2013 | 22

Working Paper

Norges Bank Research

Dissecting the 2007-2009 real estate market bust: systematic pricing correction or just a housing fad?

Daniele Bianchi, Massimo Guidolin, and Francesco Ravazzolo

NB NORGES BANK

Working papers fra Norges Bank, fra 1992/1 til 2009/2 kan bestilles over e-post:
servicesenter@norges-bank.no

Fra 1999 og senere er publikasjonene tilgjengelige på www.norges-bank.no

Working papers inneholder forskningsarbeider og utredninger som vanligvis ikke har fått sin endelige form.
Hensikten er blant annet at forfatteren kan motta kommentarer fra kolleger og andre interesserte.
Synspunkter og konklusjoner i arbeidene står for forfatternes regning.

Working papers from Norges Bank, from 1992/1 to 2009/2 can be ordered by e-mail:
servicesenter@norges-bank.no

Working papers from 1999 onwards are available on www.norges-bank.no

Norges Bank's working papers present research projects and reports (not usually in their final form)
and are intended inter alia to enable the author to benefit from the comments of colleagues and other interested parties. Views and conclusions expressed in working papers are the responsibility of the authors alone.

ISSN 1502-8143 (online)
ISBN 978-82-7553-776-6 (online)

Dissecting the 2007-2009 real estate market bust: systematic pricing correction or just a housing fad?*

Daniele Bianchi[†], Massimo Guidolin[‡], and Francesco Ravazzolo[§]

This version: September 2013

Abstract

We use Bayesian methods to estimate a multi-factor linear asset pricing model characterized by structural instability in factor loadings, idiosyncratic variances, and factor risk premia. We use such a framework to investigate the key differences in the pricing mechanism that applies to residential vs. non-residential (such as office space, industrial buildings, retail property) real estate investment trusts (REITs). Under the assumption that the subprime crisis has had its epicentre in the housing/residential sector, we interpret any differential dynamics as indicative of the propagation mechanism of the crisis towards business-oriented segments of the US real estate market. We find important differences in the structure as well as the dynamic evolution of risk factor exposures across residential vs. non-residential REITs. An analysis of cross-sectional mispricings reveals that only retail, residential, and mortgage-specialized REITs were over-priced over the initial part of our sample, i.e., 1999-2006. Moreover, residential-driven real estate has structural properties that make it different from non-residential assets.

Key words: Multi-factor models, real estate, mispricing, real estate investment trusts.

JEL codes: G11, C53.

1. **Introduction**

Most macroeconomic and policy commentaries between 2007 and 2010 have been dominated by one obsessively worrisome news item: the U.S. real estate sector was in the middle of a convulsive bust characterized by downward spiralling prices and transaction volumes. As Gleaser (2013) has recently emphasized, such a bust was not the first and possibly not even the largest among those recorded in the history of the United States, but what he calls the "Great Convulsion" was sufficiently strong to

*This working paper should not be reported as representing the views of Norges Bank. The views expressed are those of the authors and do not necessarily reflect those of Norges Bank. We would like to thank Joachim Zietz (a discussant) and seminar session participants at the 2013 American Economic Association meetings in San Diego and the 2013 European Economic Association annual conference in Goteborg. Parts of this paper have been previously circulated under the title "What Makes Residential Different from Non-Residential REITs? Evidence from Multi-Factor Asset Pricing Models."

†Bocconi University, Department of Finance, Milan. E-mail: daniele.bianchi@unibocconi.it.

‡Bocconi University, IGIER, and CAIR, Manchester Business School. E-mail: massimo.guidolin@unibocconi.it.

§Norges Bank and BI Norwegian Business School. E-mail: francesco.ravazzolo@Norges-Bank.no.

produce one of the deepest and longest recessions of the last two centuries and a full-blown financial crisis. A number of authors (see e.g., Arce and Lopez-Salido, 2011; Case and Shiller, 2003; Smith and Smith, 2006, Wheaton and Nechayev, 2008) and commentators reached a simple conclusion: the big bust was simply the epilogue of an enormous housing bubble that would have been caused by rational (see e.g., Chu, 2013; Favilukis et al., 2010) as well irrational (see e.g., Case and Shiller, 2003; Gleaser, 2013 and references therein) behaviors by households and banks. This emphasis is less than surprising because a vast literature has pointed out that, within the real estate asset class, housing would be more prone to bubbles because of an often reported psychological overconfidence bias (see e.g., Gyourko, 2009).[1] This is also consistent with a number of macroeconomic models, like Arce and Lopez-Salido's (2011), in which a rational expectations equilibrium exists in which homeowners, who extract utility from their houses, coexist with investors, who hold houses only for resale purposes and do not expect to receive any rents or direct utility from occupancy. Leveraging on specific features of the housing market, a literature has discussed the specific causes of the Great Convulsion, for instance the fact that the bubble seems to have been triggered by irrational or themselves bubbly mortgage markets (see e.g., Demyanyk and Van Hemert, 2011; Dell'Ariccia et al., 2011; Hendershott et al., 2010).

In this paper, we use state-of-the art time series methods applied to well established and flexible multi-factor asset pricing models to ask two simple questions that appear to have been neglected so far. First, we investigate whether the dominant view (often, an instinctive reflection of the ways events have unfolded and news has been broadcast during the 2007-2008 subprime crisis, see e.g., Cecchetti, 2009; Gorton, 2009; Mian and Sufi, 2009) of the 2007-2010 real estate bust as predominantly consisting of a house price deflation phenomenon has any foundations from a rational pricing perspective. Equivalently, we ask whether asset market transaction data are compatible with the hypothesis of any abnormal or exceptional dynamics having affected either the housing/residential or the mortgage financing sectors, differentially from other, non-residential segments of the U.S. real estate market. As a result, our first testable hypothesis is whether—assuming the literature has correctly identified the subprime sector as the origin of the real estate bust —residential REITs were affected by the subprime crisis earlier and more strongly than other categories.[2] The second panel of Figure 1 supports our development of formal tests of this hypothesis: the valuations of residential and mortgage real estate led other sectors between early 2007 and Summer 2008; yet, they also recovered before most other sectors after 2009 and appear to display dynamics that is different from business-related real estate indices.

[1]Using the words by Case and Shiller (2003, p. 321), "Expectations of future appreciation of the home are a motive for buying that deflects consideration from how much one is paying for housing services. That is what a bubble is all about: buying for the future price increases, rather than simply for the pleasure of occupying the home." Clearly, these two complementary motives to invest in real estate are largely absent in categories that differ from housing, when the pleasure of occupying (say) a factory building, a parcel of land, or an empty shop are generally absent. Mian and Sufi (2011) find that a large fraction of the home equity loans that were taken during the housing boom were used to finance consumption, which also appears to be a phenomenon specific to the housing choice.

[2]While residential (in particular, apartment-investing) REITs represent commercial property, the key distinction in this paper is between real estate assets that are directly related to business activities (industrial buildings, offices, shopping malls, and free-standing shops) vs. residential equity REITs that invest inmanufactured homes and apartments, as well as mortgage REITs that are involved with purchasing housing-related loans and mortgage-backed securities.

Our second question is whether the tumble in real estate prices derived from either a correction of a previous large mispricing of real estate (or parts of it) as an asset class or whether it was an irrationally precipitated event, that is difficult to rationalize using standard but flexibly implemented asset pricing models. The two perspectives show of course an interesting intersection as in this paper we also study whether any differential dynamics between the residential and the non-residential, business-specialized sectors of the U.S. real estate market may derive from a heterogeneous evolution of risk exposures and whether these implied any correction of a mispricing that had endogenously emerged in the residential sector but that had not occurred in the non-residential segment of the market.

In methodological terms, we make two key choices. First, supported by a recent real estate finance literature (see, e.g., Cotter and Roll, 2011; Gyourko, 2009) that establishes robust links between publicly traded securities and underlying real assets, we use closing market price data at monthly frequency of real estate investment trusts (REITs) to measure real estate valuations ensuring sufficient liquidity and homogeneity over time (see the discussion in Himmelberg et al., 2005).[3] Because REITs offer abundant, high-quality data for a variety of subsectors, they give us the chance to perform tests that distinguish among portfolios of residential (hence, housing-related), of mortgage, and of nonresidential real estate investments, as required by our first question. Such tests would be impossible should one use appraisal-based or repeat-sale data that are subject to upward biases and quality homogeneity issues, respectively (see e.g., Rappoport, 2007), and generally available for houses only. Moreover, the 2007-2010 downturn in REIT valuations also represents the largest bust in publicly traded real estate values in history (see Gyourko, 2009). Second, we analyze the pricing of U.S. real estate assets in an encompassing no-arbitrage multi-factor framework by training a model to jointly price stocks, government bonds, corporate bonds, as well as REITs, using driving macroeconomic forces that are capable of pricing the cross-section of U.S. securities, with or without real estate (see e.g., Bianchi et al., 2013). Because the implementation of such an APT-style framework requires data on liquid assets traded in a frictionless market, proxying real estate valuations with REITs seems natural. The model emphasizes the existence of no-arbitrage conditions between real estate and other financial assets, in the tradition of Case and Shiller (1989). As discussed by Smith and Smith (2006), to gauge the existence of misspricings in the real estate sector, it is fundamental to incorporate also cross-sectional data on the way other assets are priced.

Our estimation approach based on Bayesian Monte Carlo Markov Chain techniques allows us to entertain flexible multi-factor APT-style models in which many macroeconomic risk variables can be accommodated, risk exposures (the so-called "betas") are time-varying, idiosyncratic non-diversifiable risk follows a stochastic process (i.e., it is heteroskedastic), and also risk premia are themselves subject to instabilities. When the framework is specified to include a number of standard macroeconomic factors (the return on the market portfolio; the credit risk premium; the riskless term premium; un-

[3] The Real Estate Investment Trust Act of 1960 authorized the creation of closed-end, exhange listed funds that allow small investors to pool their holdings of commercial real estate in order to obtain the same economic benefits as might be obtained by direct ownership. REITs offer investors tax advantages but are subject to the obligation to distribute at least 90% of their taxable income to shareholders annually in the form of dividends.

expected inflation; the rate of growth of industrial production, IP; the rate of growth of real personal consumption; the 1-month real T-bill rate; one aggregate liquidity factor) that are assumed to drive the stochastic discount factor in a linear fashion, we find evidence that the model is not misspecified, in the sense that for most portfolios of equities and bonds there is no evidence of structural and persistent mispricing. A rich literature (see e.g., Iacoviello, 2005; Iacoviello and Neri, 2010) has recently endogenized the linkages between real estate prices and business cycle shocks in general equilibrium models that our empirical framework simply aims at approximating. In fact, a number of the macroeconomic factors are precisely priced in the cross-section of excess returns, with sensibly sized and signed premia. Such a flexible, empirical model captures the intuition of a number of carefully built, but tightly parameterized models (see e.g., Favilukis et al., 2010) that support a story in which gyrations in risk premia (caused by exogenous shocks) would explain the recent boom-bust pattern.

We report two novel findings. First, we find differences in the structure as well the dynamics of risk factor exposures across residential vs. industrial, office, and retail REITs. This means that indeed residential REITs, most related to housing, were "special" during our sample, and in particular during the years in which the alleged housing bubble built up. Residential REITs are characterized by negative but mildly increasing exposures to market risk, by quickly retreating exposures to business cycle risk, and by massive and quickly increasing betas vs. unexpected inflation. In fact, by 2007 residential REITs came to practically carry only unexpected inflation risk, a powerful sign of disconnect—especially at the time of stable and predictable inflation rates—from any other underlying macroeconomic forces. REITs that specialize in industrial and office investments carry instead negative exposure to real output growth risks, and positive exposure to inflation and bond market risks, as measured by Cochrane and Piazzesi's (2005) factor. Retail-specialized REITs display a negative, significant and relatively stable exposure to market risk and positive and large exposures to unexpected inflation and real interest rate risks.

Second, an analysis of cross-sectional mispricing reveals that all the indicators (Jensen's alphas) implied by REITs were positive and relatively large. Ex-post, we obtain evidence that the *entire* real estate asset class has been long and persistently over-priced in the U.S. Realized excess returns have been (on average) between 0.5 and 2 percent higher than what would have been justified by their exposure to standard risk factors between 1999 and 2011. Additionally, and with the partial exception of mortgage investments, all sector REITs describe a homogeneous dynamics over time: the alphas start out relatively low between 1999 and 2004. Between 2005 and late 2007, all alphas climb up, in some cases going from a few basis points per month in late 2004 to as high as 2.2 percent per month. This was the great U.S. real estate bubble, with trading volumes, borrowing, and prices all exploding at the same time. However, the alphas slowly decline between 2008 and 2011, settling to levels below 1% per month and often returning to zero, when macro factors perfectly explain average returns.

Our multi-factor pricing exercise reports no evidence of a *pure* housing/residential real estate bubble inflating between 2004 and 2007, to subsequently burst. All REIT subsectors record a climb-up in alphas during this period. In fact, it is the alpha of the three retail/distribution-investing REIT portfolios that shows the steepest ascent. On the one hand, U.S. real estate would have been grossly and systematically over-priced between 2004 and 2007. Over-pricing is indicated by the fact that the posterior estimates of the real estate alphas are positive, increasing, and precisely estimated; large and positive alphas signal that after taking into account the risk exposures and premia captured by the nine factors entertained in our paper, real estate yielded "too high" a return that cannot be justified. This contradicts the occasionally reported conclusions that financial models would be able to justify the real estate valuations that were witnessed between 2004 and 2007 (see e.g., Glaeser et al., 2013, Smith and Smith, 2006). In this sense, the real estate fad has been pervasive. Also the claim that the great real estate bubble would have been a debt/mortgage-fueled one is consistent with the fact that between 2001 and 2004 mortgage REITs implied the largest, positive median alphas. In fact, alongside the residential one, an even bigger real estate over-pricing occurred instead—and in the perspective of our model was potentially still under way as late as the end of 2011—in the industrial and retail real estate sectors.

The paper is structured as follows. Section 2 describes the methodology. Section 3 presents the data. Section 4 presents Bayesian posterior estimates of time-varying factor exposures and of unit risk premia. Section 5 represents the heart of the paper and contains our findings on heterogeneous mispricing across different segments of the real estate universe, with special emphasis on the dichotomy residential vs. business REITs. Section 6 performs a few robustness checks. Section 7 concludes.

2. **Research design and methodology**

2.1. *The asset pricing framework*

Our research design is based on an extension of the time-varying beta multi-factor models introduced by Ferson and Harvey (1991) that in our application reflect Case and Shiller's (1989)-style financial no-arbitrage approach, where investors earn equal risk-adjusted returns by investing across assets (see e.g., Karolyi and Sanders, 1998). A multi-factor asset pricing model (MFAPM) posits a linear relationship between asset returns and a set of macroeconomic factors that are assumed to capture business cycle effects on beliefs and/or preferences, as summarized by a pricing kernel with time-varying properties. These macroeconomic factors are typically identified with the market portfolio (i.e., aggregate wealth) returns, the credit quality spread on corporate bonds, the term spread in the riskless yield curve, the rowth of industrial production, and inflation shocks (see, e.g., Chen et al., 1986). If we define the macroeconomic factors as $F_{j,t}$ ($j = 1, ..., K$) and $r_{i,t}$ to be the *excess* return on portfolio $i = 1, ..., N$, then a MFAPM is

$$r_{i,t} = \beta_{i0,t} + \sum_{j=1}^{K} \beta_{ij,t} F_{j,t} + \epsilon_{i,t}, \tag{1}$$

where $E[\epsilon_{i,t}] = E[\epsilon_{i,t} F_{j,t}] = 0$ for all $i = 1, ..., N$ and $j = 1, ..., K$. The $r_{i,t}$ are returns in excess of the risk-free rate proxied by the 1-month T-bill. Favilukis et al. (2010) discuss the importance of focusing on risk premia instead of long-term riskless rate to characterize the recent real estate bust.

The advantage of MFAPMs such as (1) is that a number of systematic factors well below the number of test assets, $K << N$, may capture large portions of the variability in returns. Importantly, even though the notation $\beta_{ij,t}$ implies that the factor loadings are allowed to be time-varying, such patterns of variation are in general left unspecified.

One problem with (1) is the difficulty with interpreting $\beta_{i0,t}$ (often called "Jensen's alpha") when some (or all) the risk factors are not themselves traded portfolios, i.e., returns: unless all the factors are themselves tradable portfolios, it is impossible to interpret any non-zero $\beta_{i0,t}$ as an abnormal return on portfolio i "left on the table" after all risks and risk exposures have been taken into account. If some of the factors are not replicated by traded portfolios (i.e., their values cannot be written as portfolio returns), there may be an important difference between the theoretical alpha that the model uncovers, and the actual alpha that an investor may achieve by trading assets on the basis of the MFAPM. To eliminate such a possibility, we follow the literature (see e.g., Ferson and Korajczyk, 1995; Lamont 2001; Vassalou 2003) and proceed as follows. When an economic risk factor is measured or can be easily deterministically converted in the form of an excess return, such as the U.S. market portfolio, real T-bill rates, term structure spreads, and default spread variables, we use the corresponding excess returns directly as a mimicking portfolio; Shanken (1992) shows that under some conditions, such an approach delivers the most efficient estimates of the risk premia. When a factor is not an excess return, such as industrial production growth, unexpected inflation, and real consumption growth, we construct the corresponding $K' \leq K$ mimicking portfolios by estimating time-series regressions of individual portfolio returns on M economic variables and lagged instruments that are known to forecast future investment opportunities (see Section 3 for details on the choice of instruments). Using the residuals of such regressions to form a (time-varying) estimate of the $N \times N$ (conditional) idiosyncratic covariance matrix, $\mathbf{V}_t$, we then form on each month of our sample the factor-mimicking portfolios for each of the K' factors by finding a vector of weights $\mathbf{w}_{j,t}$ $(j = 1, ..., K')$ that solves

$$\min_{\mathbf{w}_{j,t}} \mathbf{w}'_{j,t}\mathbf{V}_t\mathbf{w}_{j,t} \quad \text{s.t. (i) } \mathbf{w}'_{j,t}\mathbf{B}_{[j],t} = \mathbf{0}; \text{ (ii) } \mathbf{w}'_{j,t}\mathbf{1}_N = 1,$$

where $\mathbf{B}_{[j],t}$ is the $N \times (M-1)$ matrix that excludes the jth row from the $N \times M$ matrix of slope coefficient estimates $\mathbf{B}_t$ obtained by regressing returns data on the N portfolios on the M instruments. The jth mimicking portfolio is then formed from the individual base assets/portfolios, using the time series of portfolio weights $\mathbf{w}_{j,t}$, $t = 1, 2, ..., T$.[4]

Under the framework above, in the conditional version of Merton's (1973) intertemporal CAPM (ICAPM), the expected excess return (risk premium) on asset i over the interval $[t-1, t]$ may then be related to its "betas", i.e., factor loadings measuring the exposure of asset i to each of the systematic

[4] The conditional beta of the jth mimicking portfolio on the jth factor may change as $\mathbf{B}_t$ and $\mathbf{V}_t$ change. However, such mimicking portfolios are adjusted to have constant factor betas by combining them with T-bills so that the combined portfolio has a beta equal to the time-series average of the betas from the constrained optimizations.

risk factors and the associated unit risk premia (i.e., average compensations for unit risk exposure)

$$E[r_{i,t}|\mathbf{Z}_{t-1}] = \lambda_0(\mathbf{Z}_{t-1}) + \sum_{j=1}^{K} \beta_{ij,t|t-1}\lambda_j(\mathbf{Z}_{t-1}), \tag{2}$$

where both the betas and the risk premia are conditional on the information at time $t-1$, here summarized by the $M \times 1$ vector of "instruments" $\mathbf{Z}_{t-1}$, that capture any effects of the state of the economy on the risk premia. The framework in (1)-(2) describes a general conditional pricing framework that is known to hold under a variety of alternative assumptions (see e.g., Cochrane, 2005).

2.2. *A Bayesian state-space approach*

Stochastic, time-varying betas have been recently found to be crucial ingredients of conditional asset pricing because there is a growing evidence that careful modelling the dynamics in factor exposures may provide a decisive contribution to solve the typical anomalies associated with unconditional implementations of multi-factor models. For instance, Ang and Chen (2007) and Jostova and Philipov (2005) find that in a Fama and MacBeth's style exercise (see Section 6.1 and Appendix A), the CAPM is rejected when using rolling OLS beta estimates while the opposite verdict emerges when they allow for stochastic variation (in the form of a simple AR(1) process) in the conditional CAPM betas. Moreover, a recent macroeconomic literature tends to fine discrete instability in the elasticities that connect real estate valuations to business cycle shocks (see e.g., Iacoviello and Neri, 2010). Therefore, in this paper we propose a flexible parametric model that may capture both any instability in risk exposures and in residual variances. In this section, we provide some details to allow the Reader to appreciate the key features of our methodology. Additional details appear in Appendix B.

We specify the relationship between excess returns and factors and the time-varying dynamics in factor loadings and idiosyncratic volatility in a state-space (henceforth, Bayesian time-varying stochastic volatility-with breaks, BTVSVB) form where the observation equation is the standard linear model (1),

$$r_{i,t} = \beta_{i0,t} + \sum_{j=1}^{K} \beta_{ij,t}F_{j,t} + \sigma_{it}\epsilon_{i,t}, \tag{3}$$

where $\boldsymbol{\epsilon}_t \equiv [\epsilon_{1,t}, \epsilon_{2,t}, ..., \epsilon_{N,t}]' \sim N(0, \mathbf{I}_N)$ and $E[\epsilon_{i,t}] = E[\epsilon_{i,t}F_{j,t}] = 0$ for all $i = 1, ..., N$ and $j = 1, ..., K$. The time varying parameters $\beta_{ij,t}$ and σ_{it} are described by the state equations

$$\beta_{ij,t} = \beta_{ij,t-1} + \kappa_{ij,t}\eta_{ij,t} \qquad j = 0, ..., K, \tag{4}$$

$$\ln(\sigma^2_{i,t}) = \ln(\sigma^2_{i,t-1}) + \kappa_{i\nu,t}\upsilon_{i,t} \qquad i = 1, ..., N, \tag{5}$$

where $\boldsymbol{\epsilon}_t \equiv (\epsilon_{1,t}, \epsilon_{2,t}, ..., \epsilon_{N,t})' \sim N(0, diag\{\sigma^2_{1,t}, \sigma^2_{2,t}, ..., \sigma^2_{N,t}\})$, $\boldsymbol{\eta}_{i,t} \equiv (\eta_{i0,t}, \eta_{i1,t}, ..., \eta_{iK,t}, \upsilon_{i,t})' \sim N(0, \mathbf{Q}_i)$ with $\mathbf{Q}_i$ a diagonal matrix characterized by the parameters q^2_{i0}, q^2_{i1}, ..., q^2_{iK}, q^2_{iv}. Stochastic variations (breaks) in the level of both the beta coefficients and of the idiosyncratic variance σ^2_{it} are introduced and modelled through a mixture innovation approach as in Ravazzolo et al. (2007) and Giordani and Kohn (2008). The latent binary random variables $\kappa_{ij,t}$ and $\kappa_{i\nu,t}$ are used to capture the presence of random shifts in betas and/or idiosyncratic variance and, for the sake of simplicity, these are assumed to be independent of one another (i.e., across assets and factors) and over time.

$$\Pr\left[\kappa_{ij,t}=1\right]=\pi_{ij} \qquad \Pr\left[\kappa_{i\nu,t}=1\right]=\pi_{i\nu} \qquad i=1,...,N \quad j=0,...,K. \tag{6}$$

Note that even though we allow breaks to occur independently across assets, empirically we are not restraining breaks from occurring contemporaneously across assets and/or factor exposures.

This specification is very flexible as it allows for both constant and time-varying parameters. When $\kappa_{ij,\tau}=\kappa_{i\nu,\tau}=0$ for some $t=\tau$, then (4) reduces to (1) when the factor loadings and the quantity of idiosyncratic risk are assumed to be constant, as $\beta_{ij,\tau}=\beta_{ij,\tau-1}$ and $\ln\sigma^2_{i,\tau}=\ln\sigma^2_{i,\tau-1}$. However, when $\kappa_{ij,\tau}=1$ and/or $\kappa_{iv,\tau}=1$, then a break hits either beta or idiosyncratic variance or both, according to the random walk dynamics $\beta_{ij,\tau}=\beta_{ij,\tau-1}+\eta_{ij,\tau}$ and $\ln(\sigma^2_{i,\tau})=\ln(\sigma^2_{i,\tau-1})+\upsilon_{i,t}$ (or $\sigma^2_{i,\tau}=\sigma^2_{i,\tau-1}\exp(\upsilon_{i,\tau})$). Note that because when a break affects the betas and/or variances, the random shift is measured by variables collected in $\boldsymbol{\eta}_{i,t}$, we can also interpret $\mathbf{Q}_i$ not only as a standard, "cold" measure of the covariance matrix of the random breaks in $\boldsymbol{\eta}_{i,t}$, but also of the "size" of such breaks: a large q^2_j means for instance that whenever $\beta_{ij,t}$ is hit by a break, such a shift is more likely to be large (in absolute value). The same applies to the interpretation of q^2_{iv} as the size of breaks in idiosyncratic variance. Importantly, nothing forces the changepoint indicators, $\kappa_{ij,\tau}$ and $\kappa_{i\nu,\tau}$ for $i=1,...,N$ and $j=1,...,K$, to ever imply breaks. Equivalently, the data may suggest $\kappa_{ij,\tau}=\kappa_{i\nu,\tau}=0$ $\forall\tau$ thus implying constant betas, idiosyncratic risk (and, as we shall see, risk premia).

