

# When does a central bank's balance sheet require fiscal support?

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## Yet another balance sheet simulation?

- Hall and Reis (2013), Carpenter et al. (2013), Greenlaw et al. (2013), Christensen et al. (2013) examine likely scenarios, based on historically normal behavior of interest rates and demand for central bank liabilities
- None of these papers uses a model where **inflation** and **interest rates** are **endogenously** determined
- We look at complete, though simplified, economic model in order to study why a central bank's balance sheet matters at all and the consequences of a lack of fiscal backing for the central bank

# Contribution

In our model, interest rates, inflation, and seigniorage are endogenous, hence we can answer questions such as:

- 1 Under what conditions does the central bank need fiscal support?
  - The central bank has one quasi-fiscal resource: **seigniorage**.
  - But seigniorage is endogenous. So it is possible for seigniorage to be insufficient to allow the CB to maintain policy commitments without support from the fiscal authority

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  - But seigniorage is endogenous. So it is possible for seigniorage to be insufficient to allow the CB to maintain policy commitments without support from the fiscal authority
- 2 How does the sensitivity of the CB balance sheet to shocks (changes in real rate, inflation expectations,..) depend on interest rate policy? (Berriel and Bhattarai, 2009)

# Contribution

- ③ Can **self-fulfilling solvency crises** arise when the CB holds (a lot of) long-duration assets?
- If the CB cannot rely on fiscal backing
  - ... and has to rely on seigniorage ( $\rightarrow$  higher inflation) if the value of its assets were to fall below the value of reserves
  - ... an equilibrium may arise where the public's inflation expectations become self-fulfilling:
    - $\uparrow$  future interest rates  $\rightarrow$   $\downarrow$  value of long-duration assets  $\rightarrow$  balance sheet gap that needs to be filled with future seigniorage
  - Multiplicity arises depending on 1) size/duration of balance sheet, 2) properties of currency demand function.

## Results (Preliminary)

- 1 In a calibrated model for the US economy, CB solvency would become an issue only under rather extreme scenarios, for current balance sheet size
- 2 under the baseline calibration **seigniorage is large**

PDV Seigniorage  $\gg$  Reserves

hence provides the CB with a large safety net

- .. but there is much **uncertainty** regarding the PDV of seigniorage – properties of the **currency demand** function are key (bitcoins, ...)

# The model

- Simple, perfect-foresight, non-linear, continuous-time model
  - Exogenous real interest rate  $\rho$  and income  $Y$
  - Flexible prices
- All uncertainty is revealed at time 0

# Households

- Households/**private sector** maximize

$$\int_0^{\infty} e^{\beta t} \log C dt$$

subject to

$$C(1 + \psi(v)) + \dot{F} + \frac{\dot{V} + \dot{M} + q\dot{B}^P}{P} = Y + \rho F + r \frac{V}{P} + (\chi + \delta - q\delta) \frac{B^P}{P} - \tau$$

- $F$ : storage/foreign assets paying an exogenous real rate  $\rho$
- $V$ : overnight reserves paying nominal rate  $r$ .  $M$ : currency
- $B^P$ : Woodford (2001)-style bond (depreciates at rates  $\delta$ , coupon  $\delta + \chi$ , duration  $\delta^{-1}$ )
- Population growth  $n$  and productivity growth  $\gamma$  (omitted here for simplicity)



# Transactions technology

$$v = \frac{PC}{M}$$

$$\psi(v) = \frac{\psi_0 v}{1 + \psi_1 v}$$

- $\psi_1 < 0 \Rightarrow v^{-1}$  asymptotes to  $\psi_1^{-1}$  as  $r \rightarrow \infty$
- $\psi_1 > 0 \Rightarrow$  Implies real balances go to zero for  $r > \psi_0/\psi_1^2$

# Fiscal policy

- Fiscal Authority budget constraint:

$$g + (\chi + \delta - \delta q) \frac{B}{P} = \tau + \tau^C + q \frac{\dot{B}}{P}$$

- $B = B^P + B^C$  (bonds are held either by the public or the CB)
- $\tau^C$  remittances from CB
- Passive fiscal policy:

$$\tau = \xi_0 + \xi_1 q \frac{B}{P}, \quad \xi_1 > \rho$$

# Conventional and unconventional monetary policy

- Reaction function for interest on reserves  $r$ :

$$\dot{r} = \theta_r \cdot \left( \bar{r} + \theta_\pi \left( \frac{\dot{P}}{P} - \bar{\pi} \right) - r \right), \quad r \geq \underline{r}$$

