AR(1) process.

Total labor input to firm n is an index over used labor from all households j, i.e.

$$L_{I,t}(n) = \left[\int_0^1 L_{I,t}(n,j)^{1-\frac{1}{\psi_t}} dj\right]^{\frac{\psi_t}{\psi_t - 1}}, \tag{52}$$

where ψ_t is the wage markup shock (and the elasticity of substitution between differentiated labor) and follows an AR(1) process.[4]

Let $W_{I,t}$ be the wage rate and $R_{KI,t}$ be the rental rate of capital in the intermediate goods sector. Minimizing total factor outlays gives rise to the conditional demand functions (taking factor prices as given). The Lagrangian becomes:

$$\mathcal{L} = W_{I,t}L_{I,t}(n) + R_{KI,t}\overline{K}_{I,t}(n) - \lambda\left[(1-\alpha)^{\frac{1}{\xi}} (Z_t z^L_t L_{I,t}(n))^{1-\frac{1}{\xi}} + \alpha^{\frac{1}{\xi}} \overline{K}_{I,t}(n)^{1-\frac{1}{\xi}} - T_t^{\frac{\xi-1}{\xi}}(n)\right].$$

FOC:

w.r.t. $L_{I,t}(n)$:

$$W_{I,t} - \lambda(1-\frac{1}{\xi})(1-\alpha)^{\frac{1}{\xi}} (Z_t z^L_t)^{1-\frac{1}{\xi}} L_{I,t}(n)^{-\frac{1}{\xi}} = 0$$

$$\Leftrightarrow$$

$$\frac{1}{W_{I,t}}\lambda(1-\frac{1}{\xi})(1-\alpha)^{\frac{1}{\xi}} (Z_t z^L_t)^{1-\frac{1}{\xi}} = L_{I,t}(n)^{\frac{1}{\xi}}$$

$$\Leftrightarrow$$

$$\left[\frac{1}{W_{I,t}}\lambda(1-\frac{1}{\xi})(1-\alpha)^{\frac{1}{\xi}} (Z_t z^L_t)^{1-\frac{1}{\xi}}\right]^{\xi-1} = L_{I,t}(n)^{1-\frac{1}{\xi}}. \tag{53}$$

w.r.t. $\overline{K}_{I,t}(n)$:

[4] For the model to be able to replicate the importance of the oil sector for the Norwegian economy, we have added a direct impact from the oil price and the labor demand from oil supply firms to the wage markup shock. See eq. 368 in Section 16.

$$R_{KI,t} - \lambda(1-\frac{1}{\xi})\alpha^{\frac{1}{\xi}}\overline{K}_{I,t}(n)^{-\frac{1}{\xi}} = 0$$
$$\Leftrightarrow$$
$$\left[\frac{1}{R_{KI,t}}\lambda(1-\frac{1}{\xi})\alpha^{\frac{1}{\xi}}\right]^{\xi-1} = \overline{K}_{I,t}(n)^{1-\frac{1}{\xi}}. \tag{54}$$

By inserting (53) and (54) into the production function we get

$$(1-\alpha)^{\frac{1}{\xi}}(Z_t z_t^L)^{1-\frac{1}{\xi}}\left[\frac{1}{W_{I,t}}\lambda(1-\frac{1}{\xi})(1-\alpha)^{\frac{1}{\xi}}(Z_t z_t^L)^{1-\frac{1}{\xi}}\right]^{\xi-1} + \alpha^{\frac{1}{\xi}}\left[\frac{1}{R_{KI,t}}\lambda(1-\frac{1}{\xi})\alpha^{\frac{1}{\xi}}\right]^{\xi-1} = T_t(n)^{\frac{\xi-1}{\xi}}$$
$$\Leftrightarrow$$
$$(1-\alpha)(1-\frac{1}{\xi})^{\xi-1}(\lambda Z_t z_t^L)^{\xi-1}W_{I,t}^{1-\xi} + \alpha(1-\frac{1}{\xi})^{\xi-1}\lambda^{\xi-1}(R_{KI,t})^{1-\xi} = T_t(n)^{\frac{\xi-1}{\xi}}$$
$$\Leftrightarrow$$
$$(1-\frac{1}{\xi})^{\xi-1}\lambda^{\xi-1}\left[(1-\alpha)(Z_t z_t^L)^{\xi-1}W_{I,t}^{1-\xi} + \alpha(R_{KI,t})^{1-\xi}\right] = T_t(n)^{\frac{\xi-1}{\xi}}. \tag{55}$$

To obtain an easier expression to work with, let $MC_t^{1-\xi} = \left[(1-\alpha)\left(\frac{W_{I,t}}{Z_t z_t^L}\right)^{1-\xi} + \alpha(R_{KI,t})^{1-\xi}\right]$. Inserting this in (55) gives

$$(1-\frac{1}{\xi})^{\xi-1}\lambda^{\xi-1}MC_t^{1-\xi} = T_t(n)^{\frac{\xi-1}{\xi}}. \tag{56}$$

Then, by inserting the two FOCs (53) and (54) into (56) gives the demand for labor and capital respectively:

$$L_{I,t}(n)^{1-\frac{1}{\xi}}\left[\frac{1}{W_{I,t}}(1-\alpha)^{\frac{1}{\xi}}(Z_t z_t^L)^{1-\frac{1}{\xi}}\right]^{1-\xi}MC_t^{1-\xi} = T_t(n)^{\frac{\xi-1}{\xi}}$$
$$\Leftrightarrow$$
$$L_{I,t}(n)\left[\frac{1}{W_{I,t}}(1-\alpha)^{\frac{1}{\xi}}(Z_t z_t^L)^{1-\frac{1}{\xi}}\right]^{-\xi}MC_t^{-\xi} = T_t(n)$$
$$\Leftrightarrow$$
$$L_{I,t}(n) = (1-\alpha)\left(\frac{W_{I,t}}{MC_t}\right)^{-\xi}T_t(n)(Z_t z_t^L)^{\xi-1}, \tag{57}$$

$$\overline{K}_{I,t}(n)^{1-\frac{1}{\xi}}(R_{KI,t})^{\xi-1}\alpha^{\frac{1-\xi}{\xi}}MC_t^{1-\xi} = T_t(n)^{\frac{\xi-1}{\xi}}$$
$$\Leftrightarrow$$
$$\overline{K}_{I,t}(n) = \alpha\left(\frac{R_{KI,t}}{MC_t}\right)^{-\xi}T_t(n). \tag{58}$$

In symmetric equilibrium all n firms make the same decision $L_{I,t}(n) = \overline{L}_{I,t}$, so $L_{I,t} = \int_0^1 L_{I,t}(n)dn = \int_0^1 \overline{L}_{I,t}dn = \overline{L}_{I,t}\int_0^1 1dn = \overline{L}_{I,t} = L_{I,t}(n)$. Similarly, $\overline{K}_{I,t} = \overline{K}_{I,t}(n)$ and $T_t = T_t(n)$. To find the expenditure function, insert (57) and (58) into $\Upsilon_t(W_{I,t}, R_{KI,t}, T_t) = W_{I,t}L_{I,t} + R_{KI,t}\overline{K}_{I,t}$ to get

$$\Upsilon_t = W_{I,t}(1-\alpha)\left(\frac{MC_t}{W_{I,t}}\right)^{\xi}T_t(Z_t z_t^L)^{\xi-1} + R_{KI,t}\alpha\left(\frac{MC_t}{R_t^K}\right)^{\xi}T_t$$
$$\Leftrightarrow$$
$$\Upsilon_t = (1-\alpha)\left(\frac{W_{I,t}}{Z_t z_t^L}\right)^{1-\xi}MC_t^{\xi}T_t + \alpha(R_{KI,t})^{1-\xi}MC_t^{\xi}T_t$$
$$\Leftrightarrow$$
$$\Upsilon_t = MC_t^{1-\xi}MC_t^{\xi}T_t = MC_tT_t.$$

The marginal cost function can be found by:

$$\frac{d\Upsilon_t}{dT_t} = MC_t = \left[(1-\alpha)\left(\frac{W_{I,t}}{Z_t z_t^L}\right)^{1-\xi} + \alpha(R_{KI,t})^{1-\xi}\right]^{\frac{1}{1-\xi}}. \tag{59}$$

Before we proceed with deriving firm n's optimal prices, we will first find the conditional labor demand functions facing household j, $L_{I,t}(j)$ (as this was used in the household optimization problem above). Firm n will minimize labor costs subject to the optimal level of labor input (remember that $W_{I,t}(j) = W_t(j)$): $\min_{l_{I,t}(n,j)} \int_0^1 L_{I,t}(n,j)W_t(j)dj$ subject to $\left[\int_0^1 L_{I,t}(n,j)^{1-\frac{1}{\psi_t}} dj\right]^{\frac{\psi_t}{\psi_t-1}} = L_{I,t}(n)$. This gives the following Lagrangian:

$$\mathcal{L} = \int_0^1 L_{I,t}(n,j)W_t(j)dj - \lambda\left[\int_0^1 L_{I,t}(n,j)^{1-\frac{1}{\psi_t}} dj - L_{I,t}(n)^{\frac{\psi_t-1}{\psi_t}}\right].$$

FOC w.r.t. $L_{I,t}(n,j)$:

$$W_t(j) - \lambda\left(1-\frac{1}{\psi_t}\right)L_{I,t}(n,j)^{-\frac{1}{\psi_t}} = 0$$
$$\Leftrightarrow$$
$$W_t(j)\left[\lambda\left(1-\frac{1}{\psi_t}\right)\right]^{-1} = L_{I,t}(n,j)^{-\frac{1}{\psi_t}}$$
$$\Leftrightarrow$$
$$W_t(j)^{1-\psi_t}\left[\lambda\left(1-\frac{1}{\psi_t}\right)\right]^{\psi_t-1} = L_{I,t}(n,j)^{\frac{\psi_t-1}{\psi_t}}. \tag{60}$$

Inserting the FOC into the labor constraint gives

$$\int_0^1 W_t(j)^{1-\psi_t}\left[\lambda\left(1-\frac{1}{\psi_t}\right)\right]^{\psi_t-1} dj = L_{I,t}(n)^{\frac{\psi_t-1}{\psi_t}}$$
$$\Leftrightarrow$$
$$\left[\lambda\left(1-\frac{1}{\psi_t}\right)\right]^{\psi_t-1}\int_0^1 W_t(j)^{1-\psi_t}dj = L_{I,t}(n)^{\frac{\psi_t-1}{\psi_t}}.$$

The wage rate can be defined as: $W_t \equiv \left[\int_0^1 W_t(j)^{1-\psi_t}dj\right]^{\frac{1}{1-\psi_t}}$. Inserting this, that $W_t = W_{I,t}$ and in the second line using the FOC (60) gives

$$\left[\lambda\left(1-\frac{1}{\psi_t}\right)\right]^{\psi_t-1} W_t^{1-\psi_t} = L_{I,t}(n)^{\frac{\psi_t-1}{\psi_t}}$$
$$\Leftrightarrow$$
$$L_{I,t}(n,j)^{\frac{\psi_t-1}{\psi_t}} W_t(j)^{\psi_t-1}W_t^{1-\psi_t} = L_{I,t}(n)^{\frac{\psi_t-1}{\psi_t}}$$
$$\Leftrightarrow$$
$$L_{I,t}(n,j) = \left(\frac{W_t(j)}{W_t}\right)^{-\psi_t} L_{I,t}(n)$$

Then, to find the labor demand curve facing household j, i.e. $L_{I,t}(j) = \int_0^1 L_{I,t}(n,j)dn$:

$$L_{I,t}(j) = \int_0^1 L_{I,t}(n,j)dn = \int_0^1 \left(\frac{W_t(j)}{W_{I,t}}\right)^{-\psi_t} L_{I,t}(n)dn$$

$$\Leftrightarrow$$

$$L_{I,t}(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\psi_t} \int_0^1 L_{I,t}(n)dn$$

$$\Leftrightarrow$$

$$L_{I,t}(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\psi_t} L_{I,t}. \tag{61}$$

Note that the labor demand from the oil sector $L_{O,t}(j)$ will follow similarly. Total demand facing facing household j: $L_{I,t}(j) + L_{O,t}(j) = L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\psi_t} L_t$ (where we have used that $W_{I,t} = W_{O,t} = W_t$ due to perfect labour mobility). This is used in the household section. See eq. (7).

Firms in the intermediate sector sell their goods under monopolistic competition. Each firm n charges different prices domestically and abroad: $P_t^Q(n)$ domestically and $P_t^{M^*}(n)$ abroad, where the latter is denoted in foreign currency. Profits (which are paid out as dividends to households) becomes:

$$\Psi_t(n) = P_t^Q(n) Q_t(n) + P_t^{M*}(n) S_t M_t^*(n) - W_{I,t} L_{I,t}(n) - R_{KI,t}(n)\overline{K}_{I,t}(n), \tag{62}$$

where $Q_t(n) + M_t^*(n) = T_t(n)$, and S_t is the nominal exchange rate ("NOK per foreign currency unit").

The costs of adjusting prices in the domestic and the foreign market are:

$$\gamma_{PQ,t}(n) \equiv \frac{\phi^{PQ}}{2}\left[\frac{P_t^Q(n)/P_{t-1}^Q(n)}{P_{t-1}^Q/P_{t-2}^Q} - 1\right]^2, \tag{63}$$

$$\gamma_{PM^*,t}(n) \equiv \frac{\phi^{PM^*}}{2}\left[\frac{P_t^{M^*}(n)/P_{t-1}^{M^*}(n)}{P_{t-1}^{M^*}/P_{t-2}^{M^*}} - 1\right]^2, \tag{64}$$

respectively, where $P_t^Q$and $P_t^{M^*}$are price indices ($P_t^Q = \left[\int_0^1 P_t^Q(n)^{1-\theta_t^H} dn\right]^{\frac{1}{1-\theta_t^H}}$, $P_t^{M^*} = \left[\int_0^1 P_t^{M^*}(n)^{1-\theta_t^{F^*}} dn\right]^{\frac{1}{1-\theta_t^{F^*}}}$). The costs of changing prices are governed by the parameters ϕ^{PQ} and ϕ^{PM^*}.

As shown in chapter 4, the intermediate firm n faces the following demand function from the final good sector: $Q_t(n) = \left(\frac{P_t^Q(n)}{P_t^Q}\right)^{-\theta_t^H} Q_t$ (see (85)), where θ_t^H is the elasticity of substitution between domestic goods produced by different firms in the intermediate goods sector. This parameter is assumed to follow an AR process. Correspondingly, the demand from abroad is $M_t^*(n) = \left(\frac{P_t^{M^*}(n)}{P_t^{M^*}}\right)^{-\theta_t^{F^*}} M_t^*$, where $\theta_t^{F^*}$ is the elasticity of substitution across intermediate export goods, which is also a shock that follows an AR process. Optimal price setting for firm n gives the following maximization problem:[5]

$$\max_{\{P_t^Q(n), P_t^{M^*}(n)\}} \Pi_s^{INT} =$$

$$\sum_{t=s}^{\infty} \Delta_{s,t} \left\{ \begin{array}{c} P_t^Q(n)\left(\frac{P_t^Q(n)}{P_t^Q}\right)^{-\theta_t^H} Q_t + P_t^{M^*}(n) S_t \left(\frac{P_t^{M^*}(n)}{P_t^{M^*}}\right)^{-\theta_t^{F^*}} M_t^* - MC_t\left[\left(\frac{P_t^Q(n)}{P_t^Q}\right)^{-\theta_t^H} Q_t + \left(\frac{P_t^{M^*}(n)}{P_t^{M^*}}\right)^{-\theta_t^{F^*}} M_t^*\right] \\ -\gamma_{PQ,t}(n) P_t^Q Q_t - \gamma_{PM^*,t}(n) P_t^{M^*} S_t M_t^* \end{array} \right\}.$$

FOC w.r.t. $P_t^Q(n)$:

[5] Note, that the costs of adjusting prices are linear in $P_t^Q Q_t$ and not $P_t^Q(n) Q_t(n)$.

$$
\begin{aligned}
&\left(\frac{P_t^Q(n)}{P_t^Q}\right)^{-\theta_t^H} Q_t - \frac{P_t^Q(n)}{P_t^Q}\theta_t^H \left(\frac{P_t^Q(n)}{P_t^Q}\right)^{-\theta_t^H-1} Q_t \\
&+MC_t\theta_t^H \left(\frac{P_t^Q(n)}{P_t^Q}\right)^{-\theta_t^H-1} \frac{Q_t}{P_t^Q} - \phi^{PQ}\left[\frac{P_t^Q(n)/P_{t-1}^Q(n)}{P_{t-1}^Q/P_{t-2}^Q}-1\right]\frac{1/P_{t-1}^Q(n)}{P_{t-1}^Q/P_{t-2}^Q}P_t^Q Q_t \\
&+E_t\left\{\Delta\phi^{PQ}\left[\frac{P_{t+1}^Q(n)/P_t^Q(n)}{P_t^Q/P_{t-1}^Q}-1\right]\frac{P_{t+1}^Q(n)}{P_t^Q/P_{t-1}^Q}\left(\frac{1}{P_t^Q(n)}\right)^2 P_{t+1}^Q Q_{t+1}\right\} = 0.
\end{aligned}
$$

In symmetric equilibrium, all firms will behave the same. Hence, from the definition of the price index, $P_t^Q = \left[\int_0^1 P_t^Q(n)^{1-\theta_t^H} dn\right]^{\frac{1}{1-\theta_t^H}} = \left[\overline{P}_t^{Q\,1-\theta_t^H}\int_0^1 1dn\right]^{\frac{1}{1-\theta_t^H}} = \overline{P}_t^Q = P_t^Q(n)$. Using this, the FOC can be rewritten as

$$
\begin{aligned}
&Q_t - \theta_t^H Q_t + MC_t\theta_t^H\frac{Q_t}{P_t^Q} - \phi^{PQ}\left[\frac{P_t^Q/P_{t-1}^Q}{P_{t-1}^Q/P_{t-2}^Q}-1\right]\frac{P_t^Q/P_{t-1}^Q}{P_{t-1}^Q/P_{t-2}^Q}Q_t \\
&+E_t\left\{\Delta\phi^{PQ}\left[\frac{P_{t+1}^Q/P_t^Q}{P_t^Q/P_{t-1}^Q}-1\right]\frac{(P_{t+1}^Q/P_t^Q)^2}{P_t^Q/P_{t-1}^Q}Q_{t+1}\right\} = 0.
\end{aligned} \tag{65}
$$

Similarly, the FOC w.r.t. $P_t^{M^*}(n)$ can be written as

$$
\begin{aligned}
&S_tM_t^* - \theta_t^{F^*}S_tM_t^* + MC_t\theta_t^{F^*}\frac{M_t^*}{P_t^{M^*}} - \phi^{PM^*}\left[\frac{P_t^{M^*}/P_{t-1}^{M^*}}{P_{t-1}^{M^*}/P_{t-2}^{M^*}}-1\right]\frac{P_t^{M^*}/P_{t-1}^{M^*}}{P_{t-1}^{M^*}/P_{t-2}^{M^*}}S_tM_t^* \\
&+E_t\left\{\Delta\phi^{PM^*}\left[\frac{P_{t+1}^{M^*}/P_t^{M^*}}{P_t^{M^*}/P_{t-1}^{M^*}}-1\right]\frac{\left(P_{t+1}^{M^*}/P_t^{M^*}\right)^2}{P_t^{M^*}/P_{t-1}^{M^*}}S_{t+1}M_{t+1}^*\right\} = 0.
\end{aligned} \tag{66}
$$

3.2 Making the equations stationary

3.2.1 Definitions

From (51) we get that

$$
\frac{T_t}{Z_t} = \widetilde{T}_t = \left[(1-\alpha)^{\frac{1}{\xi}}(z_t^L L_{I,t})^{1-\frac{1}{\xi}} + \alpha^{\frac{1}{\xi}}\widetilde{\overline{K}}_{I,t}^{1-\frac{1}{\xi}}\right]^{\frac{\xi}{\xi-1}}. \tag{67}
$$

To make the factor prices stationary, $\widetilde{W}_{I,t} \equiv \frac{W_{I,t}}{P_tZ_t}$ and $\widetilde{R}_{KI,t} \equiv \frac{R_{KI,t}}{P_t}$ where P_t is the price of the final good.

3.2.2 Demand equations

From (59) we have: $\frac{MC_t}{P_t} = \widetilde{MC}_t = \left[(1-\alpha)\left(\frac{\widetilde{W}_{I,t}}{z_t^L}\right)^{1-\xi} + \alpha(\widetilde{R}_{KI,t})^{1-\xi}\right]^{\frac{1}{1-\xi}}$. Use this in (57) to get that

$$
L_{I,t} = (1-\alpha)\left(\frac{\widetilde{W}_{I,t}}{\widetilde{MC}_t}\right)^{-\xi}\widetilde{T}_t(z_t^L)^{\xi-1}. \tag{68}
$$

From (58) we get that

$$
\widetilde{\overline{K}}_{I,t} = \alpha\left(\frac{\widetilde{R}_t^K}{\widetilde{MC}_t}\right)^{-\xi}\widetilde{T}_t. \tag{69}
$$

The costs of adjusting prices (equations (63) and (64)) are already stationary. Using $P_t^Q = P_t^Q(n)$, $P_t^{M^*} = P_t^{M^*}(n)$ and defining $\pi_t^Q \equiv \frac{P_t^Q}{P_{t-1}^Q}$and $\pi_t^{M^*} = \frac{P_t^{M^*}}{P_{t-1}^{M^*}}$,the equations can be rewritten as

$$\gamma_{PQ,t} = \frac{\phi^{PQ}}{2}\left[\frac{\pi_t^Q}{\pi_{t-1}^Q} - 1\right]^2, \tag{70}$$

$$\gamma_{PM^*,t} = \frac{\phi^{PM^*}}{2}\left[\frac{\pi_t^{M^*}}{\pi_{t-1}^{M^*}} - 1\right]^2. \tag{71}$$

3.2.3 First-order conditions

From (65) we get that

$$\widetilde{Q}_t - \theta_t^H \widetilde{Q}_t + \widetilde{MC}_t \theta_t^H \frac{\widetilde{Q}_t}{\widetilde{P}_t^Q} - \phi^{PQ}\left[\frac{\pi_t^Q}{\pi_{t-1}^Q} - 1\right]\frac{\pi_t^Q}{\pi_{t-1}^Q}\widetilde{Q}_t$$
$$+E_t\left\{\Lambda\phi^{PQ}\left[\frac{\pi_{t+1}^Q}{\pi_t^Q} - 1\right]\frac{(\pi_{t+1}^Q)^2}{\pi_t^Q}\widetilde{Q}_{t+1}\pi_{t+1}^z\right\} = 0. \tag{72}$$

Similarly, for (66) we get that

$$\widetilde{S}_t\widetilde{M}_t^* - \theta_t^{F*}\widetilde{S}_t\widetilde{M}_t^* + \widetilde{MC}_t\theta_t^{F*}\frac{\widetilde{M}_t^*}{\widetilde{P}_t^{M*}} - \phi^{PM*}\left[\frac{\pi_t^{M*}}{\pi_{t-1}^{M*}} - 1\right]\frac{\pi_t^{M*}}{\pi_{t-1}^{M*}}\widetilde{S}_t\widetilde{M}_t^*$$
$$+E_t\left\{\Lambda\phi^{PM*}\left[\frac{\pi_{t+1}^{M*}}{\pi_t^{M*}} - 1\right]\frac{(\pi_{t+1}^{M*})^2}{\pi_t^{M*}}\widetilde{S}_{t+1}\widetilde{M}_{t+1}^*\pi_{t+1}^z\right\} = 0. \tag{73}$$

where we define the real exchange rate as

$$\widetilde{S}_t = \frac{S_t P_t^*}{P_t}, \tag{74}$$

where S_t is the nominal exchange rate and P^* is the price level abroad.

3.2.4 Equations included in the model

The following equations are included in the model in the intermediate good sector: (67), (68), (69), (70), (71), (72) and (73).

3.3 Steady-state equations

From (67):

$$\widetilde{T}_{ss} = \left[(1-\alpha)^{\frac{1}{\xi}}(z_t^L L_{I,ss})^{1-\frac{1}{\xi}} + \alpha^{\frac{1}{\xi}}\widetilde{\widetilde{K}}_{I,ss}^{1-\frac{1}{\xi}}\right]^{\frac{\xi}{\xi-1}}. \tag{75}$$

From (68):

$$L_{I,ss} = (1-\alpha)\left(z_{ss}^L\right)^{\xi-1}\left(\frac{\widetilde{MC}_{ss}}{\widetilde{W}_{I,ss}}\right)^{\xi}\widetilde{T}_{ss}. \tag{76}$$

From (69):

$$\widetilde{\widetilde{K}}_{I,ss} = \alpha\left(\frac{\widetilde{R}_{ss}^K}{\widetilde{MC}_{ss}}\right)^{-\xi}\widetilde{T}_{ss}. \tag{77}$$

The adjustment costs (from (70) and (71)): $\gamma_{PQ,ss} = \gamma_{PM^*,ss} = 0$. Finally, the optimal prices from (72) and (73) become:

$$\widetilde{P}_{ss}^Q = \widetilde{MC}_{ss}\frac{\theta_{ss}^H}{(\theta_{ss}^H - 1)}, \tag{78}$$

$$\widetilde{P}_{ss}^{M^*} = \frac{\widetilde{MC}_{ss}}{\widetilde{S}_{ss}}\frac{\theta_{ss}^{F^*}}{\left(\theta_{ss}^{F^*} - 1\right)}. \tag{79}$$

4 Final goods sector

The final goods sector combines imported goods M_t and domestic goods Q_t to produce a final good A_t that is sold at a price P_t. The final good can be used for consumption, investments, government consumption and input to the oil supply firms.

The production function is given by

$$A_t = \left(\nu_t^{\frac{1}{\mu}} Q_t^{1-\frac{1}{\mu}} + (1-\nu_t)^{\frac{1}{\mu}} M_t^{1-\frac{1}{\mu}} \right)^{\frac{\mu}{\mu-1}}, \tag{80}$$

where ν_t is the domestic goods share (degree of home bias) and μ is the elasticity of substitution between domestic and imported goods. ν_t is regarded as an import shock and follows an AR process. The domestic goods Q_t is a composite of domestic goods produced by the different firms in the intermediate goods sector, $Q_t(n)$, i.e.

$$Q_t = \left[\int_0^1 Q_t(n)^{1-\frac{1}{\theta_t^H}} dn \right]^{\frac{\theta_t^H}{\theta_t^H - 1}}. \tag{81}$$

Here θ_t^H is the elasticity of substitution between domestic goods produced by different firms in the intermediate goods sector. The variable is a shock and assumed to follow an AR process. The imported goods M_t is a composite of imported goods produced by the different firms in the intermediate goods sector abroad, $M_t(f)$, i.e.

$$M_t = \left[\int_0^1 M_t(f)^{1-\frac{1}{\theta_t^F}} df \right]^{\frac{\theta_t^F}{\theta_t^F - 1}}, \tag{82}$$

where θ_t^F is the elasticity of substitution between imported goods produced by different firms in the intermediate goods sector abroad. It is a shock and assumed to follow an AR process.

4.1 Cost minimization

The final good sector needs to find the optimal combination of Q_t and M_t, which is found by cost minimization (note that P_t^M is in domestic currency):

$$\min_{\{Q_t, M_t\}} \left[P_t^Q Q_t + P_t^M M_t \right],$$

s.t. (80).

FOCs

$$Q_t = \nu_t \left(\frac{P_t^Q}{P_t} \right)^{-\mu} A_t, \tag{83}$$

$$M_t = (1-\nu_t) \left(\frac{P_t^M}{P_t} \right)^{-\mu} A_t, \tag{84}$$

where $P_t \equiv \left[\nu (P_t^Q)^{1-\mu} + (1-\nu)(P_t^M)^{1-\mu} \right]^{\frac{1}{1-\mu}}$. The price of the final good A_t is the marginal cost, P_t, which is the numeraire of the model.

