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Forecast Uncertainty in the Neighborhood of the Effective Lower Bound: How Much Asymmetry Should We Expect?☆

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Abstract

The lower bound on interest rates has restricted the impact of conventional monetary policies over recent years and could continue to do so in the near future, with the decline in natural real rates not predicted to reverse any time soon. A binding lower bound on interest rates has consequences not only for point forecasts but also for the entire model forecast distribution. In this paper we investigate the ramifications of the lower bound constraint on the forecast distributions from DSGE models and the implications for risk and uncertainty. To that end we start out by making the case for regime-switching as a framework for imposing the lower bound constraint on interest rates in DSGE models. We then use the framework to investigate the implications of the lower bound constraint on the forecast distributions and try to answer the question of how much asymmetry we should expect when the lower bound binds. The results suggest that: i) a lower bound constraint need not in itself imply asymmetric fan charts, ii) the degree of asymmetry of fan charts depends on various factors such as the degree of interest rate smoothing and the degree of price rigidity, and iii) different approaches to imposing the lower bound yield different results for both the width of the fan charts and their asymmetry.

Keywords: Effective Lower Bound, Regime-Switching, DSGE, Forecast Uncertainty, Fan Charts

1. Introduction

The lower bound on interest rates has restricted the impact of conventional monetary policies over recent years and could continue to do so in the near future, with the decline in natural real rates not predicted to reverse any time soon. A binding lower bound on interest

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rates has consequences not only for point forecasts but also for the entire model forecast distribution. Little work has been done to investigate the ramifications of the lower bound constraint on the forecast distributions from DSGE models and the implications for risk and uncertainty. This paper attempts to fill that gap. More precisely we investigate how the lower bound on interest rates affects the symmetry, that is the relative upside and downside risks, and the width or the overall level of uncertainty present in the forecast densities.[1]

In so doing, we challenge the widespread belief/assumption that the asymmetry in the density forecast of nominal interest rates, due to truncation at the lower bound, should translate into asymmetry in the forecast densities of other variables in the economy. For example, the IMF states that the zero lower bound is one reason they may introduce asymmetric fan charts into their World Economic Outlook analysis (see IMF, 2008, p. 42-43). The Bank of Canada makes similar assumptions when discussing how their fan charts are constructed (see Bank of Canada, 2009). These beliefs often derive from studies that have not attempted to examine the robustness of the results to alternative assumptions. Examples include Coenen & Warne (2014), Haberis et al. (2015) and Eusepi et al. (2016). One exception is Franta et al. (2014), who investigate the implications of different methods for imposing the lower bound constraint in a density forecasting context. They compare imposing the lower bound through soft conditioning and a form of hard conditioning. However, their analysis is confined to a BVAR model, making their results especially vulnerable to the Lucas critique.

An appropriate assessment of the forecast densities when the lower bound constraint on nominal interest rates binds requires a structural model where expectations are forward looking and a function of the policy environment. Many methods have previously been used to impose the lower bound constraint on interest rates mathematically, but these methods do not explain the economics behind the constraint (Fernández-Villaverde et al., 2015; Maliar & Maliar, 2015, for example). Many popular methods are inconsistent with the rational expectations solution of the nonlinear DSGE model that encompasses the problem (Guerrieri & Iacoviello, 2015b, for example). And many methods fail to recognize the effective lower bound binding as a change in monetary policy and policy objectives, riding roughshod over the Lucas critique (Guerrieri & Iacoviello, 2015b; Fernández-Villaverde et al., 2015; Maliar & Maliar, 2015, for example).

To overcome the shortcomings of the previous literature we investigate the problem through the lens of a canonical New Keynesian DSGE model where the lower bound constraint is introduced through regime-switching. More specifically, we use the model and

[1]Forecast densities, usually in the form of fan charts, are used by many inflation targeting central banks to communicate forecast uncertainty. Along with alternative (point forecast) scenarios these provide a useful tool for conveying a range of forecast outcomes and assessing their relative probabilities. However, of the the central banks that do publish fan charts, none publishes density forecasts of nominal interest rates with a binding lower bound, even though interest rates in many of these countries are at or near their lower bound (see Franta et al., 2014). In fact many of these central banks ignore the lower bound constraint completely, or they trim their fan charts at the lower bound, ignoring the consequences implied by the multivariate/general equilibrium nature of the economy. Neglecting the presence of the lower bound on interest rates results in less plausible density forecasts compared with when the constraint is imposed (see Franta et al., 2014).

modeling framework from Binning & Maih (2016). The model is solved using the perturbation methods of Maih (2015), providing a consistent approximation of the nonlinear DSGE model with the constraint imposed. Moreover, the regime-switching framework treats the lower bound constraint as a separate monetary policy regime, introduces policy parameters to explain the regimes and transitions between regimes, and thus provides some immunity from the Lucas critique, at least in theory.

Our approach allows us to investigate how the lower bound constraint affects the forecast densities of other variables in the system under a wide range of different assumptions. More specifically, we are able to investigate how different parameterizations, different orders of approximation and the nature of the transition probabilities (endogenous vs exogenous) affect the results. The flexibility of the methodology also allows us to investigate how changes in the monetary policy transmission mechanism, modeled in the same regime-switching framework, affect the results. To complete our analysis we repeat some of the simulation experiments, employing some methods popularly used in the literature for imposing occasionally binding constraints, namely: adding shocks (Lindé et al., 2016), piecewise linear solutions (Kulish et al., 2014; Guerrieri & Iacoviello, 2015a) and the extended path algorithm (Juillard & Maih, 2010; Braun & Köber, 2011; Coenen & Warne, 2014).

Three main results emerge from our investigations: i) a lower bound constraint need not in itself imply asymmetric fan charts, ii) the degree of asymmetry of fan charts depends on various factors such as the degree of interest rate smoothing and the degree of price rigidity, and iii) different approaches to imposing the lower bound yield different results for both the width of the fan charts and their asymmetry.

The remainder of this paper is structured as follows: in Section 2 we start out by making the case for using regime-switching as the most appropriate method for imposing the lower bound constraint on interest rates in DSGE models and we outline the regime-switching methodology used in this paper. We describe the model and its calibration in Section 3, and in Section 4 we present simulation results using regime-switching to impose the lower bound on interest rates. This is followed by Section 5 where we examine how the results change when alternative methods are used to enforce the effective lower bound (ELB). We discuss some of the implications for and limitations of the simulation results in Section 6. The final section concludes. All our results are computed in Matlab using the RISE toolbox.

2. Regime-Switching

In this section we make the case for using regime-switching to model the lower bound constraint on interest rates and discuss some of the advantages of modelling occasionally binding constraints in terms of regime-switching more generally. Then we describe the regime-switching methodology we use, including the use of endogenous transition probabilities.

2.1. The Case for Regime-Switching as a Means for Implementing the ELB

The lower bound constraint on nominal interest rates can be imposed mathematically in many different ways. However, all methods are not created equal and will in general have

different economic and policy implications. Each method will also have consequences for the properties of the model and ultimately the width and asymmetry of the forecast densities and fan charts near the lower bound. Here we make the case for regime-switching as the most natural way of implementing the lower bound constraint on interest rates.

Thinking of both the historical and expected future implementation of monetary policy in terms of regimes, governed by separate policy parameters, is natural. Changes in central bank governors and/or monetary policy objectives can be neatly mapped into changes in these policy parameters. Lucas (1976) has shown that ignoring (potential) changes in policy parameters can have severe consequences for forecasting and policy analysis.[2] Bianchi (2013) and Davig & Doh (2014), among others, have heeded the warning and used regime-switching DSGE models to account for changes in monetary policy regimes and parameters in the US. Davig & Doh (2014) find that changes in monetary policy parameters have occurred with these changes implying monetary policy was less aggressive in the 1970s than in other periods.

Regime-switching has also been employed in the ELB literature in various forms. The piecewise linear solutions of Eggertsson & Woodford (2003) and Braun et al. (2015) all assume the ELB period is a separate regime and that the transition back to a normal regime is determined by a Markov process. Starting from a nonlinear regime-switching DSGE model, Aruoba et al. (2013) and Gavin et al. (2015) use global approximations, while Binning & Maih (2016) use perturbation methods to model the ELB constraint. The reasons for using regime-switching to model the lower bound constraint can be summarized as follows: central banks' policy objectives as well as monetary policies can change at the ELB. In addition, the transmission mechanism itself can change at the ELB. Furthermore, the historical experience of countries that have been at the ELB seems to be consistent with some of the properties of regime-switching. We discuss each of these reasons in more detail in turn.

Spending prolonged periods with interest rates at their effective lower bound has in practise seen some central banks change their short and medium run monetary policy objectives. Buiter (2013) suggests there is evidence that the policy objectives of the Bank of England, the Fed and the ECB all changed while at the ELB. More specifically he cites evidence for the Fed's de facto inflation target having risen from 2% to 2.5%. He argues the Bank of England has introduced a de facto dual mandate objective with equal focus on unemployment and inflation. Moreover the Fed's announcement in 2012 that they would not consider raising interest rates until unemployment was at or below 6.5% could also be interpreted as a change in their short and/or medium term policy objectives. These changes in policy objectives are all consistent with changes in the underlying policy parameters governing the economy and can thus be captured by a regime-switching framework.

In addition to changing policy objectives at the ELB, monetary policies themselves have also changed. When central banks can no longer use conventional monetary polices, they resort to using unconventional monetary policies. These have included forward guidance,

[2] Lucas (1976) demonstrates this in an example where he models data using a reduced form model, where the true data-generating process is a general equilibrium model with taxation parameters that follow a Markov process.

quantitative easing and foreign exchange intervention. In fact one of the objectives of forward guidance is for central banks to explain the new monetary policy regime to market participants and outline the rules (and policy parameters) for entering and exiting this regime, which agents are unfamiliar with and have no historical experience of. Policies like quantitative easing are generally not used in normal times and a regime-switching framework introduces a means of switching off such channels or reducing their impact in normal times. Moreover, the implementation of unconventional monetary policies or alternatively fiscal policies when interest rates are at their lower bound could reduce downside risk affecting the shape and width of the model's density forecasts.

Not only can policy or policy objectives change at or near the ELB, but the monetary policy transmission mechanism itself can change. As interest rates approach the lower bound, bank margins and interest rate risk premia can start to impact and decrease interest rate pass-through, lessening the overall impact of conventional monetary policy.[3] Agents' behavior at or near the ELB could also change, especially if the ELB period is caused by precautionary savings behavior due to increased risk or uncertainty, or if the ELB period itself causes a climate of fear and uncertainty. Such a change in the monetary policy transmission mechanism can be modeled using regime switches in some of the so-called deep parameters. The change in the transmission mechanism will likely have implications for the (a)symmetry of the density forecasts.

Furthermore, the properties of the economies that have spent time at the ELB are consistent with the notion that they have multiple (steady) states. The US, Japan and the UK have all spent significant periods of time at (or near) their respective effective lower bounds (see Figure 1). This could easily qualify each ELB period to be modeled as a separate ELB steady state. Even at the beginning of the ELB period in the US, market participants had expectations that the ELB period would last for at least one year, as demonstrated by survey data from Swanson & Williams (2014) (see Figure 2). These expected durations have a natural counterpart in the regime-switching framework in terms of the transition probabilities, which reflect the expected duration of each regime. Capturing such behavior in a constant parameter framework would prove more difficult.

[3]See Agarwal & Kimball (2015) for a discussion of interest rate pass-through based on the Swedish and Danish experience with negative interest rates.

Figure 1: Policy Rates in Countries at the ELB

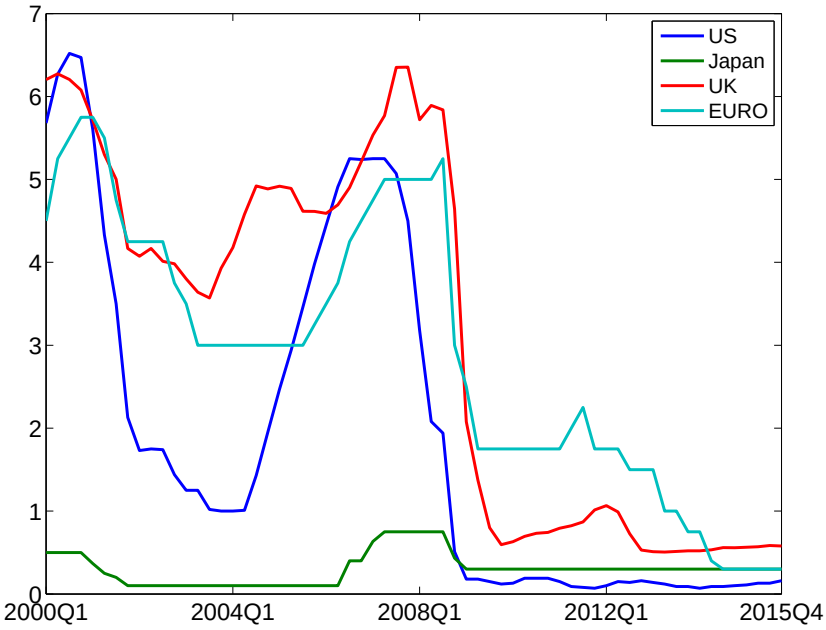


Figure 2: Expected ELB duration in the US

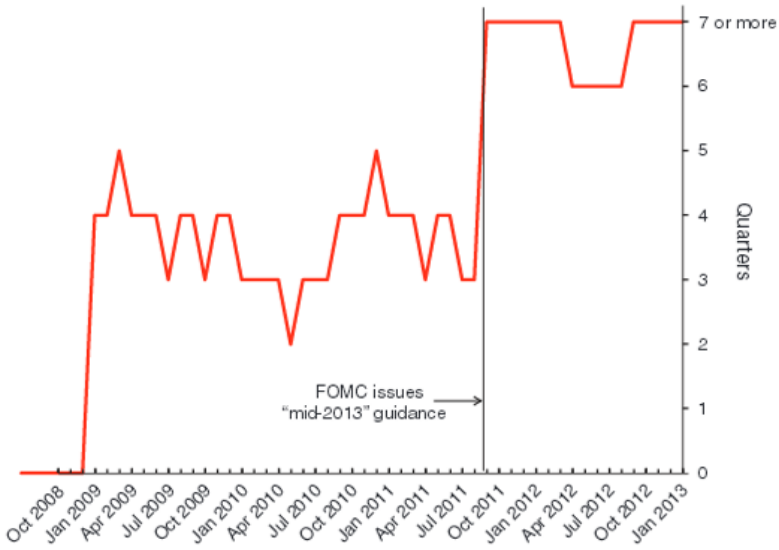


FIGURE 4. EXPECTED NUMBER OF QUARTERS
UNTIL FIRST FEDERAL FUNDS RATE INCREASE TO 25 BP OR HIGHER

Source: Blue Chip survey of forecasters. Data are top-coded at "7 or more" quarters due to the

2.2. Regime-Switching and Occasionally Binding Constraints in General

More generally, the properties and assumptions behind regime-switching solutions lend themselves well to occasionally binding constraints. We list some of these properties and assumptions. First, an approximate solution to a regime-switching DSGE model is consistent with and corresponds to the original underlying nonlinear regime-switching DSGE model. This is unlike piecewise linear solutions, which add the constraint after the model has been linearized. Such an approach is not a direct approximation of the nonlinear model with the constraint imposed. Second, solving regime-switching DSGE models using perturbation methods is fast and easily handles large models and higher order approximations. Moreover, there are computationally efficient filters for filtering and estimating regime-switching models at first and higher orders of approximation (see Alstadheim et al. (2013) and Binning & Maih (2015)). Third, agents in the model are aware of the existence of the constraint at all points in time and react accordingly. In the piecewise linear and extended path frameworks, agents are unaware of the constraint until it actually binds.[4] When the constraint no longer binds, they continue as if the constraint never existed in the first place.[5] This reduced form treatment of the problem, i.e. the constraint not entering the expectations formation process of the agents in normal times, is inconsistent with a rational expectations solution. Finally, there are parameters associated with regime-switching, the transition probabilities (endogenous and exogenous), that can be interpreted as policy parameters. This is particularly appealing when considering the implications of the Lucas critique. Many other occasionally binding constraints such as loan-to-value ratios and collateral constraints also have a policy component to them, as they are often a function of government regulation or policy.

