Where’s the Risk? The Forward Premium Bias, the Carry-Trade Premium, and Risk-Reversals in General Equilibrium

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Abstract

This paper builds a two-country dynamic stochastic general equilibrium macro model to understand three empirical facts about international currency returns. They are the downward forward premium bias, the carry trade return, and the long-run risk reversal. Cross-country heterogeneity in unit-root productivity levels generates the systematic risk priced into currency returns. The risk can be magnified through monetary policy. Both a complete markets and an incomplete markets model are qualitatively consistent with these facts. Quantitatively, the incomplete markets model performs better.

Keywords: currency risk, DSGE, monetary policy
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Introduction

The downward forward premium bias, the carry trade excess return, and the long-run risk reversal are three distinct, but related empirical regularities that have come to characterize international currency returns. The downward forward premium bias has long been a topic of study. Academic interest in the carry trade has been growing for a little more than a decade now. The long-run risk reversal, identified and studied by Engel (2016), is relatively new. In this paper, we study how these empirical patterns in currency returns might be understood in the context of a two-country dynamic stochastic general equilibrium (DSGE) macroeconomic model.

The downward forward premium bias refers to regression evidence that uncovered interest rate parity (UIP) is violated in the data. UIP says the excess return earned by going short (borrowing) the low interest rate currency and going long (lending) the high interest rate currency should be exactly offset by a loss in value of the long currency. Econometrically, UIP predicts a zero constant and a unit slope coefficient in a regression of the future currency depreciation on today’s interest rate differential. Bilson (1981) and Fama (1984) first ran these regressions, and many researchers have run them since. Almost always, the slope in the regression is less than one and often it is negative. This is what we refer to as the downward forward premium bias.\footnote{Before the global financial crisis in 2008, the covered interest arbitrage condition led the inter-bank interest rate differential to be equal to the forward contract premium on the spot exchange rate. Hence the forward premium and the interest rate differential were interchangeable. The imposition of new regulatory capital requirements on banks in 2008 caused covered interest parity to fail (Du et al. (2016) and Pinnington and Shamloo (2016)).} The downward forward premium bias is remarkably robust to the sample period and to the choice of base currency. The downward bias implies the bigger is the interest rate differential, the smaller is the subsequent currency depreciation rate, and the mainstream view is that this occurs because the two currencies have different risks and the downward bias results from the presence of a risk premium across

The carry trade is a profitable, zero-net investment strategy for currencies. It says to go short the low interest currency and to go long the high interest currency. The carry trade, while related to the forward-premium bias, is not the same thing. One notable feature of the carry trade, when formed into portfolios, is its consistent profitability. An extensive and growing literature is devoted to its study. For example, assuming no transactions costs, Lustig et al. (2014) report an excess return between portfolios of the highest and lowest interest rate countries of 6.2 percent per annum. Research aimed at understanding the cross-section of returns—why the average excess return is increasing in the average size of the interest rate differential—includes Burnside et al. (2011), Della Corte et al. (2016), Lustig and Verdelhan (2007), Menkhoff et al. (2013), and Berg and Mark (2017a, 2017b). Employing the ‘beta-risk’ framework, these authors study the cross-sectional pricing of carry trade returns where the stochastic discount factors load (depend) on risk factors constructed from macroeconomic data. This paper is more closely related to a smaller literature that studies the carry trade in general equilibrium macro models, such as Hassan (2013) who emphasizes differences in country size and Ready et al. (2017) who focus on cross-country differences in productive technology.

Engel’s (2016) risk reversal begins with the observation that a country’s currency
strength is associated with it having high relative real interest rates. Classic articles by Dornbusch (1976) and Frankel (1978) established a theoretical link between real interest rates and currency value. Empirical evidence for the link is found in Engel and West (2006), Alquist and Chinn (2008), and Mark (2009). Moreover, the idea has been put into practice by central banks to defend their currencies in times of crisis. For example, in September 1992, in an attempt to maintain the krona, Sweden’s Riksbank briefly raised its marginal lending rate to 500 percent per annum. The International Monetary Fund has historically advised its members to defend their currencies by raising interest rates—advice heeded by South Korea during the 1997 Asian Financial Crisis. Engel's (2016) argument then, is that the high interest rate country is the (relatively) safe one. It is safe because it has a strong currency. Being safe, it should pay out a negative risk premium. But observations on the carry trade and the forward premium bias says the high interest country pays a positive risk premium so it must be risky. To reconcile these contradictory predictions, the high interest rate country must undergo a risk reversal over time. In the short run, it is risky and pays a positive risk premium through the carry trade, but over time, that risk premium turns negative. Engel (2016) reports empirical evidence of these risk reversals in a sample of the G-7 countries. He then argues that the current generation of international finance models are unlikely to explain the risk reversal and what may be missing from those models is a non-pecuniary liquidity return on assets.

Is there a common source of risk that gives rise to these empirical currency return patterns? We address this question in a two-country New Keynesian DSGE model that features local currency pricing (LCP) by exporters. We evaluate both a complete markets version of the model and an incomplete markets version. Productivity is nonstationary and is driven by a stochastic trend. A central point to emerge from studies on the carry trade is that heterogeneity across countries is key to understanding currency excess returns.²

²For example, Hassan (2013) exploits differences in country size, whereas Lustig et al. (2014) exploit
Our model gives prominence to two sources of country heterogeneity. The first is heterogeneity in cross-country productivity. A central feature of total factor productivity (TFP) across countries is that they are stochastically trending. Significantly, there is little evidence of TFP convergence across countries. Divergent TFP represents a potentially significant risk factor that could be priced into currency excess returns. Our solution technique is perturbation with pruning of a third-order approximation around a nonstochastic steady state, so we cannot literally have divergent random walks in the country TFPs. Our specification of TFP features near unit root in the error-correction term so they are technically cointegrated, but are on the borderline of divergence. The TFP of one country can stay well above the other for 400 quarters or more, and these systematic differences in TFP become a source of systematic risk that is priced into international currency returns.

To create highly persistent productivity differences, we allow the error correction terms to enter the productivity processes asymmetrically. The asymmetry causes a positive productivity shock in country 1 to be reversed over time and to have a negative effect on country 2 productivity, whereas a positive productivity shock in country 2 perpetuates over time and spills over in a positive way to country 1 productivity. Hence, we have cross-country productivity heterogeneity in persistent differences in their levels, and in the nature of spillovers.

We also show that heterogeneity in monetary policy plays a role in magnifying the underlying risk, especially in generating the carry trade premium. Differences in the cyclical response of the interest rate, and differences in accommodation to inflation can be a source of currency risk. Other researchers have incorporated inflation into their analyses of currency risk, but have done so in endowment frameworks. In Bansal and Shaliastovich (2012), inflation and consumption growth are jointly governed by an exogenous long-run risk process, but people in their economy care about inflation only to the extent that exogenous differences in risk factor loadings across country stochastic discount factors (SDFs).
inflation and consumption growth are correlated. While inflation is endogenously determined in Backus et al. (2013), inflation has no effect on welfare in their endowment economy model. In our general equilibrium model, inflation and associated price dispersion do have effects on welfare.

To summarize our main results, under a benchmark specification with productivity heterogeneity and symmetric monetary policies with conventional Taylor-rule coefficient values, the incomplete markets model generates the downward forward premium bias where the implied slope coefficient in the Fama regression is less than 1 (but not negative), a carry-trade excess return of 2.8 percent per annum, and Engel’s risk-reversal. The complete markets model generates a small (66 basis points) carry-trade return and the risk-reversal, but no forward premium bias. Departures from the benchmark monetary policy by varying the degree of cyclical and/or inflation accommodation, can magnify the risk faced by agents. Here, we find both the complete markets and incomplete markets models are qualitatively consistent with the three international currency return facts, but quantitatively, the incomplete markets model performs better.

The remainder of the paper is structured as follows. The next section develops the two-country New Keynesian DSGE model with nominal rigidities and recursive utility. In Section 2, we discuss the parameterization of the model. Section 3 discusses each of the three currency return empirical regularities in some detail and assesses the model’s ability to explain the return patterns. Section 4 considers alternative specifications of productivity, Section 5 discusses the impulse response functions (IRFs), and Section 6 concludes.
1 A Two-Country Macroeconomic Model

In this section, we outline the two-country DSGE model used in our analysis. Prices are sticky and adjust through a Calvo (1983) mechanism. Exporters set nominal export prices in advance and in terms of the foreign currency—a practice called local currency pricing (LCP). Households have recursive utility (Epstein and Zin (1989) and Weil (1989)). These preferences have gained popularity in macroeconomic and financial economics research. In addition to making the current utility flow dependent on expected future utility, recursive utility generalizes power utility by regulating the intertemporal elasticity of substitution and the degree of risk aversion through separate parameters. The exogenous variables are shocks to country productivity, which themselves are unit-root processes. Productivity is cointegrated across countries, but not strongly, in the sense that the error correction term has a near unit root.