Then, to find the demand facing each intermediate firm n (and f abroad), the final good sector minimizes the cost of its inputs $Q_t(n)$ given the prices set by different firms in the intermediate goods sector:

$$\min_{\{Q_t(n)\}} \int_0^1 P_t^Q(n) Q_t(n) dn,$$

s.t. (81).

FOC

$$Q_t(n) = \left(\frac{P_t^Q(n)}{P_t^Q} \right)^{-\theta_t^H} Q_t, \tag{85}$$

where

$$P_t^Q = \left[\int_0^1 P_t^Q(n)^{1-\theta_t^H} dn \right]^{\frac{1}{1-\theta_t^H}}. \tag{86}$$

Same type of cost minimization problem for $M_t(f)$ leads to

$$M_t(f) = \left(\frac{P_t^M(f)}{P_t^M}\right)^{-\theta_t^F} M_t, \tag{87}$$

where

$$P_t^M = \left[\int_0^1 P_t^M(f)^{1-\theta_t^F} df\right]^{\frac{1}{1-\theta_t^F}}. \tag{88}$$

4.2 Making the equations stationary

$$\widetilde{A}_t = \left(\nu_t^{\frac{1}{\mu}} \widetilde{Q}_t^{1-\frac{1}{\mu}} + (1-\nu_t)^{\frac{1}{\mu}} \widetilde{M}_t^{1-\frac{1}{\mu}}\right)^{\frac{\mu}{\mu-1}}, \tag{89}$$

$$\widetilde{Q}_t = \nu_t \left(\widetilde{P}_t^Q\right)^{-\mu} \widetilde{A}_t, \tag{90}$$

$$\widetilde{M}_t = (1-\nu_t) \left(\widetilde{P}_t^M\right)^{-\mu} \widetilde{A}_t. \tag{91}$$

4.2.1 Equations included in the model

The following equations are included in the model file: (89), (90) and (91).

4.3 Steady-state equations

$$\widetilde{A}_{ss} = \left(\nu_{ss}^{\frac{1}{\mu}} \widetilde{Q}_{ss}^{1-\frac{1}{\mu}} + (1-\nu_{ss})^{\frac{1}{\mu}} \widetilde{M}_{ss}^{1-\frac{1}{\mu}}\right)^{\frac{\mu}{\mu-1}}, \tag{92}$$

$$\widetilde{Q}_{ss} = \nu_{ss} \left(\widetilde{P}_{ss}^Q\right)^{-\mu} \widetilde{A}_{ss}, \tag{93}$$

$$\widetilde{M}_{ss} = (1-\nu_{ss}) \left(\widetilde{P}_{ss}^M\right)^{-\mu} \widetilde{A}_{ss}. \tag{94}$$

5 Entrepreneurs

5.1 Maximization problem

Entrepreneurs are households, but this section focuses on a separate part of the households' budget balance (we suppress index j in this section). We distinguish the problem like this because it is clearer (alternatively this sector could have been modeled as a firm, owned by the households). Entrepreneurs rent capital to the intermediate goods sector and the oil sector gaining the rental rate $R_{K,t}$ ($= R_{KI,t} = R_{KO,t}$ due to perfect capital mobility). They rent out $\overline{K}_{I,t}$ to the intermediate goods sector and $\overline{K}_{O,t}$ to the oil supply sector. $\overline{K}_t$ is then the aggregate utilized capital rented out by the entrepreneurs. At the beginning of period t they sell the undepreciated capital $(1-\delta) K_{t-1}$ at price P_t^K to the capital producers. The latter combines it with investment goods to produce K_t to be sold back to entrepreneurs at the same price. To finance their activity, entrepreneurs borrow $B_{e,t}$ from banks at net rate r_t^e, providing capital goods as collateral. They enter in a multi-period loan contract. Finally they also decide the capital utilization rate u_t.

We define utilized capital in period t as

$$\overline{K}_t = u_t K_{t-1}. \tag{95}$$

Entrepreneurs are subject to the following budget constraint:

$$\frac{R_{K,t}}{P_t} u_t K_{t-1} + \frac{P_t^K}{P_t}(1-\delta) K_{t-1} + I_{B,t}^e = \frac{P_t^K}{P_t} K_t + (r_t^e + \delta_t^e)\frac{P_{t-1}}{P_t} B_{e,t-1} + \gamma(u_t) K_{t-1} + C_t + \frac{1}{P_t}\Xi_t, \tag{96}$$

where C_t is household consumption, $I_{B,t}^e$ is new loans, $r_t^e + \delta_t^e$ represent the interest and principal payments to banks on outstanding debt and $\gamma(u_t) K_{t-1}$ is a cost that depends positively on the utilization rate of capital:

$$\gamma(u_t) = \frac{R_{K,ss}}{P_{ss}\phi_u}\left[e^{\phi_u(u_t-1)} - 1\right], \tag{97}$$

where ϕ_u governs the cost of adjusting the utilization rate. Ξ_t represents all other terms that enter into the household budget constraint (3). Similarly, all terms in (96) (except C_t and Ξ_t) are part of DIV_t in (3).

Total utilized capital rented out must be equal to the utilized capital demanded by the intermediate goods sector and by the oil supply sector, $\overline{K}_t = \overline{K}_{I,t} + \overline{K}_{O,t}$.

Whereas households could borrow against their housing capital, the entrepreneurs can borrow against their real capital $(1-\delta) K_t$. Corresponding to the household constraint (9) and (10), this gives the following collateral constraints that need to hold:

$$B_{e,t} = \frac{(1-\delta_t^e)}{1+\phi_t^{ent}} \frac{P_{t-1}}{P_t} B_{e,t-1} + \frac{\phi_t^{ent}}{1+\phi_t^{ent}} E_t \left[\frac{P_{t+1}^K}{P_{t+1}} \frac{P_{t+1}}{P_t} (1-\delta) K_t \right], \tag{98}$$

$$\delta_{t+1}^e = (1-\delta_t^e) \frac{P_{t-1}}{P_t} \frac{B_{e,t-1}}{B_{e,t}} \left[(\delta_t^e)^{\alpha^e} - (1-\alpha^e)^{\kappa^e} \right] + (1-\alpha^e)^{\kappa^e}, \tag{99}$$

where ϕ_t^{ent} governs the constraint on new loans, and is assumed to follow an AR process. See (414) for the relationship between ϕ_t^{ent} and the loan-to-value ratio. δ_t^e is loan repayments and α^e and κ^e are exogenous parameters. See discussion on page 5, section 2 for more on these parameters. Inserting for C_t from (96), we get the following Lagrangian:

$$\mathcal{L}(B_t^e, K_t, \delta_t^e, u_t, \omega_t^e, \mu_t^e) =$$

$$E_s \sum_{t=s}^{\infty} \beta^{t-s} \left[\begin{array}{c} u\left(\frac{R_{K,t}}{P_t} u_t K_{t-1} + \frac{P_t^K}{P_t}(1-\delta) K_{t-1} + B_{e,t} - \frac{P_t^K}{P_t} K_t - R_t^e \frac{P_{t-1}}{P_t} B_{e,t-1} - \gamma(u_t) K_{t-1} - \frac{1}{P_t}\Xi \right) + \Gamma \\ -\omega_t^e \left[B_{e,t} - \frac{(1-\delta_t^e)}{1+\phi_t^{ent}} \frac{P_{t-1}}{P_t} B_{e,t-1} - \frac{\phi_t^{ent}}{1+\phi_t^{ent}} E_t \left[\frac{P_{t+1}^K}{P_{t+1}} \frac{P_{t+1}}{P_t} (1-\delta) K_t \right] \right] \\ -\mu_t^e \left[\delta_{t+1}^e - (1-\delta_t^e) \frac{P_{t-1}}{P_t} \frac{B_{e,t-1}}{B_{e,t}} \left[(\delta_t^e)^{\alpha^e} - (1-\alpha^e)^{\kappa^e} \right] - (1-\alpha^e)^{\kappa^e} \right] \end{array} \right],$$

where ω_t^e and μ_t^e are the Lagrangian multipliers and Γ includes all elements of the households maximization problem (11) not in focus here (i.e. the utility functions with respect to deposits, housing, leisure as well as the constraints).

FOC

w.r.t. K_t:

$$u'(C_t) \frac{P_t^K}{P_t} = E_t \left[\omega_t^e \frac{\phi_t^{ent}}{1+\phi_t^{ent}} \frac{P_{t+1}^K}{P_{t+1}} \frac{P_{t+1}}{P_t} (1-\delta) \right] + E_t \left[\beta u'(C_{t+1}) \left(\frac{P_{t+1}^K}{P_{t+1}} (1-\delta) + \frac{R_{K,t+1}}{P_{t+1}} u_{t+1} - \gamma(u_{t+1}) \right) \right]. \tag{100}$$

w.r.t. B_t^e (and multiplying with $\frac{B_{e,t}}{u'(C_t)}$):

$$\begin{aligned} B_{e,t} - B_{e,t} E_t \left[\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t}{P_{t+1}} \right] R_t^e - \frac{\omega_t^e}{u'(C_t)} B_{e,t} \\ +E_t \left[\frac{\omega_{t+1}^e}{u'(C_{t+1})} \frac{u'(C_{t+1})}{u'(C_t)} \beta \frac{P_t}{P_{t+1}} \frac{(1-\delta_{t+1}^e)}{1+\phi_{t+1}^{ent}} B_{e,t} \right] - \frac{\mu_t^e}{u'(C_t)} \frac{B_{e,t-1}}{B_{e,t}} \frac{P_{t-1}}{P_t} (1-\delta_t^e) \left[(\delta_t^e)^{\alpha^e} - (1-\alpha^e)^{\kappa^e} \right] \\ +E_t \left[\beta \frac{\mu_{t+1}^e}{u'(C_{t+1})} \frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t}{P_{t+1}} \frac{B_{e,t}}{B_{e,t+1}} (1-\delta_{t+1}^e) \left[(\delta_{t+1}^e)^{\alpha^e} - (1-\alpha^e)^{\kappa^e} \right] \right] = 0. \end{aligned} \tag{101}$$

w.r.t. δ_t^e:

$$-\mu_{t-1}^e + \mu_t^e \beta \frac{B_{e,t-1}}{B_{e,t}} \frac{P_{t-1}}{P_t} \left[\alpha^e (\delta_t^e)^{\alpha^e - 1} (1-\delta_t^e) - (\delta_t^e)^{\alpha^e} + (1-\alpha^e)^{\kappa^e} \right] - \omega_t^e \beta \left[\frac{B_{e,t-1}}{1+\phi_t^{ent}} \frac{P_{t-1}}{P_t} \right] = 0. \tag{102}$$

w.r.t. u_t:

$$\frac{R_{K,t}}{P_t} = \gamma'(u_{,t}). \tag{103}$$

where (recall (97))

$$\gamma'(u_t) = \frac{R_{K,ss}}{P_{ss}} e^{\phi_u(u_t - 1)}. \tag{104}$$

5.2 Making the equations stationary

First, define $\widetilde{\omega}_t^e = \frac{\omega_t^e}{u'(C_t)}$ and $\widetilde{\mu}_t^e = \frac{\mu_t^e}{Z_t u'(C_t)}$, which are both stationary.

5.2.1 First-order conditions

The stationary FOCs become:

w.r.t. K_t:

$$\widetilde{P}_t^K = E_t\left[\widetilde{\omega}_t^e \frac{\phi_t^{ent}}{1+\phi_t^{ent}} \widetilde{P}_{t+1}^K \pi_{t+1} (1-\delta)\right] + E_t\left[\Lambda \pi_{t+1}\left(\widetilde{P}_{t+1}^K (1-\delta) + \widetilde{R}_{K,t+1} u_{t+1} - \gamma(u_{t+1})\right)\right]. \tag{105}$$

w.r.t. B_t^e:

$$\begin{aligned} &\widetilde{B}_{e,t} - \widetilde{B}_{e,t} R_t^e E_t[\Lambda] - \widetilde{\omega}_t^e \widetilde{B}_{e,t} \\ &+E_t\left[\widetilde{\omega}_{t+1}^e \Lambda \frac{(1-\delta_{t+1}^e)}{1+\phi_{t+1}^{ent}}\right] B_{e,t} - \widetilde{\mu}_t^e (1-\delta_t^e) \frac{\widetilde{B}_{e,t-1}}{\widetilde{B}_{e,t}} \frac{1}{\pi_t \pi_t^{\tilde{z}}} \left[(\delta_t^e)^{\alpha^e} - (1-\alpha^e)^{\kappa^e}\right] \\ &+E_t\left[\widetilde{\mu}_{t+1}^e \Lambda (1-\delta_{t+1}^e) \frac{\widetilde{B}_{e,t}}{\widetilde{B}_{e,t+1}} \left[\left(\delta_{t+1}^e\right)^{\alpha^e} - (1-\alpha^e)^{\kappa^e}\right]\right] = 0. \end{aligned} \tag{106}$$

w.r.t. δ_t^e (leaded):

$$-\widetilde{\mu}_t^e + E_t\left[\widetilde{\mu}_{t+1}^e \Lambda \frac{\widetilde{B}_{e,t}}{\widetilde{B}_{e,t+1}} \left[\alpha^e \left(\delta_{t+1}^e\right)^{\alpha^e - 1} (1-\delta_{t+1}^e) - \left(\delta_{t+1}^e\right)^{\alpha^e} + (1-\alpha^e)^{\kappa^e}\right] - \widetilde{\omega}_{t+1}^e \Lambda \frac{\widetilde{B}_{e,t}}{1+\phi_{t+1}^{ent}}\right] = 0. \tag{107}$$

w.r.t. u_t:

$$\widetilde{R}_{K,t} = \gamma'(u_t). \tag{108}$$

5.2.2 Definitions

$$\widetilde{\widetilde{K}}_t = \frac{u \widetilde{K}_{t-1}}{\pi_t^{\tilde{z}}}, \tag{109}$$

$$\gamma(u_t) = \frac{\widetilde{R}_{K,ss}}{\phi_u}\left[e^{\phi_u (u_t - 1)} - 1\right], \tag{110}$$

$$\gamma'(u_t) = \widetilde{R}_{K,ss} e^{\phi_u (u_t - 1)}. \tag{111}$$

5.2.3 The constraints

$$\widetilde{B}_{e,t} = \frac{(1-\delta_t^e)}{1+\phi_t^{ent}} \frac{1}{\pi_t \pi_t^{\tilde{z}}} \widetilde{B}_{e,t-1} + \frac{\phi_t^{ent}}{1+\phi_t^{ent}} E_t\left[\widetilde{P}_{t+1}^K \pi_t (1-\delta) \widetilde{K}_t\right], \tag{112}$$

$$\delta_t^e = (1-\delta_{t-1}^e) \frac{1}{\pi_{t-1} \pi_{t-1}^{\tilde{z}}} \frac{\widetilde{B}_{e,t-2}}{\widetilde{B}_{e,t-1}} \left[\left(\delta_{t-1}^e\right)^{\alpha^e} - (1-\alpha^e)^{\kappa^e}\right] + (1-\alpha^e)^{\kappa^e}. \tag{113}$$

5.2.4 Equations included in the model

The following equations are included in the model file: (105), (106), (107), (108), (109), (110), (111), (112) and (113).

5.3 Steady-state equations

5.3.1 First-order conditions

FOC

w.r.t. K_t:

$$\widetilde{P}_{ss}^K = \left[1 - \pi_{ss} (1-\delta) \left(\widetilde{\omega}_{ss}^e \frac{\phi_{ss}^{ent}}{1+\phi_{ss}^{ent}} + \Lambda_{ss}\right)\right]^{-1} \Lambda_{ss} \pi_{ss} \widetilde{R}_{Kss}. \tag{114}$$

w.r.t. $B_{e,t}$:

$$\widetilde{B}_{e,ss} - \widetilde{B}_{e,ss} R^e_{ss} \Lambda_{ss} - \widetilde{\omega}^e_{ss} \widetilde{B}_{e,ss} \quad + \widetilde{\omega}^e_{ss} \Lambda_{ss} \frac{(1-\delta^e_{ss})}{1+\phi^{ent}_{ss}} B_{e,ss}$$

$$-\widetilde{\mu}^e_{ss}(1-\delta^e_{ss}) \frac{1}{\pi_{ss}\pi^z_{ss}} \left[(\delta^e_{ss})^{\alpha^e} - (1-\alpha^e)^{\kappa^e} \right] + \widetilde{\mu}^e_{ss} \Lambda_{ss} (1-\delta^e_{ss}) \left[(\delta^e_{ss})^{\alpha^e} - (1-\alpha^e)^{\kappa^e} \right] = 0. \tag{115}$$

w.r.t. δ^e_t:

$$-\widetilde{\mu}^e_{ss} + \widetilde{\mu}^e_{ss} \Lambda_{ss} \left[\alpha^e (\delta^e_{ss})^{\alpha^e - 1} (1-\delta^e_{ss}) - (\delta^e_{ss})^{\alpha^e} + (1-\alpha^e)^{\kappa^e} \right] - \widetilde{\omega}^e_{ss} \Lambda_{ss} \frac{\widetilde{B}_{e,ss}}{1+\phi^{ent}_{ss}} = 0. \tag{116}$$

w.r.t. u_t:

$$\widetilde{R}_{K,ss} = \gamma'(u_{ss}). \tag{117}$$

5.3.2 Definitions

$$u_{ss} = 1, \tag{118}$$

$$\widetilde{\overline{K}}_{ss} = \frac{\widetilde{K}_{ss}}{\pi^z_{ss}}, \tag{119}$$

$$\gamma(u_{ss}) = 0. \tag{120}$$

5.3.3 Constraints

$$\widetilde{B}_{e,ss} = \left[1 - \frac{(1-\delta^e_{ss})}{1+\phi^{ent}_{ss}} \frac{1}{\pi_{ss}\pi^z_{ss}} \right]^{-1} \frac{\phi^{ent}_{ss}}{1+\phi^{ent}_{ss}} \pi_{ss} (1-\delta) \widetilde{P}^K_{ss} \widetilde{K}_{ss}, \tag{121}$$

$$\delta^e_{ss} = (1-\delta^e_{ss}) \frac{1}{\pi_{ss}\pi^z_{ss}} \left[(\delta^e_{ss})^{\alpha^e} - (1-\alpha^e)^{\kappa^e} \right] + (1-\alpha^e)^{\kappa^e}. \tag{122}$$

6 Capital producers

Capital goods, K_t, are produced by a separate sector. At the beginning of period t the capital goods producers buy undepreciated capital $(1-\delta) K_{t-1}$ at price P^K_t from the entrepreneurs, and combine it with gross investment goods $I_{C,t}$ to produce K_t to be sold back to entrepreneurs at the same price. The capital producers operate in a perfectly competitive market, and therefore earn no profit. $I_{C,t}$ is bought from the final goods sector at a price P_t.

6.1 Maximization problem

Profit maximization for producer h leads to the following problem:

$$\max_{\{I_{C,t}(h)\}} [P^K_t K_t(h) - P^K_t (1-\delta) K_{t-1}(h) - P_t I_{C,t}(h)],$$

s.t. the capital accumulation equation:

$$K_t(h) = (1-\delta) K_{t-1}(h) + \kappa_t(h) K_{t-1}(h), \tag{123}$$

where we refer to the last term, $\kappa_t K_{t-1}$, as "net investments", that is investments net of adjustment costs:

$$\kappa_t(h) = \frac{I_{C,t}(h)}{K_{t-1}(h)} - \frac{\phi_{I1}}{2} \left[\frac{I_{C,t}(h)}{K_{t-1}(h)} - \frac{\widetilde{I}_{C,ss}\pi^z_{ss}}{\widetilde{K}_{ss}} z_{I,t} \right]^2 - \frac{\phi_{I2}}{2} \left[\frac{I_{C,t}(h)}{K_{t-1}(h)} - \frac{I_{C,t-1}}{K_{t-2}} \right]^2. \tag{124}$$

The parameters ϕ_{I1} and ϕ_{I2} govern the degree of adjustment costs, and $z_{I,t}$ is an investment shock following an AR process. Note that because of the adjustments costs, net investments are smaller than gross investments, $\kappa_t K_{t-1} < I_{C,t}$ (except in steady-state).

Maximization gives the following FOC w.r.t. $I_{C,t}$ (suppressing h):

$$\frac{P^K_t}{P_t} = \frac{1}{K_{t-1}\kappa'_t} = \frac{1}{\overline{\kappa}'_t}, \tag{125}$$

where κ_t' is the derivative of κ_t:

$$\kappa_t' = \frac{1}{K_{t-1}} - \phi_{I1}\left[\frac{I_{C,t}}{K_{t-1}} - \frac{\widetilde{I}_{C,ss}\pi^z_{ss}}{\widetilde{K}_{ss}} z_{I,t}\right]\frac{1}{K_{t-1}} - \phi_{I2}\left[\frac{I_{C,t}}{K_{t-1}} - \frac{I_{C,t-1}}{K_{t-2}}\right]\frac{1}{K_{t-1}}, \tag{126}$$

i.e.

$$\overline{\kappa}_t' = 1 - \phi_{I1}\left[\frac{I_{C,t}}{K_{t-1}} - \frac{\widetilde{I}_{C,ss}\pi^z_{ss}}{\widetilde{K}_{ss}} z_{I,t}\right] - \phi_{I2}\left[\frac{I_{C,t}}{K_{t-1}} - \frac{I_{C,t-1}}{K_{t-2}}\right]. \tag{127}$$

6.2 Making the equations stationary

Capital accumulation equation in symmetric equilibrium:

$$\widetilde{K}_t = \frac{(1-\delta)}{\pi^z_t}\widetilde{K}_{t-1} + \frac{\kappa_t}{\pi^z_t}\widetilde{K}_{t-1}. \tag{128}$$

Capital adjustment costs:

$$\kappa_t = \frac{\widetilde{I}_{C,t}\pi^z_t}{\widetilde{K}_{t-1}} - \frac{\phi_{I1}}{2}\left[\frac{\widetilde{I}_{C,t}\pi^z_t}{\widetilde{K}_{t-1}} - \frac{\widetilde{I}_{C,ss}\pi^z_{ss}}{\widetilde{K}_{ss}} z_{I,t}\right]^2 - \frac{\phi_{I2}}{2}\left[\frac{\widetilde{I}_{C,t}\pi^z_t}{\widetilde{K}_{t-1}} - \frac{\widetilde{I}_{C,t-1}\pi^z_{t-1}}{\widetilde{K}_{t-2}}\right]^2. \tag{129}$$

FOC w.r.t. $I_{C,t}$:

$$\widetilde{P}^K_t = \frac{1}{\overline{\kappa}_t'}, \tag{130}$$

where

$$\overline{\kappa}_t' = 1 - \phi_{I1}\left[\frac{\widetilde{I}_{C,t}\pi^z_t}{\widetilde{K}_{t-1}} - \frac{\widetilde{I}_{C,ss}\pi^z_{ss}}{\widetilde{K}_{ss}} z_{I,t}\right] - \phi_{I2}\left[\frac{\widetilde{I}_{C,t}\pi^z_t}{\widetilde{K}_{t-1}} - \frac{\widetilde{I}_{C,t-1}\pi^z_{t-1}}{\widetilde{K}_{t-2}}\right]. \tag{131}$$

6.2.1 Equations included in the model

The following equations are included in the model file: (128), (129), (130) and (131).

6.3 Steady-state equations

Capital accumulation equation:

$$\widetilde{K}_{ss} = \frac{(1-\delta)}{\pi^z_{ss}}\widetilde{K}_{ss} + \frac{\kappa_{ss}}{\pi^z_{ss}}\widetilde{K}_{ss}. \tag{132}$$

Capital adjustment costs:

$$\kappa_{ss} = \frac{\widetilde{I}_{C,ss}\pi^z_{ss}}{\widetilde{K}_{ss}}. \tag{133}$$

FOC w.r.t. $I_{C,t}$:

$$\widetilde{P}^K_t = \frac{1}{\overline{\kappa}_{ss}'} = 1, \tag{134}$$

where

$$\overline{\kappa}_{ss}' = 1. \tag{135}$$

7 Housing producers

At the beginning of period t the housing producer buys the undepreciated housing stock $(1-\delta_H)H_{t-1}$ at price P^H_t from households, and combines it with housing investment goods $I_{H,t}$ to produce H_t to be sold back to households at the same price. The housing producers are subject to a market with full competition, and therefore earns no profit. $I_{H,t}$ is bought from the final goods sector at a price P_t.

7.1 Maximization problem

The representative housing producer f maximizes:

$$\max_{\{I_{H,t}\}} \left[P_t^H H_t(f) - P_t^H (1-\delta_H) H_{t-1}(f) - P_t I_{H,t}(f)\right],$$

s.t. the housing accumulation equation:

$$H_t(f) = (1-\delta_H)H_{t-1}(f) + \gamma_{H,t}(f)H_{t-1}(f), \tag{136}$$

where $\gamma_{H,t}(f)H_{t-1}(f)$ is "net housing investments" and $\gamma_{H,t}(f)$ is defined as

$$\gamma_{H,t}(f) = \frac{I_{H,t}(f)}{H_{t-1}(f)Z_t^h} - \frac{\phi_{H1}}{2}\left[\frac{I_{H,t}(f)}{H_{t-1}(f)Z_t^h} - \frac{\widetilde{I}_{H,ss}\pi_{ss}^z}{\widetilde{H}_{ss}\pi_{ss}^h} z_{IH,t}\right]^2 - \frac{\phi_{H2}}{2}\left[\frac{I_{H,t}(f)}{H_{t-1}(f)Z_t^h} - \frac{I_{H,t-1}}{H_{t-2}Z_{t-1}^h}\right]^2. \tag{137}$$

The parameters ϕ_{H1} and ϕ_{H2} govern the degree of adjustment costs, and $z_{IH,t}$ is a housing investment shock following an AR process. See section 1.1 for the definition of the housing productivity parameter Z_t^h.

