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Implementing the Zero Lower Bound in an Estimated Regime-Switching DSGE Model[☆]

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Abstract

The Zero Lower Bound (ZLB) on policy rates is one of the key monetary policy issues *du jour*. In this paper we investigate the problem of modelling and estimating the ZLB in a simple New Keynesian model with regime switches. The key features of the model include switches in the time preference shock, productivity growth rate and the steady state rate of inflation leading to two steady states: a normal steady state and a ZLB steady state. The model is fitted to US data using Bayesian methods and is found to match the US experience over the great moderation and the ZLB periods very well. The key features of the model allow us to test competing theories about the determinants of the ZLB steady state. Our results suggest that the ZLB steady state is driven by precautionary savings behavior. It is also found that expectations over different regimes crucially matter for the dynamics of the system.

Keywords: Zero Lower Bound, Regime-switching, DSGE, Bayesian Estimation

1. Introduction

The Zero Lower Bound (ZLB) is one of the key policy issues *du jour*. Policymakers have been concerned about the implications of spending prolonged periods at the ZLB, the use of unconventional monetary policies, forward guidance, how to exit the ZLB and what can be done to avoid reaching the ZLB in the first place. Modellers and econometricians have been equally interested and challenged by the problems posed by modelling and estimating DSGE models at the ZLB. These problems are by no means transitory. Practitioners will

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need to continue modelling the ZLB period long after the economy has exited the ZLB if they wish to make use of all available data.

We investigate the problem of modelling and estimating the ZLB in a simple regime-switching DSGE model for the US. The model is standard along most dimensions, but features switches in the preference shock term, the productivity growth rate and the steady state rate of inflation. Crucially, we assume the ZLB is characterized by a separate steady state and derive conditions under which it is feasible. With this modelling strategy and the estimation methodology employed, the model is able to match the US experience both before and during the ZLB period and correctly determine the transition from normal times to the ZLB period. Furthermore we are able to test competing theories about the determinants of the ZLB steady state, which has important implications for policy.

The ZLB is inherently a non-linear problem due to the asymmetry it creates in interest rates. This creates challenges from both a modelling and an estimation perspective. The DSGE literature has approached this problem from several different angles. The main solution approaches considered can be summarized as follows: extended path solutions, piecewise-linear solutions, anticipated shocks, projection methods and regime-switching methods. These methods vary in their underlying assumptions, economic interpretation, computational burden and their compatibility with existing estimation techniques. We review each solution methodology, with respect to the ZLB, in turn.

[Adjemian & Juillard \(2010\)](#) and [Braun & Köber \(2011\)](#) demonstrate how the extended path solution can be used to impose a binding ZLB constraint in a DSGE model. [Adjemian & Juillard \(2010\)](#) show these models can be estimated using Simulated Method of Moments (SMM). Likelihood-based estimation and filtering in combination with these methods remains elusive, limiting their appeal.

Piecewise-linear solutions for models with occasionally binding constraints have been developed by [Jung et al. \(2005\)](#), [Cagliarini & Kulish \(2013\)](#) and [Guerrieri & Iacoviello \(2015b\)](#). Using the method of [Cagliarini & Kulish \(2013\)](#), [Kulish et al. \(2014\)](#) choose a point in the future in which the constraint is no longer binding and solve backwards to find the sequence of model solutions consistent with the binding ZLB constraint. While convenient and computationally cheap, these methods have some drawbacks. In particular these solutions are obtained using the linearized model; as a consequence they do not correspond with the original non-linear model. They assume that in normal times agents are ignorant of the existence of the constraint, agents do not assume the constrained periods to be recurrent, and the binary nature of the solution means agents' behavior in the near vicinity of the constraint is not affected by the probability of the constraint binding.

[Lindé et al. \(2015\)](#) impose the ZLB constraint in expectation using anticipated shocks. They use the conditional forecasting routines of [Maih \(2010\)](#) and [Juillard & Maih \(2010\)](#) to condition on the ZLB constraint during the ZLB period. Their approach does not take into account agents' behavior changing in the neighborhood of the ZLB, even when it is not binding. However this method could be combined with other approaches such as regime-switching for example.

Projection methods have been used by [Gust et al. \(2012\)](#), [Fernández-Villaverde et al.](#)

(2015) and [Maliar & Maliar \(2015\)](#) to impose the ZLB constraint. These methods are accurate and they take into account the probability of the constraint binding on agents' behavior. Nevertheless, these methods do not have a lot to say about how the economy enters the ZLB and why it stays at the ZLB, reducing their economic relevance. Moreover these methods are computationally expensive, limiting the size of models that can feasibly be solved. Combining estimation with projection methods puts additional restrictions on model complexity so that only toy models can realistically be estimated, further hindering their use in the policy environment.

Regime-switching methods, as advocated in this paper, are particularly appealing for modelling occasionally binding constraints for several reasons. First, agents are aware of the regime in which the constraint binds and form expectations accordingly. Second, agents are aware that the constraint binding could be a recurring event. Third, in the vicinity of the constraint binding, agents assign a significant probability of switching to the binding regime, so that the constraint impacts the system even when it is not binding. And fourth, the transition probabilities can be endogenized, allowing the probability of the constraint binding to depend on the state of the economy.¹

Regime-switching has been used to model the ZLB constraint by [Aruoba et al. \(2013\)](#), [Bianchi & Melosi \(2014\)](#) and [Gavin et al. \(2015\)](#). [Bianchi & Melosi \(2014\)](#) solve their linearized regime-switching model with a ZLB constraint around a unique deterministic steady state. In the context of a regime-switching framework, it is more natural to think of the ZLB period in the US, based on its duration, as a separate steady state the economy fluctuates around, an approach we adopt in this paper. [Aruoba et al. \(2013\)](#) and [Gavin et al. \(2015\)](#) also use models with two steady states: a normal steady state, and a deflationary steady state where nominal interest rates are zero. The choice of a deflationary steady state is motivated by [Benhabib et al. \(2002\)](#), who show that Taylor rules can admit multiple equilibria, including a deflationary ZLB equilibrium. [Aruoba et al. \(2013\)](#) perform estimation and filtering exercises on US and Japanese data. They find evidence of a deflationary ZLB steady state in Japan, but little support for a deflationary steady state in the US. This is because inflation has remained largely positive in the US over the ZLB period excluding 2009Q2 and 2009Q3.

The deflationary steady state is, however, not the only equilibria consistent with a zero nominal interest rate. [Benhabib et al. \(2002\)](#) assume the steady state real interest rate is positive when formulating their deflationary ZLB equilibria. If on the other hand the steady state real interest rate is allowed to go negative, then there exists a ZLB steady state with positive inflation. The class of piecewise linear solutions used by [Eggertsson & Woodford \(2003\)](#), [Christiano et al. \(2011\)](#) and [Braun et al. \(2015\)](#) all consider the possibility of a ZLB equilibrium with a negative steady state real interest rate. Their general methodology

¹Moreover, regime-switching models can be solved using perturbation methods which are fast and efficient allowing the possibility to solve larger models and to higher orders of approximation (see [Maih, 2015](#), for example). Regime-switching models solved using perturbation methods permit the use of efficient filtering and estimation routines (see [Alstadheim et al., 2013](#); [Binning & Maih, 2015](#), for example).

assumes the economy begins in the ZLB steady state. The parameters that govern the steady state are determined by a Markov chain where it is assumed the normal steady state is absorbing. [Christiano et al. \(2011\)](#) and [Braun et al. \(2015\)](#) in particular assume the shock process to the time discount factor can shift, generating negative steady state real interest rates, a property more consistent with the US experience at the ZLB. We use a similar mechanism in our model to explain the ZLB period in the US. [Braun et al. \(2015\)](#) provide restrictions on the size of the implied time discount factor relative to the transition probabilities to ensure households' utility and firms' pricing decisions are well defined. These conditions are easily met in their framework because the normal regime is assumed to be absorbing, a convenient although somewhat unrealistic assumption. We do not force any of the regimes to be absorbing in our model, and derive more general conditions for well-defined optimization problems.

Serious attempts at estimating DSGE models using US data over the ZLB period with binding ZLB constraints have been few and far between in the literature. This reflects the complexity of the models used to explain the ZLB, the complexity of the estimation procedures and the incongruence of the deflationary ZLB steady state assumed by some authors. Instead, many authors chop their data sets before the onset of the financial crisis, throwing away precious data points and failing to explain the most recent history (see [Gali et al., 2012](#); [Del Negro et al., 2015](#), for example).

Few papers in the literature freely estimate the ZLB period. [Gust et al. \(2012\)](#) use a particle filter which is computationally expensive, limiting the size of models that can be used. [Kulish et al. \(2014\)](#) and [Guerrieri & Iacoviello \(2015a\)](#) have to resort to non-standard filtering procedures because of the peculiarity of their solution methods.² Just like the papers mentioned above, we freely estimate the ZLB period. However, our estimation methodology differs from theirs in that we use standard filtering algorithms along the lines of [Kim & Nelson \(1999\)](#).

The paper proceeds as follows. We investigate the stylized facts of the great moderation and ZLB periods in the US to find steady states consistent with these periods (Section 2). We present a simple New Keynesian DSGE model and its derivations (Section 3) and highlight potential sources of steady state interest rate shifts (Section 4). We present a solution and estimation methodology that is both efficient and consistent with our modelling assumptions (Section 6). We test competing theories about the determinants of the ZLB steady state and find support for precautionary savings having played a key role in the fall of interest rates in the ZLB state (Sections 5 and 7). Further investigation of our model reveals the important role the expectations channel plays in determining behavior at the ZLB and the non-linear/state dependent behavior of the model (Section 8). All analysis has been carried out using the RISE toolbox in Matlab.

²[Bianchi & Melosi \(2014\)](#) estimate a model over the ZLB period, but they force regime switches in their model to occur at pre-specified points in time, so that ex post agents knew they were in each regime with complete certainty. [Aruoba et al. \(2013\)](#) estimate a surrogate model on US data up to 2007Q4 and Japanese data up to 1994Q4, before using particle filtering techniques and calibrated transition probabilities to determine the probability the economy was at a deflationary ZLB steady state during the ZLB periods.

2. Zero Lower Bound Accounting and Some Stylized Facts for the US

In this section we do two things: first, we establish the parameters responsible for determining the steady state interest rate in a standard DSGE model with balanced growth. Second, we calculate and plot some simple sample means of the fed funds rate, the per capita GDP growth rate and GDP deflator inflation, in the US both before and during the ZLB periods. Our goal is to build and estimate a simple two state regime-switching DSGE model with a separate ZLB steady state that is consistent with the US experience. With this purpose in mind, we need to know which parameters determine steady state nominal interest rates and, more importantly, which parameter shifts would be consistent with US stylized facts.

In standard DSGE models with balanced growth, the consumption euler equation determines the steady state nominal interest rate as follows

$$R_t = \frac{\Pi_t(1+g)}{\beta}. \quad (1)$$

Correspondingly, the steady state real interest rate is given by

$$\mathbb{R}_t = \frac{1+g}{\beta}, \quad (2)$$

where R_t is the steady state gross nominal interest rate, \mathbb{R}_t is the steady state gross real interest rate, Π_t is the steady state gross inflation rate, g is the growth rate of consumption and β is the time discount factor. It then follows from equations (1) and (2) that

$$\Delta R_t \approx \Delta \Pi_t + \Delta g - \Delta \beta = \Delta \Pi_t + \Delta \mathbb{R}_t. \quad (3)$$

In a standard DSGE model with balanced growth, a shift in the steady state nominal interest rate requires either a shift in the steady state rate of inflation, the growth rate of consumption, the rate of time preference, or a combination of these parameters.

We use equation (3) to carry out a very rudimentary steady state ZLB accounting exercise on US data between 1985Q1 and 2015Q2.³ More precisely, we compare the sample means for the fed funds rate, US GDP per capita growth and US GDP deflator inflation, both before and during the ZLB period in order to characterize the properties of a ZLB steady state. A full description of the data can be found in [Appendix A](#).

Figure 1 presents the quarterly gross nominal interest rate constructed using the fed funds rate.⁴ The mean of the quarterly gross interest rate fell by 0.0122 (4.97% in net annualized terms) from 1.0125 (5.09%) between 1985Q1 and 2008Q3 to 1.0003 (0.12%) between 2009Q1 and 2015Q1.

³This will also be the sample period used for estimation. This period is chosen because it contains the great moderation and ZLB periods.

⁴More precisely: $R_t \equiv 1 + FFR_t/400$, where FFR_t is the fed funds rate.

