Optimal Monetary Policy in a Model of the Credit Channel

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Abstract

We consider a simple extension of the basic New-Keynesian setup where financial markets are imperfect. In our economy, asymmetric information and default risk lead banks to optimally charge a lending rate above the risk-free rate. Our contribution is threefold. First, we show that our loglinearized model nests the case with frictionless financial markets. A key difference is that marginal costs increase with the output gap but also with the credit spread and the nominal interest rate. Second, we find that both technology and financial market shocks generate a trade-off between output and inflation stabilisation. Third, we show that the presence of financial market imperfections and endogenous variations in credit spreads can be relevant for the characterization of optimal monetary policy.

Keyworks: optimal monetary policy, financial markets, asymmetric information

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1 Introduction

Central banks devote much effort to evaluate the financial position of households, firms and financial institutions, and to monitor the evolution of credit aggregates and interest rate spreads. One reason is that these factors affect the prospects for inflation and output and are therefore important for monetary policy decisions.

During several historical episodes, central banks showed sharp reactions to changes in financial conditions. This occurred in the US during the late 1980s, when banks experienced large loan losses as consequence of the bust in the real estate market. Due to weak financial conditions, banks could not raise new capital and because of the requirement to comply with the Basel Accord, they were forced to cut back on loans. This led to a slowdown in credit growth and aggregate spending. According to Rudebush (2006), this slowdown contributed to the FOMC decision to reduce the federal funds rate well below what suggested by an estimated Taylor rule. A second example is provided by the financial market turmoil, initiated in the first half of 2007 with the deterioration in the performance of nonprime mortgages in the US. In August 2007, the FOMC justified a cut in the discount rate of 50 basis points by expressing concerns about the ongoing deterioration of financial market conditions and tightening of credit conditions, which increased appreciably the downside risks to growth.

The importance of financial conditions for the conduct of monetary policy can hardly be rationalized in the context of the frameworks commonly used for monetary policy analysis, such as the New-Keynesian model with nominal price rigidities (see e.g. Woodford (2003)). These frameworks typically rule out a role for credit aggregates and interest rate spreads by assuming frictionless financial markets.

We consider the simplest possible extension of the basic New-Keynesian setup, where financial markets are imperfect. As in Bernanke, Gertler and Gilchrist (1989) and Carltrom and Fuerst (1997, 1998), we model an environment where firms need to borrow funds in advance of production but they are credit constrained. We deviate from those papers by assuming that loans are denominated in nominal - rather than real - terms. In our economy, firms have private information about the realization of an idiosyncratic productivity shock, which banks can only monitor ex-post at a cost. The presence of asymmetric information introduces default risk, so that banks find it optimal to charge a lending rate which is above their marginal cost (the deposit rate).

The main appealing feature of our model is its analytical simplicity and the possibility to disentangle the role of financial frictions for inflation and output dynamics. We obtain three main sets of results.

First, we show that a reduced form of our model is similar in structure to the one arising in the New-Keynesian setup with frictionless financial markets. As in the standard case, it is characterized by an intertemporal IS equation, a New-Keynesian Phillips curve and a policy rule. The main difference is that marginal costs now include the output gap but also the credit spread (i.e. the difference between the loan rate and the policy instrument) and the nominal interest rate gap. These additional terms reflect the existence of information asymmetries and credit constraints. Fluctuations in the credit spread and in the policy interest rate affect inflation and output by changing the availability of credit in the economy.

Second, we find that both technology and financial market shocks act as exogenous "cost-push" factors, i.e. they generate a trade-off between output and inflation stabilisation. Moreover, the existence of asymmetric information and the nominal denomination of debt introduce a role for the credit spread and the nominal interest rate as endogenous "cost-push" factors, which do not appear in the benchmark model with frictionless financial markets.

Third, we analyze optimal monetary policy when firms are credit constrained and banks set loans and lending rates as a result of an optimal contract.[1] We show that the presence of financial market imperfections and endogenous variations in credit spreads change the characterization of optimal monetary policy. For a special case of the model, we obtain a simple quadratic approximation to the welfare function. We find that welfare is reduced by volatility of inflation and the output gap, as in the benchmark case with frictionless financial markets. In the presence of debt, however, welfare also tends to be reduced by the volatility of the nominal interest rate and of the credit spread.

[1] In ongoing work, Curdia and Woodford (2008) characterize the optimal monetary policy in a model where financial frictions matter because of heterogeneity in the spending opportunities available to different households. Our work differ in the underlying source of financial frictions and in the main findings. While the presence of financial market imperfections does not affect the optimal target criterion for monetary policy in their model, it does affect it in our environment. Other related works are Ravenna and Walsh (2006) and Faia and Monacelli (2006). The main differences are that the former paper characterizes optimal monetary policy when firms borrow in advance of production but, due to the absence of default risk, the cost of funds is the risk-free rate. The latter paper compares the welfare losses of alternative simple interest rate rules in a model similar to ours, without chracterizing optimal monetary policy.