2.3. Regime-Switching Methodology

The regime-switching methodology we employ begins with a general representation of a nonlinear DSGE model. This is augmented with regime-switching parameters and transition probabilities that govern the transition from the current regime to the next. We solve this nonlinear DSGE model using perturbation methods following Maih (2015). Although the nonlinear regime-switching DSGE model can be solved using other methods, like global and projection methods, we opt for perturbation methods because they are fast and can easily be applied to large models (and used to solve higher orders of approximation). The nonlinear

[4]The extended path algorithm is deterministic, meaning expectations do not play a role. The perfect foresight nature of the algorithm takes care of the constraint binding at present or at some point in the future, although the risk of the constraint binding due to unforseen shocks is not captured by the algorithm. As a consequence, agents' behavior will only change when the constraint actually binds.

[5]Agents are also aware of the existence of occasionally binding constraints when the model is solved using global/projection methods. However, these methods are computationally expensive, limiting their applicability to small models only. Moreover global/projection methods address the mathematical problem of imposing occasionally binding constraints, but they do not address the underlying economics of the problem.

regime-switching DSGE models we consider take the form:

$$E_t \sum_{r_{t+1}=1}^{h} p_{r_t,r_{t+1}}\left(\mathcal{I}_t\right) f_{r_t}\left(x_{t+1}\left(r_{t+1}\right), x_t\left(r_t\right), x_{t-1}, \theta_{r_t}, \theta_{r_{t+1}}, \eta_t\right) = 0, \tag{1}$$

where f_{r_t} is the equations of the model, x_t is the date t endogenous variables, r_t represents the switching process with h states, θ_{r_t} is the parameters in state r_t, $\theta_{r_{t+1}}$ is the parameters in the following period, $p_{r_t,r_{t+1}}\left(\mathcal{I}_t\right)$ is the transition probability for going from state r_t to state r_{t+1}, which depends on $\mathcal{I}_t$, the information at time t and η_t are the shocks. We note that the constant-parameter model is a special case when $h = 1$. The regime-switching DSGE model (equation (1)) is solved using the solution algorithms of Maih (2015), which gives the general solution:

$$x_t = T_{r_t}\left(x_{t-1}, \eta_t\right), \qquad p_{r_t,r_{t+1}} = Q_{r_t,r_{t+1}}\left(\mathcal{I}_t\right), \tag{2}$$

where T_{r_t} is the approximated and possibly nonlinear policy functions of the model. Agents in the model are always aware of the other regimes, regardless of which regime they are currently in. It is through the expectations channel that the dynamics of future regimes impact behavior in the current regime. This means, in our context, that agents will be aware of the lower bound regime at all times and that their behavior in the normal regime will be impacted by the presence of the lower bound constraint on interest rates. Conversely, the expected future dynamics of the normal regime will have an impact on the behavior of agents while they are in the lower bound regime.[6]

We assume the economy consists of two regimes, a normal interest rate regime and an effective lower bound regime, where interest rates are constant at a lower level.[7] Moreover, we assume each regime is characterized by a separate steady state. The effective lower bound regime can arise under several different circumstances. To illustrate, we set the time subscripts from the consumption Euler equation, in a standard DSGE model, to the current period to obtain the balanced growth interest rate:

$$R_t = \frac{\Pi_t\left(1+g\right)}{d_t \beta}, \tag{3}$$

where R_t is the gross nominal interest rate, Π_t is the gross inflation rate, g is the productivity growth rate and β is the rate of time preference. We also include d_t, which is a consumption preference shifter. Following Binning & Maih (2016) there are four different ways to describe an ELB state: i) a deflationary state (a shift down in Π_t as considered by Aruoba et al. (2013) and Gavin et al. (2015)), ii) a low (negative) growth state (a shift down in g), iii) a precautionary savings state (a shift up in d_t as considered by Christiano et al. (2011) and Braun et al. (2015)) and iv) some combination of the listed factors. Binning & Maih (2016)

[6]This is a channel that forward guidance can work through.

[7]This could be at zero (in net nominal terms), or it could be slightly above or below zero depending on the properties of the economy in question.

find that a precautionary savings state fits the US experience best at the ELB. For these reasons we use this characterization of the steady state for our ELB regime.

The transition matrix for the normal and low interest rate regimes is defined as follows:

$$Q_{r_t,r_{t+1}} = \begin{bmatrix} 1 - p_{N,L} & p_{N,L} \\ p_{L,N} & 1 - p_{L,N} \end{bmatrix},$$

where $p_{N,L}$ is the probability of transitioning from the normal regime to the lower bound regime and $p_{L,N}$ is the probability of transitioning from the lower bound regime to the normal regime. These parameters will also determine the expected duration of each regime.

2.3.1. Endogenous Transition Probabilities

We consider solving regime-switching DSGE models with endogenous transition probabilities in addition to constant transition probabilities. In principle, there are many functional forms that can be used to model endogenous transition probabilities. We restrict our attention to the class of logistic functions. We choose the sigmoid points to determine when the economy enters the lower bound regime and when the economy exits the lower bound regime. The transition matrix for the regimes now takes the form:

$$Q_{r_t,r_{t+1}} = \begin{bmatrix} 1 - p_{N_t,L_{t+1}} & p_{N_t,L_{t+1}} \\ p_{L_t,N_{t+1}} & 1 - p_{L_t,N_{t+1}} \end{bmatrix}.$$

The transition probabilities are described as follows. The probability for entering the lower bound regime is given by:

$$p_{N_t,L_{t+1}} = \frac{1}{1 + \exp\left(\kappa_1 \left(R_t^* - R_I\right)\right)},$$

The probability for exiting the lower bound regime is given by:

$$p_{L_t,N_{t+1}} = \frac{1}{1 + \exp\left(-\kappa_2 \left(R_t^* - R_O\right)\right)},$$

where R_t^* is the Taylor rule/shadow interest rate, $p_{N_t,L_{t+1}}$ is the probability of transitioning from the normal regime to the low interest rate regime, $p_{L_t,N_{t+1}}$ is the probability of transitioning from the low interest rate regime to the normal regime, κ_1 is the steepness parameter for the transition probability function for entering the lower bound regime, κ_2 is the steepness parameter for the transition probability function for exiting the lower bound regime, R_I is the interest rate threshold for entering the effective lower bound regime and R_O is the interest rate threshold for exiting the effective lower bound regime. The parameters κ_1, κ_2, R_1 and R_2 are all policy parameters. They govern when the economy enters the lower bound and when it exits the lower bound, behavior determined by monetary policy. We set $\kappa_1 = \kappa_2 = 2000$, to ensure steep logistic functions and to effectively remove the possibility

of interest rates going below their lower bound.[8] We set $R_I = R_O = 1$, where 1 is the lower bound on the gross interest rate.

3. The Model and Baseline Parametrization

We take the model from Binning & Maih (2016) as our laboratory model. It is a simple canonical New Keynesian DSGE model. The economy is closed to the rest of the world and consists of a representative household, firms and a monetary authority. The representative household receives utility from consumption and disutility from working. The household supplies labor to firms and receives labor income in return. Firms produce a differentiated product using labor and a common technology. Firms maximize profits by choosing labor inputs and prices subject to a quadratic adjustment cost on changing prices. The representative household owns firms and is paid dividends by firms. The monetary authority sets interest rates according to a Taylor-type rule when the effective lower bound on interest rates does not bind. The full set of model equations can be found in Appendix A.

Our model calibration is based on the estimation results from Binning & Maih (2016). Their paper features a regime-switching version of the model where policy can switch between a normal regime and a lower bound regime and the shock standard deviations can shift between a high volatility and low volatility regime. The model is estimated on US data, using per capita real GDP growth, GDP deflator inflation and interest rates between 1985Q1 and 2015Q2. Bayesian methods are employed, following Alstadheim et al. (2013). Our calibration is based on the normal policy and low volatility regime, which is the dominant regime as determined by the estimation procedure. Our only major deviation from the estimated parameter values in this regime is the interest rate smoothing parameter, which we set to 0.85 in our baseline calibration. We also set the lower bound on interest rates to be 1 in gross quarterly terms (0 in net quarterly terms). The parameter values for the baseline calibration are presented in Table 1 and a full description of the parameters can be found in Table A.3 in Appendix A.

[8] A steep logistic function means the chances of switching to the lower bound regime when the economy is far away from the lower bound are almost zero. It is only when the economy is in the very near vicinity of the lower bound that there is any real chance of switching to that regime.

Table 1: Baseline Calibration of the Model

Parameter	Calibration	Parameter	Calibration
β	0.9982	ξ	0.4249
χ	0.5699	ρ_r	0.85
σ	2.2207	ρ_{AR}	0
ε	6	ρ_A	0.7696
η	0.8609	σ_R	0.0038
ϕ	16.4945	σ_A	0.0269
κ_π	2.7186	σ_Z	0.0071
κ_y	0.0909	σ_π	0.0047
g_Z	0.0041	$p_{N,L}$	0.0398
$\bar{\Pi}$	1.0057	$p_{L,N}$	0.2170

4. Regime-Switching Evidence

In this section, we carry out some simulation experiments using the regime-switching framework to model the ELB. We then test, along certain dimensions, how sensitive our baseline results are to alternative assumptions, including alternative parameterizations, higher orders of approximation and endogenous transition probabilities.

4.1. Simulation Experiments

We start our simulation experiments by choosing a historical period in the US when interest rates were near the effective lower bound and treating this as the end of history (the initial conditions for our simulations). We assume the economy begins in the normal regime and is hit by a sequence of four large negative consumption preference shocks. The large negative consumption preference shocks are chosen so that they bring the economy to the effective lower bound. When the simulation violates the lower bound on interest rates, we assume the economy switches to the lower bound regime. When the shadow rate of interest returns to a level above the lower bound, we assume the economy switches back to the normal regime. We choose 2008Q3 to be the end of history/initial condition for our simulations as this was the period before the US economy hit the lower bound. Density forecasts are generated by perturbing the sequence of negative consumption preference shocks with sequences of randomly drawn i.i.d. consumption preference shocks, in addition to drawing random sequences for all other shocks. To construct the densities, we draw 2000 such random shock sequences, fixing the model calibration for all simulations.

4.2. Density Forecasts From the Baseline Model

The baseline simulations are performed using the regime-switching DSGE model with constant transition probabilities under the baseline calibration (see Table 1).[9] The results from the baseline simulation are presented in Figure 3.

[9]The size of the four consumption shocks in the central scenario is set to 3 standard deviations. The shock sizes are chosen to find the smallest sequence of shocks that brings interest rates in the central scenario comfortably to the lower bound.

Figure 3: Regime-Switching Baseline: First-order Approximation

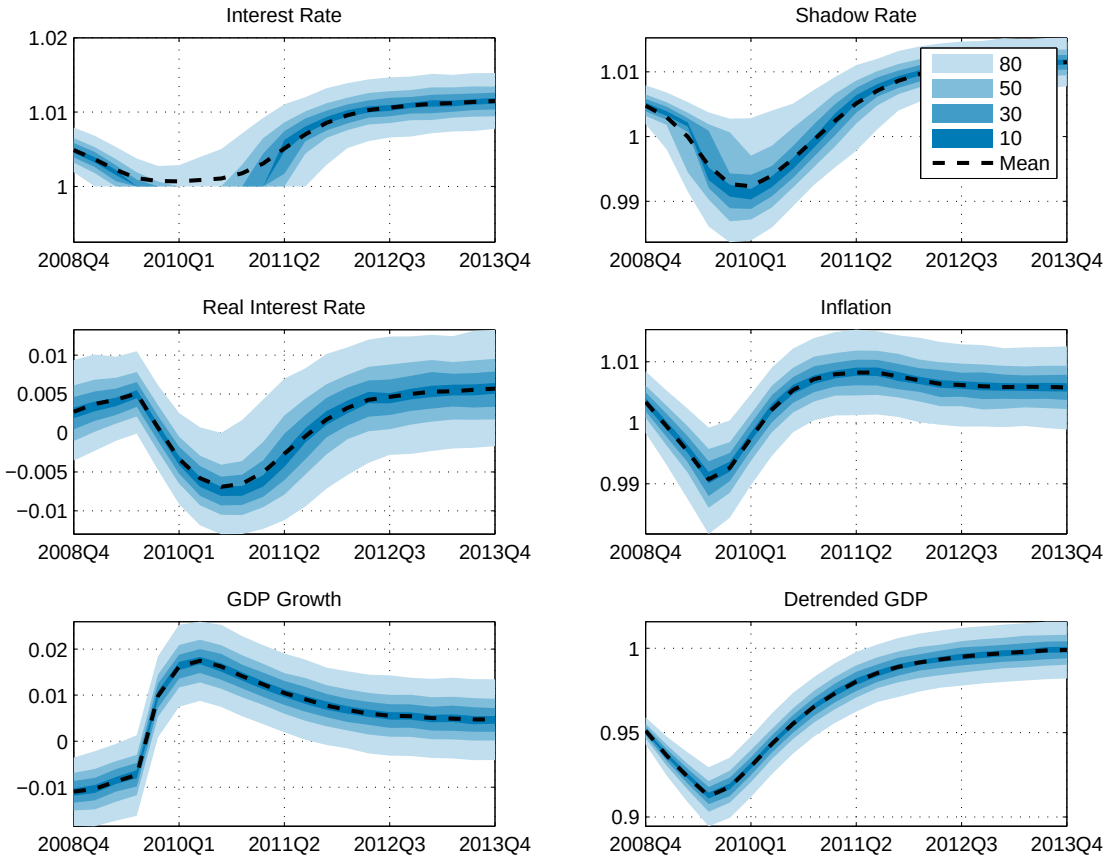


Figure Notes:

Figure Title	Model Variable
Interest Rate	R_t
Shadow Rate	R_t^*
Real Interest Rate	$E_t\left\{\frac{R_t}{\Pi_{t+1}}\right\}-1$
Inflation	Π_t
GDP Growth	$\Delta \log Y_t$
Detrended GDP	$\tilde{Y}_t$

The interest rate, the shadow rate and inflation are all in gross quarterly terms. The real interest rate and GDP growth are in net quarterly terms.