FOC w.r.t. $I_{H,t}$ (suppressing f):

$$\frac{P_t^H}{P_t} = \frac{1}{H_{t-1}\gamma'_{H,t}} = Z_t^h\left(1 - \phi_{H1}\left[\frac{I_{H,t}}{H_{t-1}Z_t^h} - \frac{\widetilde{I}_{H,ss}\pi_{ss}^z}{\widetilde{H}_{ss}\pi_{ss}^h} z_{IH,t}\right] - \phi_{H2}\left[\frac{I_{H,t}}{H_{t-1}Z_t^h} - \frac{I_{H,t-1}}{H_{t-2}Z_{t-1}^h}\right]\right)^{-1}, \tag{138}$$

as

$$\gamma'_{H,t} = \frac{1}{H_{t-1}Z_t^h} - \phi_{H1}\left[\frac{I_{H,t}}{H_{t-1}Z_t^h} - \frac{\widetilde{I}_{H,ss}\pi_{ss}^z}{\widetilde{H}_{ss}\pi_{ss}^h} z_{IH,t}\right]\frac{1}{H_{t-1}Z_t^h} - \phi_{H2}\left[\frac{I_{H,t}}{H_{t-1}Z_t^h} - \frac{I_{H,t-1}}{H_{t-2}Z_{t-1}^h}\right]\frac{1}{H_{t-1}Z_t^h}. \tag{139}$$

7.2 Making the equations stationary

First we note that

$$\frac{I_{H,t}}{H_{t-1}Z_t^h} = \frac{\frac{I_{H,t}}{Z_t}Z_t}{\frac{H_{t-1}Z_{t-1}^h}{Z_{t-1}}\frac{Z_{t-1}}{Z_{t-1}^h}Z_t^h} = \frac{\widetilde{I}_{H,t}\frac{Z_t}{Z_{t-1}}}{\widetilde{H}_{t-1}\frac{Z_t^h}{Z_{t-1}^h}} = \frac{\widetilde{I}_{H,t}\pi_t^z}{\widetilde{H}_{t-1}\pi_t^h}, \tag{140}$$

and repeat how to make the real house price stationary:

$$\widetilde{P}_t^H = \frac{P_t^H}{P_t Z_t^h}. \tag{141}$$

Housing accumulation equation:

$$\widetilde{H}_t = \frac{\pi_t^h(1-\delta_H)}{\pi_t^z}\widetilde{H}_{t-1} + \frac{\pi_t^h\gamma_{H,t}}{\pi_t^z}\widetilde{H}_{t-1}. \tag{142}$$

Housing adjustment costs:

$$\gamma_{H,t} = \frac{\widetilde{I}_{H,t}\pi_t^z}{\widetilde{H}_{t-1}\pi_t^h} - \frac{\phi_{H1}}{2}\left[\frac{\widetilde{I}_{H,t}\pi_t^z}{\widetilde{H}_{t-1}\pi_t^h} - \frac{\widetilde{I}_{H,ss}\pi_{ss}^z}{\widetilde{H}_{ss}\pi_{ss}^h} z_{IH,t}\right]^2 - \frac{\phi_{H2}}{2}\left[\frac{\widetilde{I}_{H,t}\pi_t^z}{\widetilde{H}_{t-1}\pi_t^h} - \frac{\widetilde{I}_{H,t-1}\pi_{t-1}^z}{\widetilde{H}_{t-2}\pi_{t-1}^h}\right]^2. \tag{143}$$

FOC w.r.t. $I_{H,t}$:

$$\widetilde{P}_t^H = \left(1 - \phi_{H1}\left[\frac{\widetilde{I}_{H,t}\pi_t^z}{\widetilde{H}_{t-1}\pi_t^h} - \frac{\widetilde{I}_{H,ss}\pi_{ss}^z}{\widetilde{H}_{ss}\pi_{ss}^h} z_{IH,t}\right] - \phi_{H2}\left[\frac{\widetilde{I}_{H,t}\pi_t^z}{\widetilde{H}_{t-1}\pi_t^h} - \frac{\widetilde{I}_{H,t-1}\pi_{t-1}^z}{\widetilde{H}_{t-2}\pi_{t-1}^h}\right]\right)^{-1}. \tag{144}$$

7.2.1 Equations included in the model

The following equations are included in the model file: 142, 143 and 144.

7.3 Steady-state equations

Housing accumulation equation:

$$\widetilde{H}_{ss} = \frac{\pi^h_{ss}(1-\delta_H)}{\pi^z_{ss}} \widetilde{H}_{ss} + \frac{\pi^h_{ss}\gamma_{H,ss}}{\pi^z_{ss}} \widetilde{H}_{ss}. \tag{145}$$

Housing adjustment costs:

$$\gamma_{H,ss} = \frac{\widetilde{I}_{H,ss}\pi^z_{ss}}{\widetilde{H}_{ss}\pi^h_{ss}}. \tag{146}$$

FOC w.r.t. $I_{H,t}$:

$$\widetilde{P}^H_{ss} = z_{HS,ss}. \tag{147}$$

8 Banking sector

We assume that there is an infinite number of banks, indexed by $i \in [0,1]$. Each bank consists of two retail branches and a wholesale branch. One retail branch is responsible for providing differentiated loans to households and to entrepreneurs, while the other retail branch takes care of the deposit side. Both branches set interest rates in a monopolistically competitive fashion, subject to adjustment costs. The wholesale branch manages the capital position of the bank. Its task is to choose the overall level of operations regarding deposit and lending, taking into account the risk-weighted capital requirements. We assume that banks have to adhere to a regulatory capital requirement. Failing to do so, will incur a cost proportional to total assets (lending).

Bank capital plays an important role for credit supply in the model through a feedback loop between the real and the financial side of the economy.

The balance sheet of bank i (in real terms):

$$B_t(i) = B^{TOT}_{F,t}(i) + K^B_t(i), \tag{148}$$

where $B_t(i)$ is total assets (total lending). On the liability side, $B^{TOT}_{F,t}(i)$ is total external bank funding and $K^B_t(i)$ is bank capital (equity). Total external bank funding is the sum of household deposits and foreign debt, i.e.

$$B^{TOT}_{F,t}(i) = D_t(i) + B^*_t(i). \tag{149}$$

Make note that $P_t B^*_t(i)$ is nominal foreign bank debt in domestic currency. Total lending is the sum of lending to entrepreneurs and households:

$$B_t(i) = B_{e,t}(i) + B_{h,t}(i). \tag{150}$$

If banks fail to meet their target level of risk-weighted capital requirements, ϖ_t, they incur a penalty cost. The target level of risk-weighted capital requirements consists of two elements: "hard" capital requirements, γ^b_t, and a countercyclical capital buffer, CCB^b_t, (both are AR shocks):

$$\varpi_t = \gamma^b_t + CCB^b_t.$$

Banks also face linear operational costs. Profits in period t for bank i as a whole is given by:

$$\begin{aligned} J_t(i) = r^F_t(i) B_{h,t}(i) + r^e_t(i) B_{e,t}(i) - r^d_t(i) D_t(i) - \left[\left[1-\gamma^{B^*}_t\right] R^*_t \frac{S_{t+1}}{S_t} - 1\right] B^*_t(i) \\ -\chi_o B_t(i) - \frac{\chi_c}{2}\left[\frac{K^B_t(i)}{B^{RW}_t(i)} - \varpi_t\right]^2 K^B_t(i), \end{aligned} \tag{151}$$

where $r^F_t(i)$ is the net interest rate on loans to households, $r^e_t(i)$ is the net interest rate on loans to entrepreneurs and $r^d_t(i)$ is the net deposit interest rate. The bank pays a risk premium on foreign funding. The "full" net interest rate for foreign funding hence becomes $\left[1-\gamma^{B^*}_t\right] R^*_t \frac{S_{t+1}}{S_t} - 1$, where $1-\gamma^{B^*}_t$ is the debt-elastic risk premium and R^*_t is the foreign money market rate and S_t is the nominal exchange rate. χ_o governs the operational costs, and χ_c governs the capital target costs. $B^{RW}_t(i)$ denotes the risk-weighted assets:

$$B^{RW}_t(i) = \varsigma^e B_{e,t}(i) + \varsigma^h B_{h,t}(i), \tag{152}$$

where ς^e and ς^h are the risk-weights associated with credit to entrepreneurs and households respectively. Bank capital is accumulated according to:

$$K_t^B(i) = (1-\delta^b)\frac{P_{t-1}}{P_t}K_{t-1}^B(i) + \frac{P_{t-1}}{P_t}J_{t-1}(i), \tag{153}$$

where δ^b is the dividend share of the bank capital paid out to shareholders (households).

8.1 Maximization problem

8.1.1 Wholesale branch

The wholesale branch lends to the loan branch at the interest rate $R_t^{b,e}(i)$ for entrepreneurial loans and $R_t^{b,h}(i)$ for household loans. It is funded through borrowing from the deposit branch and from abroad. The wholesale deposit rate is assumed to be equal to the money market rate R_t, which follows from a no-arbitrage condition since we assume that banks have access to unlimited financing at the money market rate. The wholesale branch takes these funding costs as given and solves the following profit maximization problem:

$$\max_{\{B_{e,t}(i),B_{h,t}(i),D_t(i),B_t^*(i)\}} E_t\left[R_t^{b,e}(i)B_{e,t}(i) + R_t^{b,h}(i)B_{h,t}(i)\right],$$

$$-E_t\left[R_tD_t(i) + \left[1-\gamma_t^{B^*}\right]R_t^*\frac{S_{t+1}}{S_t}B_t^*(i) + \chi_oB_t(i) + \frac{\chi_c}{2}\left[\frac{K_t^B(i)}{B_t^{RW}(i)} - \varpi_t\right]^2K_t^B(i)\right] \tag{154}$$

subject to the definitions in (148) - (150) and (152).

The first-order conditions for the wholesale bank becomes (in symmetric equilibrium):

$$R_t^{b,e} = R_t + \chi_o - \chi_c\varsigma^e\left[\frac{K_t^B}{B_t^{RW}} - \varpi_t\right]\left(\frac{K_t^B}{B_t^{RW}}\right)^2 \tag{155}$$

$$R_t^{b,h} = R_t + \chi_o - \chi_c\varsigma^h\left[\frac{K_t^B}{B_t^{RW}} - \varpi_t\right]\left(\frac{K_t^B}{B_t^{RW}}\right)^2 \tag{156}$$

$$R_t = E_t\left[\left[1-\gamma_t^{B^*}\right]R_t^*\frac{S_{t+1}}{S_t}\right] \tag{157}$$

where (157) is this model's version of the uncovered interest parity (UIP). It is assumed that the risk premium depends positively on the level of total foreign debt for the country as a whole, B_t^{TOT*}. See section 11.

8.1.2 Loan branch

The loan branch lends to households and entrepreneurs (at net rates $r_t^F(i)$ and $r_t^e(i)$, respectively) and borrows from the wholesale branch at the interest rates $R_t^{b,h}(i)$ and $R_t^{b,e}$, respectively. It faces costs when changing the rates, governed by the parameter ϕ^F and ϕ^e. To find the demand curve for household loans facing the loan branch i, every household will solve $\min_{B_{h,t}(i,j)}\int_0^1 r_t^F(i)B_{h,t}(i,j)\,di$ subject to $\left[\int_0^1 B_{h,t}(i,j)^{1-\frac{1}{\theta_t^{IH}}}di\right]^{\frac{\theta_t^{IH}}{\theta_t^{IH}-1}} = B_{h,t}(j)$, where θ_t^{IH} is the elasticity of substitution between loans from all loan branches (it is also a shock and is assumed to follow an AR process). This will give the demand from household j for loans from branch i : $B_{h,t}(i,j) = \left(\frac{r_t^F(i)}{r_t^F}\right)^{-\theta_t^{IH}}B_{h,t}(j)$, where r_t^F is defined as $r_t^F = \left[\int_0^1 r_t^F(i)^{1-\theta_t^{IH}}di\right]^{\frac{1}{1-\theta_t^{IH}}}$. Summing over all households gives the total demand facing the loan branch i from the household sector: $B_{h,t}(i) = \int_0^1 B_{h,t}(i,j)\,dj = \left(\frac{r_t^F(i)}{r_t^F}\right)^{-\theta_t^{IH}}\int_0^1 B_{h,t}(j)dj = \left(\frac{r_t^F(i)}{r_t^F}\right)^{-\theta_t^{IH}}B_{h,t}$.

A similar exercise for the entrepreneurs gives their corresponding demand function for $B_{e,t}(i)$, with the corresponding elasticity of substitution θ_t^e, which is also a shock in the model and assumed to follow an AR process. The maximization problem becomes:

$$\max_{\{r_t^F(i),r_t^e(i)\}} E_s\sum_{t=s}^{\infty}\Delta_{s,t}\left[\begin{array}{c}\left(r_t^F(i) - r_t^{b,h}(i)\right)B_{h,t}(i) + \left(r_t^e(i) - r_t^{b,e}(i)\right)B_{e,t}(i)\\ -\frac{\phi^F}{2}\left(\frac{r_t^F(i)}{r_{t-1}^F(i)} - 1\right)^2r_t^FB_{h,t} - \frac{\phi^e}{2}\left(\frac{r_t^e(i)}{r_{t-1}^e(i)} - 1\right)^2r_t^eB_{e,t}\end{array}\right],$$

s.t.

$$B_t(i) = B_{e,t}(i) + B_{h,t}(i), \tag{158}$$

$$B_t^h(i) = \left(\frac{r_t^F(i)}{r_t^F}\right)^{-\theta_t^{IH}} B_t^h, \tag{159}$$

$$B_t^e(i) = \left(\frac{r_t^e(i)}{r_t^e}\right)^{-\theta_t^e} B_t^e. \tag{160}$$

FOC w.r.t. $r_t^F(i)$:

$$\begin{array}{c} B_{h,t}(i) + r_t^F(i)\left[\frac{\delta B_{h,t}(i)}{\delta r_t^F(i)}\right] - r_t^{b,h}(i)\left[\frac{\delta B_{h,t}(i)}{\delta r_t^F(i)}\right] - \phi^F\left(\frac{r_t^F(i)}{r_{t-1}^F(i)} - 1\right)\frac{r_t^F}{r_{t-1}^F(i)} B_{h,t} \\ + E_t \Delta \phi^F \left(\frac{r_{t+1}^F(i)}{r_t^F(i)} - 1\right)\left(\frac{1}{r_t^F(i)}\right)^2 r_{t+1}^F(i)\, r_{t+1}^F \frac{P_{t+1}}{P_t} B_{h,t+1} = 0 \end{array}, \tag{161}$$

where

$$\frac{\delta B_{h,t}(i)}{\delta r_t^F(i)} = -\theta_t^{IH} \frac{B_{h,t}(i)}{r_t^F(i)}. \tag{162}$$

All banks will behave the same and set the same interest rate, $r_t^F(i) = \overline{r}_t^F$ for all i: $r_t^F = \left[\int_0^1 r_t^F(i)^{1-\theta_t^{IH}} di\right]^{\frac{1}{1-\theta_t^{IH}}} = \left[\int_0^1 \overline{r}_t^{F\,1-\theta_t^{IH}} di\right]^{\frac{1}{1-\theta_t^{IH}}} = \left[\overline{r}_t^{F\,1-\theta_t^{IH}} \int_0^1 1 di\right]^{\frac{1}{1-\theta_t^{IH}}} = \overline{r}_t^F = r_t^F(i)$. Using this and substitute in from (162) we can rewrite (161) as

$$1 - \theta_t^{IH} + \theta_t^{IH} \frac{r_t^{b,h}}{r_t^F} - \phi^F\left(\frac{r_t^F}{r_{t-1}^F} - 1\right)\frac{r_t^F}{r_{t-1}^F} + E_t\left[\Delta\phi^F\left(\frac{r_{t+1}^F}{r_t^F} - 1\right)\left(\frac{r_{t+1}^F}{r_t^F}\right)^2 \frac{P_{t+1}}{P_t}\frac{B_{h,t+1}}{B_{h,t}}\right] = 0. \tag{163}$$

Similarly, the FOC w.r.t. $r_t^e(i)$ becomes:

$$1 - \theta_t^e + \theta_t^e \frac{r_t^{b,e}}{r_t^e} - \phi^e\left(\frac{r_t^e}{r_{t-1}^e} - 1\right)\frac{r_t^e}{r_{t-1}^e} + E_t\left[\Delta\phi^e\left(\frac{r_{t+1}^e}{r_t^e} - 1\right)\left(\frac{r_{t+1}^e}{r_t^e}\right)^2 \frac{P_{t+1}}{P_t}\frac{B_{e,t+1}}{B_{e,t}}\right] = 0. \tag{164}$$

8.1.3 Deposit branch

The deposit branch lends to the wholesale branch at the money market interest rate r_t and pays out interest on household deposits at rate $r_t^d(i)$. It faces costs when changing the deposit rate, governed by the parameter ϕ^D. Total household demand for deposits facing branch i, $D_t(i)$, is found in the same way as the demand for loans (see above).

$$\max_{\{r_t^d(i)\}} E_s \sum_{t=s}^{\infty} \Delta_{s,t}\left[r_t D_t(i) - r_t^d(i) D_t(i) - \frac{\phi^D}{2}\left(\frac{r_t^d(i)}{r_{t-1}^d(i)} - 1\right)^2 r_t^d D_t\right],$$

s.t.

$$D_t(i) = \left(\frac{r_t^d(i)}{r_t^d}\right)^{-\theta_t^D} D_t, \tag{165}$$

where $\theta_t^D < 0$ is the elasticity of substitution between deposit services from all branches, and is a shock and assumed to follow an AR process.

The FOC w.r.t. $R_t^d(i)$ becomes:

$$-(1 - \theta_t^D) - \theta_t^D \frac{r_t}{r_t^d} - \phi^D\left(\frac{r_t^d}{r_{t-1}^d} - 1\right)\frac{r_t^d}{r_{t-1}^d} + E_t\left[\Delta\phi^D\left(\frac{r_{t+1}^d}{r_t^d} - 1\right)\left(\frac{r_{t+1}^d}{r_t^d}\right)^2 \frac{P_{t+1}}{P_t}\frac{D_{t+1}}{D_t}\right] = 0, \tag{166}$$

where we have used $r_t^d(i) = r_t^d$ for all i (analogous to the loan branches).

8.1.4 Balance sheet for the bank sector

The full balance sheet of the banking sector is then given by

$$B_t = B_{F,t}^{TOT} + K_t^B, \tag{167}$$

where B_t is total assets (total lending). On the liability side, $B_{F,t}^{TOT}$ is total external bank funding and K_t^B is bank capital (equity). Total external bank funding is the sum of household deposits and foreign debt:

$$B_{F,t}^{TOT} = D_t + B_t^*. \tag{168}$$

Total lending is the sum of lending to entrepreneurs and households:

$$B_t = B_{e,t} + B_{h,t}. \tag{169}$$

8.2 Making the equations stationary

8.2.1 Wholesale branch

$$\widetilde{J}_t = r_t^F \widetilde{B}_{h,t} + r_t^e \widetilde{B}_{e,t} - r_t^d \widetilde{D}_t - r_t \widetilde{B}_t^* - \chi_o \widetilde{B}_t - \frac{\chi_c}{2}\left[\frac{\widetilde{K}_t^B}{\widetilde{B}_t^{RW}} - \varpi_t\right]^2 \widetilde{K}_t^B, \tag{170}$$

$$\widetilde{K}_t^B = (1-\delta^b)\frac{\widetilde{K}_{t-1}^B}{\pi_t \pi_t^z} + \frac{\widetilde{J}_{t-1}}{\pi_t \pi_t^z}, \tag{171}$$

$$\widetilde{B}_t^{RW} = \varsigma^e \widetilde{B}_{e,t} + \varsigma^h \widetilde{B}_{h,t}, \tag{172}$$

$$R_t^{b,e} = R_t + \chi_o - \chi_c \varsigma^e \left[\frac{K_t^B}{B_t^{RW}} - \varpi_t\right]\left(\frac{K_t^B}{B_t^{RW}}\right)^2 \tag{173}$$

$$R_t^{b,h} = R_t + \chi_o - \chi_c \varsigma^h \left[\frac{K_t^B}{B_t^{RW}} - \varpi_t\right]\left(\frac{K_t^B}{B_t^{RW}}\right)^2 \tag{174}$$

8.2.2 Loan branch

FOC

w.r.t. r_t^F:

$$1 - \theta_t^{IH} + \theta_t^{IH}\frac{r_t^{b,h}}{r_t^F} - \phi^F\left(\frac{r_t^F}{r_{t-1}^F} - 1\right)\frac{r_t^F}{r_{t-1}^F} + E_t\left[\Delta\phi^F\left(\frac{r_{t+1}^F}{r_t^F} - 1\right)\left(\frac{r_{t+1}^F}{r_t^F}\right)^2 \pi_{t+1}^z \pi_{t+1}\frac{\widetilde{B}_{h,t+1}}{\widetilde{B}_{h,t}}\right] = 0. \tag{175}$$

w.r.t. r_t^e:

$$1 - \theta_t^e + \theta_t^e\frac{r_t^{b,e}}{r_t^e} - \phi^e\left(\frac{r_t^e}{r_{t-1}^e} - 1\right)\frac{r_t^e}{r_{t-1}^e} + E_t\left[\Lambda\phi^e\left(\frac{r_{t+1}^e}{r_t^e} - 1\right)\left(\frac{r_{t+1}^e}{r_t^e}\right)^2 \pi_{t+1}^z \pi_{t+1}\frac{\widetilde{B}_{e,t+1}}{\widetilde{B}_{e,t}}\right] = 0. \tag{176}$$

8.2.3 Deposit branch

FOC w.r.t. r_t^d:

$$-(1-\theta_t^D) - \theta_t^D\frac{r_t}{r_t^d} - \phi^D\left(\frac{r_t^d}{r_{t-1}^d} - 1\right)\frac{r_t^d}{r_{t-1}^d} + E_t\left[\Lambda\phi^D\left(\frac{r_{t+1}^d}{r_t^d} - 1\right)\left(\frac{r_{t+1}^d}{r_t^d}\right)^2 \pi_{t+1}^z \pi_{t+1}\frac{\widetilde{D}_{t+1}}{\widetilde{D}_t}\right] = 0. \tag{177}$$

8.2.4 Balance sheet

$$\widetilde{B}_{F,t}^{TOT} + \widetilde{K}_t^B = \widetilde{B}_t, \tag{178}$$

$$\widetilde{B}_t = \widetilde{B}_{e,t} + \widetilde{B}_{h,t}, \tag{179}$$

$$\widetilde{B}_{F,t}^{TOT} = \widetilde{D}_t + \widetilde{B}_t^*. \tag{180}$$

8.2.5 Equations included in the model

The following equations are included in the model file: (170), (171), (172), (173), (174), (175), (177), (178), (179) and (180).

8.3 Steady-state equations

8.3.1 Wholesale branch

$$\widetilde{J}_{ss} = r^F_{ss}\widetilde{B}_{h,ss} + r^e_t\widetilde{B}_{e,ss} - r^d_t\widetilde{D}_{ss} - r_t\widetilde{B}^*_{ss} - \chi_o\widetilde{B}_{ss}, \tag{181}$$

$$\widetilde{K}^B_{ss} = \left[1 - \frac{(1-\delta^b)}{\pi_{ss}\pi^z_{ss}}\right]^{-1}\widetilde{J}_{ss}, \tag{182}$$

$$\widetilde{B}^{RW}_{ss} = \varsigma^e\widetilde{B}_{e,ss} + \varsigma^h\widetilde{B}_{h,ss}, \tag{183}$$

$$R^{b,e}_{ss} = R_{ss} + \chi_o \tag{184}$$

$$R^{b,h}_{ss} = R_{ss} + \chi_o \tag{185}$$

$$\frac{K^B_{ss}}{B^{RW}_{ss}} = \varpi_{ss} \tag{186}$$

8.3.2 Loan branch

From the FOC w.r.t. r^F_t, we see that the lending rate to households will be set as a mark up on the wholesale lending rate in steady-state, i.e.

$$r^F_{ss} = \frac{\theta^{IH}_{ss}}{\theta^{IH}_{ss}-1}r^{b,h}_{ss}. \tag{187}$$

From the FOC w.r.t. r^e_t, we see that the lending rate to entrepreneurs will be set as a mark up on the wholesale lending rate in steady-state, i.e.

$$r^e_{ss} = \frac{\theta^e_{ss}}{\theta^e_{ss}-1}r^{b,e}_{ss}. \tag{188}$$

8.3.3 Deposit branch

From the FOC w.r.t. r^d_t, we see that the deposit rate will be set as a mark down on the money market rate in steady-state (as $\theta^D_{ss} < 0$), i.e.

$$r^d_{ss} = \frac{\theta^D_{ss}}{(\theta^D_{ss}-1)}r_{ss}. \tag{189}$$

8.3.4 Balance sheet

$$\widetilde{B}_{ss} = \widetilde{B}^{TOT}_{F,ss} + \widetilde{K}^B_{ss}, \tag{190}$$

$$\widetilde{B}_{ss} = \widetilde{B}_{e,ss} + \widetilde{B}_{h,ss}, \tag{191}$$

$$\widetilde{B}^{TOT}_{F,ss} = \widetilde{D}_{ss} + \widetilde{B}^*_{ss}. \tag{192}$$

9 Oil sector

9.1 Maximization

9.1.1 Supply firms

A continuum of oil supply firms, indexed r, combine final goods $Q_{O,t}(r)$, labor from households $L_{O,t}(r)$ and utilized capital rented from entrepreneurs $\overline{K}_{O,t}(r)$ to produce a good $Y_{R,t}(r)$ that is used for oil investments by an extraction firm and exports to a foreign oil extraction firm. P_t^{QO} is the price of $Q_{O,t}(r)$ – as it is a final good we have that $P_t^{QO} = P_t$. The wage earned by households working in the oil supply sector is $W_{O,t}$ (equal to W_t because of perfect labor mobility), while the rental price of utilized capital is $R_{KO,t}$ (equal to $R_{K,t}$ due to perfect mobility of capital).

The production function is as follows:

$$Y_{R,t}(r) = Z_{R,t} Q_{O,t}^{\alpha_q}(r)(Z_t L_{O,t}(r))^{\alpha_l} \overline{K}_{O,t}^{1-\alpha_q-\alpha_l}(r), \tag{193}$$

where α_q is the final goods share, α_l is the labor share and $1-\alpha_q-\alpha_l$ is the capital share in production. $Z_{R,t}$ is an exogenous shock, assumed to follow an AR process. Minimizing costs $(P_t^{QO} Q_{O,t}(r) + W_{O,t}L_{O,t}(r) + R_{KO,t}\overline{K}_{O,t}(r))$ subject to (193) gives rise to the following conditional demand functions and marginal cost function (as in section 3):

$$Q_{O,t}(r) = \alpha_q \left(\frac{P_t^{QO}}{MC_{R,t}}\right)^{-1} Y_{R,t}(r), \tag{194}$$

$$L_{O,t}(r) = \alpha_l \left(\frac{W_{O,t}}{MC_{R,t}}\right)^{-1} Y_{R,t}(r), \tag{195}$$

$$\overline{K}_{O,t}(r) = (1-\alpha_q-\alpha_l) \left(\frac{R_{KO,t}}{MC_{R,t}}\right)^{-1} Y_{R,t}(r), \tag{196}$$

$$MC_{R,t} = \frac{1}{Z_{R,t}} \left(\frac{P_t^{QO}}{\alpha_q}\right)^{\alpha_q} \left(\frac{W_{O,t}}{\alpha_l}\right)^{\alpha_l} \left(\frac{R_{KO,t}}{1-\alpha_q-\alpha_l}\right)^{1-\alpha_q-\alpha_l}.$$

In symmetric equilibrium all firms make the same decisions, so $Q_{O,t}(r) = Q_{O,t}$, $L_{O,t}(r) = L_{O,t}$,and $\overline{K}_{O,t}(r) = \overline{K}_{O,t}$.

Oil supply firms sell their goods under monopolistic competition. Each firm r charges different prices at home and abroad, $P_t^R(r)$ in the home market and $P_t^{R^*}(r)$ abroad, where the latter is denoted in foreign currency. Dividends (which are paid out to households) becomes:

$$\Psi_t(r) = P_t^R(r) I_{OF,t}(r) + P_t^{R*}(r) S_t M_{O*,t}(r) - MC_{R,t} Y_{R,t}(r), \tag{197}$$

where $I_{OF,t}(r)$ are goods delivered to the domestic extraction firm, $M_{O*,t}(r)$ are supply goods for exports and S_t is the nominal exchange rate. $Y_{R,t} = I_{OF,t}(r) + M_{O*,t}(r)$.

In the same way as elsewhere in this document, it can be shown that supply firm r faces the following demand functions, $I_{OF,t}(r) = \left(\frac{P_t^R(r)}{P_t^R}\right)^{-\theta^R} I_{OF,t}$ and $M_{O*,t}(r) = \left(\frac{P_t^{R*}(r)}{P_t^{R*}}\right)^{-\theta^{R^*}} M_{O*,t}$, from the domestic and foreign extraction sectors, where θ^R and θ^{R^*} are the elasticities of substitution between goods in the two markets respectively. Additionally, the costs of adjusting prices in the domestic and the foreign markets are given by

$$\gamma_{PR,t}(r) \equiv \frac{\phi^{PR}}{2}\left[\frac{P_t^R(r)/P_{t-1}^R(r)}{P_{t-1}^R/P_{t-2}^R} - 1\right]^2, \tag{198}$$

$$\gamma_{PR^*,t}(r) \equiv \frac{\phi^{PR^*}}{2}\left[\frac{P_t^{R^*}(r)/P_{t-1}^{R^*}(r)}{P_{t-1}^{R^*}/P_{t-2}^{R^*}} - 1\right]^2, \tag{199}$$

respectively, where ϕ^{PR} and ϕ^{PR^*} govern the cost of adjusting prices.

