Investment shocks and consumption

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June 2013  (First Draft: June 2009)

Abstract

Several influential papers have argued that preferences featuring a weak wealth effect on labour supply are key to generate macroeconomic co-movement across real variables in response to shocks. Using a fully general specification for the instantaneous utility function, we show that the size of the wealth effect on labour supply is largely inconsequential for macroeconomic dynamics. Instead, we find that Edgeworth complementarity between consumption and hours worked is crucial in order to obtain co-movement of key macroeconomic variables. We consider investment shocks and we show that co-movement can easily be achieved with non-separable preferences in combination with a reasonable degree of nominal rigidity. This holds even in the presence of sizeable wealth effects.

JEL classification: E32.

Keywords: investment shocks, wealth effect, complementarity, consumption, GHHH preferences, nominal rigidities, co-movement.

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1 Introduction

Since the seminal contribution by Barro and King (1984), it has been well-known that dynamic stochastic general equilibrium (DSGE) models with time-separable preferences struggle to generate the positive co-movement of key macroeconomic variables such as output, hours worked, consumption and investment that is characteristic for empirically recognisable business cycles (see, e.g., King and Rebelo, 1999). Total factor productivity shocks generate co-movement in the baseline real business cycle (RBC) model. But other disturbances such as shocks to the marginal efficiency of investment (MEI), to preferences, or to government spending fail at generating the typical patterns of business cycles, both in New Classical and New Keynesian models.

In overcoming this co-movement problem, a number of papers have highlighted the need for preferences that further restrict the labour supply decision either to be fully independent of the intertemporal consumption-savings choice as in Greenwood, Hercowitz and Huffman, henceforth GHH, (1988), or to be affected only by a limited wealth effect as in the more recent paper by Jaimovich and Rebelo (2009). Thus, GHH preferences – or more general specifications with weak wealth effects – have been used in addressing several features of business cycles, cf., e.g., Dey (2012), Dey and Tsai (2011), Monacelli and Perotti (2008), Raffo (2007) and Schmitt-Grohé and Uribe (2012).

In this paper, we reconsider the use of utility specifications with weak wealth effects on labour supply as a means to generate co-movement in DSGE models. Focusing on the MEI shocks studied by GHH, we show that the absence of a wealth effect on labour supply is not the solution to the co-movement problem posed by Barro and King (1984). The critical feature needed to generate co-movement is a complementarity in the sense of Edgeworth (1881) between consumption and hours worked, or equivalently, Edgeworth substitutability between consumption and leisure. The GHH utility specification implies both that the wealth effect on labour supply is ab-
sent and that consumption and labour effort are Edgeworth complements. It is not possible to disentangle the effect of each of them. In contrast, we work with a fully general specification of the instantaneous utility function (inspired by Bilbiie, 2011) that allows us to vary the size of the wealth effect and the degree of complementarity separately.

According to our results, GHH-type preferences have proven useful to generate co-movement not simply because they lead to weak wealth effects on labour supply, but because they imply a large degree of Edgeworth complementarity between consumption and hours worked, especially when the Frisch elasticity of labour supply with respect to the real wage is high as in GHH (1988), Jaimovich and Rebelo (2009) and Schmitt-Grohé and Uribe (2012). In addition, preferences of the King, Plosser and Rebelo, henceforth, KPR, (1988) family, which feature a positive wealth effect on labor supply, can also generate co-movement if only the Edgeworth complementarity between consumption and hours is high enough.

In general, we find that the degree of Edgeworth complementarity between consumption and hours worked is the main determinant of the sign of the impact response of consumption, whereas the size of the wealth effect on labour supply influences the magnitude of this response. But for plausible parameter values, we find this influence to be small. Therefore, the absence of a wealth effect on labour supply is largely inconsequential for the purpose of generating co-movement, while the degree of Edgeworth complementarity between consumption and hours worked is crucial.

This finding is encouraging given recent microeconometric evidence. A number of studies have found evidence in favour of non-separabilities between aggregate consumption and labour following the lead of Basu and Kimball (2002), e.g., Guerron-Quintana (2008), Kilponen (2012), Kilponen, Vilmunen and Vähämäa (2013) and Kim and Katayama (2012). Similarly, recent papers find evidence in favour of non-negligible wealth effects on labour supply, cf. Imbens, Rubin and Sacerdote (2001) and Kimball and Shapiro (2010).
Our results are in keeping with the results in Bilbiie (2011), who shows how the combination of nominal rigidities and Edgeworth complementarity between consumption and hours worked can generate co-movement after a government spending shock.\footnote{Other recent studies that discuss co-movement are Eusepi and Preston (2009) in a model with consumption heterogeneity between employed and unemployed workers, Guerrieri, Henderson and Kim (2010) in a model with a multiple sectors, Khan and Tsoukalas (2011) in an estimated model with several shocks, and Christiano, Motto and Rostagno (2012), Ajello (2012) as well as Del Negro, Eggertsson, Ferrero and Kiyotaki (2010) in models driven by financial shocks. None of these papers disentangle the different roles played by non-separability and the absence of a wealth effect.}

Our paper distinguishes from his by analysing and disentangling the different roles played by Edgeworth complementarity and wealth effects on labour supply, and by considering responses to MEI shocks in a DSGE model with endogenous capital accumulation rather than disturbances to government spending.

We believe that the study of MEI shocks, which we shall sometimes call investment shocks for short, is particularly interesting given the importance assigned to them in recent studies. First emphasised by Keynes (1936) and later reintroduced into (New Classical) macroeconomics by GHH (1988), MEI shocks have been found to be important drivers of business cycles in New Keynesian DSGE models of the US economy. For example, Justiniano, Primiceri and Tambalotti, henceforth JPT, (2010) find that MEI shocks account for about 50 per cent of output fluctuations, 80 per cent of those in investment, and 60 cent of variation in hours worked. However, the shocks can account for less than 10 per cent of the variation in consumption, and consumption fails to co-move with other macroeconomic variables subject to the shocks. Specifically, a positive MEI shock leads to a decline in consumption on impact and for the first five quarters after the shock.\footnote{Similarly, Gertler, Sala and Trigari (2008) find that the MEI shock is the most important driver of output fluctuations in a model with unemployment despite a significantly negative consumption response for almost ten quarters. The same is true for all the models, with and without financial frictions, considered in Christiano, Motto and Rostagno (2012). The decoupling between consumption and investment dynamics is even larger in the estimated model with flexible prices and wages by Schmitt-Grohé and Uribe (2012). In that model the sum of anticipated and unanticipated shocks to the marginal efficiency of investment explains 63 per cent of fluctuations in investment, but only 2 per cent of fluctuations in consumption. Comparable results are derived from an open economy model estimated by Jacob and Peersman (2013).}
Moreover, finding economic mechanisms through which consumption may increase following MEI shocks is interesting for two reasons (besides comparability with GHH, 1988). First, the lack of co-movement of consumption with other key variables in response to MEI shocks is not compensated for by other shocks in the model estimated by JPT (2010). In fact, although dynamics are driven by seven sources of aggregate fluctuations, the unconditional correlation between consumption and investment is negative in the model. Hence, while performing well in reproducing other cross-correlations, the model fails at generating the large positive correlation found in the data. Second, conditional empirical evidence based on VAR studies suggests that consumption increases significantly in response to an MEI shock, cf. Peersman and Straub (2007) for the US and the euro area, and Braun and Shioji (2007) for Japan, who both identify MEI shocks using robust sign restrictions that leave the sign of the consumption response itself unrestricted. Hence, if consumption can be made to co-move with other key macroeconomic variables after shocks to the marginal efficiency of investment in standard DSGE models, this would both improve the empirical performance of the models and provide further support to the proposition that MEI shocks are important drivers of the business cycle.³

Importantly, our results on co-movement rely on the presence of nominal price rigidity. Sticky prices lead to countercyclical mark-ups, which shift labour demand on impact of shocks other than to total factor productivity. The previous literature emphasising preferences with weak wealth effects on labour supply also rely on a labour demand shifter to generate co-movement. In GHH (1988), Jaimovich and Rebelo (2009) and Schmitt-Grohé and Uribe (2012), it is the presence of variable

³Smets and Wouters (2007) introduce a risk premium shock in the Euler equation for consumption to overcome the unconditional co-movement problem for this variable. This shock therefore opens for the possibility that a combination of shocks may generate the unconditional co-movement of key variables. While this may well be a distinguishing feature of the business cycle seen through the optics of DSGE models, we do not pursue this possibility here. Given the conditional evidence, we want to clarify whether shocks to the capital formation process itself may generate empirically recognisable business cycles in a standard DSGE model.
capacity utilisation that shifts labour demand. We argue that our combination of nominal rigidities and Edgeworth complementarity generates co-movement under more general conditions than this alternative mechanism based on variable capacity utilisation and a weak wealth effect.