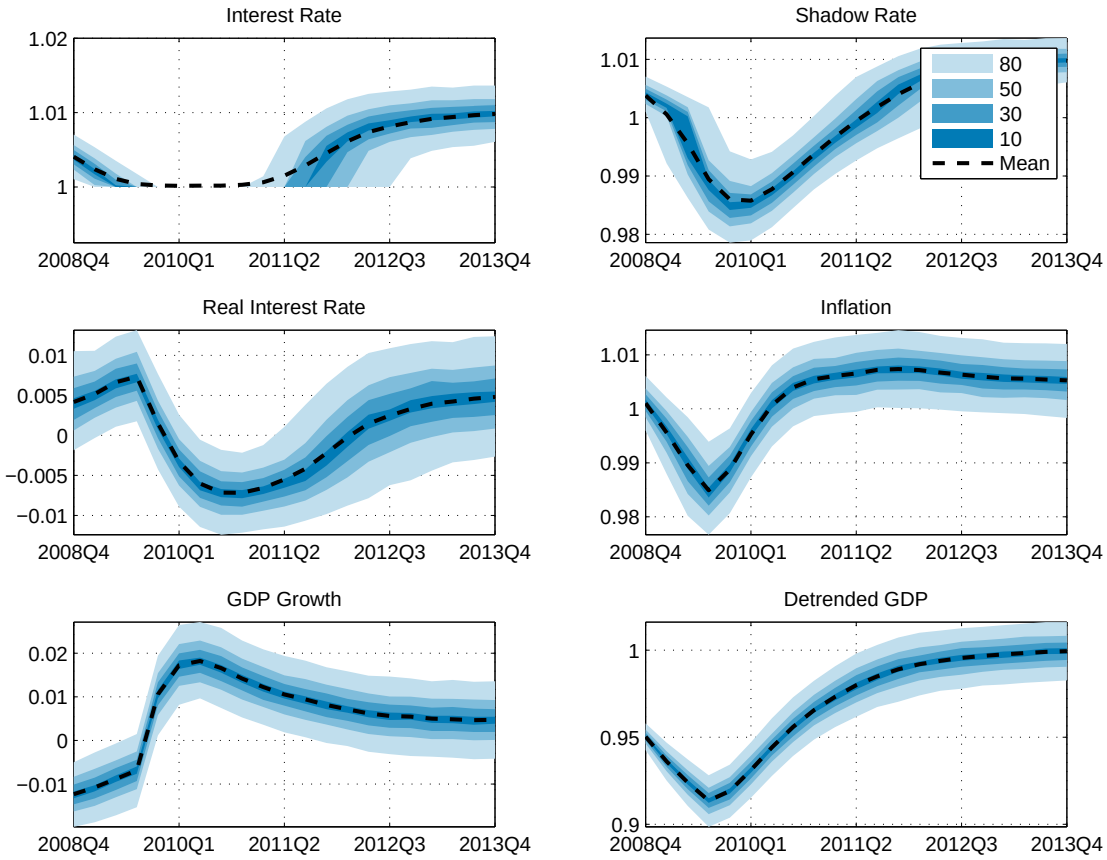
We note that while the density forecasts for the nominal interest rate, the shadow rate and the real interest rate all show varying degrees of asymmetry, very little of that asymmetry is translated into asymmetry in inflation, GDP growth or detrended GDP. This is a striking result and contrasts with results presented by Coenen & Warne (2014), Haberis et al. (2015) and Eusepi et al. (2016), who show asymmetric density forecasts when the interest rate hits the lower bound. We investigate further the conditions under which the baseline results hold true.

4.3. Higher Orders of Approximation

The first-order approximation of the regime-switching DSGE model is conditionally linear. This means that each regime, in isolation, should be linear-gaussian.[10] We investigate whether taking a nonlinear approximation of the regime-switching DSGE model results in any additional asymmetry.

The simulation results for the second-order approximation of the regime-switching model are presented in Figure 4 while the simulation results for the third-order approximation are presented in Figure C.17.

Figure 4: Regime-Switching: Second-order Approximation



The results at second and third order are very similar to the results at first order. In fact there may be less asymmetry when a second- or third-order approximation is used.[11]

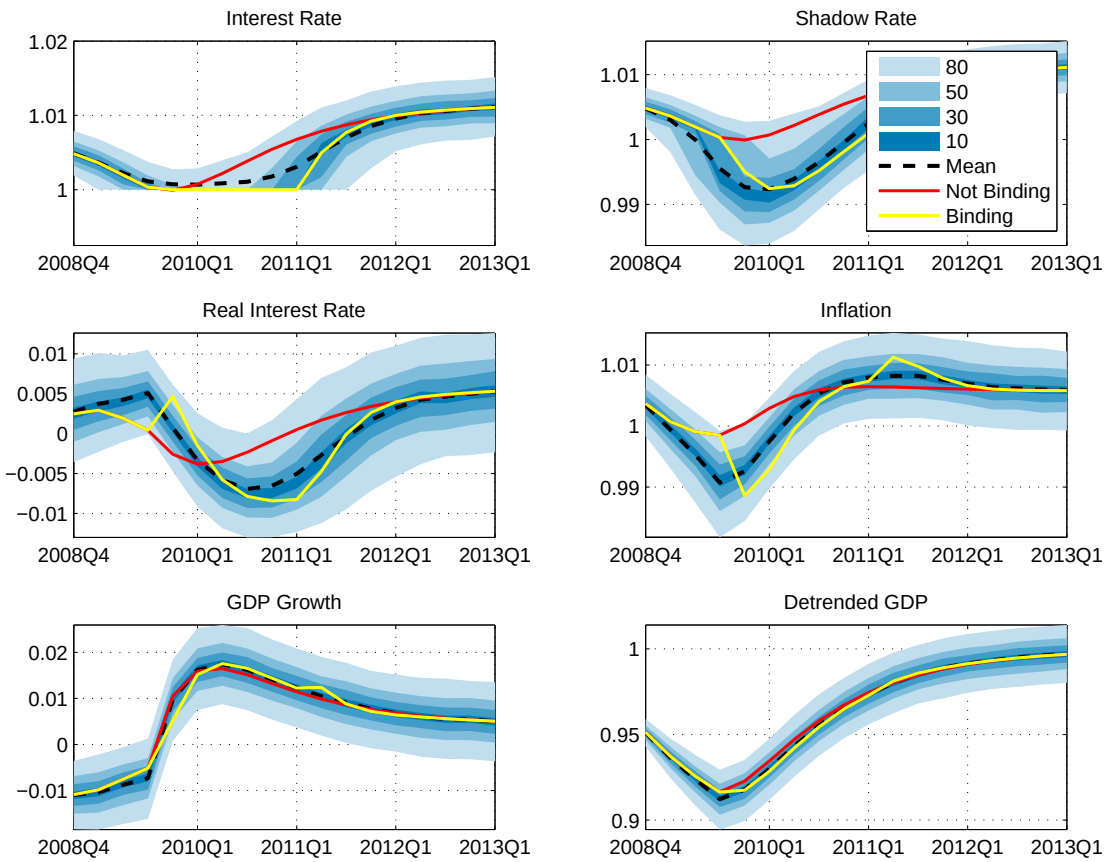
[10] Although one could consider non-gaussian shocks as well.

[11] This may be due to the consumption shocks having a larger impact on the interest rate. The consumption preference shocks in the central scenario are the same size as those used in the baseline case. However, the nonlinear solution methods endow the model with different properties. More specifically, the stochastic steady state has shifted and the model now exhibits different levels of persistence, so that the same size

4.4. Understanding the Low Degree of Asymmetry

To understand the low degree of asymmetry, we perform the same simulation in the baseline exercise, this time without perturbing the consumption preference shocks and without using random shocks for the other shock series. We also repeat the same exercise without imposing the lower bound on interest rates. These simulations are then plotted on top of the density forecasts from the baseline simulation (Figure 3) in Figure 5.

Figure 5: Regime-Switching: Comparison



The first thing we note is that the difference between the lower bound binding and not binding is quite dramatic (the difference in the yellow and red lines). When the lower bound binds the fall in inflation and the jump in real interest rates is quite sharp. This is because the Taylor rule plays an important role in stabilizing the economy. The second thing we note is the difference between the mean of the density forecasts and the simulation without the random noise (the yellow line). These are in fact quite different for all the interest rate and inflation series but similar for the GDP series. This is because the model is nonlinear in

shocks used in the baseline scenario do not have the same impact in these simulations.

nature. However, as we have observed previously, the degree of asymmetry in inflation, GDP growth and detrended GDP is quite low. We note that the degree of interest rate smoothing while plausible and consistent with other studies is at the higher end of the range. In fact, if we increased the degree of interest rate smoothing to a number close to one, the normal regime would involve an interest rate that is constant or near constant. Shifting between two conditionally linear-gaussian regimes should then not generate any asymmetry in any of the variables. To drive this point home, we investigate the role interest rate smoothing, along with some other parameters, plays in determining the shape of the density forecasts.

4.5. Sensitivity Analysis

In this section, we look at the sensitivity of our results to some key parameters in the model. More specifically, we look at the role interest rate smoothing plays in the Taylor-type rule used to set policy in normal times as well as the degree of price stickiness. Our sensitivity analysis is by no means exhaustive, there are other parameters, such as the transition probabilities, and conditions, such as the initial conditions, that can also affect the degree of asymmetry in the density forecasts (see Binning et al., 2016, for example).

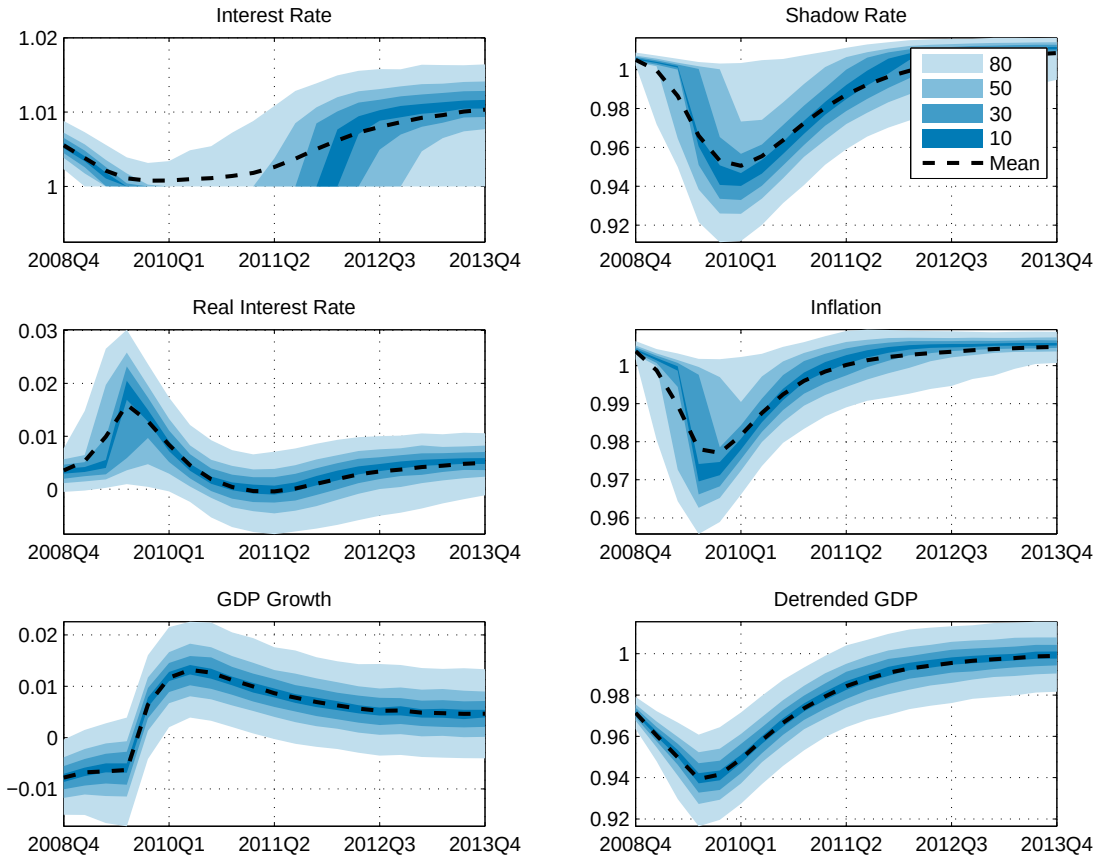
4.5.1. Less Interest Rate Smoothing

We investigate how the results change when we lower the degree of interest rate smoothing. We repeat the exercise performed in the baseline simulation, this time with the degree of interest rate smoothing set at 0.6.[12] The simulation results are presented in Figure 6.[13]

[12] In this simulation, we need to set the size of the consumption shocks equal to 1.9 standard deviations in the central scenario. Smaller consumption preference shocks are used because it is easier to bring the economy to the lower bound when the degree of interest rate smoothing is smaller.

[13] The results for the same simulations at second and third order are plotted in Figures D.18 and D.19 in Appendix D.