The maximization problem becomes

$$\max_{\{P_t^R(r), P_t^{R*}(r)\}} \Pi_s^{OS} = \sum_{t=s}^{\infty} \Delta_{s,t} \left\{ \begin{array}{c} P_t^R(r)\left(\frac{P_t^R(r)}{P_t^R}\right)^{-\theta^R} I_{OF,t} + P_t^{R*}(r) S_t \left(\frac{P_t^{R*}(r)}{P_t^{R*}}\right)^{-\theta^{R^*}} M_{O*,t} \\ -MC_{R,t}\left[\left(\frac{P_t^R(r)}{P_t^R}\right)^{-\theta^R} I_{OF,t} + \left(\frac{P_t^{R*}(r)}{P_t^{R*}}\right)^{-\theta^{R^*}} M_{O*,t}\right] \\ -\gamma_{PR,t}(r) P_t^R I_{OF,t} - \gamma_{PR^*,t}(r) P_t^{R^*} S_t M_{O*,t} \end{array} \right\}.$$

Following the same procedure as in chapter 3, and using that in symmetric eq., $P_t^R = P_t^R(r)$ and $P_t^{R*} = P_t^{R*}(r)$, the FOC w.r.t. P_t^R is given by

$$I_{OF,t} - \theta^R I_{OF,t} + MC_{R,t}\theta^R \frac{I_{OF,t}}{P_t^R} - \phi^{PR}\left[\frac{P_t^R/P_{t-1}^R}{P_{t-1}^R/P_{t-2}^R} - 1\right]\frac{P_t^R/P_{t-1}^R}{P_{t-1}^R/P_{t-2}^R} I_{OF,t}$$
$$+E_t\left\{\Delta\phi^{PR}\left[\frac{P_{t+1}^R/P_t^R}{P_t^R/P_{t-1}^R} - 1\right]\frac{(P_{t+1}^R/P_t^R)^2}{P_t^R/P_{t-1}^R} I_{OF,t+1}\right\} = 0, \tag{200}$$

w.r.t. $P_t^{R^*}$ is given by

$$S_t M_{O*,t} - \theta^{R^*} S_t M_{O*,t} + MC_{R,t}\theta^{R^*}\frac{M_{O*,t}}{P_t^{R^*}} - \phi^{PR^*}\left[\frac{P_t^{R^*}/P_{t-1}^{R^*}}{P_{t-1}^{R^*}/P_{t-2}^{R^*}} - 1\right]\frac{P_t^{R^*}/P_{t-1}^{R^*}}{P_{t-1}^{R^*}/P_{t-2}^{R^*}} S_t M_{O*,t}$$
$$+E_t\left\{\phi^{PR^*}\left[\frac{P_{t+1}^{R^*}/P_t^{R^*}}{P_t^{R^*}/P_{t-1}^{R^*}} - 1\right]\frac{\left(P_{t+1}^{R^*}/P_t^{R^*}\right)^2}{P_t^{R^*}/P_{t-1}^{R^*}} S_{t+1} M_{O,t+1}^*\right\} = 0. \tag{201}$$

9.1.2 Extraction firm, domestic

The domestic oil extraction firm is also a producer of rigs. It buys oil supply goods from the oil supply sector, which is abbreviated $I_{O,t}$. This good is then used to produce rigs $F_{O,t}$. Rigs depreciate with a constant rate δ_O, and are subject to a degree of utilization $U_{F,t}$. There will be a trade-off between raising the utilization rate (by buying more supply goods) to increase oil production, and to decrease it to reduce the wear and tear of the rigs. This unit cost of increasing the utilization rate is measured in oil supply goods, and represented by the function $a(U_{F,t})$. Total demand for supply goods from the domestic extraction firms is then given by

$$I_{OF,t} = I_{O,t} + a(U_{F,t})F_{O,t-1}. \tag{202}$$

The oil production is given by $Y_{O,t}$, which is exported at a price P_t^{O*} in foreign currency (see chapter 10 equation (273) for more on the oil price). This means that the oil price in home currency is given by $S_t P_t^{O*}$ where S_t is the nominal exchange rate. The problem at period s will then be

$$\max_{\{Y_{O,t},F_{O,t},I_{O,t},U_{F,t}\}} \sum_{t=s}^{\infty} \Delta_{s,t}\left[S_t P_t^{O*} Y_{O,t} - P_t^R I_{OF,t}\right], \tag{203}$$

where $\Delta_{s,t}$ is the stochastic discount factor between period s and t.

Rigs accumulate according to:

$$F_{O,t} = (1-\delta_O)F_{O,t-1} + Z_{IOIL,t}\left[1 - \Psi_O\left(\frac{I_{O,t}}{I_{O,t-1}}\right)\right] I_{O,t}, \tag{204}$$

where $\Psi_O\left(\frac{I_{O,t}}{I_{O,t-1}}\right)$ represents the costs of changing investment levels (defined in (213)) and $Z_{IOIL,t}$ is an oil investment productivity shock, assumed to follow an AR process. "Effective rigs usage" is defined as

$$\overline{F}_{O,t} = F_{O,t-1}U_{F,t}, \tag{205}$$

and the production function is Cobb-Douglas, i.e.

$$Y_{O,t} = Z_{O,t}(\overline{F}_{O,t})^{\alpha_o}(O_t)^{1-\alpha_o}, \tag{206}$$

where $Z_{O,t}$ is oil extraction productivity shock that follows an AR process, α_o is the rigs share and O_t is oil in the ground (which is a shock that follows an AR process).

The domestic extraction firm maximizes profits subject to (204), (205), (202) and (206), taking profits as given. Inserting from (205), (206), (202), the Lagrangian becomes:

$$\mathcal{L}(Y_{O,t},F_{O,t},I_{O,t},U_{F,t},\Omega_{O,t}) = \sum_{t=s}^{\infty}\Delta_{s,t}\left[\begin{array}{c} S_t P_t^{O*} Z_{O,t}(F_{O,t-1}U_{F,t})^{\alpha_o} O_t^{1-\alpha_o} - P_t^R(I_{O,t} + a(U_{F,t})F_{O,t-1}) \\ -\Omega_{O,t}\left[F_{O,t} - (1-\delta_O)F_{O,t-1} - Z_{IOIL,t}\left[1-\Psi_O\left(\frac{I_{O,t}}{I_{O,t-1}}\right)\right] I_{O,t}\right] \end{array}\right], \tag{207}$$

where $\Omega_{O,t}$ is the Lagrange multiplier for the rigs accumulation constraint, (204). FOC w.r.t. $F_{O,t}$:

$$\Omega_t^O = E\left[\Delta\left(\begin{array}{c} \alpha_o S_{t+1} P_{t+1}^{O*} Z_{O,t+1}(F_{O,t}U_{F,t+1})^{\alpha_o} O_{t+1}^{1-\alpha_o} F_{O,t}^{-1} \\ -P_{t+1}^R a(U_{F,t+1}) + (1-\delta_O)\Omega_{O,t+1} \end{array}\right)\right]$$

$$\Longleftrightarrow$$

$$\Omega_{O,t} = E\left[\Delta\left(\alpha_o S_{t+1} P_{t+1}^{O*} Y_{O,t+1} F_{O,t}^{-1} - P_{t+1}^R a(U_{F,t+1}) + (1-\delta_O)\Omega_{O,t+1}\right)\right]. \tag{208}$$

w.r.t. $I_{O,t}$:

$$P_t^R = \begin{array}{l} \Omega_{O,t} Z_{IOIL,t}\left[1 - \Psi_O'\left(\frac{I_{O,t}}{I_{O,t-1}}\right)\frac{I_{O,t}}{I_{O,t-1}} - \Psi_O\left(\frac{I_{O,t}}{I_{O,t-1}}\right)\right] \\ +E\left[\Delta\Omega_{O,t+1} Z_{IOIL,t+1}\Psi_O'\left(\frac{I_{O,t+1}}{I_{O,t}}\right)\left(\frac{I_{O,t+1}}{I_{O,t}}\right)^2\right] \end{array}. \tag{209}$$

w.r.t. $U_{F,t}$:

$$\alpha_o S_t P_t^{O*} \frac{Y_{O,t}}{U_{F,t}} = P_t^R a'(U_{F,t}) F_{O,t-1}. \tag{210}$$

The cost of increasing the rigs utilization rate is assumed equal to:

$$a(U_{F,t}) = a'(U_{F,ss})(U_{F,t}-1) + \frac{a'(U_{F,ss})\phi^{uf}}{2}(U_{F,t}-1)^2, \tag{211}$$

which means that

$$a'(U_{F,t}) = a'(U_{F,ss}) + a'(U_{F,ss})\phi^{uf}(U_{F,t}-1). \tag{212}$$

Furthermore, the costs of changing investment levels are

$$\Psi_O\left(\frac{I_{O,t}}{I_{O,t-1}}\right) = \frac{\phi^{RI}}{2}\left[\frac{I_{O,t}}{I_{O,t-1}} - \pi_t^z\right]^2. \tag{213}$$

It follows that

$$\Psi_O'\left(\frac{I_{O,t}}{I_{O,t-1}}\right) = \phi^{RI}\left[\frac{I_{O,t}}{I_{O,t-1}} - \pi_t^z\right]. \tag{214}$$

The cost of changing the utilization rate is governed by the parameter ϕ^{uf}, while the cost of adjusting the oil investment level is governed by the parameter ϕ^{RI}.

9.1.3 Extraction firm, abroad

The foreign extraction firm produces oil, $Y_{O*,t}$, invests, $I_{O*,t}$, and imports oil supply goods from the domestic country's oil supply sector, $M_{O*,t}$. It has the following production function:

$$Y_{O*,t} = M_{O*,t}^{\alpha_{o*}} I_{O*,t}^{\alpha_{io*}} (O_{*,t})^{1-\alpha_{io*}-\alpha_{o*}}, \tag{215}$$

where $O_{*,t}$ is oil in ground (and may be set to a shock process following an AR process, otherwise constant), and α_{o*} is the share of oil supply goods from home used in production.

Maximizing profits:

$$\max_{\{Y_{O*,t}, M_{O*,t}, I_{O*,t}\}} \sum_{t=s}^{\infty} \Delta_{s,t}^* \left[P_t^{O*} Y_{O*,t} - P_t^{R^*} M_{O*,t} - P_t^{IO^*} I_{O*,t}\right], \tag{216}$$

s.t. (215), where $\Delta_{s,t}^*$ is the foreign stochastic discount factor between period s and t.

FOC w.r.t. $M_{O*,t}$ (the export demand for oil supply goods from abroad):

$$M_{O*,t} = \alpha_{o*}\left(\frac{P_t^{R^*}}{P_t^{O*}}\right)^{-1} Y_{O*,t}, \tag{217}$$

where $Y_{O*,t}$ is an exogenous shock and is assumed to follow an AR process.

9.1.4 Oil fund

In Norway, the Government Pension Fund Act stipulates that the government's cash flow from the petroleum industry shall be transferred to the "Government Pension Fund Global" (GPFG). A fiscal rule specifies that the transfers from the GPFG to the central government's fiscal budget shall follow the expected real return on the GPFG over time. In NEMO, this relationship is simplified, as the GPFG and fiscal policy are independently treated. The full sales revenue is transferred to the GPFG in every period. Hence, the GPFG, $B_{F,t}$, accumulates according to:

$$B_{F,t} = (1 - \rho_{G_F}) \left[R_t^* \frac{P_{t-1}}{P_t} \frac{S_t}{S_{t-1}} B_{F,t-1} \right] + S_t \frac{P_t^{O*}}{P_t} Y_{O,t}, \tag{218}$$

where the amount (in real terms) transferred from the sovereign wealth fund is given by:

$$G_{F,t} = \rho_{G_F} \left[R_t^* \frac{P_{t-1}}{P_t} \frac{S_t}{S_{t-1}} B_{F,t-1} \right]. \tag{219}$$

The transfer, $G_{F,t}$, ensures that the sovereign wealth fund, $B_{F,t}$, is stationary. In the model, we assume that the transfer from the fund goes to the banking sector (and not to the government sector).

9.2 Making the equations stationary

9.2.1 Supply firms

Production function:

$$\frac{Y_{R,t}}{Z_t} = Z_{R,t} \left(\frac{Q_{O,t}}{Z_t} \right)^{\alpha_q} \left(\frac{Z_t L_{O,t}}{Z_t} \right)^{\alpha_l} \left(\frac{\overline{K}_{O,t}}{Z_t} \right)^{1-\alpha_q-\alpha_l}$$

$$\Longleftrightarrow$$

$$\widetilde{Y}_{R,t} = Z_{R,t} (\widetilde{Q}_{O,t})^{\alpha_q} (L_{O,t})^{\alpha_l} (\widetilde{\overline{K}}_{O,t})^{1-\alpha_q-\alpha_l}, \tag{220}$$

$$\widetilde{Q}_{O,t} = \alpha_q \left(\frac{\widetilde{P}_t^{QO}}{\widetilde{MC}_{R,t}} \right)^{-1} \widetilde{Y}_{R,t}. \tag{221}$$

We assume that $\widetilde{Q}_{O,t}$ is a final good, so $\widetilde{P}_t^{QO} = 1$:

$$L_{O,t} = \alpha_l \left(\frac{\widetilde{W}_{O,t}}{\widetilde{MC}_{R,t}} \right)^{-1} \widetilde{Y}_{R,t}, \tag{222}$$

$$\widetilde{\overline{K}}_{O,t} = (1 - \alpha_q - \alpha_l) \left(\frac{\widetilde{R}_{KO,t}}{\widetilde{MC}_{R,t}} \right)^{-1} \widetilde{Y}_{R,t}. \tag{223}$$

Optimal prices:

$$\begin{aligned} &\widetilde{I}_{OF,t} - \theta^R \widetilde{I}_{OF,t} + \widetilde{MC}_{R,t} \theta^R \frac{\widetilde{I}_{OF,t}}{\widetilde{P}_t^R} - \phi^{PR} \left[\frac{\pi_t^R}{\pi_{t-1}^R} - 1 \right] \frac{\pi_t^R}{\pi_{t-1}^R} \widetilde{I}_{OF,t} \\ &+ E_t \left\{ \Lambda \phi^{PR} \left[\frac{\pi_{t+1}^R}{\pi_t^R} - 1 \right] \frac{(\pi_{t+1}^R)^2}{\pi_t^R} \widetilde{I}_{OF,t+1} \pi_{t+1}^z \right\} = 0, \end{aligned} \tag{224}$$

$$\begin{aligned} &\widetilde{S}_t \widetilde{M}_{O*,t} - \theta^{R^*} \widetilde{S}_t \widetilde{M}_{O*,t} + \widetilde{MC}_{R,t} \theta^{R^*} \frac{\widetilde{M}_{O*,t}}{\widetilde{P}_t^{R^*}} - \phi^{PR^*} \left[\frac{\pi_t^{R*}}{\pi_{t-1}^{R*}} - 1 \right] \frac{\pi_t^{R*}}{\pi_{t-1}^{R*}} \widetilde{S}_t \widetilde{M}_{O*,t} \\ &+ E_t \left\{ \Lambda \phi^{PR^*} \left[\frac{\pi_{t+1}^{R*}}{\pi_t^{R*}} - 1 \right] \frac{(\pi_{t+1}^{R*})^2}{\pi_t^{R*}} \widetilde{S}_{t+1} \widetilde{M}_{O*,t+1} \pi_{t+1}^z \right\} = 0. \end{aligned} \tag{225}$$

9.2.2 Extraction firms, domestic

As the oil sector has diminishing return to scale, the stochastic trend level in this sector is $Z_t^{\alpha_o}$. We assume in this model that the value of the oil production is stationary, which means that the stochastic trend level in the oil price is $\frac{Z_{t+1}^{\alpha_o}}{Z_{t+1}}$. Therefore we define the stationary real oil price as $\widetilde{P}_t^{O*} = \frac{P_t^{O*} Z_t^{\alpha_o}}{P_t^* Z_t}$. By using this we get the following stationary solution:

$$\frac{Y_{O,t}}{Z_t^{\alpha_o}} = Z_{O,t}\left(\frac{\overline{F}_{O,t}}{Z_t}\right)^{\alpha_o}(O_t)^{1-\alpha_o}$$

$$\Longrightarrow$$

$$\widetilde{Y}_{O,t} = Z_{O,t}\left(\widetilde{\overline{F}}_{O,t}\right)^{\alpha_o}(O_t)^{1-\alpha_o}, \tag{226}$$

$$\widetilde{\overline{F}}_{O,t} = \frac{\widetilde{F}_{O,t-1}}{\pi_t^z}U_{F,t}. \tag{227}$$

By definition

$$\widetilde{I}_{OF,t} = \widetilde{I}_{O,t} + a(U_{F,t})\widetilde{F}_{O,t-1}\frac{1}{\pi_t^z}. \tag{228}$$

FOC extraction firms:

$$\frac{\Omega_{O,t}}{P_t} = E\left[\Delta\frac{P_{t+1}}{P_t}\left(\alpha_o\frac{S_{t+1}P_{t+1}^*}{P_{t+1}}\frac{P_{t+1}^{O*}Z_{t+1}^{\alpha_o}}{P_{t+1}^*Z_{t+1}}\frac{Y_{O,t+1}Z_t}{F_{O,t}Z_{t+1}^{\alpha_o}}\frac{Z_{t+1}}{Z_t} - \frac{P_{t+1}^R}{P_{t+1}}a(U_{F,t+1}) + (1-\delta_O)\frac{\Omega_{O,t+1}}{P_{t+1}}\right)\right]$$

$$\Leftrightarrow$$

$$\widetilde{\Omega}_{O,t} = E\left[\Lambda\pi_{t+1}\left(\alpha_o\widetilde{S}_{t+1}\widetilde{P}_{t+1}^{O*}\frac{\widetilde{Y}_{O,t+1}}{\widetilde{F}_{O,t}}\pi_{t+1}^z - \widetilde{P}_{t+1}^R a(U_{F,t+1}) + (1-\delta_O)\widetilde{\Omega}_{O,t+1}\right)\right], \tag{229}$$

$$\frac{P_t^R}{P_t} = \begin{array}{l} \frac{\Omega_{O,t}}{P_t}Z_{IOIL,t}\left[1-\Psi_O^{'}\left(\frac{\frac{I_{O,t}}{Z_t}}{\frac{I_{O,t-1}}{Z_{t-1}}}\frac{Z_t}{Z_{t-1}}\right)\frac{\frac{I_{O,t}}{Z_t}}{\frac{I_{O,t-1}}{Z_{t-1}}}\frac{Z_t}{Z_{t-1}} - \Psi_O\left(\frac{\frac{I_{O,t}}{Z_t}}{\frac{I_{O,t-1}}{Z_{t-1}}}\frac{Z_t}{Z_{t-1}}\right)\right] \\ +E\left[\Delta\frac{P_{t+1}}{P_t}\frac{\Omega_{O,t+1}}{P_{t+1}}Z_{IOIL,t+1}\Psi_O^{'}\left(\frac{\frac{I_{O,t+1}}{Z_{t+1}}}{\frac{I_{O,t}}{Z_t}}\frac{Z_{t+1}}{Z_t}\right)\left(\frac{\frac{I_{O,t+1}}{Z_{t+1}}}{\frac{I_{O,t}}{Z_t}}\frac{Z_{t+1}}{Z_t}\right)^2\right]\end{array}$$

$$\Leftrightarrow$$

$$\widetilde{P}_t^R = \begin{array}{l}\widetilde{\Omega}_{O,t}Z_{IOIL,t}\left[1-\Psi_O^{'}\left(\frac{\widetilde{I}_{O,t}}{\widetilde{I}_{O,t-1}}\pi_t^z\right)\frac{\widetilde{I}_{O,t}}{\widetilde{I}_{O,t-1}}\pi_t^z - \Psi_O\left(\frac{\widetilde{I}_{O,t}}{\widetilde{I}_{O,t-1}}\pi_t^z\right)\right] \\ +E\left[\Lambda\pi_{t+1}\widetilde{\Omega}_{O,t+1}Z_{IOIL,t+1}\Psi_O^{'}\left(\frac{\widetilde{I}_{O,t+1}}{\widetilde{I}_{O,t}}\pi_{t+1}^z\right)\left(\frac{\widetilde{I}_{O,t+1}}{\widetilde{I}_{O,t}}\pi_{t+1}^z\right)^2\right]\end{array}, \tag{230}$$

$$\alpha_o\frac{S_tP_t^*}{P_t}\frac{P_t^{O*}Z_t^{\alpha_o}}{P_t^*Z_t}\frac{Y_{O,t}}{Z_t^{\alpha_o}U_{F,t}} = \frac{P_t^R}{P_t}a^{'}(U_{F,t})\frac{F_{O,t-1}}{Z_{t-1}}\frac{Z_{t-1}}{Z_t}$$

$$\alpha_o\widetilde{S}_t\widetilde{P}_t^{O*}\frac{\widetilde{Y}_{O,t}}{U_{F,t}} = \widetilde{P}_t^R a^{'}(U_{F,t})\frac{\widetilde{F}_{O,t-1}}{\pi_t^z}, \tag{231}$$

$$a(U_{F,t}) = a^{'}(U_{F,ss})(U_{F,t}-1) + \frac{a^{'}(U_{F,ss})\phi^{uf}}{2}(U_{F,t}-1)^2, \tag{232}$$

$$a^{'}(U_{F,t}) = a^{'}(U_{F,ss}) + a^{'}(U_{F,ss})\phi^{uf}(U_{F,t}-1), \tag{233}$$

$$\Psi_O\left(\frac{\widetilde{I}_{O,t}}{\widetilde{I}_{O,t-1}}\pi_t^z\right) = \frac{\phi^{RI}}{2}\left[\frac{\widetilde{I}_{O,t}}{\widetilde{I}_{O,t-1}}\pi_t^z - \pi_{ss}^z\right]^2, \tag{234}$$

$$\Psi_O^{'}\left(\frac{\widetilde{I}_{O,t}}{\widetilde{I}_{O,t-1}}\pi_t^z\right) = \phi^{RI}\left[\frac{\widetilde{I}_{O,t}}{\widetilde{I}_{O,t-1}}\pi_t^z - \pi_{ss}^z\right]. \tag{235}$$

9.2.3 Producer of rigs

$$\frac{F_{O,t}}{Z_t} = (1-\delta^O)\frac{F_{O,t-1}}{Z_{t-1}}\frac{Z_{t-1}}{Z_t} + Z_{IOIL,t}\left[1-\Psi_O\left(\frac{I_{O,t}Z_{t-1}}{I_{O,t-1}Z_t}\frac{Z_t}{Z_{t-1}}\right)\right]\frac{I_{O,t}}{Z_t}$$

$$\widetilde{F}_{O,t} = (1-\delta^O)\frac{\widetilde{F}_{O,t-1}}{\pi_t^{\tilde{z}}} + Z_{IOIL,t}\left[1-\Psi_O\left(\frac{\widetilde{I}_{O,t}}{\widetilde{I}_{O,t-1}}\pi_t^{\tilde{z}}\right)\right]\widetilde{I}_{O,t}. \tag{236}$$

9.2.4 Extraction firm abroad

Lets define $Z_{*,t}$ as the stochastic trend level abroad. To get a stationary solution we must assume that $Z_{*,t} = Z_t$. This leads to

$$\frac{M_{O*,t}}{Z_t} = a_{o*}\left(\frac{\frac{P_t^{R*}}{P_t^*}}{\frac{P_t^{O*}}{P_t^*}\frac{Z_{*,t}^{\alpha_{o*}+\alpha_{io*}}}{Z_t}}\right)^{-1}\frac{Y_{O*,t}}{Z_{*,t}^{\alpha_{o*}+\alpha_{io*}}}.$$

We must assume a Cobb-Douglas production function for the foreign extraction firm and that $\alpha_{o*}+\alpha_{io*} = \alpha_o$, (where α_{o*} and α_{io*} are oil investment shares in the production function of the foreign extraction firm, see (215), and α_o is the rigs share in the domestic extraction firm, see (226)), or else we will not get a well-defined stationary model. Using this we get that

$$\frac{M_{O*,t}}{Z_t} = a_{o*}\left(\frac{\frac{P_t^{R*}}{P_t^*}}{\frac{P_t^{O*}}{P_t^*}\frac{Z_t^{\alpha_o}}{Z_t}}\right)^{-1}\frac{Y_{O*,t}}{Z_t^{\alpha_o}}$$

$$\widetilde{M}_{O*,t} = a_{o*}\left(\frac{\widetilde{P}_t^{R*}}{\widetilde{P}_t^{O*}}\right)^{-1}\widetilde{Y}_{O*,t}. \tag{237}$$

9.2.5 Oil fund

$$\widetilde{B}_{F,t} = (1-\rho_{G_F})\left[\frac{R_t^*}{\pi_t^*\pi_t^{\tilde{z}}}\frac{\widetilde{S}_t}{\widetilde{S}_{t-1}}\widetilde{B}_{F,t-1}\right] + \widetilde{S}_t\widetilde{P}_t^{O*}\widetilde{Y}_{O,t} \tag{238}$$

$$\widetilde{G}_{F,t} = \rho_{G_F}\left[\frac{R_t^*}{\pi_t^*\pi_t^{\tilde{z}}}\frac{\widetilde{S}_t}{\widetilde{S}_{t-1}}\widetilde{B}_{F,t-1}\right]. \tag{239}$$

9.2.6 Equations included in the model

The following equations are included in the model file: (220), (221), (222), (223), (224), (225), (226), (227), (228), (229), (230), (231), (232), (233), (234), (235), (236), (237), (238) and (239).