Figure 1: Quarterly Gross Interest Rate

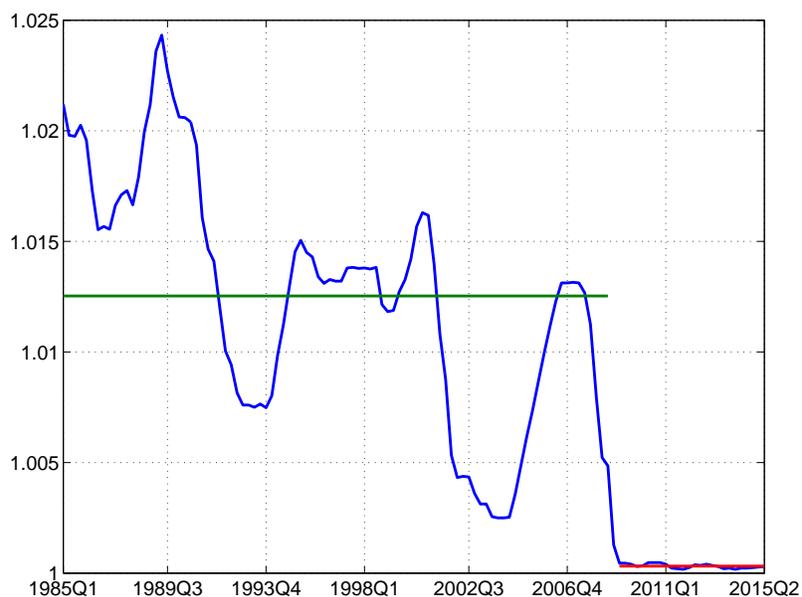
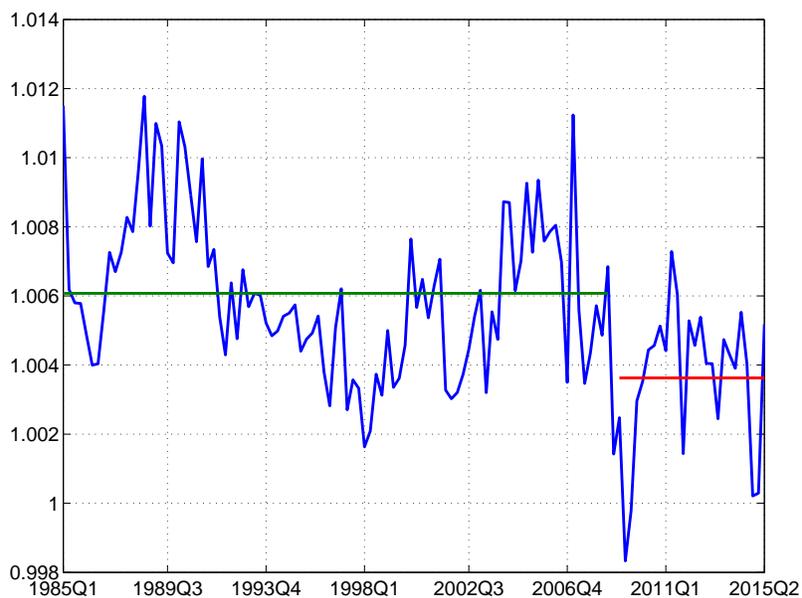
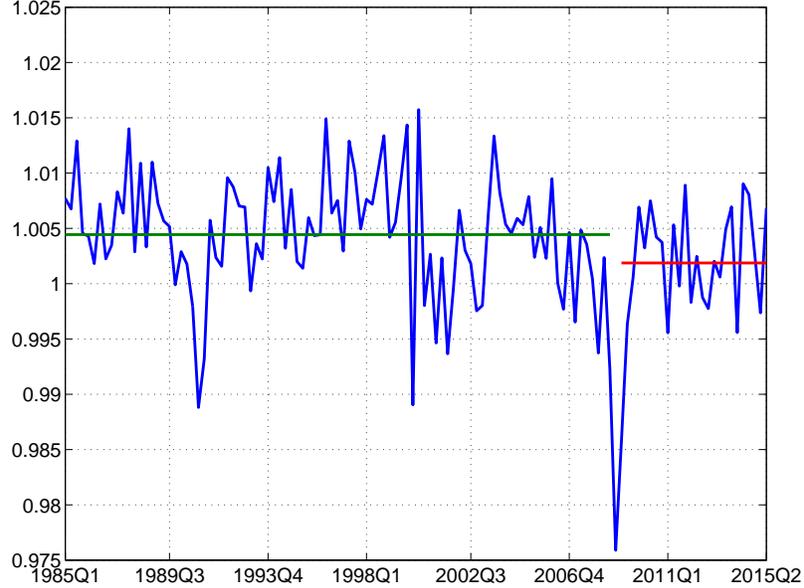


Figure 2: Quarterly Gross Inflation



The quarterly gross inflation rate is plotted in Figure 2. The mean gross inflation rate fell by 0.0024 from 1.0061 between 1985Q1 and 2008Q3 to 1.0036 between 2009Q1 and 2015Q2. The fall in the mean inflation rate by itself is not large enough to explain the fall in nominal interest rates over the same period, a result consistent with analysis by [Aruoba et al. \(2013\)](#). The implied steady state real interest rate for this period must have fallen as well.

Figure 3: Quarterly Gross Per Capita GDP Growth

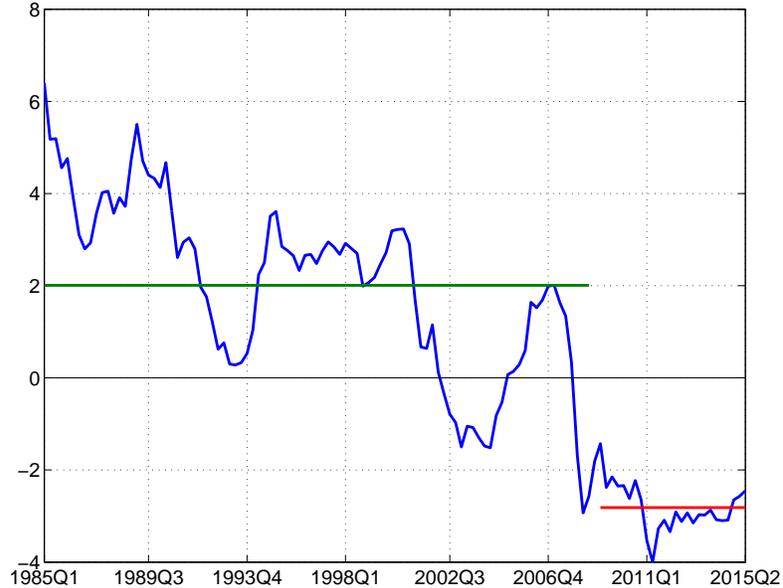


The quarterly gross per capita GDP growth rate is plotted in Figure 3. The mean per capita GDP growth rate fell by 0.0026 from 1.0044 between 1985Q1 and 2008Q3, to 1.0019 between 2009Q1 and 2015Q2. This fall in GDP growth, or more specifically the weak aggregate demand growth has been referred to as secular stagnation.⁵ The difference in the average pre and post crisis growth rates is not large enough by itself to explain the fall in nominal interest rates. The combined fall in inflation and the growth rate is 0.0050, which is still smaller than the 0.0122 fall in nominal interest rates.

Using equation (1) we can calculate the implied rate of time preference β . Between 1985Q1 and 2008Q3 β is calculated as follows $1.0061 \times 1.0044/1.0125 = 0.9981$. The implied β between 2009Q1 and 2015Q1 is $1.0036 \times 1.0019/1.0003 = 1.0052$, which violates the condition that $\beta < 1$. This also suggests the steady state gross real interest rate must have been less than one (and the net real interest rate negative) over this period. This is reflected in the one-year expected real interest rate plotted in Figure 4. The expected real interest rate is constructed using the US 1-year treasury rate and the Michigan survey of inflation expectations, one year ahead. See Appendix A for further details. The average expected real interest rate between 2009Q1 and 2015Q2 fell significantly from the average between 1985Q1 and 2008Q3.

⁵Summers (2014) has hypothesized that aggregate demand had collapsed before the financial crisis but was propped up by financial stimulus. In response to large shocks that hit the economy during the financial crisis, households saved well in excess of the investment opportunities available, causing a decline in the (real) natural rate of interest. The ZLB prevented the interest rate from falling further to a level required to clear this excess. Krugman (2013) has adopted a similar interpretation of the situation, but believes the excess saving was driven by workers' expectations of low future income growth.

Figure 4: Expected Real Interest Rate (1 Year)



If we are to think of the ZLB period as belonging to a distinct steady state, then the conventional algebra and assumptions for determining steady state interest rates in DSGE models with balanced growth need to be revisited. In the next section we build a simple regime-switching New Keynesian DSGE model, with a mechanism proposed by [Christiano et al. \(2011\)](#) and [Braun et al. \(2015\)](#), that is capable of accounting for discrepancies in conventional interest rate accounting at the ZLB. More specifically, we build a regime-switching model with two steady states; an inflation-targeting steady state and a ZLB steady state that allows for the possibility of negative (steady state) real interest rates. [Bianchi & Melosi \(2014\)](#) have proposed a similar mechanism in the context of a regime-switching model with a single steady state.

3. A Simple New Keynesian DSGE Model

We use a canonical New Keynesian DSGE model to investigate the ZLB period in the US. The model we choose is simpler than would usually be considered for policy analysis, but we have chosen this model to highlight the features that allow us to match the ZLB period. We require the model exhibit balanced growth in order to explore the role changes in productivity growth could have played in explaining the ZLB steady state. Following [Braun et al. \(2015\)](#) we introduce an additional time preference “shock” term to allow for movements in the steady state interest rate beyond those explained by the parameters governing conventional consumption euler equation dynamics.

The model consists of a representative household that derives utility from consuming final goods and disutility from working. Households supply labor services to firms and receive labor income in return. They lend to each other and receive dividends from the firms

they own. There is a continuum of firms, normalized to unit mass, producing differentiated intermediate goods using labor services and a common technology. Firms are subject to a quadratic adjustment cost à la [Rotemberg \(1982\)](#) when changing prices. The intermediate goods are aggregated into final goods by a perfectly competitive aggregator firm. The economy is closed by the monetary authority who sets policy according to a Taylor rule in the absence of the ZLB binding.

3.1. Households

The representative infinitely lived household seeks to maximize lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{j=0}^t d_j \right) \left\{ A_t \frac{(\mathcal{C}_t/Z_t)^{1-\sigma}}{1-\sigma} - \kappa \frac{N_t^{1+\eta}}{1+\eta} \right\},$$

where $\mathcal{C}_t = C_t - \chi C_{t-1}$ is consumption adjusted for the external habit stock, C_t is consumption, Z_t is the level of technology in the economy and N_t is labor. Following [Braun et al. \(2015\)](#), βd_{t+1} is the time discount factor, where d_{t+1} is a preference shock whose value is revealed at the beginning of period t and $d_0 = 1$. We investigate the process d_j follows in subsequent sections. [Gust et al. \(2012\)](#) use a similar autoregressive shock process to explain the ZLB period; its presence captures precautionary savings behaviors the model is otherwise unable to account for. [Benigno & Fornaro \(2015\)](#) provide microfoundations for d_j , explaining its presence in terms of precautionary savings behavior generated by idiosyncratic unemployment risk.⁶ σ is the inverse of the intertemporal elasticity of substitution and η is the inverse of the Frisch elasticity of labor supply. A_t is a consumption preference shock process such that

$$\log(A_t) = \rho_A \log(A_{t-1}) + \sigma_A \varepsilon_t^A. \quad (4)$$

Agents maximise their expected discounted stream of utility by choosing consumption, labor and bond holdings subject to the resource constraint

$$B_t + C_t = \frac{B_{t-1} R_{t-1}}{\Pi_t} + W_t N_t + \Phi_t,$$

⁶[Benigno & Fornaro \(2015\)](#) sketch out a stylized model with idiosyncratic unemployment risk and incomplete unemployment insurance, in the context of an economy in a stagnation trap. The fear of becoming unemployed in the next period and receiving lower income, and consuming less also raises the expected marginal utility of consuming next period. The probability of becoming unemployed is assumed constant, and the decline in income and consumption is assumed to be proportionately less than that of being employed. As a consequence the expected marginal utility of consumption increases by a proportional amount which results in an extra constant term in the consumption Euler equation. This term takes the same form as those used by [Christiano et al. \(2011\)](#) and [Braun et al. \(2015\)](#) in that it is larger than 1 and it multiplicatively augments the time discount factor.

where B_t are bond holdings, R_t is the nominal interest rate, Π_t is the gross rate of inflation, W_t is the real wage and Φ_t is profits. Setting up the Lagrangean

$$\begin{aligned} \mathcal{L}_t = E_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{j=0}^t d_j \right) & \left\{ A_t \frac{(\mathcal{C}_t/Z_t)^{1-\sigma}}{1-\sigma} - \kappa \frac{N_t^{1+\eta}}{1+\eta} \right\} \\ & - E_0 \sum_{t=0}^{\infty} \lambda_t \beta^t \left(\prod_{j=0}^t d_j \right) \left\{ B_t + C_t - \frac{B_{t-1}R_{t-1}}{\Pi_t} - W_t N_t - \Phi_t \right\}. \end{aligned}$$

This results in the following first order conditions

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial C_t} &= A_t (C_t - \chi C_{t-1})^{1-\sigma} Z_t^{\sigma-1} - \lambda_t = 0, \\ \frac{\partial \mathcal{L}_t}{\partial N_t} &= \kappa N_t^\eta - \lambda_t W_t = 0, \\ \frac{\partial \mathcal{L}_t}{\partial B_t} &= \lambda_t - E_t \left\{ \beta d_{t+1} \frac{\lambda_{t+1} R_t}{\Pi_t} \right\} = 0. \end{aligned}$$

From the first order conditions we get the marginal utility of consumption

$$\lambda_t = A_t (C_t - \chi C_{t-1})^{-\sigma} Z_t^{\sigma-1}, \quad (5)$$

the marginal rate of substitution

$$W_t = \kappa N_t^\eta / \lambda_t, \quad (6)$$

and the stochastic discount factor

$$\mathcal{M}_{t,t+1} = E_t \left\{ \beta d_{t+1} \frac{\lambda_{t+1}}{\lambda_t \Pi_{t+1}} \right\}, \quad (7)$$

where it follows

$$\mathcal{M}_{t,t+1} = \frac{1}{R_t}. \quad (8)$$

3.2. Firms

The economy consists of a continuum of firms indexed by i normalized to have a unit mass. The i th firm produces output according to the following production technology

$$Y_{it} = Z_t N_{it}, \quad (9)$$

where Y_{it} is the output produced by the i th firm, N_{it} is the labor demanded by the i th firm and technology evolves according to

$$Z_t = Z_{t-1} \exp(g_Z + \sigma_Z \varepsilon_t^Z), \quad (10)$$

where g_Z is the growth rate of productivity. Firms choose prices subject to a quadratic cost à la Rotemberg (1982). Firm i 's real profit is determined by

$$\Phi_{it} = \frac{P_{it}}{P_t} Y_{it} \exp(\sigma_\pi \varepsilon_t^\pi) - W_t N_{it} - \frac{\phi}{2} Y_t \left[\frac{P_{it}}{P_{it-1}} - \tilde{\Pi}_t \right]^2.$$