Unless it is necessary to distinguish between countries \( i = \{1, 2\} \), we will suppress the country subscript. Because a country’s productivity, \( A_t \), has a stochastic trend, level variables (except labor), will trend with \( A_t \) which causes the model to be nonstationary. The model solution requires an approximation around a non-stochastic steady state. In the text, we present the model in its original form. Numerical solution requires a stationary representation, which we obtain by dividing one-period lagged productivity into the trending variables. The stationary transformation is presented in the appendix.

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3A complete description of the model is contained in the appendix.

1.1 Households

We assume a particular functional form of recursive utility employed by Swanson (2016). Let $V_t$ be current utility, $c_t$ be the household’s real consumption and $\ell_t$ be its labor supply. Households in both countries want to maximize utility

$$V_t = (1 - \beta) \left[ \ln (c_t) - \eta \frac{\ell_t^{1+\chi}}{1+\chi} \right] - \frac{\beta}{\alpha} \ln \left[ E_t e^{-\alpha V_{t+1}} \right]$$

where $\beta \in (0, 1)$ is the subjective discount factor and $\eta > 0$, $\chi > 0$, and $\alpha \in \mathbb{R}$ are also parameters. The log form of the current utility flow of consumption fixes the intertemporal elasticity of substitution (IES) to be 1. The Frisch elasticity of labor supply is $1/\chi$. Swanson (2016) shows that relative risk aversion (RRA) is

$$RRA = \alpha + \left( \frac{1}{1 + \frac{\eta}{\chi}} \right).$$

The intertemporal marginal rate of substitution (IMRS) is, $\beta M_{t,t+1}$ where

$$M_{t,t+1} = \left( \frac{c_t}{c_{t+1}} \right) \left( \frac{e^{-\alpha V_{t+1}}}{E_t (e^{-\alpha V_{t+1}})} \right).$$

As is the convention in asset-pricing research, we refer to the IMRS as the stochastic discount factor (SDF). If $\pi_{t+1}$ is the inflation rate from $t$ to $t+1$, the nominal SDF is $\beta N_{t,t+1}$ where

$$N_{t,t+1} = M_{t,t+1} e^{-\pi_{t+1}}$$

We consider both a complete markets and an incomplete markets environment. Engel (2016) expresses doubt that complete market models can be made consistent with the

\footnote{This is a monotone transformation of the more familiar recursive utility specification, with the intertemporal elasticity of substitution (IES) set to 1. See Karantounias (2017).}
risk reversal. We include an analysis under complete markets to benchmark the results.

1.1.1 Complete markets

Denote the current state of the world by $\omega_t$ and the state history by $\omega^t = \{\omega_t, \omega_{t-1}, \ldots\}$. Households in both countries have access to a full set of nominal state-contingent securities. These securities pay one unit of currency 1 if the state occurs. Making explicit, the functional dependence on the state, let $B(\omega^t)$ be the number of state $\omega^t$ contingent bonds held by the household. The price of a bond that pays off in state $\omega_{t+1}$ is $p_\omega(\omega_{t+1}|\omega^t)$. Shares of firms are not internationally traded and are entirely owned by domestic households. Let $P(\omega^t)$ be the price level, $W(\omega^t)$ be the nominal wage, and $\Pi(\omega^t)$ be nominal firm profits. There is no physical capital in the model. Households obtain flow resources from labor income, firm profits, and state-contingent bond payoffs. Those resources are spent on consumption and a portfolio of state-contingent bonds. In country 1, the household budget constraint is,

$$c_1(\omega^t) + \sum_{\omega_{t+1}} \frac{p_\omega(\omega_{t+1}|\omega^t) B_1(\omega_{t+1})}{P_1(\omega^t)} = \frac{W_1(\omega^t)}{P_1(\omega^t)} \ell_1(\omega^t) + \frac{\Pi_1(\omega^t)}{P_1(\omega^t)} + \frac{B_1(\omega^t)}{P_1(\omega^t)}. \tag{4}$$

If $\pi(\omega_{t+1}|\omega^t)$ is the conditional probability of state $\omega_{t+1}$, the optimality conditions for the household give the Euler equation for the state-contingent bond and the labor supply equation,\(^6\)

$$p_\omega(\omega_{t+1}|\omega^t) = \beta \pi(\omega_{t+1}|\omega^t) M_1(\omega_{t+1}|\omega^t) e^{-\pi_1(\omega^t)}, \tag{5}$$

$$\eta c_1(\omega^t) \ell_1(\omega^t) = \frac{W_1(\omega^t)}{P_1(\omega^t)}. \tag{6}$$

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\(^6\)To simplify the exposition, we approximate gross inflation by the continuously compounded rate.
Summing over the prices of all state-contingent bonds gives the price of the nominally risk-free bond,

\[
\frac{1}{1 + i_1(\omega^t)} = \beta E_t \left( M_1 (\omega_{t+1}|\omega^t) e^{-\pi_1(\omega^{t+1})} \right). \tag{7}
\]

The country 2 household faces a similar environment, except the state contingent bonds are denominated in currency 1. To get real contingent bond holdings, country 2’s household revalues by the exchange rate. If \( S_{1,2} (\omega^t) \) is the nominal exchange rate (the price of currency 2), then real country 2 contingent bond holdings are \( B_2 (\omega^t) (S_{1,2} (\omega^t) P_2 (\omega^t))^{-1} \). The Euler equation for the country 2 state-contingent bond and the labor supply equation are,

\[
p_\omega (\omega^{t+1}|\omega^t) = \beta \pi (\omega_{t+1}|\omega^t) M_2 (\omega_{t+1}|\omega^t) \left( \frac{S_{1,2} (\omega^t)}{S_{1,2} (\omega^{t+1})} e^{-\pi_2(\omega^{t+1})} \right), \tag{8}
\]

\[
\eta c_2 (\omega^t) \ell_2 (\omega^t) = \frac{W_2 (\omega^t)}{P_2 (\omega^t)}. \tag{9}
\]

Equating equations (5) and (8) and rearranging, gives the gross nominal depreciation of country 1’s currency,

\[
S_{1,2} (\omega^{t+1}) = M_2 (\omega_{t+1}|\omega^t) e^{-\pi_2(\omega^{t+1})} \left( \frac{S_{1,2} (\omega^t)}{M_1 (\omega_{t+1}|\omega^t) e^{-\pi_1(\omega^{t+1})}} \right), \tag{10}
\]

and rearrangement of equation (10) gives the gross real depreciation,

\[
\frac{Q_{1,2} (\omega^{t+1})}{Q_{1,2} (\omega^t)} = \frac{M_2 (\omega_{t+1}|\omega^t)}{M_1 (\omega_{t+1}|\omega^t)}; \tag{11}
\]

where \( Q_{1,2} (\omega^t) = \frac{S_{1,2}(\omega^t)P_2(\omega^t)}{P_1(\omega^t)} \) is the real exchange rate. Exchange rates are defined so that a decline in the value of currency 1 is reflected by an increase in \( S_{1,2} \) in nominal terms and an increase in \( Q_{1,2} \) in real terms. In anticipation of future use, we define the real exchange rate from country 2’s perspective as \( Q_{2,1} = Q_{1,2}^{-1} \), and for the nominal exchange rate, \( S_{2,1} = S_{1,2}^{-1} \). Equations (10) and (11) form the basic building blocks in the stochastic
discount factor approach to the exchange rate.\footnote{Equation (11) was first derived by Backus and Smith (1993). Backus et al. (2001) further developed and refined the SDF approach to the exchange rate. See also, Lustig and Verdelhan (2013) for an exposition and survey of the approach.}

\subsection{Incomplete markets}

In the incomplete markets version of the model we can suppress the state-dependent notation. Here, each country issues a nominal non-state contingent bond denominated in their own currency. International asset trade is restricted to this pair of nominal bonds. Country 1 issues its bond at a price of 1 unit of currency 1. It pays off $1 + i_{1,t}$ units of currency 1 next period. Country 2 issues its bond at a price of 1 unit of currency 2, which pays off $1 + i_{2,t}$ units of currency 2 next period. $B_{i,j,t}$ is the number of bonds issued in currency $j$ and held by people of country $i$, ($i = \{1, 2\}, j = \{1, 2\}$). Even when productivity shocks are stationary, formulating incomplete markets this way causes net foreign bond positions to be non-stationary. To induce stationarity in international bond positions, we follow Schmidt-Grohe and Uribe (2003) and impose a small fee on the foreign bond position. We let $\tau$ be the fee paid by households for taking a long or short position in the foreign currency bond. The real cost to a country $i$ household for taking a position in the currency $j$ bond ($i \neq j$) of size $B_{i,j,t}$ is $\frac{\tau}{2} \left( \frac{S_{i,j,t} B_{i,j,t}}{P_{i,t}} \right)^2$.\footnote{Borrowing is a short position in which $B_{i,j,t} < 0$, and lending is a long position in which $B_{i,j,t} > 0$. The transaction fee is treated as a deadweight loss.} Costs for people to take positions, $B_{i,i,t}$, in their domestic currency bonds are zero.