- Unconventional monetary policy: path for  $B^C$  (Treasuries held by CB)
- The central bank budget constraint is:

$$q \frac{\dot{B}^C}{P} - \frac{\dot{V} + \dot{M}}{P} = (\chi + \delta - \delta q) \frac{B^C}{P} - r \frac{V}{P} - \tau^C$$

## A present value take on CB solvency and remittances

$$\underbrace{q \frac{B_0^C}{P_0} - \frac{V_0}{P_0}}_{\text{Mkt value of assts - reserves}} + \underbrace{\int_0^\infty \frac{\dot{M}_t}{P_t} e^{-\int_0^t \rho_s ds} dt}_{\text{PDV seigniorage}} = \underbrace{\int_0^\infty \tau_t^C e^{-\int_0^t \rho_s ds} dt}_{\text{PDV remittances}}$$

- if LHS > 0 CB is “solvent”: it needs no fiscal backing

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- if LHS > 0 CB is “solvent”: it needs no fiscal backing
- PDV of remittances does not depend on:
  - ① Future path of  $B^C$  (whether  $B^C$  is held to maturity or not)
  - ② Accounting rules, e.g., rule for remittances, deferred asset ...
- Time path of  $\tau^C$  will
- See also Berriel and Bhattarai (2009), Hall and Reis (2013), Bassetto and Messer (2013)...

## Rule for remittances

- As long as  $\tau^C \geq 0$ , rule for remittances does not matter for equilibrium
- Two principles: i) remittances cannot be negative, ii) whenever positive, remittances are such that the central bank capital measured at historical costs remains constant over time:

$$\tilde{K} = (\tilde{q}B^C - V - M) e^{nt} = \text{constant.}$$

where  $\tilde{q}$  evolves according to

$$\dot{\tilde{q}} = (q - \tilde{q}) \max \left\{ 0, \frac{\dot{B}^C}{B^C} + \delta \right\}$$

yielding

$$\tau^C = \max \left\{ 0, (\chi - \delta(\tilde{q} - 1)) \frac{B^C}{P} + \left( \dot{\tilde{q}} - (q - \tilde{q}) \left( \frac{\dot{B}^C}{B^C} + (\delta + n) \right) \frac{B^C}{P} \right) - r \frac{V}{P} \right\} \underbrace{\mathcal{I}_{\{\tilde{K} \geq \tilde{K}_0\}}}_{\text{Deferred Asset}}$$

# Equilibrium

- Fisher eq. + Taylor rule yield a solution for interest rates:

$$r_t = \int_0^{\infty} e^{-(\theta_{\pi}-1)\theta_r s} \theta_r (\theta_{\pi} \rho_{t+s} - (\bar{r} - \theta_{\pi} \bar{\pi})) ds + \kappa e^{(\theta_{\pi}-1)\theta_r t}$$

- This solution is not unique. In the paper we show simulations under explosive paths, but our results do not hinge on non-explosiveness

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- This solution is not unique. In the paper we show simulations under explosive paths, but our results do not hinge on non-explosiveness
- $v_t^2 \psi'(v_t) = r_t$ ,  $v_t = \frac{P_t C_t}{M_t}$ , defines money demand  $\Rightarrow$  seigniorage
  - CB determines the size of the balance sheet, but its composition between interest-bearing liabilities ( $V$ ) and currency ( $M$ ) is determined by the private sector

- No arbitrage condition for long term bonds:  $\frac{\chi + \delta}{q} - \delta + \frac{\dot{q}}{q} = r \Rightarrow$

$$q_0 = (\chi + \delta) \int_0^{\infty} e^{-(\int_0^t r_s ds + \delta t)} dt$$



# Three levels of CB balance sheet problems

*Level 1:* Accounting capital (book value of  $B^C - V - M) < 0$

- Rule for remittances implies  $\tau^C = 0$  for some period.
- No fiscal support ( $\tau^C < 0$ ) required. In fact, PDV of  $\tau^C$  may actually increase

*Level 2:*  $qB^C - V < 0$ : Market value of assets  $<$  interest-bearing liabilities

