NORGES BANK

Zero coupon yields – explanation of the calculations

The individual zero coupon yields or the curve that plots these yields are often referred to as the yield term structure. The zero coupon yield curve shows an *unambiguous relationship* between the effective yield (yield to maturity) and term to maturity and is not affected by the fact that individual bonds are issued with differing coupons. The published zero coupon yields are estimated values based on observed effective yields on Norwegian Treasury bills (zero coupon securities) and government bonds (coupon securities).

Norges Bank calculates zero coupon yields using a parametric method developed by Nelson and Siegel[1] and developed further by Svensson[2], referred to as the NSS method.

The forward interest rate function in the NSS method contains six parameters to be estimated. These parameters are $\beta_0, \beta_1, \beta_2, \beta_3, \tau_1$ and τ_2 in the following forward interest rate function:

$$f_t(m) = \beta_{0t} + \beta_{1t} e^{\left(\frac{-m}{\tau_1}\right)} + \beta_{2t} \frac{m}{\tau_1} e^{\left(\frac{-m}{\tau_1}\right)} + \beta_{3t} \frac{m}{\tau_2} e^{\left(\frac{-m}{\tau_2}\right)}$$

When the six parameters have been estimated, the forward rate function generates zero coupon yields for all future points in time (continuous function). The zero coupon yield can be derived by integrating the forward interest rate. Let i_m be the zero coupon rate with term to maturity equal to m:

$$i_m = \beta_{0t} + \beta_{1t} \frac{1 - e^{\left(\frac{-m}{\tau_1}\right)}}{\frac{m}{\tau_1}} + \beta_{2t} \left(\frac{1 - e^{\left(\frac{-m}{\tau_1}\right)}}{\frac{m}{\tau_1}} - e^{\left(\frac{-m}{\tau_1}\right)} \right) + \beta_{3t} \left(\frac{1 - e^{\left(\frac{-m}{\tau_2}\right)}}{\frac{m}{\tau_2}} - e^{\left(\frac{-m}{\tau_2}\right)} \right)$$

Since the function in the NSS method is non-linear, certain upper and lower limits are imposed, as well as starting parameter values. Norges Bank estimates zero coupon yields by using to different sets of starting values and threshold values for the parameters, as shown in the table below.

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[1] Nelson and Siegel: "Parsimonious Modeling of Yield Curves", *The Journal of Business*, Vol. 60, No. 4 (Oct., 1987).
[2] Svensson: "Estimating and interpreting forward interest rates: Sweden 1992-1994", *NBER Working Paper Series*, No. 4871, September 1994.

Table: Overview of threshold and starting values for the parameters for estimating zero coupon yields using the NSS method. Percent for all β_i. τ_i are time constants.

Model		Parameter					
		$\boldsymbol{\beta_0}$	$\boldsymbol{\beta_1}$	$\boldsymbol{\beta_2}$	$\boldsymbol{\beta_3}$	$\boldsymbol{\tau_1}$	$\boldsymbol{\tau_2}$
1	Upper threshold	B1+	30	30	30	30	30
	Lower threshold	B1-	-30	-30	-30	0.0001	0.0001
	Starting values	B1***	S0-	1	1	1	1
2	Upper threshold	B1+	30	30	30	30	30
	Lower threshold	B1-	-30	-30	-30	0.0001	0.0001
	Starting values	β_0(t-1)	β_1(t-1)	β_2(t-1)	β_3(t-1)	τ_1(t-1)	τ_2(t-1)

Symbol	Explanation
B1+	Effective yield (in percent) on the government bond with the longest residual term to maturity plus 2 percentage points.
B1-	Effective yield (in percent) on the government bond with the longest residual term to maturity minus 2 percentage points, but not lower than 0.0001 percent.
B1* **	Average of the effective yield (in percent) of the three government bonds with the longest residual term to maturity.
S0-	Effective yield (in percent) on government securities with the shortest residual term to maturity, minus B1***.
Infinite	Infinite (no limit).
[Parameter](t-1)	Estimated parameter from the previous trade date.

Zero coupon yields are estimated daily. The estimation of zero coupon yields also provides a theoretical price for each security in the underlying data. This theoretical price is equal to the sum of all future cash flows associated with the security discounted to today using the model's estimated zero coupon yields. The method that minimises the squared error between the observed and theoretical prices (lowest *root-mean-square error*, RMSE) on the day in question will be the method used in the calculation of the published zero coupon yields.

All government bonds outstanding are included in the calculations, except for government bonds with less than one year to maturity.

From 1 July 2020, the source for the individual bonds included in the calculations: E-Bond (Bloomberg), primary dealers' price quotations under the primary dealer agreement (Source through 30 June 2020: Oslo Børs).