Figure 6: Regime-Switching: First-order Approximation with $\rho_r = 0.6$



Reducing the degree of interest rate smoothing results in noticeable asymmetry in interest rates, the shadow rate and the real interest rate, as we observed in the baseline calibration. However, in this scenario we observe sizeable asymmetry in inflation as well, which we did not observe at the higher value of interest rate smoothing. GDP growth and detrended GDP remain relatively symmetric.[14] This supports our hypothesis that interest rate smoothing plays an important role in determining the degree of asymmetry in the forecast densities one would expect at the lower bound.[15]

[14] In this simple model, the path for GDP is pinned down by the IS curve, the Taylor-type rule and the NK Phillips curve. Agents' response to interest rate movements, which remain constant across the regimes, are dampened by a low intertemporal elasticity of substitution and habit formation. This results in very little asymmetry in GDP. This result is model-specific and a not necessarily a property of DSGE models in general.

[15] We note that the asymmetry in inflation is to the upside (the mean of inflation is higher than the median during the peak of the downturn). This is not necessarily a general property of this calibration; alternative initial conditions and alternative shock sizes could easily result in increased downside risk.

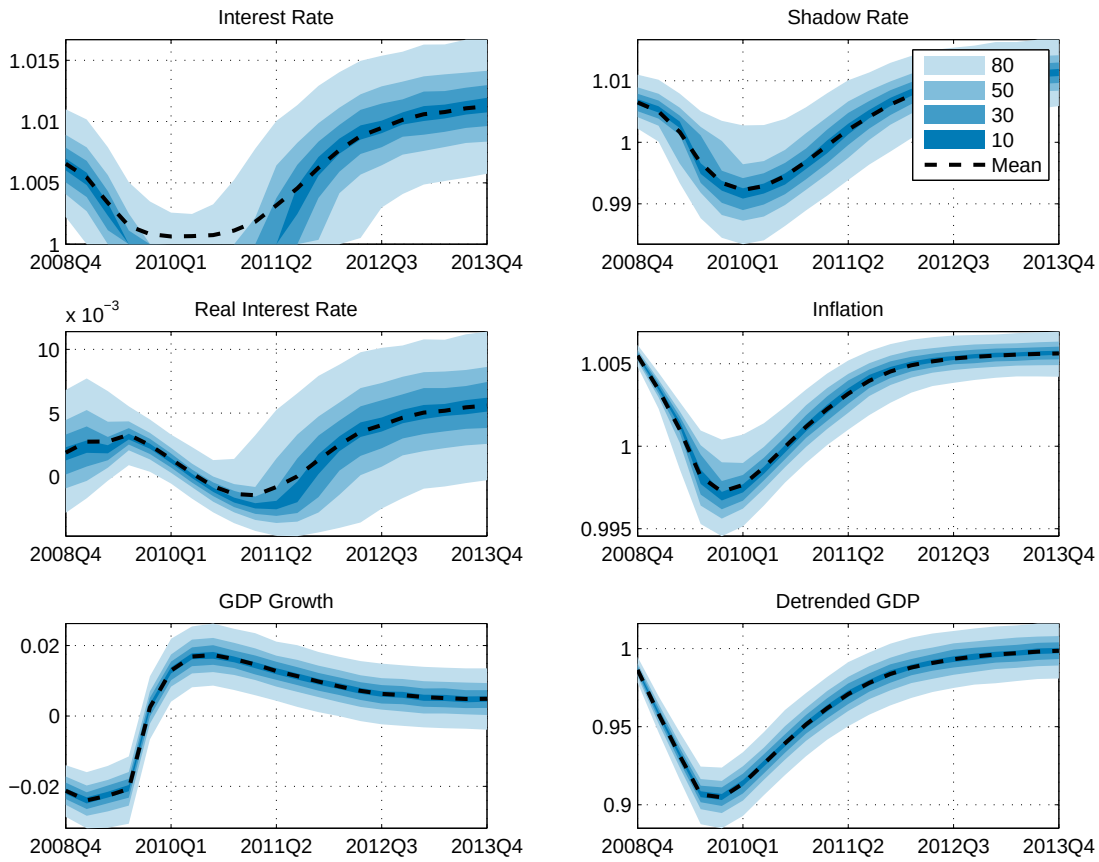
4.5.2. Less Interest Rate Smoothing and More Price Rigidity

We also investigate the role price rigidity plays in affecting the degree of asymmetry in the density forecasts. In recent work, Lindé et al. (2016) has shown that increased price rigidity when imposing the lower bound on interest rates, using anticipated monetary policy shocks, results in less asymmetry and, more importantly, it helps remove sign reversals. Sign reversals are a phenomenon that occurs when anticipated monetary policy shocks are used to enforce the lower bound constraint. In the face of large negative demand shocks, the ELB is generally seen as a contractionary monetary policy since the interest rate is not able to decrease any further in order to give a boost to the economy. In that case, the anticipated shocks required to keep the interest rate from going below its lower bound are also expected to be positive. The positive monetary policy shocks then act as contractionary policy shocks. In some cases, however, if the ELB is expected to last for a very long time, some of the shocks in the sequence of shocks required to keep the interest rate at the ELB may be negative. Negative monetary policy shocks are expansionary, which leads to an improvement of economic conditions at the ELB. Lindé et al. (2016) refer to this as sign reversals; instead of being contractionary, monetary policy shocks can become expansionary at the ELB. This may also lead to asymmetry on the upside and could manifest in different methods in different ways.

We repeat the simulation exercise with the interest rate smoothing parameter set at 0.6 to ensure that we are able to generate asymmetry in inflation. We also raise the Rotemberg adjustment cost to 200, ensuring a high degree of price rigidity.[16] The results of the simulation are presented in Figure 7.

[16] We need to increase the size of the consumption shocks in the central scenario to 4 standard deviations in order to bring the economy down to the lower bound.

Figure 7: Regime-Switching: First-order Approximation with $\rho_r = 0.6$, $\phi = 200$

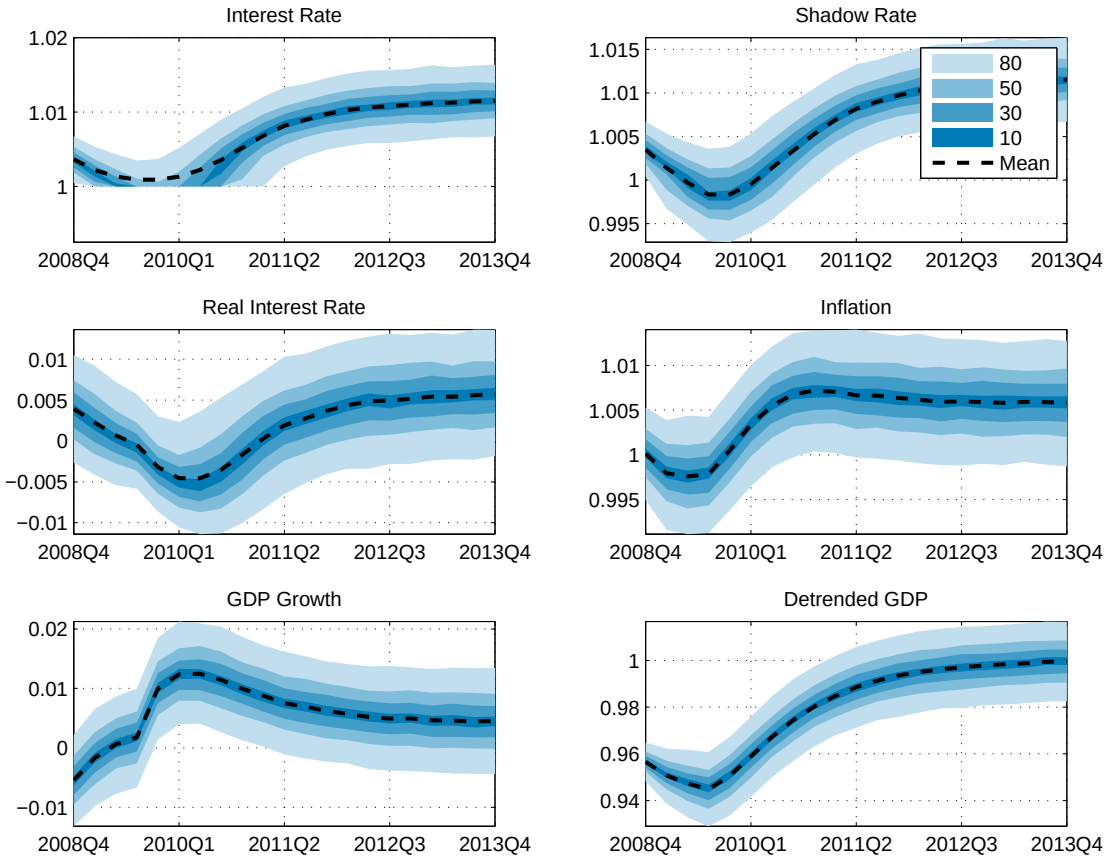


While we still see quite a lot of asymmetry in nominal interest rates, the shadow rate and the real interest rate, the degree of asymmetry in inflation has been significantly diminished by increasing the degree of price rigidity. The increased price rigidity means that it is harder to move prices and as a result a lot of the upside/downside risk is removed from the simulation. This is in line with results from Lindé et al. (2016).

4.6. Endogenous Switching

We repeat the simulations, this time using endogenous transition probabilities as described in Section 2.3.1. The results for the simulation with ρ_r set at 0.85 are presented in Figure 8.

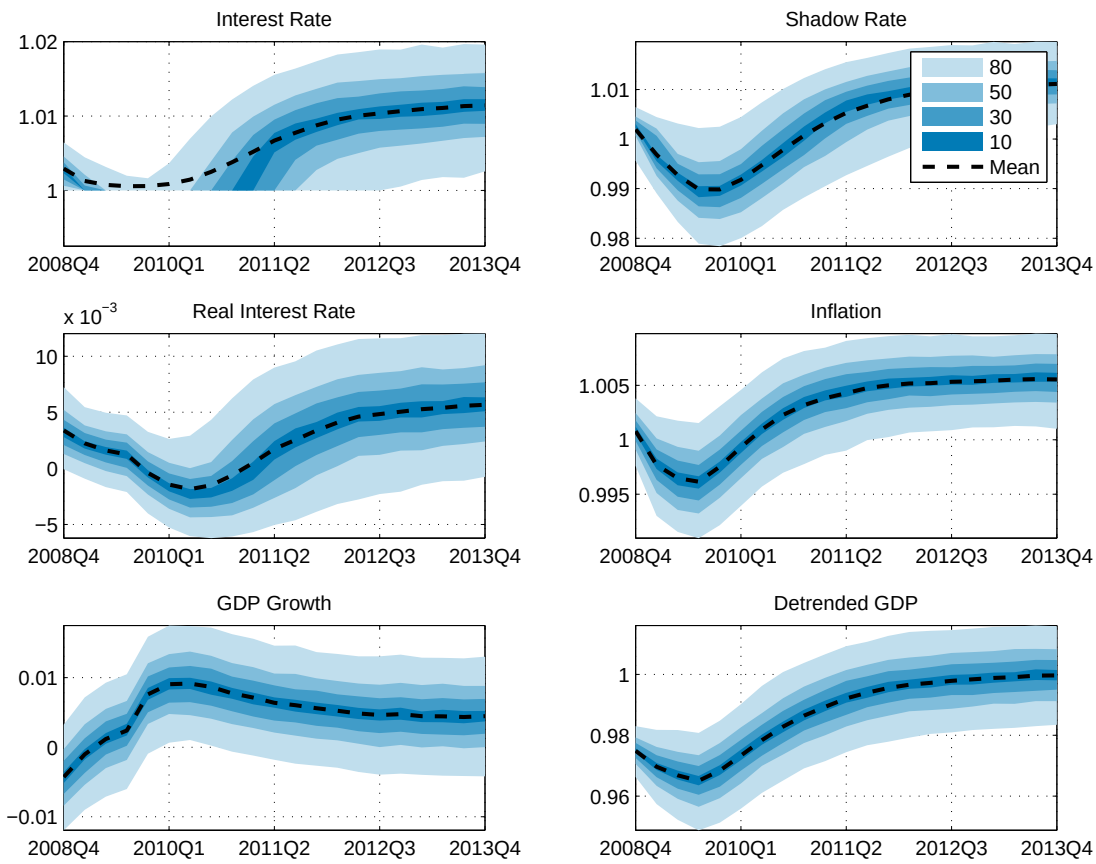
Figure 8: Endogenous Switching: First-order with $\rho_r = 0.85$



Despite the asymmetry present in the nominal interest rate, we see very little asymmetry in the shadow rate or real interest rates. We see no asymmetry in inflation, GDP growth or detrended GDP. We repeat the exercise with ρ_r set at 0.6.[17] The simulation results are presented in Figure 9.

[17] We use consumption shocks that are 1.9 standard deviations to bring the economy to the ELB.

Figure 9: Endogenous Switching: First-order with $\rho_r = 0.6$



In this simulation, we see quite a bit of asymmetry in the nominal interest rate. More of this asymmetry is translated into the shadow rate and real interest rates when ρ_r is equal to 0.6 than when it is equal to 0.85. However very little of this asymmetry is translated into inflation, GDP growth or detrended GDP.