The cross-sectional equilibrium restrictions derived from (2) are then imposed as

$$r_{i,t}=\lambda_{0,t}+\sum_{j=1}^{K}\lambda_{j,t}\beta_{ij,t|t-1}+e_{i,t} \qquad i=1,...,N, \tag{7}$$

where $e_{i,t}\sim N(0,\psi^2)$ and $\beta_{ij,t|t-1}$ represents a draw from the predictive distribution in the state dynamics (4). This is obtained by integrating out both the probability of having a structural break as well as the uncertainty about the size of the break itself. This is the exact analog of the logic emphasized by Ferson and Harvey (1991), namely, that the time t excess return on asset i can be determined by investors with reference only to information available up to time $t-1$. Moreover, even though the time-varying betas, $\beta_{ij,t|t-1}$, clearly depend only on the information up to time $t-1$ so that the spirit of (2) applies, (7) avoids any parameterization of the dependence of the betas from the instruments in $\mathbf{Z}_{t-1}$ which may be advantageous. For instance, Ghysels (1998) has noted that the estimates of factor loadings obtained from the explicit use of instrumental variables are very sensitive to the specific variables considered.

Following McCulloch and Rossi (1991) and Geweke and Zhou (1996) the risk premia $\boldsymbol{\lambda}_t\equiv(\lambda_{0,t},\lambda_{1,t},...,\lambda_{K,t})'$ are estimated jointly with the loadings $\mathbf{B}_t\equiv\left\{\beta_{ij,t}\right\}_{i=1}^{N}{}_{j=0}^{K}$ the idiosyncratic variances $\boldsymbol{\sigma}^2_t\equiv$

$\left(\sigma_{1t}^2, \sigma_{2t}^2, ..., \sigma_{Nt}^2\right)'$, as well as the other parameters $\mathbf{\Theta} = \{\boldsymbol{\theta}_i\}_{i=1}^N$ with $\boldsymbol{\theta}_i \equiv (\mathbf{q}_i^2, \boldsymbol{\pi}_i)$, where $\mathbf{q}_i^2 \equiv (q_{i0}^2, q_{i1}^2, ..., q_{iK}^2, q_{iv}^2)'$ is the vector of conditional volatilities of the factor loadings and the idiosyncratic risks. By fully characterizing the joint posterior distribution of both betas and risk premia we avoid the "error in variables" problem that otherwise affects the standard two-step estimation procedure (see Section 6.1).

2.3. *Posterior simulation*

We estimate (4) using a Bayesian approach, which probably represents the only feasible estimation method for a model with the features of the BTVSVB framework.[5] Such a Bayesian setting also allows us to account for parameter uncertainty when estimating both states and parameters. This is particularly relevant because this implies that we can characterize the posterior probabilities for the unobserved binary states $\kappa_{ij,t}$ and $\kappa_{i\nu,t}$ for $t = 1, ..., T$. These can then be used to incorporate uncertainty regarding the timing of the structural breaks in the joint posterior of the state dynamics. For the Bayesian algorithm illustrated in Appendix B to work, we need to specify the prior distributions of each of the parameters. Appendix A illustrates such priors.

Posteriors are then characterized through the Gibbs sampler algorithm developed in Geman and Geman (1984), in combination with the data augmentation technique by Tanner and Wong (1987). The latent variables $\beta_{ij,t}$, σ_{it}^2 and $\kappa_{ij,t}$, $\kappa_{i\nu,t}$ for each of the $i = 1, ..., N$ assets, each of the $j = 1, ..., K$ factors and at each time $t = 1, ..., T$, are simulated alongside the model parameters $\boldsymbol{\theta}_i$ and the equilibrium risk premia $\boldsymbol{\lambda}_t$. One can think of the latent variables as nuisance parameters that are "integrated out" by the Gibbs sampler. However, to apply the Gibbs sampler we need to write down the complete likelihood function, the joint density of data and state variables. Defining $\boldsymbol{\theta} \equiv \{\boldsymbol{\theta}_i\}_{i=1}^N$, $\mathbf{B}_t \equiv \{\boldsymbol{\beta}_{it}\}_{i=1}^T$, $\mathbf{B} \equiv \{\mathbf{B}_t\}_{t=1}^T$, $\mathbf{R} \equiv \{r_{it}\}_{i=1}^T {}_{t=1}^N$, $\mathbf{F} \equiv \{\boldsymbol{F}_t\}_{t=1}^T$, $\boldsymbol{\lambda} \equiv \{\boldsymbol{\lambda}_t\}_{t=1}^T$, $\mathcal{K}_\beta \equiv \{\kappa_{ij,t}\}_{j=1}^K {}_{i=1}^N {}_{t=1}^T$, $\mathcal{K}_\sigma \equiv \{\kappa_{iv,t}\}_{i=1}^N {}_{t=1}^T$, $\mathbf{\Sigma} = \left\{\boldsymbol{\sigma}_{it}^2\right\}_{i=1}^N {}_{t=1}^T$, the likelihood is

$$p(\mathbf{R}, \mathbf{B}, \mathcal{K}, \mathbf{\Sigma}, \boldsymbol{\lambda}|\boldsymbol{\theta}, \mathbf{F}) = \prod_{t=1}^T \left\{ \prod_{i=1}^N p(r_{it}|\mathbf{F}_t, \boldsymbol{\beta}_{it}, \sigma_{it}^2) p(\sigma_{it}^2|\sigma_{it-1}^2, \kappa_{i\nu,t}, q_{iv}^2) \pi_{i\nu}^{\kappa_{i\nu t}} (1 - \pi_{i\nu})^{1-\kappa_{i\nu t}} \times \right. \tag{8}$$

$$\left. \times \left[\prod_{j=0}^K p(\beta_{ij,t}|\beta_{ij,t-1}, \kappa_{ij,t}, q_{ij}^2) \times \pi_{ij}^{\kappa_{ijt}} (1 - \pi_{ij})^{1-\kappa_{ijt}} \right] p\left(\boldsymbol{\lambda}_t, \psi^2|\mathbf{B}_t, \mathbf{R}_t\right) \right\},$$

where $\mathcal{K} \equiv (\mathcal{K}_\beta, \mathcal{K}_\sigma)$ and $\boldsymbol{F}_t = (F_{1,t}, F_{2,t}, ..., F_{K,t})'$. Combining the prior specifications (21)-(23) with the complete likelihood, we obtain the posterior density $p(\boldsymbol{\theta}, \mathbf{B}, \mathcal{K}, \mathbf{\Sigma}, \boldsymbol{\lambda}|\mathbf{R}, \mathbf{F}) \propto p(\boldsymbol{\theta}) p(\mathbf{R}, \mathbf{B}, \mathcal{K}, \mathbf{\Sigma}, \boldsymbol{\lambda}|\boldsymbol{\theta}, \mathbf{F})$. Our Gibbs sampler is a combination of the Forward Filtering Backward Sampling of Carter and Kohn (1994), Omori et al. (2007) and the efficient sampling algorithm for the random breaks proposed in Gerlach et al. (2000). At each iteration of the sampler we sequentially cycle through the following steps:

[5] In a frequentist framework it would be hard to separately identify the stochastic shifts represented by the variables $\kappa_{ij,t}$ and $\kappa_{iv,t}$ from the continuous shocks in $\eta_{ij,t}$ and $\upsilon_{i,t}$ without specifying some *ad-hoc* parametric process for $\kappa_{ij,t}$ and $\kappa_{iv,t}$.

1. Draw $\mathcal{K}_\beta$ conditional on $\mathbf{\Sigma}, \mathcal{K}_\sigma, \boldsymbol{\theta}$, $\mathbf{R}$ and $\mathbf{F}$.

2. Draw $\mathbf{B}$ conditional on $\mathbf{\Sigma}, \mathcal{K}, \boldsymbol{\theta}$, $\mathbf{R}$ and $\mathbf{F}$.

3. Draw $\mathcal{K}_\sigma$ conditional on $\mathbf{B}, \mathcal{K}_\beta, \boldsymbol{\theta}$, $\mathbf{R}$ and $\mathbf{F}$.

4. Draw $\mathbf{R}$ conditional on $\mathbf{B}, \mathcal{K}, \boldsymbol{\theta}$, $\mathbf{R}$ and $\mathbf{F}$.

5. Draw $\boldsymbol{\lambda}$ conditional on $\mathbf{B}, \mathcal{K}, \boldsymbol{\theta}$, $\mathbf{R}$ and $\mathbf{\Sigma}$.

6. Draw $\boldsymbol{\theta}$ conditional on $\mathbf{B}, \mathcal{K}$, $\mathbf{R}$ and $\mathbf{F}$.

We use a burn-in period of 1,000 and draw 5,000 observations storing every second observation to simulate the posterior of parameters and latent variables. The autocorrelations of the draws are low.

2.4. *Restricted models*

The BTVSVB model presented in (3)-(6) is the most general specification we consider in this paper. However, such a framework is richly parameterized and we cannot rule out that issues related to over-parameterization may arise. Therefore, for benchmarking purposes, we also estimated models derived by imposing a number of restrictions on the dynamics of the state equation:

1. $\kappa_{i\nu,t} = 0 \ \forall i, t$, i.e. a constant idiosyncratic volatility model:

$$
\begin{aligned}
r_{i,t} &= \beta_{i0,t} + \sum_{j=1}^{K} \beta_{ij,t} F_{j,t} + \sigma_i \epsilon_{i,t} \qquad i = 1, ..., N, \\
\beta_{ij,t} &= \beta_{ij,t-1} + \kappa_{ij,t} \eta_{ij,t} \qquad j = 0, ..., K,
\end{aligned} \tag{9}
$$

   under the same distributional assumption as (3)-(7). We will call this model a Bayesian homoskedastic time-varying betas model, i.e., BTVB.

2. $\kappa_{ij,t} = 1 \ \forall i, j, t$ and $\kappa_{i\nu,t} = 1 \ \forall i, t$, a time-varying parameter model (TVPM) in which both the betas and idiosyncratic risk follow random walk specifications common to the applied econometrics literature (Koop and Potter, 2007):

$$
\begin{aligned}
r_{i,t} &= \beta_{i0,t} + \sum_{j=1}^{K} \beta_{ij,t} F_{j,t} + \sigma_{it} \epsilon_{i,t} \\
\beta_{ij,t} = \beta_{ij,t-1} + \eta_{ij,t} \quad j = 0, ..., K, &\qquad \ln(\sigma_{i,t}^2) = \ln(\sigma_{i,t-1}^2) + \upsilon_{i,t} \quad i = 1, ..., N.
\end{aligned} \tag{10}
$$

   The TVPM assumes a unit probability of breaks (even though these are of a small size) in the dynamics of the states $\beta_{ij,t}$ and $\sigma_{i,t}^2$ at each point in time. This is indeed a fairly strict assumption which is not necessarily supported by the data, as we will document in our empirical analysis. Note that even though we simply name the model TVPM, it still features stochastic volatility as $\kappa_{iv,t} = 0$ is not imposed.

3. Trivially, the symmetric case of $\kappa_{ij,t} = \kappa_{iv,t} = 0\ \forall t$ implies that $\beta_{ij,t} = \beta_{ij,t-1} = \beta_{ij}$ and $\ln(\sigma^2_{i,t}) = \ln(\sigma^2_{i,t-1}) = \ln(\sigma^2_i)$ and consists of the classical case with constant betas and idiosyncratic variances. Section 6.1 shows how such a model may be simply estimated using OLS, according a the two-step Fama-MacBeth approach.

The constant volatility BTVB specification is used to highlight the effects of instabilities in residual variances. The TVPM is used as a competing specification to show the benefits of considering the parsimony of occasionally large breaks in (4)-(6) as opposed to small, frequent breaks. For all these restricted specifications, the choice of priors and the MCMC methods are the same as in Sections 2.3-2.4, with suitable adaptations required by the simpler structure of the constrained models.

2.5. *Pricing errors*

We follow Geweke and Zhou (1996) and measure the closeness of the pricing approximation provided by (7), $E_{t-1}[r_{i,t}] \simeq \lambda_{0,t} + \sum_{j=1}^{K} \beta_{ij,t|t-1}\lambda_{j,t}$, by computing at each time t the average squared recursive pricing error across all the N test assets/portfolios,

$$Q^2_{t,N} = \frac{1}{N}\left[\boldsymbol{\beta}'_{0,t}\left(\mathbf{I}_N - \mathbf{B}_t\left(\mathbf{B}'_t\mathbf{B}_t\right)^{-1}\mathbf{B}'_t\right)\boldsymbol{\beta}_{0,t}\right] \qquad t = 1, ..., T, \tag{11}$$

where $\boldsymbol{\beta}_{0,t}$ is the $N \times 1$ vector of intercepts, $\mathbf{I}_N$ is an N-dimensional identity matrix, and $\mathbf{B}_t \equiv (\boldsymbol{\iota}_N, \boldsymbol{\beta}_{1,t}, ..., \boldsymbol{\beta}_{K,t})$ is a $N \times K$ matrix collecting vectors of time t betas of all the assets/portfolios vs. each of the K risk factors, with $\boldsymbol{\beta}_{j,t} \equiv \left(\beta_{1j,t},\ ..., \beta_{Nj,t}\right)'$ a $N \times 1$ vector of factor loadings on the jth risk factor. These pricing errors are recursive because at each point in time they are obtained using only information available up to that point. Because our Gibbs sampling scheme derives posteriors for all the objects that enter $\boldsymbol{\beta}_{0,t}$ and $\mathbf{B}_t$, we compute the posterior density of the average (squared) pricing error statistic.

2.6. *Decomposition tests*

Independently of the estimation methods employed, we use the estimated time series of posterior factor loadings and risk premia to perform a number of "economic" tests. We use (7) to decompose excess asset returns on each time period in a component related to risk, represented by the term $\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}$ plus a residual $\lambda_{0,t} + e_{i,t}$. In principle, a multi-factor model is as good as the implied percentage of total variation in excess returns explained by the first component, $\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}$. However, we should emphasize that even though (7) refers to excess returns, these are simply statistical implementations of the asset pricing framework in (1). This implies that in practice it may be excessive to expect that $\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}$ is able to explain most (or even much) of the variability in excess returns. A more sensible goal seems that $\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}$ ought to at least explain the *predictable* variation in excess returns (see the discussion in Ferson and Korajczyk, 1995)

We therefore adopt the following approach. First, the excess return on each asset is regressed onto a set of M instrumental variables that proxy for available information at time $t-1$, $\mathbf{Z}_{t-1}$,

$$r_{i,t} = \theta_{i0} + \sum_{m=1}^{M} \theta_{im} Z_{m,t-1} + \xi_{i,t}, \tag{12}$$

to compute the sample variance of the resulting fitted values,

$$Var[P(r_{it}|\mathbf{Z}_{t-1})] \equiv Var\left[\hat{\theta}_{i0} + \sum_{m=1}^{M} \hat{\theta}_{im} Z_{m,t-1}\right], \tag{13}$$

where the notation $P(r_{it}|\mathbf{Z}_{t-1})$ means "linear projection" of r_{it} on a set of instruments, $\mathbf{Z}_{t-1}$. Second, for each asset $i = 1, ..., N$, a time series of fitted risk compensations, $\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}$, is derived and regressed onto the instrumental variables,

$$\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1} = \theta'_{i0} + \sum_{m=1}^{M} \theta'_{im} Z_{m,t-1} + \xi'_{i,t} \tag{14}$$

to compute the sample variance of fitted risk compensations:

$$Var\left[P\left(\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right] \equiv Var\left[\hat{\theta}'_{i0} + \sum_{m=1}^{M} \hat{\theta}'_{im} Z_{m,t-1}\right]. \tag{15}$$

The predictable component of excess returns in (12) not captured by the model is then the sample variance of the fitted values from the regression of the residuals $\hat{\xi}_{i,t}$ on the instruments, $Var[P(r_{i,t} - \sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1})]$. At this point, it is informative to compute two variance ratios, commonly called $VR1$ and $VR2$, after Ferson and Harvey (1991):

$$VR1 \equiv \frac{Var\left[P\left(\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right]}{Var[P(r_{it}|\mathbf{Z}_{t-1})]} > 0 \tag{16}$$

$$VR2 \equiv \frac{Var\left[P\left(r_{i,t} - \sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right]}{Var[P(r_{it}|\mathbf{Z}_{t-1})]} > 0. \tag{17}$$

VR1 should be equal to 1 if the multi-factor model is correctly specified, which means that all the predictable variation in excess returns ought to be captured by variation in risk compensations; at the same time, VR2 should be equal to zero if the multi-factor model is correctly specified.[6]

When these tests are implemented using the estimation outputs obtained from the BTVSVB framework, we preserve complete consistency with our Bayesian framework: drawing from the joint posterior densities of the factor loadings $\beta_{ij,t|t-1}$ and the implied risk premia $\lambda_{j,t}$, $i = 1, ..., N$, $j = 1, ..., K$, and $t = 1, ..., T$, and holding the instruments fixed over time, it becomes possible to actually compute VR1

[6] $VR1 = 1$ does not imply that $VR2 = 0$ and viceversa, because $Var[P(r_{it}|\mathbf{Z}_{t-1})]$ is not simply $Var[P(\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1})] + Var[P(r_{i,t} - \sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1})]$ as it also reflects a covariance effect.

and VR2 in correspondence to each of such draws. This means that any large set S of draws from the (matching) posterior distributions for the $\{\beta_{ij,t|t-1}\}$ and $\{\lambda_{j,t}\}$ generates a posterior distribution for the statistics $VR1$ and $VR2$. This makes it possible to conduct standard Bayesian "inferences" concerning the properties of $VR1$ and $VR2$ in our sample.

Finally, the predictable variation of returns due to the MFAPM may be decomposed into components imputed to each of the individual systematic risk factors, by factoring as in

$$
\begin{aligned}
Var[P(\sum_{j=1}^{K}\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1})] &= \sum_{j=1}^{K} Var\left[P\left(\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right] + \\
&+\sum_{j=1}^{K}\sum_{k=1}^{K} Cov[P\left(\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right), P\left(\lambda_{k,t}\beta_{ik,t|t-1}|\mathbf{Z}_{t-1}\right)]
\end{aligned} \tag{18}
$$

and tabulating and reporting $Var\left[P\left(\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right]$ for $j = 1, ..., K$ as well as the residual term $\sum_{j=1}^{K}\sum_{k=1}^{K} Cov[P\left(\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right), P\left(\lambda_{k,t}\beta_{ik,t|t-1}|\mathbf{Z}_{t-1}\right)]$ to measure any interaction terms. Note that because of the existence of the latter term, the equality

$$
\sum_{j=1}^{K}\frac{Var\left[P\left(\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right]}{Var\left[P\left(\sum_{j=1}^{K}\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1}\right)\right]} = 1 \tag{19}
$$

fails to hold, i.e., the sum of the K risk compensations is not guaranteed to equal the total predictable variation from the asset pricing model because of the covariance among individual risk compensations.

3. Data and summary statistics

Our paper is based on a large panel of monthly time series (45) sampled over the period 1994:01-2011:12. Although the choice of portfolios or individual securities in tests of multi-factor models is a researched topic in the empirical finance literature, in our case it is the economic questions that best advise us to use portfolios of securities. The 1994:01 starting date derives from the availability of monthly return series for all the sector REIT total return indices used in this paper. An initial five-year worth of observations is used to set priors and the analysis is implemented over the remaining 156 observations, per each series, over the interval 1999:01-2011:12. The series belong to three main categories. The first group, "Portfolio Returns", includes several asset classes like stocks, bonds and real estate, organized in portfolios, a procedure that is useful to tame the contribution of non-diversifiable risk. The stocks are publicly traded firms listed on the NYSE, AMEX and Nasdaq (from CRSP) and sorted according to two criteria. First, we form 10 industry portfolios by sorting firms according to their four-digit SIC code. Second, we form 10 additional portfolios by sorting (at the end of every year, and recursively updating this sorting at an annual frequency) NYSE, AMEX and Nasdaq stocks according to their size, as measured by the aggregate market value of the company's equity. Industry- and size-sorting criteria are sufficiently unrelated to make it plausible that the corresponding portfolios may contain different and non-overlapping information on the underlying factors and risk premia.

Data on long- (10-year) and medium-term (5-year) government bond returns are from Ibbotson and available from CRSP. Data on 1-month T-bill and 10-year government bond yields are from FREDII® at the Federal Reserve Bank of St. Louis and from CRSP. Data on below investment grade bond returns are approximated from Moody's (10-to-20 year maturity) Baa average corporate bond yields and converted into returns using Shiller's (1979) approximation formula. The data on sector tax-qualified REIT total returns come from the North American Real Estate Investment Trust (NAREIT) Association and consist of data on 8 portfolios, i.e., Industrial, Office, Shopping Centers, Regional Malls, Free Standing shops, Apartments, Manufactured Homes, and mortgage REITs. Apartments and Manufactured Homes represent the "Residential" real estate sector. These seven portfolios are formed when REITs are classified on the basis of their main focus of activity (with "other", residual REITs not considered). Mortgage REITs specialize in mortgage-backed security (MBS) investments. These are breakdowns common in the literature (see e.g., Payne and Waters, 2007). All excess return series are computed as the difference between total returns and 1-month T-bill returns, as usual.

Finally, we use a range of macroeconomic variables as standard proxies for the systematic, economy-wide risk factors potentially priced in asset returns. Lagged values of these risk factors (or simple transformation of the factors) are also used as "instruments" when relevant in our methodology, our logic being that all these variables belonged to the information set of the investors when they had made their portfolio decisions. In practice, we employ nine factors: the excess return on a value-weighted market portfolio that includes all stocks traded on the NYSE, AMEX, and Nasdaq; the default risk premium measured as the difference between Baa Moody's *yields* and yields on 10-year Treasuries; the change in the term premium, the difference between 10-year and 1-month Treasury yields; the unexpected inflation rate, computed as the residual of a simple ARIMA(0,1,1) model applied to (seasonally adjusted) CPI inflation; the rate of growth of (seasonally adjusted) industrial production (IP); the rate of growth of (seasonally adjusted) real personal consumption growth; the 1-month real T-bill rate of return computed as the difference between the 1-month T-bill nominal return and realized CPI inflation rate (not seasonally adjusted); the traded Liquidity factor from Pastor and Stambaugh (2003); the Bond premium factor from Cochrane and Piazzesi (2005).[7] Using a large number of factors is typical of the literature.[8]

Table 1 presents summary statistics for the time series under investigation over our overall 1994-2011 sample. The summary statistics in Table 1 show no unexpected stylized facts. Starting with the four REIT sectors, the three equity groups imply largely similar sample means, medians, and standard deviations of returns; these yield comparable monthly Sharpe ratios that fall between 0.12 and 0.15

[7] The traded liquidity factor consists of value-weighted returns on a high-minus-low exposure portfolio on an aggregate liquidity risk factor that sorts stocks on the basis of liquidity measures on stocks listed on the NYSE and AMEX. The bond risk premium factor is constructed as the projection of the equally weighted average of one-year excess holding period return on bonds with maturities of two, three, four, and five years on a constant, the one-year yield, and the two-through five-year forward rates. The bond risk factor is the fitted value of this regression.

[8] For instance Connor and Korajczyk (1988) find there are more than five factors at work in the economy; Ludvigson and Ng (2009) find evidence in favor of eight latent factors.

(here residential REITs display the highest Sharpe ratio of 0.148, as a result of a sample standard deviation that is slightly smaller than in the case of other sectors). As one would expect, mortgage REITs are characterized by lower mean and median returns; however, because their volatility is similar to that of equity REITs, their realized sample Sharpe ratio is relatively low, only 0.06 per month.