The paper proceeds as follows. In section 1, we describe the environment. In section 2, we derive the conditions characterizing the equilibrium of the economy when financial contracts are written in nominal terms. In section 3, we log-linearize the model, we derive its reduced form and we compare it to the reduced form arising in the efficient equilibrium. This enables us to highlight the effect of financial market frictions on inflation and output dynamics. In section 4, we characterize the optimal monetary policy. For a particular case of our model economy, we obtain a simple quadratic approximation of the social welfare, which we compare to the one arising under frictionless financial markets. We also derive the first-order conditions of the social planner problem under discretion and we discuss the role of financial frictions for the optimal conduct of monetary policy. In section 5, we characterize numerically optimal monetary policy under commitment. In section 6, we conclude.

2 The environment

The economy is inhabited by a representative infinitely-lived household, a continuum of wholesale firms producing a homogeneous good and owned by risk-neutral entrepreneurs, a continuum of monopolistically competitive retail firms producing differentiated goods and owned by the households, a zero-profit financial intermediary, and a central bank.

In the first part of the period, households decide how to allocate their nominal wealth among existing nominal assets, namely money, a portfolio of nominal state-contingent bonds, and one-period deposits. In the second part of the period, they receive wage income and purchase consumption goods.

Wholesale firms are endowed with a linear production technology that uses labor as the only input. They need to pay workers in advance of production. At the beginning of the period, each firm receives from the government an exogenous nominal endowment that is used as the firm's internal funds. However, these funds are not sufficient to finance the wage bill, so firms need to raise external finance. In their productive activity, wholesale firms face idiosyncratic productivity shocks and thus default risk. Lending occurs through perfectly competitive banks, which are able to ensure a safe return by providing funds to the continuum of firms facing idiosyncratic shocks. Bank loans take the form of risky debt, which is the optimal contractual arrangement between lenders and borrowers in the presence of asymmetric information and costly state verification.

Firms in the retail sector buy the homogeneous good from wholesale firms in a competitive market and use them to produce differentiated goods at no costs. Because of this product differentiation, retail firms acquire some market power and become price makers. In their price-setting activity, however, they are not free to change their price at will, because prices are subject to Calvo contracts. Retail firms are owned by the households, who receive their profits.

2.1 Households

At the beginning of period t, the financial market opens. First, the interest on nominal financial assets acquired at time $t-1$ is paid. The households, holding an amount W_t of nominal wealth, choose to allocate it among existing nominal assets, namely money M_t, a portfolio of nominal state-contingent bonds A_{t+1} each paying a unit of currency in a particular state in period $t+1$, and one-period deposits denominated in units of currency D_t paying back $R_t^d D_t$ at the end of the period.

In the second part of the period, the goods market opens. Households' money balances are increased by the nominal amount of their revenues and decreased by the value of their expenses. Taxes are also paid or transfers received. The amount of nominal balances brought into period $t+1$ is equal to

$$M_t + P_t w_t h_t + Z_t - P_t c_t - T_t, \tag{1}$$

where h_t is hours worked, w_t is the real wage, Z_t are nominal profits transferred from retail producers to households, and T_t are lump-sum nominal taxes collected by the government. c_t denote a CES aggregator of a continuum $j \in (0,1)$ of differentiated consumption goods produced by retail firms,

$$c_t = \left[\int_0^1 c_t\left(j\right)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

with $\varepsilon > 1$. $P_t\left(j\right)$ denotes the price of good j, and $P_t = \left[\int_0^1 P_t\left(j\right)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}}$ is the price of the CES aggregator.

Nominal wealth at the beginning of period $t+1$ is given by

$$W_{t+1} = A_{t+1} + R_t^d D_t + R_t^m \left\{M_t + P_t w_t h_t + Z_t - P_t c_t - T_t\right\}, \tag{2}$$

where R_t^m denotes the interest paid on money holdings.