The paper is organised as follows. Section 2 presents the model whereas section 3 discusses the parametrisation. Results are presented and analysed in section 4. In section 5, we discuss some special cases and we compare our results to the previous literature. Some concluding remarks are given in section 6.

2 The model

The model is a standard New Keynesian model with endogenous capital accumulation. The economy consists of a representative household, a continuum of firms, and an inflation-targeting central bank. There is monopolistic competition and nominal rigidities in goods markets, and perfect competition in labour and capital rental markets.

The representative household consumes a bundle of the intermediate goods produced by individual firms. Each period, it chooses how much to consume of this final good (in addition to its composition) and how much to invest in one-period risk-free bonds. As in Christiano, Eichenbaum and Evans (2005), it also chooses how much to invest in new capital subject to investment adjustment costs.

Each firm combines rented capital with labour services supplied by the representative household to produce a differentiated intermediate good. It sets the price of its good according to a Calvo price-setting mechanism and stands ready to satisfy demand at the chosen price. Given this demand, and given wages and rental rates, the firm chooses factor inputs to production to minimise its costs.

Each period begins with the realisation of shocks to the economy. We are interested only in MEI shocks, i.e., shocks to the extent to which output devoted to investment increases the capital stock available for use in future production, and so
we ignore other shocks that may affect economy.

2.1 Households

The representative household maximises its expected life-time utility defined as

\[ E_t \sum_{k=0}^{\infty} \beta^k U(C_{t+k}, L_{t+k}) \]  

(1)

where \( \beta \) is the subjective discount factor, \( L_t \) denotes leisure in period \( t \), and \( C_t \) is the period’s final goods consumption given as a Dixit-Stiglitz aggregate of the differentiated intermediate goods supplied by firms

\[ C_t \equiv \left( \int_0^1 C_t(i) \frac{\varepsilon_p^{-1}}{\varepsilon_p} \, di \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \]  

(2)

Here, \( \varepsilon_p \) is the elasticity of substitution between product varieties, and \( C_t(i) \) represents consumption of the good produced by firm \( i \).

With this specification, we restrict preferences to be additively separable over time (and weakly if not additively separable in consumption and leisure) as is common in macroeconomics, cf., e.g., King and Rebelo (1999) and Christiano, Trabandt and Walentin (2011).\(^4\) But we deviate from most of the literature by working with a general Edgeworthian specification of the instantaneous utility function \( U(\cdot) \), imposing only a few desirable restrictions at this stage. First, we assume that the instantaneous utility function is increasing in both arguments, concave and twice continuously differentiable. That is, using subscripts to denote partial derivatives

\(^4\)See Deaton and Muellbauer (1980) for a discussion of the limits that this specification imposes on the substitutability between consumption and leisure across time and states of nature (and between intermediate consumption goods and leisure within a period). By implication, we do not explore the possibility that ‘exotic preferences’ of the kind discussed by Backus, Routledge and Zin (2005) that relaxes these restrictions may provide solutions to the co-movement problem. It will be of considerable interest, however, to consider preferences like those proposed by Greenwood, Hercowitz and Huffman (1988) which imposes additional restrictions on the substitutability between consumption and leisure within the period by making it independent of the intertemporal consumption allocation.
of $U(\cdot)$, we impose $U_C > 0$, $U_L > 0$, $U_{CC} \leq 0$, $U_{LL} \leq 0$ and $U_{CC}U_{LL} - (U_{CL})^2 \geq 0$. This is to ensure that the utility function represents preferences that are monotone and convex, see, e.g., Mas-Colell, Whinston and Green (1995), while allowing for substitutability or complementarity between consumption and leisure in the sense of Edgeworth (1881) with $U_{CL} \neq 0$. Second, we pay heed to Bilbiie’s (2009) warning against the potential inferiority of consumption and leisure in business cycle models with a general specification for $U(\cdot)$ by imposing the conditions he derives to ensure that they are normal goods, cf. section 3 for more details.

Maximisation follows a two-stage budgeting procedure. Denoting the price demanded by firm $i$ by $P_t(i)$, expenditure minimisation by the household at the lower stage (for a given level of final goods consumption) leads to a downward-sloping demand schedule for the intermediate good produced by this particular firm

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon_p} C_t$$

where $\varepsilon_p$ now represents the elasticity of demand. $P_t$ is the price index defined as

$$P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon_p} di\right)^{\frac{1}{1-\varepsilon_p}}$$

This price index has the property that the minimum expenditure required to purchase $C_t$ units of the composite good is given by $P_tC_t$.

At the higher stage, the household takes the price level ($P_t$), the wage rate ($W_t$), and the real rental rate of capital ($R^K_t$) as given and chooses total consumption and leisure as well as bond-holdings, investment and capital to maximise (1) subject to a sequence of budget and capital accumulation constraints.

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5Strictly speaking, this would only require that we let $U(\cdot)$ be strictly increasing and quasi-concave. The slightly stricter assumptions are for mathematical convenience.
The budget constraints take the form

\[ P_t C_t + P_t I_t + R_t^{-1} B_{t+1} \leq B_t + W_t N_t + T_t + P_t R^K_t K_t \]  \hspace{1cm} (5)

The left-hand side gives the allocation of resources to consumption, investment \((I_t)\) and to one-period risk-free bonds \((B_{t+1})\). \(R_t^{-1}\) is the inverse of the risk-free (gross) nominal interest rate and represents the price of a bond that pays one unit of account in period \(t+1\). The right-hand side gives available resources as the sum of bond holdings, labour income from hours worked \((N_t)\), dividends from firms \((T_t)\), and rental income from the capital stock \((K_t)\) owned by the household. Hours worked are given as \(N_t = 1 - L_t\), where the time endowment is normalised to one.

Capital accumulation is restricted to follow a law of motion given by

\[ K_{t+1} = (1 - \delta) K_t + Z_t \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) I_t \]  \hspace{1cm} (6)

where \(I_t\) is the amount of the final good acquired by the household for investment purposes, \(\delta\) represents the depreciation rate of capital, and \(S(.)\) is a function representing investment-adjustment costs. Following Christiano, Eichenbaum and Evans (2005), we assume that \(S(1) = S'(1) = 0\) and \(S''(1) > 0\). \(Z_t\) is the MEI shock, which affects the extent to which resources allocated to investment (net of investment-adjustment costs) increase the capital stock available to be rented out to firms for use in production in the next period. It is therefore a shock to the marginal efficiency of investment; it affects the productivity of new installations while leaving the productivity of the existing capital stock unaffected. Thus, it captures the fact that new technologies often have to be adopted through investment. But it may also reflect disturbances to the capital accumulation process more generally, for instance related to the financing, the regulatory framework, or even to fluctuations in the weather that may affect the installation process.
First-order conditions with respect to consumption and bond holdings give rise to an Euler equation summarising the intertemporal consumption allocation choice. It takes the standard form

$$ R_t^{-1} = E_t \Lambda_{t,t+1} $$

(7)

where

$$ \Lambda_{t,t+1} = \beta \frac{U_{C,t+1}}{U_{C,t}} \frac{P_t}{P_{t+1}} $$

(8)

is the household’s stochastic discount factor.

The first-order condition with respect to consumption and leisure gives the condition

$$ \frac{W_t}{P_t} = \frac{U_{L,t}}{U_{C,t}} $$

(9)

equalising the real wage to the marginal rate of substitution of leisure for consumption.