Following Lombardo & Vestin (2008), $\exp(\sigma_\pi \varepsilon_t^\pi)$ is a stochastic subsidy to firms. The quadratic price adjustment cost is relative to the inflation index

$$\tilde{\Pi}_t = \Pi_t^\xi \bar{\Pi}^{1-\xi}.$$

Firm i 's expected discounted sum of future profits is given by

$$\Xi_{it} = E_0 \sum_{t=0}^{\infty} \mathcal{M}_{t,t+1} P_t \left[\frac{P_{it}}{P_t} Y_{it} \exp(\sigma_\pi \varepsilon_t^\pi) - W_t N_{it} - \frac{\phi}{2} Y_t \left[\frac{P_{it}}{P_{it-1}} - \tilde{\Pi}_t \right]^2 \right]. \quad (11)$$

Firms maximize profits by choosing prices subject to the demand constraint

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t,$$

and the production function. Substituting these into equation (11) gives

$$\Xi_{it} = E_0 \sum_{t=0}^{\infty} \mathcal{M}_{t,t+1} P_t \left[\left(\frac{P_{it}}{P_t} \right)^{1-\varepsilon} Y_t \exp(\sigma_\pi \varepsilon_t^\pi) - \frac{W_t}{Z_t} \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t - \frac{\phi}{2} Y_t \left[\frac{P_{it}}{P_{it-1}} - \tilde{\Pi}_t \right]^2 \right].$$

The firm's first order condition with respect to prices is given by

$$(1 - \varepsilon) Y_{it} \exp(\sigma_\pi \varepsilon_t^\pi) + \varepsilon \frac{W_t}{Z_t} Y_{it} \frac{P_t}{P_{it}} - \phi \frac{P_t Y_t}{P_{it-1}} \left[\frac{P_{it}}{P_{it-1}} - \tilde{\Pi}_t \right] + \dots \\ \dots + E_t \left\{ \phi \mathcal{M}_{t,t+1} P_{t+1} Y_{t+1} \frac{P_{it+1}}{P_{it}^2} \left[\frac{P_{it+1}}{P_{it}} - \tilde{\Pi}_{t+1} \right] \right\}.$$

Assuming symmetry of the firms allows us to write the non-linear Philips curve as follows

$$\left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{W_t}{Z_t} - \exp(\sigma_\pi \varepsilon_t^\pi) - \left(\frac{\phi}{\varepsilon - 1} \right) \Pi_t \left[\Pi_t - \tilde{\Pi}_t \right] + \dots \\ \dots + E_t \left\{ \left(\frac{\phi}{\varepsilon - 1} \right) \mathcal{M}_{t,t+1} \Pi_{t+1}^2 \frac{Y_{t+1}}{Y_t} \left[\Pi_{t+1} - \tilde{\Pi}_{t+1} \right] \right\}. \quad (12)$$

3.3. Monetary Policy

The monetary authority sets policy according to the rule

$$R_t = \max(R_t^E, R_t^*), \quad (13)$$

where R_t^E is the interest rate at the effective lower bound and R_t^* is a Taylor-type rule of the form

$$R_t^* = R_{t-1}^{*\rho_r} \left(\bar{R}^* \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\kappa_\pi} \left(\frac{Y_t Z_{t-1}}{Y_{t-1} Z_t} \right)^{\kappa_y} \right)^{1-\rho_r} \exp(A_t^R), \quad (14)$$

where

$$A_t^R = \rho_{AR} A_{t-1}^R + \sigma_r \varepsilon_t^R. \quad (15)$$

Persistence is added to the monetary policy shock to remove serial correlation. The interest rate at the effective lower bound is determined according to

$$R_t^E = K + \sigma_E \varepsilon_t^E, \quad (16)$$

where K is equal to the interest rate at the effective lower bound and the shock $\sigma_E \varepsilon_t^E$ allows for a small amount of variation in the interest rate preventing a stochastic singularity. [Bianchi & Melosi \(2014\)](#) make a similar assumption about interest rates at the effective lower bound, however they assume that interest rates follow an autoregressive process.⁷

3.4. Market Clearing

The goods market clearing condition is given by

$$Y_t = C_t + \frac{\phi}{2} Y_t \left[\Pi_t - \tilde{\Pi}_t \right]^2. \quad (17)$$

In a symmetric equilibrium; $Y_t = \int_0^1 Y_{it} di$, $N_t = \int_0^1 N_{it} di$, $\Phi_t = \int_0^1 \Phi_{it} di$, and $B_t = 0$. The model variables,

$$x_t = [C_t, N_t, R_t, \Pi_t, W_t, \lambda_t, \mathcal{M}_{t,t+1}, Y_t, R_t^*, R_t^E, A_t^R, Z_t, A_t]^',$$

are described by equations (4) - (10) and (12) - (17).

3.5. Stationarizing the Model

The model exhibits trend growth, so the non-stationary variables need to be stationarized in order to solve the model. There is only one trend process in the model so we stationarize everything relative to the productivity stock as follows; $\tilde{\lambda}_t = \lambda_t Z_t$, $\tilde{C}_t = C_t/Z_t$, $\tilde{Y}_t = Y_t/Z_t$,

⁷[Kulish et al. \(2014\)](#) take a different approach and assume the interest rate is exactly equal to zero over the ZLB period, removing the need to add a measurement error.

$\tilde{W}_t = W_t/Z_t$, $\mu_t = \exp(g_Z + \sigma_Z \varepsilon_t^Z)$. Equations (5), (6), (7), (9), (12), (14) and (17) become

$$\tilde{\lambda}_t = A_t \left(\tilde{C}_t - \chi \tilde{C}_{t-1} / \mu_t \right)^{-\sigma}, \quad (18)$$

$$\tilde{W}_t = \kappa N_t^\eta / \tilde{\lambda}_t, \quad (19)$$

$$\mathcal{M}_{t,t+1} = E_t \left\{ \beta d_{t+1} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t \Pi_{t+1} \mu_{t+1}} \right\}, \quad (20)$$

$$\tilde{Y}_t = N_t, \quad (21)$$

$$\begin{aligned} & \left(\frac{\varepsilon}{\varepsilon - 1} \right) \tilde{W}_t - \exp(\sigma_\pi \varepsilon_t^\pi) - \left(\frac{\phi}{\varepsilon - 1} \right) \Pi_t \left[\Pi_t - \tilde{\Pi}_t \right] + \dots \\ & \dots + E_t \left\{ \left(\frac{\phi}{\varepsilon - 1} \right) \mathcal{M}_{t,t+1} \Pi_{t+1}^2 \frac{\tilde{Y}_{t+1}}{\tilde{Y}_t} \mu_{t+1} \left[\Pi_{t+1} - \tilde{\Pi}_{t+1} \right] \right\}, \quad (22) \end{aligned}$$

$$R_t^* = R_{t-1}^{*\rho_r} \left(\bar{R}^* \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\kappa_r} \left(\frac{\tilde{Y}_t}{\tilde{Y}_{t-1}} \right)^{\kappa_y} \right)^{1-\rho_r} \exp(A_t^R), \quad (23)$$

$$\tilde{Y}_t = \tilde{C}_t + \frac{\phi}{2} \tilde{Y}_t \left[\Pi_t - \tilde{\Pi}_t \right]^2. \quad (24)$$

We also add the following measurement equation to the model

$$\Delta \log(Y_t) = \log(\tilde{Y}_t) - \log(\tilde{Y}_{t-1}) + \log(\mu_t). \quad (25)$$

The variables in the model economy,

$$\tilde{x}_t = \left[\tilde{C}_t, N_t, R_t, \Pi_t, \tilde{W}_t, \tilde{\lambda}_t, \mathcal{M}_{t,t+1}, \tilde{Y}_t, R_t^*, R_t^E, \Delta \log(Y_t), A_t^R, A_t \right]',$$

are now described by equations (4), (8), (13), (15), (16), and (18) - (25).

3.6. Steady State

We set all the time subscripts equal to t and solve for the deterministic steady state.

$$A_t = 1, \quad (26)$$

$$A_t^R = 1, \quad (27)$$

$$\Pi_t = \bar{\Pi}, \quad (28)$$

$$\tilde{Y}_t = \tilde{Y}, \quad (29)$$

$$\tilde{C}_t = \tilde{Y}_t, \quad (30)$$

$$N_t = \tilde{Y}_t, \quad (31)$$

$$R_t^* = \frac{\Pi_t \exp(g_z)}{\beta}, \quad (32)$$

$$R_t^E = K, \quad (33)$$

$$R_t = \max(R_t^E, R_t^*), \quad (34)$$

$$d_{t+1} = \frac{\Pi_t \exp(g_z)}{R_t \beta}, \quad (35)$$

$$\mu_t = \exp(g_z), \quad (36)$$

$$\tilde{\lambda}_t = \left(\tilde{C}_t (1 - \chi/\mu_t) \right)^{-\sigma}, \quad (37)$$

$$\mathcal{M}_{t,t+1} = 1/R_t, \quad (38)$$

$$\tilde{W}_t = \left(\frac{\varepsilon - 1}{\varepsilon} \right), \quad (39)$$

$$\kappa = \tilde{\lambda} \tilde{W}_t / N_t^\eta, \quad (40)$$

$$\Delta \log(Y_t) = \log(\mu_t). \quad (41)$$

4. A Zero Lower Bound Steady State

We interpret the world in which the ZLB binds some of the time as a regime-switching environment. More specifically, we consider an environment that consists of two steady states, a normal or inflation-targeting steady state and a ZLB steady state. The normal and ZLB steady states differ in terms of the interest level, and what the interest rate implies for other parameters and steady states in the model. To better understand the problem requires a dissection of the steady state nominal interest rate at the ZLB. Combining the steady state consumption euler equation (32) with the steady state interest rate at the effective lower bound (equation 33) gives

$$K = \frac{\exp(g_z) \bar{\Pi}}{\beta d}. \quad (42)$$

There are four parameters in (42) that could be allowed to switch to generate a ZLB steady state. In particular, the following parameter and parameter combinations would be consistent with a ZLB steady state:

1. A fall in $\bar{\Pi}$ relative to normal times;
2. A fall in g_z relative to normal times;
3. A rise in β relative to normal times;
4. A rise in d relative to normal times;

5. Some combination of scenarios 1 to 4.

Scenario 3 and 4 are equivalent. However, scenario 4 is more appealing because $\beta < 1$ and the constraints on d implied by regime-switching are less restrictive. For this reason we rule out scenario 3 as a candidate for achieving a ZLB steady state. Scenarios 2 to 4 are all consistent with a decline in the steady state real interest rate. Based on the rudimentary accounting exercise in Section 2, a shift in g_Z by itself is unlikely to be large enough to account for the observed shift in steady state nominal interest rates in the US. As a consequence we rule this out as a candidate. Likewise, based on the analysis in Section 2, a shift in $\bar{\Pi}$ alone is unlikely to be large enough to generate the observed shift in steady state nominal interest rates. Moreover [Aruoba et al. \(2013\)](#) find little empirical support for a deflationary ZLB steady state in the US. Therefore we rule this out as a candidate. This leaves two options; a shift in d alone, and a simultaneous shift in d , g_Z and $\bar{\Pi}$.⁸ We investigate these competing scenarios empirically and evaluate their plausibility in subsequent sections.

5. Candidate ZLB Models

In this section, we describe the candidate ZLB models for estimation with a special emphasis on the parameters related to regime-switching. In the previous section, we established that the US experience at the ZLB is consistent with two scenarios: a shift in time preferences, or a combination of factors. It is also possible that shock standard deviations have fluctuated over time. We allow for this by estimating models with switching shock standard deviations in addition to models with constant shock standard deviations. This results in a total of four candidate models for estimation and evaluation.

5.1. States

When considering monetary policy states, we consider a two-state world with the following states

$$s_t = L, N$$

where L represents the low interest rate state (the ZLB state) and N represents the normal state (inflation-targeting/Taylor rule). We assume the following transition matrix

$$Q = \begin{bmatrix} 1 - p_{L,N} & p_{L,N} \\ p_{N,L} & 1 - p_{N,L} \end{bmatrix}, \quad (43)$$

where $p_{L,N}$ is the probability of going from the ZLB state in period t to the normal state in period $t + 1$. $p_{L,L} = 1 - p_{L,N}$ is the probability of remaining in the ZLB state. $p_{N,L}$ is the probability of going from the normal state in period t to the ZLB state in period $t + 1$, and $p_{N,N} = 1 - p_{N,L}$ is the probability of remaining in the normal state.

⁸Note that the second candidate nests the steady state inflation only shift studied by [Aruoba et al. \(2013\)](#) and [Gavin et al. \(2015\)](#).

When we relax the assumption of constant shock standard deviations, we take a somewhat naïve approach and allow all shock standard deviations to switch on a second Markov chain, where $s_t = 1, 2$,

$$H = \begin{bmatrix} 1 - p_{1,2} & p_{1,2} \\ p_{2,1} & 1 - p_{2,1} \end{bmatrix},$$

is the transition matrix and $p_{1,2}$ is the probability of going from state 1 in period t to state 2 in period $t + 1$. For identification purposes, we assume $\sigma_R(1) < \sigma_R(2)$.