The household's problem is to maximize preferences, defined as

$$E_o\left\{\sum_0^{\infty}\beta^t\left[u\left(c_t\right)+\kappa\left(m_t\right)-v\left(h_t\right)\right]\right\}, \tag{3}$$

where $u_c > 0$, $u_{cc} < 0$, $\kappa_m \geq 0, \kappa_{mm} < 0$ and $v_h > 0$, $v_{hh} > 0$, and $m_t \equiv M_t/P_t$ denotes real balances. The problem is subject to the budget constraint

$$M_t + D_t + E_t\left[Q_{t,t+1}A_{t+1}\right] \leq W_t, \tag{4}$$

Define $\pi_t \equiv \frac{P_t}{P_{t-1}}$ and $\Delta_{m,t} \equiv \frac{R_t - R_t^m}{R_t}$. The optimality conditions can be written as

$$\frac{v_h\left(h_t\right)}{u_c\left(c_t\right)} = w_t \tag{5}$$

$$\frac{1}{R_t} = E_t\left[Q_{t,t+1}\right] \tag{6}$$

$$R_t = R_t^d$$

$$u_c\left(c_t\right)+\kappa_m\left(m_t\right) = \beta R_t E_t\left\{\frac{u_c\left(c_{t+1}\right)+\kappa_m\left(m_{t+1}\right)}{\pi_{t+1}}\right\}$$

$$\frac{\kappa_m\left(m_t\right)}{u_c\left(c_t\right)} = \frac{\Delta_{m,t}}{1-\Delta_{m,t}}. \tag{7}$$

Moreover, the optimal allocation of expenditure between the different types of goods lead to the demand functions

$$c_t\left(j\right) = \left(\frac{P_t\left(j\right)}{P_t}\right)^{-\varepsilon} c_t, \tag{8}$$

where $P_t\left(j\right)$ is the price of good j.

2.2 Wholesale firms

The wholesale sector consists of a continuum of competitive firms, indexed by i, owned by infinitely lived entrepreneurs. Each firm produces the amount $y_{i,t}$ of a homogeneous good, using a linear technology

$$y_{i,t} = A_t \omega_{i,t} l_{i,t}. \tag{9}$$

Here A_t is an aggregate exogenous productivity shock and $\omega_{i,t}$ is an iid productivity shock with distribution function Φ and density function ϕ.

At the beginning of the period, each firm receives a nominal endowment $P_t\tau$ that can be used as internal funds.[2] Since these funds are not sufficient to finance the firm's desired level of production, firms need to raise external finance.

[2] As τ has no implication for the dynamics of the model, we assume for simplicity that the transfer is fixed across firms and time periods.

The assumption that firms receive an endowment from the government at the beginning of the period is made for simplicity as it enables to provide an analytical characterization of the optimal policy in the presence of credit constraints and information asymmetry. It is standard in this literature to assume that entrepreneurs decide in period t how to allocate their profits to consumption and investment expenditures (see e.g. Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999)) . The value of the stock of capital available to firms in period $t+1$ provides the firm with a certain net worth (internal funds) that can be used in that period production. In that environment, aggregate shocks affect the evolution of firms' net worth, thus creating endogenous persistence. In our environment, the absence of capital accumulation implies that the persistence of the endogenous variables merely reflects the persistence of the exogenous shocks. Nonetheless, financial frictions still affect the economy through the credit constraint and the endogenous spread over the risk-free rate charged by financial intermediaries to firms, which reflects the existence of default risk.

2.2.1 Factor demand

Firms need to raise external finance to pay for labor services. Before observing the idiosyncratic productivity shock $\omega_{i,t}$, firms sign a contract with the financial intermediary to raise the amount $P_t\left(x_{i,t}-\tau\right)$, for total funds at hand $P_t x_{i,t}$, where

$$x_{i,t} \geq w_t l_{i,t}. \tag{10}$$

We assume that entrepreneurs sell output only to retailers. Let $\overline{P}_t$ be the price of the wholesale homogenous good, and $\frac{\overline{P}_t}{P_t} = \chi_t^{-1}$ the relative price of wholesale goods to the aggregate price of retail goods. Each firm i's demand for labor is derived by solving the problem

$$\max \left[\frac{\overline{P}_t}{P_t}\mathcal{E}\left[A_t\omega_{i,t}l_{i,t}\right]-w_t l_{i,t}\right]$$

subject to the financing constraint (10). Here the expectation $\mathcal{E}[\cdot]$ is taken with respect to the idiosyncratic shock unknown at the time of the factor hiring decision, and w_t denotes the payment of labor services measured in terms of the final consumption good. Denote the Lagrange multiplier on the financing constraint as $(q_{i,t}-1)$. Optimality requires that

$$q_{i,t} = q_t = \frac{A_t}{w_t\chi_t} \tag{11}$$

$$x_{i,t} = w_t l_{i,t} \tag{12}$$

implying that

$$\mathcal{E}\left[y_{i,t}\right] = A_t l_{i,t} = A_t \frac{x_{i,t}}{w_t} = \chi_t q_t x_{i,t}. \tag{13}$$

Equation (13) states that, as the production function is CRS, wholesale firms must sell at a mark-up $\chi_t q_t$ over firms' production costs. This allows them to cover for the presence of credit frictions and for the monopolistic distortion in the retail sector. This latter matters for firms in the wholesale sector because P_t is the deflator of the nominal wage, and thus affects the real marginal cost faced by wholesale producers.