First-order conditions with respect to investment and capital equates marginal cost and benefits of additional investment and capital so that

$$ 1 = Q_t Z_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] $$

$$ + E_t \left[ \Lambda_{t,t+1} \frac{P_{t+1}}{P_t} Q_{t+1} Z_{t+1} S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right] $$

(10)

and

$$ Q_t = \beta E_t \left\{ \Lambda_{t,t+1} \frac{P_{t+1}}{P_t} \left[ R_{t+1}^K + Q_{t+1} (1 - \delta) \right] \right\} $$

(11)

where $Q_t$, representing Tobin’s $q$, is equal to the ratio of the Lagrange multipliers attached to the capital accumulation equation and the budget constraint, respectively.

We solve the model by log-linearising the equilibrium conditions around the steady state.\(^6\) The log-linearised Euler equation (7) becomes

\(^6\)The steady state is solved in appendix A. Small case variables denote variables expressed in
\[ \sigma c_t = -\nu n_t + \sigma E_t c_{t+1} + \nu E_t n_{t+1} + (r_t - E_t \pi_{t+1}) \]  \hspace{1cm} (12)

with

\[ n_t = -\frac{1 - N}{N} l_t \]

The parameter \( \sigma \) represents the inverse of the elasticity of intertemporal substitution so that

\[ \sigma = -\frac{U_{CCC}}{U_C} \geq 0 \]

while \( \nu \) given as

\[ \nu = \frac{U_{CLN}}{U_C} \]

represents utility acceleration in the terminology of Bilbiie (2011) and Frisch (1959). Note that consumption and leisure are Edgeworth substitutes (complements) if \( \nu < 0 \) (\( \nu > 0 \)) since, in this case, \( U_{CL} < 0 \) (\( U_{CL} > 0 \)). We say that consumption and hours worked are Edgeworth complements (substitutes) if consumption and leisure are Edgeworth substitutes (complements).

The log-linearised (9) gives a standard labour supply schedule

\[ w_t - p_t = \varphi n_t + \gamma c_t \]  \hspace{1cm} (13)

where \( \varphi \) given as

\[ \varphi = \frac{U_{CLN}}{U_C} - \frac{U_{LLN}}{U_L} \]

represents the inverse of the constant-consumption labour supply elasticity with respect to the real wage, and \( \gamma \) given as

\[ \gamma = \frac{U_{CLC}}{U_L} - \frac{U_{CCC}}{U_C} \]

percentage deviations from the steady state with the exception of the nominal interest rate and the inflation rate, which are absolute deviations.
measures the effect on labour effort from the intertemporal allocation of consumption. Since this reflects the shadow price of consumption at time $t$, we follow the literature in referring to $\gamma$ as a measure of the wealth effect on labour supply, cf., e.g., Jaimovich and Rebelo (2009).

The linearised first-order conditions with respect to investment and capital read

\begin{align}
 i_t &= \frac{1}{1+\beta} (\beta E_t i_{t+1} + i_{t-1} + \lambda_s (q_t + z_t)) \\
 q_t &= - (r_t - E_t \pi_{t+1}) + \left(1 - \beta (1 - \delta)\right) E_t r_{t+1}^k + \beta (1 - \delta) E_t q_{t+1} \tag{15}
\end{align}

where the value of $\lambda_s^{-1} \equiv S''(1) > 0$ governs investment-adjustment costs, while capital accumulation becomes

\begin{align}
 k_{t+1} &= (1 - \delta) k_t + \delta (i_t + z_t) \tag{16}
\end{align}

We assume that the MEI shock that affects the economy through these household equilibrium conditions evolves according to the first-order autoregressive process

\begin{align}
 z_t &= \rho_z \log z_{t-1} + \epsilon_{z,t} \tag{17}
\end{align}

where $0 < \rho_z < 1$, and $\epsilon_{z,t} \sim (0, \sigma_z^2)$ is white noise.

### 2.2 Firms

Each firm $i \in [0,1]$ produces a differentiated good, $Y_t(i)$, according to

\begin{align}
 Y_t(i) &= K_t(i)^{\alpha} N_t(i)^{1-\alpha} \tag{18}
\end{align}

where $K_t(i)$ denotes the period-$t$ capital stock rented by firm $i$, and $N_t(i)$ is the number of hours worked in the production process of firm $i$.

Firm $i$’s marginal cost can be found as the Lagrange multiplier from the firm’s
cost minimisation problem

\[ MC_t(i) = \frac{W_t/P_t}{(1 - \alpha) \left( K_t(i)/N_t(i) \right)^\alpha} = \frac{R_t^K}{\alpha (N_t(i)/K_t(i))^{1-\alpha}} \]  

(19)

where \( R_t^K \) denotes the real rental rate of capital. Conditional factor demand schedules imply that firm \( i \) will choose factor inputs such that

\[ \frac{K_t(i)}{N_t(i)} = \alpha \frac{W_t/P_t}{1 - \alpha \frac{R_t^K}{R_t^K}} \]  

(20)

This equation implies that, on the margin, the cost of increasing capital in production equals the cost of increasing labour. Since all firms have to pay the same wage for the labour they employ, and the same rental rate for the capital they rent, it follows that marginal costs (of increasing output) are equalised across firms regardless of any heterogeneity in output induced by differences in prices. Hence, \( MC_t(i) = MC_t \) \( \forall i \) where

\[ MC_t = \frac{1}{1 - \alpha} \left( \frac{\alpha}{1 - \alpha} \right)^{-a} \left( \frac{W_t}{P_t} \right)^{1-a} \left( R_t^K \right)^{\alpha} \]  

(21)

follows from combining (19) and (20).

Firms follow a Calvo price-setting mechanism when setting prices. Each period, a measure \((1 - \theta_p)\) of randomly selected firms get to post new prices, while remaining firms must keep their prices constant. A firm allowed to choose a new price at time \( t \) sets \( P_t(i) = P_t^* \) to maximise the value of the firm to its owners, the households. At time \( t \), this value is given by

\[ \sum_{k=0}^\infty E_t \left\{ \Lambda_{t,t+k} [P_{t+k}(i) Y_{t+k}(i) - \Psi (Y_{t+k}(i))] \right\} \]  

(22)

where \( \Lambda_{t,t+k} \) is the stochastic discount factor, and \( \Psi(.) \) is the cost function (i.e. the value function from the cost minimisation problem described above). Optimisation
is subject to the demand for the firm’s product given by

$$Y^d_t (i) \equiv C_t (i) + I_t (i) = \left( \frac{P_t (i)}{P_t} \right)^{-\varepsilon_p} (C_t + I_t) \quad (23)$$

its production technology, (18), and the restriction from the Calvo mechanism that

$$P_{t+k+1} (i) = \begin{cases} P^*_t (i+1) \quad \text{w.p.} \quad (1 - \theta_p) \\ P_{t+k} (i) \quad \text{w.p.} \quad \theta_p \end{cases} \quad (24)$$

The first-order condition is given by

$$\sum_{k=0}^{\infty} \theta_p^k E_t \{ \Lambda_{t+1} Y_{t+k} (i) \left[ P^*_t - \mu_p P_{t+k} MC_{t+k} \right] \} = 0 \quad (25)$$

where $\mu_p \equiv \varepsilon_p (\varepsilon_p - 1)^{-1}$ is the desired mark-up of price over nominal marginal cost. This condition reflects the forward-looking nature of price-setting; firms take not only current but also future expected marginal costs into account when setting prices.