Shares of the firms continue to be domestically owned and not internationally traded. The household draws flow resources from labor income, firm profits, and net payoffs from positions in the domestic and foreign currency bonds. Those resources are spent on consumption, new net positions in domestic and foreign currency bonds, and the transaction fee on net foreign currency bond positions. The country $i = \{1, 2\}$ ($i \neq j$) household
budget constraint is,

\[
c_{i,t} + \frac{B_{i,i,t}}{P_{i,t}} + \frac{Q_{i,j,t}B_{i,j,t}}{P_{j,t}} + \frac{\tau}{2} \left( \frac{Q_{i,j,t}B_{i,j,t}}{P_{j,t}} \right)^2 = W_{i,t} \ell_{i,t} + \Pi_{i,t} + \frac{(1 + i_{i,t-1}) B_{i,i,t-1}}{P_{i,t}} + \frac{(1 + i_{j,t-1}) Q_{i,j,t}B_{i,j,t-1}}{P_{j,t}}.
\]

As long as \( \tau > 0 \), households will want \( B_{i,j} = 0 \) in the steady state.

The Euler equations associated with optimal bond holdings for country \( i \) are,

\[
\frac{1}{1 + i_{i,t}} = \beta E_t \left( M_{i,t,t+1} e^{-\pi_{i,t+1}} \right),
\]

\[
\left( \frac{1 + \tau (Q_{i,j,t}B_{i,j,t}/P_{j,t})}{1 + i_{j,t}} \right) = \beta E_t \left( M_{i,t,t+1} e^{\Delta \ln(Q_{i,j,t+1})} e^{-\pi_{j,t+1}} \right).
\]

Looking at equation (14), the effect of the transactions fee is to raise the net price paid for the foreign currency bond when the household has a long position, \( B_{i,j,t} > 0 \), and to lower the return. The price is increasing in the long position. Conversely, if the household has a short position, \( B_{i,j,t} < 0 \), the effect of the transaction fee is to lower the net issue price and to increase the cost of the foreign currency loan.

The optimality conditions for the labor-leisure choice are unaffected by the change to incomplete markets and continue to be described by equations (6) and (9).

### 1.2 Goods Demand

In each country, a continuum of firms, indexed by \( f \in [0,1] \) each produce a differentiated product. \( c_{i,j,t} \) is the consumption good produced in country \( j \) and consumed in country \( i \). The aggregate consumption by country \( i \) households of goods produced in country \( j \) is

\[
c_{i,j,t} = \left[ \int_0^1 c_{i,j,t}(f) \frac{\sigma - 1}{\sigma} df \right]^{\frac{\sigma}{\sigma - 1}},
\]

11
where $\sigma$ is the elasticity of substitution between varieties $f$. When $i = j$, this is ‘home’ goods consumption of ‘domestically’ produced goods, and for $i \neq j$, $c_{i,j,t}$ are imports. The price index associated with the bundle $c_{i,j,t}$ is

$$P_{i,j,t} = \left[ \int_{0}^{1} p_{i,j,t}(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}}. \quad (16)$$

Aggregate consumption in country $i$ is the constant elasticity of substitution (CES) index,

$$c_{i,t} = \left( d^\frac{1}{\mu} c_{i,i,t}^{\mu} + (1-d)^\frac{1}{\mu} c_{i,j,t}^{\mu} \right)^{\frac{\mu}{\mu-1}}. \quad (17)$$

The elasticity of substitution between ‘home’ and ‘foreign’ goods is $\mu$, and home-bias in consumption is represented by $d > 1/2$. The aggregate price level associated with equation (17) is

$$P_{i,t} = [dP_{i,i,t}^{1-\mu} + (1-d)P_{i,j,t}^{1-\mu}]^{\frac{1}{1-\mu}}. \quad (18)$$

### 1.3 Intermediate Goods Production

Firm $f \in [0, 1]$ is able to distinguish between domestic and foreign shoppers and can charge them different prices. Country $i$ firms set prices of their exports in country $j$’s currency. The production function for a firm is,

$$y_{i,t}(f) = A_{i,t} \ell_{i,t}(f). \quad (19)$$

The firm’s total costs are

$$\frac{W_{i,t}}{P_{i,t}} \ell_{i,t}(f).$$
Output is demand determined, \( y_{i,t}(f) = c_{i,i,t}(f) + c_{j,i,t}(f) \), where home and foreign demands are, respectively,

\[
c_{i,i,t}(f) = d \left( \frac{p_{i,i,t}(f)}{P_{i,t}} \right)^{-\sigma} \left( \frac{P_{i,i,t}}{P_{i,t}} \right)^{-\mu} c_{i,t}, \tag{20}
\]

\[
c_{j,i,t}(f) = (1 - d) \left( \frac{p_{j,i,t}(f)}{P_{j,t}} \right)^{-\sigma} \left( \frac{P_{j,i,t}}{P_{j,t}} \right)^{-\mu} c_{j,t}. \tag{21}
\]

It follows that labor employed by firm \( f \) is

\[
\ell_{i,t}(f) = \frac{c_{i,i,t}(f) + c_{j,i,t}(f)}{A_{i,t}}. \tag{22}
\]

Prices are sticky in the sense of Calvo (1983). Each period, the firm is allowed to change its price with probability \( 1 - \alpha_c \). If the firm is chosen to reset prices, it adjusts both the price for domestic market, \( p_{i,i,t}(f) \), which is set in country \( i \)'s currency, and the price of exports, \( p_{j,i,t}(f) \), set in units of country \( j \)'s currency. During the life of the contract, price is indexed to the continuously compounded steady state inflation rate (\( \bar{\pi}_i \) for domestic or \( \bar{\pi}_j \) for exports). These prices are set to maximize expected present value of future profits with prices fixed at the optimal choices. Formally, the problem is to maximize

\[
E_t \sum_{s=0}^{\infty} (\alpha_c \beta)^s \left[ \frac{p_{i,i,t}(f)e^{s\bar{\pi}_i}}{P_{i,t+s}} c_{i,i,t+s}(f) + \frac{Q_{i,j,t+s}p_{j,i,t}(f)e^{s\bar{\pi}_j}}{P_{j,t+s}} c_{j,i,t+s}(f) - \frac{W_{i,t+s}}{P_{i,t+s}} \ell_{i,t+s}(f) \right],
\]

subject to the output demand equations (20) and (21) and the labor demand equation (22).
1.4 Aggregation, Equilibrium, and Monetary Policy

We obtain aggregate domestic demand for domestically produced goods in country $i$ by equating firm $f$’s supply to demand,

$$A_{i,t}\ell_{i,t}(f) = d\left(\frac{P_{i,i,t}}{P_{i,t}}\right)^{-\mu} c_{i,t} - \sigma \left(\frac{P_{i,i,t}(f)}{P_{i,t}}\right)^{-\sigma} \left(\frac{P_{j,i,t}}{P_{j,t}}\right)^{-\mu} c_{j,t}, \quad (24)$$

then integrating equation (24) to get,

$$A_{i,t}\ell_{i,t} = c_{i,i,t}v_{i,i,t} + c_{j,i,t}v_{j,i,t}, \quad (25)$$

where $\ell_{i,t} = \int_0^1 \ell_{i,t}(f) \, df$ is total country 1 employment, and

$$c_{i,i,t} = d\left(\frac{P_{i,i,t}}{P_{i,t}}\right)^{-\mu} \left(\int_0^1 c_{i,i,t}(f)^{\frac{\sigma-1}{\sigma}} \, df\right)^{\frac{\sigma}{\sigma-1}}, \quad (26)$$

$$c_{j,i,t} = (1 - d)\left(\frac{P_{j,i,t}}{P_{j,t}}\right)^{-\mu} \left(\int_0^1 c_{j,i,t}(f)^{\frac{\sigma-1}{\sigma}} \, df\right)^{\frac{\sigma}{\sigma-1}}, \quad (27)$$

are aggregate domestic demand and export demand.