- No fiscal support required as long as

$$\int_0^{\infty} \frac{\dot{M}_t}{P_t} e^{-\int_0^t \rho_s ds} dt > V - qB^C$$

- High expected inflation  $\rightarrow q \downarrow$  but PDV of seigniorage holds up/increases (stream of seigniorage provides a *hedge* against drops in  $q$ )

Level 3: 
$$\frac{qB^C - V}{P_0} + \int_0^\infty \frac{\dot{M}_t}{P_t} e^{-\int_0^t \rho_s ds} dt < 0$$

- For very high  $r$ /inflation economy may move to the other side of the Laffer curve:  $\rightarrow \int_0^\infty \frac{\dot{M}_t}{P_t} e^{-\int_0^t \rho_s ds} dt \downarrow$
- $\int_0^\infty \tau_t^C e^{-\int_0^t \rho_s ds} dt < 0 \rightarrow \tau_t^C < 0$  for some  $t \rightarrow$  fiscal support required

# Parameters

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normalization, foreign assets

$$Y - G = 1$$

$$F_0 = 0$$

discount rate, reversion to st.st., population and productivity growth

$$\beta = 0.01$$

$$\gamma = 0.01$$

$$n = 0.0075$$

monetary policy

$$\theta_\pi = 2$$

$$\theta_r = 1$$

$$\bar{\pi} = 0.02$$

initial assets (par value), reserves, and currency as of Jan 2 2014

$$\frac{B^C}{P} = .327$$

$$\frac{V}{P} = .207$$

$$\frac{M}{P} = .104$$

bonds: duration and coupon

$$\delta^{-1} = 6$$

$$\chi = 0.035$$

money demand

$$\psi_0 = 2.31 \times 10^{-6}$$

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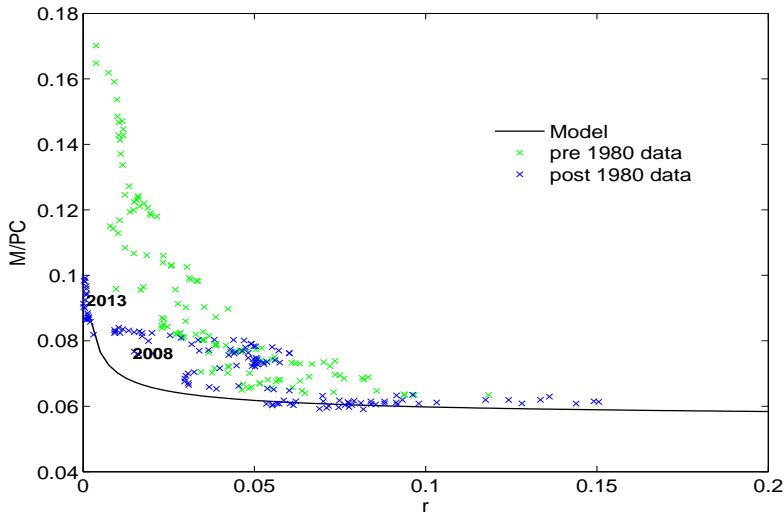
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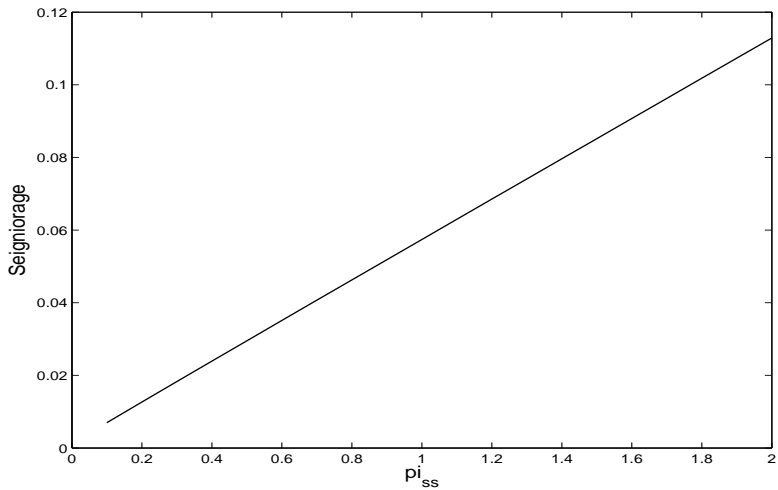
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# A scatter plot of short term interest rates and M/PC