9.3 Steady-state equations

9.3.1 Supply firms

Production function:

$$\widetilde{Y}_{R,ss} = (\widetilde{Q}_{O,ss})^{\alpha_q}(L_{O,ss})^{\alpha_l}(\widetilde{\overline{K}}_{O,ss})^{1-\alpha_q-\alpha_l}. \tag{240}$$

Marginal cost function,

$$\widetilde{MC}_{R,ss} = \left(\frac{\widetilde{P}_{ss}^{QO}}{\alpha_q}\right)^{\alpha_q}\left(\frac{\widetilde{W}_{O,ss}}{\alpha_l}\right)^{\alpha_l}\left(\frac{\widetilde{R}_{KO,ss}}{1-\alpha_q-\alpha_l}\right)^{1-\alpha_q-\alpha_l}. \tag{241}$$

Factor demand:

$$\widetilde{Q}_{O,ss} = \alpha_q \left(\frac{\widetilde{P}^{QO}_{ss}}{\widetilde{MC}_{R,ss}} \right)^{-1} \widetilde{Y}_{R,ss}, \tag{242}$$

$$L_{O,ss} = \alpha_l \left(\frac{\widetilde{W}_{O,ss}}{\widetilde{MC}_{R,ss}} \right)^{-1} \widetilde{Y}_{R,ss}, \tag{243}$$

$$\widetilde{\widetilde{K}}_{O,ss} = (1 - \alpha_q - \alpha_l) \left(\frac{\widetilde{R}_{KO,ss}}{\widetilde{MC}_{R,ss}} \right)^{-1} \widetilde{Y}_{R,ss}. \tag{244}$$

Optimal prices:

$$\widetilde{P}^{R}_{ss} = \widetilde{MC}_{R,ss} \frac{\theta^R}{(\theta^R - 1)}, \tag{245}$$

$$\widetilde{P}^{M^*}_{ss} = \frac{\widetilde{MC}_{R,ss}}{\widetilde{S}_{ss}} \frac{\theta^{R^*}}{\left(\theta^{R^*} - 1\right)}. \tag{246}$$

9.3.2 Extraction firms, domestic

Production function:

$$\widetilde{Y}_{O,ss} = Z_{O,ss} \left(\widetilde{\widetilde{F}}_{O,ss} \right)^{\alpha_o} (O_{ss})^{1-\alpha_o}, \tag{247}$$

$$Z_{O,ss} = \text{constant} \;\text{ and } O_{ss} = \text{constant}, \tag{248}$$

$$\widetilde{\widetilde{F}}_{O,ss} = \frac{\widetilde{F}_{O,ss}}{\pi^z_{ss}}. \tag{249}$$

FOC extraction firms:

$$\widetilde{\Omega}_{O,ss} = \pi_{ss} \Lambda_{ss} \left(\alpha_o \widetilde{S}_{ss} \widetilde{P}^{O*}_{ss} \frac{\widetilde{Y}_{O,ss}}{\widetilde{F}_{O,ss}} \pi^z_{ss} - \widetilde{P}^{R}_{,ss} a(U_{F,ss}) + (1 - \delta_O) \widetilde{\Omega}_{O,ss} \right), \tag{250}$$

$$U_{F,ss} = 1 \text{ and } \; a(U_{F,ss}) = 0, \tag{251}$$

$$\widetilde{P}^{R}_{t} = \begin{array}{l} \widetilde{\Omega}_{O,ss} Z_{IOIL,ss} \left[1 - \Psi^{'}_{O} \left(\frac{\widetilde{I}_{O,ss}}{\widetilde{I}_{O,ss}} \pi^z_{ss} \right) \frac{\widetilde{I}_{O,ss}}{\widetilde{I}_{O,ss}} \pi^z_{ss} - \Psi_O \left(\frac{\widetilde{I}_{O,ss}}{\widetilde{I}_{O,ss}} \pi^z_{ss} \right) \right] \\ + \pi_{ss} \Lambda_{ss} \widetilde{\Omega}_{O,ss} \Psi^{'}_{O} \left(\frac{\widetilde{I}_{O,ss}}{\widetilde{I}_{O,ss}} \pi^z_{ss} \right) \left(\frac{\widetilde{I}_{O,ss}}{\widetilde{I}_{O,ss}} \pi^z_{ss} \right)^2 \end{array}, \tag{252}$$

$$Z_{IOIL,ss} = 1, \; \Psi_O \left(\pi^z_{ss} \right) = 0 \text{ and } \Psi^{'}_{O} \left(\pi^z_{ss} \right) = 0, \tag{253}$$

$$\alpha_o \widetilde{S}_{ss} \widetilde{P}^{O*}_{ss} \frac{\widetilde{Y}_{O,ss}}{U_{F,ss}} = \widetilde{P}^{R}_{ss} a^{'} (U_{F,ss}) \frac{\widetilde{F}_{O,ss}}{\pi^z_{ss}}, \tag{254}$$

$$\widetilde{I}_{OF,ss} = \widetilde{I}_{O,ss}. \tag{255}$$

9.3.3 Producer of rigs

$$\widetilde{F}_{O,ss} = (1 - \delta^O) \frac{\widetilde{F}_{O,ss}}{\pi^z_{ss}} + Z_{IOIL,ss} \left[1 - \Psi_O \left(\frac{\widetilde{I}_{O,ss}}{\widetilde{I}_{O,ss}} \pi^z_{ss} \right) \right] \widetilde{I}_{O,ss}. \tag{256}$$

9.3.4 Oil price

$$\widetilde{P}^{O*}_{ss} = \text{constant}. \tag{257}$$

9.3.5 Extraction firms abroad

$$\widetilde{M}_{O*,ss} = a_{o*}\left(\frac{\widetilde{P}^{R*}_{ss}}{\widetilde{P}^{O*}_{ss}}\right)^{-1}\widetilde{Y}_{O*,ss}, \tag{258}$$

$$\widetilde{Y}_{O*,ss} = \text{constant}. \tag{259}$$

9.3.6 Oil fund

$$\widetilde{G}_{F,ss} = \rho_{G_F}\left[\frac{R^*_{ss}}{\pi^*_{ss}\pi^z_{ss}}\widetilde{B}_{F,ss}\right]. \tag{260}$$

$$\begin{aligned}
\widetilde{B}_{F,ss} &= (1-\rho_{G_F})\left[\frac{R^*_{ss}}{\pi^*_{ss}\pi^z_{ss}}\widetilde{B}_{F,ss}\right] + \widetilde{S}_{ss}\widetilde{P}^{O*}_{ss}\widetilde{Y}_{O,ss} \\
&\Leftrightarrow \\
&= \frac{R^*_{ss}}{\pi^*_{ss}\pi^z_{ss}}\widetilde{B}_{F,ss} - \rho_{G_F}\frac{R^*_{ss}}{\pi^*_{ss}\pi^z_{ss}}\widetilde{B}_{F,ss} + \widetilde{S}_{ss}\widetilde{P}^{O*}_{ss}\widetilde{Y}_{O,ss} \\
&\Leftrightarrow \\
&= \frac{R^*_{ss}}{\pi^*_{ss}\pi^z_{ss}}\widetilde{B}_{F,ss} - \widetilde{G}_{F,ss} + \widetilde{S}_{ss}\widetilde{P}^{O*}_{ss}\widetilde{Y}_{O,ss} \\
&\Leftrightarrow \\
&= \left[1-\frac{R^*_{ss}}{\pi^*_{ss}\pi^z_{ss}}\right]^{-1}\left[\widetilde{S}_{ss}\widetilde{P}^{O*}_{ss}\widetilde{Y}_{O,ss} - \widetilde{G}_{F,ss}\right]
\end{aligned} \tag{261}$$

10 Foreign sector

As the intermediate sector abroad is symmetric to the intermediate sector home we can in the same way as in section 3 derive the optimal price setting rule for the imported real price $\widetilde{P}^M_t$ (that goes into the final good sector):

$$\begin{aligned}
&\widetilde{M}_t - \theta^F_t\widetilde{M}_t + \widetilde{S}_t\widetilde{MC}^*_t\theta^F_t\frac{\widetilde{M}_t}{\widetilde{P}^M_t} - \phi^{PM}\left[\frac{\pi^M_t}{\pi^M_{t-1}}-1\right]\frac{\pi^M_t}{\pi^M_{t-1}}\widetilde{M}_t \\
&+E_t\left\{\Lambda^*\phi^{PM}\left[\frac{\pi^M_{t+1}}{\pi^M_t}-1\right]\frac{(\pi^M_{t+1})^2}{\pi^M_t}\widetilde{M}_{t+1}\pi^z_{t+1}\frac{\widetilde{S}_t}{\widetilde{S}_{t+1}}\right\} = 0,
\end{aligned} \tag{262}$$

where Λ^* is the foreign stochastic discount factor in stationary terms (and assumed equal to $(R^*_t)^{-1}$ for simplicity), ϕ^{PM} is a parameter that captures the cost of changing the price of exported goods from home and θ^F_t is the substitution elasticity between imported goods from home. In steady-state we get that

$$\widetilde{P}^M_{ss} = \frac{\theta^F_{ss}}{\theta^F_{ss}-1}\widetilde{S}_{ss}\widetilde{MC}^*_{ss}. \tag{263}$$

Marginal costs abroad is a shock and follow an exogenous AR process.

Tuning to exports, the demand from (the final goods sector) abroad for exports (facing the domestic intermediate sector) is given by (symmetric to the domestic final goods sector):

$$\widetilde{M}^*_t = (1-\nu^*_t)\left(\widetilde{P}^{M*}_t\right)^{-\mu^*}\widetilde{Y}^*_{NAT,t}, \tag{264}$$

where ν^*_t is the domestic share abroad, assumed to follow an AR process. $Y^*_{NAT,t}$ is output abroad. In steady-state we get that

$$\widetilde{M}^*_{ss} = (1-\nu^*_{ss})\left(\widetilde{P}^{M*}_{ss}\right)^{-\mu^*}\widetilde{Y}_{NAT,ss}. \tag{265}$$

Foreign output, money market interest rates, inflation and the international oil price are modeled as a block exogenous system of equations, based on a simple New Keynesian model with added backward looking terms to add more dynamics and realism. Foreign output is divided into trading partners (a predefined list of Norway's closest trading partners) and non-trading partners (the rest of the world). The model variables are in gap form and stationary.

The output gap for trading partners ($\widehat{\widetilde{Y}^*_{NAT,t}}$) is partly backward looking (controlled by a parameter ϕ^{Y*}), and partly equal to $\widehat{\widetilde{Y}^*_{FNAT,t}}$ (defined below) by $(1-\phi^{Y*})$. Additionally it is affected negatively by the oil price gap ($\widehat{P^{O*}_t}$), as this increases costs, and positively by the output gap among non-trading partners, $\widehat{\widetilde{Y}^{NTP}_{NAT,t}}$ (ϕ^{O*} and ϕ^{YNTP*} are positive parameters):

$$\widehat{\widetilde{Y}^*_{NAT,t}} = \phi^{Y*}\widehat{\widetilde{Y}^*_{NAT,t-1}} + (1-\phi^{Y*})\widehat{\widetilde{Y}^*_{FNAT,t}} - \phi^{O*}\widehat{P^{O*}_t} + \phi^{YNTP*}\widehat{\widetilde{Y}^{NTP}_{NAT,t}} + \widehat{z_{U*,t}}. \tag{266}$$

$\widehat{z_{U*,t}}$ is a shock that follows an AR process and $\widehat{\widetilde{Y}^*_{FNAT,t}}$ is specified as a dynamic IS curve:

$$\widehat{\widetilde{Y}^*_{FNAT,t}} = \widehat{\widetilde{Y}^*_{FNAT,t+1}} - \psi^{R*}(\widehat{R^*_t} - \widehat{\pi^*_{t+1}}), \tag{267}$$

where ψ^{R*} relates the real interest rate to output. Output gap for non-trading partners, $\widehat{\widetilde{Y}^{NTP}_{NAT,t}}$, is assumed to follow:

$$\widehat{\widetilde{Y}^{NTP}_{NAT,t}} = \lambda^{YNTP}\widehat{\widetilde{Y}^{YNTP}_{NAT,t-1}} - \phi^{ONTP}\widehat{P^{O*}_t} + \phi^{YNTP}\widehat{\widetilde{Y}^*_{NAT,t}} + \widehat{z_{YNTP,t}}. \tag{268}$$

where $\widehat{z_{YNTP,t}}$ is a shock, and λ^{YNTP} ($\in [0,1]$), ϕ^{ONTP} ($\in (0,\infty)$) and ϕ^{YNTP} ($\in (0,\infty)$) are parameters. The total global output gap is a weighted sum of trading partners and non-trading partners' output:

$$\widehat{\widetilde{Y}^{GLOB}_{NAT,t}} = \alpha^{GLOB}\widehat{\widetilde{Y}^*_{NAT,t}} + (1-\alpha^{GLOB})\widehat{\widetilde{Y}^{NTP}_{NAT,t}}, \tag{269}$$

where α^{GLOB} is the weight on trading partners output gap in the global output gap. The inflation gap for trading partners is given by

$$\widehat{\pi^*_t} = \phi^{P*}\widehat{\pi^*_{t-1}} + (1-\phi^{P*})\widehat{\pi^*_{F,t}} + \phi^{OP*}\widehat{P^{O*}_t} + \widehat{z_{\theta^{H*},t}}, \tag{270}$$

where some agents are backward looking (controlled by a parameter ϕ^{P*}), ϕ^{OP*} is a positive parameter picking up the effect that increasing real oil prices increases real marginal cost for trading partner firms, $\widehat{z_{\theta^{H*},t}}$ is a shock that follows an AR process and $\widehat{\pi^*_{F,t}}$ is specified according to a Phillips curve:

$$\widehat{\pi^*_{F,t}} = \alpha^{P*}\widehat{\pi^*_{F,t+1}} + \alpha^{Y*}\widehat{\widetilde{Y}^*_{NAT,t}}, \tag{271}$$

where α^{P*} and α^{Y*} are parameters. The foreign monetary policy rate (equal to the money market interest rate) is given by a Taylor rule with smoothing:

$$\widehat{R^*_t} = \omega^{R*}\widehat{R^*_{t-1}} + (1-\omega^{R*})\left[\omega^{P*}\widehat{\pi^*_t} + \omega^{Y*}\widehat{\widetilde{Y}^*_{NAT,t}}\right] + \widehat{z_{R*,t}}, \tag{272}$$

where $\widehat{z_{R*,t}}$ is a shock that follows an AR process, the parameter ω^{R*} governs interest rate smoothing, and ω^{P*} and ω^{Y*} are weights on inflation and output respectively. Lastly, the oil price gap is given by

$$\widehat{P^{O*}_t} = \beta^O\widehat{P^{O*}_{t+1}} + \kappa^O\widehat{\widetilde{Y}^{GLOB}_{NAT,t}} + \widehat{z_{PO*,t}}, \tag{273}$$

where $\widehat{z_{PO*,t}}$ is an oil supply shock that follows an AR process. β^O and κ^O are parameters.

10.1 Equations included in the model

The following equations are included in the model file: (262), (264), (266), (267), (270), (271), (272) and (273).

11 The aggregate resource constraint and the UIP

The uncovered interest rate parity was derived in Section 8.1.1 and repeated here:

$$R_t = E_t\left[\left[1-\gamma^{B^*}_t\right]R^*_t\frac{S_{t+1}}{S_t}\right],$$

where $1-\gamma_t^{B^*}$ is the debt-elastic risk premium and R_t^* is the foreign money market rate and S_t is the nominal exchange rate. It is assumed that the risk premium depends positively on the country's net foreign debt position (B_t^{TOT*}, defined below) and the anticipated growth rate of the exchange rate:

$$1-\gamma_t^{B^*} = \exp\left[\phi^B\left(\widetilde{B}_t^{TOT*} - \widetilde{B}_{ss}^{TOT*}\right) + \phi^S\left(E_t\widetilde{S}_{t+1}\widetilde{S}_t - \widetilde{S}_{ss}^2\right)\right] + z_t^B,$$

where z_t^B is an exogenous exchange rate risk premium shock following an AR(1) process and ϕ^B and ϕ^S are non-negative parameters. Note that the risk premium is defined in stationary terms ($\widetilde{B}_t^{TOT*} \equiv \frac{B_t^{TOT*}}{Z_t}$ and $\widetilde{S}_t = \frac{S_tP_t^*}{P_t}$).

The country's net foreign debt position (B_t^{TOT*}) is equal to mainland private foreign debt, B_t^* (held by banks), less government claims, $B_{F,t}$:

$$B_t^{TOT*} = B_t^* - B_{F,t}.$$

$B_{F,t}$ was derived in Section 9.1.4. Taking the household budget constraint as the point of departure and inserting for profits, dividends and lump-sum taxes, we will now derive mainland Norway's debt accumulation expression, B_t^*. We start out from the housholds' budget constraint (removing all indices):

$$\begin{aligned} &P_tC_t + P_tD_t + P_t^HH_t + \left(r_{t-1}^F + \delta_t^B\right)P_{t-1}B_{h,t-1} \\ &= W_tL_t\left[1-\gamma_t\right] + P_tI_{B,t} + R_{t-1}^dP_{t-1}D_{t-1} + \left(1-\delta_H\right)P_t^HH_{t-1} + DIV_t - TAX_t. \end{aligned} \tag{274}$$

First, define TAX_t :

$$TAX_t = P_tG_t. \tag{275}$$

Then, we need to find DIV_t. Starting with intermediate sector profits, Π_t^{INT}:

$$\Pi_t^{INT} = P_t^QQ_t + P_t^{M^*}S_tM_t^* - W_{I,t}L_{I,t} - R_{KI,t}\overline{K}_{I,t} - \gamma_{PQ,t}P_t^QQ_t - \gamma_{PM^*,t}P_t^{M^*}S_tM_t^*. \tag{276}$$

From (151), bank sector profits (including adjustment costs):

$$P_{t-1}J_{t-1} = r_{t-1}^FP_{t-1}B_{h,t-1} + r_{t-1}^eP_{t-1}B_{e,t-1} - r_{t-1}^dP_{t-1}D_{t-1} - r_{t-1}P_{t-1}B_{t-1}^* - P_{t-1}ADJ_{t-1}^{BANK}, \tag{277}$$

where we have defined bank adjustment costs

$$\begin{aligned} P_{t-1}ADJ_{t-1}^{BANK} &= \chi_oP_{t-1}B_{t-1} + \frac{\chi_c}{2}\left[\frac{K_{t-1}^B}{B_{t-1}^{RW}} - \varpi_{t-1}\right]^2P_{t-1}K_{t-1}^B \\ &+\frac{\phi^F}{2}\left(\frac{r_{t-1}^F}{r_{t-2}^F}-1\right)^2r_{t-1}^FP_{t-1}B_{h,t-1} + \frac{\phi^e}{2}\left(\frac{r_{t-1}^e}{r_{t-2}^e}-1\right)^2r_{t-1}^eP_{t-1}B_{e,t-1} + \frac{\phi^D}{2}\left(\frac{r_{t-1}^d}{r_{t-2}^d}-1\right)^2r_{t-1}^dP_{t-1}D_{t-1} \end{aligned}$$

Also remembering dividends from eq. (153):

$$\begin{aligned} P_tK_t^B &= (1-\delta^b)P_{t-1}K_{t-1}^B + P_{t-1}J_{t-1} \\ &\Leftrightarrow \\ \delta^bP_{t-1}K_{t-1}^B &= P_{t-1}K_{t-1}^B - P_tK_t^B + P_{t-1}J_{t-1}. \end{aligned} \tag{278}$$

Oil supply profits:

$$\Pi_t^{OS} = P_t^R I_{OF,t} + P_t^{R*} S_t M_{O*,t} - PQ_{O,t} - W_{O,t} L_{O,t} - R_{KO,t}\overline{K}_{O,t} - \gamma_{PR,t} P_t^R I_{OF,t} - \gamma_{PR^*,t} P_t^{R^*} S_t M_{O*,t}. \tag{279}$$

.

Capital producers' profits while inserting from eq. (123), and the zero profit condition:

$$P_t^K K_t - P_t^K (1-\delta) K_{t-1} = P_t I_{C,t} \tag{280}$$

In similar fashion for housing profit:

$$P_t^H H_t - P_t^H (1-\delta_H) H_{t-1} = P_t I_{H,t}. \tag{281}$$

Entrepreneurs' surplus (S_t^{ent}):

$$P_t S_t^{ent} = R_{K,t} u_t K_{t-1} + P_t^K (1-\delta) K_{t-1} + P_t I_{B,t}^e - P_t^K K_t - (r_t^e + \delta_t^e) P_{t-1} B_{e,t-1} - P_t \gamma(u_t) K_{t-1}. \tag{282}$$

Inserting for intermediate profit, bank dividends, oil supply profits and entrepreneurs' surplus for DIV_t and for TAX_t into (274) gives:

$$\begin{gathered}
P_t C_t + P_t D_t + P_t^H H_t + \left(r_{t-1}^F + \delta_t^B\right) P_{t-1} B_{h,t-1} \\
= W_t L_t - \gamma_t W_t L_t + P_t I_{B,t} + R_{t-1}^d P_{t-1} D_{t-1} + (1-\delta_H) P_t^H H_{t-1} \\
-TAX_t + DIV_t
\end{gathered} \tag{283}$$

$$\Leftrightarrow$$

$$\begin{gathered}
P_t C_t + P_t D_t + P_t^H H_t + \left(r_{t-1}^F + \delta_t^B\right) P_{t-1} B_{h,t-1} \\
= W_t L_t - \gamma_t W_t L_t + P_t I_{B,t} + R_{t-1}^d P_{t-1} D_{t-1} + (1-\delta_H) P_t^H H_{t-1} \\
-P_t G_t. + P_t^Q Q_t + P_t^{M^*} S_t M_t^* - W_{I,t} L_{I,t} - R_{KI,t}\overline{K}_{I,t} - \gamma_{PQ,t} P_t^Q Q_t - \gamma_{PM^*,t} P_t^{M^*} S_t M_t^* \\
+\delta^b P_{t-1} K_{t-1}^B \\
+P_t^R I_{OF,t} + P_t^{R*} S_t M_{O*,t} - PQ_{O,t} - W_{O,t} L_{O,t} - R_{KO,t}\overline{K}_{O,t} - \gamma_{PR,t} P_t^R I_{OF,t} - \gamma_{PR^*,t} P_t^{R^*} S_t M_{O*,t} \\
+R_{K,t} u_t K_{t-1} + P_t^K (1-\delta) K_{t-1} + P_t I_{B,t}^e - P_t^K K_t - (r_t^e + \delta_t^e) P_{t-1} B_{e,t-1} - P_t \gamma(u_t) K_{t-1}.
\end{gathered} \tag{284}$$

Then we define all adjustment costs including capital requirement costs as:

$$\begin{aligned}
ADJ_t \quad = \quad & \gamma_t W_t L_t + \gamma_{PQ,t} P_t^Q Q_t + \gamma_{PM^*,t} P_t^{M^*} S_t M_t^* + \gamma_{PR,t} P_t^R I_{OF,t} + \gamma_{PR^*,t} P_t^{R^*} S_t M_{O*,t} \\
& + P_{t-1} ADJ_{t-1}^{BANK} + P_t \gamma(u_t) K_{t-1}.
\end{aligned} \tag{285}$$

We insert (285) and (??) in (284) and also substitute the expression for bank dividends from eq. (278)

$$\begin{gathered}
P_t C_t + P_t D_t + P_t^H H_t + \left(r_{t-1}^F + \delta_t^B\right) P_{t-1} B_{h,t-1} \\
= W_t L_t + P_t I_{B,t} + R_{t-1}^d P_{t-1} D_{t-1} + (1-\delta_H) P_t^H H_{t-1} \\
-P_t G_t. + P_t^Q Q_t + P_t^{M^*} S_t M_t^* - W_{I,t} L_{I,t} - R_{KI,t}\overline{K}_{I,t} \\
+P_{t-1} K_{t-1}^B - P_t K_t^B + P_{t-1} J_{t-1} \\
+P_t^R I_{OF,t} + P_t^{R*} S_t M_{O*,t} - PQ_{O,t} + W_{O,t} L_{O,t} + R_{KO,t}\overline{K}_{O,t} \\
+R_{K,t} u_t K_{t-1} + P_t^K (1-\delta) K_{t-1} + P_t I_{B,t}^e - P_t^K K_t - (r_t^e + \delta_t^e) P_{t-1} B_{e,t-1} - (ADJ_t - P_{t-1} ADJ_{t-1}^{BANK}).
\end{gathered} \tag{286}$$

Inserting for bank profits, $P_{t-1}J_{t-1}$:

$$
\begin{aligned}
P_tC_t + P_tD_t + P_t^H H_t + \left(r_{t-1}^F + \delta_t^B\right) P_{t-1}B_{h,t-1} \\
= W_tL_t + P_tI_{B,t} + R_{t-1}^d P_{t-1}D_{t-1} + (1-\delta_H) P_t^H H_{t-1} \\
-P_tG_{t.} + P_t^Q Q_t + P_t^{M^*} S_t M_t^* - W_{I,t}L_{I,t} - R_{KI,t}\overline{K}_{I,t} \\
+P_{t-1}K_{t-1}^B - P_tK_t^B + r_{t-1}^F P_{t-1}B_{h,t-1} + r_{t-1}^e P_{t-1}B_{e,t-1} - r_{t-1}^d P_{t-1}D_{t-1} - r_{t-1}P_{t-1}B_{t-1}^* - P_{t-1}ADJ_{t-1}^{BANK} \\
+P_t^R I_{OF,t} + P_t^{R*} S_t M_{O*,t} - PQ_{O,t} + W_{O,t}L_{O,t} + R_{KO,t}\overline{K}_{O,t} \\
+R_{K,t}u_tK_{t-1} + P_t^K(1-\delta)K_{t-1} + P_tI_{B,t}^e - P_t^K K_t - (r_t^e + \delta_t^e)P_{t-1}B_{e,t-1} - (ADJ_t - P_{t-1}ADJ_{t-1}^{BANK}).
\end{aligned}
\tag{287}
$$

Replacing gross interest rate $R_t^x = 1 + r_t^x$ and cancelling terms:

$$
\begin{aligned}
P_tC_t + P_tD_t + P_t^H H_t + \delta_t^B P_{t-1}B_{h,t-1} \\
= W_tL_t + P_tI_{B,t} + P_{t-1}D_{t-1} + (1-\delta_H) P_t^H H_{t-1} \\
-P_tG_{t.} + P_t^Q Q_t + P_t^{M^*} S_t M_t^* - W_{I,t}L_{I,t} - R_{KI,t}\overline{K}_{I,t} \\
+P_{t-1}K_{t-1}^B - P_tK_t^B - r_{t-1}P_{t-1}B_{t-1}^* \\
+P_t^R I_{OF,t} + P_t^{R*} S_t M_{O*,t} - PQ_{O,t} - W_{O,t}L_{O,t} - R_{KO,t}\overline{K}_{O,t} \\
+R_{K,t}u_tK_{t-1} + P_t^K(1-\delta)K_{t-1} + P_tI_{B,t}^e - P_t^K K_t - \delta_t^e P_{t-1}B_{e,t-1} - ADJ_t.
\end{aligned}
\tag{288}
$$

We then make use of the borrowing constraints for households, (4), and similar for entrepreneurs and the banks sector's balance sheet :

$$
P_tI_{B,t} = P_tB_{h,t} - P_{t-1}B_{h,t-1} + \delta_t^B P_{t-1}B_{h,t-1}, \tag{289}
$$

$$
P_tI_{B,t}^e = P_tB_{e,t} - P_{t-1}B_{e,t-1} + \delta_t^e P_{t-1}B_{e,t-1}, \tag{290}
$$

$$
P_tK_t^B + P_tB_t^* = P_tB_{e,t} + P_tB_{h,t} - P_tD_t. \tag{291}
$$

Inserting these and cancelling more terms:

$$
\begin{aligned}
P_tC_t + P_t^H H_t + P_{t-1}K_{t-1}^B + P_{t-1}B_{t-1}^* \\
= W_tL_t + (1-\delta_H) P_t^H H_{t-1} \\
-P_tG_{t.} + P_t^Q Q_t + P_t^{M^*} S_t M_t^* - W_{I,t}L_{I,t} - R_{KI,t}\overline{K}_{I,t} \\
+P_{t-1}K_{t-1}^B - P_tK_t^B - r_{t-1}P_{t-1}B_{t-1}^* + P_tK_t^B + P_tB_t^* \\
+P_t^R I_{OF,t} + P_t^{R*} S_t M_{O*,t} - PQ_{O,t} - W_{O,t}L_{O,t} - R_{KO,t}\overline{K}_{O,t} \\
+R_{K,t}u_tK_{t-1} + P_t^K(1-\delta)K_{t-1} - P_t^K K_t - ADJ_t.
\end{aligned}
\tag{292}
$$

Using (280) and (281) and $L_t = L_{I,t} + L_{O,t}$ and $\overline{K}_{O,t} + \overline{K}_{I,t} = \overline{K}_t = u_tK_{t-1}$ and cancelling terms

$$
\begin{aligned}
P_tC_t + P_{t-1}B_{t-1}^* + P_tI_{C,t} + P_tI_{H,t} \\
= \\
-P_tG_{t.} + P_t^Q Q_t + P_t^{M^*} S_t M_t^* \\
-r_{t-1}P_{t-1}B_{t-1}^* + P_tB_t^* \\
+P_t^R I_{OF,t} + P_t^{R*} S_t M_{O*,t} - PQ_{O,t} \\
-ADJ_t.
\end{aligned}
\tag{293}
$$

Then inserting for the final goods zero profit condition and the market clearing condition, eq. (310):

$$P_tA_t - P_t^M M_t = P_t^Q Q_t, \tag{294}$$

$$P_tA_t = P_tC_t \ + P_tI_{C,t} + P_tI_{H,t} + P_tG_t + P_tQ_{O,t}$$

gives

$$\begin{gathered} P_{t-1}B_{t-1}^* \\ = \\ -P_t^M M_t + P_t^{M^*} S_t M_t^* \\ -r_{t-1}P_{t-1}B_{t-1}^* + P_tB_t^* \\ +P_t^R I_{OF,t} + P_t^{R*} S_t M_{O*,t} \\ -ADJ_t. \end{gathered} \tag{295}$$

Rearranging:

$$P_tB_t^* = R_{t-1}P_{t-1}B_{t-1}^* + P_t^M M_t - P_t^{R*} S_t M_{O*,t} - P_t^{M^*} S_t M_t^* - P_t^R I_{OF,t} + ADJ_t. \tag{296}$$

In the model, we set $ADJ_t = 0$, and subtract $G_{F,t}$ to get:

$$P_tB_t^* = R_{t-1}P_{t-1}B_{t-1}^* + P_t^M M_t - P_t^{R*} S_t M_{O*,t} - P_t^{M^*} S_t M_t^* - P_t^R I_{OF,t} - G_{F,t}. \tag{297}$$

11.1 Making the equations stationary

The UIP condition:

$$E_t \left[\frac{R_t^*}{R_t} \frac{\pi_{t+1}}{\pi_{t+1}^*} \frac{\widetilde{S}_{t+1}}{\widetilde{S}_t} \left[1 - \gamma_t^{B^*} \right] \right] = 1, \tag{298}$$

Private foreign debt:

$$\widetilde{B}_t^{TOT*} = \widetilde{B}_t^* - \widetilde{B}_{F,t}. \tag{299}$$

Mainlands's net foreign debt position

$$\widetilde{B}_t^* = \frac{R_{t-1}}{\pi_t^{\tilde{z}} \pi_t} \widetilde{B}_{t-1}^* + \left[\widetilde{P}_t^M \widetilde{M}_t - \widetilde{S}_t \widetilde{P}_t^{R^*} \widetilde{M}_{O*,t} - \widetilde{S}_t \widetilde{P}_t^{M*} \widetilde{M}_t^* - \widetilde{P}_t^R \widetilde{I}_{OF,t} - \widetilde{G}_{F,t} \right]. \tag{300}$$

11.1.1 Equations included in the model

The following equations are included in the model file: (298), (299) and (300).