In equation (25), $v_{i,i,t}^p = \int_0^1 \left(\frac{p_{i,i,t}(f)}{P_{i,t}}\right)^{-\sigma} \, df$ is a measure of price dispersion for goods in the domestic market, and $v_{j,i,t}^p = \int_0^1 \left(\frac{p_{j,i,t}(f)}{P_{j,t}}\right)^{-\sigma} \, df$ is import price dispersion in foreign country $j$. The recursive representation for the price dispersion terms, $v_{i,j,t}^p (i = \{1, 2\}, j = \{1, 2\})$, is obtained by noting that a fraction $\alpha_c$ of these firms are stuck with last period’s price, $p_{i,j,t-1}(f)$. Since there are a large number of firms charging what they charged last period, it will also be the case that $\int_0^\alpha p_{i,j,t-1}(f)^{-\sigma} \, df = \alpha_c P_{i,j,t-1}^{-\sigma}. \quad (9)$ The complementary measure of firms $(1 - \alpha_c)$ are able to reset price for exports and the domestic market. They

$$P_{i,j,t}^{1-\sigma} = (1 - \alpha_c) p_{i,j,t}^* + \alpha_c P_{i,j,t-1}^{1-\sigma}. \quad (10)$$

Now the price dispersion term is defined to be

\[v_{i,j,t}^p = \int_0^1 \left(\frac{p_{i,j,t}(f)}{P_{i,t}}\right)^{-\sigma} \, df = \int_0^1 \left(\frac{p_{i,j,t}(f)}{P_{i,j,t}}\right)^{-\sigma} \, df \quad (11)\]
all reset to the same price, $p^*_{i,j,t}$. The result is the recursive representation,

$$v^p_{i,j,t} = (1 - \alpha_c) \left( \frac{p^*_{i,j,t}}{P_{i,j,t}} \right)^{-\sigma} + \alpha_c \left( \frac{P_{i,j,t-1}}{P_{i,j,t}} \right)^{-\sigma} v^p_{i,j,t-1} e^{-\sigma \tilde{\pi}_i} \quad (28)$$

Finally, to close the model, we specify the interest rate rule followed by the monetary authorities. The natural level of output is an infinite-dimensional moving average of past output,

$$\ln (\bar{y}_{j,t}) = \rho_y \ln (\bar{y}_{j,t-1}) + (1 - \rho_y) \ln (y_{j,t}) .$$

We take the deviation $(\ln (y_{j,t}) - \ln (\bar{y}_{j,t}))$ to measure the output gap. Let $\pi^*_j$ be country $j$’s inflation target. The monetary authorities in country $j$ set the short-term interest rate according to a Taylor (1993) type feedback rule with interest rate smoothing,

$$i_{j,t} = (1 - \delta_j) \bar{i} + \delta_j i_{j,t-1} + (1 - \delta_j) \left( \xi_j \left( \pi_{j,t} - \pi^*_j \right) + \zeta_j \left( \ln (y_{j,t}) - \ln (\bar{y}_{j,t}) \right) \right) , \quad (29)$$

where $\bar{i} = 1/\beta - 1$ is the steady state interest rate in the normalized model.
The Productivity Process and Parameter Values

The exogenous shocks driving the model are from productivity. As motivation for our productivity processes, we construct quarterly total factor productivity observations for Australia, Japan, and the US, from GDP, investment, and employment. Observations for Australia and the US extend from 1973Q1 through 2014Q4. Observations for Japan begin in 1979Q4. GDP and investment are from Datastream. Employment is also from Datastream for Japan and the US. Australian employment from 1973Q1-1978Q1 is from FRED, and from 1978Q2-2014Q4 it is from Datastream. Capital is imputed by the perpetual inventory method. A 4-quarter backward looking moving average is used to seasonally adjust the observations. Investment and GDP for Australia and Japan were converted to real 2013 US dollars to facilitate comparison across countries. One reason for looking at Japan and Australia is that they form a typical country pair in the carry trade with Japan serving as the funding source and Australia as the destination.

Figure 1 plots log TFP for the US, Japan, and Australia. To facilitate comparison, the Australian and US series are normalized to be 1 in 1973, and the Japanese series normalized to be 1 in 1979 when its series begins. Japan’s TFP grew rapidly in the 1980s but slowed down significantly in the 1990s, following the collapse of the Japanese stock and housing markets. A less pronounced slowdown for the US, and a more pronounced slowdown for Australia occurs in the early 2000s. Notably, log TFP \( a_t = \ln (A_t) \) for all three countries appear to be stochastically trending within our observational time-frame, and show no evidence of converging toward each other. To capture these features in productivity in our two-country model, we assume that each country’s log TFP has a unit root, and that the tendency for them to converge is weak. We cannot, however, let the TFP series diverge.

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10 We consider Australia, Japan, and the US because the model is suitable for developed economies.
11 Ready et al. (2016) emphasize Australia and Japan, whereas Backus et al. (2013) focus on Australia and the US.
from each other. A specification that achieves this is,\(^{12}\)
\[
\begin{align*}
\Delta a_{1,t} &= -\psi_1 (a_{1,t-1} - a_{2,t-1}) + \sigma_1 \epsilon_{1,t} \\
\Delta a_{2,t} &= -\psi_2 (a_{1,t-1} - a_{2,t-1}) + \sigma_2 \epsilon_{2,t}
\end{align*}
\] (30)
(31)

where \(\epsilon_{i,t} \sim \text{i.i.d} \sim N(0, 1)\) and \(\sigma_i > 0\), for \(i = \{1, 2\}\), and \(0 < \psi_2 < \psi_1 < 1\). Setting \(\psi_1\) to be a small positive number, and setting \(\psi_2\) slightly below \(\psi_1\) gives persistence to deviations between \(a_{1,t}\) and \(a_{2,t}\), while maintaining the technical requirements of cointegration.\(^{13}\)

(Figure 1 Here)

Figure 6 plots a long realization of the simulated process. We set \(\psi_1 = 0.003, \psi_2 = 0.0027\), and the innovation standard deviations \(\sigma_1 = \sigma_2 = 0.01\), which approximately matches the volatility of TFP growth (0.0073 for Australia and 0.0110 for Japan) in the data. The simulated log productivity processes cross only once in 4200 periods but do not diverge.

(Figure 6 Here)

The Calvo (1983) probability is set at \(\alpha_c = 0.8\), which implies an average contract duration of 3 quarters. Home bias is assumed to be \(d = 0.85\), and \(\sigma = 10\) which implies a mark up of 11 percent. The elasticity of substitution between domestic goods and imports is \(\mu = 1.5\).

To parameterize the utility function, we follow Swanson (2016) in setting \(\chi = 3\), which implies a Frisch elasticity of labor supply of \(1/3\) and set \(\eta\) to generate a steady state labor

\(^{12}\)Let \(z_t = a_{1,t} - a_{2,t}\) be the error-correction term. Subtracting equation (31) from equation (30) gives
\[
z_t = (1 + \psi_2 - \psi_1) z_{t-1} + (\sigma_1 \epsilon_{1,t} - \sigma_2 \epsilon_{2,t})
\]
The autoregressive coefficient, \(1 + \psi_2 - \psi_1\), is close to, but less than 1.

\(^{13}\)Kollmann (2016) works with a similar process in a two-country endowment economy model.
supply of $\bar{\ell}_1 = \bar{\ell}_2 = 1$. We consider a range of relative risk aversion values of 10, 20, 30, and 60. High degrees of risk aversion are typically needed to explain asset returns data.

The log consumption part of the utility function implies that the intertemporal elasticity of substitution is 1. This is lower than the values, ranging between 1.5 and 2, typically assumed in asset pricing research (e.g., Bansal and Yaron (2004), Colacito and Croce (2011), and Bansal and Shaliastovich (2012)). Empirical estimates of recursive utility functions that parameterize the intertemporal elasticity of substitution and employ both consumption and asset price data (e.g., Chen et al. (2007)), estimate the elasticity to lie between 1.11 and 2.22.