Equation (12) states that the financing constraint is always binding. Given the contract stipulated by the firm with the financial intermediary (which sets the amount of funds $x_{i,t}$ and the repayment on these funds), the firm always find it profitable to use the entire amount of funds and produce, also when expected productivity is low. This way, it can minimize the probability of default.

2.2.2 The financial contract

Loans are stipulated in units of currency after all aggregate shocks have occurred, and repaid at the end of the same period. Lending occurs through the financial intermediary, which collects deposits from households and use them to finance loans to firms.

Firms face an idiosyncratic productivity shock, whose realization is observed at no costs only by the entrepreneur. The financial intermediary can monitor its realization but a fraction of the firm's output is consumed in the monitoring activity. If the realization of the idiosyncratic shock is sufficiently low, the value of the firm's production is not sufficient to repay the loan and the firm defaults. Households lend to firms through a financial intermediary, which is able to ensure a safe return. This is possible because by lending to the continuum of firms $i \in (0, 1)$ producing the wholesale good, the financial intermediary can differentiate the risk due to the presence of idiosyncratic shocks.

The informational structure corresponds to a standard costly state verification (CSV) problem. The solution is a debt contract (see e.g. Gale and Hellwig (1985)) that is characterized by three properties. First, the repayment to the FI is constant in states when monitoring does not occur. Second, the firm is declared bankrupt if and only if the fixed repayment cannot be honoured. Third, in case of bankruptcy, the FI commits to monitor and completely seizes the output in the hands of the firm.

Recall that the presence of agency costs implies that $y_{i,t} = \omega_{i,t}\chi_t q_t x_{i,t}$. Define

$$f(\overline{\omega}) \equiv \int_{\overline{\omega}}^{\infty} \omega\Phi(d\omega) - \overline{\omega}\left[1 - \Phi(\overline{\omega})\right]$$

$$g(\overline{\omega};\mu) \equiv \int_{0}^{\overline{\omega}} \omega\Phi(d\omega) - \mu\Phi(\overline{\omega}) + \overline{\omega}\left[1 - \Phi(\overline{\omega})\right]$$

as the expected shares of output accruing respectively to an entrepreneur and to a lender, after stipulating a contract that sets the fixed repayment at $\overline{P}_t\chi_t q_t\overline{\omega}_{it}x_{i,t}$ units of money. In case of default, a stochastic fraction μ_t of the input costs $x_{i,t}$, measured in units of money, is used in monitoring. We assume that μ_t follows a AR1 process given by $\mu_t = \alpha_\mu\mu_{t-1} + \epsilon_{\mu,t}$.[3] At the individual firm level, total output and the government subsidy are split between the entrepreneur, the lender, and monitoring costs so that

$$f(\overline{\omega}_t) + g(\overline{\omega}_t) = 1 - \mu_t\Phi(\overline{\omega}_t).$$

The optimal contract is the pair $(x_{i,t}, \overline{\omega}_{i,t})$ that solves the following CSV problem:

$$\max\ \overline{P}_t\chi_t q_t f(\overline{\omega}_{i,t})x_{i,t}$$

subject to

$$\begin{aligned}
\overline{P}_t\chi_t q_t g(\overline{\omega}_{i,t})x_{i,t} &\geq R_t^d P_t\left(x_{i,t} - \tau\right) &\quad (14)\\
\overline{P}_t\left[f(\overline{\omega}_{i,t}) + g(\overline{\omega}_{i,t};\mu_t) - 1 + \mu_t\Phi(\overline{\omega})\right] &\leq 0 &\quad (15)\\
\overline{P}_t\chi_t q_t f(\overline{\omega}_{i,t})x_{i,t} &\geq P_t\tau &\quad (16)
\end{aligned}$$

The optimal contract maximizes the entrepreneur's expected profits subject to the lender being willing to lend out funds, (14), the feasibility condition, (15), and the entrepreneur being willing to sign the contract, (16). Notice that the intermediary needs to pay back to the household a gross return equal to the safe interest on deposits, R_t^d. Since in equilibrium $R_t = R_t^d$, the financial intermediary's expected return on each unit of loans cannot be lower than R_t.

The optimality conditions can be written as

$$\begin{aligned}
q_t &= \frac{R_t}{1 - \mu_t\Phi(\overline{\omega}_{i,t}) + \frac{\mu_t f(\overline{\omega}_{i,t})\phi(\overline{\omega}_{i,t})}{f_{\overline{\omega}}(\overline{\omega}_{i,t})}}, &\quad (17)\\
x_{i,t} &= \left\{\frac{R_t}{R_t - q_t g(\overline{\omega}_{i,t};\mu_t)}\right\}\tau. &\quad (18)
\end{aligned}$$

[3] This assumption reflects the large time variation in bankruptcy costs documented by Natalucci et al (2004).