We only consider exogenous transition probabilities, but the solution and estimation tools are flexible enough to allow for endogenous transition probabilities, something we leave for future work.

5.2. A Time Preference Shift

In the models where only the time preference “shock” can shift, we can characterize the steady state nominal interest rate in the ZLB and normal states as follows

$$\bar{R}(L) = K, \quad \bar{R}(N) = \frac{\bar{\Pi} \exp(g_Z)}{\beta}.$$

This leads to the following restrictions on the time preference shock or shift term

$$d(L) = \frac{\bar{\Pi} \exp(g_Z)}{K\beta} > 1, \quad d(N) = 1.$$

We interpret shifts in d in a similar fashion to [Gust et al. \(2012\)](#) so that $d(L) > d(N)$ represents an increase in precautionary saving in the ZLB state. We also require the following two conditions to hold

$$p_{L,L}\beta d(L) < 1. \tag{44}$$

$$0 \leq \frac{(1 - p_{L,L})\beta d(L)}{(1 - p_{L,L}\beta d(L))} \frac{(1 - p_{N,N})\beta}{(1 - p_{N,N}\beta)} < 1, \tag{45}$$

These conditions are derived in [Appendix B](#) and imposed during the estimation procedure. Equation (44) is the same condition derived by [Braun et al. \(2015\)](#), but they do not need condition (45) because they assume the normal state is absorbing. Condition (45) allows for the possibility that real interest rates can be negative in the ZLB state, where this possibility depends on the size of $d(L)$ relative to β and the duration of the ZLB state relative to the normal state.

5.3. A Combination of Factors

In a second class of models we allow for steady state inflation, productivity growth and the preference shock term to shift when in the ZLB state. We impose the following restrictions on the steady state growth and inflation terms

$$g_Z(L) < g_Z(N), \\ \bar{\Pi}(L) < \bar{\Pi}(N).$$

The steady state interest rate in the ZLB and normal states must obey the following restrictions

$$\bar{R}(L) = K, \quad \bar{R}(N) = \frac{\bar{\Pi}(N) \exp(g_Z(N))}{\beta}.$$

These restrictions lead to the following relations for the time preference shift term in the ZLB and normal states

$$d(L) = \frac{\bar{\Pi}(L) \exp(g_Z(L))}{K\beta} > 1, \quad d(N) = 1.$$

As outlined in the previous model, we require conditions (44) and (45) to hold to ensure the household's utility and the firm's pricing problem are well defined. Again, these are imposed during estimation. In this example, negative real interest rates can occur in the ZLB steady state by shifting g_Z , $\bar{\Pi}$ and d .

6. Solution and Estimation

We solve our models using a perturbation method for regime-switching rational expectations models. While we only consider a first-order approximation, this approach is easily extended to higher orders of approximation when appropriate, as in [Maih \(2015\)](#).⁹

The model is estimated on US data using Bayesian methods following the methodology outlined in [Alstadheim et al. \(2013\)](#).¹⁰ The sample period considered spans 1985Q1 to 2015Q2. This period is chosen to cover the great moderation, financial crisis and ZLB periods. The procedure easily handles longer data samples, but we focus on this period to reduce the number of regimes required, making the exposition clearer. The model is estimated using quarterly data on US per capita GDP growth, GDP deflator inflation and the fed funds rate. See [Appendix A](#) for a full description of the data. We do not detrend the data because we want to examine the role that trend shifts in inflation and GDP growth may have played in explaining the ZLB period.

All the parameters are estimated except for ε , K and σ_E . The priors for the estimated parameters are presented in [Tables 2, C.3, C.4 and C.5](#). The elasticity of substitution between differentiated intermediate goods, ε , is set to 6, implying a markup of 20%. K and σ_E determine the interest rate dynamics at the effective lower bound and need to be calibrated carefully to enable the procedure to find a distinct ZLB regime. We calibrate K to be 1.0003, which is the average of the fed funds rate between 2009Q1 and 2015Q2. We calibrate σ_E to be 0.0001. Our experience leads to the following observations: setting σ_E too high results in some uncertainty as to which regime the economy is in over the ZLB period.

⁹The methodology is fast and efficient, and scales easily to larger models and higher orders of approximation without adding too much additional computational burden. This leaves the possibility of exploring the impact of fiscal policy and unconventional monetary policies in a richer DSGE model with an occasionally binding ZLB constraint for future work.

¹⁰If the model were solved to higher orders of approximation, we could use the methods in [Binning & Maih \(2015\)](#) to estimate the model.

Setting σ_E too low means the procedure never wants to spend any time in the ZLB regime at any point over history.

There may potentially be issues identifying some of the key parameters of interest. Because this is a regime-switching model, we can no longer use some of the usual machinery, as do [Iskrev \(2008\)](#), [Iskrev \(2010\)](#) and [Komunjer & Ng \(2011\)](#), to determine which parameters are identified and to what degree they are identified. However, we impose some parameter restrictions (see equation (45)) during estimation which are no doubt helpful in improving identification.

7. Results

In this section we summarize the estimation results for all models considered, highlighting any features of note. We rank the models in terms of their relative plausibility using the marginal data densities (Laplace approximation). The estimation results from the most plausible model are then discussed in further detail and in the following section. Table 1 below summarizes the key results from the estimation.

Table 1: Summary of Estimation Results

Model	Log MDD (Laplace)	Expected ZLB Duratation	Features of Note
Preference Shift	1503.796	4.6104	$d(L) = 1.0088$
Combination	1506.361	4.1649	$d(L) = 1.0052$, $g_Z(L, N) = 0.0022, 0.0035$, $\bar{\Pi}(L, N) = 1.0017, 1.0018$
Preference Shift + Shock Std.	1523.520	4.6083	$d(L) = 1.0114$
Combination + Shock Std.	1509.381	4	$d(L) = 1.0088$, $g_Z(L, N) = 0.0029, 0.0046$, $\bar{\Pi}(L, N) = 1.0046, 1.0055$

All estimated models have an expected ZLB duration of between 4 and 4.61 quarters. These are lower than the mean expected durations found by [Kulish et al. \(2014\)](#), which range between 7 and 11 quarters. However these durations are more in line with analysis by [Swanson & Williams \(2014\)](#), who use survey data to find an expected ZLB duration of 4 quarters before late 2011 and 6 quarters thereafter. Our estimates are affected by the requirement that

mean square stability be satisfied.¹¹ The ZLB regime is in isolation an unstable regime, but in combination with an inflation-targeting regime, there exist parameter values that result in a stable system. In the absence of other stabilizing channels like unconventional monetary policies, forward guidance and fiscal policy, for example, the expected ZLB duration is likely to be shortened to avoid instabilities.

When we allow the ZLB steady state to be determined by a combination of factors, we find limited evidence for a shift in inflation rates and no evidence of a deflationary ZLB steady state, which is in line with [Aruoba et al. \(2013\)](#) for the US. There is, however, more support for a fall in productivity growth rates over the ZLB period in these models. [Gust et al. \(2012\)](#) find that negative productivity shocks have played an important role over the ZLB period, a result that is qualitatively in line with a shift in the productivity growth rate.

The model with the ZLB steady state determined by shifts in the time preference and switching shock standard deviations is most preferred by the data according to the marginal data densities. This is followed by the model where the ZLB steady state is determined by a combination of factors and where shock standard deviations are allowed to switch. Amongst the constant shock standard deviation models, the model where the ZLB regime is determined by a combination of factors is preferred to the model with the ZLB regime determined by a shift in time preferences alone.

This implies that when standard deviations are constant, switches to a ZLB regime are more likely explained by shifts in productivity growth rates in addition to shifts in the time preference shock. However, the data prefers models with switching shock standard deviations, and in that category the model where the ZLB steady state is solely determined by a preference shift dominates the model where the ZLB steady state is determined by a combination of factors. It is likely that the increase in shock volatility at the beginning of the ZLB period is incorrectly picked up by the shift in the productivity growth rate in the absence of switching shock volatilities.

All subsequent analysis in this section and the next will be conducted using the preferred model, namely the two chain regime-switching DSGE model where the time preference shock and the shock standard deviations switch. The complete estimation results from this model are presented in [Table 2](#). The complete estimation results for the other models can be found in [Appendix C](#).

¹¹See [Gupta et al. \(2003\)](#) and [Costa et al. \(2005\)](#) for an explanation of mean square stability.

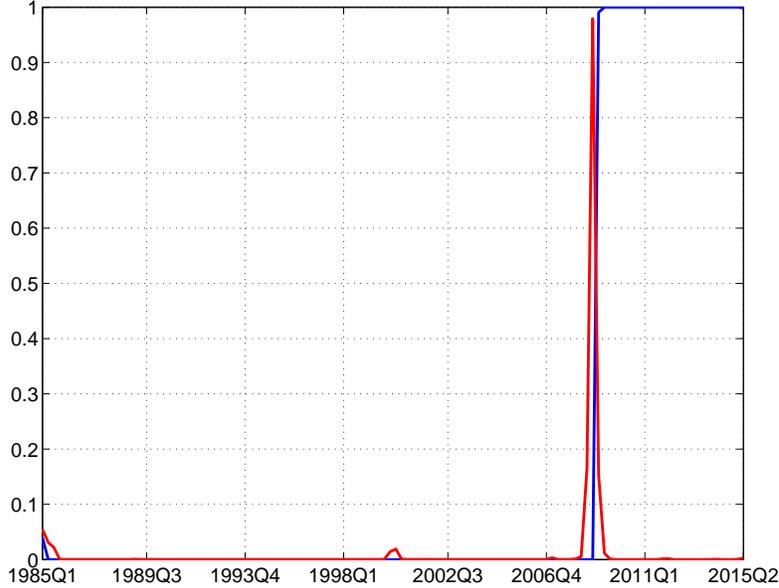
Table 2: Parameter Estimates

Parameter	Distribution	Prior Mean	Prior Std Dev	Lower Bound	Upper Bound	Posterior Mode
β	uniform			0.9800	0.9985	0.9982
χ	beta	0.5000	0.1500			0.5699
σ	normal	2.0000	0.3000			2.2207
η	normal	2.0000	0.5000			0.8609
ϕ	normal	10.0000	2.0000			16.4945
κ_π	normal	1.5000	0.2500			2.7186
κ_y	normal	0.1200	0.0500			0.0909
g_Z	uniform			0.0000	0.0500	0.0041
$\bar{\Pi}$	uniform			1.0000	1.1000	1.0057
ξ	beta	0.5000	0.1500			0.4249
ρ_r	beta	0.5000	0.1500			0.6039
ρ_{AR}	beta	0.5000	0.1500			0.4806
ρ_A	beta	0.5000	0.1500			0.7696
$\sigma_R(1)$	inverse gamma	0.1000	2.0000			0.0038
$\sigma_R(2)$	inverse gamma	0.1000	2.0000			0.0226
$\sigma_A(1)$	inverse gamma	0.1000	2.0000			0.0269
$\sigma_A(2)$	inverse gamma	0.1000	2.0000			0.0278
$\sigma_Z(1)$	inverse gamma	0.1000	2.0000			0.0071
$\sigma_Z(2)$	inverse gamma	0.1000	2.0000			0.0469
$\sigma_\pi(1)$	inverse gamma	0.1000	2.0000			0.0047
$\sigma_\pi(2)$	inverse gamma	0.1000	2.0000			0.0482
$p_{N,L}$	beta	0.0500	0.0200			0.0398
$p_{L,N}$	beta	0.0500	0.0200			0.2170
$p_{1,2}$	beta	0.0500	0.0200			0.0258
$p_{2,1}$	beta	0.0500	0.0200			0.0510

All shock standard deviations are higher in the second state and for this reason we refer to this as the high volatility state. The consumption preference shock standard deviation (σ_A) is only slightly higher in the high volatility state than it is in the low volatility state. In contrast the stochastic subsidy shock (σ_π), the productivity shock (σ_Z) and the monetary policy shock (σ_R) are all substantially larger in the high volatility state compared to the low volatility state.

While not directly comparable, we compare our parameter estimates with the 1984Q1:-2004Q4 subsample estimates of [Smets & Wouters \(2007\)](#). We find estimates of habit formation (χ) that are slightly lower (0.5699 compared with 0.68) and estimates of the inverse of the intertemporal elasticity of substitution (σ) that are slightly higher (2.2207 compared with 1.47). Our estimate of the inverse of the Frisch elasticity of labor supply is 0.8609, which is quite a bit lower than their estimate of 2.3. We estimate a response to inflation in the Taylor rule of 2.2207, which is a bit higher than their estimate of 1.77, and a response to output of 0.0909, which is in line with [Smets & Wouters \(2007\)](#). It is likely that the fulfilment of mean square stability has pushed up the inflation response in the Taylor rule.