In our benchmark monetary rule, the parameters are symmetric across countries. The coefficient on the lagged interest rate is $\delta_j = 0.9$, and the inflation and output gap response coefficients, $\xi_j = 1.5$ and $\zeta_j = 0.5$, conform to the Taylor (1983) rule.

A third-order approximation of the model to its non-stochastic steady state is numerically solved with pruning, using Dynare 4.3.3. The third-order approximation is necessary in order to generate time-variation in risk premia, and pruning is required for non-explosive simulations. Kim et al. (2005) discuss how recursively built observations in second-ordered approximations introduce higher-ordered terms in the expansion that do not correspond to higher-order coefficients in the Taylor expansion. These higher-ordered terms generate explosive time paths in simulations, and a stable solution is obtained by pruning the extraneous higher order terms. These explosive elements are also present in third-order approximated simulations.

3 International Currency Returns

This section discusses the three empirical regularities on international currency returns in some detail, and reports the model’s contribution towards understanding them. Subsec-
tion 3.1 discusses the long-standing issue of the downward forward premium bias. Subsection 3.2 analyzes the carry trade return and Subsection 3.3 takes up Engel’s risk-reversal.

3.1 The Downward Forward Premium Bias

UIP says excess currency returns are zero in expectation. Because the difference between bond yields across countries is expected to be offset by a loss in the value of the high interest rate currency, UIP also says the interest rate differential is an unbiased predictor of the future change in the spot exchange rate. UIP implies $\alpha_0 = 0$ and $\beta_0 = 1$, in the regression of the depreciation rate of currency 1 on the interest rate differential,

$$
\Delta \ln (S_{1,2,t+1}) = \alpha_0 + \beta_0 (i_{1,t} - i_{2,t}) + \epsilon_{t+1}.
$$

(32)

Equation (32) is referred to as the Fama regression. Fama (1984) ran these regressions and reported estimates of $\beta_0$ that not only differed from 1, but were negative. This empirical pattern has been found to be pervasive and robust over time. The near universal estimates of $\beta_0 < 1$ is what we are calling the downward forward premium bias. Froot and Thaler (1990) distinguish between the forward premium puzzle (or anomaly), when $\beta_0 < 0$, and the forward premium bias, when $0 < \beta_0 < 1$. The ‘forward premium’ terminology stems from the epoch before the global financial crisis (2008), when the covered interest parity arbitrage condition held. At that time, there was an equivalence between the interest rate differential and the forward premium.\footnote{The forward premium is the percentage difference between the forward price of the foreign currency and the current spot price. Here, we use inter-bank interest rates. Interest rates are from Datastream. Exchange rates are from Bloomberg.}

If the forward premium puzzle is present, an excess return from going short the low interest rate currency and going long the high interest rate might be expected to be enhanced by an increase in the value of the long currency. If only a negative bias is present
0 < β₀ < 1, the excess yield differential might be expected to be less than fully offset by a loss in the long currency value. The dominant hypothesis for the downward forward premium bias is that excess currency returns are available to investors as compensation for differential currency risk.

Since the downward forward premium bias is well-known, has been shown to be remarkably robust over time, and has been extensively documented, it is perhaps not necessary to report here. Nevertheless, Table 1 shows estimates of equation (32) using low-inflation and developed countries with the US, Australia, and Japan alternately serving as the base country. Observations are quarterly and span from 1973Q1 to 2014Q4. The forward premium puzzle is, by no means universal. With the US as the base country, negative estimates of β₀ are obtained for 6 cases and significantly negative only once. When Japan is the base country, the puzzle is present in only 2 of 9 cases and never significantly negative. In 2 cases (Norway and Sweden), the slope exceeds 1. Similarly, with Australia as the base country, the puzzle is present in 5 cases but never significantly negative. However, the downward bias is pervasive. For each choice of base country, slope estimates are significantly less than 1 in 5 cases with the US as base country, 6 cases for Australia as base country, and 5 cases with Japan as base country.

Two other features of the table are worthy of note. The first is that R² are nearly zero. The second is that many of the regressions show evidence consistent with the exchange rate following a random walk (α₀ = β₀ = 0). For example, with Japan as the base country, neither the constant (α₀) nor slope (β₀) are ever individually significantly different from zero.

It is also worth mentioning that the forward premium puzzle does not say that a positive 

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15 See the surveys by Engel (1996, 2014) and Lewis (1995).
country 1-2 interest rate differential predicts an appreciation of currency 1. It says that the higher is \( i_{1,t} - i_{2,t} \), the smaller is the depreciation in currency 1. The exchange rate can still be increasing, but does so at a decreasing rate as the interest differential rises. It does not necessarily imply that the exchange rate is expected to decline. To infer the predicted direction of change, one needs also to properly account for the size and sign of the constant.

Table 2 shows the implied slope coefficient (\( \beta_0 \)) in the Fama regression generated by the model. In Panel A, monetary policy is set to our benchmark specification where the response coefficients on inflation are 1.5 and 0.5 on the output gap. Higher risk aversion allows the model to generate the forward premium bias. Under complete markets, the bias occurs with risk aversion as low as 20, and under incomplete markets, the model generates a bias, with an implied slope of 0.84 when risk aversion is 60.\(^{16}\)

\(<Table 2 Here>\)

In the model results shown in Panel A, the only differences between countries is in productivity. An important insight from international currency return research (e.g. Lustig and Verdelhan (2007) and Lustig et al. (2011)), is that differences across countries are essential for understanding risk premiums. Under the benchmark monetary policies, differences between country productivity processes alone are not sufficient to generate a sizable downward forward premium bias in the complete markets model.

Backus et al. (2013) study the effect of heterogeneous monetary policies in generating risk and the forward premium bias in a complete markets endowment model with recursive preferences and with endogenous inflation. In their endowment economy framework, the nominal interest rate is generated by a feedback rule from inflation and exogenous consumption growth. The nominal interest rate also has to equal (minus) the log of the

\(^{16}\)In the incomplete markets model, symmetric bond holding costs are assumed, \( \tau_{1,2} = \tau_{2,1} = \tau \). The results are robust to alternative settings of the international bond holding cost of \( \tau = 0.005, 0.01, 0.03, \) and 0.05. In the reported calculations, we set \( \tau = 0.01 \).
expected nominal SDF, which follows from the household’s Euler equation. They obtain a closed form solution for the model from imposing the restrictions implied by these two equations. Even though the model’s agents do not care about inflation per se, in the sense that it has no effect on welfare, inflation is generated endogenously in their framework, and the correlation between inflation and consumption growth and hence, the characteristics of currency returns, can be altered by varying the coefficients in the interest rate rule. They find that a combination of relative inflation accommodation (small $\xi_i$) and relative procyclicality (large $\zeta_i$) generates currency risk which produces the downward forward premium bias.\footnote{In Bansal and Shaliastovich (2012), exogenous inflation and consumption growth are governed by a joint process which they estimate from data. But because inflation is exogenous, it has no effect on welfare in their model.}

Panel B considers departures from the benchmark monetary policy. The productivity processes are unchanged and risk aversion is set at 60. In the first instance, country 2 is more accommodating of inflation ($\xi_2 = 1.2$). This moderately attenuates the complete markets bias but intensifies the incomplete markets bias. The results are similar when country 2 is accommodating and country 1 is a pure inflation targeter ($\xi_1 = 1.5, \xi_2 = 1.2, \zeta_1 = 0, \zeta_2 = 0.5$). The most pronounced downward bias is obtained, with an implied slope of 0.56, under incomplete markets when country 1 responds aggressively to inflation.

### 3.2 The Carry Trade Return

The carry trade is a rule that says to go short the low interest currency (e.g. the yen) and to go long the high interest currency (e.g. the Australian dollar). The carry trade places primary emphasis on interest rates. Exchange rate considerations are secondary. It can be seen how the carry would generate positive returns on average if the exchange rate followed a random walk, since on average, the carry trader is earning the interest rate differential. If a forward premium puzzle is present, the subsequent exchange rate movements might be
expected to further enhance the return from the interest differential, adding to the positive expected excess return. If only the downward forward premium bias is present, the carry trade might still yield positive profits because the yield differential is expected to be only partially offset by subsequent exchange rate movements.