11.2 Steady-state equations

Risk premium

$$1 - \gamma_{ss}^{B^*} = z_{ss}^B = 0 \Rightarrow \gamma_{ss}^{B^*} = 1, \tag{301}$$

The UIP condition

$$\frac{R_{ss}^*}{\pi_{ss}^*} = \frac{R_{ss}}{\pi_{ss}}, \tag{302}$$

Private foreign debt

$$\widetilde{B}^{TOT*}_{ss} = \widetilde{B}^{*}_{ss} - \widetilde{B}_{F,ss}, \tag{303}$$

Mainlands Norway's net foreign debt position

$$\widetilde{B}^{*}_{ss} = \left[1 - \frac{R_{ss}}{\pi^{z}_{ss}\pi_{ss}}\right]^{-1}\left[\widetilde{P}^{M}_{ss}\widetilde{M}_{ss} - \widetilde{S}_{ss}\widetilde{P}^{R^*}_{ss}\widetilde{M}_{O*,ss} - \widetilde{S}_{ss}\widetilde{P}^{M*}_{ss}\widetilde{M}^{*}_{ss} - \widetilde{P}^{R}_{ss}\widetilde{I}_{OF,ss} - \widetilde{G}_{F,ss}\right]. \tag{304}$$

12 Monetary policy

Monetary policy can either follow a Taylor type rule:

$$R_t = (R_{t-1})^{\omega_R}\left(R_{ss}\left(\frac{\pi_t}{\pi_{ss}}\right)^{\omega_P}\left(\frac{\widetilde{Y}_{NAT,t}}{\widetilde{Y}_{NAT,ss}}\right)^{\omega_Y}\right)^{1-\omega_R} e^{Z_{RN3M,t}}, \tag{305}$$

where ω_R governs interest rate persistence and ω_P and ω_Y are the weights on inflation and output respectively, while $Z_{RN3M,t}$ represents a monetary policy shock that follows an AR process, or it can minimize a loss function (either under commitment or discretionary policies), i.e.

$$\min_{\{\widehat{R}_{P,t}\}}\sum_{t=s}^{\infty}\beta_p^{t-s}\left[(\widehat{\pi}_{pol,t})^2 + \lambda_y\left(\widehat{\widetilde{Y}}_{NAT,t}\right)^2 + \lambda_{dr}\left(\triangle R_{P,t}\right)^2 + \lambda_{lr}\left(\widehat{R}^{YEAR}_{P,t}\right)^2\right], \tag{306}$$

where β_p is the central bank's discount factor, $\widehat{x}_t$ denotes a variable deviation from steady-state, $\widehat{R}_{P,t}$ is the key policy rate gap and $\widehat{\widetilde{Y}}_{NAT,t}$ is the output gap (see section 14). Furthermore, the annualized key policy rate change is defined as

$$\triangle R_{P,t} = 4\left(R_{P,t} - R_{P,t-1}\right), \tag{307}$$

the annualized key policy rate gap is defined as

$$\widehat{R}^{YEAR}_{P,t} = 4\widehat{R}_{P,t},$$

and the 4-quarter inflation rate gap is given by

$$\widehat{\pi}_{pol,t} = \widehat{\pi}_t + \widehat{\pi}_{t-1} + \widehat{\pi}_{t-2} + \widehat{\pi}_{t-3} + \log\left(\frac{z_{inf,t}}{z_{inf,ss}}\right), \tag{308}$$

where $z_{inf,t}$ is a monetary policy preference shock that follows an AR process. λ_y, λ_{dr} and λ_{lr} are the corresponding weights in the loss function.

13 Market clearing

Intermediate goods market:

$$T_t = Q_t + M^*_t. \tag{309}$$

Final goods market:

$$A_t = C_t + I_{C,t} + I_{H,t} + G_t + Q_{O,t}. \tag{310}$$

Total investment:

$$I_t = I_{C,t} + I_{H,t}. \tag{311}$$

Capital goods sector:

$$\overline{K}_t = \overline{K}_{O,t} + \overline{K}_{I,t}. \tag{312}$$

Labor market:

$$L_t = L_{O,t} + L_{I,t}. \tag{313}$$

Oil supply sector:

$$Y_{R,t} = I_{OF,t} + M_{O*,t}. \tag{314}$$

13.1 Making the equations stationary

Intermediate goods market:

$$\widetilde{T}_t = \widetilde{Q}_t + \widetilde{M}_t^*. \tag{315}$$

Final goods market:

$$\widetilde{A}_t = \widetilde{C}_t + \widetilde{I}_{C,t} + \widetilde{I}_{H,t} + \widetilde{G}_t + \widetilde{Q}_{O,t}. \tag{316}$$

Total investment:

$$\widetilde{I}_t = \widetilde{I}_{C,t} + \widetilde{I}_{H,t}. \tag{317}$$

Capital goods sector:

$$\widetilde{\overline{K}}_t = \widetilde{\overline{K}}_{O,t} + \widetilde{\overline{K}}_{I,t}. \tag{318}$$

Labor market:

$$L_t = L_{O,t} + L_{I,t}. \tag{319}$$

Oil supply sector:

$$\widetilde{Y}_{R,t} = \widetilde{I}_{OF,t} + \widetilde{M}_{O*,t}. \tag{320}$$

13.1.1 Equations included in the model

The following equations are included in the model file: (315), (316), (317), (318), (319) and (320).

13.2 Steady-state equations

Intermediate goods market:

$$\widetilde{T}_{ss} = \widetilde{Q}_{ss} + \widetilde{M}_{ss}^*. \tag{321}$$

Final goods market:

$$\widetilde{A}_{ss} = \widetilde{C}_{ss} + \widetilde{I}_{C,ss} + \widetilde{I}_{H,ss} + \widetilde{G}_{ss} + \widetilde{Q}_{O,ss}. \tag{322}$$

Total investment:

$$\widetilde{I}_{ss} = \widetilde{I}_{C,ss} + \widetilde{I}_{H,ss}. \tag{323}$$

Capital goods sector:

$$\widetilde{\overline{K}}_{ss} = \widetilde{\overline{K}}_{O,ss} + \widetilde{\overline{K}}_{I,ss}. \tag{324}$$

Labor market:

$$L_{ss} = L_{O,ss} + L_{I,ss}. \tag{325}$$

Oil supply sector:

$$\widetilde{Y}_{R,ss} = \widetilde{I}_{OF,ss} + \widetilde{M}_{O*,ss}. \tag{326}$$

14 Extra definitions

14.1 Interest rates

The policy rate, $R_{P,t}$, is the product of the money market interest rate, R_t, and the money market risk premium, $Z_{prem,t}$ (which is assumed to be a shock that follows an AR process):

$$R_{P,t} = R_t Z_{prem,t}. \tag{327}$$

14.2 Prices

Inflation:

$$\pi_t \equiv \frac{P_t}{P_{t-1}}. \tag{328}$$

Inflation abroad:

$$\pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}.$$

Imported inflation:

$$\pi_t^M = \pi_t \frac{\widetilde{P}_t^M}{\widetilde{P}_{t-1}^M}. \tag{329}$$

Exported inflation:

$$\pi_t^{M*} = \pi_t^* \frac{\widetilde{P}_t^{M*}}{\widetilde{P}_{t-1}^{M*}}. \tag{330}$$

Inflation of domestically produces goods:

$$\pi_t^Q = \pi_t \frac{\widetilde{P}_t^Q}{\widetilde{P}_{t-1}^Q}. \tag{331}$$

Oil supply goods domestic inflation:

$$\pi_t^R = \pi_t \frac{\widetilde{P}_t^R}{\widetilde{P}_{t-1}^R}. \tag{332}$$

Oil supply goods export inflation:

$$\pi_t^{R*} = \pi_t^* \frac{\widetilde{P}_t^{R*}}{\widetilde{P}_{t-1}^{R*}}. \tag{333}$$

House price inflation:

$$\pi_t^H = \frac{\widetilde{P}_t^H}{\widetilde{P}_{t-1}^H} \pi_t \pi_t^h. \tag{334}$$

Wage inflation ($\widetilde{W}_{I,t} = \widetilde{W}_{O,t} = \widetilde{W}_t$):

$$\pi_t^W = \frac{\widetilde{W}_t}{\widetilde{W}_{t-1}} \pi_t \pi_t^z. \tag{335}$$

Oil supply sector wage inflation:

$$\pi_t^{WO} = \pi_t^W. \tag{336}$$

Intermediate sector wage inflation:

$$\pi_{,t}^{WI} = \pi_t^W. \tag{337}$$

4-quarter inflation:

$$\widehat{\pi}_t^{YEAR} = \sum_{i=0}^{3} \widehat{\pi}_{t-i}. \tag{338}$$

4-quarter inflation abroad,

$$\widehat{\pi}_t^{*YEAR} = \sum_{i=0}^{3} \widehat{\pi}_{t-i}^{*}. \tag{339}$$

4-quarter inflation for imported goods:

$$\widehat{\pi}_t^{M,YEAR} = \sum_{i=0}^{3} \widehat{\pi}_{t-i}^{M}. \tag{340}$$

4-quarter inflation for exported goods:

$$\widehat{\pi}_t^{M*,YEAR} = \sum_{i=0}^{3} \widehat{\pi}_{t-i}^{M*}. \tag{341}$$

14.3 Variables in CPI units

To aggregate variables we need them to be given in same unit of account. We choose to cast all variables in CPI units. The steady-state versions are straight forward.

The intermediate goods export is given by

$$\widetilde{X}_{NAT,t}^{I} = \widetilde{S}_t \widetilde{P}_t^{M^*} \widetilde{M^*}_{,t}. \tag{342}$$

The oil supply goods export:

$$\widetilde{X}_{NAT,t}^{O} = \widetilde{S}_t \widetilde{P}_t^{R*} \widetilde{M}_{O*,t}. \tag{343}$$

Total export:

$$\widetilde{X}_{NAT,t} = \widetilde{X}_{NAT,t}^{I} + \widetilde{X}_{NAT,t}^{O}. \tag{344}$$

Total import, with measurement error/shock (that follows an AR process):

$$\widetilde{M}_{NAT,t} = \widetilde{P}_t^{M} \widetilde{M}_t. \tag{345}$$

Oil investment:

$$\widetilde{I}_{NAT,t}^{OIL} = \widetilde{P}_t^{R} \widetilde{I}_{O,t}. \tag{346}$$

Oil production:

$$\widetilde{OIL}_{NAT,t} = \widetilde{S}_t \widetilde{P}_t^{O*} \widetilde{Y}_{O,t}. \tag{347}$$

Oil supply goods:

$$\widetilde{Y}_{R,NAT,t} = \widetilde{P}_t^{R} \widetilde{I}_{OF,t} + \widetilde{S}_t \widetilde{P}_t^{R*} \widetilde{M}_{O*,t}. \tag{348}$$

Output (mainland economy) is then given by (inserting for A from 322)

$$\widetilde{Y}_{NAT,t} = \left(\widetilde{A}_t - \widetilde{Q}_{O,t} + \widetilde{I}_{NAT,t}^{OIL} + \widetilde{X}_{NAT,t} - \widetilde{M}_{NAT,t} \right) \frac{1}{1 - \log(z_{x,t})} \tag{349}$$

$$\Leftrightarrow$$

$$\widetilde{Y}_{NAT,t} = \left(\widetilde{C}_t + \widetilde{I}_{C,t} + \widetilde{I}_{H,t} + \widetilde{G}_t + \widetilde{I}_{NAT,t}^{OIL} + \widetilde{X}_{NAT,t} - \widetilde{M}_{NAT,t} \right) \frac{1}{1 - \log(z_{x,t})},$$

where $z_{x,t}$ is an inventory shock to the mainland economy, that follows an AR process. Total output is then given by

$$\widetilde{Y}_{NAT,t}^{OIL} = \widetilde{Y}_{NAT,t} + \widetilde{OIL}_{NAT,t}. \tag{350}$$

15 Measurement equations

The following equations measure the growth rate gaps of key variables. We do not include measurement errors. (Note that $\triangle$ does *not* relate to the variable Δ, where the latter being the stochastic discount rate.)

$$\triangle\widetilde{C}_t = \log\left(\frac{\widetilde{C}_t}{\widetilde{C}_{t-1}}\right) + \log\left(\frac{\pi^z_t}{\pi^z_{ss}}\right), \tag{351}$$

$$\triangle\widetilde{G}_t = \log\left(\frac{\widetilde{G}_t}{\widetilde{G}_{t-1}}\right) + \log\left(\frac{\pi^z_t}{\pi^z_{ss}}\right), \tag{352}$$

$$\triangle\widetilde{I}_{C,t} = \log\left(\frac{\widetilde{I}_{C,t}}{\widetilde{I}_{C,t-1}}\right) + \log\left(\frac{\pi^z_t}{\pi^z_{ss}}\right), \tag{353}$$

$$\triangle\widetilde{I}_{H,t} = \log\left(\frac{\widetilde{I}_{H,t}}{\widetilde{I}_{H,t-1}}\right) + \log\left(\frac{\pi^z_t}{\pi^z_{ss}}\right), \tag{354}$$

$$\triangle\widetilde{Y}_{NAT,t} = \log\left(\frac{\widetilde{Y}_{NAT,t}}{\widetilde{Y}_{NAT,t-1}}\right) + \log\left(\frac{\pi^z_t}{\pi^z_{ss}}\right), \tag{355}$$

$$\triangle\widetilde{M}^*_t = \log\left(\frac{\widetilde{M}^*_t}{\widetilde{M}^*_{t-1}}\right) + \log\left(\frac{\pi^z_t}{\pi^z_{ss}}\right), \tag{356}$$

$$\triangle\widetilde{M}_t = \log\left(\frac{\widetilde{M}_t}{\widetilde{M}_{t-1}}\right) + \log\left(\frac{\pi^z_t}{\pi^z_{ss}}\right), \tag{357}$$

$$\triangle\widetilde{W}_t = \log\left(\frac{\widetilde{W}_t}{\widetilde{W}_{t-1}}\right) + \log\left(\frac{\pi^z_t}{\pi^z_{ss}}\right). \tag{358}$$

16 Shock processes

There are various shocks in the model. All are assumed to be AR(1) processes, where the λ's govern the persistence of the processes (i.e. autocorrelation), the ϵ's are normally distributed white noise innovations and the σ's are parameters governing the standard deviations of the respective shocks. Most shock processes are modeled as deviations from the steady state. The shocks included in the model are the following:

House price stochastic trend inflation shock:

$$\log(\pi^h_t) = (1-\lambda_{\pi^h})\log(\pi^h_{ss}) + \lambda_{\pi^h}\log(\pi^h_{t-1}) + \epsilon_{\pi^h,t}\sigma_{\pi^h}. \tag{359}$$

Shock to productivity in the intermediate sector:

$$\log(z^L_t) = (1-\lambda_{z^L})\log(z^L_{ss}) + \lambda_{z^L}\log(z^L_{t-1}) + \epsilon_{z^L,t}\sigma_{z^L}. \tag{360}$$

Stochastic trend growth shock:

$$\log(\pi^z_t) = (1-\lambda_{\pi^z})\log(\pi^z_{ss}) + \lambda_{\pi^z}\log(\pi^z_{t-1}) + \epsilon_{\pi^z,t}\sigma_{\pi^z}. \tag{361}$$

Government expenditure shock:

$$\log(\widetilde{G}_t) = (1-\lambda_G)\log(\widetilde{G}_{ss}) + \lambda_G\log(\widetilde{G}_{t-1}) + \epsilon_{G,t}\sigma_G. \tag{362}$$

Export demand shock:

$$\log(\nu^*_t) = (1-\lambda_{\nu^*})\log(\nu^*_{ss}) + \lambda_{\nu^*}\log(\nu^*_{t-1}) + \epsilon_{\nu^*,t}\sigma_{\nu^*}. \tag{363}$$

Import demand shock:

$$\log(\nu_t) = (1 - \lambda_\nu)\log(\nu_{ss}) + \lambda_\nu \log(\nu_{t-1}) + \epsilon_{\nu,t}\sigma_\nu. \tag{364}$$

Oil in the ground shock:

$$\log(O_t) = (1 - \lambda_O)\log(O_{ss}) + \lambda_O \log(O_{t-1}) + \epsilon_{O,t}\sigma_O. \tag{365}$$

Shock to a parameter that can be mapped to the corporate loan-to-value ratio, see (414):

$$\log(\phi_t^{ent}) = (1 - \lambda_{\phi^{ent}})\log(\phi_{ss}^{ent}) + \lambda_{\phi^{ent}} \log(\phi_{t-1}^{ent}) + \epsilon_{\phi^{ent},t}\sigma_{\phi^{ent}}. \tag{366}$$

Shock to a parameter that can be mapped to the household loan-to-value ratio, see (476):

$$\log(\phi_t) = (1 - \lambda_\phi)\log(\phi_{ss}) + \lambda_\phi \log(\phi_{t-1}) + \epsilon_{\phi,t}\sigma_\phi. \tag{367}$$

Shock to the competition in the labor market (wage markup shock):

$$\log(\psi_t) = (1 - \lambda_\psi)\log(\psi_{ss}) + \lambda_\psi \log(\psi_{t-1}) + \epsilon_{\psi,t}\sigma_\psi - \rho_{ffm} FFM_t, \tag{368}$$

where we have that $FFM = 0.75\widehat{L}_{O,t} + 0.25\widehat{P}_t^{O*}$.

Shock to banks' capital requirement:

$$\log(\gamma_t^B) = (1 - \lambda_{\gamma^B})\log(\gamma_{ss}^B) + \lambda_{\gamma^B} \log(\gamma_{t-1}^B) + \epsilon_{\gamma^B,t}\sigma_{\gamma^B}. \tag{369}$$

Shock to the competition in the deposit market:

$$\log(\theta_t^D) = (1 - \lambda_{\theta^D})\log(\theta_{ss}^D) + \lambda_{\theta^D} \log(\theta_{t-1}^D) + \epsilon_{\theta^D,t}\sigma_{\theta^D}. \tag{370}$$

Shock to the competition in the loan market for the entrepreneurs:

$$\log(\theta_t^e) = (1 - \lambda_{\theta^e})\log(\theta_{ss}^e) + \lambda_{\theta^e} \log(\theta_{t-1}^e) + \epsilon_{\theta^e,t}\sigma_{\theta^e}. \tag{371}$$

Shock to the competition in the market for exports $\widetilde{M}_t^*$:

$$\log(\theta_t^{F*}) = (1 - \lambda_{\theta^F})\log(\theta_{ss}^{F*}) + \lambda_{\theta^F} \log(\theta_{t-1}^{F*}) + \epsilon_{\theta^{F*},t}\sigma_{\theta^{F*}}. \tag{372}$$

Shock to the competition in the market for imports $\widetilde{M}_t$:

$$\log(\theta_t^F) = (1 - \lambda_{\theta^{F*}})\log(\theta_{ss}^F) + \lambda_{\theta^{F*}} \log(\theta_{t-1}^F) + \epsilon_{\theta^F,t}\sigma_{\theta^F}. \tag{373}$$

Shock to the competition in the market for the domestic good $\widetilde{Q}_t$:

$$\log(\theta_t^H) = (1 - \lambda_{\theta^H})\log(\theta_{ss}^H) + \lambda_{\theta^H} \log(\theta_{t-1}^H) + \epsilon_{\theta^H,t}\sigma_{\theta^H}. \tag{374}$$

Shock to the competition in the market for household lending:

$$\log(\theta_t^{IH}) = (1 - \lambda_{\theta^{IH}})\log(\theta_{ss}^{IH}) + \lambda_{\theta^{IH}} \log(\theta_{t-1}^{IH}) + \epsilon_{\theta^{IH},t}\sigma_{\theta^{IH}}. \tag{375}$$

Exogenous exchange rate risk premium shock:

$$z_t^B = (1 - \lambda_B) z_{ss}^B + \lambda_B z_{t-1}^B + \epsilon_{B,t}\sigma_B. \tag{376}$$

Shock to the households preferences for deposits:

$$\log(z_t^d) = (1 - \lambda_d)\log(z_{ss}^d) + \lambda_d \log(z_{t-1}^d) + \epsilon_{d,t}\sigma_d. \tag{377}$$

Shock to the productivity of oil investments:

$$\log(Z_{IOIL,t}) = (1 - \lambda_{IOIL})\log(Z_{IOIL,ss}) + \lambda_{IOIL}\log(Z_{IOIL,t-1}) + \epsilon_{IOIL,t}\sigma_{IOIL}. \tag{378}$$

Shock to the households preferences for housing:

$$\log(z_t^h) = (1 - \lambda_h)\log(z_{ss}^h) + \lambda_h \log(z_{t-1}^h) + \epsilon_{h,t}\sigma_h. \tag{379}$$

Investment shock:

$$\log(z_{I,t}) = (1 - \lambda_I)\log(z_{I,ss}) + \lambda_I \log(z_{I,t-1}) + \epsilon_{I,t}\sigma_I. \tag{380}$$

Housing investment shock:

$$\log(z_{IH,t}) = (1 - \lambda_{IH})\log(z_{IH,ss}) + \lambda_{IH}\log(z_{IH,t-1}) + \epsilon_{IH,t}\sigma_{IH}. \tag{381}$$

Oil price shock (in gap-form):

$$\widehat{z_{PO*,t}} = \lambda_{PO*}\widehat{z_{PO*,t-1}} + \epsilon_{PO*,t}\sigma_{PO*}. \tag{382}$$

Oil production shock abroad:

$$\log(\widetilde{Y}_{O*,t}) = (1 - \lambda_{YO*,t})\log(\widetilde{Y}_{O*,ss}) + \lambda_{YO*,t}\log(\widetilde{Y}_{O*,t-1}) + \epsilon_{YO*,t}\sigma_{YO*}. \tag{383}$$

Oil extraction firms productivity shock:

$$\log(Z_{O,t}) = (1 - \lambda_{OIL})\log(Z_{O,ss}) + \lambda_{OIL}\log(Z_{O,t-1}) + \epsilon_{OIL,t}\sigma_{OIL}. \tag{384}$$

Money market risk premium shock:

$$\log(z_{prem,t}) = (1 - \lambda_{prem})\log(z_{prem,ss}) + \lambda_{prem}\log(z_{prem,t-1}) + \epsilon_{prem,t}\sigma_{prem}. \tag{385}$$

Oil supply firms productivity shock:

$$\log(Z_{R,t}) = (1 - \lambda_R)\log(Z_{R,ss}) + \lambda_R \log(Z_{R,t-1}) + \epsilon_{R,t}\sigma_R. \tag{386}$$

Monetary policy preference shock (when solved under optimal policy):

$$\log(z_{inf,t}) = (1 - \lambda_{rnfolio})\log(z_{inf,ss}) + \lambda_{rnfolio}\log(z_{inf,t-1}) + \epsilon_{inf,t}\sigma_{inf}. \tag{387}$$

Monetary policy shock (when solved with a Taylor rule):

$$z_{RN3M,t} = \lambda_{RN3M} z_{RN3M,t-1} + \epsilon_{RN3M,t}\sigma_{RN3M}. \tag{388}$$

Shock to the households preferences for consumption:

$$\log(z_t^u) = (1 - \lambda_u)\log(z_{ss}^u) + \lambda_u \log(z_{t-1}^u) + \epsilon_{u,t}\sigma_u. \tag{389}$$

Inventory shock:

$$\log(z_{x,t}) = (1 - \lambda_{wedge})\log(z_{x,ss}) + \lambda_{wedge}\log(z_{x,t-1}) + \epsilon_{wedge,t}\sigma_{wedge}. \tag{390}$$

Shock to foreign money market interest rates (in gap-form):

$$\widehat{z_{R*,t}} = \lambda_{R*}\widehat{z_{R*,t-1}} + \epsilon_{R*,t}\sigma_{R*}. \tag{391}$$

Shock to output abroad, trading partners (in gap-form):

$$\widehat{z_{U*,t}} = \lambda_{U*}\widehat{z_{U*,t-1}} + \epsilon_{U*,t}\sigma_{U*}. \tag{392}$$

Shock to output abroad, non-trading partners (in gap-form):

$$z_{\widehat{YNTP},t} = \epsilon_{YNTP,t}\sigma_{YNTP}. \tag{393}$$

Shock to inflation abroad (in gap-form):

$$\widehat{z_{\theta^{H*},t}} = \lambda_{\theta^{H*}}\widehat{z_{\theta^{H*},t-1}} + \epsilon_{\theta^{H*},t}\sigma_{\theta^{H*}}. \tag{394}$$

Shock to marginal costs abroad:

$$\log(\widetilde{MC}^*_t) = (1-\lambda_{MC*})\log(\widetilde{MC}^*_{ss}) + \lambda_{MC*}\log(\widetilde{MC}^*_{t-1}) + \epsilon_{MC*,t}\sigma_{MC*}. \tag{395}$$

17 The steady state of the model

This chapter derives the steady state of NEMO.