The REIT panel of Table 1 reveals few differences between Industrial and Office REITs (but the former are more volatile than the latter). On the contrary, the realized risk-return performance of Retail REITs appears to be driven by Free Standing REITs with a monthly Sharpe ratio of 0.18, to be contrasted to the comparably poor performance of Shopping Center-related REITs, 0.11. Finally, and in spite of the recent housing bust, the Residential sector reveals a good risk-reward trade-off, mostly driven by the Apartment-specialized sector, as it is characterized by strong average realized returns (1.2% per month), in spite of its high volatility (6% per month); Manufactured Home REIT returns give instead more stable, but lower returns. Most equity Sharpe ratios are in the 0.10-0.15 range. Bond Sharpe ratios are relatively high, due to the fact that our sample contains the massive flight-to-quality into Treasuries that has occurred during the financial crisis.

Figure 1 provides a visual summary of the movements of the REIT total return indices under investigation. As a benchmark, we also plot the total return index for the value-weighted market portfolio. To favor comparability across different sectors and sectors, all total return indices are standardized to equal 100 in correspondence with the end of January 2007. This date is chosen because most of the literature (see e.g., Aït-Sahalia et al., 2009) has dated the onset of the subprime crisis to early to mid-2007. To limit the number of series plotted, Industrial and Office REITs are aggregated in a "Industrial and Office" (I&O) sector, Shopping Centers, Regional Malls, Free Standing shops REITs into a "Retail" sector, and Apartments, Manufactured Homes into a"Residential" one. The top panel of Figure 1 provides motivation for our analysis because it shows that the residential sector exactly peaks in correspondence to the end of 2006 and leads the remaining two equity REIT sectors through all of 2007 and 2008. In fact, the mortgage REIT sector had already boomed between 2003 and 2005, but had also reached a new, local peak in early 2007 and—consistently with most anecdotal accounts of the onset of the subprime crisis (see e.g., Mian and Sufi, 2009)—subsequently tumbled starting in late Spring 2007. Interestingly however, from Fall 2008—approximately after the demise of Lehmann Brothers—the I&O and retail sectors started to lead (and fall at higher rate than) residential and mortgage REITs. This is consistent with the policy debate and the financial press accounts of the time (see e.g., Greenlee, 2009). Starting in Spring 2009, all four sectors recovered somewhat, with their total return indices approximately returning to the levels of late 2003, but the residential REIT index displays a "V-shaped" bounce-back that has no equivalent in the case of the other sectors. In fact, a simple calculation for the period January 2007 - December 2011 reveals that residential REIT is the only portfolio in Figure 1 for which average returns are positive, albeit small. Our goal in this paper is to explain these differential dynamics.

The bottom panel of Figure 1 presents similar information with reference to the raw eight REIT sectors. On the one hand, the picture that emerges is qualitatively similar to the one commented already. For instance, both apartments and manufactured homes follow the lead-lag-lead pattern observed for the aggregated data, even though the recovery of apartment-investing REITs appears to be slower than for manufactured homes. On the other hand, a few additional patterns are visible. For instance, REITs specialized in free-standing retail units have been hardly affected by the crisis, while REITs specialized in industrial buildings seem to have suffered the most, arguably as a result of the deep recession and of the structural over-capacity accumulated between 2005 and 2007, with the result that gross valuations as of the end of 2011 still lagged behind the levels last observed in 1999.

3.1. *Can REITs represent valuations in the real estate market?*

One crucial assumption that backs our empirical investigation is that REITs may be used to proxy the valuations in the U.S. real estate market. Even though testing this connection is beyond the scope of our paper, luckily there is a well developed real estate finance literature that has examined exactly this research question. The most recent conclusions of this literature are largely consistent with the claim that REITs are informative of the state of the real estate market in its various components and disaggregations. While the early literature had reported mixed findings (see e.g., Clayton and MacKinnon, 2003; Ling and Naranjo, 2003; Seck, 1996; but see Gyourko and Keim, 1992, for early findings that the public market reliably leads the private market in commercial real estate over the cycle), recent results support instead the thesis that REITs would accurately reflect, or even forecast, underlying property values. For instance, Chiang (2009) shows that past returns on public markets can forecast returns in real, physical markets: This result is consistent with the notion that public markets are more efficient in processing information than private markets. Moreover, the early literature had relied almost exclusively on appraisal-based measures of private real estate returns. Recent research by Boudry et al. (2012) using the novel NCREIF (National Council of Real Estate Investment Fiduciaries) MIT transaction-based indices (developed by Fisher et al., 2007), show that the relation between REIT and direct (privately-held properties) real estate returns appears to be strong, at least at long horizons.[9] More specifically, using a cointegration framework, they find robust evidence that REITs and the underlying real estate are related and that they share a long run equilibrium; both REITs and direct real estate returns adjust towards this long run relationship. Gyourko (2009) also finds considerable statistical association in the way housing, residential commercial real estate, and non-residential income-producing properties behave over time. He also notices a deterioration in underwriting standards similar to what has been reported for the housing sector. These results motivate our use of residential vs. non-residential REIT valuations in our paper as representative of the general, aggregate conditions in the U.S. real estate market.

[9] Additionally, since REITs tend to invest in institutional quality real estate, an ideal index would be constructed based on a similar set of properties. In this regard, the NCREIF universe of properties would make an excellent match to the set of REIT properties, since both groups tend to invest in institutional quality real estate.

4. **Empirical results**

4.1. *Posterior model likelihoods*

The first question we ask, as is common in all empirical papers, concerns the model that should be used in the investigation. We use the marginal likelihood of different models to perform a comparison that takes into account their overall (in-sample) statistical performance and not only their asset pricing plausibility, as described in Sections 2.5 and 2.6. The marginal likelihood of a model is known to take into account both the uncertainty about the size and the presence of structural breaks and the uncertainty concerning the parameters in (3)- (6). The marginal likelihood of each model is computed as

$$p(\mathbf{R}|\mathbf{F};\mathcal{M}_i) = \int ... \int \sum_{\mathcal{K}} p(\mathbf{R}|\mathbf{B},\mathcal{K},\mathbf{\Sigma},\boldsymbol{\lambda},\boldsymbol{\theta},\mathbf{F};\mathcal{M}_i) \times p(\boldsymbol{\theta},\mathbf{B},\mathcal{K},\mathbf{\Sigma},\boldsymbol{\lambda}|\mathbf{R},\mathbf{F};\mathcal{M}_i)d\mathbf{B}d\mathbf{\Sigma}d\boldsymbol{\theta}d\mathbf{\Sigma}, \quad (20)$$

where $\mathcal{M}_i$ identifies the ith model and the posterior density $p(\mathbf{R},\mathbf{B},\mathcal{K},\mathbf{\Sigma},\boldsymbol{\lambda}|\boldsymbol{\theta},\mathbf{F};\mathcal{M}_i)$ is given by (8). Following Chib (1995), we compute the marginal likelihood by replacing the unobservable breaks and parameters in the likelihood of the data generating process defined by (3)- (6) for each draw.

Table 2 reports the marginal log-likelihoods for each of the specifications in Sections 2.2-2.5 as well as the Bayes factors opposing each of the restricted frameworks to the BTVSVB one. However, because these are marginal log-likelihoods, one can easily compute Bayes factors comparing the remaining two models by difference of factors computed with reference to BTVSVB. The BTVSVB model implies the highest marginal log-likelihood across *all* of the portfolios under consideration, as well as the highest overall marginal likelihood, -193 vs. -386 for the homoskedastic BTVB model, -909 for TVPM, and -1492 for the two-step traditional implementation described in Section 6.1. Surprisingly, the TVP model ranks second across stocks outperforming the BTVB. This result indicates that fully acknowledging instability in the idiosyncratic risks plays a relevant role that cannot be simply surrogated by latent change-point models for the betas, similarly to the findings in Bianchi et al. (2013) and Nardari and Scruggs (2007) with reference to alternative applications. As one would expect, the classical Fama-MacBeth procedure ranks last with an overall marginal likelihood around 15 times lower than under BTVSVB. The Bayes factors, by construction, largely confirm these results: by exceeding 100, all of them are highly significant (see Kass and Raftery, 1995, for a justification of this scale/threshold). Interestingly, even the lowest among the asset/portfolio-specific Bayes factors in Table 2 is 125.

4.2. *Comparing pricing error performance*

Table 2 establishes the superiority of the more flexible BTVSVB model over all other competitors using a statistical metric. Yet, one would also like to have the comfort of some economic distance measure. We therefore extend our model comparison in Section 4.1 to include a different criterion, which is luckily rather straightforward: given our objectives, a model is as good as its realized pricing performance. In particular, because Section 5 will examine our research questions using a common notion of mispricing index (Jensen's alpha), it is crucial that a model be able to price our test portfolios as well as possible before any mispricings (i.e., $\beta_{i0,t}$s) are estimated.

Table 3 reports time averages of posterior mean pricing error $Q_{t,N}$ for each of our four models (to include the Fama-MacBeth two-step approach) across different sub-samples. The BTVSVB model yields the lowest average pricing error across the whole sample period (Panel A), with an average posterior error of 0.61% per month. As one would expect, because pricing financial portfolios in times of crisis is harder than in "normal" times, the mean pricing error increases towards the end of our sample (panel C, that refers to the 2007-2011 sub-sample), reaching 0.66%. However, the BTVSVB model keeps substantially out-performing all other models, often cutting average errors by almost half (this is for instance the case in panel C vs. the second best model, the homoskedastic BTVB, with an average of 1.01% per month). The naive two-step Fama-MacBeth approach turns out to be largely superior to a TVPM, in which random walk dynamics in betas and idiosyncratic variance are (incorrectly) imposed. For instance, over the full sample, while TVPM yields a mean pricing error of 1.99% per month (basically, the model has no explanatory power), the two-step approach yields a 1.05%. However, both statistics are grossly inferior to the 0.61% achieved by BTVSVB. Median pricing error statistics in the rightmost columns of Table 3 confirm our earlier remarks. Given the evidence in Sections 4.1 and 4.2, in the rest of the paper we exclusively devote our attention to the economic insights derived from the BTVBSV model, even though Section 6 performs a few robustness checks concerning the three alternatives introduced in Section 2.5.

4.3. *Factor loadings*

Figures 2-6 report selected plots of the posterior medians of the factor loadings $\beta_{ij,t}$ for each of the eight REIT sectors at the heart of our empirical investigation, along with the loadings for other four portfolios—two equity portfolios for deciles 10 (the largest stocks covered by CRSP) and 1 (the smallest stocks), and two bond portfolios (10-year Treasuries and long term corporate bonds below the investment grade threshold)—to be taken as representative of the range of results we have obtained for stocks and bonds.[10] Although we have performed estimation with reference to nine factors, we have tabulated results for only 5 of them to save space. In each plot, besides the posterior medians estimated over time, we also show the associated 95 percent Bayesian confidence bands.[11] A time t, the 95% credibility interval is characterized by the 2.5th and 97.5th percentiles of the posterior density of $\beta_{ij,t}$. In what follows we limit ourselves to two types of comments: a general comparison of the real estate asset class to stocks and bonds; a comparison within the real estate asset class among different types of portfolios to answer our key questions concerning the differences between residential

[10] Our plots never specifically focus on results for the industry portfolios, even though these have been used in estimating the MFAPM and especially its implied risk premia. Complete results are available upon request. Bianchi et al. (2013) focusses on related results for stocks and bonds, obtained using similar methods.

[11] Pinning down the "statistical significance" of coefficients (betas or lambdas) on the basis of 95% credibility intervals represents a rather stringent criterion because the Bayesian posterior density will reflect not only the uncertainty on the individual coefficient but also the overall uncertainty on the entire model (e.g., the uncertainty on structural instability of all the coefficients).

and retail/industrial REITs.

Figures 2-6 show that the real estate asset class shows particular exposure to risk factors that differ from those typical of stocks and bonds. For instance, in Figure 2 over our 1999-2011 sample, (most) REIT portfolios have a market beta that is lower (often close to zero or negative) than stocks have, similarly to what Cotter and Roll (2011) and Lee et al. (2008) have recently reported. Moreover, although for most REIT portfolios the general dynamics of the exposures to market risk follows a similar path over time, residential (in particular manufactured homes) and mortgage REITs show a marked increase between 2005 and 2007, and the corresponding 95% confidence regions all come to include zero by 2008. This indicates that while before the collapse of the subprime market, all REITs offered some partial hedging against aggregate market risk, this property is lost by residential and mortgage REITs as a result of the boom, making them market-neutral; in the case of equity portfolios (both reported and unreported), any oscillations tend to occur instead around a stationary but positive level. However, REITs also display market exposures that are similar to those of bonds, which also oscillate over our estimation sample.

One may wonder whether Figure 2 can be interpreted to mean that U.S. real estate is not exposed to business cycle risks, given the small or negative market betas. Figure 3, with reference to IP betas, shows that this is not the case. Most REIT indices—among them this is especially clear in the case of apartments, offices, and regional malls—display an upward trending and eventually "significantly" positive IP beta until 2004-2005. However, starting in 2006, this beta declines for all equity REIT portfolios at least until late 2008, when the beta resumes an upward trend. Interestingly, this occurs when exposure to market risk grows. Also in this respect, though fluctuations are more obvious, real estate shows dynamic properties that are more comparable to bonds (both corporate and Treasuries) than to stocks that, with few exceptions, tend to display positive and large exposure to real output.

However, the major differences between REITs and stocks and bonds emerge in the context of one traditional and here important macroecomic factor, unexpected inflation. In Figure 4, all sector REIT portfolios display a strongly time-varying and statistically significantly positive exposure to unexpected inflation. By contrast, equities and bonds generally show small, occasionally negative and often downward trending betas vs. unexpected inflation. This finding is consistent with two traditional views often discussed in the literature. First, that real estate would represent a "composite" asset class that inherits mixed features (here, factor exposures) from both stocks and bonds, see e.g., Simpson et al. (2007) and references therein. The second view is that real estate may correlate strongly with inflationary shocks (i.e., unexpected inflation, see e.g., Hoesli et al., 2008).[12]

Figures 5 and 6 show that U.S. real estate is not significantly exposed to either liquidity or short-term (real) interest rate risks. This is interesting because there is instead a well developed literature that has related real estate valuations to interest rate levels and monetary policy (see e.g., Iacoviello, 2005;

[12] Differences across asset classes appear instead to be weaker in the case of the credit risk premium, real consumption growth, Cochrane and Piazzesi's bond risk factor, and the riskless yield spread (unreported). This also occurs because the $\beta_{ij,t}$ posteriors are centered around generally small coefficient values and their 95% confidence bands often include zero. Complete plots of the beta posteriors are available upon request.

Iacoviello and Neri, 2010) and that has often emphasized the special character of real estate because of its illiquidity. However, we should bear in mind that we are investigating REITs, i.e., publicly traded vehicles that may be seen as derivatives linked to actual properties. In the case of the liquidity factor (Figure 5), it is evident that the betas are so small that the 95% confidence band tends to include the zero in more than 80% of our sample. In fact, this type of dynamics characterizes most other portfolios, including equities, as well. As far as the real rate factor is concerned, important differences across real estate portfolios emerge. Albeit, on average, all sectors are characterized by modest real rate betas, industrial, free-standing shops and regional malls-investing REITs are characterized by betas that, starting from zero or negative exposures, smoothly increase over time to become positive and precisely estimated by the end of our sample. Manufactured homes imply instead an opposite trend.

Treasury bond portfolios carry essentially nil exposure to market risk once the eight additional macroeconomic factors are controlled for, but gives negative betas on the credit risk factor, as one would expect. However, in line with basic principles, the beta of non-investment grade quality bonds is positive and precisely estimated. All bond portfolios have negative exposures to the slope of the yield curve; these betas are large with a posterior distribution tilted away from zero, especially in the case of 10-year Treasuries, which capture flight-to-quality effects, in the sense that Treasuries command high prices and low risk premia exactly when the riskless yield curve is flat or inverted, as typical of the early stages of recessions. Treasury bonds, especially long-term ones, have a positive and precisely estimated exposure to unexpected inflation, which is sensible because government securities are notoriously exposed to inflationary shocks. Treasury returns have weak exposure to IP growth and real consumption growth factors. Finally, the BTVSVB model allows us to infer considerable instability in the betas of all Treasuries with respect to IP growth, credit spread, and liquidity risks, with rather heterogeneous trends.

Figures 2-6 also show that for most factors and portfolios, including real estate, the BTVSVB model reveals interesting variation in betas. However, we emphasize that such time variation is not forced upon the data, in the sense that a casual look at the plots reveals that combinations of test assets and factors can be found for which the $\beta_{ij,t}$s implies little or no instability. For instance, in the left column of Figure 6, concerning the exposure of apartment REITs to the real short rate factor, the plot reveals a posterior median of $\beta_{apts,TB,t}$ that is flat at approximately -1.1 throughout the entire sample period. Interestingly, for most factors the eight REIT sectors tend to share a common dynamics in their exposures, even when such betas are characterized by different means. For instance, in Figure 4, the $\beta_{ij,t}$s with respect to unexpected inflation all generally increase (the only exception is mortgage REITs), but a comparison between the industrial and manufactured homes sectors reveals that while the former climbs from 0.4 in 1999 to almost 8 at the end of 2011, the latter increases from 0.8 in 1999 to 3.5 at the end of the sample.

The third feature of the factor exposures that deserves comment is that different REITs are characterized by a substantially heterogeneous dynamics in estimated beta posteriors. Moreover, residential REITs are clearly different from retail and industrial REITs; mortgage REITs have a risk factor structure that is very specific and that diverges from equity REITs. In this case it is useful to express comments on factor exposures across plots, but for each REIT portfolio. Residential, housing-driven REITs are characterized by negative but mildly increasing exposure to market risk, by quickly retreating exposures to industrial production growth, term premium, and real interest rate risks, and by massive and quickly increasing betas vs. unexpected inflation.[13] Interestingly, if one disregards betas whose confidence region includes a zero exposure, by 2007 residential REITs came to carry only unexpected inflation risk.

REITs that specialize in I&O investments carry instead negative exposure to real output growth risk, and positive exposure to inflation and bond market risks, as measured by Cochrane and Piazzesi's factor (the plot is unreported); moreover, during the peak of the real estate "bubble" (2003-2006), I&O portfolios came to display negative and accurately estimated exposures with respect to real consumption risks, which—with the same caveats used above with reference to market portfolio and IP risks—which adds to a characterization of this period of intense price growth. Retail- specialized REITs provide a negative, significant and relatively stable exposure to market risk and positive and large beta vs. unexpected inflation and short rate risks (increasing in the former case and time-varying with a trough in 2004-2005 in the latter). A comparison between residential on the one hand, and I&O and retail REITs on the other, sheds light on one potential cause of the differential behavior in the aftermath of the 2007-2009 crisis: the residential sector no longer has any exposure to general market dynamics and its upward swing is then explained by an increasing risk of unexpected inflation that represents a sensible story in the presence of massive quantitative easing interventions by the Federal Reserve.

REITs that specialize in mortgages have a large, negative and progressively declining exposure to inflation risk (the corresponding Bayesian confidence bands touch zero around mid-2009), and by positive beta with respect to the riskless term spread and the bond factor. Given the nature of mortgage REITs, that systematically invest in mortgage-backed securities, these fixed income-type results are expected. In this sense, mortgage REITs appear to be different from the equity REIT sectors analyzed above. However, similarly to all the equity REIT portfolios, mortgage REITs are also exposed to unexpected inflation risk, which is obviously one of the characterizing features of real estate as an asset class.

4.4. *Risk premia estimates*

Table 4 shows results for the posterior densities of the time series of risk premia estimates $\{\hat{\lambda}_{j,t}\}$ $(j = 1, ..., K)$.[14] The table reports summary statistics for both the full sample as well as for the recent, financial crisis subsample, 2007-2011. The table shows results that have to be interpreted with caution. If one applies standard (but frequentist in nature) statistical inference to the time series

[13] The apparence of negative betas vs. the market portfolio should not be surprising but one has to keep in mind that while an expectation of positive market beta is typical of the CAPM where the market is the only factor picking up aggregate risk, in our extended MFAPM there are in principle another eight factors that may capture business cycle effects.

[14] Plots are available from the Authors upon request. However, risk premia are sufficiently variable over time that in this case plots are particularly revealing, especially because the size of the 95% confidence bands is often volatile.

of *mean* posterior estimates of the risk premia $\{\hat{\lambda}_{j,t}\}$ and computes standard t-tests, then we have evidence in favor of as many as four priced risk factors in the cross-section of excess asset returns: using a 10% threshold for p-values, the market, IP growth, inflation and liquidity risks appear to be priced; in three cases, the finding holds with a p-value of 5% or less. Despite the small sample size, a similar result obtains in the case of the crisis sample. Interestingly, the market factor stops being precisely estimated—at least as far as the posterior mean reveals—and it is replaced in this role by Cochrane and Piazzesi's factor.

In particular, market risk carries a mean posterior price of 0.33% per month with a "frequentist-type" p-value of 0.009; IP growth implies a risk premium of 0.19% with a p-value of 0.084; liquidity risk carries a mean posterior price of 0.32%, again with a p-value of 0.046; finally, inflation risk commands a premium of -0.14% with a p-value of 0.030. While the finding of a significantly priced market factor may be not surprising, the result that also typical macroeconomic risks be priced is consistent with earlier evidence centered on real estate data (see e.g., Ling and Naranjo, 1997). Moreover, the evidence of liquidity being a priced macro-style factor is consistent with the findings in Næs et al. (2011). The negative sign for the inflation risk factor is consistent with earlier evidence in Chen et al. (1986), Ferson and Harvey (1991), and Lamont (2001). Finally, the time series mean of the intercept $\lambda_{0,t}$—which should be zero if the assumed MFAPM held—is relatively small (0.19 percent) and not precisely estimated. In the crisis sub-sample, the posterior mean of $\lambda_{0,t}$ gets larger and yet it is imprecisely estimated.[15]

4.5. *Economic tests*

So far our discussion has focused on the statistical performance of the model, with emphasis on whether there was evidence of either the $\lambda_{0,t}$s coefficients being different from zero and whether there was any evidence that the assumed risk factors were priced in the cross-section of risky portfolios. We have uncovered encouraging evidence that the BTVSVB model may be consistent with the data. We now discuss the empirical results obtained from variance decomposition tests applied to the BTVSVB framework. In particular, we compute the VR1 and VR2 ratios and proceed to decompose $Var[P(\sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1})]$ as the sum of the contributions given by each of the factors.[16]

Table 5 shows posterior medians and 95% Bayesian confidence intervals derived under all the models entertained in this paper, including the TVPM and homoskedastic BTVB models introduced in Section 2.5. However, in this Section we focus on columns 3-8 concerning results for the BTVSVB framework only, while Section 6 performs robustness checks. In particular, columns 3 and 6 of Table 5 show posterior medians of $VR1$ and $VR2$ for each of the 31 portfolios under examination. Variance ratio

[15] The evidence turns inconclusive in the full sample if we use (averages over time of) 95% Bayesian credibility intervals built using posterior densities. The reason is all these densities attach a non-negligible probability to zero or small risk premia on the different factors. These differences in results derived from means vs. quantiles of the posterior densities of risk premia are possible because the posterior densities have a highly non-normal, non-symmetric shape.

[16] In what follows, the information at time $t-1$ ($\mathbf{Z}_{t-1}$) is proxied by the instrumental variables listed in Section 3, plus a dummy variable to account for the "January effect" in the cross-section of stock returns, a widely documented calendar anomaly that investors are likely to take into account.

results are encouraging, although there is some difference between the $VR1$ vs. the $VR2$ dimensions. Under a VR1 perspective, approximately 75% of the predictable variation in excess returns is captured by the MFAPM. Such percentages are in fact high for the sectorial REIT data, still considerable but more heterogeneous (ranging from 49 to 90 percent) for equity portfolios, and even higher in the case of bond portfolios.

However, because $VR1+VR2 = 1$ does not hold, the finding of good VR1 ratios fails to imply that the VR2 ratios are always as close to zero as much as we would like them to be. Even though VR2 is generally at or below 20% only, in Table 5 we occasionally notice portfolios for which more than a third of total predictable variation cannot be explained by the macroeconomic risk factors assumed in our MFAPM, so that it is time-varying idiosyncratic variances that pick up the slack. As far as real estate assets are concerned, these relatively high VR2 ratios characterize only apartment-specialized REITs, with a VR2 ratio of 45%. However, and consistent with the empirical findings in Bianchi et al. (2013), some equity portfolios (e.g., high tech, retail and residual, "other" stocks) occasionally spike up, signalling a less satisfactory fit offered by the BTVSVB framework.

Table 6 disentangles the sources of predictable variation in excess returns that the BTVSVB model seems apt to capture. Table 6 shows that the predictable variation in excess REIT returns is mostly explained by exposure to the inflation risk factor (its contribution is 34 percent on average), followed by market risk (32 percent), and liquidity (22 percent). Also IP growth risk contributes a non-negligible 16 percent.[17] Although there is no precise connection between the sign and accuracy of estimation of the risk premia and the decomposition of risk sources in Table 6, the four factors giving substantial contributions all imply precisely estimated risk premia. These results contribute to the idea that public real estate portfolios are mostly priced off pure aggregate real activity, liquidity and inflation shocks, more than on the basis of typical financial factors such as market or credit default risk (though the CAPM-style market portfolio retains a role). Moreover, residential real estate remains an asset class dominated by inflation concerns, not only in terms of (posterior median) exposures and risk premia, but also in terms of the contribution of these concerns to explain the predictable variation of realized excess returns.