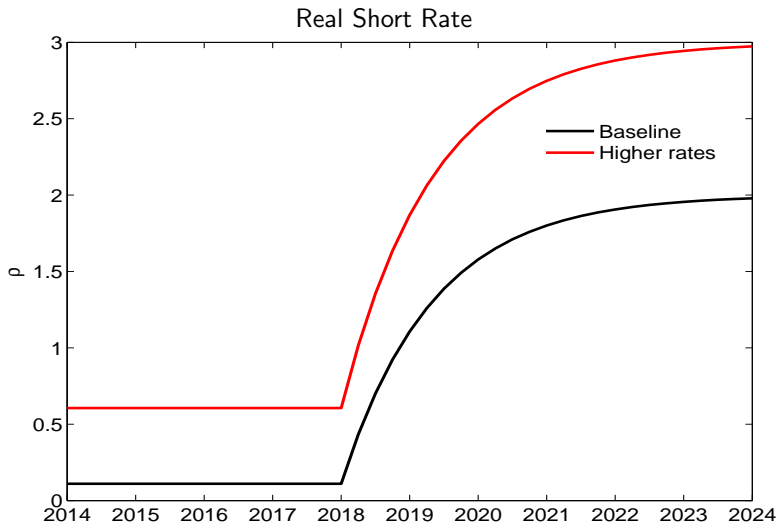
# Laffer curve



# Baseline and scenarios

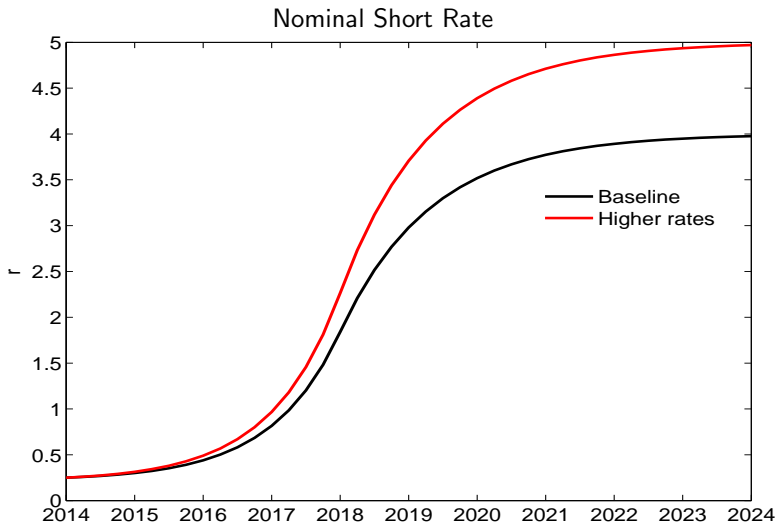
- Baseline: real rate path ( $\rho$ ) chosen so that  $r$  path roughly matches Carpenter et al.
- Scenarios (time 0 “surprise”, perfect foresight afterwards)
  - ① *Exogenous “shocks”*
    - “Higher rates” Carpenter et al. scenario (with the CB recognizing the change, or not)
    - 10 year (exogenous) “inflation scare”
    - Explosive paths (hyperinflations)
  - ② *Self-fulfilling crises*