From equation (17), it follows that the terms of the contract depend on the state of the economy only through the aggregate mark-ups χ_t and q_t and the return R_t. Hence, they are the same for all firms, $\overline{\omega}_{i,t} = \overline{\omega}_t$. Since initial wealth is also the same across firms, it follows from equation (18) that the size of the project is the same across firms. The conditions can thus be rewritten as

$$q_t = \frac{R_t}{1 - \mu_t \Phi\left(\overline{\omega}_t\right) + \frac{\mu_t f(\overline{\omega}_t)\phi(\overline{\omega}_t)}{f_{\overline{\omega}}(\overline{\omega}_t)}} \tag{19}$$

$$x_t = \left\{ \frac{R_t}{R_t - q_t g\left(\overline{\omega}_t; \mu_t\right)} \right\} \tau. \tag{20}$$

Notice that the gross interest rate on the loan extended to firm i, $R^l_{i,t}$, can be backed up from the debt repayment. It is given by

$$\overline{P}_t \overline{\omega}_t \chi_t q_t x_t = R^l_{i,t} P_t \left(x_t - \tau\right)$$

implying that $R^l_{i,t} = R^l_t$, for all i.

We can use the expression above to obtain the spread between the loan rate and the risk-free rate, $\Delta_t \equiv \frac{R^l_t}{R^d_t}$,

$$\Delta_t = \frac{\overline{\omega}_t}{g(\overline{\omega}_t; \mu_t)}. \tag{21}$$

2.2.3 Entrepreneurs

Entrepreneurs have linear preferences on consumption and are infinitely lived. They consume a CES basket of differentiated goods similar to that of households.

At the end of each period, entrepreneurs sell their output to the retail sector and, if they do not default, repay the debt. If they default, the bank completely seizes firm's production, sells it to the retail sector and pays an amount $\mu \chi_t q_t x_t$ in monitoring costs.

Profits of entrepreneurs are entirely allocated to final consumption goods

$$\int_0^1 P_t\left(j\right) e_{i,t}\left(j\right) dj = \overline{P}_t \left(\omega_{i,t} - \overline{\omega}_t\right) \chi_t q_t x_t,$$

where $e_{i,t}\left(j\right)$ is firm i's consumption of good j. Notice that $\int_0^1 P_t\left(j\right) e_{i,t}\left(j\right) = P_t e_{i,t}$,where $e_{i,t}$ is the demand of the final consumption good of entrepreneur i. Aggregating across firms, we get

$$e_t = f\left(\overline{\omega}_t\right) q_t x_t, \tag{22}$$

where $e_t = \int_0^1 e_{i,t} di$ is the aggregate entrepreneurial consumption of the final consumption good. Using equations (19)-(20), we can rewrite aggregate entrepreneurial consumption as

$$e_t = \tau R_t H(\mu_t, \overline{\omega}_t) \tag{23}$$

where

$$H(\mu_t, \overline{\omega}_t) \equiv \frac{1}{1 + \frac{\mu_t \phi(\overline{\omega}_t)}{f_{\overline{\omega}}(\overline{\omega}_t)}}.$$

2.3 Retail firms

As in Bernanke, Gertler and Gilchrist (1999), monopolistic competition occurs at the "retail" level. More specifically, a continuum of monopolistically competitive retailers buy wholesale output from entrepreneurs in a competitive market and then differentiate it at no cost. Because of product differentiation, each retailer has some market power. Profits are distributed to the households, who own firms in the retail sector.

Let $Y_t(j)$ be the quantity of output sold by retailer j. This quantity can be used for households' consumption, $c_t(j)$, and for entrepreneurs' consumption, $e_t(j)$. Hence,

$$Y_t(j) = c_t(j) + e_t(j).$$

The final good Y_t is a CES composite of individual retail goods

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{24}$$

with $\varepsilon > 1$.

2.3.1 Price setting

We assume that each retailer can change its price with probability $1-\theta$, following Calvo (1983). Let $P_t^*(j)$ denote the price for good j set by retailers that can change the price at time t, and $Y_t^*(j)$ the demand faced given this price. Then each retailer chooses its price to maximize expected discounted profits, given by

$$E_t\left[\sum_{k=0}^{\infty} \theta^k \overline{Q}_{t,t+k} \frac{P_t^* - \overline{P}_{t+k}}{P_{t+k}} Y_{t+k}^*(j)\right],$$

where $\overline{Q}_{t,t+k} = \beta \frac{u_c(c_{t+1}) + \kappa_m(m_{t+1})}{u_c(c_t) + \kappa_m(m_t)}$.