An expanding literature has advanced our understanding of the carry trade. Many recent empirical studies focus on portfolios of the carry trade and investigate the cross-sectional variation of carry trade returns in relation to their exposure to risk factors (see Burnside et al. (2011), Lustig and Verdelhan (2007), Lustig et al. (2011), Menkhoff et al. (2013), Della Corte et al. (2013), and Berg and Mark (2017b)). Formation of portfolios enhances identification of systematic components of the returns by diversifying away idiosyncratic risks.

An example of the kinds of excess returns found among developed countries with similar (and relatively low) inflationary experiences is given in Table 3, which we take from Berg and Mark (2017a). Each period, they sort countries by interest rate from low to high and compute excess currency returns using the USD as the funding currency. The excess returns are divided into 6 portfolios and the average of the equally weighted portfolio returns are shown in the table. Also shown, are the mean interest differentials between the portfolios and the US, and the currency appreciation on the portfolio of foreign currencies.\(^{18}\)

\(<\text{Table 3 Here}>\)

Countries with the lowest interest rates \(P_1\), pay a carry return of -1.19 percent per annum. Their interest rates lie 2.9 percent below the US interest rate. If UIP held, the -2.9 percent interest differential would be offset by an +2.9 percent loss on the destination currencies, but instead, they gain on average 1.7 percent. The high interest rate portfolio \(P_5\), pays an average carry currency excess return of 3.2 percent. The 2.6 percent interest

\(^{18}\)Positive mean exchange rate returns mean the portfolios of currencies is rising in value relative to the US dollar.
rate differential is enhanced by an additional 67 basis point appreciation of the foreign currency. There is a forward premium puzzle present in the $P_3$ portfolio of currencies. In the portfolio of the highest interest currencies ($P_6$), the 6.7 percent interest rate differential is partially offset by a 2.9 percent loss on the exchange rate. The highest interest rate currencies depreciate on average, against the USD.

The carry trade return and the downward forward premium bias are related, but distinct phenomena. The point was made empirically by Hassan and Mano (2014). We illustrate the distinction using the complete markets specification outlined in Subsection 1.1.1. Here, we temporarily ignore the unit root in productivity (i.e., set $G_1 = G_2 = 1$) and $\beta$. Let $m_{t+1} = \ln (M_{t+1})$ be the log real SDF. Similarly, let the log nominal SDF be $n_{t+1} = \ln (N_{t+1}) = m_{t+1} - \pi_{t+1}$. Representing currency 1’s depreciation with equation (10) and interest rates with equation (7), the Fama regression can be expressed in terms of the SDFs as

$$n_{2,t+1} - n_{1,t+1} = \alpha_0 + \beta_0 (E_t (n_{2,t+1} - n_{1,t+1})) + \epsilon_{t+1}.$$ 

The forward premium puzzle is why the correlation between the relative log nominal SDFs are negatively correlated with expected relative log nominal SDFs. $E_t n_{t+1}$ is the conditional entropy of $N_{t+1}$. If the nominal SDFs are log-normally distributed, the conditional entropy of $N_{t+1}$ is $E_t (n_{t+1}) + \frac{1}{2} Var_t (n_{t+1})$.\(^{19}\) Exploiting these results, the country 1 currency risk premium can be represented as

$$\left( i_{1,t} - i_{2,t} - E_t \Delta \ln (S_{1,2,t+1}) \right) = \frac{1}{2} (Var_t (n_{2,t+1}) - Var_t (n_{1,t+1})).$$

The carry trade return then is, an issue about differences in SDF volatility across countries. If country 1 systematically pays a positive carry trade return, it has the smoother, lower variance nominal SDF. Country 1 pays the excess return presumably because it is the risky

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\(^{19}\)See Backus at al. (2001).
country. What is an explanation that reconciles country 1 being risky and also having the less volatile SDF? One explanation is the riskiness of country 1 induces its residents to save heavily through the precautionary motive. Over time, they have accumulated a large buffer stock of saving which they use to insulate the SDF from shocks. Countries with high interest rates also exhibit high saving rates, so the empirical patterns seem to fit the story.

Table 4 shows the results for the carry trade. The model specifications are the same as those in the analysis of the forward premium bias. The gross carry currency return at \( t + 1 \) is the excess return from going long currency 1 and going short currency 2 if \( i_{1,t} > i_{2,t} \). If, at \( t \), country 2 has the higher interest rate, the excess return is calculated as shorting currency 1 and going long currency 2. Panel A shows the mean gross carry currency excess returns generated under the benchmark monetary policy, for different values of risk aversion. Under symmetric monetary policy, neither the complete markets or incomplete markets model are generally able to generate positive carry trade excess returns. The carry returns also become increasingly negative as risk aversion is increased.

<Table 4 Here>

Under incomplete markets, people are subject to transactions costs involving international borrowing and lending. The gross carry calculations ignore these costs. The net carry figures are carry trade excess returns for country 1 and country 2 individuals after accounting for the international bond positional fees. Net carry returns are obtained as follows: A country 1 individual who goes long currency 1 by shorting currency 2 realizes the net excess return

\[
(1 + i_{1,t}) - \left( \frac{1 + i_{2,t}}{1 + \tau B_{1,2,t} Q_{1,2,t}} \right) \left( \frac{S_{1,2,t+1}}{S_{1,2,t}} \right),
\]
whereas a country 2 individual realizes the net excess return

\[
\left( \frac{1 + i_{1,t}}{1 + \tau B_{2,1,t}Q_{2,1,t}} \right) \left( \frac{S_{2,1,t+1}}{S_{2,1,t}} \right) - (1 + i_{2,t}).
\]

As a result of the positional fees losses in net carry excess returns, shown in Panel A, exceed the losses from the gross carry, as expected.

In Panel B, we again show the results from variations in monetary policy. A positive gross carry is generated under complete markets ranging between 2.4 percent and 2.9 percent, depending on the policy specification. Asymmetries in monetary policy also generate gross carry excess returns of similar magnitudes under incomplete markets, and net carry excess returns from the point of view of both country 1 and country 2 residents are positive.

How might monetary policy create systematic risks that are compensated? To economize on notation, let \( \sigma^2_t(x_{t+1}) = Var_t(x_{t+1}) \) denote the conditional variance of \( x_{t+1} \) and \( \sigma_t(x_{t+1}, u_{t+1}) \) be the conditional covariance between \( x_{t+1} \) and \( u_{t+1} \). Under complete markets, the nominal carry trade return is

\[
i_{1,t} - i_{2,t} - E_t \Delta \ln (S_{1,2,t+1}) = \frac{1}{2} \left\{ \begin{array}{c}
[\sigma^2_t(m_{2,t+1}) + \sigma^2_t(\pi_{2,t+1})] \\
- [\sigma^2_t(m_{1,t+1}) + \sigma^2_t(\pi_{1,t+1})] \\
+ 2 [\sigma_t(m_{1,t+1}, \pi_{1,t+1}) - \sigma_t(m_{2,t+1}, \pi_{2,t+1})]
\end{array} \right\}
\]

Monetary policy can increase the carry return paid by country 1 by lowering the conditional variance of the log real SDF, the conditional variance of inflation, and by increasing the conditional covariance between the log real SDF and inflation. In the case of log utility, this would mean generating a negative covariance between consumption growth and inflation.
3.3 Risk-Reversals

The conventional wisdom is that the high real-interest rate country should have a strong currency. IMF advice that countries defend against currency depreciation during foreign exchange crises by raising interest rates are founded on this view. This view is an implication of classic exchange rate models, such as Dornbusch (1976) and Frankel (1979). Engel (2016) observes that the positive relation between the real interest rate and currency strength seems to contradict the empirical evidence on the forward premium bias.

The argument, according to Engel (2016), is this. Say the real interest rate in country 2 is high. Country 2 should have the strong currency, and a strong currency means country 2 is safe. Its risk is low and the risk premium paid out over time should be negative. But if the downward forward premium bias is present (or if there is a carry trade return) in the short run, country 2 pays a positive risk premium to those who go long currency 2 and short currency 1. The question is how can country 2 be the risky country in the short run and the safe country in the long run? Engel’s answer is that there must be a risk reversal for country 2 over time.

Engel (2016) characterizes the issue in terms of real interest rate differentials and the real exchange rate. Let the ex post real excess return on a long position in currency 2 be

$$\rho_{t+1} = r_{2,t} - r_{1,t} + \Delta \ln (Q_{1,2,t+1})$$

He characterizes the downward forward premium bias as a positive correlation between the ex ante excess return and the real interest rate differential,

$$E_S \equiv Corr ((E_t \rho_{t+1}), (r_{2,t} - r_{1,t})) > 0.$$ 

When country 2 has the higher real interest rate, it is expected to pay a positive currency
excess return in the short run.