# Baseline and Higher Rates Scenario





## Baseline and “Higher Rates” Scenario



## Balance Sheet Implications

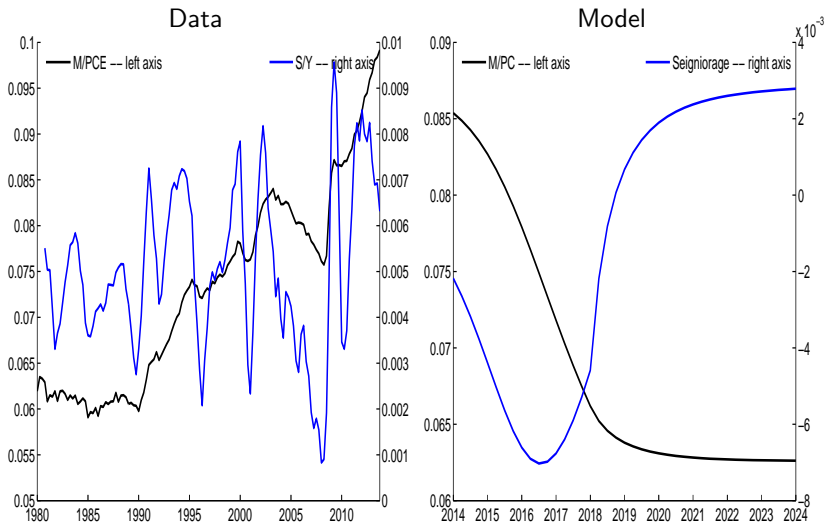
(1)	(2)	(3)	(4)	(5)	(6)	(7)
$qB/P$ $-V/P$	PDV seig.	(1)+(2)	$\bar{\tau}^C$	q	$\Delta M/P$	$\bar{B}/B$

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Baseline scenario	0.146	0.998	1.144	0.0026	1.08	
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# seigniorage and M/PC



## Balance Sheet Implications

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	$qB/P$	PDV	(1)+(2)	$\bar{\tau}^C$	$q$	$\Delta M/P$	$\bar{B}/B$
	$-V/P$	seig.					

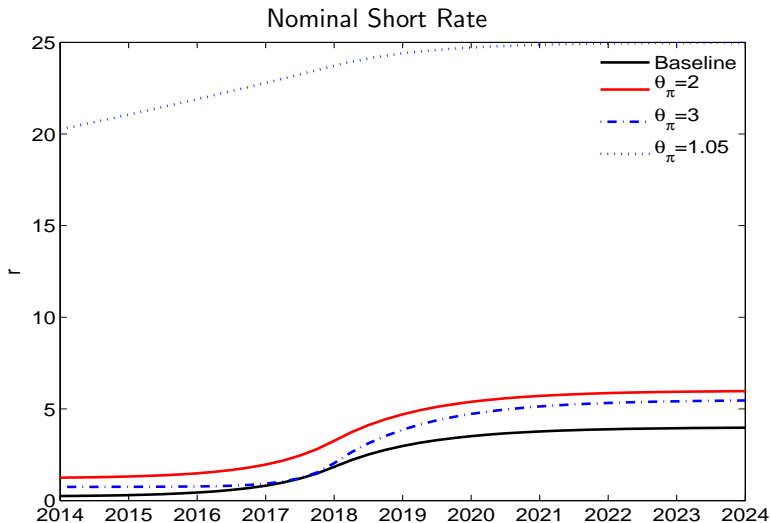
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Baseline scenario	0.146	0.998	1.144	0.0026	1.08		
Higher rates ( $\gamma$ )	0.142	1.287	1.429	0.0031	1.06	0.003	64.59

## Balance Sheet Implications

	(1) $qB/P$ $-V/P$	(2) PDV seig.	(3) (1)+(2)	(4) $\bar{\tau}^C$	(5) q	(6) $\Delta M/P$	(7) $\bar{B}/B$
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Higher rates ( $\beta$ )	0.133	0.169	0.302	0.0033	1.06	-0.006	14.42

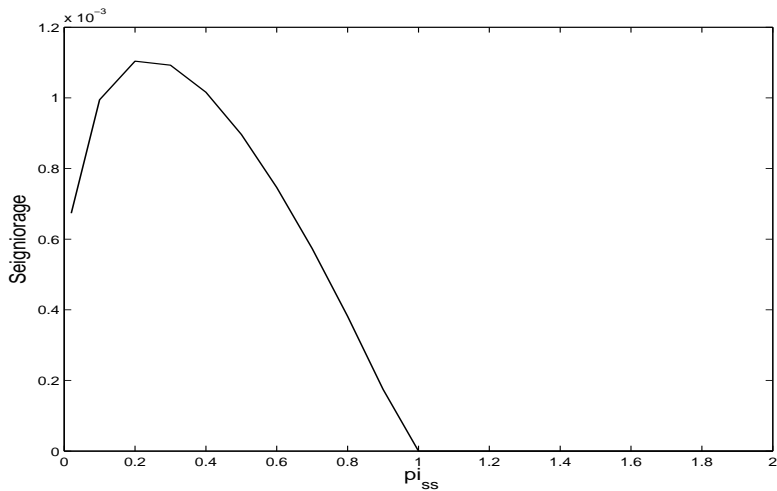
# Baseline vs different $\theta_\pi$ – Higher $\beta$ /same intercept



