A comment on “Linear-Quadratic Approximation to Unconditionally Optimal Policy: The Distorted Steady-State”

Christian Jensen
University of South Carolina

Context

- Most problems are not LQ, so LQ approximations
- But, *naive* LQ approximation can yield wrong answers - Judd (96, 98)
- Correct LQ requires approximating around steady state with optimal policy - Fleming (71), Magill (77)
  - Benigno and Woodford (08) do corrected LQ approximations around steady state with TP policy
  - Damjanovic, Damjanovic and Nolan (08) do corrected LQ around steady state with UO or OUC policy
  - Correction necessary when steady state is distorted
- Question remains: What is best way for commitment to achieve *continuation*, TP or OUC?

TP vs OUC

- In forward-looking models, optimal commitment is not time-invariant: discretion today, promise commitment rule in future
- Woodford (99) argues commitment more credible if same equation applied at all times - continuation
- What is optimal continuation policy under commitment?
  - Woodford (99) suggests always applying optimal commitment rule
  - Blake (01) and Jensen and McCallum (02) suggest optimizing *unconditional* expected value of objective subject to time-invariant policy - OUC policy
  - Optimizing original objective subject to time-invariant policy does not yield continuation
  - Jensen and McCallum (08) show no optimal continuation policy exists for conditional objective in forward-looking model

TP vs OUC with corrected LQ approximation

- LQ approximated objectives differ, making it harder to compare
- If TP or OUC steady state is not well-defined, approach unusable
- Using non-LQ objective to compare might not be a good idea, policies are just linear approximations to fully optimal - BW (08)
- TP steady state matches that of optimal commitment, but know from golden vs modified golden rule that it is a bad idea to choose policy based on steady state

Correcting LQ approximations does not resolve TP vs OUC, enhances its importance

- Choice is not facilitated by computational burden - DDN(08, 08)

Why do TP and OUC differ?

- TP chooses today's policy *as if* it had been committed to a long time ago, i.e. as if affected past expectations
- OUC chooses today's policy taking into account it was not committed to a long time ago, so cannot affect past expectations, but takes into account effect on expectations today and in future due to commitment to time-invariant rule

Why do TP and OUC differ?

- TP ignores that policy expectations for the present period are given, and pretends they are not
- OUC exploits that policy expectations for current period are given, to the degree possible with a time-invariant policy equation
  - Since this will always be true when reoptimizing, OUC policy satisfies continuation, which is what is required
  - OUC exploits initial expectations as optimal commitment does, but not to the same degree, due to the time-invariance constraint
  - TP does not exploit this, which is why it does worse on average, by not achieving continuation in the cheapest possible way

Why do TP and OUC differ?

- TP is suboptimal in initial period(s), but optimal for all later
- OUC is suboptimal for all periods, but chosen so as to be optimally suboptimal (unconditionally) given continuation constraint

Example highlighting TP weakness

Minimize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \alpha y_t^2\right) \tag{1}$$

subject to

$$\pi_t = \beta^J E_t \pi_{t+J} + \lambda y_t + u_t,\ t = 0,\ 1,\ 2,... \tag{2}$$

$$\alpha, \lambda > 0,\ 1 > \beta > 0 \tag{3}$$

Sbordone (2007):

$$\hat{\pi}_t = \tilde{\varrho}\hat{\pi}_{t-1} + \zeta_t \hat{s}_t + b_{1t} E_t \hat{\pi}_{t+1} + b_{2t} \sum_{j=2}^{\infty} \varphi_{1t}^{j-1} E_t \hat{\pi}_{t+j} + u_t \tag{4}$$

log-linearizing around a steady state with a time-varying trend inflation

Example highlighting TP weakness

Optimal commitment policy is

$$\pi_t = -\frac{\alpha}{\lambda} y_t,\ t = 0,\ 1, \ldots,\ J-1 \tag{5}$$

$$\pi_t = -\frac{\alpha}{\lambda} y_t + \frac{\alpha}{\lambda} y_{t-J},\ t = J,\ J+1,\ J+2, \ldots \tag{6}$$

$$\text{TP suggests } \pi_t = -\frac{\alpha}{\lambda} y_t + \frac{\alpha}{\lambda} y_{t-J},\ t = 0,\ 1,\ 2, \ldots \tag{7}$$

$$\text{OUC suggests } \pi_t = -\frac{\alpha}{\lambda} y_t + \beta^J \frac{\alpha}{\lambda} y_{t-J},\ t = 0,\ 1,\ 2, \ldots \tag{8}$$

As $J \to \infty$, optimal commitment converges to

$$\pi_t = -\frac{\alpha}{\lambda} y_t \tag{9}$$

OUC does too, but TP does not

Disagreements with DDN and steady state concern

- Should not use unconditional optimization in purely backward-looking conditional problems
  - Unnecessary for continuation of optimal plan
  - Just as TP never optimal for unconditional problems

- Not desirable that discounting in objective becomes irrelevant with OUC

- Not desirable that end up at "wrong" steady state with OUC
  - But should not choose policy based just on steady state properties

But last two acceptable if OUC is cheapest way to achieve continuation, on average