We also find differences between the percentage contribution of the liquidity and IP growth risks to explain residential REIT excess returns vs. non-residential REITs: in the former case, the two factors play a dominant role, with contributions of 26% from liquidity (averaging across manufactured homes and apartments), and of 20% from IP growth; in the latter case, these two factors provide more modest contributions, 17% in the case of liquidity and 16% in the case of IP growth. As the financial crisis has revealed, it is in general the housing sector that is most affected by business cycle downturns (even though the relationship is clearly endogenous) and may suffer from liquidity dry-ups.[18]

[17] The sum of these contributions may exceed 100% because of the negative contributions given by the interaction effects.

[18] Because business-related REITs are explained "less" by liquidity and IP growth concerns than residential REITs are, one may wonder what makes up for the difference. Leaving aside the problematic interaction effects, office and most of the retail REITs are explained by the inflation factor with above-average contributions.

Let's also ask again what makes (if anything) REITs different from other asset classes. Table 6 reveals another simple and yet powerful answer: in the case of stocks, the leading risk factor that explains the dynamics of asset prices is represented by liquidity and the market factors, with average contributions to $Var[P(\sum_{j=1}^{7}\lambda_{j,t}\beta_{ij,t|t-1}|\mathbf{Z}_{t-1})$ of 36 and 31 percent, respectively, and several peaks well in excess of 50% (40%) in the case of the market portfolio (liquidity) factor. Conversely, stocks are affected by inflation risk on a scale that is inferior to what we have observed for REITs. One may say that while in the case of equities, general aggregate risk is mostly represented by the value-weighted market and liquidity mimicking portfolios, on the contrary, macroeconomic factors play this role in the case of REITs.

5. **Heterogeneous mispricing in REIT sectors**

Figure 7 reports (posterior median) estimates of $\beta_{i0,t}$. In a ICAPM interpretation of (1), $\beta_{i0,t} \neq 0$ represents evidence of non-zero excess returns for a portfolio i with zero exposures to the K risk factors, which implies the existence of an arbitrage opportunity and it is inconsistent with first principles (e.g., non-satiation). Equivalently, the Jensen's alphas, $\beta_{i0,t}$s, are measures of abnormal (excess) returns. The figure starts by presenting medians of $\beta_{i0,t}$ posteriors as well as 95 percent confidence intervals computed in the usual way, with reference to the eight REIT sectors as well as a few other stock and bond portfolios representative of the overall universe of 31 portfolios used in the estimation.

If one ignores the considerable uncertainty in the data (both objective and related to parameter estimation), Figure 7 offers a rather stark view of a number of asset pricing trends that have involved real estate over the past decade and a half: all the Jensen's alpha related to REITs are positive and relatively large (these are monthly mispricing estimates). Ex-post, we have evidence that—even in the light of a no-arbitrage multi-factor model driven by macroeconomic risks—real estate as an asset class has been long and persistently over-priced in the U.S., in the sense that realized excess returns have been (on average) between 0.5 and 2 percent higher than would have been justified by their exposure to systematic risk between 1999 and 2011. Additionally, and with the partial exception of mortgage investments, all REIT portfolios describe rather homogeneous dynamics over time: the alphas start out relatively low (in fact, near zero in the case of retail-investing REITs and manufactured homes) between 1999 and 2004. Between 2005 and late 2007, all alphas climb up, in some cases (apartments and manufactured homes) going from a few basis points per month in late 2004 to as high as 2.2 percent per month (which is a massive annualized abnormal performance in excess of 25%). This was the great U.S. real estate bubble, with trading, borrowing volumes and prices all exploding at the same time. However, the alphas for most sectors then slowly declined between 2008 and 2011, settling to levels below 1% per month and often returning to zero percent, when macro factors can perfectly explain average excess returns. Finally, mortgage REITs show a rather peculiar behavior over time: although the mispricing of mortgages seems to have been rather large and accurately estimated with reference to the 2001-2003 period (when the corresponding posterior median $\beta_{mortgages,0,t}$ touched 3% per month), since 2004 the mortgage alphas have been declining to reach on average just a few dozen basis points above zero between 2005 and 2011.

Figure 7 shows no evidence of a *pure* housing/residential real estate bubble—as measured by the mispricing of apartment and manufactured home-investing REITs—inflating between 2004 and 2007, to subsequently burst. All REIT sectors record a climb-up in alphas during this period. In fact, it is the alphas of the three retail/distribution-investing REIT sectors that show the steepest ascent, with an increase in posterior medians between 2004 and 2007 of 1.2, 1.5, and 1.6 percent for regional malls, free-standing, and apartment REITs, respectively. However, the I&O's estimated alphas are somewhat smaller, although they have also never declined below 40 basis points per month in the post-2004 period. Interestingly, all alphas are estimated to have rather tight posterior distributions, even though the 95% confidence intervals tend to widen towards the very end of our sample (2011), when they include zero (absence of mispricing) for five REIT sectors out of eight. Yet, Figure 7 shows that two of out three sectors that were still characterized by some (modest, the lower 2.5% bound is 0.2% at most) mispricing at the end of our sample, were apartment- and manufactured home-investing REITs, i.e., again residential REITs whose process of adjustment may have been slowed down, possibly by the massive interventions in the mortgage-backed securities market by the Federal Reserve between 2009 and 2011.

These structurally high alphas for REITs of all kinds differ from the typical alphas estimated for stock and bond portfolios, when the posteriors for the $\beta_{i0,t}$s tend to yield medians that are small, often negative, and whose sign changed several times between 1999 and 2011, consistent with the evidence in Bianchi et al. (2013). For instance, even in the case of small capitalization stocks—that the empirical finance literature has long characterized as a case of mispricing by standard factor models—their alphas start out at 3% in the late 1990s to then decline to zero by 2003. Another example is 10-year Treasuries, for which the estimated alpha has a posterior median that systematically cycles around zero over our sample, an indication of no persistent mispricing. Moreover, most of the non-REIT alpha posteriors are estimated with very poor precision, in the sense that for at least 19 of the 23 stock and bond portfolios under examination, one can always draw a straight horizontal line at zero alpha that is contained in the 95% Bayesian interval throughout our sample.

In conclusion, Figure 7 tells us a story that only partially matches the tale of the financial crisis often reported by the popular press and portions of the literature (see the Introduction). On the one hand, and ignoring confidence regions for the time being, it is a fact that—differently from most other portfolios (stocks and bonds)—U.S. real estate would have been grossly and systematically over-priced between 2004 and 2007. Over-pricing is indicated by the fact that the posterior estimates of the real estate alphas are positive, climbing, and precisely estimated; large and positive alphas signal that, after taking into account the risk exposures and premia captured by the rich set of nine macroeconomic factors entertained in our paper, real estate yielded "too high" a return, which cannot be rationally justified. In this sense, the real estate fad has been pervasive. Also the claim that the real estate bubble would have been a debt/mortgage-fueled one (see e.g., Brueckner, Calem and Nakamura, 2012; Coleman, LaCour-Little and Vandell, 2008; Pavlov and Wachter, 2011) is consistent with the fact that between 2001 and 2004 mortgage REITs implied the largest median alphas among the eight plots in the figure. On the other hand, there is no evidence of a larger bubble in the residential vs. the O&I and retail sectors because the alphas of manufactured homes and apartment-investing REITs are actually estimated to be slightly negative and declining as late as in early 2005; the mispricing of apartment-investing REITs did turn out to become positive and large (exceeding 2 percent per month) in 2007, but such alphas were quickly corrected during 2008. The actual real estate overpricing occurred instead—and in the perspective of our model is indeed potentially under way—in the industrial and retail sectors: in particular, the posterior median alphas of industrial and regional malls-specialized REITs remain persistently at levels around 1 percent per month throughout our sample, including the 2009-2011 period. In this sense, the 2007-2008 real estate bust did not simply consist of a temporary residential real estate (housing) and mortgage-driven fad, but occurred as a result of a large-scale, widespread correction of substantial mispricings of the entire real estate asset class, a correction partially still under way as of the end of 2011.

As a final note of caution, we emphasize that a finding of positive alphas on any portfolios does not imply that they would have yielded high or positive realized, observed returns during all or parts of our sample. From Figure 1, we know this was not the case between 2007 and 2009 for all REIT sectors, while the recovery in O&I and regional malls valuations over 2010-2011 has also been muted. A positive alpha simply means that realized excess returns on these REITs should on average have been even lower than what the data reveal, based on their exposures to priced risk factors.[19] Equivalently, particularly the valuations of industrial and regional malls should have dropped even more than they did. This ongoing poor pricing of portions of the real estate universe represented by REITs would therefore concern industrial factory space and large-scale regional mall properties that have been affected by the deep 2008-2009 recession more severely than other asset types, and with effects that may still linger to rationally depress their valuations.

6. **Robustness checks**

What portion of the earlier economic insights is driven by our modelling choices, concerning the structure and parameterization of the model in (3)-(7)? Sections 6.1 and 6.2 tackle exactly this question and reach a simple conclusion: using a flexible model that provides a good fit to the cross-section of asset returns plays a major role. Absent the BTVSVB model from our analysis, there is no ability to detect real estate mispricings. This means that no evidence of a real estate "fad" could be found, which would represent a rather finding "orthogonal" to the existing literature and policy commentary.

[19] This is similar to a point made by Himmelberg et al. (2005), who have emphasized that high price growth is not evidence per se that housing is overvalued: when price growth is supported by basic economic factors (e.g., low real long-term interest rates and high income growth), then no misspricing may be detected; conversely, real estate may be over-valued also in times of falling prices. In fact, Favilukis et al. (2010) have argued that between 2000 and 2006, a widespread relaxation of collateralized borrowing constraints, declining transaction costs, and a sustained depression of long-term interest rates that coincided with a vast inflow of foreign capital in the U.S. bond markets would have contributed to drive real estate risk premia down. In this case, large alphas could derive from dropping betas relative to macroeconomic risks that leave an increasing portion of average, realized excess returns on real estate unexplained.

6.1. *Traditional two-stage Fama-MacBeth estimation*

The framework in (1)-(2) just describes a general conditional pricing framework that is actually known to hold under a variety of assumptions and conditions. A variety of alternative methodologies have been proposed to estimate the factor loadings $\{\beta_{ij,t}\}$ and the risk premia λ_{jt}. These tasks are logically distinct from the formulation of the MFAPM and they have an exquisite statistical nature. A number of papers have pursued a rather simple, one would say "seminonparametric", rolling window approach that consists of a two-stage testing procedure *à* la Fama and Mac Beth (1973) first applied by Ferson and Harvey (1991) to the estimation of linear multi-factor models. Appendix C summarizes the methodology.

The rightmost column of plots in Figures 8-10 and the bottom panel in Table 4 report results from a Fama-MacBeth two-step strategy. To save space, we have been selective when reporting plots as these refer only to office, regional malls, apartments, and mortgage REITs. A comparison with earlier findings reveals important differences in the estimates of the loadings produced by the standard procedure, both in comparison to the BTVSVB and the TVP models. Although such a judgement is subjective and cannot replace the hard evidence against TVPM and the Fama-MacBeth's approach commented in Sections 4.1 and 4.2, the results in Figures 8-10 are implausible. First, in a number of combinations of test portfolios and factors, Figures 8 and 9 display jagged shapes characterized by pervasive time variation and wide confidence intervals that not only differ from those in Section 4.3, but also from the TVPM ones that are plotted in the leftmost column. However, a different dynamics in factor loadings is by itself hardly problematic, because given our ignorance on the structure of the data generating process, it is legitimate for alternative estimation strategies to return different results in small samples.

What is more of a concern is the fact that in Figure 10 and in Table 2 the results of the Fama-MacBeth strategy are economically implausible. In Figure 10, all the alphas that have been plotted are strongly gyrating, they repeatedly change sign even though there is little evidence of statistically significant mispricings and—which is more puzzling—the alphas assume relatively large values that are hard to justify in economic terms, similarly to what has been observed already by Bianchi et al. (2013) in a similar two-step application: alphas exceeding 4 or 5 percent *per month* are frequently inferred for long subsamples. Such large alphas can only give indications as to the poor specification of the model and not on the underlying phenomenon.

According to Figure 10 none of the REIT portfolios (residential or non-residential, plotted or unreported to save space) would have been systematically over- or under-priced during our sample period. For instance, the plot of the estimated alphas for regional mall-specialized REITs exceeds the incredible level of 3% per month (i.e., 36 percent in annualized terms) on a few occasions between 2003 and 2004. Such a level is approached again around the end of our sample. However, the same estimated alpha persistently drops below -1% per month during 2008-2009. Results get even more difficult to interpret in the second panel of Table 4. Over the full sample, only two factors appear to be significantly priced, these are the default spread and liquidity with very high risk premia of 0.88 and

1.18 percent, respectively (p-values are 0.002 and 0.076), while there is evidence of a highly significant and absurdly large cross-sectional intercept λ_0, 0.85% per month with a p-value that is essentially nil.

6.2. *A TVP model*

The leftmost column of plots in Figures 8-10 and the bottom panel in Table 4 report selected estimation results from the TVPM estimated using Bayesian methods adapted from Sections 2.3-2.4 to this special case. The results refer again to four REIT sectors only—the ones for which the most interesting differences had emerged in Section 5—i.e., office, regional malls, apartments, and mortgage-specialized REITs. A comparison with Figures 2-7 and the qualitative comments expressed in Section 4.3 reveal important differences in the shape of the posteriors for the factor loadings produced by the BTVSVB vs. the TVPM. Moreover, confidence regions around median posteriors tend to be narrower than in the case in which discrete breakpoints in factor loadings were allowed. The downside is that, because variation in the parameters is a built-in feature deriving from the assumed random walk process $\beta_{ij,t} = \beta_{ij,t-1} + \eta_{ij,t}$, all the examples in Figures 8 and 9 show pervasive instability in the loadings, although their variation tends to occur smoothly over time. For instance, in Figure 8, equity REITs were significantly negatively exposed to IP growth risk between 2006 and 2008, to then swing to a zero exposure between 2010 and 2011. Such wild swings, implying posterior median $\beta_{ij,t}$s that are often of large magnitude do not appear to be completely realistic because the real estate and finance literature offer little in the way of rational justifications for these patterns.[20]

Interestingly, a TVPM implies precisely estimated but also strongly time-varying posterior densities for the factor loadings accompanied by rather constant, and spread out posterior estimates for the Jensen's alphas, the $\beta_{i0,t}$s in Figure 10. Under a TVPM, there is no story to be told about mispricing in REITs and the recent subprime bust: all REIT categories, both those plotted in Figure 10 and those that are available on request, imply flat alphas over time and the corresponding Bayesian confidence regions systematically include zero for most of our sample (always, after 2001). Although the finding of imprecisely estimated $\beta_{i0,t}$s tends to be associated with the idea that a MFAPM should not be rejected, in this case a number of doubts remain. These are strengthened by the fact that in the bottom panel of Table 2, with reference to the full sample, most of the risk premia posterior medians are not precisely estimated, so that only the market and liquidity factors are significant; the default premium factor yields instead a precisely estimated but negative premium, which is in turn problematic in terms of interpretation.

[20] More generally, the finding of $\beta_{ij,t}$s (for instance, with respect to inflation risk in Figure 9) with posterior medians that all average between 2 and 5 and hence levels that are easily between 50 and 200 percent higher than those found in Section 4.3 is also unrealistic because in the presence of the type of estimated risk premia reported in Table 2, these translate into large expected monthly excess returns (in absolute value) that often exceed the very variance of realized excess returns.

7. Conclusions

In this paper we have asked a simple question: can a rational multi-factor asset pricing model in which macroeconomic factors measure risk shed any light on the actual or alleged differences in the pricing mechanism underlying residential vs. non-residential real estate? Equivalently, has it been fair to place most of the burden of the recent real estate bust on over-pricing and misconduct that would have taken place mainly in the private, residential housing sector? To provide an answer to this question, we have made two critical choices. First, we have estimated using Bayesian methods a rich multi-factor stochastic volatility model with time-varying factor loadings and discrete breakpoints. Such a choice is intended to deal with the widespread evidence that asset pricing relationships are unstable, in the sense that the exposures of different portfolios to risk variables change over time, and that the price of such exposures may be unstable too (see Ferson and Harvey, 1991). Second, we have addressed this question resorting to abundant and detailed data on publicly traded REIT total return indices for disaggregated sector portfolios to distinguish between residential, business-related (i.e., industrial, office, and retail) investments, and mortgage specializations.

We uncover two key results. First, there are differences in the structure and dynamic evolution of risk factor exposures across residential and non-residential REITs. Residential REITs—according to most of the literature, the area from which the subprime crisis would have originated—are characterized by a negative but mildly increasing exposure to market risk, by quickly retreating exposures to industrial production growth, term premium, and real interest rate risk, and by massive and quickly increasing beta on unexpected inflation. REITs that specialize in industrial and office investments carry instead negative exposure to real output growth risks, and positive exposure to inflation and bond market risk. Retail- (shopping, regional malls and free-standing) specialized REITs display a negative, significant and stable exposure to market risk and positive and large exposures to unexpected inflation and real interest rate risks. A comparison among residential on the one hand, and I&O and retail REITs on the other hand, sheds light on one potential cause of their diverging behavior in the aftermath of the 2007-2009 crisis: the residential sector no longer has any exposure to general market dynamics and its upward swing is then explained by increasing inflationary risk.

Second, an analysis of cross-sectional mispricing reveals that all the Jensen's alpha implied by REITs were positive and relatively large over parts of our sample. Additionally, and with the partial exception of mortgage investments, all sector REITs described a homogeneous dynamics over time: the alphas start out relatively low between 1999 and 2004. Between 2005 and late 2007, all alphas climb up, in some cases going from a few basis points per month in late 2004 to as high as 2.2 percent per month. This was the great U.S. real estate bubble. However, the alphas of most sectors then decline between 2008 and 2011, settling to levels below 1% per month and often returning to zero percent, when macro risk factors can perfectly explain average excess returns. Real estate is special among all other asset classes: none of the equity portfolios appears to have been persistently mispriced. Moreover, the claim that the real estate "bubble" would have been a debt/mortgage-fueled one is consistent with our result that between 2001 and 2004 mortgage REITs implied the largest, positive median alphas. Yet, there is no evidence of a pure housing/residential real estate bubble inflating between 2004 and 2007, to subsequently burst. All REIT sectors record a climb-up in alphas during this period. In fact, it is the alpha of the three retail/distribution-investing REIT portfolios that shows that steepest ascent. However, the real estate over-pricing occurred across the board and also involved the industry and office sectors.

This finding of a deeply rooted and persistent overpricing of specific types of commercial real estate properties, has important policy implications. On the one hand, should the current regime of low rates of growth in the U.S. economy and of low inflation risk persist, the progressive removal of any residual mispricing in Figure 7 may translate into future, low, potentially negative realized real estate returns. On the other hand, in the measure in which—as sometimes discussed in policy circles (see e.g., Bernanke, 2012, Greenlee, 2009; Gyourko, 2009)—industrial and regional mall property investments sit in large amounts on the balance sheets of nationally- and regionally-relevant U.S. banks, their exposure to macroeconomic and inflation risks may end up hindering the correct transmission of monetary policy impulses.

References

[1] Aït-Sahalia J. Andritzky .J. Jobst A. Nowak S. and Tamirisa N., "How to Stop a Herd of Running Bears? Market Response to Policy Initiatives during the Global Financial Crisis." IMF working paper 2009/204.

[2] Ang A. and Chen J., "CAPM Over the Long Run: 1926-2001", *Journal of Empirical Finance*, 2007, 14, 1-40.

[3] Arce Ó. and López-Salido D., "Housing Bubbles", *American Economic Journal: Macroeconomics*, 2011, 3 ,212-241.

[4] Bernanke B., "Banks and Bank Lending: The State of Play", Remarks to the 48th Annual Conference on Bank Structure and Competition Sponsored by the Federal Reserve Bank of Chicago, 2012.

[5] Bianchi D. Guidolin M. and Ravazzolo F., "Macroeconomic Factors Strike Back: A Bayesian Change-Point Model of Time-Varying Risk Exposures and Premia in the U.S. Cross-Section", Norges Bank Research working paper No. 2013-19.

[6] Boudry W.I. Coulson N.E. Kallberg J.G. and Liu C.H., "On the Hybrid Nature of REITs", *Journal of Real Estate Finance and Economics*, 2012, 44, 230-249.

[7] Brueckner J.K. Calem P.S. and Nakamura L.I., "Subprime Mortgages and the Housing Bubble", *Journal of Urban Economics*, 2012, 71, 230-243.

[8] Carter C. and Kohn R., "On the Gibbs Sampling for State-Space Models", *Biometrika*, 1994, 81, 541-553.

[9] Case K.E. and Shiller R.J., "The Efficiency of the Market for Single-Family Homes", *American Economic Review*, 1989, 79, 125-137.

[10] Case K.E. and Shiller R.J., "Is There a Bubble in the Housing Market?" *Brookings Papers on Economic Activity*, 2003, 2, 299-342.

[11] Cecchetti S., "Crisis and Responses: The Federal Reserve in the Early Stages of the Financial Crisis." *Journal of Economic Perspectives*, 2009, 23, 51-75.

[12] Chen N.-F. Roll R. and Ross S., "Economic Forces and the Stock Market", *Journal of Business*, 1986, 59, 383-403.

[13] Chiang K., "Discovering REIT Price Discovery: A New Data Setting", *Journal of Real Estate Finance and Economics*, 2009, 39, 74-91.

[14] Chib S., "Marginal Likelihood from the Gibbs Output", *Journal of the American Statistical Association*, 1995, 432, 1313-1321.

[15] Clayton J. and MacKinnon G., "The Relative Importance of Stock, Bond and Real Estate Factors in Explaining REIT Returns", *Journal of Real Estate Finance and Economics*, 27, 39-60.

[16] Cochrane J.H. and Piazzesi M., "Bond Risk Premia", *American Economic Review*, 2005, 94, 138-160.

[17] Coleman M. LaCour-Little M. and Vandell K.D., "Subprime Lending and the Housing Bubble: Tail Wags Dog?", *Journal of Housing Economics*, 2008, 17, 272-290.

[18] Connor G. and Korajczyk R., "Risk and Return in an Equilibrium APT: Application of a New Test Methodology", *Journal of Financial Economics*, 1998, 21, 255-289.

[19] Cotter J. and Roll R., "A Comparative Anatomy of REITs and Residential Real Estate Indexes: Returns, Risks and Distributional Characteristics", 2011, working paper, UCLA.

[20] Dell'Ariccia G. Igan D. and Laeven L., "Credit Boom and Lending Standards: Evidence from the Subprime Mortgage Market", *Journal of Money, Credit and Banking*, 2011, 44, 367-384.

[21] Demyanyk Y.S. and Van Hemert O., "Understanding the Subprime Mortgage Crisis", *Review of Financial Studies*, 2011, 24,1848-1880.

[22] Fama E. and MacBeth J., "Risk, Return and Equilibrium: Empirical Tests", *Journal of Political Economy*, 1973, 81, 607-636.

[23] Favilukis J. Ludvigson S.C. and Van Nieuwerburgh S., "The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk-Sharing in General Equilibrium", 2011, working paper, New York University.

[24] Ferson W. and Harvey C.R., "The Variation of Economic Risk Premiums", *Journal of Political Economy*, 1991, 99, 385-415.

[25] Ferson W. and Korajczyk R., "Do Arbitrage Pricing Models Explain the Predictability of Stock Returns?", 1995, *Journal of Business*, 68, 309-349.

[26] Geman S. and Geman D., "Stochastic Relaxation, Gibbs Distributions, and the Bayesian Restoration of Images" *IEEE Transactions*, 1984, 6, 721-741.

[27] Gerlach R. Carter C. and Kohn R., "Efficient Bayesian Inference for Dynamic Mixture Models" , *Journal of the American Statistical Association*, 2000, 95, 819-828.

[28] Geweke J. and Zhou G., "Measuring the Pricing Error of the Arbitrage Pricing Theory ", *Review of Financial Studies*, 1996, 9, 557-587.

[29] Ghysels E., "On Stable Factor Structure in the Pricing: Do Time-Varying Betas Help or Hurt?", *Journal of Finance*, 1998, 53, 549-573.

[30] Giordani P. and Kohn R., "Efficient Bayesian Inference for Multiple Change-Point and Mixture Innovation Models", *Journal of Business and Economic Statistics*, 2008, 26, 1, 66-77.