## Balance Sheet Implications/Higher Rates

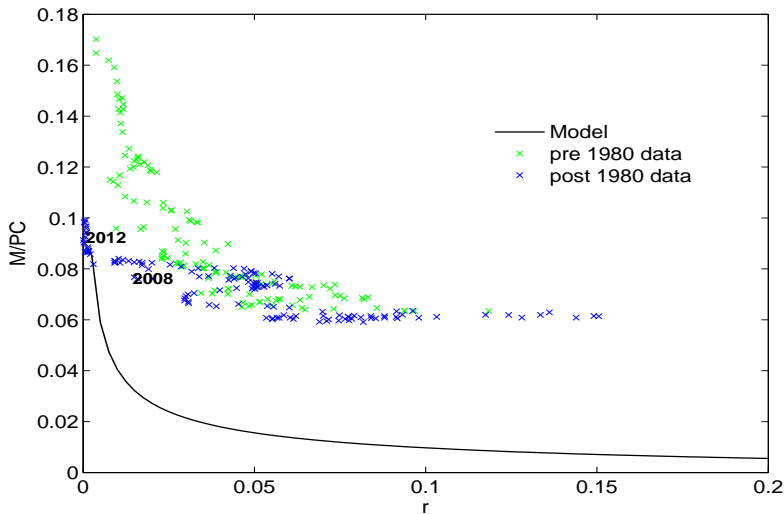
	(1) $qB/P$ $-V/P$	(2) PDV seig.	(3) (1)+(2)	(4) $\bar{\tau}^C$	(5) q	(6) $\Delta M/P$	(7) $\bar{B}/B$
Baseline scenario	0.146	0.998	1.144	0.0026	1.08		
Higher rates ( $\beta$ )	0.133	0.169	0.302	0.0033	1.06	-0.006	14.42
Higher rates ( $\beta$ ) /same intercept Lower $\theta_\pi$	-0.055	1.162	1.107	0.0132	0.52	-0.024	3.38

# Laffer curve/Alternative Money Demand





# A scatter plot of short term interest rates and M/PC /Alternative Money Demand



# Balance Sheet Implications / Alternative Money Demand

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	$qB/P$	PDV	(1)+(2)	$\bar{\tau}^C$	q	$\Delta M/P$	$\bar{B}/B$
	$-V/P$	seig.					
Baseline scenario	0.146	0.214	0.360	0.0008	1.08		
Higher rates ( $\beta$ ) /same intercept	-0.111	0.095	-0.016	-0.0002	0.52	-0.080	.97
Lower $\theta_\pi$							

## Self-fulfilling CB Solvency Crises

- Can multiple equilibria arise in absence of fiscal backing?
- If  $\tau_t^C \geq 0$  for all  $t$ , then the CB *must rely on seigniorage* if  $qB - V < 0$

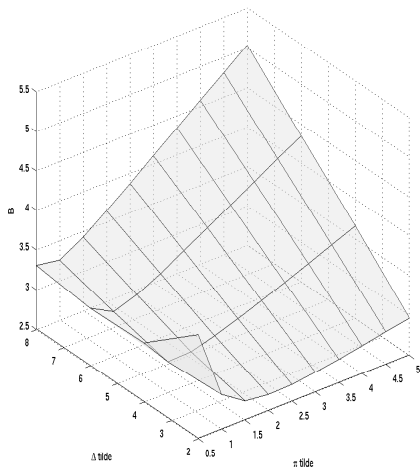
## Self-fulfilling CB Solvency Crises

- Can multiple equilibria arise in absence of fiscal backing?
- If  $\tau_t^C \geq 0$  for all  $t$ , then the CB *must rely on seigniorage* if  $qB - V < 0$
- Expectations that the CB will switch to a new rule with target  $\tilde{\pi} > \bar{\pi}$  will imply  $q_0(\tilde{\pi}) \downarrow$  and produce a gap  $\frac{q_0(\tilde{\pi})B_{0-}^C - V_0}{P_0(\tilde{\pi})} < 0$
- This gap will have to be filled with future seigniorage  $PDVS_0(\tilde{\pi})$ , thereby validating the initial expectations
- We consider equilibria where the central bank will change its inflation target to  $\tilde{\pi} > \bar{\pi}$  at time  $t = \tilde{T}$  for a period  $\tilde{\Delta}$  and find solutions of