Figure 5: Volatility vs. ZLB States

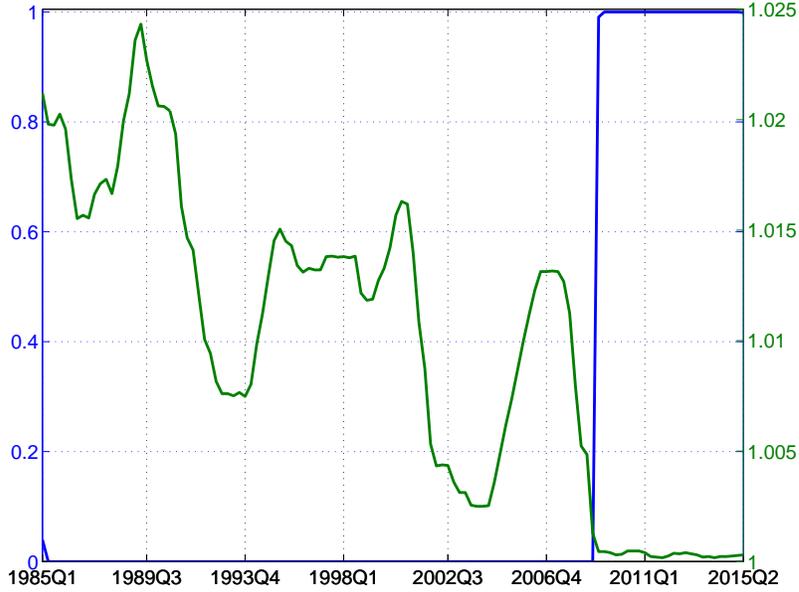


Notes: The blue line is the estimated probability of being in the ZLB state, the red line is the estimated probability of being in the high volatility state.

In Figure 5 we plot the smoothed probability of being in the high volatility state against the smoothed probability of being in the ZLB state. We make two key observations. First there is strong evidence that the shocks that hit during the start of the financial crisis came from a higher volatility state, but were short-lived. Secondly, allowing for changes in shock volatility helps identify the ZLB regime. Compared to the model without changes in shock volatility (see Figures C.13 and C.15), the transition to the ZLB state occurs much earlier in 2009Q1, and it is a much sharper transition, occurring in the space of a single period.

The increase in shock volatility occurs in the same period as the switch from the normal state to the ZLB state. This lends support to our interpretation of d as representing precautionary savings behavior. Increases in shock volatility could lead to increases in uncertainty and precautionary savings behavior which, if large enough, could send the economy to the ZLB. While the increase in shock volatility was only transitory, households' precautionary savings behavior could persist if there is a backward-looking component to how they form beliefs about uncertainty under the ZLB regime.

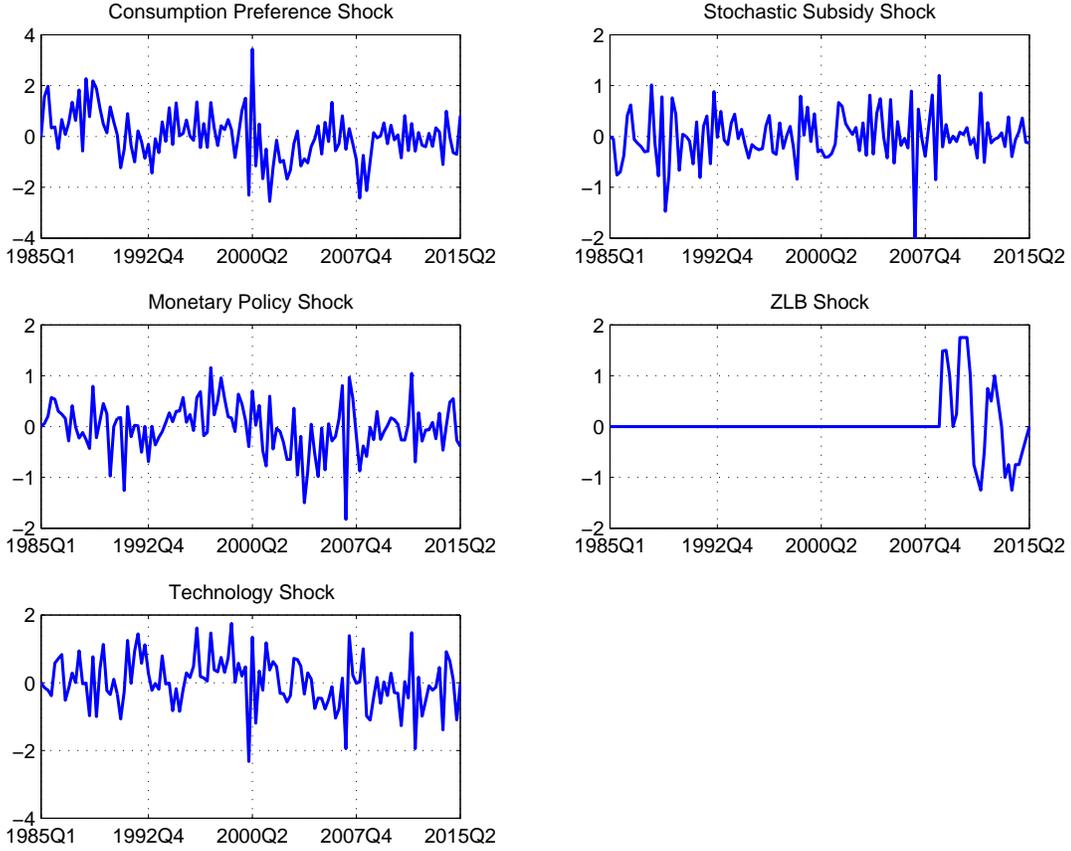
Figure 6: Interest Rate vs. Probability of ZLB Regime



Notes: The blue line with y axis on the left represents the probability of being in a ZLB regime, while the green line with y axis on the right, is the fed funds rate in gross quarterly terms.

We plot the probability of being in the ZLB state against the nominal interest rate in Figure 6. This figure reiterates an important point: the shift to the ZLB regime is more pronounced and occurs earlier when a change in volatility is controlled for. The sharp change in the probability of being in the ZLB state occurs when very sharp cuts were made, sending the nominal interest rate to its effective lower bound.

Figure 7: Model Shocks



Notes: Shocks are assumed to be distributed $\sim iid N(0, 1)$ in the RISE Toolbox.

We plot the smoothed shocks in Figure 7. Both the consumption and stochastic subsidy shocks appear to be smaller in 2009Q1 compared with the results obtained under constant shock volatility (see Figures C.14 and C.16). This is a consequence of allowing the shock volatilities to change over time. Otherwise, the shocks appear to be reasonably Gaussian.

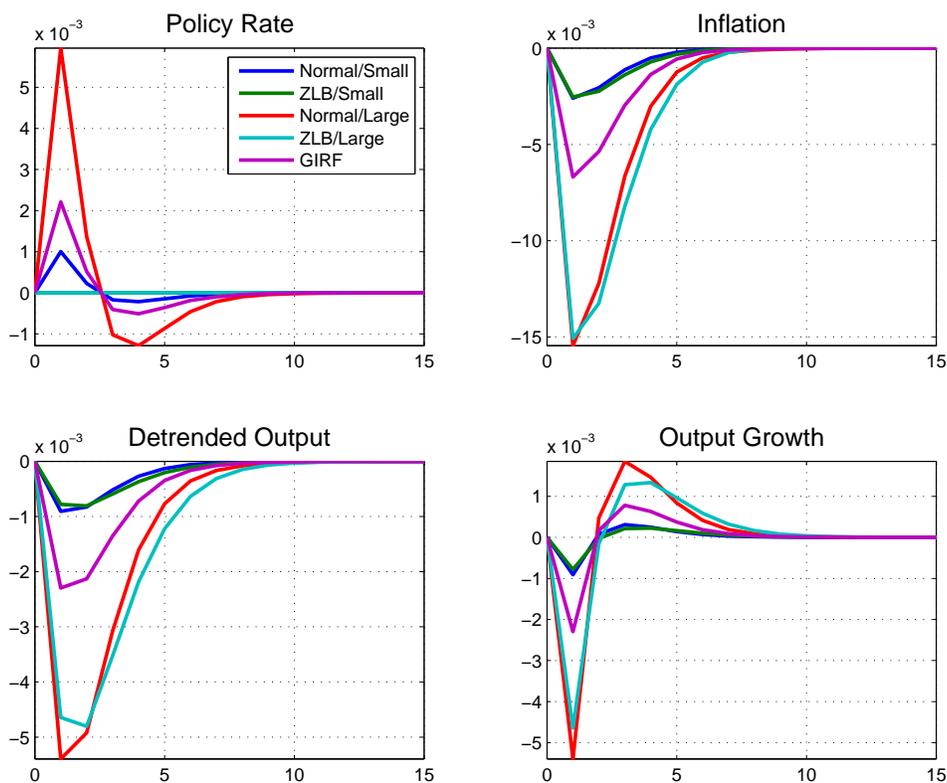
8. Economic and Policy Implications

In this section we investigate some of the model properties of our preferred model and dig deeper into the economics behind the model and estimation results to understand any potential implications for policy. We begin by investigating the model responses to a monetary policy shock and a consumption preference shock. The impulse responses for the productivity and stochastic subsidy shocks can be found in Appendix D.

We look at both regime-specific impulse responses and generalized impulses. The regime-specific impulses are constructed by conditioning on a single regime. Ex ante agents believe at each point in time over the impulse response period that there is a positive probability

they can switch to a different regime in the following period. Ex post, they remain in that regime for the duration of the impulse response. As a consequence, the behavior of agents in the regime-specific impulse responses is influenced by the model dynamics in the other regimes through the expectations channel.

Figure 8: Impulse Response Function: Monetary Policy Shock



Notes:

Normal/Small	Taylor rule and small shock standard deviation regime
ZLB/Small	ZLB and small standard deviation regime
Normal/Large	Taylor rule and large shock regime
ZLB/Large	ZLB and large standard deviation regime
Policy Rate	R_t
Inflation	Π_t
Detrended Output	\tilde{Y}_t
Output Growth	$\Delta \log Y_t$

Figure 8 presents impulse responses for a monetary policy shock. The policy rate does not respond to a monetary policy shock when the economy is in a ZLB regime. However inflation, detrended output and output growth in the ZLB regimes all respond to a monetary policy shock. As we mentioned earlier in this section, each regime does not exist in isolation, but is directly influenced by the other regimes through the expectations channel. Inflation, detrended output and output growth all respond to a monetary policy shock because agents

are forward-looking and take into account the probability that they could switch into an inflation-targeting regime in subsequent periods, in which case the dynamics of the model in the inflation-targeting regimes affect the dynamics of the economy when in the ZLB regime. In fact the responses for inflation, output and output growth in both large shock regimes and both small shock regimes are very similar. In this case the generalized impulse responses average through the regime-specific impulse responses.

Figure 9: Impulse Response Function: Consumption Preference Shock

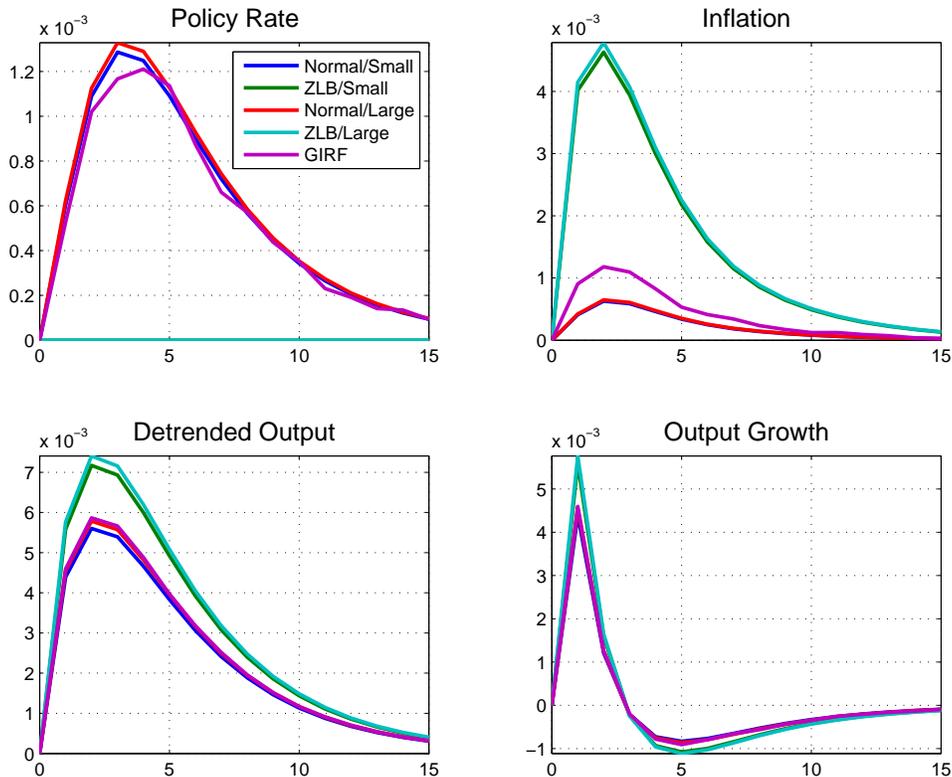
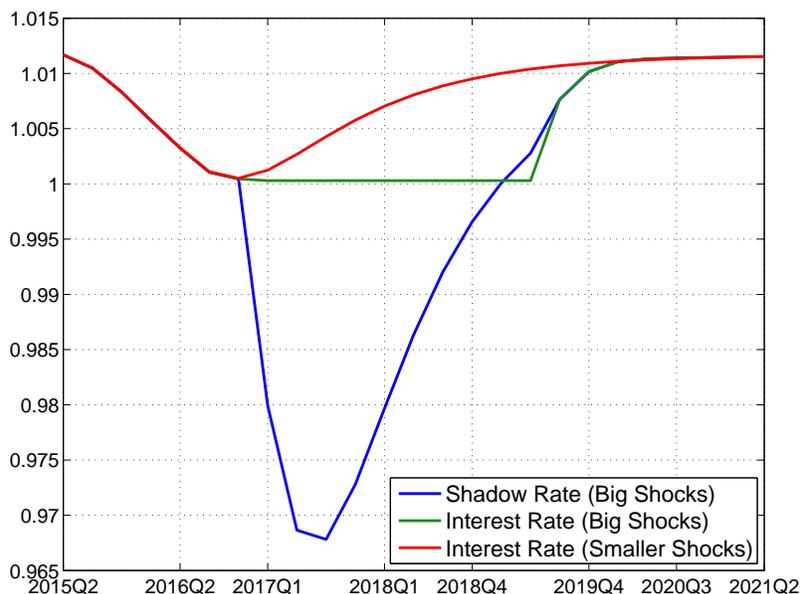


Figure 9 presents the model responses to a consumption preference shock. We should first note that because the difference between the standard deviations of the consumption preference shock in the large shock and small shock regimes is quite small, the differences between the regime-specific impulses along the shock standard deviation dimension are also small. However, what is of interest is the difference between the inflation response in the ZLB regimes and the inflation-targeting regimes. More specifically the inflation response is much larger at the ZLB than it is when in an inflation-targeting regime. This highlights the non-linear nature of the economy with an occasionally binding ZLB constraint, the impulse responses are state dependent as opposed to linearized models with constant parameters where impulse responses are the same regardless of the state of economy.