In log-linear form, aggregate production (18) becomes

$$y_t = \alpha k_t + (1 - \alpha) n_t \quad (26)$$

while log-linearizing (25) and combining it with the law of motion of the price index results in the standard New Keynesian Phillips curve

$$\pi^p_t = \beta E_t \pi^p_{t+1} + \kappa_p mc_t \quad (27)$$

where $\kappa_p = (1 - \beta \theta_p) (1 - \theta_p) \theta_p^{-1}$ and

$$mc_t = (1 - \alpha) (w_t - p_t) + \alpha r_t^h \quad (28)$$
The factor input relation (20) becomes

\[ r^k_t = (w_t - p_t) + n_t - k_t \]  

(29)

2.3 Monetary policy and market clearing

We let central bank responds to inflation and output growth according a simple log-linear monetary policy rule with interest rate smoothing:

\[ r_t = \rho_t r_{t-1} + (1 - \rho_t) \phi_x \pi_t + (1 - \rho_t) \phi_y (y_t - y_{t-1}) \]  

(30)

The model is closed by the aggregate resource constraint which takes the log-linear form

\[ y_t = C Y c_t + I Y i_t \]  

(31)

3 Parameterisation

We parameterise the log-linear model in the previous section in order to study the impulse responses of key variables to MEI shocks. Our objective is not to conduct a calibration exercise to match a number of moments in the model with those in the data. Such an exercise would require the unrealistic assumption that the business cycle is driven only by investment shocks. More modestly, our objective is to illustrate some simple but potentially important economic mechanisms under a plausible parameterisation.

Standard parameters are set at values common in calibrations of macroeconomic models to the US economy. We consider the length of a period to be one quarter, and we let \( \beta = 0.99 \) implying that the annual interest rate is about 4 per cent in steady state. We set the depreciation rate to \( \delta = 0.025 \) and the capital share to \( \alpha = 0.33 \). Desired mark-ups in the goods market are assumed to be 20 per cent (as in Bilbiie, 2011), which we achieve by setting \( \varepsilon_p = 6 \). We set the inverse of the second
derivative of the investment adjustment cost function to $\lambda_v = 0.37$, smaller than the 0.4 estimated by Christiano, Eichenbaum and Evans (2005), but larger than the 0.34 estimated by JPT (2010) and the 0.17 estimated by Smets and Wouters (2007).

The benchmark model features price rigidity and we set $\theta_p = 0.8$ corresponding to five quarters of expected price duration. This is longer than suggested by the microdata evidence provided by Bils and Klenow (2004) and Nakamura and Steinsson (2008). But our model does not feature the kind of strategic complementarities in price setting studied by Ball and Romer (1990) and Sbordone (2002) that would allow us to set the Calvo parameter lower in our macromodel. This value brings the slope of the New Keynesian Phillips curve more in line with reduced-form estimates of this composite parameter, cf. Gali, Gertler and Lopez-Salido (2005) and Levin, Lopez-Salido and Yun (2007). Our results still go through qualitatively with a lower value of $\theta_p$ in line with the microevidence, as shown in the sensitivity analysis in section 3. In addition, wages are flexible in our model. Introducing wage stickiness would allow us to obtain the same results with a much lower degree of price stickiness.\footnote{It is the joint level of nominal rigidity that matter for our results as we discuss in the working paper version of this paper (Furlanetto and Seneca, 2010).}

In some comparisons to the benchmark model, we let prices be flexible by setting $\theta_p = 0$.

In the monetary policy rule, we use estimates from Galí and Rabanal (2005) and we set $\rho_r = 0.69$, $\phi_p = 1.35$ and $\phi_y = 0.26$. Finally, the shock we consider is moderately persistent with $\rho_z = 0.7$. This is in line with values estimated by JPT (2010) and Smets and Wouters (2007).

The remaining four parameters $\sigma, \nu, \varphi$ and $\gamma$ are related to household preferences. Since they differ from those present in models with specific functional forms for the felicity function, we discuss them in more detail. Specifically, the presence of Edgeworth substitutability or complementarity, which is reflected in a non-zero value for the utility acceleration parameter $\nu$, has important implications for both the labour supply elasticity with respect to the real wage and the wealth effect on
labour supply. We discuss each of these in turn.

First, a non-zero \( \nu \) generally drives a wedge between the constant-consumption labour supply elasticity, \( \varphi \), and the Frisch labour supply elasticity defined as the elasticity of labour supply with respect to the real wage when the marginal utility of consumption is constant. To see this, note that the Frisch elasticity, evaluated in the steady state, can be found from (9) as

\[
\psi \equiv \left. \frac{dW}{dN} \right|_{dU_c=0} \frac{N}{W} = \frac{(U_{CL})^2 N}{U_L U_{CC}} - \frac{U_{LL} N}{U_L}
\]

so that

\[
\psi = \varphi - \frac{\nu \gamma}{\sigma}
\]

This means that \( \psi = \varphi \) if and only if \( \nu = 0, \gamma = 0 \) or \( \sigma \to 0 \). Using the Frisch elasticity, an alternative version of the labour supply relation (13) reads

\[
w_t - p_t = \psi n_t - \frac{\gamma}{\sigma} u_{C,t}
\]

Second, a non-zero \( \nu \) drives a wedge between the inverse of the elasticity of intertemporal substitution and the size of the wealth effect on labour supply. From the definitions of \( \nu, \sigma \) and \( \gamma \) it follows directly that \( \gamma = \sigma \) if and only if \( \nu = 0 \). In general, the model’s steady state implies that

\[
\gamma = \sigma + \frac{U_{CL} C}{U_L} = \sigma + \nu \frac{PC}{WN}
\]

cf. appendix A, where \( WN/PC = 0.7 \) with our parameterisation. This means that a small wealth effect on labour supply is associated with a high elasticity of intertemporal substitution, a high degree of Edgeworth substitutability of consumption and leisure, or a combination of the two.

To the best of our knowledge, there is very limited empirical evidence on the
size of the utility acceleration parameter, \( \nu \), and the wealth effect parameter, \( \gamma \).\(^8\) A number of papers have emphasised the importance of non-separable instantaneous preferences following the seminal contribution by Basu and Kimball (2002), see e.g. Guerron-Quintana (2008), Hall (2009), Kilponen (2012), Kilponen, Vilmunen and Vähämaa (2013) and Kim and Katayama (2012). But as they are all based on specific assumptions about functional forms, they provide little direct evidence on the size of \( \nu \). For \( \gamma \), the empirical studies by Imbens, Rubin and Sacerdote (2001) and Kimball and Shapiro (2010) find substantial effects on labour supply from unearned income. This suggests that \( \gamma \) should be positive, but it does not provide direct guidance on the value. Schmitt-Grohé and Uribe (2012) and Dey (2012) provide estimates for \( \gamma \) in the context of DSGE models with Jaimovich-Rebelo preferences. They find values very close to zero. However, both studies calibrate the elasticity of intertemporal substitution to 1. Khan and Tsoukalas (2011), who estimate this elasticity along with \( \gamma \) in a similar framework, find very different results. They arrive at a low elasticity of intertemporal substitution (\( \sigma = 2.45 \)) and an intermediate value for the wealth effect on labor supply (\( \gamma = 0.53 \)).

Given the lack of direct knowledge about \( \nu \) and \( \gamma \), we base our benchmark calibration on empirical evidence on \( \sigma \), \( \varphi \) and \( \psi \), using the steady-state restriction (33) and the definition of the Frisch elasticity (32) to recover values for \( \nu \) and \( \gamma \).

We set \( \varphi^{-1} = 1.5 \) and \( \psi = 1 \) in keeping with the baseline estimates for the these two parameters in Kimball and Shapiro (2010). These values are also close to the estimates provided by Kilponen, Vilmunen and Vähämaa (2013), namely \( \varphi^{-1} = 1.32 \) and \( \psi = 0.96 \). Bilbiie (2011) uses \( \varphi^{-1} = 2 \) and \( \psi = 1 \). Since \( \varphi < \psi \), this implies that \( \nu < 0 \) by (32) so that consumption and hours worked are Edgeworth complements.