[31] Glaeser E.L., "A Nation of Gamblers: Real Estate Speculation and American History", *American Economic Review*: Papers and Proceedings, 2013, 103, 1-42.

[32] Glaeser E.L. Gottlieb J.D. and Gyourko J., "Can Cheap Credit Explain the Housing Boom?" In *Housing and the Financial Crisis*, edited by Edward Glaeser and Todd Sinai, 2013. Chicago: University of Chicago Press.

[33] Gorton G., "The Panic of 2007," *European Financial Management*, 2010, 15, 10-46.

[34] Greenlee J., 2009, Testimony on Residential and Commercial Real Estate before the Subcommittee on Domestic Policy, Committee on Oversight and Government Reform, U.S. House of Representatives, Atlanta, GA, November 2, 2009.

[35] Gyourko J., "Understanding Commercial Real Estate: Just How Different from Housing Is It?", 2009, NBER working paper No. 14708.

[36] Gyourko J. and Keim D., "What Does the Stock Market Tell Us about Real Estate Returns?", *AREUEA Journal*, 1992, 20, 457-485.

[37] Hendershott P. Hendershott R. and Shilling J., "The Mortgage Finance Bubble: Causes and Corrections", *Journal of Housing Research*, 2010, 19, 1-16.

[38] Himmelberg C. Mayer C. and Sinai T., "Assessing High House Prices: Bubbles, Fundamentals and Misperceptions" *Journal of Economic Perspectives*, 2005, 19, 67-92.

[39] Hoesli M. Lizieri C.M. and MacGregor B.D., "The Inflation Hedging Characteristics of U.S. and U.K. Investments: A Multi-Factor Error Correction Approach", *Journal of Real Estate Finance and Economics*, 2008, 38, 183-206.

[40] Iacoviello M., "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle", *American Economic Review*, 2005, 95, 739-764.

[41] Iacoviello M. and Neri S., "Housing Market Spillovers: Evidence from an Estimated DSGE Model", *American Economic Journal: Macroeconomics*, 2010, 2, 125-164.

[42] Jostova G. and Philipov A., "Bayesian Analysis of Stochastic Betas", *Journal of Financial and Quantitative Analysis*, 2005, 40, 4, 747-778.

[43] Karolyi G.A. and Sanders A., "The Variation of Economic Risk Premiums in Real Estate Returns", *Journal of Real Estate Finance and Economics*, 1998, 17, 245-262.

[44] Koop G. and Potter S., "Estimation and Forecasting with Multiple Breaks", *Review of Economic Studies*, 2007, 74, 763-789.

[45] Lamont O., "Economic Tracking Portfolios", *Journal of Econometrics*, 2001, 105 161-184.

[46] Lee M. Lee M. and Chiang K., "Real Estate Risk Exposure of Equity Real Estate Investment Trusts", *Journal of Real Estate Finance and Economics*, 2008, 36, 165-181.

[47] Ling, D.C. and Naranjo A., "Economic Risk Factors and Commercial Real Estate Returns", *Journal of Real Estate Finance and Economics*, 1997, 15, 3, 283-307.

[48] Ling, D.C. and Naranjo A., "The Dynamics of REIT Capital Flows and Returns", *Real Estate Economics*, 2003, 31, 405-434.

[49] Ludvigson S.C. and Ng S., "Macro Factors in Bond Risk Premia", *Review of Financial Studies*, 2009, 22, 5027-5067.

[50] McCulloch R. and Rossi P., "A Bayesian Approach to Testing the Arbitrage Pricing Theory", *Journal of Econometrics*, 1991, 49, 141-168.

[51] Merton R., "An Intertemporal Capital Asset Pricing Model", *Econometrica*, 1973, 41, 867-887.

[52] Mian A. and Sufi A., "The Consequences of Mortgage Credit Expansion: Evidence from the U.S. Mortgage Default Crisis", *Quarterly Journal of Economics*, 2010, 124, 1449-1496.

[53] Mian A. and Sufi A., " House Prices, Home-Equity Based Borrowing, and the U.S. Household Leverage Crisis", *American Economic Review*, 2011, 101, 2132-2156.

[54] Omori. Y. Chib S. Shepard N. and Nakajima J., "Stochastic Volatility with Leverage: Fast and Efficient Likelihood Inference", *Journal of Econometrics*, 2007, 140, 425-449.

[55] Pavlov A. and Wachter S., "Subprime Lending and Real Estate Prices", *Real Estate Economics*, 2011, 39, 1-17.

[56] Payne J. and Waters G., "Have Equity REITs Experienced Periodically Collapsing Bubbles?", *Journal of Real Estate Finance and Economics*, 2007, 34, 207-224.

[57] Pastor L. and Stambaugh R.F., "Liquidity Risk and Expected Stock Returns", *Journal of Political Economy*, 2003, 111, 642-685.

[58] Rappaport J., "A Guide to Aggregate House Price Measures", Federal Reserve Bank of Kansas City *Economic Review*, 2007, 92, 41-71.

[59] Seck D., "The Substitutability of Real Estate Assets", *Real Estate Economics*, 1996, 24, 75-96.

[60] Shanken J., "On the Estimation of Beta Pricing Models", *Review of Financial Studies*, 1992, 5, 1-34.

[61] Shiller R.J., "The Volatility of Long-Term Interest Rates and Expectations Models of the Term Structure", *Journal of Political Economy*, 1979, 87, 1190-1219.

[62] Simpson M.W. Ramchander S. and Webb J.R., "The Asymmetric Response of Equity REIT Returns to Inflation", *Journal of Real Estate Finance and Economics*, 2007, 34, 513-529.

[63] Smith M.H. Smith G., "Bubble, Bubble, Where is the Housing Bubble?", *Brookings Papers on Economic Activity*, 2006, 1, 1-67.

[64] Tanner M.A. and Wong W.H., "The Calculation of Posterior Distributions by Data Augmentation", *Journal of the American Statistical Association*, 1987, 82, 528-550.

[65] Wheaton W.C. and Nechayev G., "The 1998-2005 Housing 'Bubble' and the Current 'Correction': What's Different This Time?", *Journal of Real Estate Research*, 2008, 30, 1-26.

Appendix A: prior specification

We choose a conjugate prior structure to keep the numerical analysis as simple as possible. As far as the structural break probabilities are concerned, we assume simple Beta distributions:

$$\pi_{ij} \sim Beta(a_{ij}, b_{ij}) \qquad \pi_{i\nu} \sim Beta(a_{i\nu}, b_{i\nu}) \qquad \text{for } i = 1, ...N \quad j = 1, ...K. \tag{21}$$

The parameters a_{ij}, b_{ij} and $a_{i\nu}, b_{i\nu}$ represent the shape hyperparameters and can be set according to our prior beliefs about the occurrence of structural breaks in $\beta_{ij,t}$ and $\ln(\sigma_{i,t}^2)$, respectively.[21]

As for the conditional variance parameters $\mathbf{q}_i^2$, which reflect our prior beliefs about the size of the structural breaks, we assume an inverted Gamma prior,

$$q_{ij}^2 \sim IG(\gamma_{ij}, \delta_{ij}) \qquad q_{i\nu}^2 \sim IG(\gamma_{i\nu}, \delta_{i\nu}) \quad \text{for } \ i = 1, ...N \quad j = 1, ...K, \tag{22}$$

where $\gamma_{ij} > 0$, $\gamma_{i\nu} > 0$ and $\delta_{ij} > 2$, $\delta_{i\nu} > 2$ are the scale and degrees of freedom parameters for the factor loadings and the (log-) variances.[22] Finally, the prior distribution for the risk premia $\boldsymbol{\lambda}_t$ is characterized as a standard multivariate normal distribution with independent priors:

$$\boldsymbol{\lambda}_t \sim N_K(\underline{\boldsymbol{\lambda}}, \underline{\mathbf{V}}) \qquad \psi^2 \sim IG(\psi_0, \Psi_0) \quad \text{for} \quad t = 1, ...T. \tag{23}$$

The parameters $\underline{\boldsymbol{\lambda}}$ and $\underline{\mathbf{V}}$ represent the location vector and the scale matrix for the K-dimensional multivariate normal distribution; ψ_0 and Ψ_0 are the scale and degrees of freedom of the conditional variance ψ^2 parameters, respectively, in (7). Because these priors are independent of one another, the density of the joint prior distribution $p(\boldsymbol{\Theta})$ is given by the product of the prior specifications (21)-(23).

Realistic values for the different prior distributions obviously depend on the application. We use weak priors, excluding the size of the breaks $\mathbf{Q}_i$ and the probabilities $\Pr(\kappa_{ij,\tau} = 1)$ and $\Pr(\kappa_{iv,\tau} = 1)$ for which our priors are informative (see Appendix B). We set the prior hyperparameters to imply, on average, breaks in $\beta_{ij,t}$ and $\sigma_{i,t}^2$ approximately 5% and 2% of the time. Priors are instead uninformative for breaks with prior mean for the size of the break smaller than 0.3. All other priors are "flat" in the sense that they imply posteriors that tend to be centered around their maximum likelihood estimates.

Appendix B: the Gibbs sampling algorithm

In this section we derive the full conditional posterior distributions of the latent variables and the parameters discussed in Section 2.4. For ease of exposition, we report the results for the ith asset. We use a burn-in period of 1,000 and draw 5,000 observations storing every other of them to simulate the posterior distribution. The resulting autocorrelations of the draws are very low.

[21] Under a Beta distribution, the unconditional expected prior probability of a structural break for the ith asset beta relative to the jth factor is defined as $a_{ij}/(a_{ij}+b_{ij})$ while in the case of idiosyncratic variance, this is equal to $a_{i\nu}/(a_{i\nu}+b_{i\nu})$.

[22] Under an Inverted Gamma prior, the expected size of a break for, say, the exposure of ith asset to the jth factor is $\gamma_{ij}/(\delta_{ij} - 2)$ for $\delta_{ij} > 2$.

Step 1. Sampling K_β.

The structural breaks in the conditional dynamics of the factor loadings measured by the latent binary state $\kappa_{i0t,...,}\kappa_{iKt}$, are drawn using the algorithm of Gerlach et al. (2000). This algorithm increases the efficiency of the sampling procedure since allows to generate $\kappa_{it} = (\kappa_{i0t,...,}\kappa_{iKt})$, without conditioning on the relative regression parameters $\beta_{it} = (\beta_{i0t}, ..., \beta_{iKt})$. The conditional posterior density of κ_{it}, $t = 1, ..., T, i = 1, ..., N$, for each of ith asset/portfolio is defined as

$$p\left(\kappa_{i0t,...,}\kappa_{iKt}|\mathcal{K}_{i\beta[-t]}, \mathcal{K}_{i\sigma}, \mathbf{\Sigma}_i, \theta_i, \mathbf{R}_i, \mathbf{F}\right) \propto p\left(\mathbf{R}_i|\mathcal{K}_{i\beta}, \mathcal{K}_{i\sigma}, \mathbf{\Sigma}_i, \theta_i, \mathbf{F}\right) p\left(\kappa_{i0t,...,}\kappa_{iKt}|\mathcal{K}_{i\beta[-t]}, \mathcal{K}_{i\sigma}, \mathbf{\Sigma}_i, \theta_i, \mathbf{F}\right)$$
$$\propto p\left(r_{i,t+1,...,}r_{i,T}|r_{i,1,...,}r_{i,t}, \mathcal{K}_{i\beta}, \mathcal{K}_{i\sigma}, \mathbf{\Sigma}_i, \theta_i, \mathbf{F}\right) p\left(r_{i,t}|r_{i,1,...,}r_{i,t-1}, \mathcal{K}_{i\beta[1:t]}, \mathcal{K}_{i\sigma}, \mathbf{\Sigma}_i, \theta_i, \mathbf{F}\right)$$
$$p\left(\kappa_{i0t,...,}\kappa_{iKt}|\mathcal{K}_{i\beta[-t]}, \mathcal{K}_{i\sigma}, \mathbf{\Sigma}_i, \theta_i, \mathbf{F}\right) \tag{24}$$

where $\mathcal{K}_{i\beta[-t]} = \left\{\{\kappa_{ijs}\}_{j=0}^{K}\right\}_{s=1,s\neq t}^{T}$, $\mathcal{K}_{i\beta[1:t]} = \left\{\{\kappa_{ijd}\}_{j=0}^{K}\right\}_{d=1}^{t}$ and $\mathcal{K}_{i\sigma} = \{\kappa_{i\nu,t}\}_{t=1}^{T}$. We assume that each of the κ_{ijs} breaks are independent from each other such that the joint density is defined as $\prod_{j=0}^{K} \pi_{ij}^{\kappa_{ijs}} (1-\pi_{ij})^{1-\kappa_{ijs}}$. The remaining densities $p\left(r_{i,t+1,...,}r_{i,T}|r_{i,1,...,}r_{i,t}, \mathcal{K}_{i\beta}, \mathcal{K}_{i\sigma}, \mathbf{\Sigma}_i, \theta_i, \mathbf{F}\right)$ and $p\left(r_{it}|r_{i1,...,}r_{it-1}, \mathcal{K}_{i\beta t}, \mathcal{K}_{i\sigma}, \mathbf{\Sigma}_i, \theta_i, \mathbf{F}\right)$ are evaluated as in Gerlach et al. (2000). Notice that, since κ_{ijt} is a binary state the integrating constant is easily evaluated.

Step 2. Sampling the factor loadings **B**.

The full conditional posterior density for the time-varying factor loadings is computed using a standard forward filtering backward sampling as in Carter and Kohn (1994). For each of the $i = 1, ..., N$ assets, the prior distribution of the $\beta_{i0}, ..., \beta_{iK}$ loadings is a multivariate normal with the location parameters corresponding to OLS estimates and a covariance structure which is diagonal and defined by the OLS variances. The initial priors are sequentially updated via the Kalman Filtering recursion, then the parameters are drawn from the posterior distribution which is generated by a standard backward recursion (see Fruhwirth-Schnatter 1994, Carter and Kohn 1994, and West and Harrison 1997).

Step 3 and 4. Sampling the breaks and the values of the idiosyncratic volatility.

In order to draw the structural breaks $\mathcal{K}_{i\sigma}$ and the idiosyncratic volatilities $\mathbf{\Sigma}_i$ for each of the *ith* portfolios, we follow a similar approach as in step 1. The stochastic breaks $\mathcal{K}_{i\sigma}$ are drawn by using the Gerlach et al. (2000) algorithm. The conditional variances $\ln \sigma_{it}^2$, does not show a linear structure even though still preserving the standard properties of state space models. The model is rewritten as

$$\ln\left(r_{i,t} - \beta_{i0t} - \sum_{j=1}^{K} \beta_{ijt} F_{jt}\right)^2 = \ln \sigma_{it}^2 + u_t \qquad \ln \sigma_{it}^2 = \ln \sigma_{it-1}^2 + \kappa_{\nu it}\nu_{it} \tag{25}$$

where $u_t = \ln \varepsilon_t^2$ has a $\ln \chi^2(1)$. Here we follow Omori et al. (2010) and approximate the $\ln \chi^2(1)$ distribution with a finite mixture of ten normal distributions, such that the density of u_t is given by

$$p(u_t) = \sum_{l=1}^{10} \varphi_l \frac{1}{\sqrt{\varpi_l^2 2\pi}} \exp\left(-\frac{(u_t - \mu_l)^2}{2\varpi_l}\right) \tag{26}$$

with $\sum_{l=1}^{10} \varphi_l = 1$. The appropriate values for μ_l, φ_l and ϖ_l^2 can be found in Omori et al. (2010).

Mechanically, in each step of the Gibbs Sampler, we simulate at each time t a component of the mixture. Given the mixture component, we can apply the standard Kalman filter method, such that $\mathcal{K}_{i\sigma}$ and $\boldsymbol{\Sigma}_i$ can be sampled in a similar way as $\mathcal{K}_{i\beta[t]}$ and $\beta_{i0[t]}, ..., \beta_{iK[t]}$ in the first and second step. The initial prior of the log idiosyncratic volatility $\ln \sigma_{i0}^2$ is normal with mean -1 and conditional variance equal to 0.1.

Step 5a. Sampling the time-varying risk premia .

The cross-sectional equilibrium restriction in (2) is satisfied at each time t conditional on the latent states $\mathbf{B}_{t|t-1} = \left\{ \left\{ \beta_{ijt|t-1} \right\}_{i=1}^{N} \right\}_{j=0}^{K}$ and $\boldsymbol{\Sigma}_t = \left\{ \boldsymbol{\sigma}_{it}^2 \right\}_{i=1}^{N}$. Given an initial normal-inverse gamma prior, the full conditional of the equilibrium risk premia $\lambda_t = (\lambda_{0t}, ..., \lambda_{Kt})$ at time t, is defined as

$$p(\lambda_t|\tau, \mathbf{B}_{t|t-1}, \boldsymbol{\Sigma}_t, R_t) \propto |\Sigma_t^*|^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2} \left(R_t - \mu_t^*\right)^\top \left(\Sigma_t^*\right)^{-1} \left(R_t - \mu_t^*\right) \right\} \tag{27}$$

where $R_t = (r_{1t,...,}r_{N,t})$ and Σ_0, μ_0 respectively the prior mean and variance of λ_t, such that the conditional (ex-ante time-varying) risk premia can be sampled at each time t by a normal distribution with $\mu_t^* = \Sigma_t^*(\Sigma_0^{-1}\mu_0 + \tau^{-2} X_{t-1}^\top R_t)$ and $\Sigma_t^* = (\Sigma_0^{-1} + \tau^{-2} X_{t-1}^\top X_{t-1})^{-1}$, $X_{t-1} = \left[\iota, \mathbf{B}_{t|t-1}\right]$, respectively as location and scale parameters. The conditional posterior for the variance of the risk premia τ^2 is an inverse gamma distribution

$$p(\tau^2|\lambda_t, \mathbf{B}_{t|t-1}, \boldsymbol{\Sigma}_t, R_t) \propto \tau^{-a_0} \exp\left(-\frac{b_0}{2\tau}\right) \prod_{i=1}^{N} \frac{1}{\tau} \exp\left(\frac{-\left(r_{it} - \lambda_{0t} - \sum_{j=1}^{K} \beta_{ijt|t-1}\lambda_{jt}\right)^2}{2\tau^2} \right) \tag{28}$$

such that τ^2 can be sampled from an inverse-gamma distribution with scale parameter $b = b_0 + \sum_{i=1}^{N} \left(r_{it} - \lambda_{0t} - \sum_{j=1}^{K} \beta_{ijt|t-1}\lambda_{jt}\right)^2$ and degrees of freedom $a = a_0 + N$.

Step 5b. Sampling the stochastic breaks probabilities.

The full conditional posterior densities for the breaks probabilities $\pi = (\pi_{i1,...,}\pi_{iK})$ is given by

$$p\left(\pi|q^2, \mathbf{B}, \boldsymbol{\Sigma}, \mathcal{K}_\beta, \mathbf{R}, \mathbf{F}\right) \propto \prod_{j=0}^{K} \pi_{ij}^{a_{ij}-1} \left(1 - \pi_{ij}\right)^{b_{ij}-1} \prod_{t=1}^{T} \pi_{ij}^{\kappa_{ijt}} (1 - \pi_{ij})^{1-\kappa_{ijt}} \tag{29}$$

and hence the individual π_{ij} parameter can be sampled from a Beta distribution with shape parameters $a_{ij} + \sum_{t=1}^{T} \kappa_{ijt}$ and $b_{ij} + \sum_{t=1}^{T} (1 - \kappa_{ijt})$ for $j = 0, ..., K$. Likewise the full conditional posterior distribution for the breaks probabilities in the idiosyncratic volatilities π_ν is given by

$$p\left(\pi_\nu|q^2, \mathbf{B}, \boldsymbol{\Sigma}, \mathcal{K}_\sigma, \mathbf{R}, \mathbf{F}\right) \propto \pi_{i\nu}^{a_{i\nu}-1} \left(1 - \pi_{i\nu}\right)^{b_{i\nu}-1} \prod_{t=1}^{T} \pi_{i\nu}^{\kappa_{i\nu t}} (1 - \pi_{i\nu})^{1-\kappa_{i\nu t}}$$

such that the individual $\pi_{i\nu}$ can be sampled from a Beta distribution with shape parameters $a_{i\nu} + \sum_{t=1}^{T} \kappa_{i\nu t}$ and $b_{i\nu} + \sum_{t=1}^{T} (1 - \kappa_{i\nu t})$ for $i = 1, ..., N$.

Step 5c. Sampling the conditional variance of the states.

The prior distributions for the conditional volatilities of the factor loadings β_{ijt} for $j = 0, ..., K$ are inverse-gamma

$$p\left(q_{ij}^2|\pi, \mathbf{B}, \mathbf{\Sigma}, \mathcal{K}_\beta, \mathcal{K}_\sigma, \mathbf{R}, \mathbf{F}\right) \propto q_{ij}^{-\nu_{ij}} \exp\left(-\frac{\delta_{ij}}{2q_{ij}^2}\right) \prod_{t=1}^{T} \left(\frac{1}{q_{ij}} \exp\left(\frac{-\left(\beta_{ijt} - \beta_{ijt-1}\right)^2}{2q_{ij}^2}\right)\right)^{\kappa_{ijt}} \tag{30}$$

hence q_{ij}^2 is sampled from an inverse-gamma distribution with scale parameter $\nu_{ij}+\sum_{t=1}^{T} \kappa_{ijt} \left(\beta_{ijt} - \beta_{ijt-1}\right)^2$ and degrees of freedom equal to $\nu_{ij} + \sum_{t=1}^{T} \kappa_{ijt}$. Likewise the full conditional of the variance for the idiosyncratic log volatility $q_{i\nu}^2$ is defined as

$$p\left(q_{i\nu}^2|\pi, \mathbf{B}, \mathbf{\Sigma}, \mathcal{K}_\beta, \mathcal{K}_\sigma, \mathbf{R}, \mathbf{F}\right) \propto q_{i\nu}^{-\nu_{i\nu}} \exp\left(-\frac{\delta_{i\nu}}{2q_{i\nu}^2}\right) \prod_{t=1}^{T} \left(\frac{1}{q_{i\nu}} \exp\left(\frac{-\left(\ln\sigma_{it}^2 - \ln\sigma_{it-1}^2\right)^2}{2q_{i\nu}^2}\right)\right)^{\kappa_{i\nu t}} \tag{31}$$

such that $q_{i\nu}^2$ is sampled from an inverted Gamma distribution with scale parameter $\nu_{i\nu}+\sum_{t=1}^{T} \kappa_{i\nu t}(\ln\sigma_{it}^2$ $-\ln\sigma_{it-1}^2)^2$ and degrees of freedom equal to $\nu_{i\nu} + \sum_{t=1}^{T} \kappa_{i\nu t}$.

Choice of priors

Realistic values for the different prior distributions obviously depend on the problem at hand (see Groen, Paap and Ravazzolo, 2012). In general, we use weak priors, excluding the size of the breaks $\mathbf{Q}_i$ and the probabilities $\Pr(\kappa_{1ij,\tau} = 1)$ and $\Pr(\kappa_{2i,\tau} = 1)$ for which our priors are quite informative. This is also important because these priors restrict the maximum number of breaks of maximum magnitude and therefore help to identify the factor exposures, which is otherwise rather problematic because linear multifactor models are subject to well-known indeterminacy problems upon rotations of factors and risk premia (see e.g., McCulloch and Rossi, 1991). The prior shape parameters for the probability of breaks in the dynamics of the price sensitivities are set to be $a_{ij} = 3.2$ and $b_{ij} = 60$. As such,

$$E\left[\pi_{ij}\right] = \frac{3.2}{3.2+60} = 0.05 \qquad \text{and} \qquad Std\left[\pi_{ij}\right] = \left(\frac{3.2 \times 60}{(3.2+60)^2(3.2+60+1)}\right)^{1/2} = 0.03$$

which means an expected 5% prior probability of a random shock in the dynamics of the loadings. With respect to idiosyncratic volatility, the shape hyperparameters are $a_{i\nu} = 1$ and $b_{i\nu} = 99$, so that

$$E\left[\pi_{i\nu}\right] = \frac{1}{1+99} = 0.01 \qquad \text{and} \qquad Std\left[\pi_{i\nu}\right] = \left(\frac{99}{100^2 \times 101}\right)^{1/2} = 0.01$$

which set the expected prior probability of having a break in the dynamics of idiosyncratic risks to be equal to 1%. These small prior probabilities make the modelling dynamics more parsimonious, mitigating the magnitude of prior information, letting the data speak about the likelihood of random breaks. The marginal (expected) posterior probability of random breaks both in the factor loadings and idiosyncratic risks are reported in a separate appendix. The prior beliefs on the size of the breaks are inverse-gamma distributed. The prior scale hyper-parameters $\gamma_{ij}, \gamma_{i\nu}$ and the $\delta_{ij}, \delta_{i\nu}$ degrees of freedom are calibrated supporting a prior view for premiums to be normally distributed with zero mean and variance such that there is 95% probability that annualized premia are smaller in absolute value than the larger of the absolute value between the minimum and the maximum return observed in the sample for all the assets. Finally, the prior residual variance is centered at about 10, a value that appeared in the higher range of the maximum likelihood estimates. All other priors imply that the posteriors tend to be centered around their maximum likelihood estimates which eases comparisons with the existing literature.