The long-run risk premium paid out by country 2 is
\[
\sum_{j=0}^{\infty} \rho_{t+1+j} = \sum_{j=0}^{\infty} (r_{2,t+j} - r_{1,t+j}) + \ln (Q_{1,2,\infty}) - \ln (Q_{1,2,t}),
\]
where \(Q_{1,2,\infty}\) denotes the long-run real exchange rate.\(^{20}\) If country 2, being the high real interest rate country, is safe in the long-run, the long-run risk premium should be negatively correlated with the current real interest rate differential,

\[
E_L \equiv Corr \left( E_t \left( \sum_{j=0}^{\infty} \rho_{t+1+j} \right), (r_{2,t} - r_{1,t}) \right) < 0.
\]

Engel (2016) estimates a three-variable vector error correction model and uses the estimated model to compute \(E_t \rho_{t+1}, E_t \left( \sum_{j=0}^{\infty} \rho_{t+1+j} \right),\) and \(Q_{1,2,\infty}\) for the G7 countries with the USD serving as the base currency. In every instance, he finds \(E_L < 0.\)\(^{21}\) He then studies log-linearized versions of long-run risk models under complete markets, considering differences across countries arising from differences in country-specific shocks and by considering heterogeneous dependence on the same global shocks. He concludes that these models are consistent with the forward premium bias, \(E_S > 0,\) but they cannot generate the risk reversal, \(E_L < 0.\) His suggestion is to introduce non-pecuniary liquidity returns on assets as an avenue to explain the risk-reversal. Valchev (2015) pursued just such a suggestion.

Table 5 reports the \(E_S\) and \(E_L\) correlations implied by our model. In Panel A, monetary policies are symmetric and set at the benchmark Taylor rule values. Risk aversion varies from 10 to 60. The complete markets model generates \(E_S\) and \(E_L\) with the correct signs, while the incomplete markets model does so only when risk aversion is 60.

\(<\text{Table 5 Here}>\)

Panel B allows variations in monetary policy with risk aversion of 60. Here, both models generally imply a risk-reversal. When there are monetary policy asymmetries, the strength

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\(^{20}\)Engel (2016) characterizes the puzzle in terms of covariances. Here, we work with correlations.

\(^{21}\)He also finds a downward forward premium bias for Canada and Italy, and a forward premium puzzle for France, Germany, Japan, and the UK.
of the risk reversal, as indicated by the size of the correlations, is stronger in the incomplete markets model.

4 Symmetrically Cointegrated TFP, Stationary TFP, and Decomposition

The previous section shows that the general equilibrium model can qualitatively produce the three international currency return facts under our nonstationary specification of TFP, and that monetary policy can magnify or dampen the underlying systematic risk priced into currency excess returns. This section asks how important is it for the model to display TFP heterogeneity, and how important is it for TFP to be non-stationary.

In Table 6, risk aversion is 60 and the error-correction coefficients in equations (30) and (31) are $\psi_1 = 0.003$ and $\psi_2 = -0.003$. This gives a symmetric log TFP processes with AR(1) coefficient on the error correction term of 0.994. The policy variations in this table conform to those considered previously with risk aversion of 60. Both the complete and incomplete market models generate a modest forward premium bias. Neither model generates a positive carry trade excess return. Complete markets generates the risk reversal, but the $E_S$ and $E_L$ correlations under incomplete markets have the wrong sign.

<Table 6 Here>

In Table 7, log TFP is the symmetric stationary AR(1) process, $a_t = \rho_A a_{t-1} + \epsilon_t$. Risk aversion is 60, and asymmetric monetary policy has country 1 both responding aggressively to inflation and relatively pro-cyclical ($\xi_1 = 2.0, \xi_2 = 1.2, \zeta_1 = 0.9, \zeta_2 = 0.5$). Variations in productivity persistence of $\rho_A$ ranging from 0.9 to 0.99 have little effect on the model’s ability to generate the return facts. In each case, a very small downward forward premium bias and carry trade excess return is generated. However, neither complete nor incomplete
markets models can generate the risk reversal when productivity is stationary. Stationarity in log TFP is inconsistent with the data on TFP and the model generally does not explain well the currency return facts when TFP is assumed to be stationary.

Table 7 Here

Table 8 presents a decomposition of the returns moments from each country’s productivity process. We do this for policy parameters $\xi_1 = 2.0, \xi_2 = 1.2, \zeta_1 = 0.9, \zeta_2 = 0.5$. The ‘Benchmark’ column replicates the earlier results where both productivity shocks are active. In the column $\sigma_2 = 0$, the productivity shocks for country 2 have been shut down. The results are weakened when uncertainty is driven only by country 1 productivity. There is much less forward premium bias, the carry trade excess return is much smaller, and the incomplete markets model does not explain the risk reversal. Productivity shocks for country 1 are shut down for the column labeled $\sigma_1 = 0$. As can be seen, the risk priced into returns by the model is generated in large part by shocks to country 2 productivity.

Table 8 Here

5 Impulse Response Functions

To illustrate some of the workings of the model, this section plots impulse responses to exogenous shocks to TFP growth in the incomplete markets model. Risk aversion is 60 and monetary policies are asymmetric with coefficients set at $\xi_1 = 1.5, \xi_2 = 1.2, \zeta_1 = \zeta_2 = 0.5$. Recall that the log TFP are unit-root processes so the shocks result in a permanent change in productivity. Due to the asymmetric ways the the error correction terms enter, a positive country 1 growth shock initially increases it’s TFP and output, raising $a_{1,t}$ but having no initial effect on $a_{2,t}$. At $t + 1$, the difference between $a_{1,t+1} - a_{2,t+1} = \sigma_1 > 0$, the standard deviation of the $\Delta a_{1,t}$ process. From $t + 1$ onward, $\Delta a_{1,t+1}$ and $\Delta a_{2,t+1}$ decline at the same,
very low rate \( \psi_1(a_{1,t} - a_{2,t}) = 3e - 5 \). A positive country 2 growth shock causes \( a_{1,t} - a_{2,t} < 0 \), which causes \( a_{1,t} \) and \( a_{2,t} \) to increase over time.

Figure 6 plots exchange rate and interest rate responses to productivity shocks. Figure 6 plots responses of the (ex ante) risk premium \( (i_{1,t} - i_{2,t} - E_t \Delta s_{1,2,t+1}) \) and the ex post realization excess return, \( (i_{1,t} - i_{2,t} - \Delta s_{1,2,t+1}) \). Looking at the responses to country 2 productivity, the shock to \( A_2 \) initially causes the risk premium to increase. Both \( i_1 \) and \( i_2 \) increase, but the differential \( i_1 - i_2 \) initially declines. Country 1’s currency appreciation \( \Delta s_{1,2,t} < 0 \) more than offsets the change in the interest differential, resulting in an increase in the excess return. The implied slope in the Fama regression is not negative because both \( \Delta s_{1,2,t+1} \) and \( i_{1,t} - i_{2,t} \) decline.

A country 1 productivity growth shock makes country 1 risky in the short run. The impact effect is to generate a positive expected excess return from a long position in country 1’s currency, but this expected excess return turns negative shortly afterward. A country 2 productivity growth shock makes country 2 risky, generating a positive expected excess return from a long position in country 2’s currency. The dynamical response of the risk premium in Figure 6 illustrates the risk-reversal at work. Figure 6 plots labor and consumption responses. Productivity shocks affect aggregate consumption in both countries to jump up on impact, resulting in positively correlated consumption growth rates. On the other hand, labor is negatively correlated. A positive country 1 shock initially reduces country 1 labor and increases it in country 2.
6 Conclusion

This paper has shown how a two-country dynamic stochastic general equilibrium macro model can explain three empirical facts that characterize international currency returns—the downward forward premium bias, the carry trade return, and the long-run risk reversal. Previous research has typically employed endowment models with exogenous consumption and has not jointly addressed all three aspects of returns. Our model lays the foundation for a unified framework for understanding empirical patterns in international currency returns.

Some sort of heterogeneity across countries is an essential element in understanding international currency returns. In this reasonably standard two-country macro model, nonstationarity and cross-country heterogeneity in productivity was this element. Cross-country heterogeneity in monetary policy alone is not enough, although monetary policy heterogeneity can accentuate the risks being priced into international currency returns. Both a complete markets version and an incomplete markets version of the model are generally consistent with the downward forward premium bias, the carry trade return, and the risk reversal.