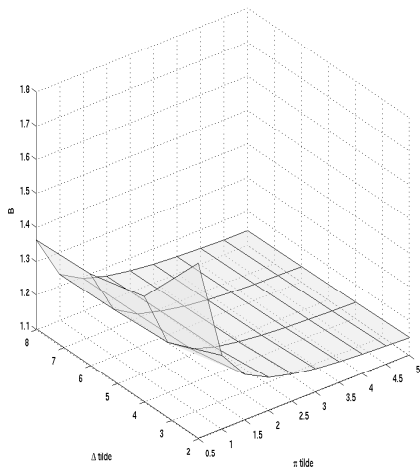
$$\frac{q_0(\tilde{\pi})B_{0-}^C - V_0}{P_0(\tilde{\pi})} + PDVS_0(\tilde{\pi}) = 0$$

# For What Level of $B^C$ Are Self-fulfilling Crises Possible?

Baseline



Alternative money demand calibration



# Conclusions

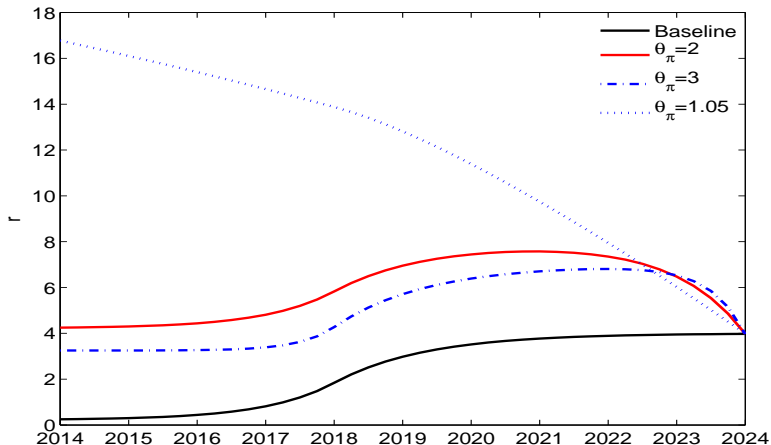
- CB solvency would become an issue only under rather extreme scenarios, for current balance sheet size
- These conclusions hinge on the properties of **currency demand** and **seigniorage**
  - ... on which there is considerable uncertainty
- Implications: Fiscal backing for the CB (e.g., Bank of England) allows it to pursue its mandate without being concerned about its balance sheet
  - ... and such backing seems unlikely to be needed in equilibrium on the basis of our model

## Inflation Scare – different $\theta_\pi$ 's

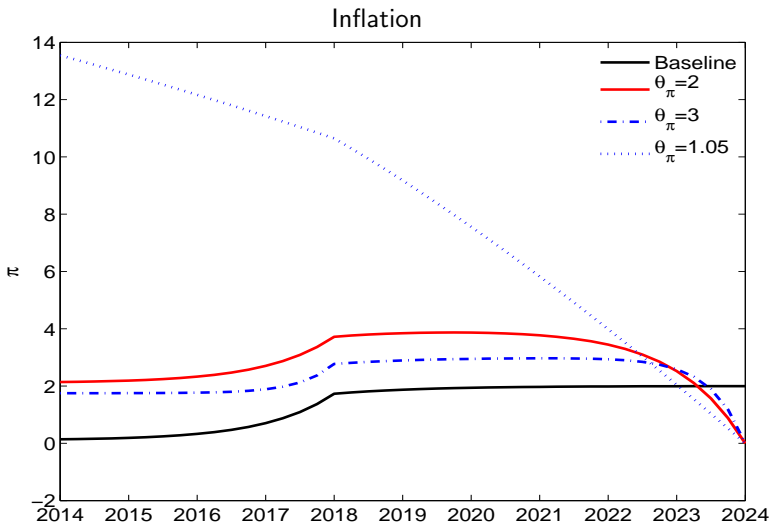
- Fisher equation:

$$r_t = \rho_t + \frac{\dot{P}_t}{P_t} + x$$

for  $t \in [0, 10]$ , where  $x = P(\Delta P) \times \Delta P = 2\%$   
Nominal Short Rate