Appendix C: traditional two-stage Fama-MacBeth estimation

Here we outline the classical, two-stage procedure *à* la Fama and MacBeth (1973) also used by Ferson and Harvey (1991) and very popular in the empirical finance literature. This represents the standard benchmark for the estimation of the equilibrium asset pricing model shown in (1)-(2).

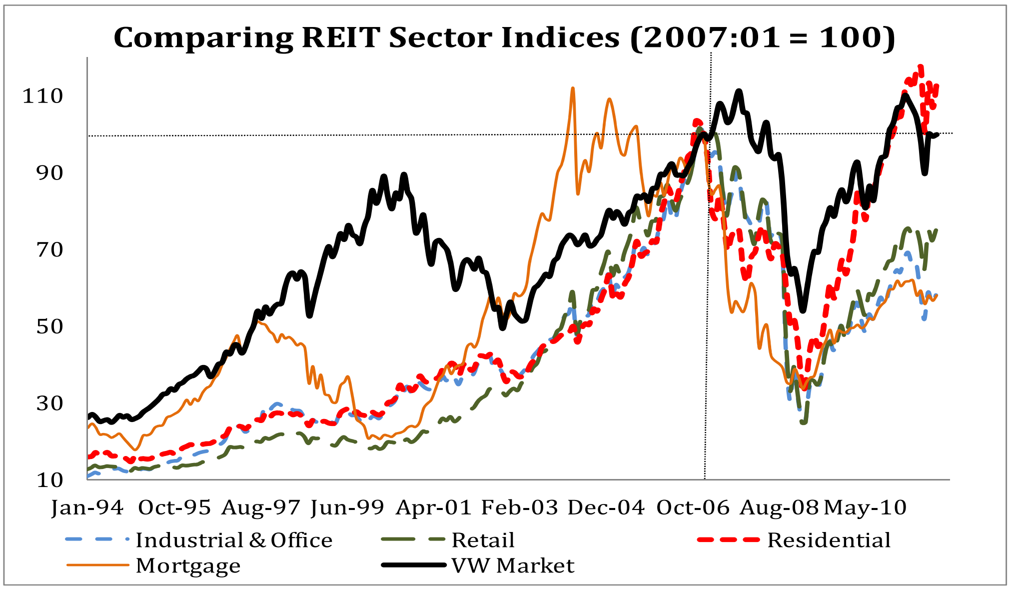
In the first stage, for each of the assets, the factor betas are estimated using time-series regressions from historical excess returns on the assets and economic factors. That is, for month t, (1) is estimated using the previous sixty months (ranging from $t-61$ to $t-1$) in order to obtain estimates for the betas, $\hat{\beta}_{ij,t}^{60}$. This time-series regression is updated each month. In this paper—also to preserve asymmetry with the heteroskedastic modelling choices made in Section 2.2—we model each of the asset specific variances as following a univariate EGARCH(1,1) process. The choice of a 60-month rolling window scheme is typical of the literature.[23] In the second stage, the equilibrium restriction (2) is estimated for each of the periods in our sample as a cross-sectional regression using ex-post realized excess returns:

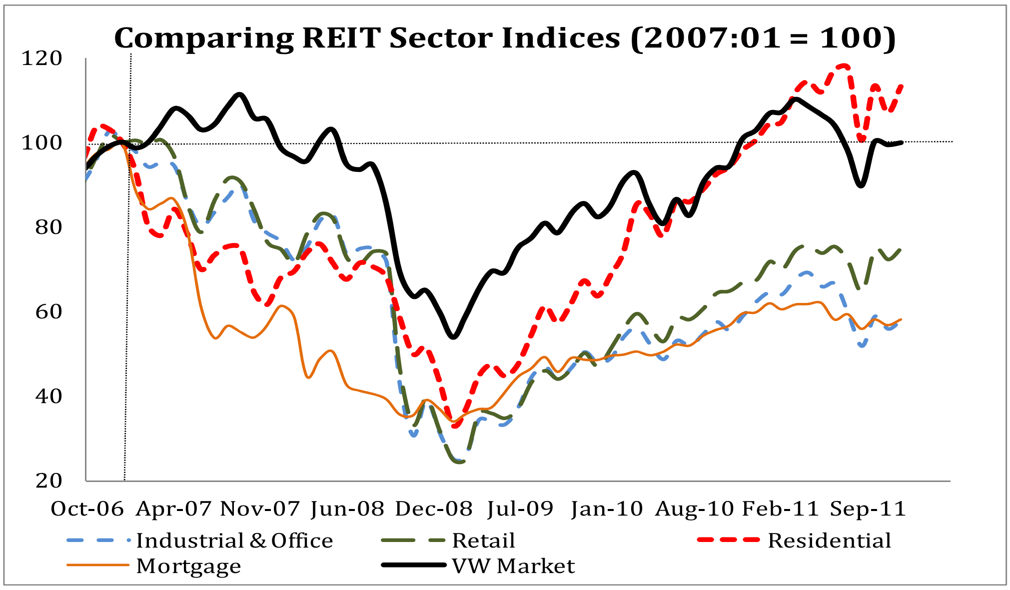
$$r_{i,t} = \lambda_{0,t} + \sum_{j=1}^{K} \lambda_{j,t} \hat{\beta}_{ij,t}^{60} + \zeta_{i,t} \qquad i = 1, ..., N, \quad t = 61, ..., T. \tag{32}$$

Clearly, these T cross-sectional regressions implement (2) in a nonparametric fashion, in the sense that any resulting time variation in the $\lambda_{0,t}$ and $\lambda_{j,t}$ coefficients fails to be explicitly and parametrically related to any of the instruments assumed by the researcher. In (32) $\lambda_{0,t}$ is the zero-beta (abnormal) excess return and the $\lambda_{j,t}$s are proxies for the factor risk premiums on each month, $j = 1, ..., K$. This derives from the fact that if one considers a portfolio κ such that $\hat{\beta}_{\kappa j,t}^{60} = 0$ for all $j \neq \kappa$ and $\hat{\beta}_{\kappa\kappa,t}^{60} = 1$, then $\lambda_{\kappa,t}$ is simply the conditional mean of $r_{\kappa,t} - \lambda_{0,t}$. Notice that $\lambda_{0,t}$ should equal zero $\forall t$ if the model is correctly specified because in the absence of arbitrage, all zero-beta assets should command a rate of return that equals the short-term rate. Tests of multi-factor models evaluate the importance of the economic risk variables by evaluating whether their risk premiums are priced or whether, on average, the (second-stage, estimated) coefficients $\hat{\lambda}_{j,t}$ are significantly different from zero.

[23] In unreported tests we have attempted to optimize this choice by picking the sliding window that produced the lowest average information criterion, such as the BIC. We find that a 5-year window gives at all times a BIC which is sensibly lower than any other window in the range [3, 10] years.

Figure 1
Comparing the Dynamics of Sector and Subsector REIT Indices Over Time





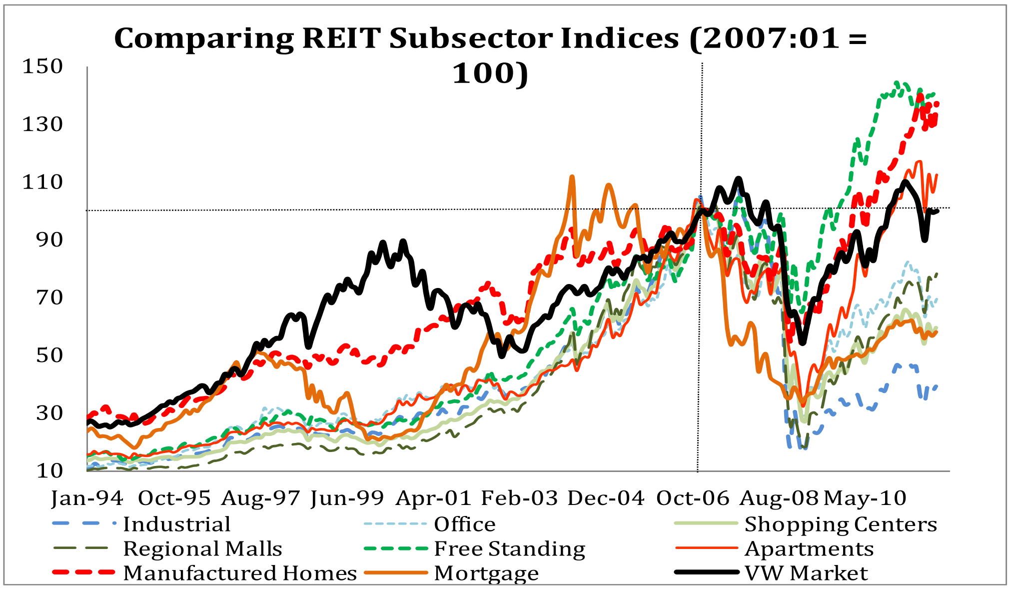


Figure 2: Factor Loadings: Value-Weighted Market Portfolio Factor

This figure reports the time series of the posterior medians loadings on the market risk factor estimated from a dynamic Bayesian model with latent breakpoints in betas and idiosyncratic risk. The sample period is 1994:01 - 2011:12. The first 60 monthly observations are used as a training sample in order to calibrate the prior distribution for both the latent states and parameters. The gray area surrounding posterior median plots represent 95% confidence intervals.

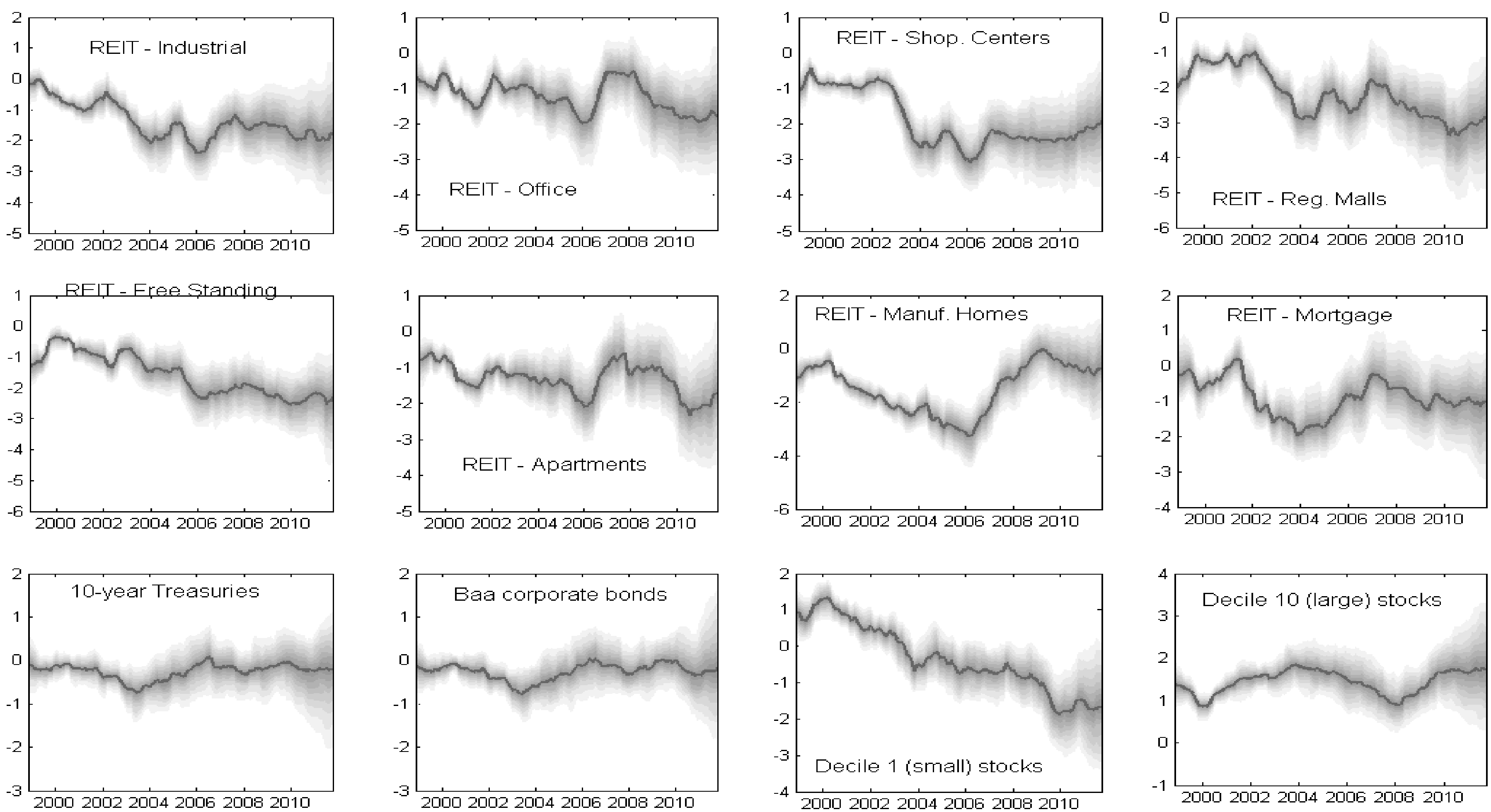


Figure 3: Factor Loadings: Industrial Production

This figure reports the time series of the posterior median loadings on the industrial production growth factor estimated from a dynamic Bayesian model with latent breakpoints in betas and idiosyncratic risk. The sample period is 1994:01 - 2011:12. The first 60 monthly observations are used as a training sample in order to calibrate the prior distribution for both the latent states and parameters. The gray area surrounding posterior median plots represent 95% confidence intervals.

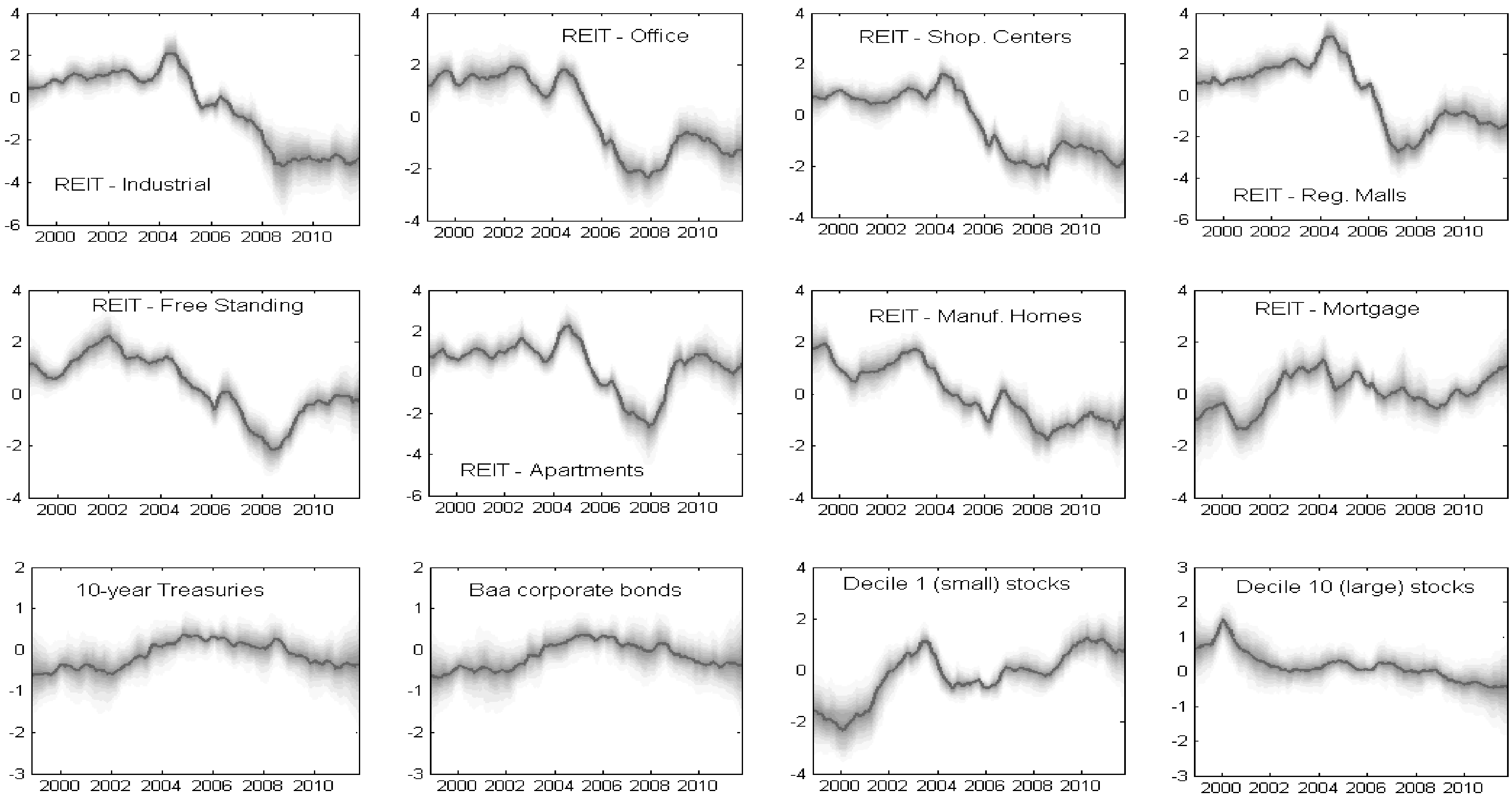


Figure 4: Factor Loadings: Unexpected Inflation

This figure reports the time series of the posterior median loadings on the unexpected inflation factor estimated from a dynamic Bayesian model with latent breakpoints in betas and idiosyncratic risk. The sample period is 1994:01 - 2011:12. The first 60 monthly observations are used as a training sample in order to calibrate the prior distribution for both the latent states and parameters. The gray area surrounding posterior median plots represent 95% confidence intervals.

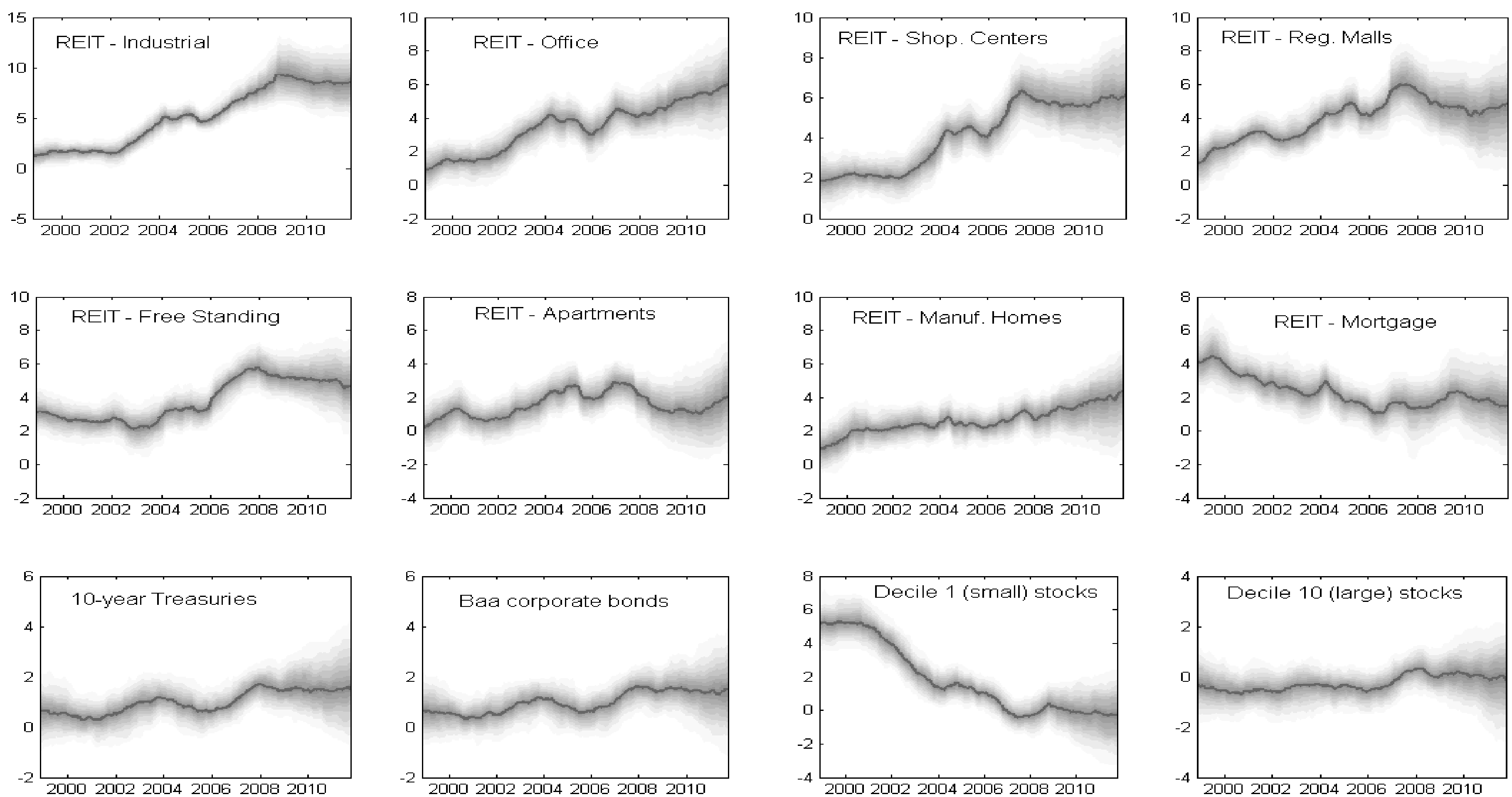


Figure 5: Factor Loadings: Liquidity Risk

This figure reports the time series of the posterior median loadings on the liquiduty risk factor estimated from a dynamic Bayesian model with latent breakpoints in betas and idiosyncratic risk. The sample period is 1994:01 - 2011:12. The first 60 monthly observations are used as a training sample in order to calibrate the prior distribution for both the latent states and parameters. The gray area surrounding posterior median plots represent 95% confidence intervals.

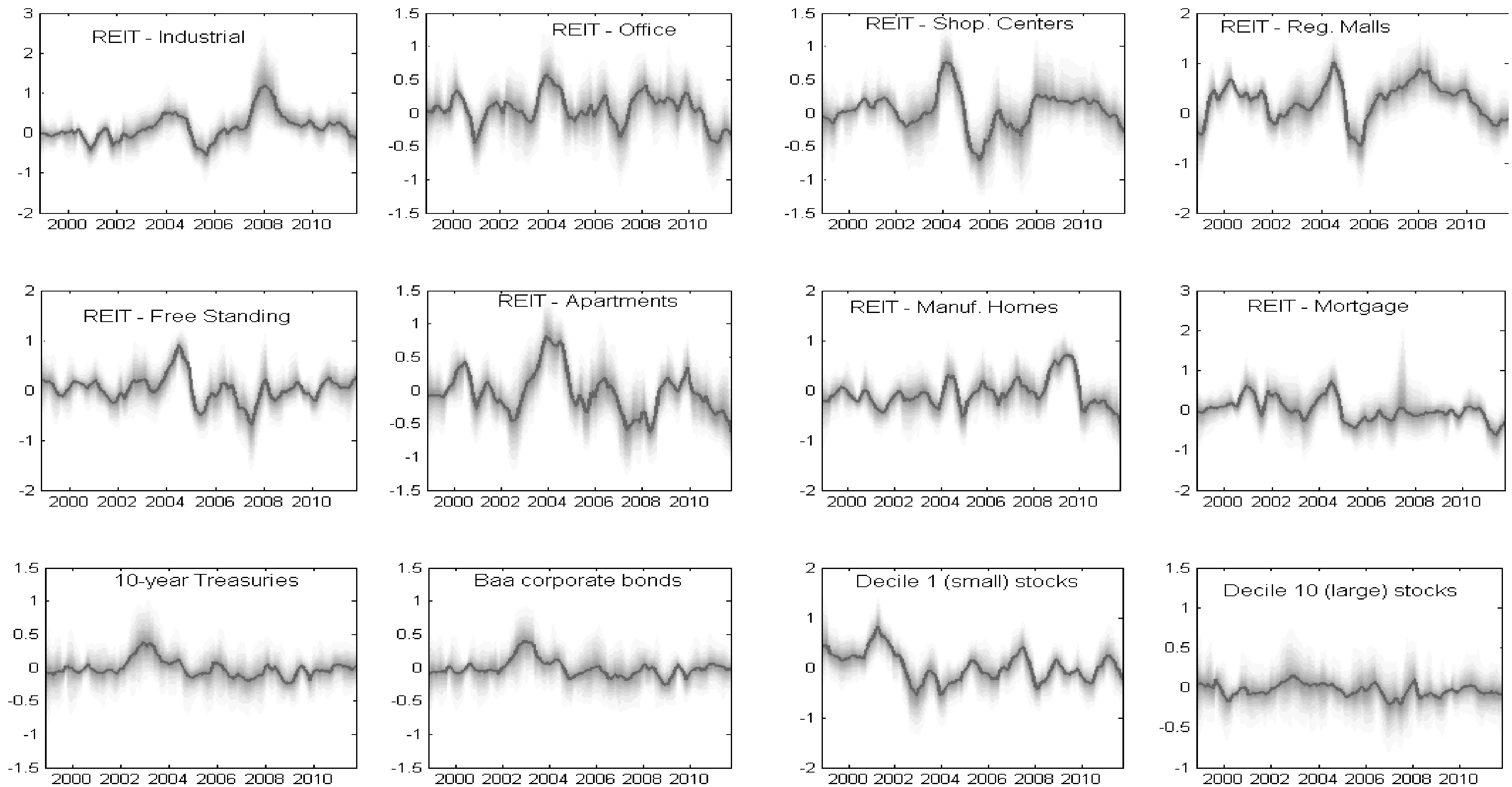


Figure 6: Factor Loadings: Real Short-Term Rates

This figure reports the time series of the posterior medians loadings on the real T-Bill interest rate factor estimated from a dynamic Bayesian model with latent breakpoints in betas and idiosyncratic risk. The sample period is 1994:01 - 2011:12. The first 60 monthly observations are used as a training sample in order to calibrate the prior distribution for both the latent states and parameters. The gray area surrounding posterior median plots represent 95% confidence intervals.