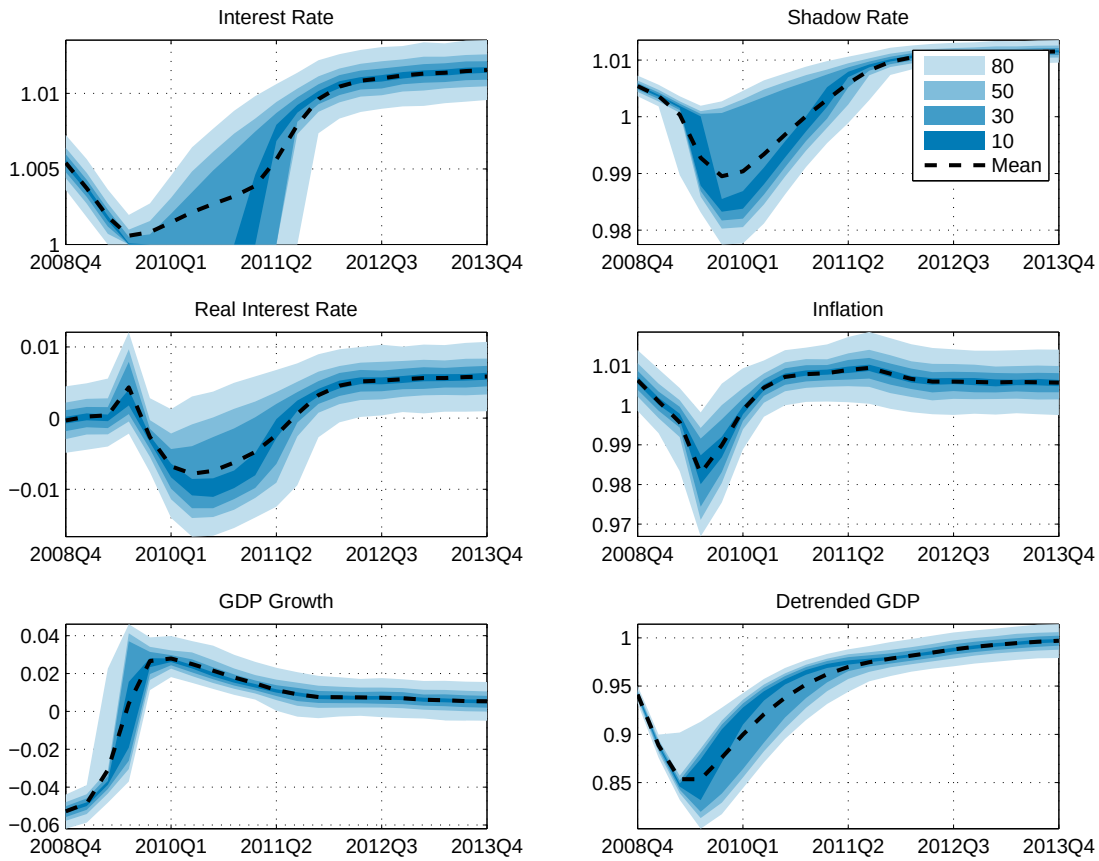
4.7. Engineering Asymmetry at the ELB

From the baseline simulation scenario we have learned that high degrees of interest rate smoothing result in a relatively low degree of asymmetry in inflation and GDP. In the baseline case we assumed that the only parameter that switched when we were at the effective lower bound was the preference shifter term (d_t in equation 3). However, it is likely that the transmission mechanism for monetary policy could also change, at or near the effective lower bound. We perform a simulation experiment where we allow the inverse of the intertemporal elasticity of substitution (σ) to switch from 2.2207 in normal times to 10 when at the lower bound, in addition to the preference shifter switching.[18] This would imply that agents

[18]In order for the simulations to reach the lower bound, we need to increase the size of the consumption shocks to 8 standard deviations.

become less responsive to interest rates in the ELB regime. The results from this experiment are plotted in Figure 10.

Figure 10: Endogenous Switching: First-order with $\rho_r = 0.85$, $\sigma(N) = 2.2207$, $\sigma(L) = 10$



Comparing the results with the baseline case (Figure 3) where the intertemporal elasticity of substitution does not switch, we see that there is much more asymmetry in GDP growth and detrended GDP. This is because the transmission mechanism at the lower bound has changed, agents' behavior has changed at the lower bound because they are much less sensitive to changes in the interest rate.[19]

[19] While each regime in isolation is linear-gaussian and symmetrically distributed, the difference in distributions, due to the change in the transmission mechanism, is sufficient to result in the weighted average of the distributions being asymmetric. In the baseline case, the transmission mechanism remained the same between the regimes so that the distributions for each regime were much more similar, resulting in minimal asymmetry. In this case, the switch in the inverse of the intertemporal elasticity of substitution changes agents' behavioral rules between regimes and makes the IS curve nonlinear.

4.8. Summary

We summarize the results from imposing the lower bound constraint using regime-switching as follows: first, the degree of interest rate smoothing in the Taylor-type rule plays an important role in determining the shape and width of the density forecasts when the economy is in the vicinity of the lower bound. More specifically, the fan charts are generally wider (more uncertainty) and exhibit more asymmetry (more up/down side risk) when the degree of interest rate smoothing is reduced. Second, increasing the degree of price rigidity decreases the degree of asymmetry in the density forecasts for inflation. Third and finally, we can engineer asymmetry by changing the transmission mechanism at the lower bound. This can be done by changing so-called deep parameters in the ELB regime in addition to the policy parameters. In the next section, we look at some alternative methods for imposing the lower bound constraint on interest rates.

5. Alternative Methods

Having investigated the properties of density forecasts produced using regime-switching to enforce the lower bound on interest rates, we now focus our attention on the properties of density forecasts when alternative methods are used to impose the ELB constraint. In particular, we look at the density forecasts when the following methods are used: a method of adding anticipated monetary policy shocks, piecewise linear solutions and the extended path algorithm. Global/projection methods can also be used to implement the lower bound constraint although we do not consider them here because they are computationally expensive and generally used with small models.

5.1. Anticipated Monetary Policy Shocks

Holden & Paetz (2012) and Lindé et al. (2016) use shocks to impose the lower bound constraint on interest rates. In the case of Holden & Paetz (2012) a sequence of news shocks, referred to as "shadow price shocks", are found that prevent interest rates going below the lower bound. We apply the method of Lindé et al. (2016), which involves adding anticipated monetary policy shocks to the Taylor rule to prevent interest rates from violating the lower bound constraint. The duration of the time spent at the lower bound is calculated using a shooting algorithm.

To implement the algorithm, the linearized DSGE model is solved for anticipated shocks for a given horizon. This can be done in the same way as Beneš et al. (2008), Maih (2010), Laséen & Svensson (2011) and Leeper et al. (2013). The solved system of equations then has the VARMA representation:[20]

$$x_t = A(\theta)\, x_{t-1} + \sum_{j=0}^{n} B_j(\theta)\, Z \eta_{t+j}, \tag{4}$$

[20] Maih (2015) generalizes this solution to higher orders of approximation.

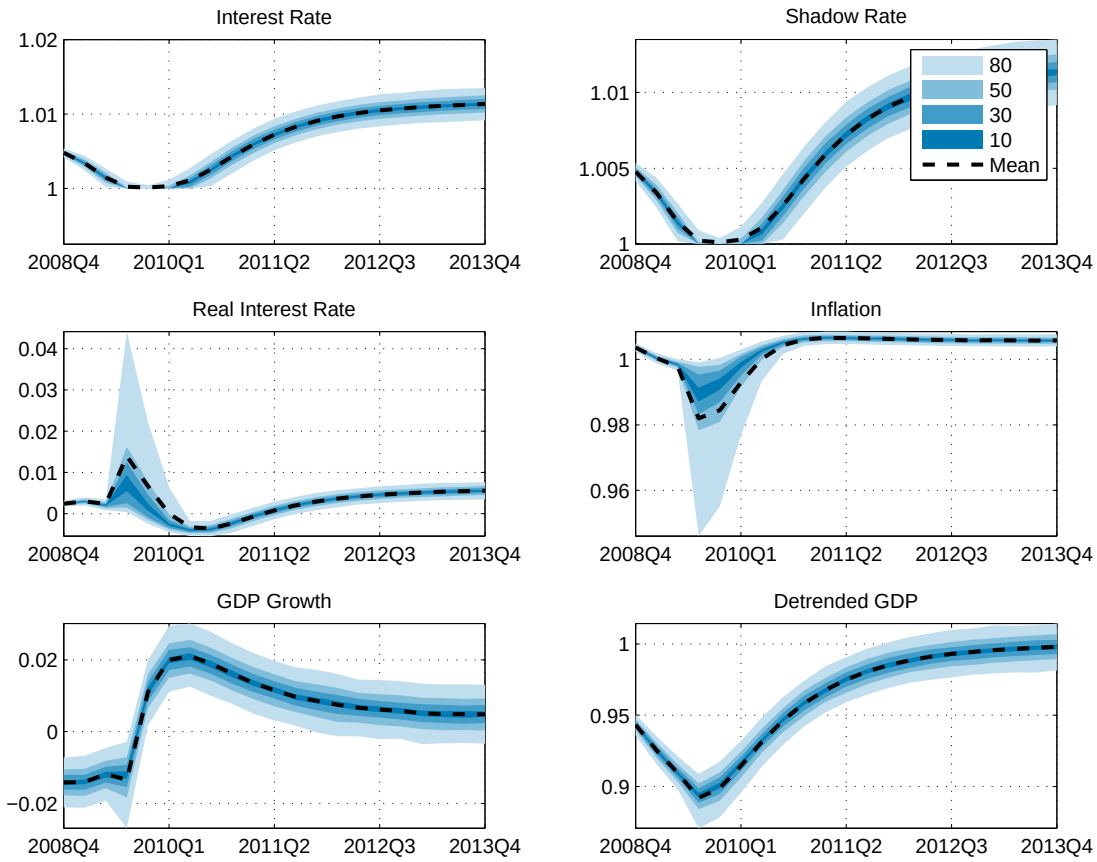
where x_t is a vector of endogenous variables, $A(\theta)$ is a system matrix, $B_j(\theta)$ is a system matrix for the contemporary and anticipated shocks, η_{t+j} are the (anticipated) shocks in period $t+j$, n is the anticipation horizon and Z is a selection matrix that selects the monetary policy shock.

The algorithm proceeds as follows: the model is simulated forward through time, although monetary policy shocks are not used in the simulation. When the lower bound constraint on interest rates is violated, an anticipated monetary policy shock is added at that point in time to prevent interest rates going below the lower bound. The model is then simulated forward for an additional period. If the constraint is not violated in this period, the simulation continues as normal and no further monetary policy shocks are added. If the lower bound constraint is violated in this period, we return to the first period and find a sequence of two monetary policy shocks, such that interest rates do not violate the lower bound constraint. We continue in such a fashion until the constraint no longer binds, or we are unable to find a sequence of shocks equal to or shorter than the anticipation horizon, in which case there is no solution.

The simulation results for the algorithm when the anticipation horizon is set at three quarters and ρ_r is set at 0.85 are presented in Figure 11.[21]

[21] The consumption shocks in the central scenario are set to be 3 times their standard deviation.

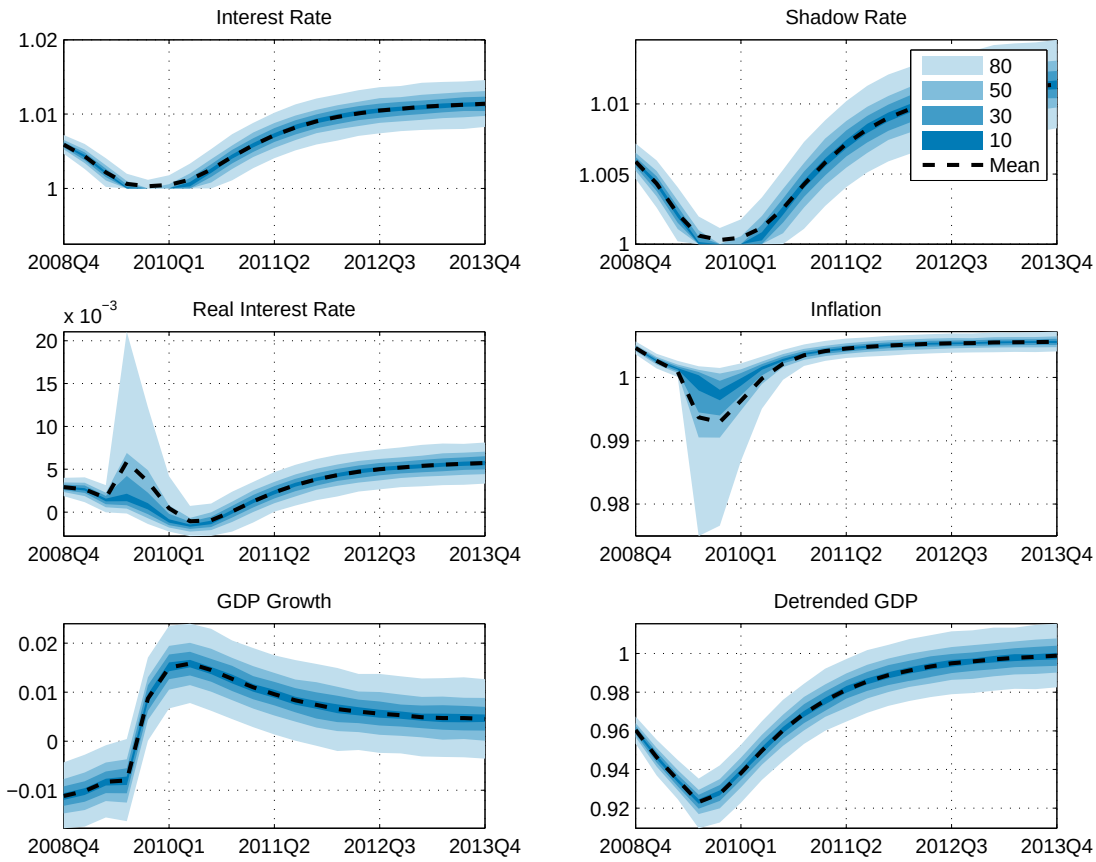
Figure 11: Anticipated Monetary Policy Shocks: $\rho_r = 0.85$



In this simulation, we see quite a lot of asymmetry in the real interest rate and inflation. There is also noticeable asymmetry in GDP growth and detrended GDP, something that was much less evident when using regime-switching. More generally, we observe that the shape and width of the density forecasts differ quite a bit from our experience when using regime-switching to impose the lower bound constraint.

We repeat the exercise, this time with the interest rate smoothing parameters set at 0.6. The results are plotted in Figure 12.

Figure 12: Anticipated Monetary Policy Shocks: $\rho_r = 0.6$



Again we note there is considerable asymmetry in the density forecasts for real interest rates and inflation. However, the degree of asymmetry appears to decline when we decrease the degree of interest rate smoothing. This is in contrast to our experience with regime-switching, where a decrease in interest rate smoothing resulted in an increase in the asymmetry of inflation.[22]

We should also note that the asymmetry in these simulations may arise for the wrong reasons. This is because shocks are being added to the model to prevent interest rates going below zero and this changes the shock distributions. Of course if the shocks are no longer gaussian, then it would be natural to assume the density forecasts will no longer be symmetric. The algorithm itself may be prone to sign reversals, which could also introduce asymmetry for spurious reasons. Furthermore, this method is quite sensitive to the shock anticipation horizon. We used 3 quarters, but using more than this makes the results more

[22] Conventional consumption Euler equations as used in this model do not discount future anticipated shocks. As a consequence these shocks can have a very large impact on today's activity. The higher degree of interest rate smoothing prevents the monetary authority from moving rates enough to respond to these shocks and hence results in more instability and asymmetry as these shocks are amplified.

explosive due to the extremely forward looking nature of the consumption Euler equation.[23]

5.2. Piecewise Linear Solution

Kulish et al. (2014) and Guerrieri & Iacoviello (2015a) have imposed the lower bound constraint on interest rates using a piecewise linear solution. Piecewise linear solutions involve linearizing the DSGE model sans the constraint. The constraint is then imposed on the model after it has been linearized. In periods where the constraint does not bind, the linearized model without the constraint is used. In periods where the constraint binds, a period in the not too distant future where the constraint does not bind is chosen and the model is solved backwards to obtain the sequence of policy functions consistent with the constraint binding in the current period. The algorithm we use in this exercise follows closely the one developed by Guerrieri & Iacoviello (2015b).