Finally, we set \( \sigma \) equal to 2.5. This is in line with results from Basu and Kimball (2002), who find a value around 2 using a single-equation approach, and Vissing-Jorgensen (2002), who finds a value around 2.5 for stock-holders. Similarly, Khan and Tsoukalas (2011) and Kim and Katayama (2012) find a value of 2.45 and 2.20,

\(^8\)The same point is stressed in Bilbiie (2011), Hall (2009) and Schmitt-Grohé and Uribe (2012).
respectively, employing a full information DSGE approach. Higher values are found by Kilponen, Vilmunen and Välimaa (2013) and by Guerron-Quintana (2008). Hall (1988) summarises the evidence up to the 1980s by concluding that the elasticity of intertemporal substitution is unlikely to be much above 0.1 (which implies a $\sigma$ lower than 10). Yogo (2002) conduct a cross-country analysis to estimate the elasticity of intertemporal substitution when instruments are weak and confirms Hall’s (1988) result for the US. A low elasticity of intertemporal substitution is found also by Bilbiie and Straub (2013) with a particular focus on the stability of the parameter estimate. Our choice of a value towards the lower end of the range of estimates for $\sigma$ is a conservative one in that larger values of $\sigma$ would imply a larger degree of Edgeworth complementarity between consumption and hours worked for given values of $\gamma$, $\varphi$ and $\psi$.

With these choice, $\nu = -1.29$ and $\gamma = 0.65$ in our baseline calibration. This compares with Bilbiie’s choice of $\nu = -5$ and $\gamma = 1$, and with $\gamma = 0$ for the utility function specified by GHH and $\gamma = 1$ for the KPR form. Hence, our benchmark specification implies a moderate degree of complementarity between consumption and hours worked and a substantial wealth effect on labour supply as well as plausible values for the elasticity of intertemporal substitution and the wage elasticity of labour supply. We take this as a useful starting point for disentangling the roles played by $\nu$ and $\gamma$ in the transmission mechanism of investment shocks.

We remark that the chosen parameter values ensure that the instantaneous utility function is concave and that consumption and leisure are non-inferior. Applying proposition 1 in Bilbiie (2011) imposes the restrictions that $\tilde{\gamma} \geq 0$, $\varphi \geq 0$ and $\nu \leq (\tilde{\gamma} \varphi) / (\tilde{\gamma} + \varphi)$, where for our model $\tilde{\gamma} \equiv \gamma \left( W N / P C \right)$. These are clearly satisfied

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9 These studies infer the value of the elasticity of intertemporal substitution directly from estimates of the consumption (or output) elasticity to the interest rate. This elasticity may be influenced by the presence of non-separable preferences but also from the presence of habit persistence in consumption or rule-of-thumb consumers. The absence of these features in our model favours the use of a higher value for the elasticity of intertemporal substitution. For a discussion of rule-of-thumb consumers in the response to investment shocks, see Furlanetto, Natvik and Seneca (2013).

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both for our baseline calibration and in our sensitivity analyses, where we restrict attention to cases where $\gamma \geq 0$, $\varphi > 0$ and $\nu \leq 0$.

The general preference specification proposed in the previous section nests some special cases that we consider in the discussion of the results. The standard functional form for the separable utility function is

$$U(C, L) = \overline{U}(C) + V(1 - N)$$

(34)

In this case, $\nu = 0$, $\gamma = \sigma$ and $\varphi = \psi$. A second case is the KPR (1988) functional form

$$U(C, L) = \frac{1}{1-\sigma} C^{1-\sigma} V(1 - N)$$

(35)

It is straightforward to show that $U_{CL}/U_L = 1 - \sigma$ in this case, so that $\gamma = 1$. By the steady-state restriction on $U_{CL}/U_L$, this means that $\nu = -1.04$ when $\sigma = 2.5$, which in turn implies that $\psi^{-1} = 0.92$ when $\varphi^{-1} = 1.5$. A third special case is the functional form proposed by GHH (1988)

$$U(C, L) = \overline{U}(C + V(1 - N))$$

(36)

It is straightforward to show that $U_{CL}/U_L = -\sigma$ in this case, so that $\gamma = 0$. By the steady-state restriction on $U_{CL}/U_L$, this means that $\nu = -1.74$ when $\sigma = 2.5$, and that $\varphi = \psi$.

4 Results

4.1 The baseline in comparison

In figure 1 we plot impulse responses to an investment shock for six main macroeconomic variables. To assess the role of the degree of Edgeworth complementarity between consumption and leisure and the strength of the wealth effect on labour supply, we compare responses under our baseline calibration with three alternative
calibrations for $\nu$ and $\gamma$.

To set the scene, we consider one such alternative first, in which the degree of complementarity is set to zero, leaving the size of the wealth effect at its baseline value. That is, the felicity function takes the additively separable form (34). This makes the model very similar to the one considered in JPT (2010), and not surprisingly, the impulse responses - shown as dashed lines in figure 1 - are similar to the ones reported in that paper. In particular, the response of consumption is negative on impact and for the first five quarters after the shock has occurred.

To provide intuition for this, we follow Barro and King (1984) and JPT (2010) by considering the labour market equilibrium condition. With sticky prices, mark-ups in the goods market will generally deviate from their desired levels. Defining the economy’s average mark-up implicitly as

$$\mu_{p,t} = \frac{MPN_t}{W_t/P_t}$$

where $MPN_t$ is the marginal product of labour, the labour market equilibrium condition can be written as

$$\mu_{p,t}^{-1} MPN_t \left(\frac{N_t}{C_t, N_t}\right) = MRS_t \left(\frac{C_t, N_t}{C_t, N_t}\right)$$

(37)

Here, $\mu_{p,t}$ represents the time-varying wedge driven between the marginal rate of substitution of leisure for consumption, $MRS_t$, and the marginal product of labour, as a consequence of monopolistic competition and nominal rigidities. Note that the marginal rate of substitution depends positively both on consumption and hours worked when consumption and leisure are non-inferior, while the marginal product of labour is decreasing in hours worked.

Now consider the effect of an investment shock and suppose for a moment that

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10Given the steady state restrictions described in the previous section, the elasticity of intertemporal substitution ($\sigma$) and the (inverse of the) Frisch elasticity of labour supply ($\psi$) are affected whenever we depart from the baseline parameterisation for the parameters $\nu$ and $\gamma$. Other parameters remain unchanged. We evaluate the implications for $\sigma$ and $\psi$ later in this section.
prices are flexible so that $\mu_{p,t}$ is constant. By increasing the marginal efficiency of capital, the shock increases the rate of return on investment. As a consequence, households shift demand away from consumption towards investment as part of a process of intertemporal substitution. The decline in consumption works to shift the labour supply curve, i.e. the right-hand side of (37), down in $(N, W/P)$ space. This is the wealth effect on labour supply. Effectively, some of the substitution works through a reduction also in the consumption of leisure. Since capital is predeter-
mined, the economy moves down along a fixed labour demand curve on impact of the shock when prices are flexible. As a result, while consumption declines, hours increase to produce more investment goods. This is an illustration of Barro and King’s (1984) insight, following from (37), that the co-movement of consumption and employment requires shocks directly to the marginal product of labour (or the marginal rate of substitution) when preferences are time-separable and prices are flexible. The MEI shock, which affects the marginal product of labour only indirectly through capital accumulation, is unable to generate such co-movement in this environment.

But when prices are sticky as in figure 1, this is not the only effect at play. With nominal price rigidity, some firms are unable to increase their prices in response to the increase in demand stemming from the investment boom induced by the shock. Consequently, the economy’s average mark-up falls, effectively shifting the labour demand curve, i.e. the left-hand side of (37), up. Now, by (37), consumption and hours may co-move through intratemporal substitution from leisure to consumption. However, in figure 1 (dashed lines) – as in JPT (2010) – this effect is found to be too small to overturn the intertemporal substitution from consumption to investment in the version of model without utility acceleration.

Now consider the responses to the investment shock in the model with our base-
line calibration shown with solid lines in figure 1. While the wealth effect on labour supply is the same as before, the model now features Edgeworth complementar-
ity between consumption and hours worked. In this case, the model delivers co-
movement across real variables, and the response of output and hours is larger than in the previous case. The reason is that, with $v < 0$, an increase in hours worked has a positive effect on the marginal utility of consumption. Consequently, unless monetary policy is very aggressive in increasing interest rates, households will find it optimal to increase consumption along with hours worked as can be seen from the Euler equation (12). In addition, complementarity between consumption and hours worked lowers the degree of intertemporal substitution by increasing $\sigma$. This favours current consumption by reducing size of the intertemporal substitution from consumption to investment. Now that consumption increases, the upward shift in labour demand stemming from price rigidity is accompanied by an upward shift in the labour supply curve. This works to dampen the increase in hours and therefore consumption. The size of the shift is determined by the size of the wealth effect, i.e. by the parameter $\gamma$.