# Inflation Score

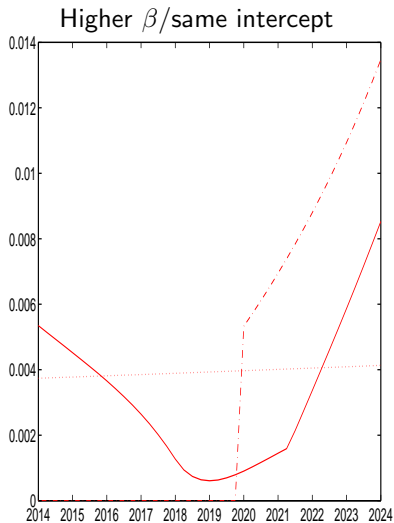
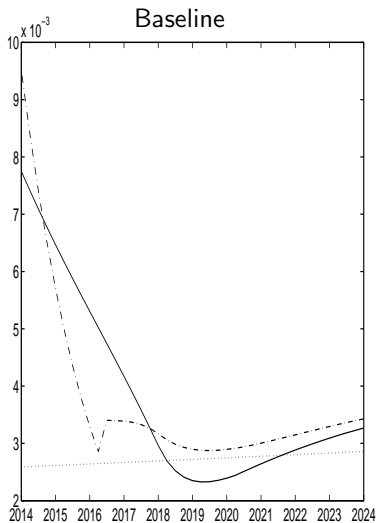




## Balance Sheet Implications / Inflation Score

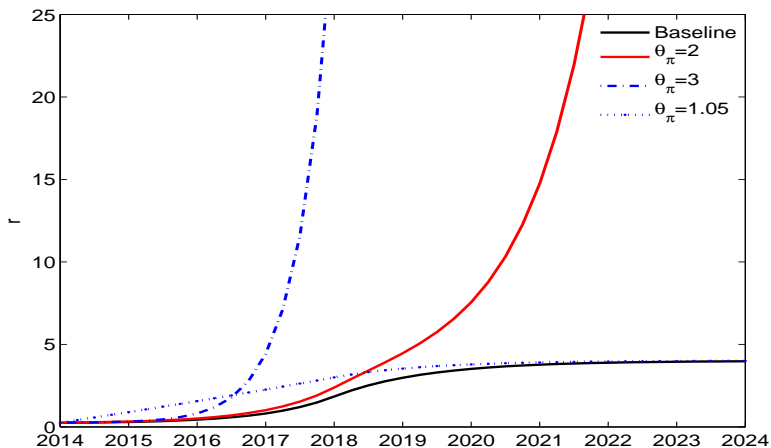
	(1) $qB/P$ $-V/P$	(2) PDV seig.	(3) (1)+(2)	(4) $\bar{\tau}^C$	(5) q	(6) $\Delta M/P$	(7) $\bar{B}/B$
Baseline scenario	0.133	0.937	1.070	0.0024	1.11		
Inflation score	0.067	0.943	1.010	0.0023	0.94	-0.023	7.16
Higher $\theta_\pi$							
Inflation score	0.081	0.928	1.009	0.0022	0.98	-0.020	9.10
Lower $\theta_\pi$							
Inflation score	0.004	0.968	0.972	0.0023	0.68	-0.020	3.65

# Paths for remittances



## Explosive paths – different $\theta_\pi$ 's

- Explosive paths:  $r_t = \text{stable solution} + \kappa e^{\theta_r(\theta_\pi - 1)t}$



## Balance Sheet Implications / Explosive path

	(1) $qB/P$ $-V/P$	(2) PDV seig.	(3) (1)+(2)	(4) $\bar{\tau}^C$	(5) q	(6) $\Delta M/P$	(7) $\bar{B}/B$
Baseline scenario	0.146	0.998	1.144	0.0026	1.08		
Explosive path	3.124	Inf	Inf	Inf	0.88	3.043	Inf
Higher $\theta_\pi$							
(10) Explosive path	6.124	Inf	Inf	Inf	0.64	6.120	Inf
Lower $\theta_\pi$							
(13) Explosive path	0.241	Inf	Inf	Inf	1.05	0.106	Inf