17.1 Setting exogneous parameters

We start with some calibrated steady-state values. As the Norwegian inflation target has up until recently been 2.5%:

$$\pi_{ss} = (1.025)^{\frac{1}{4}} = \pi_t^M = \pi_{ss}^{M*} = \pi_t^Q. \tag{396}$$

Weighted foreign inflation is assumed to be 2%:

$$\pi^*_{ss} = (1.02)^{\frac{1}{4}}. \tag{397}$$

Average trend growth (technology growth) in Norway (and abroad) is assumed to be 1%:

$$\pi^z_{ss} = (1.01)^{\frac{1}{4}}. \tag{398}$$

Nominal wage growth must be the product of inflation and technology growth:

$$\pi_{W,ss} = \pi_{WI,ss} = \pi_{WO,ss} = \pi_{ss}\pi^z_{ss}. \tag{399}$$

House prices are assumed to grow by approximately 4.6 % above inflation (annualized), which leads to:

$$\pi^h_{ss} = (1.0460)^{\frac{1}{4}}. \tag{400}$$

The weighted foreign interest rate is assumed to be 3.44 % on annual basis:

$$R^*_{ss} = \frac{\pi^*_{ss}\pi^z_{ss}}{\beta^*} \approx (1.0344)^{\frac{1}{4}}. \tag{401}$$

Government's share of final goods is assumed to be $100 \cdot og$%:

$$og = \frac{\widetilde{G}_{ss}}{\widetilde{A}_{ss}} = calibrated. \tag{402}$$

The share of final goods used as input to the oil supply sector, is assumed to be $100 \cdot oq$%:

$$oq = \frac{\widetilde{S}_{ss}\widetilde{P}^{O*}_{ss}\widetilde{Y}_{O,ss}}{\widetilde{A}_{ss}} = calibrated. \tag{403}$$

Maximum loan-to-value ratio for entrepreneurs is assumed to be $100 \cdot LTV^e_{ss}$%:

$$LTV^e_{ss} = calibrated. \tag{404}$$

Maximum loan-to-value ratio for households is assumed to be $100 \cdot LTV_{ss}\%$:

$$LTV_{ss} = calibrated. \tag{405}$$

The stationary real price level of houses:

$$\widetilde{P}^H_{ss} = 1. \tag{406}$$

17.2 Solution

From the UIP condition (302) we get that

$$R_{ss} = \frac{R^*_{ss}}{\pi^*_{ss}}\pi_{ss}. \tag{407}$$

From definition of the policy rate (327) we have that

$$R_{P,ss} = \frac{R_{ss}}{Z_{prem,ss}}. \tag{408}$$

From (189) we can see that the deposit branch sets the deposit rate as a mark-down on the money market rate in steady-state (as $\theta^D_{ss} < 0$):

$$r^d_{ss} = \frac{\theta^D_{ss}}{\theta^D_{ss} - 1} r_{ss}. \tag{409}$$

From equations (184) and (185) we can find the steady-state values of the wholesale lending rates ($R^{b,h}_{ss}$, $R^{b,e}_{ss}$). Then from (187) we see that the loan branch sets the loan rate to households as a mark up on the wholesale rate, $R^{b,h}_{ss}$, in steady-state:

$$r^F_{ss} = \frac{\theta^{IH}_{ss}}{\theta^{IH}_{ss} - 1} r^{b,h}_{ss}. \tag{410}$$

From (188) we see that loan branch sets the loan rate to entrepreneurs as a mark up on the wholesale rate, $R^{b,e}_{ss}$, in steady-state:

$$r^e_{ss} = \frac{\theta^e_{ss}}{1 - \theta^e_{ss}} r^{b,e}_{ss}. \tag{411}$$

From the optimal total investment condition (134) we get that

$$\widetilde{P}^K_{ss} = \frac{1}{\overline{K}'_{ss}} = 1. \tag{412}$$

Next, we want to find $\widetilde{R}_{K,ss}$, i.e. the real return on capital, but first we need to find the δ^e_{ss} and ϕ^{ent}_{ss}. By using (122) we can find δ^e_{ss} by fixed-point iterations, and by using the assumed value of $LTV^e_{ss} \equiv \frac{P_{ss}B_{e,ss}}{P^K_{ss}(1-\delta)K_{ss}}$ and (121) we get the steady-state loan to value ratio:

$$LTV^e_{ss} = \left[1 - \frac{(1-\delta^e_{ss})}{1+\phi^{ent}_{ss}} \frac{1}{\pi_{ss}\pi^z_{ss}}\right]^{-1} \frac{\phi^{ent}_{ss}}{1+\phi^{ent}_{ss}}\pi_{ss} \tag{413}$$

$$\Longleftrightarrow$$

$$LTV^e_{ss} = \left[\left(1 - \frac{1}{1+\phi^{ent}_{ss}} \frac{(1-\delta^e_{ss})}{\pi_{ss}\pi^z_{ss}}\right) \frac{1+\phi^{ent}_{ss}}{\phi^{ent}_{ss}\pi_{ss}}\right]^{-1}, \tag{414}$$

$$1 - LTV^e_{ss}\left(\frac{1+\phi^{ent}_{ss} - \frac{(1-\delta^e_{ss})}{\pi_{ss}\pi^z_{ss}}}{\phi^{ent}_{ss}\pi_{ss}}\right) = 0, \tag{415}$$

which means that we can find ϕ^{ent}_{ss} by fixed-point iterations of (415) until convergence. From the first-order condition w.r.t. δ^e_t (116) we get that

$$\widetilde{\omega}^e_{ss}\widetilde{B}_{e,ss} = \mu^{HELP1}_e \widetilde{\mu}^e_{ss}, \tag{416}$$

where (remember that we can find the steady-state value of the stochastic discount factor from (40)), i.e.

$$\mu_e^{HELP1} = \left[\frac{\left(\Lambda_{ss}\delta^{e,prime} - 1\right)\left(1 + \phi_{ss}^{ent}\right)}{\Lambda_{ss}}\right], \tag{417}$$

where we define

$$\delta^{e,prime} = \alpha^e \left(\delta_{ss}^e\right)^{\alpha^e - 1}\left(1 - \delta_{ss}^e\right) - \left(\delta_{ss}^e\right)^{\alpha^e} + \left(1 - \alpha^e\right)^{\kappa^e}. \tag{418}$$

Rearrange the FOC w.r.t. $B_{e,t}$ (115):

$$\begin{gathered}\left[1 - \Lambda_{ss}R_{ss}^e\right]\widetilde{B}_{e,ss} + \left[\Lambda_{ss}\left[\frac{(1-\delta_{ss}^e)}{1+\phi_{ss}^{ent}}\right] - 1\right]\widetilde{\omega}_{ss}^e\widetilde{B}_{e,ss}, \\ -\frac{(1-\delta_{ss}^e)}{\pi_{ss}\pi_{ss}^z}\left[\left(\delta_{ss}^e\right)^{\alpha^e} - \left(1-\alpha^e\right)^{\kappa^e}\right]\widetilde{\mu}_{ss}^e + \Lambda_{ss}(1-\delta_{ss}^e)\left[\left(\delta_{ss}^e\right)^{\alpha^e} - \left(1-\alpha^e\right)^{\kappa^e}\right]\widetilde{\mu}_{ss}^e = 0 \\ \Longleftrightarrow \\ \left[1 - \Lambda_{ss}R_{ss}^e\right]\widetilde{B}_{e,ss} + \mu_e^{HELP2}\widetilde{\omega}_{ss}^e\widetilde{B}_{e,ss} + \mu_e^{HELP3}\widetilde{\mu}_{ss}^e = 0,\end{gathered} \tag{419}$$

where

$$\mu_e^{HELP2} = \left[\Lambda_{ss}\left[\frac{(1-\delta_{ss}^e)}{1+\phi_{ss}^{ent}}\right] - 1\right], \tag{420}$$

$$\mu_e^{HELP3} = \left[\Lambda_{ss}\pi_{ss}\pi_{ss}^z - 1\right]\left[\delta_{ss}^e - \left(1-\alpha^e\right)^{\kappa^e}\right], \tag{421}$$

and we have used that

$$\delta_{ss}^e - \left(1-\alpha^e\right)^{\kappa^e} = \frac{1}{\pi_{ss}\pi_{ss}^z}\left(1-\delta_{ss}^e\right)\left[\left(\delta_{ss}^e\right)^{\alpha^e} - \left(1-\alpha^e\right)^{\kappa^e}\right]. \tag{422}$$

Insert for $\widetilde{\omega}_{ss}^e\widetilde{B}_{e,ss}$ from (416) into (419) to get that

$$\begin{gathered}\left[1 - \Lambda_{ss}R_{ss}^e\right]\widetilde{B}_{e,ss} + \mu_e^{HELP2}\mu_e^{HELP1}\widetilde{\mu}_{ss}^e + \mu_e^{HELP3}\widetilde{\mu}_{ss}^e = 0 \\ \Longleftrightarrow \\ \widetilde{\mu}_{ss}^e = \left[\mu_e^{HELP2}\mu_e^{HELP1} + \mu_e^{HELP3}\right]^{-1}\left[1 - \Lambda_{ss}R_{ss}^e\right]\widetilde{B}_{e,ss}.\end{gathered} \tag{423}$$

Insert this into (416):

$$\widetilde{\omega}_{ss}^e = \frac{\mu_e^{HELP1}}{\mu_e^{HELP2}\mu_e^{HELP1} + \mu_e^{HELP3}}\left[\Lambda_{ss}R_{ss}^e - 1\right]. \tag{424}$$

Then by using (40), (424), δ_{ss}^e, ϕ_{ss}^{ent} and FOC w.r.t. $\widetilde{K}_t$, (114) we get that

$$\widetilde{R}_{K,ss} = \left[1 - (1-\delta)\,\pi_{ss}\left(\widetilde{\omega}_{ss}^e\frac{\phi_{ss}^{ent}}{1+\phi_{ss}^{ent}} + \Lambda_{ss}\right)\right]\frac{\widetilde{P}_{ss}^K}{\Lambda_{ss}\pi_{ss}}. \tag{425}$$

From (117):

$$\gamma_{ss}'(u_{ss}) = \widetilde{R}_{K,ss}. \tag{426}$$

Insert the demand function for Q_t (93) and M_t (94) into the final goods production function (92) to find the steady-state value of real domestic prices (using $\widetilde{P}_{ss}^M = 1$):

$$\begin{gathered}\widetilde{A}_{ss} = \left[\nu_{ss}^{\frac{1}{\mu}}\left(\nu_{ss}\left(\widetilde{P}_{ss}^Q\right)^{-\mu}\widetilde{A}_{ss}\right)^{1-\frac{1}{\mu}} + (1-\nu_{ss})^{\frac{1}{\mu}}\left((1-\nu_{ss})\left(\widetilde{P}_{ss}^M\right)^{-\mu}\widetilde{A}_{ss}\right)^{1-\frac{1}{\mu}}\right]^{\frac{\mu}{\mu-1}} \\ \Longleftrightarrow \\ 1 = \left[\nu_{ss}\left(\widetilde{P}_{ss}^Q\right)^{1-\mu} + (1-\nu_{ss})\left(\widetilde{P}_{ss}^M\right)^{1-\mu}\right]^{\frac{\mu}{\mu-1}} \\ \Longleftrightarrow \\ \widetilde{P}_{ss}^Q = \left(\frac{1-(1-\nu_{ss})\left(\widetilde{P}_{ss}^M\right)^{1-\mu}}{\nu_{ss}}\right)^{\frac{1}{1-\mu}}.\end{gathered} \tag{427}$$

From the FOC of optimal domestic price setting (78):

$$\widetilde{MC}_{ss} = \frac{\theta_{ss}^H - 1}{\theta_{ss}^H}\widetilde{P}_{ss}^Q. \tag{428}$$

From the FOC of optimal imported price setting (263):

$$\widetilde{S}_{ss} = \frac{\theta^F - 1}{\theta^F} \widetilde{P}^M_{ss} \frac{1}{\widetilde{MC}^*_{ss}}. \tag{429}$$

From the FOC of optimal exported price setting (79):

$$\widetilde{P}^{M*}_{ss} = \frac{\theta^{F*}_{ss}}{\theta^{F*}_{ss} - 1} \frac{\widetilde{MC}_{ss}}{\widetilde{S}_{ss}}. \tag{430}$$

Using the intermediate goods production function (75), demand for labor (76) and the demand for utilized capital (77):

$$\widetilde{T}_{ss} = \left[(1-\alpha)^{\frac{1}{\xi}} \left(z^L_{ss} L_{I,ss} \right)^{1-\frac{1}{\xi}} + \alpha^{\frac{1}{\xi}} \left(\widetilde{K}_{I,ss} \right)^{1-\frac{1}{\xi}} \right]^{\frac{\xi}{\xi-1}}. \tag{431}$$

$$L_{I,ss} = (1-\alpha) \left(z^L_{ss} \right)^{\xi-1} \left(\frac{\widetilde{MC}_{ss}}{\widetilde{W}_{I,ss}} \right)^{\xi} \widetilde{T}_{ss}. \tag{432}$$

$$\widetilde{K}_{I,ss} = \alpha \left(\frac{\widetilde{MC}_{ss}}{\widetilde{R}_{K,ss}} \right)^{\xi} \widetilde{T}_{ss}. \tag{433}$$

Insert (433) and (432) into (431) to get:

$$\widetilde{T}_{ss} = \left[\begin{array}{l} (1-\alpha)^{\frac{1}{\xi}} \left(z^l_{ss} (1-\alpha) \left(z^l_{ss} \right)^{\xi-1} \left(\frac{\widetilde{MC}_{ss}}{\widetilde{W}_{I,ss}} \right)^{\xi} \widetilde{T}_{ss} \right)^{\frac{\xi-1}{\xi}} \\ +\alpha^{\frac{1}{\xi}} \left(\alpha \left(\frac{\widetilde{MC}_{ss}}{\widetilde{R}_{K,ss}} \right)^{\xi} \widetilde{T}_{ss} \right)^{\frac{\xi-1}{\xi}} \end{array} \right]^{\frac{\xi}{\xi-1}}$$

$$\Longleftrightarrow$$

$$1 = \left[\begin{array}{l} (1-\alpha)^{\frac{1}{\xi}} \left((1-\alpha) \left(z^l_{ss} \frac{\widetilde{MC}_{ss}}{\widetilde{W}_{I,ss}} \right)^{\xi} \right)^{\frac{\xi-1}{\xi}} \\ +\alpha^{\frac{1}{\xi}} \left(\alpha \left(\frac{\widetilde{MC}_{ss}}{\widetilde{R}_{K,ss}} \right)^{\xi} \right)^{\frac{\xi-1}{\xi}} \end{array} \right]^{\frac{\xi}{\xi-1}}$$

$$\Longleftrightarrow$$

$$1 = \left[(1-\alpha) \left(\frac{z^l_{ss}}{\widetilde{W}_{I,ss}} \right)^{\xi-1} + \alpha \left(\frac{1}{\widetilde{R}_{K,ss}} \right)^{\xi-1} \right] \left(\widetilde{MC}_{ss} \right)^{\xi-1}$$

$$\Longleftrightarrow$$

$$(1-\alpha) \left(\frac{\widetilde{W}_{I,ss}}{z^l_{ss}} \right)^{1-\xi} = \left(\widetilde{MC}_{ss} \right)^{1-\xi} - \alpha \left(\widetilde{R}_{K,ss} \right)^{1-\xi} \tag{434}$$

$$\Longleftrightarrow$$

$$\widetilde{W}_{I,ss} = \left(\frac{\left(\widetilde{MC}_{ss} \right)^{1-\xi} - \alpha \left(\widetilde{R}_{K,ss} \right)^{1-\xi}}{1-\alpha} \right)^{\frac{1}{1-\xi}} z^l_{ss}.$$

Where $z^L_{ss} = 1$. From the wage setting rule by households (47):

$$\widetilde{W}_{I,ss} = \widetilde{W}_{ss} = \frac{\psi_{ss}}{\psi_{ss} - 1} MRS(L_{ss}, \widetilde{C}_{ss}), \tag{435}$$

At this stage we know $\widetilde{W}_{ss}$ (from (435)):

$$MRS(L_{ss}, \widetilde{C}_{ss}) = \widetilde{W}_{ss} \frac{\psi_{ss} - 1}{\psi_{ss}}. \tag{436}$$

To proceed, we need to switch our attention to the oil sector. First we assume that $Z_{R,ss} = 1$, $Z_{O,ss} = 1$, $\widetilde{P}^{O*}_{ss} = 1$, $\widetilde{Y}_{O*,ss} = 1$, $O_{ss} = 0.1011$. Given that $\widetilde{Q}_{O,t}$ is bought from the final goods sector we have that

$$\widetilde{P}^{QO}_{ss} = 1. \tag{437}$$

From the production of rigs (256) we get:

$$\frac{\widetilde{F}_{O,ss}}{\widetilde{I}_{O,ss}} = \frac{\pi^z_{ss}}{\pi^z_{ss} - 1 + \delta^O}. \tag{438}$$

From the production function of the extraction firm (247) and using that $\widetilde{F}_{O,ss} = \widetilde{\widetilde{F}}_{O,ss}\pi^z_{ss}$ (249):

$$\widetilde{Y}_{O,ss} = Z_{O,ss}\left(\frac{\widetilde{F}_{O,ss}}{\pi^z_{ss}}\right)^{\alpha_o}(O_{ss})^{1-\alpha_o}. \tag{439}$$

From the FOC of extraction firms w.r.t. F^O_t (250), remember that we know Λ_{ss} and $\widetilde{S}_{ss}$:

$$\widetilde{\Omega}_{O,ss} = \pi_{ss}\Lambda_{ss}\left(\alpha_o\widetilde{S}_{ss}\widetilde{P}^{O*}_{ss}\pi^z_{ss}\frac{\widetilde{Y}_{O,ss}}{\widetilde{F}_{O,ss}} + (1-\delta_O)\widetilde{\Omega}_{O,ss}\right) \tag{440}$$

$$\Longrightarrow$$

$$\frac{\widetilde{Y}_{O,ss}}{\widetilde{F}_{O,ss}} = \widetilde{\Omega}_{O,ss}\frac{\left(\delta_O + \frac{1}{\pi_{ss}\Lambda_{ss}} - 1\right)}{\alpha_o\widetilde{S}_{ss}\widetilde{P}^{O*}_{ss}\pi^z_{ss}}. \tag{441}$$

From the FOC of extraction firms w.r.t. $I_{O,t}$ (252):

$$\widetilde{P}^R_{ss} = \widetilde{\Omega}_{O,ss}. \tag{442}$$

From the FOC of extraction firms w.r.t. $U_{F,t}$ (254):

$$\frac{\widetilde{Y}_{O,ss}}{\widetilde{F}_{O,ss}} = \frac{1}{\alpha_o\widetilde{S}_{ss}\widetilde{P}^{O*}_{ss}\pi^z_{ss}}\widetilde{P}^R_{ss}a^{'}(U_{F,ss}). \tag{443}$$

$$\Longrightarrow$$

$$\frac{\widetilde{Y}_{O,ss}}{\widetilde{F}_{O,ss}} = \frac{\widetilde{\Omega}_{O,ss}a^{'}(U_{F,ss})}{\alpha_o\widetilde{S}_{ss}\widetilde{P}^{O*}_{ss}\pi^z_{ss}}. \tag{444}$$

Combining (441) and (444) we get that

$$a^{'}(U_{F,ss}) = \left(\delta_O + \frac{1}{\pi_{ss}\Lambda_{ss}} - 1\right). \tag{445}$$

From the supply firm production function (240):

$$\widetilde{Y}_{R,ss} = (\widetilde{Q}_{O,ss})^{\alpha_q}(L_{O,ss})^{\alpha_l}(\widetilde{\widetilde{K}}_{O,ss})^{1-\alpha_q-\alpha_l}. \tag{446}$$

From (241), we get that marginal costs is given by

$$\widetilde{MC}_{R,ss} = \left(\frac{\widetilde{P}^{QO}_{ss}}{\alpha_q}\right)^{\alpha_q}\left(\frac{\widetilde{W}_{O,ss}}{\alpha_l}\right)^{\alpha_l}\left(\frac{\widetilde{R}_{KO,ss}}{1-\alpha_q-\alpha_l}\right)^{1-\alpha_q-\alpha_l}. \tag{447}$$

Supply firm factor demands ((242), (243) and (244)):

$$\frac{\widetilde{Q}_{O,ss}}{\widetilde{Y}_{R,ss}} = \alpha_q\left(\frac{\widetilde{P}^{QO}_{ss}}{\widetilde{MC}_{R,ss}}\right)^{-1}, \tag{448}$$

$$\frac{L_{O,ss}}{\widetilde{Y}_{R,ss}} = \alpha_l\left(\frac{\widetilde{W}_{O,ss}}{\widetilde{MC}_{R,ss}}\right)^{-1}, \tag{449}$$

$$\frac{\widetilde{\widetilde{K}}_{O,ss}}{\widetilde{Y}_{R,ss}} = (1-\alpha_q-\alpha_l)\left(\frac{\widetilde{R}_{KO,ss}}{\widetilde{MC}_{R,ss}}\right)^{-1}. \tag{450}$$

Then, from market clearing in the oil supply goods market ((255) and (326)):

$$\widetilde{I}_{OF,ss} = \widetilde{I}_{O,ss}, \tag{451}$$

$$\widetilde{Y}_{R,ss} = \widetilde{I}_{OF,ss} + \widetilde{M}_{O*,ss}. \tag{452}$$

From perfect factor mobility we get that

$$\widetilde{W}_{O,ss} = \widetilde{W}_{ss}. \tag{453}$$

Now we have 4 unknowns and 4 equations (446), (448), (449) and (450):

$$1 = \left(\frac{\widetilde{Q}_{O,ss}}{\widetilde{Y}_{R,ss}}\right)^{\alpha_q} \left(\frac{L_{O,ss}}{\widetilde{Y}_{R,ss}}\right)^{\alpha_l} \left(\frac{\widetilde{\widetilde{K}}_{O,ss}}{\widetilde{Y}_{R,ss}}\right)^{1-\alpha_q-\alpha_l}, \tag{454}$$

$$1 = \left(\alpha_q \frac{\widetilde{MC}_{R,ss}}{\widetilde{P}^{QO}_{ss}}\right)^{\alpha_q} \left(\alpha_l \frac{\widetilde{MC}_{R,ss}}{\widetilde{W}_{O,ss}}\right)^{\alpha_l} \left((1-\alpha_q-\alpha_l)\frac{\widetilde{MC}_{R,ss}}{\widetilde{R}_{KO,ss}}\right)^{1-\alpha_q-\alpha_l}, \tag{455}$$

$$1 = \alpha_q^{\alpha_q}\alpha_l^{\alpha_l}\left(1-\alpha_q-\alpha_l\right)^{(1-\alpha_q-\alpha_l)} \left(\widetilde{P}^{QO}_{ss}\right)^{-\alpha_q} \left(\widetilde{W}_{O,ss}\right)^{-\alpha_l} \left(\widetilde{R}_{KO,ss}\right)^{-(1-\alpha_q-\alpha_l)} \widetilde{MC}_{R,ss}, \tag{456}$$

$$\widetilde{MC}_{R,ss} = \alpha_q^{-\alpha_q}\alpha_l^{-\alpha_l}\left(1-\alpha_q-\alpha_l\right)^{\alpha_q+\alpha_l-1} \left(\widetilde{P}^{QO}_{ss}\right)^{\alpha_q} \left(\widetilde{W}_{O,ss}\right)^{\alpha_l} \left(\widetilde{R}_{KO,ss}\right)^{1-\alpha_q-\alpha_l}, \tag{457}$$

We can then solve for $\widetilde{MC}_{R,ss}$, $\frac{\widetilde{Q}_{O,ss}}{\widetilde{Y}_{R,ss}}$, $\frac{L_{O,ss}}{\widetilde{Y}_{R,ss}}$ and $\frac{\widetilde{\widetilde{K}}_{O,ss}}{\widetilde{Y}_{R,ss}}$. From (245) and (246):

$$\widetilde{P}^{R}_{ss} = \widetilde{MC}_{R,ss}\frac{\theta^R}{(\theta^R-1)}, \tag{458}$$

$$\widetilde{P}^{R^*}_{ss} = \frac{\widetilde{MC}_{R,ss}}{S_{ss}}\frac{\theta^{R^*}}{\left(\theta^{R^*}-1\right)}. \tag{459}$$

From (441) we get (since $\widetilde{P}^R_{ss} = \widetilde{\Omega}_{O,ss}$) that

$$\frac{\widetilde{F}_{O,ss}}{\widetilde{Y}_{O,ss}} = \left[\widetilde{P}^R_{ss}\frac{a'\left(U_{F,ss}\right)}{\alpha_o\widetilde{S}_{ss}\widetilde{P}^{O*}_{ss}\pi^z_{ss}}\right]^{-1}. \tag{460}$$

Then by using (438):

$$\frac{\widetilde{I}_{O,ss}}{\widetilde{Y}_{O,ss}} = (\pi^z_{ss} - 1 + \delta^O)\frac{\widetilde{F}_{O,ss}}{\widetilde{Y}_{O,ss}}. \tag{461}$$

We know $\widetilde{S}_{ss}$, $\widetilde{P}^{O*}_{ss}$, $\widetilde{P}^{R*}_{ss}$ and we set $\widetilde{Y}_{O*,ss}$ so from the demand from extraction firms abroad (258) we get that

$$\widetilde{M}_{O*,ss} = a_{o*}\left(\frac{\widetilde{P}^{R*}_{ss}}{\widetilde{P}^{O*}_{ss}}\right)^{-1}\widetilde{Y}_{O*,ss}. \tag{462}$$

From (439), we find that

$$\widetilde{Y}_{O,ss} = \left[Z_{O,ss}\left(\frac{\widetilde{F}_{O,ss}}{\pi^z_{ss}\widetilde{Y}_{O,ss}}\right)^{\alpha_o}\right]^{\frac{1}{1-\alpha_o}} O_{ss}, \tag{463}$$

which means we can back out $\widetilde{F}_{O,ss}$, $\widetilde{I}_{O,ss}$ and get $\widetilde{\widetilde{F}}_{O,ss}$ from:

$$\widetilde{\widetilde{F}}_{O,ss} = \frac{\widetilde{F}_{O,ss}}{\pi^z_{ss}}. \tag{464}$$

From (452) we can then back out $\widetilde{Y}_{R,ss}$, and we can therefore solve for $\widetilde{\widetilde{K}}_{O,ss}$, $L_{O,ss}$ and $\widetilde{Q}_{O,ss}$. From the definition of oq (403), we get that

$$\widetilde{A}_{ss} = \frac{\widetilde{S}_{ss}\widetilde{P}^{O*}_{ss}\widetilde{Y}_{O,ss}}{oq}. \tag{465}$$

Imports ($\widetilde{M}_{ss}$) and $\widetilde{Q}_{ss}$ can then be found from eq. (94), (93) and (427) ($\widetilde{P}^M_{ss}$ is calibrated to 1):

$$\widetilde{M}_{ss} = (1-\nu_{ss})\left(\widetilde{P}^{M}_{ss}\right)^{-\mu}\widetilde{A}_{ss},$$
$$\widetilde{Q}_{ss} = \nu_{ss}\left(\widetilde{P}^{Q}_{ss}\right)^{-\mu}\widetilde{A}_{ss}.$$

To find traditional exports ($\widetilde{M}^*_{ss}$) we use the trade balance. Using eq. (304) and (261) in (303), setting $\widetilde{B}^{TOT*}_{ss}=0$, gives:

$$\widetilde{B}^{TOT*}_{ss} = \widetilde{B}^{*}_{ss} - \widetilde{B}_{F,ss}$$
$$\Leftrightarrow$$
$$\widetilde{B}^{TOT*}_{ss} = \left[1-\frac{R_{ss}}{\pi^z_{ss}\pi_{ss}}\right]^{-1}\left[\widetilde{P}^{M}_{ss}\widetilde{M}_{ss} - \widetilde{S}_{ss}\widetilde{P}^{R^*}_{ss}\widetilde{M}_{O*,ss} - \widetilde{S}_{ss}\widetilde{P}^{M*}_{ss}\widetilde{M}^*_{ss} - \widetilde{P}^{R}_{ss}\widetilde{I}_{OF,ss} - \widetilde{G}_{F,ss}\right]$$
$$-\left[1-\frac{R^*_{ss}}{\pi^*_{ss}\pi^z_{ss}}\right]^{-1}\left[\widetilde{S}_t\widetilde{P}^{O*}_{ss}\widetilde{Y}_{O,ss} - \widetilde{G}_{F,ss}\right] = 0$$
$$\Leftrightarrow \text{using } \frac{R^*_{ss}}{\pi^*_{ss}} = \frac{R_{ss}}{\pi_{ss}}$$
$$\widetilde{P}^{M}_{ss}\widetilde{M}_{ss} - \widetilde{S}_{ss}\widetilde{P}^{R^*}_{ss}\widetilde{M}_{O*,ss} - \widetilde{S}_{ss}\widetilde{P}^{M*}_{ss}\widetilde{M}^*_{ss} - \widetilde{P}^{R}_{ss}\widetilde{I}_{OF,ss} = \widetilde{S}_t\widetilde{P}^{O*}_{ss}\widetilde{Y}_{O,ss}$$
$$\Leftrightarrow$$
$$\widetilde{M}^*_{ss} = \left[\widetilde{P}^{M}_{ss}\widetilde{M}_{ss} - \widetilde{S}_{ss}\widetilde{P}^{R^*}_{ss}\widetilde{M}_{O*,ss} - \widetilde{P}^{R}_{ss}\widetilde{I}_{OF,ss} - \widetilde{S}_t\widetilde{P}^{O*}_{ss}\widetilde{Y}_{O,ss}\right]\frac{1}{\widetilde{S}_{ss}\widetilde{P}^{M*}_{ss}}.$$