Notice that complementarity only works to generate co-movement when interacting with the effects of nominal rigidity. Absent variable mark-ups, the tight relationship between the marginal product of labour and the marginal rate of substitution of leisure for consumption emphasised by Barro and King (1984) would still prevent consumption and hours worked from co-moving. Therefore, it is the combination of sticky prices and non-separable preferences that allows for macroeconomic co-movement in response to investment shocks. This is in line with the findings of Bilbiie (2011) in his analysis of fiscal shocks.

We now turn to the role played by the wealth effect on labour supply. A large literature has argued that a small wealth effect is needed to achieve macroeconomic co-movement, cf. GHH (1988), Jaimovich and Rebelo (2009) and Schmitt-Grohé and Uribe (2012) among others. In contrast, our baseline model features a sizeable wealth effect, but it is nonetheless able to generate macroeconomic co-movement also of consumption with other real variables.

The x-marked lines in figure 1 show responses when we keep the degree of complementarity between consumption and hours worked as in the baseline calibration,
but reduce the size of the wealth effect to zero. In this case, the increase in consumption brought about by the complementarity between consumption and hours worked does not lead to an upward shift in the labour supply curve. This means that hours and output will increase more, which in turn leads to a further increase in consumption through the effects of complementarity. The reduction in $\gamma$ leads to a lower value of $\sigma$, which works to increase the size of the intertemporal substitution effect towards investment. But the main effect is to amplify the increase in consumption and hours, though this effect is seen to be relatively small, particularly on impact.

A similar mechanism operates in the consumption response when the wealth effect is reduced to zero in the version of the model without Edgeworth complementarity. In this case, a positive wealth effect would work to limit the fall in consumption by a downward shift in the labour supply curve inducing substitution from leisure to consumption as discussed above. When this effect is absent, as shown in the dotted lines in figure 1, all of the intertemporal substitution into investment comes from consumption. Moreover, since $\sigma = 0$ in this case, the intertemporal substitution effect is large. Consequently, consumption falls more when the wealth effect is zero in the case without complementarity between consumption and hours worked, as can be seen by comparing the dotted with the dashed lines in figure 1. In this special case, it turns out that the assumed monetary policy is not sufficiently accommodating given the size of the nominal rigidity to support an increase in output and hours worked on impact of the shock.\footnote{We remark that consumption eventually turns positive while investment turns negative as the transitional growth dynamics kick in as a response to the higher level of capital brought about by the initial investment boom (see, e.g. King and Rebelo, 1999). This is particularly clearly visible in this special case.}

In sum, the responses in figure 1 suggest that the sign of the consumption response is determined mainly by the degree of complementarity between consumption and employment, while the absolute size of the effect is influenced by the strength of the wealth effect. By implication, macroeconomic co-movement is fully compatible
with a positive wealth effect on labour supply. To investigate this link further, we now turn to a more careful analysis of the sensitivity of the consumption response to variation in $\gamma$ and $\nu$.

### 4.2 Sensitivity

The left panel of figure 2 shows how the impact response of consumption varies with the degree of complementarity keeping the size of the wealth effect fixed at the baseline $\gamma = 0.65$. The figure’s right panel shows the effect of varying the size of the wealth effect keeping utility acceleration at its benchmark $\nu = -1.29$. As the figure shows, the consumption response is much more sensitive to the degree of complementarity than to the size of the wealth effect. In particular, the consumption response is negative for values of $\nu$ higher than about $-0.6$, while the response stays positive even for large values of $\gamma$.\(^{12}\) As noted above, there is little direct empirical evidence on what could be a plausible value for $\nu$. However, the choice of $\nu$ has implications for the value of $\sigma$. When $\nu = -0.6$, the steady state restriction in (33) implies that $\sigma = 1.5$ so that the elasticity of intertemporal substitution is high compared to the empirical evidence discussed in section 3. As this favours the higher values of $\sigma$ that are implied by lower values of $\nu$, a positive consumption response does not appear to rely on implausibly large degrees of Edgeworth complementarity between consumption and employment.

In figure 3 we show how the impact response of consumption changes as we vary both the size of the wealth effect (right horizontal axis) and the strength of complementarity (left horizontal axis).\(^{13}\) The impact response under our benchmark calibration is marked with a dot on the response surface. The shape of the curve in the left (right) panel of figure 2 can be recognised by moving along the left (right) horizontal axis from this point.

\(^{12}\)The size of the wealth effect is commonly assumed to be between 0 (as with GHH preferences) and 1 (as with KPR preferences), see e.g. Jaimovich and Rebelo (2009).

\(^{13}\)Note that we graph the negative of $\nu$ on the left horizontal axis. Moving from 0 to 5 on the axis increases the strength of the complementarity.
More generally, the figure shows that the impact response of consumption is very sensitive to the degree of complementarity except for very large values for the wealth effect parameter. For the commonly assumed values between zero and one, the response surface increases steeply as we lower the utility acceleration parameter. The response is negative when complementarity is weak, and positive when complementarity is strong.

With our baseline value for $\nu$, the effect on the impact response from changing $\gamma$ is limited. But when the complementarity is strong, reducing the wealth effect on labour supply greatly amplifies the positive response of consumption. In this region of the parameter space (the western corner of the surface), the elasticity of intertemporal substitution is low, and the upward shift in the labour supply curve is small. Both factors favour a large expansion in consumption on impact of the shock. Alternatively, when complementarity is weak, reducing the wealth effect on labour supply greatly amplifies the negative response of consumption. In this region (the southern corner), the elasticity of intertemporal substitution is high, and the shift to right of the labour supply curve that works to mitigate the negative consumption response is small. Thus, in this case, a low wealth effect on labour supply is outright detrimental for the purpose of generating co-movement in the model.

This brings us to our key point. The degree of Edgeworth complementarity between consumption and hours worked is the main determinant of the sign of the impact response of consumption, whereas the size of the wealth effect on labour supply influences the magnitude of this response. In our baseline model, with what we take to be a plausible parameterisation, we find the influence of the wealth effect to be small. If the complementarity is stronger or weaker than we assume in our baseline, the size of the wealth effect may have a larger effect on the size but not the sign of the consumption response.\textsuperscript{14}

\textsuperscript{14}A discussed above, whenever we change $\nu$ and $\gamma$, we are also changing the values of $\sigma$ and $\psi$, set at 2.5 and 1, respectively, in the baseline calibration. Toward the southern tip of the surface in figure 3, when $\gamma = 0.1$ and $\nu = -0.1$ e.g., we have $\sigma = 0.24$ and $\psi = 0.7$. Towards the western tip, e.g. when $\gamma = 0.1$ and $\nu = -5$, we have $\sigma = 7.3$ and $\psi = 0.74$. When $\gamma = 5$ and $\nu = -5$ (to the north), $\sigma = 12.2$ and $\psi = 2.72$, and when $\gamma = 5$ and $\nu = -0.1$ (to the east), $\sigma = 5.14$ and
In figure 4, we present the sensitivity of the consumption impact response to the degree of nominal rigidity \( \theta_p \), monetary policy responsiveness to inflation \( \phi_p \) and the persistence of the shock \( \rho_z \). As discussed above, nominal rigidity is necessary to render the consumption response positive on impact of the shock. The left panel of figure 4 shows that, with our baseline calibration, the degree of price rigidity has to be larger than about 0.6 corresponding to an expected duration of price contract of two and a half quarters\(^{15}\). Similarly, if monetary policy is very aggressive in fighting inflation to undo the effects of nominal rigidity, consumption will fall even with the degree of complementarity assumed in our baseline, as can be seen in the middle panel of figure 4. Finally, the right panel shows that a very persistent investment shock drives consumption down with our baseline calibration. This is because the intertemporal substitution effect is very strong when the shock is very persistent\(^{16}\).