The piecewise linear solution starts with the set of linearized equations from a nonlinear DSGE model without the occasionally binding constraint:

$$\mathscr{A}_0(\theta) E_t x_{t+1} + \mathscr{A}_1(\theta) x_t + \mathscr{A}_2(\theta) x_{t-1} + \mathscr{A}_3(\theta) \eta_t = 0. \tag{5}$$

Linearizing the system when the constraint binds gives an analogous system of equations

$$\mathscr{A}_0^*(\theta) E_t x_{t+1} + \mathscr{A}_1^*(\theta) x_t + \mathscr{A}_2^*(\theta) x_{t-1} + \mathscr{D}(\theta) + \mathscr{A}_3^*(\theta) \eta_t = 0, \tag{6}$$

where $\mathscr{A}_i$, $\mathscr{A}_i^*$ for $i = 0, 1, 2, 3$ and $\mathscr{D}(\theta)$ are system matrices that are potentially nonlinear functions of the structural parameters of the original model. The term $\mathscr{D}(\theta)$ appears when the constraint binds because the system is linearized around the steady state in normal times. Solving the system backwards for a series of time-varying policy functions gives the following set of policy functions:

$$x_t = P_t(\theta) x_{t-1} + R_t(\theta) + Q_1(\theta) \eta_1 \text{ for t} = 1, \tag{7}$$

$$x_t = P_t(\theta) x_{t-1} + R_t(\theta) \quad \forall t \in \{2, \infty\}, \tag{8}$$

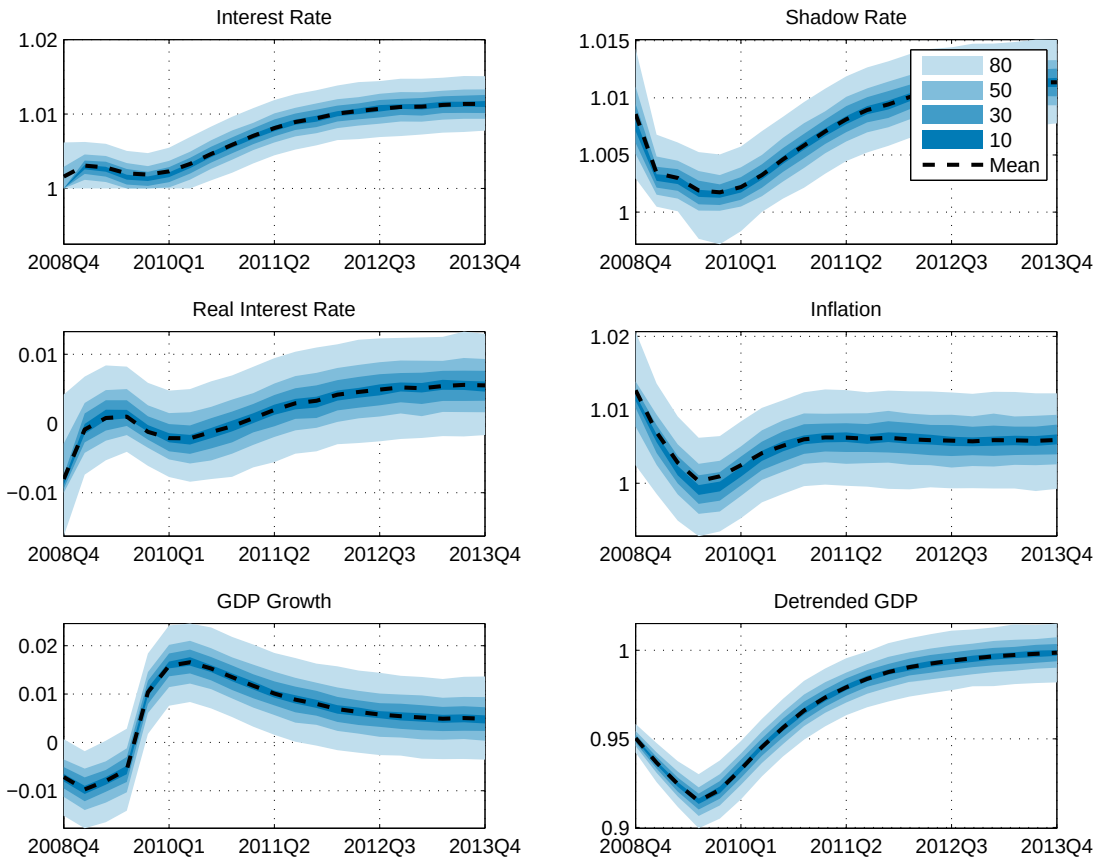
where $P_t(\theta)$, $R_t(\theta)$ and $Q_1(\theta)$ will be functions of $\mathscr{A}_0(\theta)$, $\mathscr{A}_1(\theta)$, $\mathscr{A}_2(\theta)$, $\mathscr{A}_3(\theta)$, $\mathscr{D}(\theta)$, $\mathscr{A}_0^*(\theta)$, $\mathscr{A}_1^*(\theta)$, $\mathscr{A}_2^*(\theta)$, $\mathscr{A}_3^*(\theta)$ and x_t. Guerrieri & Iacoviello (2015b) have noted that their piecewise linear solution is equivalent to the extended path solution when used with a linearized model that is subject to an occasionally binding constraint.

We present the results for the simulations where ρ_r is equal to 0.85 in Figure 13.[24]

[23] This issue that is dealt with by McKay et al. (2015).

[24] We use consumption shocks in the central scenario that are 3 times their shock standard deviations.

Figure 13: Piecewise Linear: $\rho_r = 0.85$



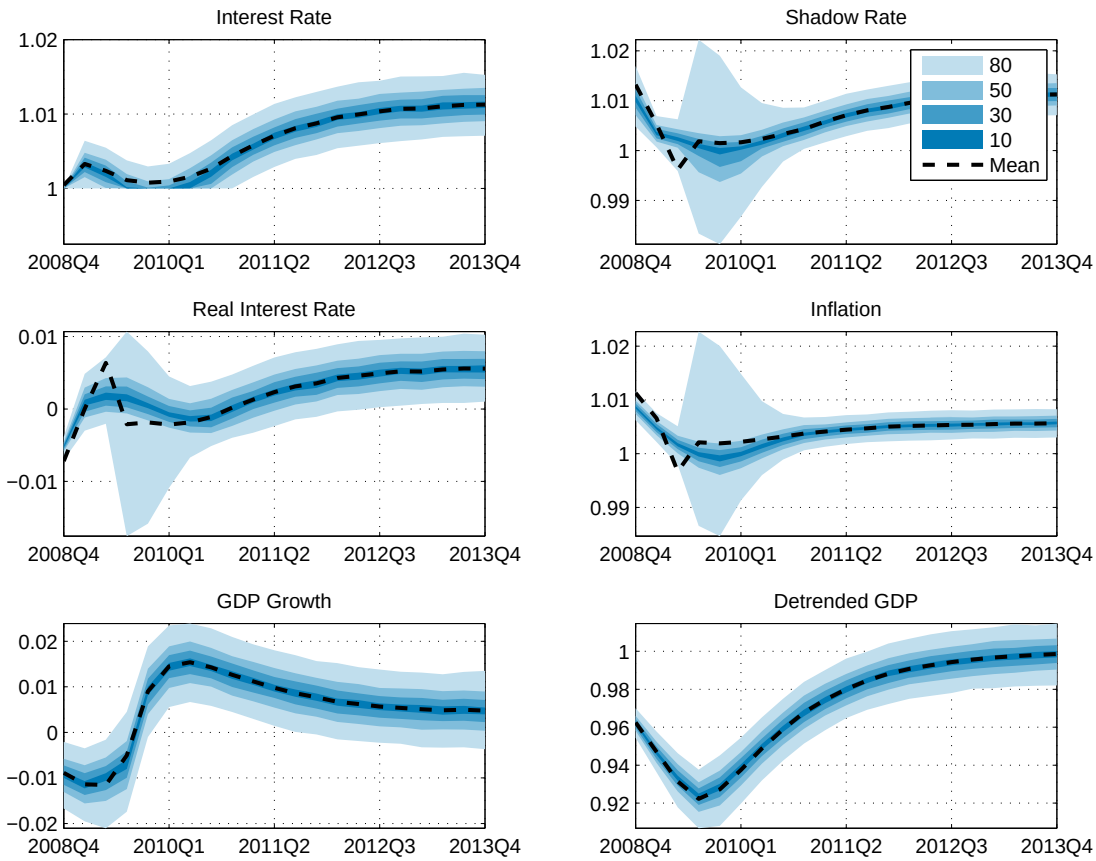
When ρ_r is set at 0.85 we see that some of the asymmetry in interest rates translates into the shadow rate. There is also a small amount of asymmetry in inflation, with the mean being higher than the median in the first two years of the simulation. We note that the initial increase in the nominal interest rate is due to the initial conditions.[25]

We repeat the simulation exercise, this time with ρ_r set at 0.6. The results are presented in Figure 14.[26]

[25] Each solution method changes the properties of the model. For the piecewise linear model we were unable to find a reasonable sequence of four equally sized negative consumption shocks that removed this initial hump in interest rates. Larger negative shocks resulted in numerical instabilities.

[26] The consumption shocks in the central scenario are 0.6 times their shock standard deviations.

Figure 14: Piecewise Linear: $\rho_r = 0.6$



When the piecewise linear solution is used with ρ_r set at 0.6 the degree of asymmetry increases quite substantially. There is now noticeable asymmetry in the shadow rate, the real interest rate and inflation, where there was very little with ρ_r equal to 0.85. Qualitatively, this is similar to our experience with regime-switching, that is a decrease in the degree of interest rate smoothing results in an increase in the degree of asymmetry present in inflation and GDP. Our experience with this algorithm when interest rate smoothing is set at 0.6 or even lower is that it can become unstable for some of the draws and that this instability may contribute to the asymmetry, just as the sign reversals may contribute to the asymmetry with the anticipated shocks.

5.3. Extended Path Solution

The extended path algorithm, originally due to Fair & Taylor (1983), has been used by Adjemian & Juillard (2010), Braun & Köber (2011) and Coenen & Warne (2014) to impose the lower bound constraint on interest rate projections. The extended path algorithm assumes that the DSGE model can be written in the following form:

$$E_t f\left(x_{t+1}, x_t, x_{t-1}, \eta_t\right) = 0, \tag{9}$$

where $f(\bullet)$ is a potentially nonlinear function that describes the equations of the system, x_t is a vector of date t endogenous variables and η_t is a vector of structural shocks. The extended path algorithm is deterministic in nature.[27] This means agents have perfect foresight and that we can bring the expectations operator inside the nonlinear function as follows:

$$E_t f\left(x_{t+1}, x_t, x_{t-1}, \eta_t\right) = f\left(E_t\left\{x_{t+1}\right\}, x_t, x_{t-1}, \eta_t\right) = f\left(x_{t+1}, x_t, x_{t-1}, \eta_t\right) = 0. \tag{10}$$

The extended path algorithm involves solving the entire forward path of the model variables at a given point in time by stacking the nonlinear equations for T (large) future time periods as represented by the system of equations:

$$\begin{aligned} f\left(x_{t+1}, x_t, x_{t-1}, \eta_t\right) &= 0, \\ \vdots \quad & \quad \vdots \\ f\left(x_T, x_{T-1}, x_{T-2}, \eta_{T-1}\right) &= 0, \end{aligned} \tag{11}$$

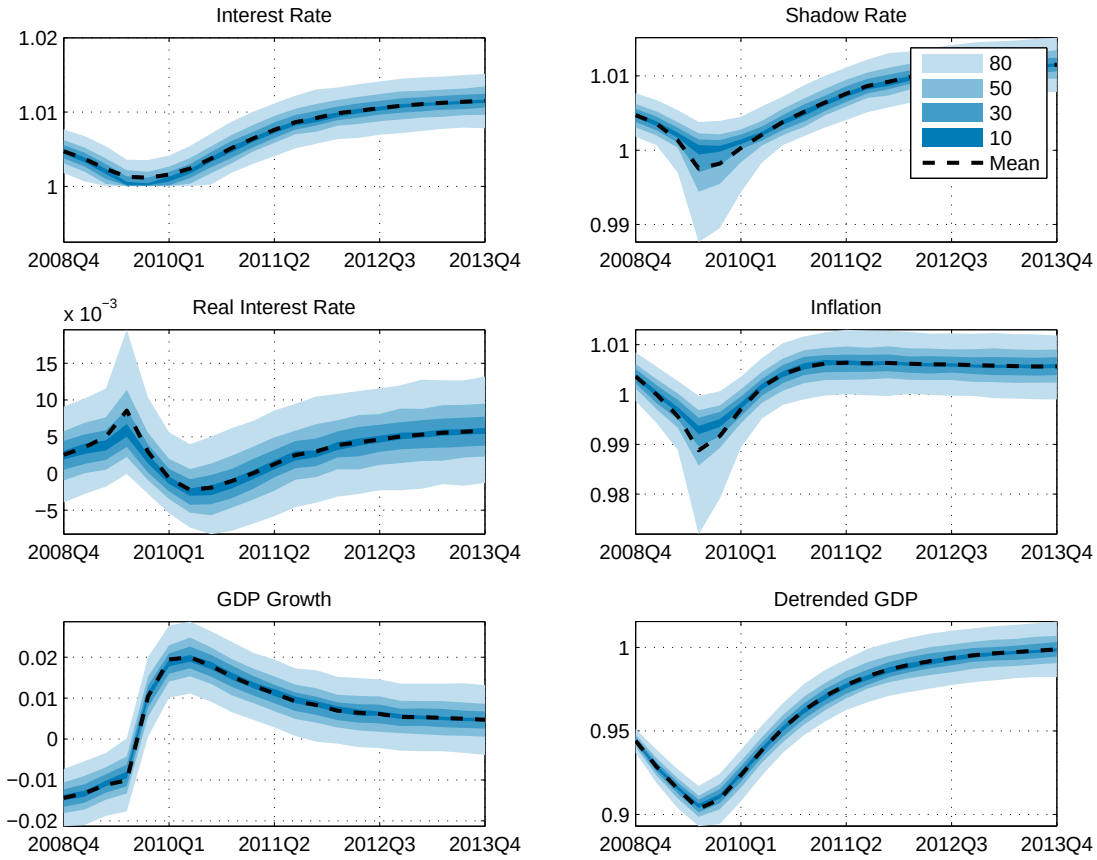
where $\eta_{t+1} = \eta_{t+2} = \ldots = \eta_{T-1} = 0$. After supplying a sequence of shocks and imposing the constraint(s) on the linear/nonlinear model, the algorithm proceeds through the following steps: starting at some initial condition, the current period's set of shocks are added to the model (all future shocks are assumed to be equal to zero), the model equations (equation (11)) are stacked together and the paths for the endogenous variables are solved using a Newton type algorithm. The forecasts for this period are retained and time is updated (the model is rolled forward one period). The current period's shock is added to the problem and the system of stacked equations (equation (11)) are solved again using a Newton type algorithm. This process is repeated until the simulation is finished.