References


Figure 1: Log TFP

Log TFP

USA
JPN
AUS

Date
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
Table 1: Fama Regression–Forward Premium Puzzle/Bias in the Data

<table>
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<tr>
<th>Country 2</th>
<th>α₀</th>
<th>t-stat</th>
<th>β₀</th>
<th>t-stat</th>
<th>R²</th>
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<tbody>
<tr>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Australia</td>
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<td>-0.487</td>
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<td>0.003</td>
</tr>
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<td>-0.854</td>
<td>0.006</td>
</tr>
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<td>0.007</td>
<td>1.244</td>
<td>-0.171</td>
<td>-0.411</td>
<td>0.001</td>
</tr>
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<td>0.558</td>
<td>1.173</td>
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<td>0.346</td>
<td>0.363</td>
<td>0.002</td>
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<td>0.819</td>
<td>0.520</td>
<td>0.011</td>
</tr>
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<td>-0.012</td>
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<td>1.074</td>
<td>0.278</td>
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<td></td>
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<td>-1.074</td>
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<td>-1.244</td>
<td>-0.171</td>
<td>-0.411</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Notes: The regression is $\Delta \ln(S_{1,2,t+1}) = \alpha_0 + \beta_0(i_{1,t} - i_{2,t}) + \epsilon_{t+1}$. Data is quarterly, from 1973Q1 through 2014Q4. T-ratios are constructed with Newey-West (1983) standard errors.
Table 2: Implied Slope in Fama Regression–Forward Premium Puzzle/Bias

<table>
<thead>
<tr>
<th>Risk Aversion</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete</td>
<td>1.046</td>
<td>0.965</td>
<td>0.880</td>
<td>0.705</td>
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<tr>
<td>Incomplete</td>
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<td>1.162</td>
<td>1.069</td>
<td>0.839</td>
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</table>

B. Alternative Monetary Policies

<table>
<thead>
<tr>
<th>Policy Parameters</th>
<th>ξ_1</th>
<th>ξ_2</th>
<th>ζ_1</th>
<th>ζ_2</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.5</td>
<td>1.2</td>
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</tr>
<tr>
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<td>0.0</td>
<td>1.2</td>
<td>0.5</td>
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</tbody>
</table>

Risk Aversion is 60

Complete | 0.794 | 0.834 | 0.775 | 0.761
Incomplete | 0.646 | 0.643 | 0.563 | 0.659

Notes: The regression is \( \Delta \ln(S_{2,t+1}) = \alpha_0 + \beta_0(i_{1,t} - i_{2,t}) + \epsilon_{t+1} \). Under the benchmark monetary policy, inflation response coefficients are \( \xi_1 = \xi_2 = 1.5 \) and output-gap response coefficients are \( \zeta_1 = \zeta_2 = 0.5 \).


<table>
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<tr>
<th></th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
<th>( P_4 )</th>
<th>( P_5 )</th>
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<tr>
<td>Mean Currency Excess Return</td>
<td>-1.188</td>
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<td>0.828</td>
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<td>Mean Exchange Rate Return</td>
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<td>0.816</td>
<td>1.287</td>
<td>-0.316</td>
<td>0.674</td>
<td>-2.886</td>
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</tbody>
</table>

Notes: This table is taken from Berg and Mark (2017a). Developed countries include Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Netherlands, New Zealand, Norway, Poland, Portugal, Singapore, South Korea, Spain, Sweden, Switzerland, Taiwan, United Kingdom, and United States.
Table 4: Model Implied Carry Trade Returns

<table>
<thead>
<tr>
<th>A. Benchmark Monetary Policies</th>
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<td>Risk Aversion</td>
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</tr>
<tr>
<td>Gross Carry</td>
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<tr>
<td>Incomplete Markets</td>
<td></td>
</tr>
<tr>
<td>Risk Aversion</td>
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</tr>
<tr>
<td>Net Carry 1</td>
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<tr>
<td>Net Carry 2</td>
<td>-0.087</td>
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</table>

<table>
<thead>
<tr>
<th>B. Alternative Monetary Policies</th>
<th></th>
</tr>
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<tr>
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<td>$\zeta_1$</td>
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<tr>
<td>$\xi_2$</td>
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<tr>
<td>Gross Carry</td>
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</tr>
<tr>
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<td>Net Carry 2</td>
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Table 5: Model Implied Risk Reversal

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<th>30</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
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</tr>
<tr>
<td>$E_S$</td>
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<td>0.241</td>
<td>0.401</td>
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<tr>
<td>$E_L$</td>
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<td>-0.210</td>
<td>-0.387</td>
</tr>
<tr>
<td>Incomplete Markets</td>
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<td></td>
</tr>
<tr>
<td>$E_S$</td>
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<tr>
<td>$E_L$</td>
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<td>-0.032</td>
<td>-0.250</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Risk Aversion</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>60</th>
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<tbody>
<tr>
<td>Complete Markets</td>
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<tr>
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<td>-0.235</td>
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<tr>
<td>$E_L$</td>
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<td>-0.417</td>
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Table 6: Symmetrically Cointegrated TFP under Alternative Monetary Policies

<table>
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<tr>
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<td>0.5</td>
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</tr>
<tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fama</td>
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<td>0.883</td>
<td>0.879</td>
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<tr>
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<tr>
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<td>-0.007</td>
<td>-0.041</td>
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<td>-0.640</td>
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<tr>
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<td>Eₛ</td>
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<td>-0.548</td>
<td>-0.530</td>
<td>-0.486</td>
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<tr>
<td>Eₗ</td>
<td>0.418</td>
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<td>0.444</td>
<td>0.408</td>
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</table>

Note: ψ₁ = 0.003, ψ₂ = −0.003. Risk Aversion is 60.

Table 7: Stationary TFP under Monetary Policy Asymmetry and Varying Persistence

<table>
<thead>
<tr>
<th>ρₐ</th>
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</thead>
<tbody>
<tr>
<td>0.90</td>
</tr>
</tbody>
</table>

| Complete Markets |     |     |     |     |
| Fama            | 0.986 | 0.987 | 0.985 | 0.979 |
| Carry           | 0.014 | 0.040 | 0.085 | 0.155 |
| Eₛ              | -0.185 | -0.229 | -0.304 | -0.403 |
| Eₗ              | -0.239 | -0.196 | -0.122 | -0.037 |
| Incomplete Markets |     |     |     |     |
| Fama            | 0.997 | 0.999 | 0.999 | 0.990 |
| Gross Carry     | 0.092 | 0.093 | 0.078 | 0.146 |
| Net Carry 1     | 0.067 | 0.031 | -0.057 | -0.187 |
| Net Carry 2     | 0.066 | 0.026 | -0.060 | -0.141 |
| Eₛ              | -0.196 | -0.243 | -0.311 | -0.425 |
| Eₗ              | -0.104 | -0.048 | 0.003 | 0.037 |

Note: Risk Aversion is 60. Monetary policy parameters are ξ₁ = 2.0, ξ₂ = 1.2, ζ₁ = 0.9, ζ₂ = 0.5.
<table>
<thead>
<tr>
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<th>$\sigma_1 = 0$</th>
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<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>Fama</td>
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<td>1.007</td>
<td>0.849</td>
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<tr>
<td>Carry</td>
<td>2.876</td>
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<td>3.179</td>
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<td>$E_S$</td>
<td>0.220</td>
<td>0.387</td>
<td>-0.034</td>
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<tr>
<td>$E_L$</td>
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<td>-0.241</td>
<td>-0.023</td>
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<tr>
<td><strong>Incomplete Markets</strong></td>
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<td></td>
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</tr>
<tr>
<td>Fama</td>
<td>0.659</td>
<td>0.944</td>
<td>0.617</td>
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<td>Gross Carry</td>
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<td>3.631</td>
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<td>Net Carry 1</td>
<td>1.240</td>
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<td>Net Carry 2</td>
<td>1.217</td>
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<td>3.394</td>
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<td>0.407</td>
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<td>0.448</td>
</tr>
<tr>
<td>$E_L$</td>
<td>-0.417</td>
<td>0.027</td>
<td>-0.449</td>
</tr>
</tbody>
</table>

Note: Monetary policy parameters, $\xi_1 = 2.0, \xi_2 = 1.2, \zeta_1 = 0.9, \zeta_2 = 0.5.$
Figure 2: Simulated Log TFP
Figure 3: Impulse Responses
Figure 4: Impulse Responses
Figure 5: Impulse Responses