5 Our results in perspective

In this section, we provide further detail by comparing the results presented in the previous section with those obtained with two widely used special cases for the instantaneous utility function. Moreover, we compare the mechanism highlighted in this paper as a way to obtain co-movement with the alternative proposed by GHH (1988) in a flexible-price environment.

\[ \psi = 0.76. \] These values for \( \sigma \) and \( \psi \) implied by large departures from our baseline calibration of \( \gamma \) and \( \nu \) are far from the values found in the empirical literature, cf. section 3.

\(^{15}\)In our model wages are flexible. Wage stickiness would act as a second shifter of labour demand. In the presence of wage rigidity, the model would rely on a much lower degree of price rigidity to obtain co-movement. Similarly, the presence of rule-of-thumb consumers would favour a positive consumption response for a given degree of nominal rigidity. We consider a model with sticky wages in the working paper version of this article (Furlanetto and Seneca, 2010) and one with rule-of-thumb consumers in Furlanetto, Natvik and Seneca (2013).

\(^{16}\)These results are in line with the Bilbiie’s (2011) findings for fiscal policy shocks. For his model without capital accumulation, he is able to derive a threshold value for \( \nu \) determining the sign of the consumption response, which depends on the degree of price rigidity, the aggressiveness of monetary policy in responding to consumption, and the degree of persistence of the shock. Interestingly, this threshold does not depend on the size of the wealth effect in keeping with our results.
5.1 Two special cases

In figure 5, we show responses to output, consumption, investment and hours worked for our baseline model (solid lines) along with a model with KPR preferences (dashed lines) as in (35) and GHH preferences (dotted lines) as in (36).

KPR preferences are characterised by a somewhat larger wealth effect on labour supply ($\gamma = 1$) and a slightly weaker complementarity ($\nu = -1.04$) than our baseline specification. But these differences are quite small (moving along a relatively flat segment of the surface in figure 3), and the responses remain similar to the baseline case. The impact response of consumption is only slightly lower with KPR preferences as a result of the weaker complementarity. Since KPR preferences are consistent with balanced growth, this makes them an appealing alternative to the fully general utility specification used here.

With GHH preferences, the wealth effect is absent ($\gamma = 0$) and the complementarity is stronger ($\nu = -1.74$). Therefore, the impact response of consumption is larger in this case. In keeping with the discussion in the previous section, this is due both to the higher degree of complementarity implied by GHH preferences and to the absence of a wealth effect. The stronger complementarity both increases the response directly (by about a third of the total difference), and it increases the amplification from reducing the wealth effect on labour supply. In terms of figure 3, the reduction in $\nu$ in isolation moves the economy to a part of the response surface which is steeper in the direction of lower $\gamma$. While the two effects go hand in hand with GHH preferences making it impossible to distinguish between them, our general utility specification allows us to disentangle and vary them separately.

5.2 GHH and variable capacity utilisation

The co-movement problem of consumption following MEI shocks was first addressed by GHH (1988).\textsuperscript{17} More recently, Jaimovich and Rebelo (2009) have analysed the

\textsuperscript{17}GHH (1988) assess the co-movement of consumption by its correlation with output. They do not report impulse response functions. We are able to reproduce the correlations of output with
issue in a similar neoclassical model with both contemporaneous and news shocks. Both papers suggest the combination of preferences with a low or zero wealth effect and variable capacity utilisation as a way to obtain procyclical consumption responses in an RBC model with flexible wages and prices. Similarly to countercyclical mark-ups, variable capacity utilisation works to shift labour demand through its effect on the marginal product of labour. This opens for co-movement by breaking the tight restriction on consumption and hours worked when prices are flexible, cf. (37).

We now further investigate this alternative mechanism to obtain macroeconomic co-movement, and we compare it to the one proposed in our baseline model.

To do so, we introduce variable capacity utilisation into the model in section 2, and we let preferences take the special GHH form to facilitate comparison with the existing literature. We compare two alternative specifications for the cost of changing the utilisation of the capital stock. The first follows the 'maintenance cost' specification of Christiano, Eichenbaum and Evans (2005). The idea behind this specification is that an intensified utilisation of capital increases the cost of maintaining the capital stock. It therefore enters the household budget constraint directly. The second follows the 'user cost' specification of GHH (1988). The idea behind this specification is that an increase in utilisation increases the rate of depreciation of the capital stock. It therefore enters the capital accumulation relation instead of the budget constraint. Details are provided in appendix B.

In figure 6, we compare the model in section 2 with GHH preferences (solid lines) to two alternative versions with variable capacity utilisation. The first alternative has a maintenance cost specification (dashed lines) and the second user costs (dotted lines). Variable capacity utilisation acts an second shifter of labour demand in addition to the movement in average mark-ups resulting from price rigidity. As the investment shock increases output, the household finds it optimal also to increase the utilisation of its capital stock. This increases the marginal prod-
uct of labour, and the labour demand curve shifts up. But as shown in figure 6, the household’s ability to increase utilisation leads to substitution into leisure. By the Edgeworth substitutability between leisure and consumption, this effect favours lower consumption. Therefore, the effect of introducing variable capacity utilisation is largely inconsequential in our model with sticky prices, at least with the maintenance cost specification. With user costs, variable capacity utilisation actually weakens the positive response of consumption.

In figure 7, we consider the effects of introducing variable capacity utilisation into an RBC version of our model with flexible prices ($\theta_p = 0$) and GHH preferences. The solid lines show responses with fixed utilisation as in our benchmark model. As the marginal rate of substitution is independent of consumption and $\mu_{p,t}$ is fixed in this case, it follows from (37) that hours and so (because capital is predetermined) also output stay constant on impact of the shock. Following an MEI shock, consumption will therefore decline enough to exactly offset the increase in investment brought about by the shock. Only as the new investments increase the capital stock will the labour demand schedule gradually shift out, increasing hours, output and the real wage, besides allowing consumption to recover.

Dashed lines show the effect of introducing variable capacity utilisation with maintenance costs. Again hours and output stay constant and consumption falls to off-set the increase in investment despite the fact that an increase in capital utilisation may shift the labour demand curve up in this case. We can show analytically, that this does not happen. By combining linearised first-order conditions, we obtain the following expression linking hours worked to the accumulated capital stock, $k_t$:

$$\left[\left(1 + \frac{1}{\lambda_a}\right)\left(1 - \frac{\alpha}{\alpha}\right) \varphi + 1 + \varphi\right] n_t = k_t$$

(38)

where $\lambda_a$ is the elasticity of the marginal utilisation cost with respect to the rental rate of capital. As the accumulated capital stock is a predetermined variable that

18Specifically, we combine (13), (27), (28), (29), (48) and (49).
cannot respond when the shock hits, it follows that hours worked will stay constant on impact of the shock. From the labour supply relation (which with GHH preferences is independent of consumption) and the condition determining relative factor inputs, it follows that the real wage and the rental rate of capital remain unaffected too. Now, in the absence of movements in the rental rate, households keep the utilisation of capital unchanged, and as this keeps all inputs into production constant, output also remains unaffected on impact of the shock. But then, equilibrium in the goods market will be achieved through intertemporal substitution of consumption and investment only as in the case with fixed capital utilisation.\footnote{Notice that the zero impact response of output depends neither on the calibration nor on the type of shock hitting the economy.}

Dotted lines in figure 7 show responses with the alternative user cost specification. In this case, the tight restriction in (38) no longer holds, and hours worked are free to move on impact of the shock also in this version of the model with GHH preferences and flexible prices. When parameters are kept at their baseline values (as in the dotted lines), we find that hours increase only marginally, while consumption falls. Only when we let both the capital utilisation and the labour input margin be very elastic, e.g. by setting $\varphi^{-1} = 2.5$ and $\lambda_a = 0.15$ (dashed-dotted lines) as in Jaimovich and Rebelo (2009) instead of our baseline $\varphi^{-1} = 1.5$ and $\lambda_a = 1.17$, are we able to generate a positive response of consumption in the RBC model of a magnitude comparable to that of our baseline model. Note, however, that this comes about in the RBC model only with a substantially higher degree of complementarity ($\nu = -1.74$) than in our New Keynesian model ($\nu = 1.29$).