The first-order conditions of the firm's profit maximization problem imply that

$$\frac{P_t^*}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t\left\{\sum_{k=0}^{\infty} \theta^k \overline{Q}_{t,t+k} \frac{\overline{P}_{t+k}}{P_{t+k}^{1-\varepsilon}} P_t^{-\varepsilon} Y_{t+k}\right\}}{E_t\left\{\sum_{k=0}^{\infty} \theta^k \overline{Q}_{t,t+k} \frac{P_t^{1-\varepsilon}}{P_{t+k}^{1-\varepsilon}} Y_{t+k}\right\}}$$

Now define

$$\begin{aligned}
\Theta_{1,t} &\equiv \frac{\overline{P}_t}{P_t} Y_t + E_t\left\{\sum_{k=1}^{\infty} \theta^k \overline{Q}_{t,t+k} \frac{\overline{P}_{t+k}}{P_{t+k}^{1-\varepsilon}} P_t^{-\varepsilon} Y_{t+k}\right\} \\
\Theta_{2,t} &\equiv Y_t + E_t\left\{\sum_{k=1}^{\infty} \theta^k \overline{Q}_{t,t+k} \frac{P_t^{1-\varepsilon}}{P_{t+k}^{1-\varepsilon}} Y_{t+k}\right\}
\end{aligned}$$

Using the expression for the aggregate price index, $P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P_t^*\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$, and substituting out $\frac{P_t^*}{P_t}$, we obtain the following conditions

$$\begin{aligned}
1 &= \theta \pi_t^{\varepsilon-1} + (1-\theta)\left(\frac{\varepsilon}{\varepsilon-1} \frac{\Theta_{1,t}}{\Theta_{2,t}}\right)^{1-\varepsilon} \\
\Theta_{1,t} &= \frac{1}{\chi_t} Y_t + \theta E_t\left[\pi_{t+1}^{\varepsilon} \overline{Q}_{t,t+1} \Theta_{1,t+1}\right] \\
\Theta_{2,t} &= Y_t + \theta E_t\left[\pi_{t+1}^{\varepsilon-1} \overline{Q}_{t,t+1} \Theta_{2,t+1}\right].
\end{aligned}$$

2.3.2 Price dispersion

Recall that the aggregate retail price level is given by $P_t = \left[\int_0^1 P_t(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}}$. Define the relative price of differentiated good j as $p_t(j) \equiv \frac{P_t(j)}{P_t}$ and divide both sides by P_t to express everything in terms of relative prices, $1 = \int_0^1 (p_t(j))^{1-\varepsilon} dj$.

Define also the relative price dispersion term as

$$s_t \equiv \int_0^1 (p_t(j))^{-\varepsilon} dj.$$

This equation can be written in recursive terms as

$$s_t = (1-\theta)\left(\frac{1-\theta \pi_t^{\varepsilon-1}}{1-\theta}\right)^{-\frac{\varepsilon}{1-\varepsilon}} + \theta \pi_t^{\varepsilon} s_{t-1}.$$

2.4 Market clearing

Market clearing conditions are listed below.

Money:

$$M_t^s = M_t,$$

Bonds:

$$A_t = 0$$

Labor:

$$h_t = l_t$$

Loans:

$$d_t = x_t - \tau$$

Wholesale goods:

$$y_t = \int_0^1 Y_t(j)\, dj$$

Retail goods:

$$Y_t(j) = c_t(j) + e_t(j)\,, \text{ for all } j.$$

2.5 Competitive equilibrium

The central bank needs to specify an additional rule for either R_t^m or M_t^s. It is convenient to express this rule in terms of $\Delta_{m,t}$. In order to facilitate the comparison of our model with the standard New-Keynesian setup, we assume a monetary policy such that

$$\Delta_{m,t} = \Delta_m,$$

for all i. Then,

$$\kappa_m(m_t) = \frac{\Delta_m}{1-\Delta_m} u_c(c_t)$$

and we can define

$$U(c_t, \Delta_{m,t}) \equiv u_c(c_t)\left(1 + \frac{\Delta_m}{1-\Delta_m}\right).$$

Under a policy of constant $\Delta_{m,t}$, money demand becomes recursive and can therefore be neglected for the solution of the system.

We assume a functional form $U(c_t; \Delta_m) - v(h_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \psi \frac{h_t^{1+\varphi}}{1+\varphi}$ and we define $\widehat{\pi}_{t+1} \equiv \log \pi_{t+1}$, $\widehat{p}_t(j) = \log p_t(j)$, $a_t = \log A_t$, and $\widehat{\mu}_t = \log \mu_t$.

The system of equilibrium conditions can be written in log-linearized form as reported in Appendix A.