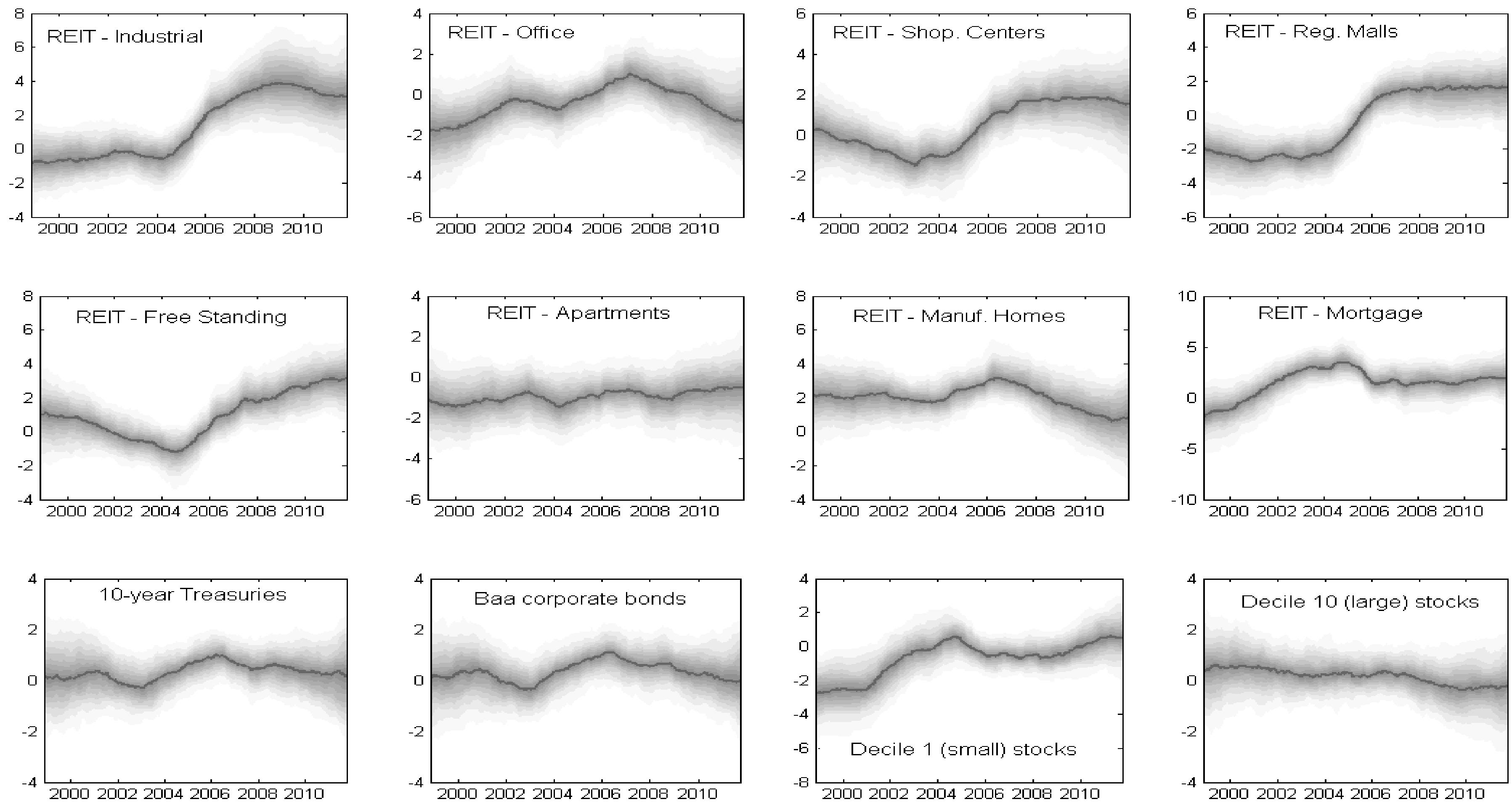


Figure 7: Jensen's Alphas

This figure reports the time series of the posterior medians of the Jensen's alpha from a dynamic Bayesian model with latent breakpoints in betas and idiosyncratic risk. The sample period is 1994:01 - 2011:12. The first 60 monthly observations are used as a training sample in order to calibrate the prior distribution for both the latent states and parameters. The gray area surrounding posterior median plots represent 95% confidence intervals.

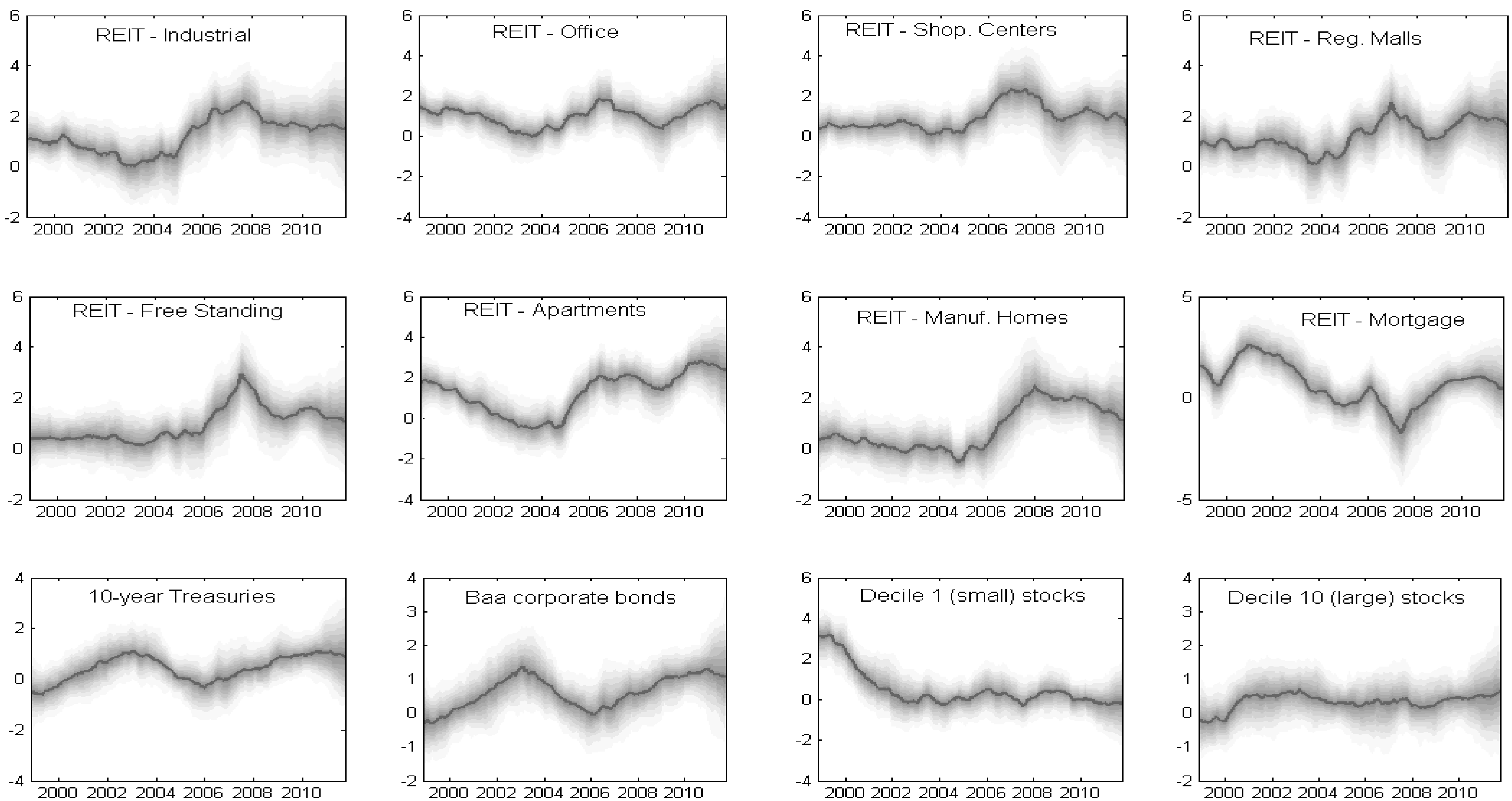


Figure 8: Selected REIT Loadings under Alternative Model Specifications: Ind. Production

This figure reports the time series of the loadings on the industrial production factor estimated from two alternative model specifications. The left column shows the posterior medians estimated from a dynamic Bayesian model with time-varying parameters. The right column shows the median estimates of the loadings from a naive rolling-window Fama-MacBeth method. The sample period is 1994:01 - 2011:12. The first 60 monthly observations are used respectively as a training sample for the time-varying parameter model, and as size of the rolling-window for the Fama-MacBeth. The gray areas on the left surrounding posterior median plots represent 95% confidence intervals. The red lines on the right represent the confidence intervals under the asymptotic distribution of the betas.

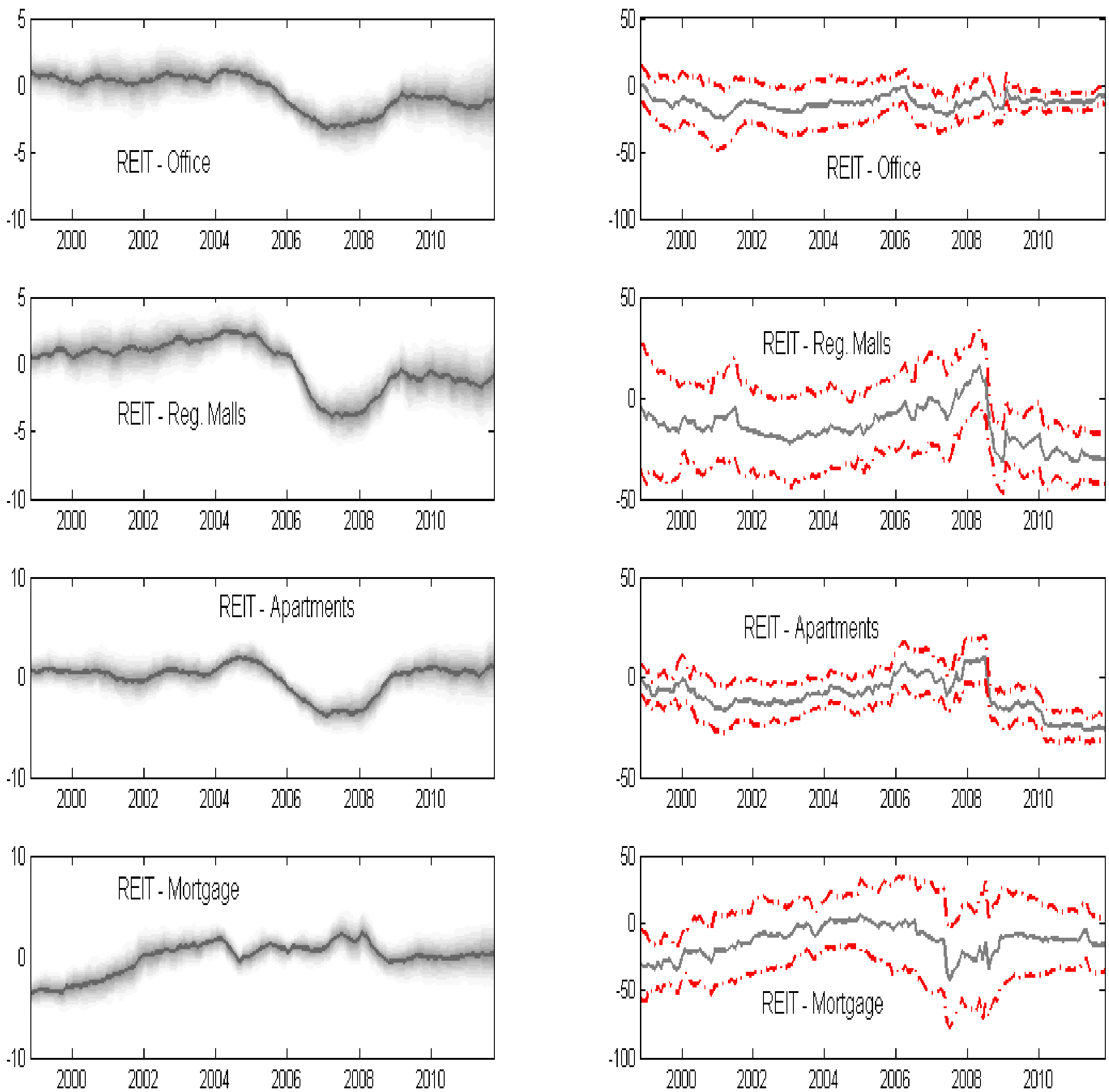


Figure 9: Selected REIT Loadings under Alternative Model Specifications: Unexpected Inflation

This figure reports the time series of the loadings on the unexpected inflation risk factor estimated from two alternative model specifications. The left column shows the posterior medians estimated from a dynamic Bayesian model with time-varying parameters. The right column shows the median estimates of the loadings from a naive rolling-window Fama-MacBeth method. The sample period is 1994:01 - 2011:12. The first 60 monthly observations are used respectively as a training sample for the time-varying parameter model, and as size of the rolling-window for the Fama-MacBeth. The gray areas on the left surrounding posterior median plots represent 95% confidence intervals. The red lines on the right represent the confidence intervals under the asymptotic distribution of the betas.

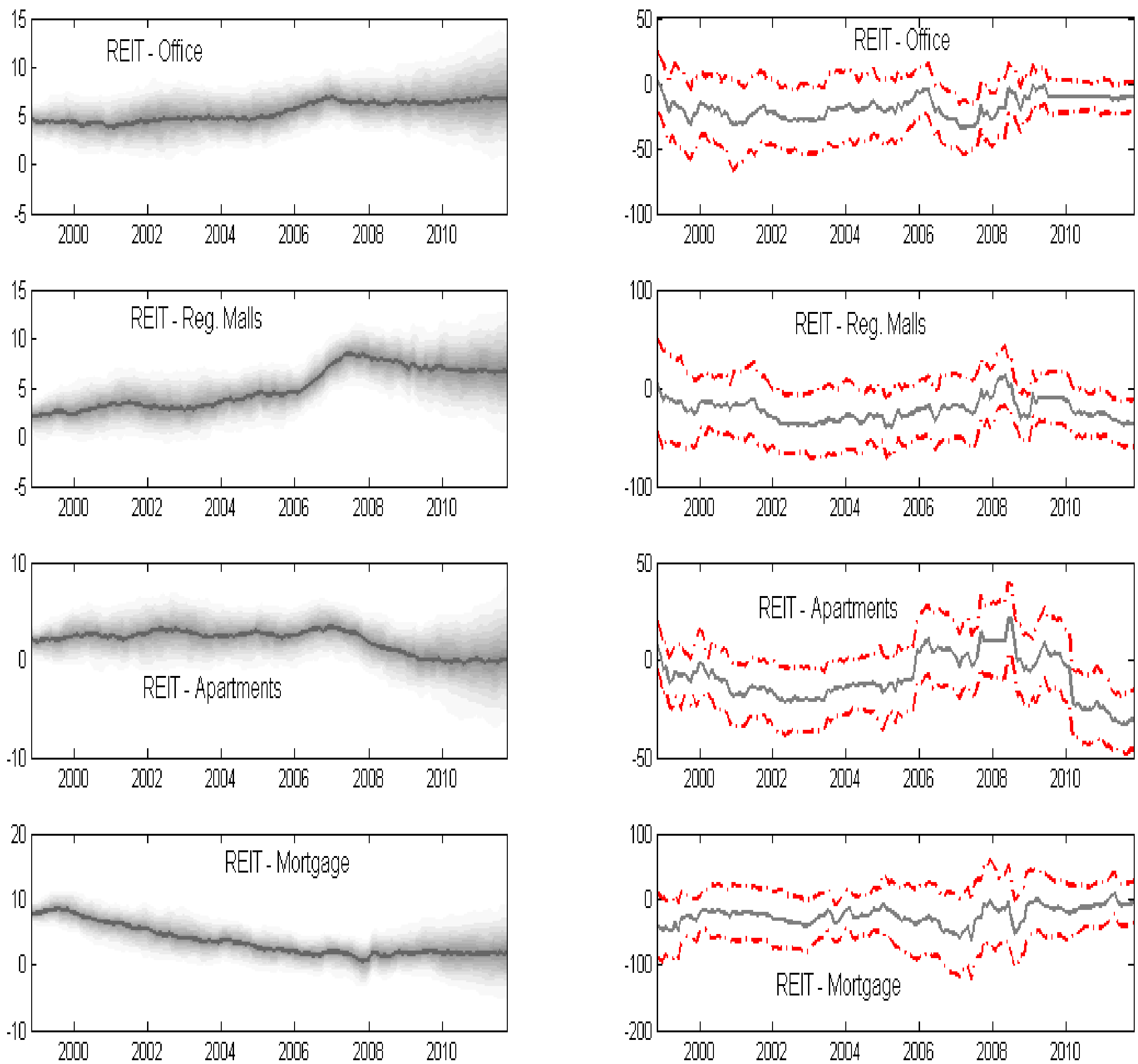


Figure 10: Selected REIT Loadings under Alternative Model Specifications: Jensen's Alphas

This figure reports the time series of the Jensen's alphas estimated from two alternative model specifications. The left column shows the posterior medians estimated from a dynamic Bayesian model with time-varying parameters. The right column shows the median estimates of the loadings from a naive rolling-window Fama-MacBeth method. The sample period is 1994:01 - 2011:12. The first 60 monthly observations are used respectively as a training sample for the time-varying parameter model, and as size of the rolling-window for the Fama-MacBeth. The gray areas on the left surrounding posterior median plots represent 95% confidence intervals. The red lines on the right represent the confidence intervals under the asymptotic distribution of the betas.

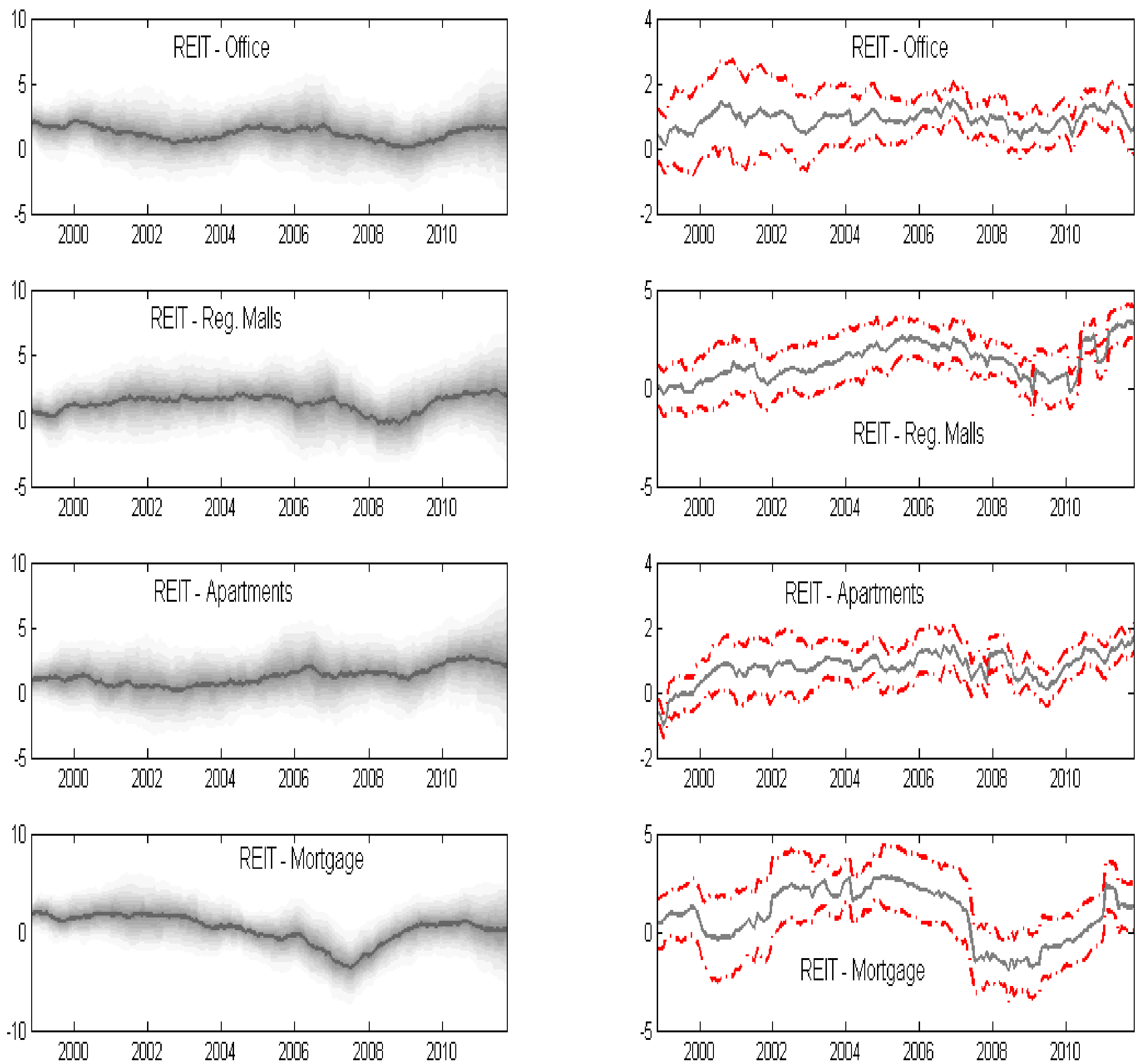


Table 1: Descriptive Statistics

This table reports the descriptive statistics for each of the 31 portfolios used in the empirical analysis as well as the risk factors and the instrumental variables. Data are monthly and cover the sample period 1994:01 - 2011:12.