Hence, while the combination of nominal rigidities and Edgeworth complementarity delivers a positive response of consumption, allowing it to co-move with other key variables, under very general conditions, the ability of the combination of GHH preferences and variable capacity utilisation to generate a positive response in an economy with flexible prices is sensitive to the choice of specification and of parameter values. In particular, it relies on the user cost specification of variable capacity
utilisation costs and highly elastic labour and utilisation margins.

6 Conclusion

We have shown that the strength of the wealth effect on labour supply is largely inconsequential for macroeconomic dynamics in response to a MEI shock, whereas the degree of Edgeworth complementarity between consumption and hours worked is key to obtain macroeconomic co-movement. The degree of complementarity determines the sign of the initial consumption response, while the strength of the wealth effect influences the magnitude of the response. Preferences with a wealth effect similar to the one implied by the KPR family can easily deliver co-movement when they are non-separable and prices are sufficiently sticky. An alternative mechanism proposed by GHH (1988) in a flexible price environment achieves co-movement only under stricter conditions.

Given the importance of the interactions between the degree of Edgeworth substitutability between consumption and leisure on one hand, and the strength of the wealth effect on the other, providing more knowledge about the magnitude of these characteristics of household preferences is an important topic for future empirical work. In addition, in future work it would be interesting to further analyse the role of non-separable preferences and wealth effects in the context of a medium scale DSGE model, possibly estimated on data for key macroeconomic variables. One interesting question relates to the labour market. Galí, Smets and Wouters (2011) have argued that a preference specification separable in consumption and labour and with a very low wealth effect on labour supply is very useful in order to reconcile the sluggish behaviour of the average real wage with a (mildly) procyclical labour force participation. However, they do not discuss the implications for the investment-consumption correlation. According to our analysis, separable preferences with a low wealth effect are unlikely to generate co-movement. However, the Galí, Smets and Wouters (2011) model features more frictions (including habit persistence) than
our baseline model, which could potentially affect the co-movement properties of the model. We plan to investigate this issue in future research.
Acknowledgements

For excellent comments and discussion, we thank Susanto Basu, Florin Bibilee, Efrem Castelnovo, Marco Del Negro, Hanna Freystatter, Jordi Galí, Nicolas Groshenny, Jean Imbs, Güneş Kamber, Tim Kehoe, Juha Kilponen, Mariano Kulish, Gisle Natvik, Gert Peersman, Giorgio Primiceri, Peter Sinclair, Tommy Sveen and Rafael Wouters, as well as seminar participants at the Bank of England, Central Bank of Iceland, Reserve Bank of Australia, Reserve Bank of New Zealand, University of New South Wales, University of Padova, ASSET conference 2009 in Istanbul, SAEe 2009 in Valencia, Zeuthen workshop 2010 in Copenhagen, SWIM workshop 2010 in Auckland, NBRE 2010 in Venastul, Dynare conference 2010 in Helsinki, and the CEF conference 2010 in London. Martin Seneca thanks the Central Bank of Iceland for hospitality while the first version of this paper was being written. The opinions expressed here are solely those of the authors and do not necessarily reflect the views of Norges Bank.
A The steady state

Steady-state variables are indicated by omission of time subscripts. In steady state we have $\Pi^p = 0$ where $\Pi^p$ represents steady-state price inflation. From (6) we get $I = \delta K$ and from (7) $R = \beta^{-1}$. From (10) we get $Q = 1$ and so from (11) $R^K = (\beta^{-1} - 1 + \delta)$. (25) implies $MC = \mu_p^{-1}$.

Combining (18) and (19) then gives the restriction

$$\gamma_k \equiv \frac{K}{Y} = \frac{\alpha MC}{R^K}$$

(39)

so that

$$\gamma_i \equiv \frac{I}{Y} = \frac{\delta \alpha}{\mu (\beta^{-1} - 1 + \delta)}$$

(40)

Then, from (31) we get

$$\gamma_c \equiv \frac{C}{Y} = 1 - \gamma_i$$

(41)

Combining (18) and (6) gives

$$Y = N \left( \gamma_i \delta^{-1} \right)^{\frac{\alpha}{1-\alpha}}$$

(42)

and consequently

$$C = \gamma_c Y$$

(43)

while (20) now gives

$$\frac{W}{P} = (1 - \alpha) MC \frac{Y}{N}$$

(44)

Taking $N$ as given, (9) leads to the steady-state restriction

$$\frac{W}{P} = \frac{U_L}{U_C}$$

(45)

By implication,

$$\frac{U_{CL}C}{U_L} = \frac{U_{CL}N}{U_C} \frac{PC}{WN} = \nu \frac{PC}{WN}$$

(46)
Combining (42), (43) and (44) shows that

\[
\frac{WN}{PC} = \frac{1 - \alpha}{\mu_p \gamma_c}
\]

(47)

**B Variable capacity utilisation**

With variable capacity utilisation, each household chooses the rate at which its capital stock is utilised, \( U_t \), which transforms the accumulated capital stock, \( K_t \), into effective capital rented by firms and used in the production process. In logs, we have

\[
k_t = u_t + \bar{k}_t
\]

(48)

Following Christiano, Eichenbaum and Evans (2005), the cost of utilisation is determined by the increasing and convex function \( a(\cdot) \) so that the maintenance cost term \( M_t = a(U_t) \bar{K}_t \) enters the household budget constraint. The first-order condition equates the marginal benefit of raising utilisation with the marginal cost. The log-linear form is given as

\[
r^k_t = \lambda_a u_t
\]

(49)

where \( \lambda_a \) is the elasticity of the marginal cost of utilisation evaluated in the steady state. We set the value of this parameter to 1.17, which is the value estimated by Smets and Wouters (2007). The economy’s resource constraint becomes

\[
y_t = C \frac{c_t}{Y} + I \frac{i_t}{Y} + K \left( \beta^{-1} - 1 + \delta \right) u_t
\]

(50)

Following the user costs specification of GHH (1988), the rate of depreciation is increasing and convex in utilisation so that \( \delta_t = \delta(U_t) \) enters the law of motion for capital instead of the usual depreciation parameter. The first-order condition again equates the marginal benefit with the marginal cost of increasing utilisation. The
log-linear form is

\[ r_t^k = \lambda_u u_t + q_t \] (51)

where \( \lambda_u \) now represents the elasticity of the marginal rate of depreciation. The resource constraint takes the form (31), but the log-linear capital accumulation relation is now

\[ \bar{k}_{t+1} = (1 - \delta) \bar{k}_t + \delta (i_t + z_t) - (\beta^{-1} - 1 + \delta) u_t \] (52)
References


Kimball, M., Shapiro, M., 2010. Labor supply: are the income and substitution effects both large or both small? Manuscript.


Figure 1: Responses to a shock to the marginal efficiency of investment in the baseline model with $\nu = -1.29$ and $\gamma = 0.65$ (solid lines) and in three alternative parameterisations with $\nu = 0$ and $\gamma = 0.65$ (dashed lines), $\nu = -1.29$ and $\gamma = 0$ (x-marked lines), and $\nu = \gamma = 0$ (dotted lines).
Figure 2: Sensitivity of the impact response of consumption to utility acceleration \((\nu)\) and the wealth effect on labour supply \((\gamma)\).
Figure 3: Sensitivity of the impact response of consumption to the degree of Edgeworth complementarity ($-\nu$) and the wealth effect on labour supply ($\gamma$).
Figure 4: Sensitivity of the impact response of consumption to the degree of nominal rigidity ($\theta_p$), the responsiveness of monetary policy to inflation ($\phi_p$) and the persistence of the shock to the marginal efficiency of investment ($\rho_z$).

Figure 5: Responses to a shock to the marginal efficiency of investment in the baseline model (solid lines) and in two alternative parameterisations with KPR (dashed lines) and GHH preferences (dotted lines), respectively.
Figure 6: Responses to a shock to the marginal efficiency of investment in the baseline model (solid lines) and in two alternative version with variable capacity utilisation with maintenance (dashed lines) and user costs (dotted lines), respectively.
Figure 7: Responses to a shock to the marginal efficiency of investment in a RBC version of the baseline model (solid lines) and in three alternative versions with variable capacity utilisation.