The results from the simulations using the extended path algorithm with interest rate smoothing set to 0.85 are plotted in Figure 15.[28]

[27] Adjemian & Juillard (2013) have developed a stochastic version of the extended path algorithm that breaks certainty equivalence. The method combines the conventional extended path algorithm with the third-order perturbation solution and a Gaussian quadrature method for calculating expectations. Their method takes into account the lower bound constraint at all points in time, although it adds significant computational cost, limiting its applicability.

[28] The consumption shocks in the central scenario are 4 times their calibrated shock standard deviations.

Figure 15: Extended Path: $\rho_r = 0.85$

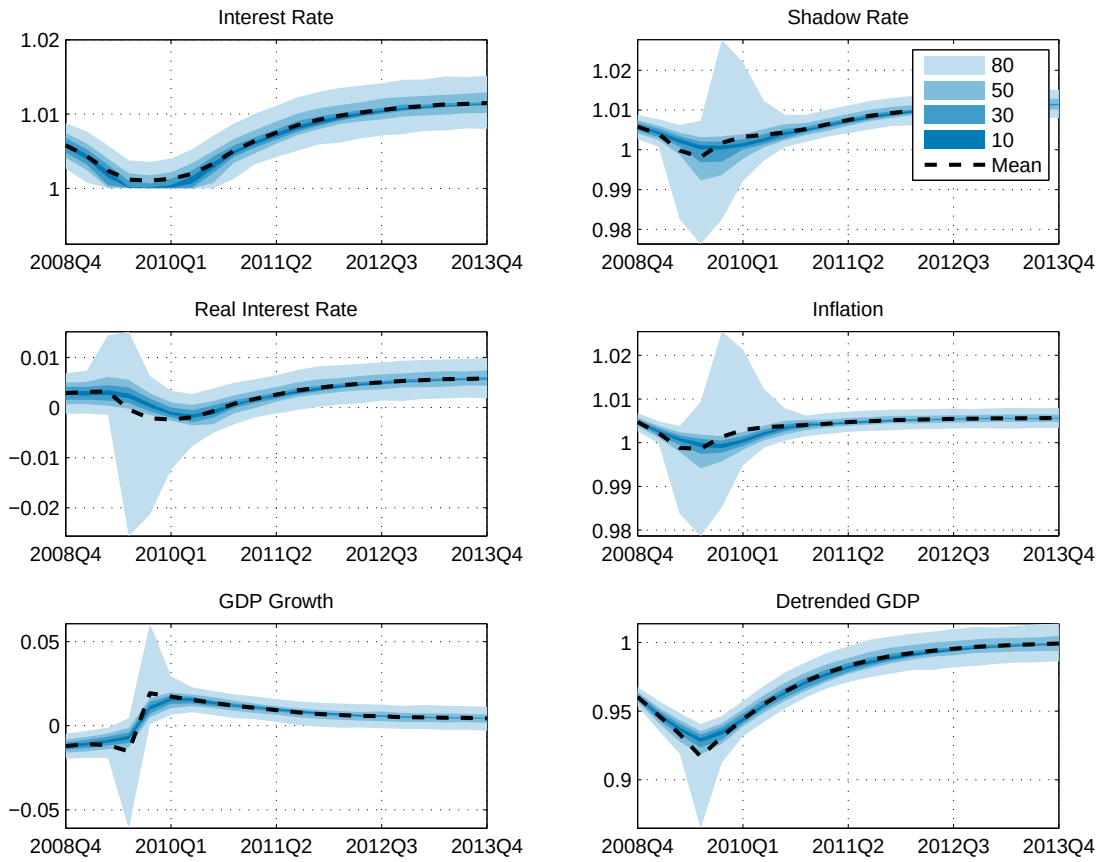


From Figure 15 we can see that the lower bound on interest rates translates into noticeable asymmetry in the real interest rate. This in turn translates into noticeable asymmetry in inflation and a small amount of asymmetry in real GDP.

We rerun the simulations this time with ρ_r set at 0.6. The results are presented in Figure 16.[29]

[29]The consumption shocks in the central scenario are 3 times their calibrated shock standard deviations.

Figure 16: Extended Path: $\rho_r = 0.6$



We notice that the imposition of the lower bound results in sizeable asymmetries in the shadow rate and the real interest rate. We also note that this translates into quite sizeable asymmetries in inflation, GDP growth and detrended GDP. As we have noticed with some of the other solution methods, a lower degree of interest rate smoothing results in a higher degree of asymmetry in inflation and GDP. Our experience with this algorithm is similar to our experience with the piecewise linear algorithm in the sense that lower degrees of interest rate smoothing lead to more unstable draws and it is this instability that may introduce additional asymmetry into the forecast densities. As a consequence, some of the asymmetry could be arising for the wrong reasons.

5.4. Summary and Discussion

We summarize and discuss the results from the alternative methods for enforcing the lower bound on interest rates. In general the results differ quite a bit from the results we obtained using regime-switching. We note that all methods introduce varying degrees of asymmetry into the inflation and GDP projections and that in general the shape, width and degree of upside/downside risk differ across the different methodologies when compared with regime-switching. Moreover, interest rate smoothing seems to work in the same direction when

comparing the regime-switching results with the results obtained from the piecewise linear solution and the extended path algorithm: that is, an increase in interest rate smoothing tends to decrease the asymmetry in inflation and GDP. However, we note that the opposite appears to hold true when anticipated monetary policy shocks are used to enforce the lower bound constraint.

The alternative methods considered in this section all provide mathematical tools for imposing the lower bound constraint on interest rates. However, each of these methods has its limitations. Piecewise linear solutions are constructed from the linearized model so they are not consistent with the original nonlinear DSGE model with the constraint imposed. Extended path solutions are deterministic and certainty equivalent. Adding anticipated monetary policy shocks to prevent interest rates from going below zero deforms the shock distributions in the model, which can in turn artificially introduce asymmetry into the forecast distributions. This method is highly sensitive to the shock anticipation horizon, due to the extremely forward-looking nature of the consumption Euler equation.

Moreover, we conjecture that sign reversals, or the equivalent instabilities introduced with piecewise linear and extended path solutions, could affect the results of these alternative methods. As a consequence this could introduce spurious asymmetry into the forecast densities. The testing of mean square stability with regime-switching eliminates this instability from the resulting density forecasts.

Furthermore, anticipated monetary policy shocks, piecewise linear solutions and the extended path algorithm all ignore the presence of the lower bound constraint when it does not bind. This means that even in the near vicinity of the constraint binding, agents' behavior is not affected by the lower bound until it binds. This is incompatible with a rational expectations solution of the underlying nonlinear model. In addition to ignoring the lower bound constraint, these methods ignore the economics of the underlying problem, namely the changes in monetary policy, monetary policy objectives and the monetary policy transmission mechanism that can occur at or near the lower bound.

6. Discussion/Economic & Policy Implications/Limitations

We discuss further some of the parameters that play an important role in determining the width and shape of the density forecasts when the lower bound on interest rates binds. We also discuss some of the limitations of the exercises we have performed and some factors not accounted for in our simulations that could influence the shape of the density forecasts.

From our analysis using regime-switching, we find that one of the most important parameters for determining the degree of asymmetry for the inflation density forecasts, when interest rates are at their lower bound, is the degree of interest rate smoothing in the Taylor rule. We have noticed that this also plays an important role in determining the properties of the density forecasts when alternative methods are used to impose the lower bound constraint. In particular we find that an increase in the degree of interest rate smoothing results in a reduction in the degree of asymmetry in inflation. We examine several interrelated reasons for this.

When a high degree of interest rate smoothing is estimated in a Taylor-type rule, this suggests that interest rates do not have to move much to keep inflation in check.[30] This implies monetary policy is quite effective or credible. It is agents' fear of the repercussions of inflation dynamics and expectations getting out of kilter that keeps the economy functioning with what appears to be very little effort from the monetary authority. Moreover, Billi (2013) has described a high degree of interest rate smoothing as capturing inflation expectations that are well anchored. This is true in the sense that inflation dynamics (in a DSGE model) will be a function of interest rate smoothing and that inflation expectations will inherit a lot of persistence from monetary policy.

In similar work, Woodford (2003) has investigated the role of interest rate smoothing in optimal policy and found high degrees of smoothing to be optimal. When interest rates are very persistent, as is the case when interest rate smoothing is high, moving interest rates by a small amount affects the implied long rates and shifts the entire yield curve. In DGSE models, the implied long rate is a key determinant of consumption dynamics through the consumption Euler equation.[31] In fact, one could think of high degrees of interest smoothing as a form of commitment, namely to move interest rates in a very smooth fashion.

Finally, a high degree of interest rate smoothing implies that interest rates do not move much. In the limit, a high degree of interest rate smoothing will mean interest rates do not move at all. In such an economy, the responses to policy in the normal regime should then be quite similar to the lower bound regime, reducing the amount of asymmetry we should expect to see. In fact, if we move between two regimes with constant interest rates, and each regime is linear, then we should not expect to see any asymmetry in the other variables in the system.

In other work, Binning et al. (2016) have found that the transition probabilities, in the case of exogenous transition probabilities, and the initial conditions also play an important role in determining the shape and asymmetry of the model's density forecasts. More specifically, if the probability of remaining in the lower bound regime increases, then so does the asymmetry in inflation. This is because the stabilizing effect of the Taylor rule in the normal regime is diminished in the lower bound regime, resulting in greater downside risk. Likewise, the initial conditions also play an important role. If the economy is too near the lower bound on interest rates, then most draws enter the lower bound very quickly, in which case if almost all the draws are in the same regime at the same time, and the regimes are conditionally linear-gaussian, then we should not see much asymmetry.

The analysis we have presented is far from exhaustive. We have primarily used one calibration of a simple canonical New Keynesian DSGE model. In this model, the Taylor

[30] This assumes that inflation and real GDP dynamics are under control.

[31] This can be seen by solving the consumption Euler equation forward in time. For example, take a simple consumption Euler equation without habit formation: $C_t^{-\sigma} = E_t\left\{\frac{\beta R_t}{\Pi_{t+1}} C_{t+1}^{-\sigma}\right\}$. Log-linearizing and solving it forward gives: $\hat{c}_t = E_t\left\{\hat{c}_{t+1+\infty} - \frac{1}{\sigma}\sum_{j=0}^{\infty}\left(\hat{r}_{t+j} - \pi_{t+1+j}\right)\right\}$, where $E_t\left\{\sum_{j=0}^{\infty}\left(\hat{r}_{t+j} - \pi_{t+1+j}\right)\right\}$ is the implied (infinite period) long real rate according to the expectations hypothesis. $E_t\left\{\hat{c}_{t+1+\infty}\right\} = 0$, so that consumption today is determined by this long rate.

rule is the only stabilizing force in the economy. When the economy hits the lower bound, it is temporarily destabilized, causing various degrees of asymmetry in the density forecasts of other variables in the model. However, this is not how the economy works in reality; policy makers have other means (with various degrees of effectiveness) at their disposal for stabilizing the economy. More specifically, central banks can use unconventional monetary policies such as quantitative easing and forward guidance to try and stabilize the economy when they can no longer cut interest rates. The government can also use active fiscal policy to stabilize the economy and keep inflation under control according to the fiscal theory of the price level. Incorporating these mechanisms into a DSGE model will have implications for the shape and width of the model's density forecasts when the lower bound binds. More specifically, additional sources of stability in the economy could reduce the downside risk and the asymmetry in the forecast uncertainty.[32] Our analysis has also ignored the open economy consequences of a binding lower bound and the implications this will have on the exchange rate and its role as a shock absorber.

7. Conclusion

This paper makes the case for regime-switching as the natural framework for modeling the ELB and investigating the width and shape of fan charts in the vicinity of the ELB. It then applies the regime-switching methodology to a small DSGE model calibrated to fit the US economy. The following results emerge from that investigation: first, the introduction of a lower bound constraint does not necessarily result in significant asymmetry in the fan charts. Second, a reduction in interest rate smoothing results in increased asymmetry in the density forecasts for inflation. Third, an increase in price stickiness results in a decrease in forecast asymmetry. Fourth, changes in the monetary policy transmission mechanism can create asymmetry in the fan charts.