Portfolio/Factor	**Mean**	**Median**	**Std. Dev.**	**Sharpe Ratio**
	10 Industry Portfolios, Value-Weighted			
Non Durable Goods	0.901	1.220	3.773	0.170
Durable Goods	0.557	0.940	7.520	0.039
Manufacturing	0.983	1.420	5.150	0.140
Energy	1.144	0.890	5.723	0.154
High Tech	1.052	1.420	7.916	0.100
Telecommunications	0.529	1.040	5.651	0.047
Shops and Retail	0.782	1.110	4.693	0.111
Healthcare	0.909	1.140	4.351	0.149
Utilities	0.793	1.290	4.286	0.124
Other	0.644	1.260	5.468	0.070
	10 Size-Sorted Portfolios, Value-Weighted			
Decile 1	1.002	1.360	6.567	0.113
Decile 2	1.006	1.270	6.864	0.109
Decile 3	0.973	1.590	6.354	0.112
Decile 4	0.891	1.560	6.067	0.104
Decile 5	0.924	1.650	5.991	0.111
Decile 6	0.924	1.560	5.426	0.122
Decile 7	0.991	1.540	5.323	0.137
Decile 8	0.899	1.390	5.330	0.120
Decile 9	0.889	1.660	4.808	0.131
Decile 10	0.702	1.160	4.510	0.098
	Bond Returns			
10-Year T-Notes	0.554	0.630	2.090	0.141
5-Year T-Notes	0.493	0.556	1.300	0.178
Baa Corporate Bonds (10-20 years)	0.746	0.854	2.717	0.179
	Real Estate Returns - Subsectors			
NAREIT - Industrial	1.083	1.365	9.428	0.087
NAREIT - Office	1.109	1.584	6.590	0.129
NAREIT - Shopping Centers	0.955	1.304	6.642	0.105
NAREIT - Regional Malls	1.335	1.584	7.970	0.135
NAREIT - Free Standing	1.188	1.607	5.115	0.181
NAREIT - Apartments	1.153	1.331	6.013	0.148
NAREIT - Manufactured Homes	0.883	0.987	5.317	0.117
NAREIT - Mortgage TR	0.638	1.743	6.407	0.059
	Economic Risk Variables			
Excess Value-Weighted Mkt	0.489	1.070	4.704	0.104
Default Premium	2.373	2.210	0.871	
Term Spread	-0.004	-0.030	0.254	
Industrial Production Growth	0.001	0.004	0.254	
Real Per Capita Consumption Growth	0.176	0.218	0.701	
Real T-Bill Interest Rate	0.154	0.165	0.247	
Unexpected Inflation	0.057	0.061	0.317	
Bond Risk Factor	0.525	0.546	1.354	0.195
Liquidity Factor	0.776	0.576	4.063	0.127
	Instrumental Variables			
Term Yield Spread	1.756	1.720	1.183	
Credit Yield Spread	0.970	0.860	0.464	
Dividend Yield	1.840	1.810	0.469	

Table 2: Marginal Likelihoods and Bayes Factors Across Alternative Model Specifications

This table reports the values of the marginal log-likelihoods and the relative Bayes Factors for different model specifications. The values reported are also disaggregated in the contributions coming from each of the portfolios under investigation. *BTVSVB* stands for Bayesian time-varying betas, stochastic volatility model, while *BTVB* and *TVPM* are, respectively, the dynamic Bayesian model restricted to have constant volatility and random-walk betas. *Fama-MacBeth* is the standard two-step procedure. *BF1* is the Bayes Factor of the BTVSVB model vs. the homoskedastic volatility restriction. Likewise, *BF2* and *BF3* are the Bayes Factors comparing the BTVSVB model with the TVPM and the Fama-MacBeth approaches, respectively.

	BTVSVB	**TVPM**	**BTVB**	**Fama-MacBeth**	**BF1**	**BF2**	**BF3**
				10 Industry Portfolios, Value-Weighted			
Non Durable Goods	**-190.57**	-926.84	-383.07	-788.55	736.27	192.49	597.98
Durable Goods	**-268.62**	-1028.21	-403.24	-1958.54	759.59	134.61	1689.92
Manufacturing	**-155.60**	-862.27	-378.23	-1746.62	706.67	222.63	1591.02
Energy	**-280.75**	-1018.37	-405.95	-1117.97	737.62	125.21	837.22
High Tech	**-210.55**	-983.32	-388.35	-3323.45	772.77	177.79	3112.89
Telecom	**-198.17**	-961.15	-386.33	-1796.91	762.98	188.15	1598.74
Shops and Retail	**-182.93**	-958.60	-383.97	-1048.66	775.67	201.04	865.73
Health	**-203.86**	-952.85	-384.40	-656.81	748.98	180.54	452.95
Utilities	**-192.60**	-953.47	-389.87	-998.45	760.86	197.27	805.85
Other	**-148.71**	-905.96	-381.15	-1894.60	757.25	232.44	1745.90
				10 Size-sorted Portfolios, Value-Weighted			
Decile 1	**-227.57**	-1000.03	-407.44	-1532.34	772.46	179.87	1304.77
Decile 2	**-225.66**	-974.12	-399.95	-1959.37	748.46	174.29	1733.71
Decile 3	**-180.30**	-1008.04	-385.80	-2038.83	827.74	205.50	1858.53
Decile 4	**-180.16**	-950.51	-380.15	-1944.03	770.35	199.98	1763.87
Decile 5	**-149.07**	-949.84	-375.99	-2203.63	800.77	226.92	2054.56
Decile 6	**-136.95**	-922.25	-374.22	-1902.32	785.30	237.28	1765.37
Decile 7	**-153.80**	-801.33	-375.15	-2241.10	647.53	221.35	2087.30
Decile 8	**-138.34**	-636.46	-374.46	-3401.11	498.12	236.12	3262.76
Decile 9	**-109.38**	-858.20	-374.72	-2792.95	748.82	265.35	2683.57
Decile 10	**-101.85**	-505.96	-371.93	-2348.01	442.92	227.75	7718.52
				Real Estate (REITs)			
NAREIT - Industrial	**-278.86**	-1131.12	-402.02	-1003.97	852.26	123.16	725.12
NAREIT - Office	**-241.04**	-1086.78	-388.99	-1030.04	845.74	147.95	788.99
NAREIT - Shopping Centers	**-241.13**	-1059.56	-392.08	-969.19	818.43	150.95	728.06
NAREIT - Regional Malls	**-273.79**	-1115.65	-400.68	-1122.50	841.86	126.89	848.71
NAREIT - Free Standing	**-206.28**	-1035.04	-384.89	-638.89	828.75	178.60	432.60
NAREIT - Apartments	**-259.65**	-1065.16	-390.85	-849.43	805.51	131.20	589.78
NAREIT - Manufactured Homes	**-224.30**	-1065.24	-389.42	-681.12	840.95	165.13	456.83
NAREIT - Mortgage TR	**-296.79**	-1068.81	-409.77	-821.61	772.02	112.98	524.82
				Bond Returns			
10 - Year Treasury	**-115.47**	-483.88	-370.87	-401.70	368.41	255.40	286.23
5 - Year Treasury	**-101.53**	-380.29	-371.72	-242.97	278.76	270.19	141.45
Baa Corporate Bonds (10-20 years)	**-113.72**	-528.18	-371.09	-786.38	414.46	257.36	672.66
Overall	**-193.16**	-908.95	-386.35	-1491.68	717.04	191.82	1475.05

Table 3: Average Pricing Errors from Alternative Models

This table reports the average pricing errors of each of the models developed in Section 2 of the paper and across different sub-samples as well as in the full sample. *BTVSVB* stands for Bayesian time-varying betas, stochastic volatility model, while *BTVB* and *TVPM* are, respectively, the dynamic Bayesian model restricted to have constant conditional volatility and random-walk betas. Fama-MacBeth is the standard two-step procedure. The table reports the posterior mean (over time), the posterior standard deviation as well as the confidence interval at the 95% level.

Average Pricing Errors					
	Mean %	**Std %**	**2.5 %**	**50 %**	**97.5 %**
Panel A: 1999:01 - 2011:12					
BTVSVB	**0.613**	0.119	0.426	0.612	0.803
BTVB	0.954	0.065	0.861	0.943	1.059
TVPM	1.989	0.435	1.200	2.044	2.653
Fama-MacBeth	1.052	0.093	0.954	1.023	1.286
Panel B: 1999:01 - 2007:01					
BTVSVB	**0.584**	0.116	0.417	0.571	0.783
BTVB	0.913	0.037	0.856	0.911	0.982
TVPM	1.882	0.453	1.155	1.902	2.614
Fama-MacBeth	0.996	0.036	0.952	0.991	1.056
Panel C: 2007:01 - 2011:11					
BTVSVB	**0.658**	0.113	0.463	0.671	0.819
BTVB	1.023	0.027	0.987	1.024	1.073
TVPM	2.158	0.345	1.610	2.254	2.660
Fama-MacBeth	1.139	0.088	1.018	1.119	1.320

Table 4: Risk Premia

This table reports statistics describing the posterior distribution of the risk premia on each factor across different model specifications. Data are monthly and cover the sample period 1994:01 - 2011:12. The first five years of monthly data are used to calibrate the priors for all the models except for the two-step Fama-MacBeth method.

	Full sample (Jan. 1999 - Dec. 2011)					Sub-sample (Jan. 2007 - Dec. 2011)				
	Mean	**Std. Error**	**t-stat**	**p-value**	**Median**	**Mean**	**Std. Error**	**t-stat**	**p-value**	**Median**
Bayesian model with instability in betas and idiosyncratic variances										
Intercept	0.1903	0.1408	1.3512	0.1766	0.7012	0.2797	0.2237	1.2503	0.2112	0.7864
Excess VW Mkt	0.3292	0.1252	2.6292	**0.0086**	0.2151	0.0656	0.2498	0.2627	0.7928	-0.0153
Default Premium	0.0394	0.1562	0.2526	0.8006	0.0184	-0.2376	0.1175	-2.0209	**0.0433**	-0.2542
Term Spread	0.0148	0.0309	0.4802	0.6311	0.0108	0.0529	0.0348	1.5191	0.1287	0.0411
Ind. Production Growth	0.1940	0.1124	1.7260	**0.0843**	0.1794	0.2220	0.1104	2.0108	**0.0443**	0.2405
Real Cons. Growth	-0.0169	0.0985	-0.1713	0.8640	-0.0032	-0.0437	0.2029	-0.2155	0.8294	0.0000
Real T-Bill Int. Rate	-0.0493	0.0423	-1.1661	0.2436	-0.0502	-0.0387	0.0748	-0.5178	0.6046	-0.0286
Unexpected Inflation	-0.1439	0.0663	-2.1694	**0.0301**	-0.1108	-0.0741	0.1692	-0.4376	0.6617	-0.0260
Bond Risk Factor	-0.0708	0.0865	-0.8177	0.4135	-0.0707	-0.2688	0.1446	-1.8581	**0.0632**	-0.2721
Liquidity Factor	0.3191	0.1597	1.9984	**0.0457**	0.2131	0.5571	0.2016	2.7635	**0.0057**	0.5567
Two-step Fama-MacBeth method										
Intercept	0.8473	0.1914	4.4274	**0.0000**	1.1065	0.7984	0.3031	2.6338	**0.0084**	1.0247
Excess VW Mkt	-0.4215	0.4008	-1.0515	0.2930	0.2438	-0.6223	0.7768	-0.8011	0.4231	0.1555
Default Premium	0.8830	0.2857	3.0905	**0.0020**	0.9611	0.7805	0.3954	1.9737	**0.0484**	0.8664
Term Spread	0.0497	0.0636	0.7811	0.4347	-0.0016	0.0825	0.1165	0.7083	0.4788	-0.0013
Ind. Production Growth	-0.0229	0.0331	-0.6930	0.4883	-0.0014	-0.0442	0.0631	-0.7000	0.4839	-0.0066
Real Cons. Growth	-0.0002	0.0098	-0.0242	0.9807	0.0101	-0.0013	0.0172	-0.0756	0.9397	0.0071
Real T-Bill Int. Rate	-0.0378	0.0916	-0.4132	0.6795	-0.1398	0.0754	0.1626	0.4638	0.6428	-0.0075
Unexpected Inflation	0.0105	0.0194	0.5422	0.5877	-0.0229	0.0335	0.0367	0.9120	0.3618	-0.0212
Bond Risk Factor	-0.5956	0.4785	-1.2447	0.2132	-0.1987	-0.6036	0.7208	-0.8374	0.4024	0.1117
Liquidity Factor	1.1758	0.6632	1.7729	**0.0762**	0.5457	1.2697	1.2650	1.0038	0.3155	0.8025
Bayesian time-varying parameter model										
Intercept	0.7555	0.4155	1.8183	**0.0690**	0.7591	0.8607	0.2716	3.1686	**0.0015**	0.8675
Excess VW Mkt	0.1906	0.0808	2.3592	**0.0183**	0.1853	0.0143	0.1315	0.1087	0.9134	0.0000
Default Premium	-0.1553	0.0806	-1.9273	**0.0539**	-0.1496	-0.2146	0.1106	-1.9406	**0.0523**	-0.2071
Term Spread	0.0090	0.0226	0.3993	0.6897	0.0080	0.0068	0.0293	0.2330	0.8158	0.0036
Ind. Production Growth	0.0343	0.0758	0.4519	0.6513	0.0273	0.2600	0.1260	2.0635	**0.0391**	0.2675
Real Cons. Growth	-0.0174	0.0660	-0.2633	0.7923	-0.0110	-0.0312	0.1098	-0.2844	0.7761	-0.0131
Real T-Bill Int. Rate	-0.0152	0.0259	-0.5858	0.5580	-0.0143	0.0009	0.0367	0.0243	0.9806	0.0000
Unexpected Inflation	0.0131	0.0607	0.2165	0.8286	0.0116	-0.0391	0.1024	-0.3818	0.7026	-0.0200
Bond Risk Factor	-0.0267	0.0408	-0.6541	0.5130	-0.0248	-0.1584	0.0670	-2.3646	**0.0181**	-0.1738
Liquidity Factor	0.1510	0.0789	1.9140	**0.0556**	0.1442	0.0556	0.1440	0.3860	0.6995	0.0313
Homoskedastic Bayesian model with instability in betas										
Intercept	0.7147	0.3606	1.9819	**0.0495**	0.7178	0.7793	0.4181	1.8639	**0.0623**	0.7954
Excess VW Mkt	0.0183	0.0653	0.2802	0.7793	-0.0111	0.0074	0.0764	0.0970	0.9227	0.0000
Default Premium	0.0142	0.0831	0.1710	0.8642	0.0123	-0.2233	0.1005	-2.2227	**0.0262**	-0.2242
Term Spread	0.0534	0.0265	2.0154	**0.0439**	0.0421	-0.0081	0.0383	-0.2105	0.8333	-0.0052
Ind. Production Growth	0.0347	0.0674	0.5149	0.6066	0.0286	0.0208	0.0824	0.2525	0.8007	0.0080
Real Cons. Growth	-0.0280	0.0657	-0.4266	0.6697	-0.0254	0.0133	0.0832	0.1596	0.8732	0.0000
Real T-Bill Int. Rate	0.0279	0.0190	1.4649	0.1429	0.0275	0.0219	0.0278	0.7866	0.4315	0.0189
Unexpected Inflation	0.0165	0.0525	0.3141	0.7534	0.0127	-0.0154	0.0697	-0.2203	0.8256	-0.0070
Bond Risk Factor	0.0541	0.0680	0.7945	0.4269	0.0507	-0.0107	0.0715	-0.1492	0.8814	0.0000
Liquidity Factor	0.1266	0.0660	1.9193	**0.0549**	0.1361	0.1825	0.0890	2.0510	**0.0403**	0.1573

Table 5: Variance Decomposition Tests Across Models

This table reports the results of variance decomposition tests across models. The first two columns show the values for the standard two-step Fama-MacBeth methodology. All rates are in excess of the holding period return on a 1-month T-Bill. VR1 is the ratio of the variance of a model's predicted returns and the variance of expected returns estimated from a projection on a set of instruments. VR2 is the ratio of the variance of the predictable part of returns not explained by a model and the variance of projected returns. The instrumental variables are the lagged monthly dividend yield on the NYSE/AMEX calculated as in Campbell and Beeler (2012), the lagged yield of a Baa corporate bond, and the lagged spread of long- vs. short-term government bond yields. *BTVSVB* stands for Bayesian time-varying betas with stochastic volatility, while *TVPM* and *BTVB* are, respectively, a random-walk time-varying parameter and homoskedastic time-varying betas model.

	Fama-MacBeth		**BTVSVB**					
	VR1	**VR2**	**VR1**			**VR2**		
			2.5%	**50%**	**97.5%**	**2.5%**	**50%**	**97.5%**
			10 Industry Portfolios, Value-Weighted					
Non Durable Goods	0.310	0.775	0.348	0.672	0.822	0.179	0.323	0.634
Durable Goods	0.026	0.915	0.414	0.751	0.901	0.107	0.245	0.549
Manufacturing	0.136	0.806	0.466	0.767	0.845	0.110	0.187	0.530
Energy	0.166	0.827	0.483	0.780	0.972	0.038	0.200	0.531
High Tech	0.131	0.753	0.597	0.844	0.929	0.059	0.165	0.325
Telecommunications	0.122	0.751	0.305	0.488	0.569	0.425	0.498	0.671
Shops and Retail	0.125	0.814	0.274	0.620	0.885	0.143	0.344	0.766
Health	0.235	0.778	0.417	0.754	0.945	0.047	0.270	0.600
Utilities	0.189	0.786	0.519	0.721	0.923	0.054	0.212	0.476
Other	0.125	0.859	0.304	0.646	0.860	0.148	0.348	0.663
			10 Size-Sorted Portfolios, Value-Weighted					
Decile 1	0.213	0.736	0.607	0.862	0.940	0.069	0.149	0.335
Decile 2	0.089	0.948	0.606	0.898	0.982	0.013	0.126	0.416
Decile 3	0.106	0.834	0.500	0.813	0.985	0.014	0.165	0.513
Decile 4	0.038	0.935	0.532	0.876	0.961	0.033	0.158	0.473
Decile 5	0.195	0.759	0.472	0.769	0.872	0.136	0.262	0.513
Decile 6	0.203	0.749	0.438	0.712	0.854	0.172	0.294	0.540
Decile 7	0.197	0.853	0.504	0.701	0.824	0.159	0.369	0.529
Decile 8	0.125	0.836	0.499	0.734	0.836	0.118	0.266	0.454
Decile 9	0.096	0.933	0.553	0.797	0.848	0.121	0.262	0.485
Decile 10	0.182	0.810	0.452	0.734	0.884	0.127	0.285	0.534
			Real Estate (REITs)					
NAREIT - Industrial	0.266	0.750	0.502	0.733	0.906	0.101	0.242	0.509
NAREIT - Office	0.165	0.811	0.577	0.715	0.902	0.114	0.201	0.433
NAREIT - Shopping Centers	0.041	0.963	0.777	0.911	0.969	0.030	0.105	0.188
NAREIT - Regional Malls	0.161	0.856	0.430	0.723	0.821	0.179	0.190	0.492
NAREIT - Free Standing	0.174	0.816	0.548	0.705	0.881	0.135	0.205	0.413
NAREIT - Apartments	0.151	0.892	0.322	0.542	0.747	0.255	0.454	0.696
NAREIT - Manufactured Homes	0.218	0.851	0.718	0.810	0.917	0.079	0.151	0.234
NAREIT - Mortgage TR	0.159	0.875	0.422	0.743	0.876	0.126	0.214	0.520
			Bond Returns					
10 - Year Treasury	0.356	0.705	0.506	0.644	0.889	0.127	0.356	0.505
5 - Year Treasury	0.318	0.686	0.695	0.956	1.029	0.008	0.032	0.275
Baa Corp Bonds (10-20 years)	0.415	0.623	0.594	0.811	0.916	0.076	0.186	0.407

Table 6: Sources of Risk

This table reports the results of variance decomposition tests for each of the risk factors from the BTVBSV model. All rates of return are monthly and in excess of the holding period return on a 1-month T-Bill.VR1 is the ratio of the variance of a model's predicted returns and the variance of expected returns estimated from a projection on a set of instruments. VR2 is the ratio of the variance of the predictable part of returns not explained by a model and the variance of projected returns. The instrumental variables are the lagged monthly dividend yield on the NYSE/AMEX calculated as in Campbell and Beeler (2012), the lagged yield of a Baa corporate bond, and the lagged spread of long- vs. short-term government bond yields. The first two columns report the overall posterior medians of VR1 and VR2. From the third column, we report the contributions of each factor in percentage terms. The last column reports the posterior median of the interaction effect.

	VR1	VR2	Market	Credit Risk	Term Spread	IP Growth	Real Cons.	Real T-Bill	Unexp. Infl.	Bond Factor	Liquidity	Interaction
						10 Industry Portfolios, Value-Weighted						
Non Durable Goods	0.672	0.323	0.2486	0.0276	0.0176	0.1773	0.0484	0.0334	0.2416	0.0607	0.4088	-0.2641
Durable Goods	0.751	0.245	0.2783	0.0146	0.0104	0.2402	0.0257	0.0167	0.1290	0.0374	0.3063	-0.0586
Manufacturing	0.767	0.187	0.3065	0.0148	0.0160	0.0762	0.0233	0.0096	0.1555	0.0391	0.4432	-0.0843
Energy	0.780	0.200	0.5840	0.0232	0.0397	0.1947	0.0202	0.0126	0.2682	0.0178	0.3991	-0.5596
High Tech	0.844	0.165	0.7143	0.0091	0.0049	0.0343	0.0146	0.0049	0.0633	0.0142	0.1860	-0.0456
Telecommunications	0.488	0.498	0.3429	0.0208	0.0110	0.0715	0.0217	0.0095	0.1083	0.0238	0.4042	-0.0137
Shops and Retail	0.620	0.344	0.2836	0.0361	0.0163	0.2773	0.0413	0.0160	0.1962	0.0512	0.3277	-0.2457
Health	0.754	0.270	0.4426	0.0178	0.0132	0.1686	0.0368	0.0201	0.3070	0.0226	0.3218	-0.3504
Utilities	0.721	0.212	0.3172	0.0201	0.0131	0.2925	0.0421	0.0155	0.2901	0.0561	0.2770	-0.3238
Other	0.646	0.348	0.4431	0.0127	0.0176	0.0790	0.0535	0.0116	0.1185	0.0263	0.2978	-0.0602
						10 Size-sorted Portfolios, Value-Weighted						
Decile 1	0.862	0.149	0.2185	0.0167	0.0168	0.1753	0.0307	0.0165	0.2614	0.0470	0.3741	-0.1569
Decile 2	0.898	0.126	0.4108	0.0086	0.0168	0.1029	0.0150	0.0073	0.3049	0.0208	0.3036	-0.1907
Decile 3	0.813	0.165	0.3726	0.0119	0.0106	0.1181	0.0224	0.0072	0.2772	0.0316	0.3341	-0.1856
Decile 4	0.876	0.158	0.4246	0.0115	0.0112	0.0909	0.0208	0.0076	0.2186	0.0265	0.4382	-0.2498
Decile 5	0.769	0.262	0.3453	0.0104	0.0093	0.1129	0.0175	0.0085	0.1247	0.0228	0.3601	-0.0116
Decile 6	0.712	0.294	0.2632	0.0194	0.0245	0.1132	0.0337	0.0094	0.2135	0.0348	0.2333	0.0549
Decile 7	0.701	0.369	0.2527	0.0160	0.0181	0.1029	0.0286	0.0144	0.1818	0.0406	0.1898	0.1550
Decile 8	0.734	0.266	0.2645	0.0113	0.0126	0.1632	0.0203	0.0078	0.2871	0.0241	0.3731	-0.1642
Decile 9	0.797	0.262	0.3890	0.0110	0.0114	0.1480	0.0225	0.0115	0.1691	0.0200	0.3542	-0.1369
Decile 10	0.734	0.285	0.4050	0.0094	0.0095	0.1386	0.0150	0.0075	0.1619	0.0200	0.1496	0.0834
							Real Estate (REITs)					
NAREIT - Industrial	0.733	0.242	0.2457	0.0055	0.0089	0.1145	0.0117	0.0135	0.4221	0.0072	0.2336	-0.0628
NAREIT - Office	0.715	0.201	0.3977	0.0149	0.0093	0.1478	0.0150	0.0062	0.2535	0.0218	0.1097	0.0242
NAREIT - Shopping Centers	0.911	0.105	0.3635	0.0064	0.0040	0.1908	0.0140	0.0097	0.4745	0.0121	0.1704	-0.2454
NAREIT - Regional Malls	0.723	0.190	0.3258	0.0076	0.0119	0.1618	0.0222	0.0094	0.3469	0.0098	0.2146	-0.1100
NAREIT - Free Standing	0.705	0.205	0.3019	0.0094	0.0051	0.1584	0.0090	0.0141	0.4951	0.0101	0.1068	-0.1100
NAREIT - Apartments	0.542	0.454	0.3719	0.0294	0.0233	0.2055	0.0258	0.0107	0.2067	0.0191	0.3720	-0.2644
NAREIT - Manufactured Homes	0.810	0.151	0.2748	0.0204	0.0090	0.1787	0.0301	0.0303	0.3553	0.0178	0.2218	-0.1382
NAREIT - Mortgage TR	0.743	0.214	0.3252	0.0248	0.0258	0.1914	0.0326	0.0361	0.2045	0.0409	0.3292	-0.2107
							Bond Returns					
10 - Year Treasury	0.644	0.356	0.2438	0.0652	0.0371	0.1006	0.0394	0.0210	0.3182	0.0522	0.2253	-0.1029
5 - Year Treasury	0.956	0.032	0.2262	0.0592	0.0374	0.2718	0.0690	0.0269	0.4209	0.0865	0.2593	-0.4572
Baa Corporate Bonds (10-20 years)	0.811	0.186	0.2559	0.1768	0.0350	0.1335	0.0419	0.0242	0.3765	0.0424	0.2880	-0.3742