To further understand the impact of hitting the ZLB, we carry out some simulation experiments. The rules of our experiments are as follows: if a shock is sufficiently large

to send interest rates below the effective lower bound, we assume the economy switches to the ZLB regime and remains there as long as the shadow rate remains below the effective ZLB: while the shadow rate remains below the effective lower bound, the economy is in the ZLB regime; once the shadow rate is larger than the effective lower bound, the economy switches back to the normal regime. We carry out two simulations: in the first we assume the economy only just hits the ZLB, in the second we assume the economy narrowly misses hitting the ZLB. In both exercises, we assume the economy begins at its normal regime steady state. It is then hit by a sequence of negative consumption preference shocks for five periods. In the first simulation, the shock sizes are chosen to be twice the size of the estimated shock standard deviations, which is just large enough for the economy to enter the ZLB regime. In the second example, we choose shocks that are slightly smaller, so that the economy narrowly misses hitting the ZLB.

Figure 10: Negative Consumption Shocks: Interest Rates



Figures 10, 11 and 12 plot the quarterly gross interest rate, quarterly gross inflation rate and detrended output for both simulations. The first simulation is labeled “Big Shocks” and the second simulation is labeled “Smaller Shocks”. Both simulations are nearly identical until interest rates hit the ZLB in the first simulation. When the economy hits the ZLB, there is a near catastrophic fall in inflation. The decline in output is also slightly longer lasting, resulting in a greater cumulative output loss. In the second simulation, the economy comes within a few basis points of hitting the effective lower bound, inflation falls by a much smaller amount, while the output profile returns to the normal-regime steady state slightly faster. This illustrates the important role interest rates play in stabilizing the economy and returning inflation to target after the economy has been perturbed. Hitting the ZLB

Figure 11: Negative Consumption Shocks: Inflation

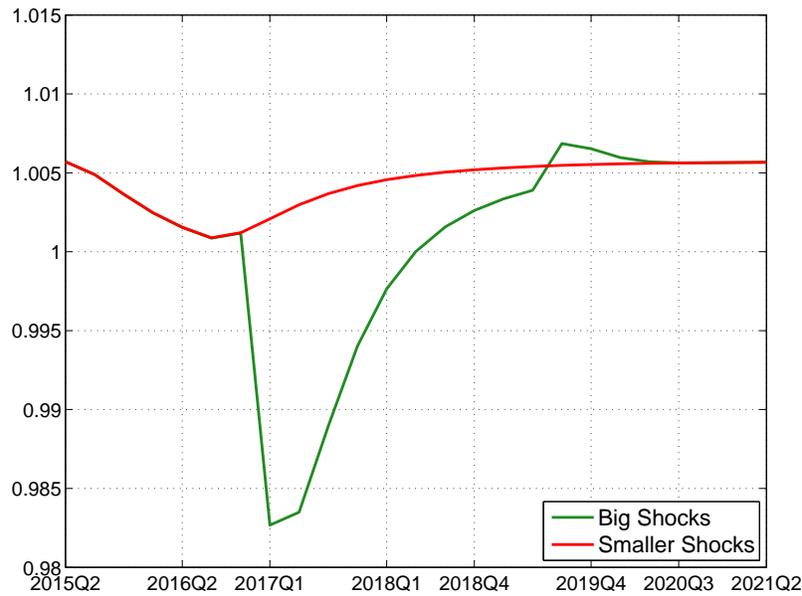
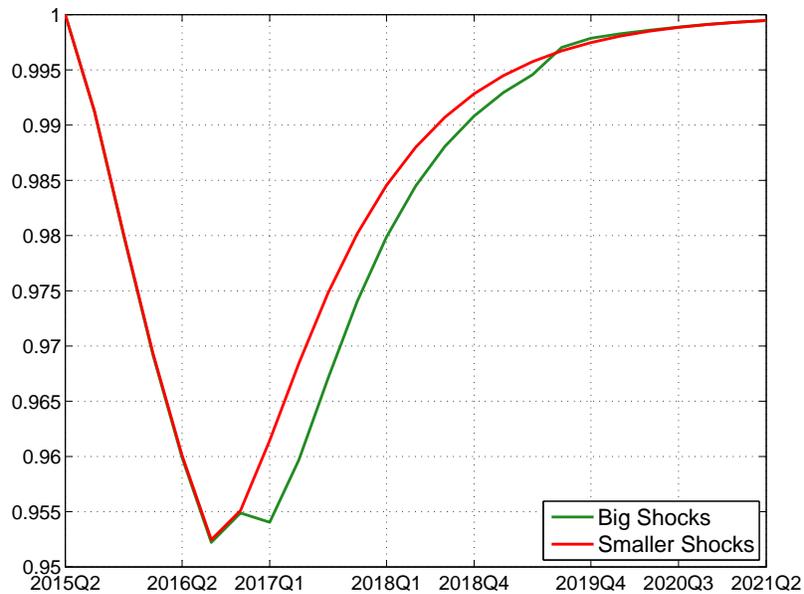


Figure 12: Negative Consumption Shocks: Detrended Output



temporarily destabilizes the economy by preventing interest rates adjusting to keep inflation under control.

From the impulse responses and simulation exercises we take two lessons. First the expectations channel matters, or more precisely agents' expectations about how the economy

will exit the ZLB regime matter and have implications for the behavior of the economy when in the ZLB regime. In practical terms this has consequences for central bank communication about the exit strategy from the ZLB while the economy is still at the ZLB. Second, this analysis confirms the non-linear nature of an economy facing a ZLB constraint. Failing to take the ZLB constraint into account could result in incorrect analysis and policy conclusions. This is reinforced by the different responses of inflation at the ZLB when hit by a consumption preference shock.

The properties of the ZLB steady state are worthy of further investigation. In particular, the economic interpretation of d and what it means for policy. As said earlier we interpret the time preference shift term d , as proxying precautionary savings behavior, following [Gust et al. \(2012\)](#). Consequently an increase in d results in a decrease in the propensity to consume and a decline in interest rates.

Conventional DSGE models only exhibit precautionary behavior when solved using non-linear methods. Linearization means our model is certainty equivalent and unable to directly capture this type of precautionary savings behavior.¹² The inclusion of d provides a shortcut to this channel that is missing from the model. The spike in shock volatility at the beginning of the ZLB period lends some support to d capturing precautionary savings due to a general increase in uncertainty.

There are, however, alternative precautionary savings stories that revolve around the labor market. [Benigno & Fornaro \(2015\)](#) derive microfoundations for the precautionary savings behavior in a simple DSGE model with unemployment, idiosyncratic unemployment risk and incomplete unemployment insurance. The behavior represented by d arises because the probability of becoming unemployed is assumed constant and unemployment insurance is assumed to be proportional to, although less than, the income of those in employment. More importantly, this type of precautionary savings behavior survives the model's linearization.

Using a similar intuition to [Benigno & Fornaro \(2015\)](#), [Ragot & Challe \(2011\)](#) and [Ragot et al. \(2015\)](#) derive fully fledged DSGE models that exhibit time-varying precautionary savings behavior when linearized to investigate the linkages between aggregate demand and precautionary savings. Moreover, their models explain the linkages between aggregate demand, unemployment risk, incomplete unemployment insurance and precautionary savings. Crucially, they assume that the probability of becoming unemployed is endogenous and a function of the state of the economy. [Ragot et al. \(2015\)](#) describe a powerful feedback loop between precautionary savings and aggregate demand: negative aggregate demand shocks lower aggregate demand, which in turn discourages job creation so that unemployment and unemployment risk both rise; imperfectly insured households increase their precautionary wealth in response to the rise in idiosyncratic unemployment risk; this in turn leads to cuts in consumption and further declines in aggregate demand which, of course, has further consequences for unemployment and precautionary savings. [Ragot et al. \(2015\)](#) estimate their

¹²If we were to solve our model using a certainty non-equivalent solution method, it is unclear whether the model would accurately account for all precautionary savings behavior in the data due to model misspecification.

model and find evidence that precautionary saving amplified the downturn during the financial crisis. Neither [Ragot & Challe \(2011\)](#) nor [Ragot et al. \(2015\)](#) consider the implications of this type of precautionary savings behavior on the interest rate at the ZLB. It is, however, highly plausible that a similar mechanism has contributed to the US economy hitting the ZLB and that our framework is able to capture this from the evidence in the data.

If d is capturing precautionary savings behavior and this precautionary savings behavior is driving the economy towards the ZLB, then we pose the question: what can policy do to reduce the impact of this precautionary behavior? If this is the type of precautionary savings behavior that occurs because of unemployment risk and incomplete unemployment insurance and we take the model of [Ragot et al. \(2015\)](#) seriously, then precautionary savings behavior could be reduced by decreasing the chances of becoming unemployed, increasing the unemployment benefit, or by increasing the job finding rate. This ignores any disincentives that closing the gap between the income received while employed and the benefit received while unemployed may have on workers' employment decisions. In the event that it is uncertainty that is driving this precautionary behavior, policies like forward guidance could be employed to reduce uncertainty (see [Filardo & Hofmann, 2014](#)).

9. Conclusion

We present a combined modelling, solution and estimation methodology that is capable of accounting for the ZLB period in the US. More specifically, we assume the economy is subject to regime switches which lead the economy to alternate between fluctuating around a normal times steady state and a ZLB steady state. In normal times, monetary policy is governed by a Taylor-type rule. We assume that the ZLB steady state is characterized by negative real interest rates and derive conditions to ensure well-defined optimization problems for both firms and households. The model is solved using a perturbation method and estimated using Bayesian methods with a regime-switching Kalman filter. Our framework correctly detects the presence of and transition to a ZLB steady state.

Our methodology allows us to test competing theories about the determinants of the ZLB steady state. We find it more likely that the ZLB regime is characterized by an increase in precautionary savings and that shock volatilities increased temporarily at the beginning of the financial crisis. Further investigation of the model properties reinforces the importance of the expectations channel and how the dynamics of the normal regime play an important role for agents' behavior while at the ZLB. Moreover, the model dynamics can change quite radically while in the ZLB regime.

Our method is appealing because it can easily be scaled to larger models and higher orders of approximation. We leave for future work the investigation of the role that fiscal policy and unconventional monetary policies played over this period and the incorporation of endogenous transition probabilities in the modelling framework. The solution and estimation codes are available as part of the RISE toolbox in Matlab.

Appendix A. Data

All data is taken from the St. Louis Federal Reserve's FRED database. We report the FRED pneumonics in brackets. The data is quarterly and spans 1985Q1 to 2015Q2. We used the log change in per capita US GDP, constructed using Real Gross Domestic Product (GDP1) and the Civilian Noninstitutional Population (CNP16OV), the percentage change in the US GDP deflator (GDPDEF) and the quarterly average of the monthly fed funds rate (FEDFUNDS). The data is not detrended. The expected real interest rate plotted in Figure 4. is constructed by subtracting the University of Michigan inflation expectations data for inflation one year ahead (MICH) from the 1-year treasury rate (DGS1).