Investigation of the same issues using alternative methodologies to the regime-switching framework reveal that: first, the results are quantitatively quite different; second, they can generate asymmetries where asymmetry is not expected due to sign reversals and other numerical instabilities; third, an increase in interest rate smoothing reduces asymmetry in the forecast densities produced by the piecewise linear and extended path solutions. All simulations were carried out in Matlab using the RISE toolbox.[33]

Appendix A. Baseline Model: Binning and Maih (2016)

$$\log(A_t) = \rho_A \log(A_{t-1}) + \sigma_A \varepsilon_t^A, \tag{A.1}$$

$$\mathscr{M}_{t,t+1} = \frac{1}{R_t}, \tag{A.2}$$

[32]Similar points have been raised by Knüppel & Schultefrankenfeld (2012).
[33]The RISE toolbox can be downloaded from https://github.com/jmaih/RISE_toolbox.

$$R_t = \max\left(R_t^E, R_t^*\right), \tag{A.3}$$

$$A_t^R = \rho_{A^R} A_{t-1}^R + \sigma_r \varepsilon_t^R. \tag{A.4}$$

$$R_t^E = K, \tag{A.5}$$

$$\tilde{\lambda}_t = A_t \left(\tilde{C}_t - \chi \tilde{C}_{t-1}/\mu_t\right)^{-\sigma}, \tag{A.6}$$

$$\tilde{W}_t = \kappa N_t^{\eta}/\tilde{\lambda}_t, \tag{A.7}$$

$$\mathscr{M}_{t,t+1} = E_t \left\{ \beta d_{t+1} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t \Pi_{t+1} \mu_{t+1}} \right\}, \tag{A.8}$$

$$\tilde{Y}_t = N_t, \tag{A.9}$$

$$\left(\frac{\varepsilon}{\varepsilon - 1}\right) \tilde{W}_t - \exp\left(\sigma_\pi \varepsilon_t^\pi\right) - \left(\frac{\phi}{\varepsilon - 1}\right) \Pi_t \left[\Pi_t - \tilde{\Pi}_t\right] + \ldots$$
$$\ldots + E_t \left\{ \left(\frac{\phi}{\varepsilon - 1}\right) \mathscr{M}_{t,t+1} \Pi_{t+1}^2 \frac{\tilde{Y}_{t+1}}{\tilde{Y}_t} \mu_{t+1} \left[\Pi_{t+1} - \tilde{\Pi}_{t+1}\right] \right\}, \tag{A.10}$$

$$R_t^* = R_{t-1}^{*\rho_r} \left(\bar{R}^* \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\kappa_\pi} \left(\frac{\tilde{Y}_t}{\tilde{Y}_{t-1}}\right)^{\kappa_y} \right)^{1-\rho_r} \exp\left(A_t^R\right), \tag{A.11}$$

$$\tilde{Y}_t = \tilde{C}_t + \frac{\phi}{2} \tilde{Y}_t \left[\Pi_t - \tilde{\Pi}_t\right]^2, \tag{A.12}$$

$$\Delta \log\left(Y_t\right) = \log\left(\tilde{Y}_t\right) - \log\left(\tilde{Y}_{t-1}\right) + \log\left(\mu_t\right), \tag{A.13}$$

where $\mu_t = \exp(g_Z + \sigma_Z \varepsilon_t^Z)$ and $\tilde{\Pi}_t = \Pi_{t-1}^{\xi} \bar{\Pi}^{1-\chi}$. The complete set of variables in the model is defined as follows

$$\tilde{x}_t = \left[\tilde{C}_t,\, N_t,\, R_t,\, \Pi_t,\, \tilde{W}_t,\, \tilde{\lambda}_t,\, \mathscr{M}_{t,t+1},\, \tilde{Y}_t,\, R_t^*,\, R_t^E,\, \Delta \log\left(Y_t\right),\, A_t^R,\, A_t\right]',$$

Table A.2: Variable Description

Variable	Description
$\tilde{C}_t$	Detrended consumption
N_t	Hours worked
R_t	Interest rate
Π_t	Inflation rate
$\tilde{W}_t$	Real detrended wage
$\tilde{\lambda}_t$	Marginal utility of consumption
$\mathscr{M}_{t,t+1}$	Stochastic discount factor
$\tilde{Y}_t$	Effective output
R_t^*	Shadow/Taylor rule interest rate
R_t^E	Effective lower bound interest rate
$\Delta \log (Y_t)$	Log change in per capita GDP
A_t^R	Autoregressive monetary policy shock term
A_t	Autoregressive consumption preferences shock
ε_t^A	Consumption preference shock
ε_t^R	Monetary policy shock
ε_t^π	Stochastic subsidy shock
ε_t^Z	Productivity shock

Table A.3: Parameter Description

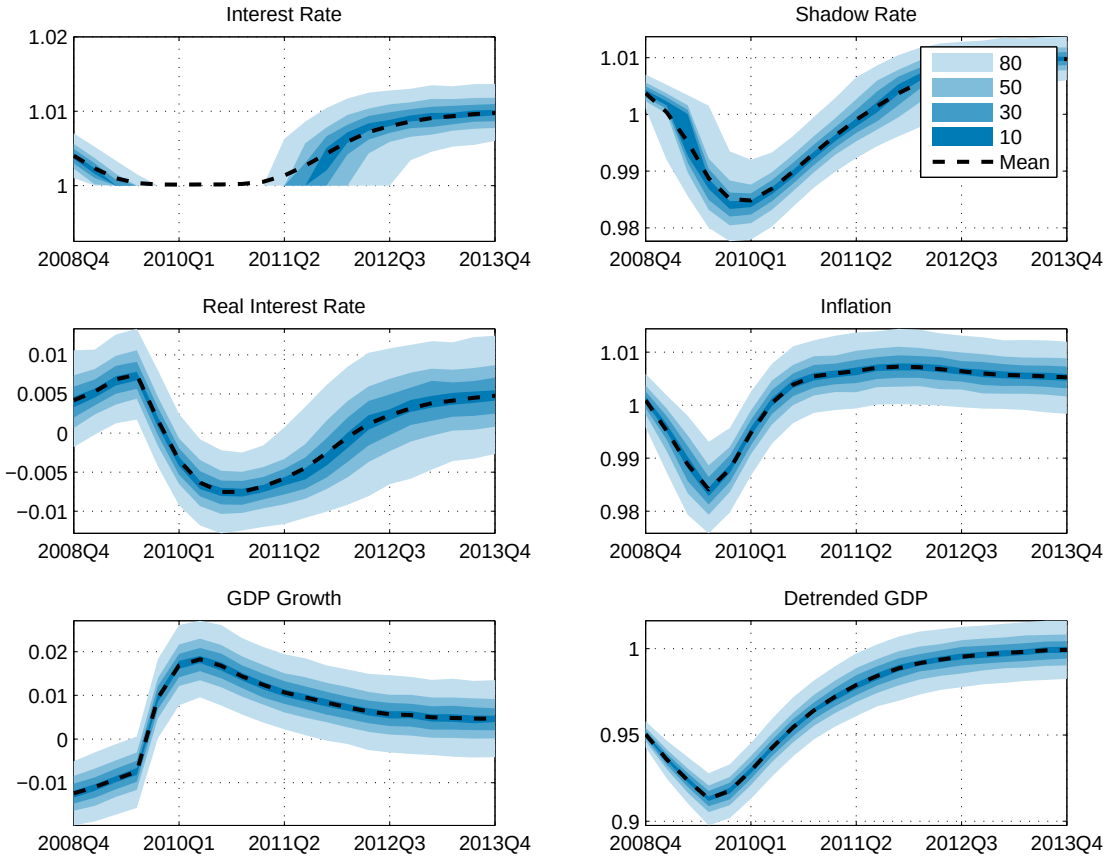
Parameter	Description
β	Discount factor
χ	Habit formation parameter
σ	Inverse of the intertemporal elasticity of substitution
ϵ	Elasticity of substitution between differentiated final goods
η	Inverse of Frisch elasticity of labor supply
ϕ	Rotemberg cost of adjusting prices
κ_π	Weight on inflation in the Taylor rule
κ_y	Weight on growth in the Taylor rule
g_Z	Productivity growth rate
$\bar{\Pi}$	Steady state rate of inflation
ξ	Degree of inertia in inflation
ρ_r	Interest rate smoothing parameter
ρ_{AR}	Monetary policy shock persistence term
ρ_A	Consumption preference shock persistence term
σ_R	Monetary policy shock std deviation
σ_A	Consumption preference shock std deviation
σ_Z	Technology shock std deviation
σ_π	Stochastic subsidy shock std deviation
$p_{N,L}$	Transition probability from normal to low interest rate regime
$p_{L,N}$	Transition probability from low to normal interest rate regime

Appendix B. Data

All data is taken from the St. Louis Federal Reserve's FRED database. We report the FRED pneumonics in brackets. The data is quarterly and spans 1985Q1 to 2015Q2. We used the log change in per capita US GDP, constructed using Real Gross Domestic Product (GDP1) and the Civilian Noninstitutional Population (CNP16OV), the percentage change in the US GDP deflator (GDPDEF), and the quarterly average of the monthly federal funds rate (FEDFUNDS). The data are not detrended.

Appendix C. Nonlinear Regime-Switching

Figure C.17: Regime-Switching: Third-order Approximation



Appendix D. Sensitivity Analysis

Figure D.18: Regime-Switching: Second-order Approximation with $\rho = 0.6$

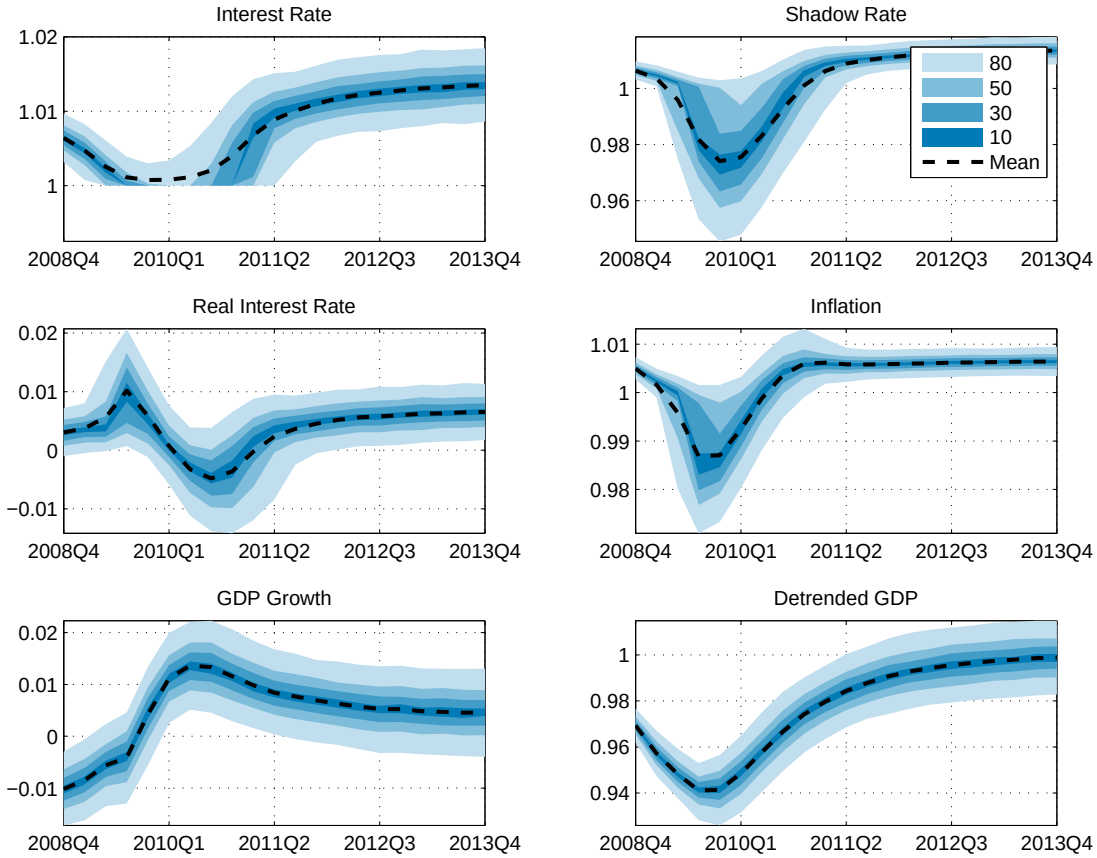
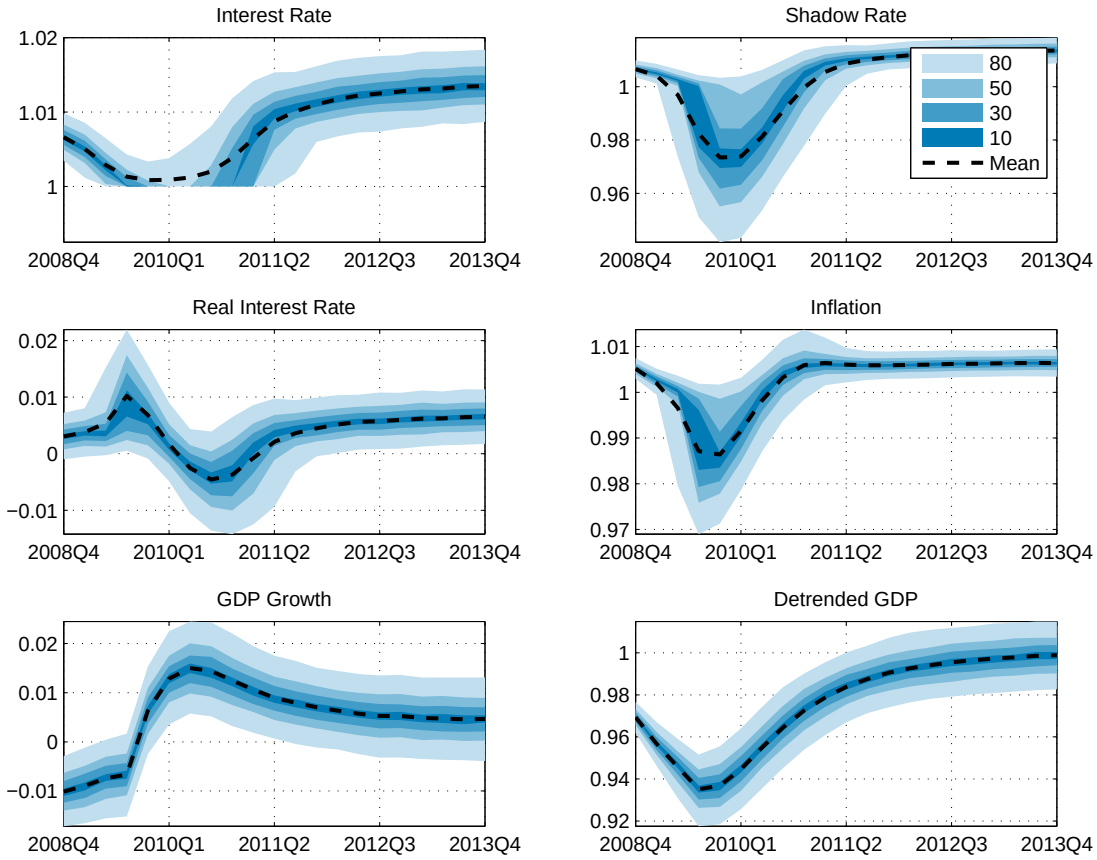


Figure D.19: Regime-Switching: Third-order Approximation with $\rho = 0.6$



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