2.6 The system in reduced form

After some algebra, the system of equilibrium conditions that characterizes the evolution of the aggregate variables (together with an appropriate monetary policy rule) can be written as

$$\begin{aligned}
\left(1+\sigma\frac{e}{c}\right)\widehat{\chi}_t &= (1+\varphi)\ a_t - (\sigma\alpha_1+\alpha_2)\,\widehat{\Delta}_t - (\sigma+\varphi)\,\widehat{Y}_t - \widehat{R}_t + \left(\alpha_3 - \sigma\alpha_1\frac{g_\mu\mu}{g}\right)\widehat{\mu}_t \qquad (25)\\
\delta_1\widehat{\Delta}_t &= \left(1+\varphi+\sigma\frac{Y}{c}\right)\widehat{Y}_t - \sigma\frac{e}{c}\widehat{R}_t - (1+\varphi)\ a_t - \delta_2\widehat{\mu}_t \qquad (26)\\
\widehat{Y}_t &= E_t\widehat{Y}_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right) + \alpha_1 E_t\left(\widehat{\Delta}_{t+1} - \widehat{\Delta}_t\right) \qquad (27)\\
&\quad + \frac{e}{c}E_t\left(\widehat{\chi}_{t+1} - \widehat{\chi}_t\right) + \alpha_1\frac{g_\mu\mu}{g}E_t\left(\widehat{\mu}_{t+1} - \widehat{\mu}_t\right)\\
\widehat{\pi}_t &= -\lambda\widehat{\chi}_t + \beta E_t\widehat{\pi}_{t+1} \qquad (28)
\end{aligned}$$

where the coefficients $\alpha_1, \alpha_2, \alpha_3, \delta_1$ and δ_2 are defined in Appendix B. Notice that α_1 and α_2 are positive coefficients.

In our economy, the mark-up is inversely related to the marginal costs of retail firms. Indeed, an increase in the input cost of retail production, i.e. a higher price of wholesale goods, generates a fall in the mark-up $\chi_t = P_t/\overline{P}_t$. Equation (25) shows that the markup is negatively related to three factors. The first is the spread between the loan rate and the policy rate. An increase in the spread implies a higher cost of external finance for wholesale firms, which then need to increase the price of intermediate goods, $\overline{P}_t$. The second is the demand of final goods. In the presence of higher demand for retail goods, and correspondingly of intermediate goods to be used as production inputs, wholesale firms need to pay a higher real wage to workers to induce them to supply the required labor services. This increases the price of wholesale goods, $\overline{P}_t$, relative to the price of retail goods, P_t. The third is the nominal interest rate. Wholesale firms must borrow funds to finance production through nominal loans. As a result, any increase in the policy rate represents an additional cost which is covered by charging a higher price of wholesale goods.

Equation (26) shows that the spread between the loan rate and the policy rate increases with aggregate demand. An increase in the demand of retail (and thus also of wholesale) goods implies an implicit tightening of the credit constraint, since the exogenously given amount of internal funds must now be used to finance a higher level of debt. The increased default risk generates a larger spread. For the same reasons, the spread decreases with the nominal interest rate. An increase in this latter generates a reduction in the demand of final goods and thus in the demand of input for their production (wholesale goods). For a given amount of internal funds, leverage and the risk of default fall, reducing the spread.

Equation (27) is the IS curve. The change in the demand of final consumption goods react to the real interest rate, as in the standard case with frictionless financial market. However, it also reacts to changes in the spread and in the mark-up.

Equation (28) is the expectation-augmented Phillips curve, representing aggregate supply. It differs from the formulation with frictionless financial markets because $-\widehat{\chi}_t$ appears instead of aggregate demand, $\widehat{Y}_t$. However, we have already noticed that the markup varies inversely with aggregate demand. A decrease in the markup signals excess demand and generates inflationary pressures.

2.6.1 Case with frictionless financial markets

We consider the special case when monitoring costs are zero, i.e. $\mu_t = 0$, for all t. In this case, firms still need to borrow in advance of production. However, the information asymmetry concerning wholesale firms' productivity disappears because banks can monitor at no cost.

When $\mu_t = 0$, for all t, $f(\overline{\omega}_t) + g(\overline{\omega}_t; \mu_t) = 1$. Also, since there are no monitoring costs, banks set $\overline{\omega}_t$ as high as possible subject to the constraint that the firm is willing to sign the contract, i.e.

$$f(\overline{\omega}_t) = \frac{\tau}{q_t x_t}$$

This maximizes banks' profits, as they can size the production of all defaulting firms at no cost. In such equilibrium,

$$\begin{aligned} g(\overline{\omega}_t; \mu_t) &= 1 - \frac{\tau \chi_t}{y_t} \\ e_t &= \tau \\ Y_t &= c_t + \tau. \end{aligned}$$