From the demand for export (265), we must have:

$$\nu^*_{ss} = 1 - \frac{\widetilde{M}^*_{ss}\left(\widetilde{P}^{M*}_{ss}\right)^{\mu^*}}{\widetilde{Y}_{NAT*,ss}}. \quad (466)$$

We can then substitute $\widetilde{M}^*_{ss}$ into the market clearing equation for the intermediate good (321) to find $\widetilde{T}_{ss}$, i.e.

$$\widetilde{T}_{ss} = \widetilde{Q}_{ss} + \widetilde{M}^*_{ss}. \quad (467)$$

As we now have $\widetilde{T}_{ss}$, we can use (432) and (433) to find $L_{I,ss}$ and $\widetilde{\overline{K}}_{I,ss}$ respectively. So from (324) we get that

$$\widetilde{\overline{K}}_{ss} = \widetilde{\overline{K}}_{I,ss} + \widetilde{\overline{K}}_{O,ss}. \quad (468)$$

By the definitions of utilized capital (119) we then have

$$\widetilde{K}_{ss} = \widetilde{\overline{K}}_{ss}\pi^z_{ss}, \quad (469)$$

i.e. we have found the aggregate capital stock in steady-state. Then, by using the expression for capital accumulation (132) and the definition of capital adjustment costs (133):

$$\widetilde{K}_{ss} = \frac{(1-\delta)}{\pi^z_{ss}}\widetilde{K}_{ss} + \widetilde{I}_{C,ss}$$
$$\Longleftrightarrow$$
$$\widetilde{I}_{C,ss} = \left[1-\frac{(1-\delta)}{\pi^z_{ss}}\right]\widetilde{K}_{ss}. \quad (470)$$

Then by using this and (133), we get that

$$\kappa_{ss} = \frac{\widetilde{I}_{C,ss}\pi^z_{ss}}{\widetilde{K}_{ss}} = \pi^z_{ss} - 1 - \delta.$$

As we now know both $L_{I,ss}$ (from 432) and $L_{O,ss}$ (from 243), we can find aggregate labor from (325):

$$L_{ss} = L_{I,ss} + L_{O,ss}. \tag{471}$$

To proceed, we can use the definition of MRS (48):

$$\begin{aligned} MRS(L_{ss}, \widetilde{C}_{ss}) &= \tfrac{v'(L_{ss})}{\widetilde{u}'(\widetilde{C}_{ss})} \\ &\Longleftrightarrow \\ \widetilde{u}'\left(\widetilde{C}_{ss}\right) &= \tfrac{v'(L_{ss})}{MRS(L_{ss},\widetilde{C}_{ss})}. \end{aligned} \tag{472}$$

By using the calibrated value of z^u_{ss} and (36), we can find $\widetilde{C}_{ss}$:

$$\widetilde{C}_{ss} = \frac{\widetilde{u}'\left(\widetilde{C}_{ss}\right)}{z^u_{ss}}. \tag{473}$$

The government expenditure can be found by using the definition of og (402):

$$\widetilde{G}_{ss} = og\widetilde{A}_{ss}. \tag{474}$$

We now switch our attention to the households FOC's. By using (50), we can find δ^B_{ss} by fixed-point iterations, and by using the definition of $LTV_{ss} \equiv \frac{P_{ss}B_{h,ss}}{P^H_{ss}H_{ss}}$ and (49), we get the loan to value ratio in steady-state:

$$LTV_{ss} = \left[1 - \frac{(1-\delta^B_{ss})}{1+\phi_{ss}} \frac{1}{\pi_{ss}\pi^z_{ss}}\right]^{-1} \frac{\phi_{ss}}{1+\phi_{ss}} \pi_{ss}\pi^h_{ss} \tag{475}$$

$$\Longleftrightarrow$$

$$LTV_{ss} = \left[\left(1 - \frac{1}{1+\phi_{ss}} \frac{(1-\delta^B_{ss})}{\pi_{ss}\pi^z_{ss}}\right) \frac{1+\phi_{ss}}{\phi_{ss}\pi_{ss}\pi^h_{ss}}\right]^{-1}, \tag{476}$$

$$1 - LTV_{ss}\left(\frac{1+\phi_{ss} - \frac{(1-\delta^B_{ss})}{\pi_{ss}\pi^z_{ss}}}{\phi_{ss}\pi_{ss}\pi^h_{ss}}\right) = 0. \tag{477}$$

We can find ϕ_{ss} by fixed-point iterations of (477) until convergence. From the first-order condition w.r.t. δ^B_t (44), we get that

$$\widetilde{\omega}_{ss}\widetilde{B}_{h,ss} = \mu^{HELP1}\widetilde{\mu}_{ss}, \tag{478}$$

where, remember that we can find the steady-state value of the stochastic discount factor from (40):

$$\mu^{HELP1} = \left[\frac{\left(\Lambda_{ss}\delta^{prime} - 1\right)(1+\phi_{ss})}{\Lambda_{ss}}\right], \tag{479}$$

where we define

$$\delta^{prime} = \alpha^h \left(\delta^B_{ss}\right)^{\alpha^h - 1} (1-\delta^B_{ss}) - \left(\delta^B_{ss}\right)^{\alpha^h} + (1-\alpha^h)^{\kappa^h}. \tag{480}$$

Rearrange the FOC w.r.t. $\widetilde{B}_{h,t}$ (41):

$$\begin{gathered} \left[1 - \Lambda_{ss}R^F_{ss}\right]\widetilde{B}_{h,ss} + \left[\Lambda_{ss}\left[\tfrac{(1-\delta^B_{ss})}{1+\phi_{ss}}\right] - 1\right]\widetilde{\omega}_{ss}\widetilde{B}_{h,ss} \\ -\tfrac{(1-\delta^B_{ss})}{\pi_{ss}\pi^z_{ss}}\left[\left(\delta^B_{ss}\right)^{\alpha^h} - \left(1-\alpha^h\right)^{\kappa^h}\right]\widetilde{\mu}_{ss} + \Lambda_{ss}(1-\delta^B_{ss})\left[\left(\delta^B_{ss}\right)^{\alpha^h} - \left(1-\alpha^h\right)^{\kappa^h}\right]\widetilde{\mu}_{ss} = 0 \\ \Longleftrightarrow \\ \left[1 - \Lambda_{ss}R^F_{ss}\right]\widetilde{B}_{h,ss} + \mu^{HELP2}\widetilde{\omega}_{ss}\widetilde{B}_{h,ss} + \mu^{HELP3}\widetilde{\mu}_{ss} = 0, \end{gathered} \tag{481}$$

where

$$\mu^{HELP2} = \left[\Lambda_{ss}\left[\frac{(1-\delta^B_{ss})}{1+\phi_{ss}}\right] - 1\right], \tag{482}$$

$$\mu^{HELP3} = \left[\Lambda_{ss}\pi_{ss}\pi^z_{ss} - 1\right]\left[\delta^B_{ss} - \left(1-\alpha^h\right)^{\kappa^h}\right], \tag{483}$$

and we have used that

$$\delta_{ss}^{B}-\left(1-\alpha^{h}\right)^{\kappa^{h}}=\frac{1}{\pi_{ss}\pi_{ss}^{z}}(1-\delta_{ss}^{B})\left[\left(\delta_{ss}^{B}\right)^{\alpha^{h}}-\left(1-\alpha^{h}\right)^{\kappa^{h}}\right]. \tag{484}$$

Insert for $\widetilde{\omega}_{ss}\widetilde{B}_{h,ss}$ from (478) into (481) to get:

$$\begin{gathered}\left[1-\Lambda_{ss}R_{ss}^{F}\right]\widetilde{B}_{h,ss}+\mu^{HELP2}\mu^{HELP1}\widetilde{\mu}_{ss}+\mu^{HELP3}\widetilde{\mu}_{ss}=0\\ \Longleftrightarrow\\ \widetilde{\mu}_{ss}=\left[\mu^{HELP2}\mu^{HELP1}+\mu^{HELP3}\right]^{-1}\left[1-\Lambda_{ss}R_{ss}^{F}\right]\widetilde{B}_{h,ss}.\end{gathered} \tag{485}$$

Insert this into (478) to get:

$$\widetilde{\omega}_{ss}=\frac{\mu^{HELP1}}{\mu^{HELP2}\mu^{HELP1}+\mu^{HELP3}}\left[\Lambda_{ss}R_{ss}^{F}-1\right]. \tag{486}$$

We have now the possibility to solve the steady state of the housing market. From market clearing in the final goods market (322) we can now find $\widetilde{I}_{H,ss}$:

$$\widetilde{I}_{H,ss}=\widetilde{A}_{ss}-\widetilde{C}_{ss}\ -\widetilde{I}_{C,ss}-\widetilde{G}_{ss}-\widetilde{Q}_{O,ss}. \tag{487}$$

We now have both $\widetilde{I}_{H,ss}$ and $\widetilde{I}_{C,ss}$, so $\widetilde{I}_{ss}$ can be found from (323). From the housing accumulation equation (145), the housing adjustment costs (146) and the solution of $\widetilde{I}_{H,ss}$ from (487) we have that

$$\widetilde{H}_{ss}=\left[1-\frac{\pi_{ss}^{h}(1-\delta_{H})}{\pi_{ss}^{z}}\right]^{-1}\widetilde{I}_{H,ss}, \tag{488}$$

which means that

$$\gamma_{H,ss}=\frac{\pi_{ss}^{z}}{\pi_{ss}^{h}}-1+\delta_{H}. \tag{489}$$

Given the assumption that the steady-state value of $\widetilde{P}_{ss}^{H}=1$, we use (40), (486), ϕ_{ss} and FOC w.r.t. to $\widetilde{H}_{t}$ (43) to find that

$$\widetilde{w}'(\widetilde{H}_{ss})=\widetilde{P}_{ss}^{H}\widetilde{u}'\left(\widetilde{C}_{ss}\right)\left[1-(1-\delta_{H})\Lambda_{ss}\pi_{ss}\pi_{ss}^{h}-\frac{\phi_{ss}}{1+\phi_{ss}}\widetilde{\omega}_{ss}\pi_{ss}\pi_{ss}^{h}\right], \tag{490}$$

i.e. (from eq. (39))

$$z_{ss}^{h}=\widetilde{w}'(\widetilde{H}_{ss})\widetilde{H}_{ss}. \tag{491}$$

From the FOC w.r.t. to housing (147), we then get that

$$z_{HS,ss}=\widetilde{P}_{ss}^{H}. \tag{492}$$

Now we can then find $\widetilde{B}_{h,ss}$ from the constraint on households loans (49) and therefore $\widetilde{\mu}_{ss}$ from (485). Likewise we can solve for $\widetilde{B}_{e,ss}$ from the constraint on entrepreneurs loans (121), and the fact that we now know ϕ_{ss}^{ent}, $\widetilde{K}_{I,ss}$ and $\widetilde{R}_{K,ss}$. Given this we can find $\widetilde{\mu}_{ss}^{e}$ from (423).

Then we turn to the banking sector. Total loans are given by (191):

$$\widetilde{B}_{ss}=\widetilde{B}_{e,ss}+\widetilde{B}_{h,ss}. \tag{493}$$

By using the FOC w.r.t. to deposits (42) and (37) and setting $z_{ss}^{d}=0.1889$:

$$\begin{gathered}\Lambda_{ss}R_{ss}^{d}=1-\frac{\widetilde{d}'(\widetilde{D}_{ss})}{\widetilde{u}'(\widetilde{C}_{ss})}\\ \Longleftrightarrow\\ \widetilde{d}'\left(\widetilde{D}_{ss}\right)=\widetilde{u}'\left(\widetilde{C}_{ss}\right)\left[1-\Lambda_{ss}R_{ss}^{d}\right]\\ \Longleftrightarrow\\ \widetilde{D}_{ss}=\frac{z_{ss}^{d}}{\widetilde{u}'(\widetilde{C}_{ss})[1-\Lambda_{ss}R_{ss}^{d}]}\end{gathered} \tag{494}$$

Now we can find the value of the risk-weighted lending by using (183). Then by the definition of the capital requirement and by given value of ϖ_{ss} (186) we get:

$$K_{ss}^{B}=\varpi_{ss}B_{s}^{RW} \tag{495}$$

By using total assets and bank capital, we can find banks' total external funding from (190) :

$$\widetilde{B}^{TOT}_{F,ss} = \widetilde{B}_{ss} - \widetilde{K}^{B}_{ss}. \tag{496}$$

Then by using this and deposits above, we can find banks' foreign funding from (192):

$$\widetilde{B}^{*}_{ss} = \widetilde{B}^{TOT}_{F,ss} - \widetilde{D}_{ss}. \tag{497}$$

By using (181) we can find $\widetilde{J}_{ss}$. To make the $\widetilde{J}_{ss}$ and $\widetilde{K}^{B}_{ss}$ consistent with (182), we need to set the value of δ^b as follows:

$$\delta^b = \frac{\widetilde{J}_{ss}}{\widetilde{K}^{B}_{ss}} - \pi_{ss}\pi^{z}_{ss} + 1. \tag{498}$$

Using that $\widetilde{B}^{TOT*}_{ss} = 0$ and (303) we get:

$$\widetilde{B}_{F,ss} = \widetilde{B}^{*}_{ss}. \tag{499}$$

Lastly, where we have set $\widetilde{Y}_{NAT*,ss} = 1$.

We can now find the transfer from the oil fund to the banks (ρ_{G_F}) by using and (261):

$$\begin{aligned}
\widetilde{B}_{F,ss} &= (1-\rho_{G_F})\left[\frac{R^{*}_{ss}}{\pi^{*}_{ss}\pi^{z}_{ss}}\widetilde{B}_{F,ss}\right] + \widetilde{S}_{ss}\widetilde{P}^{O*}_{ss}\widetilde{Y}_{O,ss} \\
&\Leftrightarrow \\
1-\rho_{G_F} &= \left[\widetilde{B}_{F,ss} - \widetilde{S}_{ss}\widetilde{P}^{O*}_{ss}\widetilde{Y}_{O,ss}\right]\left[\frac{R^{*}_{ss}}{\pi^{*}_{ss}\pi^{z}_{ss}}\widetilde{B}_{F,ss}\right]^{-1} \\
&\Leftrightarrow \\
\rho_{G_F} &= 1 - \left[1 - \widetilde{S}_{ss}\widetilde{P}^{O*}_{ss}\frac{\widetilde{Y}_{O,ss}}{\widetilde{B}_{F,ss}}\right]\left[\frac{R^{*}_{ss}}{\pi^{*}_{ss}\pi^{z}_{ss}}\right]^{-1}
\end{aligned} \tag{500}$$

And then from (260):

$$\widetilde{G}_{F,ss} = \rho_{G_F}\left[\frac{R^{*}_{ss}}{\pi^{*}_{ss}\pi^{z}_{ss}}\widetilde{B}_{F,ss}\right]. \tag{501}$$

The rest is found by trivial substitution.
- *We did it!*
- *Now what?*

18 List of all parameters

Dynamic parameters

Parameter	Value	Description
Households		
α^h	0.9959	Parameter governing the dynamics of the amortization rate for households
κ^h	1.0487	Parameter governing the dynamics of the amortization rate for households
b^{sa}	0.6393	Share of households with backward-looking expectations regarding house prices
b^c	0.9384	Habit persistence in consumption
b^d	0.4813	Habit persistence in deposits
β	0.99	Discount factor
b^h	0.9867	Habit persistence in housing services
b^l	0.5862	Habit persistence in leisure
δ^H	0.0228	Depreciation rate for housing capital
λ^{sa}	0.9495	Degree of how backward-looking agents are while forming house price expectations
ϕ^W	666.92	Cost of changing wages
z^d	0.1889	Deposit preferences
ζ	3	The inverse of the Frisch elasticity of labor supply
Intermediate goods sector		
α	0.256	Share of capital used in intermediate-goods production
ϕ^{PM^*}	285.60	Cost of changing export prices in foreign currency
ϕ^{PQ}	669	Costs of changing domestic prices
θ^{F*}	6	Elasticity of substitution between exported goods
ξ	0.929	Elasticity of substitution between capital and labor
Final goods		
μ	0.5	Elasticity of substitution between domestic and imported goods
Entrepreneurs		
α^e	0	Parameter governing the dynamics of the amortization rate for entrepreneurs
κ^e	0.9977	Parameter governing the dynamics of the amortization rate for entrepreneurs
ϕ_u	0.2192	Cost of changing the utilization rate for entrepreneurs
Capital producers		
δ	0.0108	Depreciation rate of capital
ϕ_{I1}	12.5432	Cost of changing business investment from its steady-state value
ϕ_{I2}	165.6624	Cost of changing business investment from the previous period's value
Housing sector		
ϕ_{H1}	60.7278	Cost of changing housing investment from its steady-state value
ϕ_{H2}	199.6549	Cost of changing housing investment from the previous period's value
Banking sector		
χ_c	10	Cost of deviating from the target capital-to-assets ratio
χ_o	0.0046	Fixed operational cost of banks (spread btw wholesale lending rate and money market rate)
δ^b	0.0161	Fraction of bank capital that is paid as dividends to shareholders
ϕ^B	0.0016	Elasticity of interest rate risk premium w.r.t. net foreign assets
ϕ^D	0.0732	Adjustment costs for changing deposit rate
ϕ^e	18.5013	Adjustment costs for changing business loan rate
ϕ^F	18.3597	Adjustment costs for changing household loan rate
ϕ^S	0	Elasticity of interest rate risk premium w.r.t. real exchange rate
θ^D	7.007	Elasticity of substitution between household deposits
ς^h	0.4	Risk weight on loans to households
ς^e	0.8	Risk weights on loans to households
Oil sector		
α_l	0.28	Share of labor used in oil supply goods production
α_{o*}	0.15	Foreign oil extractor's share of oil supply goods from home country in production
α_o	0.55	Share of rigs used in oil production

α_q	0.69	Share of final goods used in oil supply goods production
δ_O	0.021	Depreciation rate of oil rigs
γ^O	0.0336	Cost of changing utilization rate for oil extractors (first)
ϕ^{uf}	17.7955	Cost of changing utilization rate for oil extractors (second)
ϕ^{PR}	1245.6	Cost of adjusting oil supply goods prices in the domestic market
ϕ^{PR^*}	1723.1	Cost of adjusting oil supply goods prices in the foreign market
ϕ^{RI}	8.2151	Cost of changing oil investment
ρ_{G_F}	0.0501	The fraction of transfer from GPFG to banking sector
θ^R	400	Elasticity of substitution between oil supply goods domestically
θ^{R^*}	5	Elasticity of substitution between oil supply goods for export
Z_O	1	Oil extraction productivity
Z_R	1	Oil supply productivity
Foreign sector		
α^{P*}	0.1497	Parameter governing how expected inflation affects current inflation of trading partners
α^{GLOB}	0.1	Weight of trading partner's output gap in global output gap
α^{Y*}	0.0462	Parameter governing how trading partners' output affects current inflation of trading partners
β^O	0.2026	Parameter governing how expected real oil price affects current real oil price
β^*	0.999	Discount factor abroad
κ^O	4.0027	Parameter governing how global output gap affects real oil price
λ^{YNTP}	0.9258	Persistence of non-trading partners output gap
μ^*	0.5	Foreigners' elasticity of substitution between foreign and exported goods
ω^{P*}	1.4606	Response coefficient to inflation in the Taylor rule for trading partners
ω^{R*}	0.8414	Interest rate smoothing in the Taylor rule for trading partners
ω^{Y*}	0.04	Response coefficient to output in the Taylor rule for trading partners
ϕ^{P*}	0.8862	Persistence of trading partners' inflation process
ϕ^{OP*}	0.0006	Parameter governing how real oil price affects trading partners' inflation
ϕ^{PM}	830.10	Cost of changing prices of imported goods
ϕ^{ONTP}	0.0012	Parameter governing how real oil price affects non-trading partners' output gap
ϕ^{YNTP}	0.0114	Parameter governing how trading partners' output gap affects non-trading partners' output gap
ϕ^{Y*}	0.6146	Persistence of trading partners' output gap
ϕ^{O*}	0.0048	Parameter governing how real oil price affects trading partners' output gap
ϕ^{YNTP*}	1.0994	Parameter governing how non-trading partners' output gap affects trading partners' output gap
ψ^{R*}	0.7569	Parameter governing how foreign real interest rate affects trading partners' output gap
θ^F	6	Elasticity of substitution between imported goods
Monetary policy		
β_p	0.99	Discount factor of the central bank
λ_{dr}	0.4	Weight on the annualized policy rate change in the loss function
λ_{lr}	0.02	Weight on the annualized interest rate gap in the loss function
λ_y	0.3	Weight on the output gap in the loss function
ω_P	0	Response coefficient to inflation in the mimicking rule
ω_{P1}	0.2921	Response coefficient to future expected inflation in the mimicking rule
ω_{PREM}	0	Response coefficient to money market premium in the mimicking rule
ω_R	0.6663	Interest rate smoothing in the mimicking rule
ω_{RF}	0	Response coefficient to foreign interest rate in the mimicking rule
ω_S	0.0159	Response coefficient to real exchange rate in the mimicking rule
ω_W	0.8705	Response coefficient to wage inflation in the mimicking rule
ω_Y	0.2417	Response coefficient to output in the mimicking rule

Shock-related parameters

Parameter	Value	Description
Persistence parameters		
λ_B	0.737	Exchange rate risk premium shock
λ_G	0.9145	Government spending shock
λ_h	0.6938	Housing preferences shock
λ_I	0.6457	Shock to business investment adjustment costs
λ_{IH}	0.8608	Shock to housing investment adjustment costs
λ_{inf}	0.75	Monetary policy shock (Optimal policy)
λ_{IOIL}	0.834	Shock to oil investment technology
λ_{z^L}	0.804	Temporary productivity shock in the intermediate goods sector
λ_{MC*}	0.0965	Shock to marginal costs abroad
λ_{ν}	0.9336	Import demand shock
λ_{ν_*}	0.9238	Export demand shock
λ_{PO*}	0.8736	Real oil price (supply) shock
$\lambda_{\phi^{ent}}$	0.9102	Shock to LTV ratio of entrepreneurs
λ_{ϕ}	0.783	Shock to LTV ratio of households
λ_{prem}	0.8168	Shock to money market premium
λ_{ψ}	0.2797	Wage markup shock
λ_{RN3M}	0.7919	Monetary policy shock (Taylor rule)
λ_{R*}	0.3222	Monetary policy shock (trading partners)
λ_{θ^e}	0.9641	Markup shock to lending rate for loans to entrepreneurs
λ_{θ^H}	0.4347	Price markup shock
$\lambda_{\theta^{H*}}$	0.0523	Trading partners' price markup shock
$\lambda_{\theta^{IH}}$	0.8895	Markup shock to lending rate for loans to households
λ_u	0.7248	Consumption preference shock
λ_{U*}	0.7825	Trading partners' demand shock
λ_{wedge}	0.838	Inventory shock
λ_{YO*}	0.7458	Shock to the oil production abroad
St.dev. (multiplied by 100)		
σ_B	0.6178	Exchange rate risk premium shock
σ_G	0.3806	Government spending shock
σ_h	28.6767	Housing preferences shock
σ_I	23.0179	Shock to business investment adjustment costs
σ_{IH}	2.575	Shock to housing investment adjustment costs
σ_{inf}	1	Monetary policy shock (Optimal policy)
σ_{IOIL}	2.6119	Shock to oil investment technology
σ_{z^L}	0.598	Temporary productivity shock in the intermediate goods sector
σ_{MC*}	34.629	Shock to marginal costs abroad
σ_{ν}	0.4277	Import demand shock
σ_{ν_*}	4.2376	Export demand shock
σ_{PO*}	7.9181	Real oil price (supply) shock
$\sigma_{\phi^{ent}}$	2.5902	Shock to LTV ratio of entrepreneurs
σ_{ϕ}	25.4232	Shock to LTV ratio of households
σ_{prem}	0.0372	Shock to money market premium
σ_{ψ}	63.3097	Wage markup shock
σ_{RN3M}	0.0302	Monetary policy shock (Taylor rule)
σ_{R*}	0.0841	Monetary policy shock (Trading partners)
σ_{θ^e}	84.8579	Markup shock to lending rate for loans to entrepreneurs
σ_{θ^H}	20.1448	Price markup shock
$\sigma_{\theta^{H*}}$	0.8327	Trading partners' price markup shock
$\sigma_{\theta^{IH}}$	167.9416	Markup shock to lending rate for loans to households
σ_u	3.0209	Consumption preference shock
σ_{U*}	1.1147	Trading partners' demand shock
σ_{wedge}	0.1844	Inventory shock
σ_{YNTP}	0.1828	Global demand shock
σ_{YO*}	3.4093	Shock to oil production abroad
Other		
ρ_{ffm}	0.5269	Oil price-induced effects on wage bargaining

Steady-state parameters

Parameter	Value	Description
π_{ss}	1.0062	Gross inflation (quarterly)
π^*_{ss}	1.005	Gross foreign inflation (quarterly)
π^h_{ss}	1.0113	Relative house price trend inflation (quarterly)
π^z_{ss}	1.0025	Trend productivity growth (quarterly)
G_{ss}	1.5484	Government spending as a share of output
$z_{inf,ss}$	1	Inflation target shock
$M_{O*,ss}$	0.11	Oil supply goods export volume
MC^*_{ss}	1	Foreign marginal cost
γ^b_{ss}	0.136	Bank capital-to-risk-weighted assets ratio target
CCB^b_{ss}	0.02	Countercyclical capital buffer
$Y_{NAT*,ss}$	1	Foreign output
ν_{ss}	0.65	Share of domestic goods in final goods production
ν^*_{ss}	0.212	Share of domestic goods abroad (export demand)
O_{ss}	0.1011	Oil in the ground
P^{O*}_{ss}	1	Real oil price
ϕ^{ent}_{ss}	0.9917	Collateral coefficient governing LTV ratio for entrepreneurs
ϕ_{ss}	0.1095	Collateral coefficient governing LTV ratio for households
ψ_{ss}	2.5	Elasticity of substitution between differentiated labor
R^*_{ss}	1.005	Money market interest rate abroad (quarterly)
θ^e_{ss}	13.95	Elasticity of substitution between loans to entrepreneurs
θ^H_{ss}	6	Elasticity of substitution between intermediate goods
θ^{IH}_{ss}	29.2846	Elasticity of substitution between loans to households
$Z_{B,ss}$	0	Risk premium shock
z^h_{ss}	0.523	Housing preference shock
$z_{I,ss}$	1	Shock to business investment adjustment costs
$z_{IH,ss}$	1	Shock to housing investment adjustment costs
$Z_{IOIL,ss}$	1	Shock to oil investment specific technology
z^L_{ss}	1	Temporary productivity shock in the intermediate goods sector
$Z_{prem,ss}$	1.0012	Shock to money market risk premium
z^u_{ss}	2.6315	Consumption preference shock
$z_{x,ss}$	1.0772	Inventory shock

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