Appendix B. Conditions

In this appendix, we derive some conditions on the transition probabilities, the time discount factor and time preference shifter to ensure utility is well defined. The representative household's lifetime utility at time 0 is given by

$$V_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{j=0}^t d_j \right) \left\{ A_t \frac{(\mathcal{C}_t/Z_t)^{1-\sigma}}{1-\sigma} - \kappa \frac{N_t^{1+\eta}}{1+\eta} \right\}, \quad (\text{B.1})$$

Assume the economy is in the ZLB steady state, where no shocks can hit the economy, but it is still possible for the economy to switch to the normal steady state, then

$$V(L) = u(L) + p_{L,L} \beta d(L) V(L) + (1 - p_{L,L}) \beta d(L) V(N) \quad (\text{B.2})$$

where $u(L)$ is the period utility in state L . We can rearrange equation (B.2) to get

$$V(L) = \frac{u(L) + (1 - p_{L,L}) \beta d(L) V(N)}{1 - p_{L,L} \beta d(L)} \quad (\text{B.3})$$

this is the condition in [Braun et al. \(2015\)](#), because the N state is absorbing in their model. From this equation we get the condition that

$$p_{L,L} \beta d(L) < 1 \quad (\text{B.4})$$

We do not force the N state to be absorbing, so we need to find the steady state value of $V(N)$ to solve for $V(L)$. $V(N)$ is given by

$$V(N) = u(N) + (1 - p_{N,N}) \beta V(L) + p_{N,N} \beta V(N) \quad (\text{B.5})$$

Rearranging gives

$$V(N) = \frac{u(N) + (1 - p_{N,N}) \beta V(L)}{1 - p_{N,N} \beta} \quad (\text{B.6})$$

Substituting equation (B.6) into (B.3) gives

$$\begin{aligned} V(L) &= \frac{u(L)}{(1 - p_{L,L}\beta d(L))} + \frac{(1 - p_{L,L})\beta d(L)}{(1 - p_{L,L}\beta d(L))} \left(\frac{u(N) + (1 - p_{N,N})\beta V(L)}{1 - p_{N,N}\beta} \right) \\ &= \frac{u(L)}{(1 - p_{L,L}\beta d(L))} + \frac{(1 - p_{L,L})\beta d(L)u(N)}{(1 - p_{L,L}\beta d(L))(1 - p_{N,N}\beta)} + \frac{(1 - p_{L,L})\beta d(L)(1 - p_{N,N})\beta}{(1 - p_{L,L}\beta d(L))(1 - p_{N,N}\beta)} V(L) \end{aligned}$$

which can be rewritten as

$$V(L) = \frac{A_1 u(L) + A_2 u(N)}{1 - A_3} \quad (\text{B.7})$$

where

$$\begin{aligned} A_1 &= \frac{1}{(1 - p_{L,L}\beta d(L))}, & A_2 &= \frac{(1 - p_{L,L})\beta d(L)}{(1 - p_{L,L}\beta d(L))(1 - p_{N,N}\beta)}, \\ A_3 &= \frac{(1 - p_{L,L})\beta d(L)(1 - p_{N,N})\beta}{(1 - p_{L,L}\beta d(L))(1 - p_{N,N}\beta)} \end{aligned}$$

Likewise for $V(N)$:

$$V(N) = \frac{u(N)}{(1 - p_{N,N}\beta)} + \frac{(1 - p_{N,N})\beta}{(1 - p_{N,N}\beta)} \left(\frac{u(L) + (1 - p_{L,L})\beta d(L)V(N)}{1 - p_{L,L}\beta d(L)} \right)$$

which can be written as

$$V(N) = \frac{B_1 u(N) + B_2 u(L)}{1 - A_3} \quad (\text{B.8})$$

where

$$B_1 = \frac{1}{(1 - p_{N,N}\beta)}, \quad B_2 = \frac{(1 - p_{N,N})\beta}{(1 - p_{N,N}\beta)(1 - p_{L,L}\beta d(L))}$$

This leaves us with two conditions on the transition probabilities, the discount factor and discount shift term:

$$p_{L,L}\beta d(L) < 1 \quad (\text{B.9})$$

$$0 \leq \frac{(1 - p_{L,L})\beta d(L)(1 - p_{N,N})\beta}{(1 - p_{L,L}\beta d(L))(1 - p_{N,N}\beta)} < 1 \quad (\text{B.10})$$

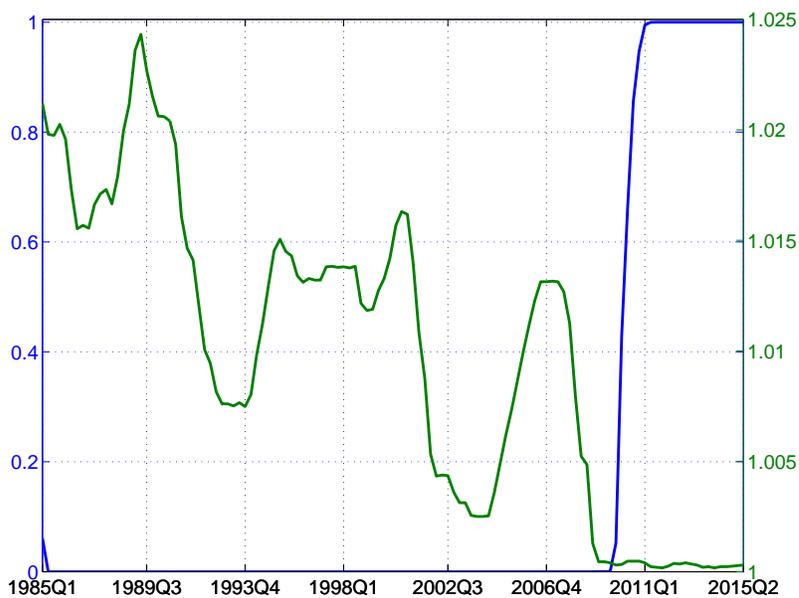
Appendix C. Alternate Estimation Results

A Time Preference Shift

Table C.3: Parameter Estimates

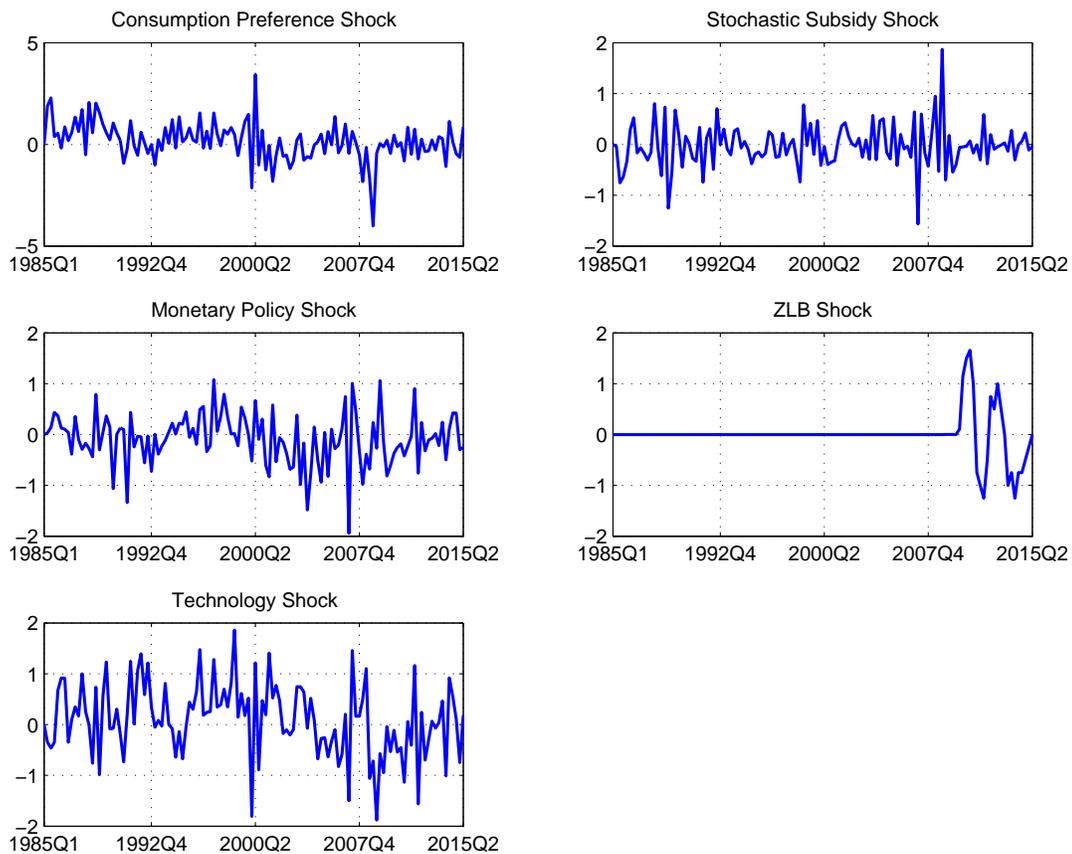
Parameter	Distribution	Prior Mean	Prior Std Dev	Lower Bound	Upper Bound	Posterior Mode
β	uniform			0.9800	0.9985	0.9980
χ	beta	0.5000	0.1500			0.6722
σ	normal	2.0000	0.3000			2.2637
η	normal	2.0000	0.5000			0.7219
ϕ	normal	10.0000	2.0000			17.0730
κ_π	normal	1.5000	0.2500			2.6512
κ_y	normal	0.1200	0.0500			0.0526
g_Z	uniform			0.0000	0.0500	0.0029
$\bar{\Pi}$	uniform			1.0000	1.1000	1.0042
ξ	beta	0.5000	0.1500			0.4734
ρ_r	beta	0.5000	0.1500			0.5964
ρ_{AR}	beta	0.5000	0.1500			0.5838
ρ_A	beta	0.5000	0.1500			0.7532
σ_R	inverse gamma	0.1000	2.0000			0.0038
σ_A	inverse gamma	0.1000	2.0000			0.0420
σ_Z	inverse gamma	0.1000	2.0000			0.0086
σ_π	inverse gamma	0.1000	2.0000			0.0048
$p_{N,L}$	beta	0.0500	0.0200			0.0610
$p_{L,N}$	beta	0.0500	0.0200			0.2169

Figure C.13: Interest Rate vs. Probability of ZLB Regime



Notes: The blue line with y axis on the left represents the probability of being in a ZLB regime, while the green line with y axis on the right is the fed funds rate in gross quarterly terms.

Figure C.14: Model Shocks



A Combination of Factors

Table C.4: Parameter Estimates

Parameter	Distribution	Prior Mean	Prior Std	Lower Bnd	Upper Bnd	Post Mode
β	uniform			0.9800	0.9985	0.9985
χ	beta	0.5000	0.1500			0.9305
σ	normal	2.0000	0.3000			2.0935
η	normal	2.0000	0.5000			0.5892
ϕ	normal	10.0000	2.0000			17.0401
κ_π	normal	1.5000	0.2500			2.5958
κ_y	normal	0.1200	0.0500			0.0369
$g_Z(L)$	uniform			-0.0500	0.0500	0.0022
$g_Z(N)$	uniform			0.0000	0.0500	0.0035
$\bar{\Pi}(L)$	uniform			1.0000	1.1000	1.0017
$\bar{\Pi}(N)$	uniform			1.0000	1.1000	1.0018
ξ	beta	0.5000	0.1500			0.4843
ρ_r	beta	0.5000	0.1500			0.6070
ρ_{AR}	beta	0.5000	0.1500			0.6126
ρ_A	beta	0.5000	0.1500			0.5428
σ_R	inverse gamma	0.1000	2.0000			0.0038
σ_A	inverse gamma	0.1000	2.0000			0.1794
σ_Z	inverse gamma	0.1000	2.0000			0.0105
σ_π	inverse gamma	0.1000	2.0000			0.0047
$p_{N,L}$	beta	0.0500	0.0200			0.0660
$p_{L,N}$	beta	0.0500	0.0200			0.2401

Figure C.15: Interest Rate vs. Probability of ZLB Regime

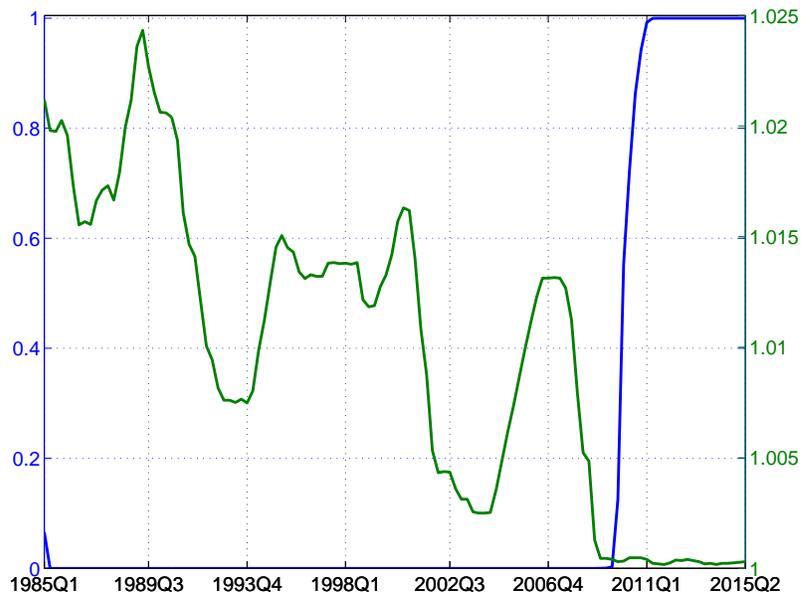
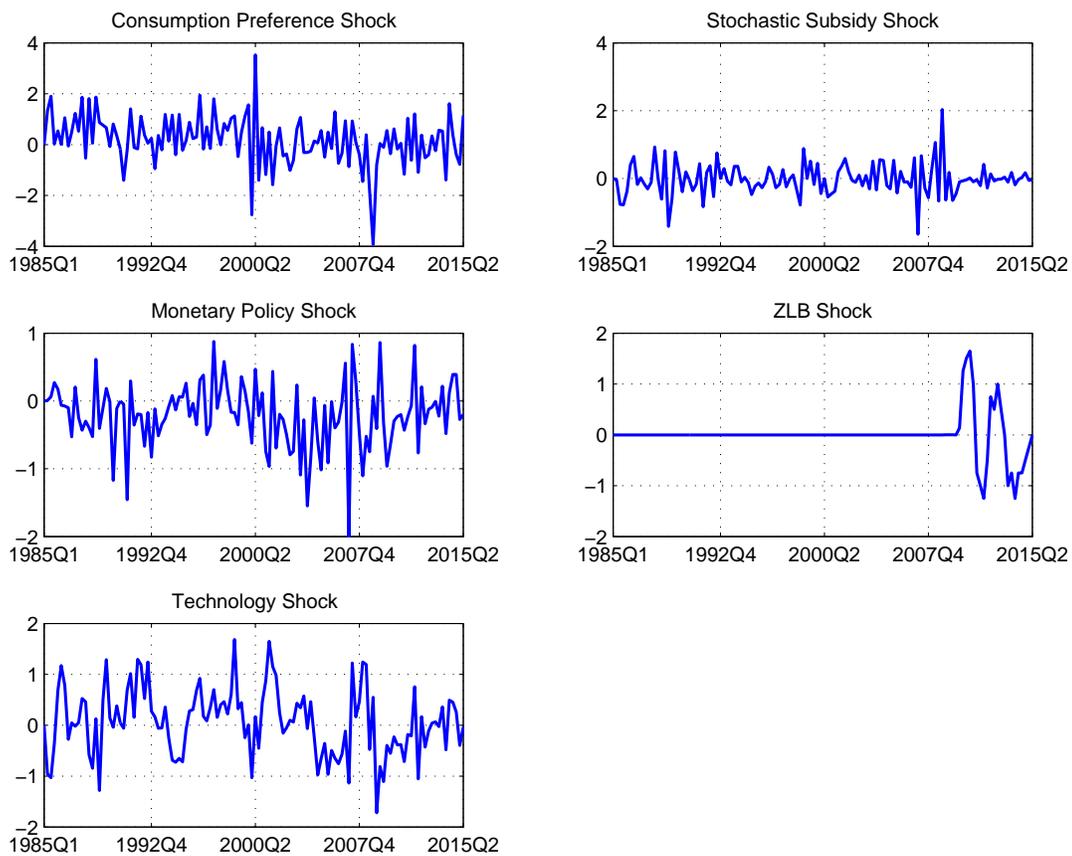


Figure C.16: Model Shocks



A Combination of Factors with Switching Shock Standard Deviations

Table C.5: Parameter Estimates

Parameter	Distribution	Prior Mean	Prior Std Dev	Lower Bound	Upper Bound	Posterior Mode
β	uniform			0.9800	0.9985	0.9985
χ	beta	0.5000	0.1500			0.8450
σ	normal	2.0000	0.3000			2.8349
η	normal	2.0000	0.5000			1.0558
ϕ	normal	10.0000	2.0000			15.7179
κ_π	normal	1.5000	0.2500			3.0749
κ_y	normal	0.1200	0.0500			0.1033
$g_Z(L)$	uniform			-0.0500	0.0500	0.0029
$g_Z(N)$	uniform			0.0000	0.0500	0.0046
$\bar{\Pi}(L)$	uniform			1.0000	1.1000	1.0046
$\bar{\Pi}(N)$	uniform			1.0000	1.1000	1.0055
ξ	beta	0.5000	0.1500			0.4115
ρ_r	beta	0.5000	0.1500			0.5989
ρ_{AR}	beta	0.5000	0.1500			0.5630
ρ_A	beta	0.5000	0.1500			0.5251
$\sigma_R(1)$	inverse gamma	0.1000	2.0000			0.0038
$\sigma_R(2)$	inverse gamma	0.1000	2.0000			0.0272
$\sigma_A(1)$	inverse gamma	0.1000	2.0000			0.0964
$\sigma_A(2)$	inverse gamma	0.1000	2.0000			0.0297
$\sigma_Z(1)$	inverse gamma	0.1000	2.0000			0.0057
$\sigma_Z(2)$	inverse gamma	0.1000	2.0000			0.0790
$\sigma_\pi(1)$	inverse gamma	0.1000	2.0000			0.0038
$\sigma_\pi(2)$	inverse gamma	0.1000	2.0000			0.0629
$p_{N,L}$	beta	0.0500	0.0200			0.0292
$p_{L,N}$	beta	0.0500	0.0200			0.2500
$p_{1,2}$	beta	0.0500	0.0200			0.0234
$p_{2,1}$	beta	0.0500	0.0200			0.0522

Figure C.17: Volatility vs. ZLB States

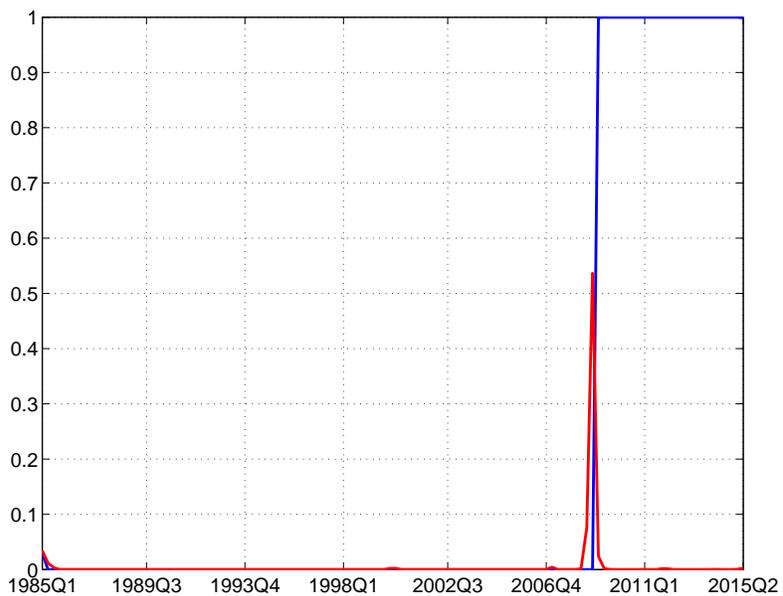


Figure C.18: Interest Rate vs. Probability of ZLB State

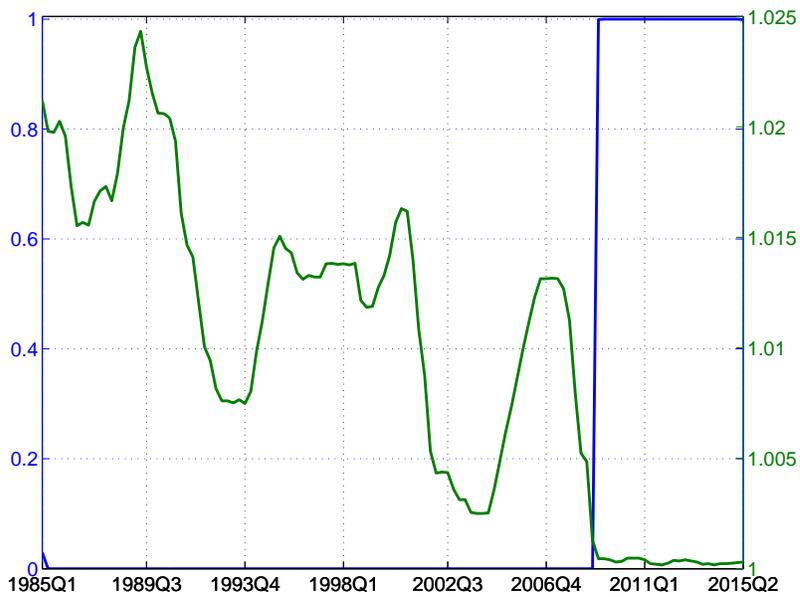
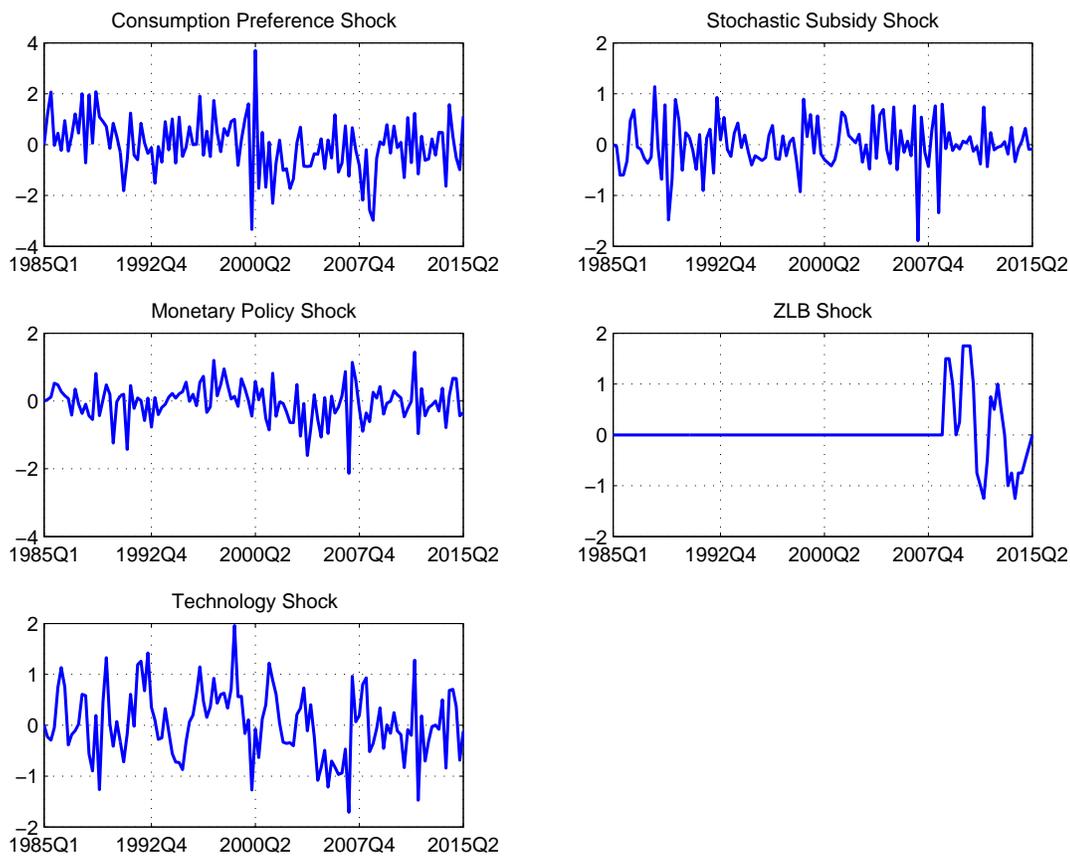


Figure C.19: Model Shocks



Appendix D. Impulse Responses

Figure D.20: Impulse Response Function: Productivity Shock

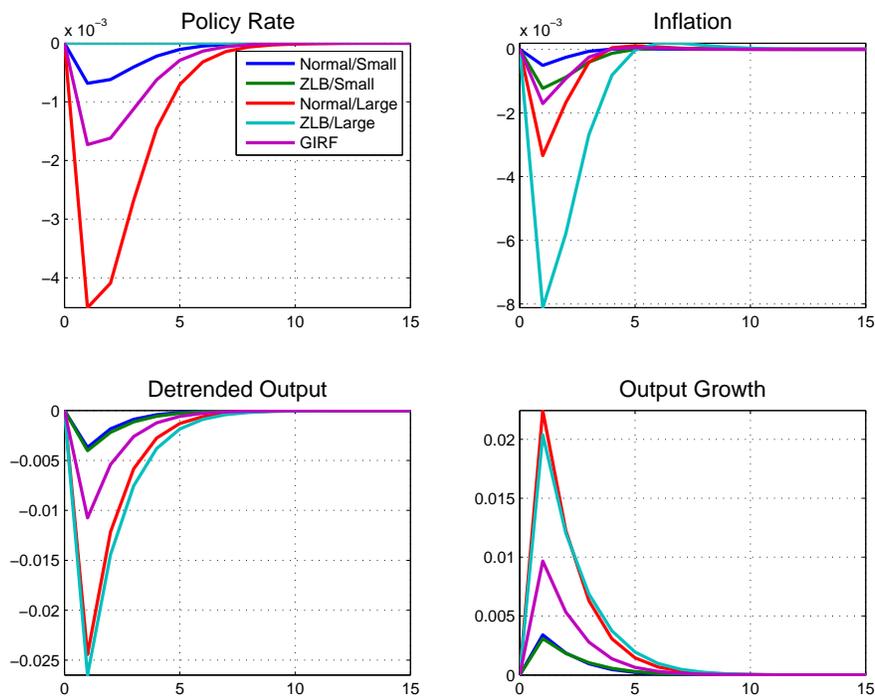
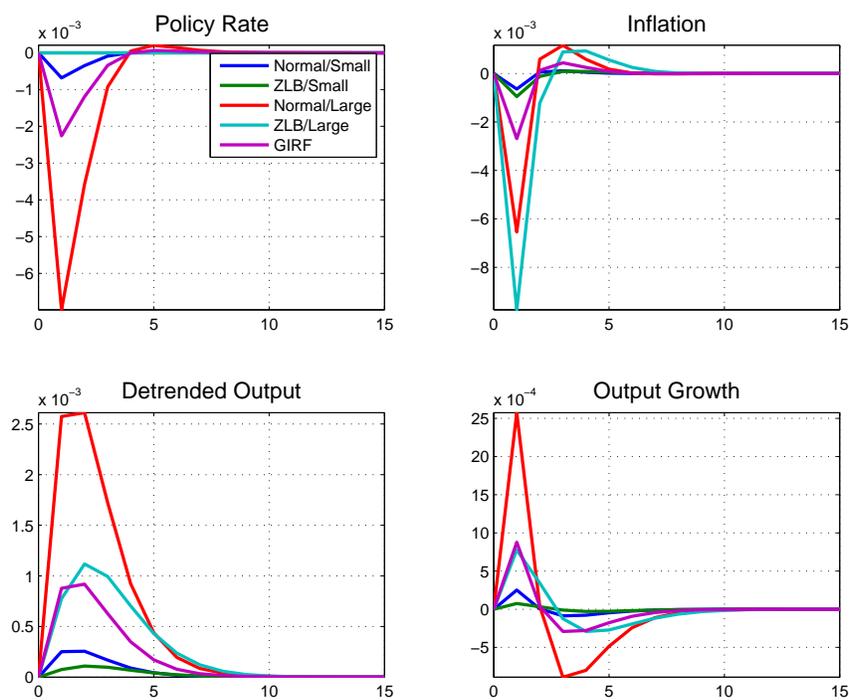


Figure D.21: Impulse Response Function: Stochastic Subsidy Shock



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