Moreover, from the bank's zero profit condition, we have

$$x_t = \frac{R_t \tau}{R_t - q_t \left(1 - \frac{\tau \chi_t}{y_t}\right)}.$$

The log-linearized system can then be written as

$$\widehat{\chi}_t = -\left[\frac{(R-1)\chi\tau}{y}(1+\sigma+\varphi) + (\sigma+\varphi)\right]\widehat{Y}_t + \left(\frac{\chi\tau}{y} - \frac{1}{q}\right) R\widehat{R}_t + \left[\frac{(R-1)\chi\tau}{y} + 1\right](1+\varphi)\, a_t$$

$$\widehat{Y}_t = E_t\widehat{Y}_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right)$$

$$\widehat{\pi}_t = -\lambda\widehat{\chi}_t + \beta E_t\widehat{\pi}_{t+1}$$

In the limiting case where $\tau \approx 0$, $R_t = q_t$. The system then becomes

$$\widehat{\chi}_t = -(\sigma+\varphi)\widehat{Y}_t - \widehat{R}_t + (1+\varphi)a_t$$

$$\widehat{Y}_t = E_t\widehat{Y}_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right)$$

$$\widehat{\pi}_t = -\lambda\widehat{\chi}_t + \beta E_t\widehat{\pi}_{t+1}$$

The equations above coincide with the reduced-form system of equilibrium conditions obtained by Ravenna and Walsh (2006) in their model of the "cost-channel," where firms borrow in advance of production but, since there is no asymmetric information nor default risk, they simply pay the risk-free rate on these funds.

3 The system in deviation from the efficient equilibrium

In order to characterize the optimal response of monetary policy, it is convenient to write the reduced form of the system (25)-(28) in terms of gaps from the efficient equilibrium, in which $\mu_t = 0$ and prices are flexible.

We the gap between actual and efficient output as $\widetilde{Y}_t = \widehat{Y}_t - \widehat{Y}^e_t$. The system can then be rewritten as

$$\delta_1\widehat{\Delta}_t = \left(1+\varphi+\sigma\frac{Y}{c}\right)\widetilde{Y}_t - \sigma\frac{e}{c}\widetilde{R}_t + \widehat{\xi}_{2,t}$$

$$\begin{aligned}\widetilde{Y}_t &= E_t\widetilde{Y}_{t+1} - \frac{c+\sigma e}{c-\varphi e}\frac{1}{\sigma}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right) \\ &\quad + \frac{\alpha_1 c - \alpha_2 e}{c-\varphi e}E_t\left(\widehat{\Delta}_{t+1} - \widehat{\Delta}_t\right) - \frac{e}{c-\varphi e}E_t\left(\widehat{R}_{t+1} - \widehat{R}_t\right) + \upsilon_t \qquad (29)\end{aligned}$$

$$\widehat{\pi}_t = \lambda(\sigma+\varphi)\widetilde{Y}_t + \lambda(\sigma\alpha_1+\alpha_2)\widehat{\Delta}_t + \lambda\widetilde{R}_t + \beta E_t\widehat{\pi}_{t+1} - \lambda\widehat{\xi}_{1,t} \qquad (30)$$

where

$$\widehat{\xi}_{1,t} \equiv -\sigma\left(\frac{1+\varphi}{\sigma+\varphi}\right)(E_t a_{t+1} - a_t) + \left(\alpha_3 - \sigma\alpha_1\frac{g_\mu\mu}{g}\right)\widehat{\mu}_t \qquad (31)$$

$$\widehat{\xi}_{2,t} \equiv -\sigma\frac{e}{c}\sigma\left(\frac{1+\varphi}{\sigma+\varphi}\right)(E_t a_{t+1} - a_t) + \left(1+\sigma\frac{e}{c}\right)\left(\frac{1+\varphi}{\sigma+\varphi}\right)a_t - \delta_2\widehat{\mu}_t \qquad (32)$$

$$\upsilon \equiv \frac{e}{c-\varphi e}E_t\left(\widehat{\xi}_{1,t+1} - \widehat{\xi}_{1,t}\right) + \alpha_1\frac{c+\sigma e}{c-\varphi e}\frac{g_\mu\mu}{g}E_t\left(\widehat{\mu}_{t+1} - \widehat{\mu}_t\right) \qquad (33)$$

Equation (29) is a forward-looking IS-curve describing the determinants of the gap between actual output and its efficient level. The first line of the expression shows that, as in the standard new-Keynesian model, the gap is affected by its expected future value and by the real interest rate. In this model, however, the output gap depends also on the expected change in the nominal interest rate and in the credit spread, as well as on the composite shock υ_t.

In the Phillips curve (30), the first determinant of inflation is the output gap. This is a standard term, but enters here with a different coefficient reflecting the presence of entrepreneurs in the economy. The term $\widehat{\xi}_{1,t}$ denotes a "cost-push" factor, which is a combination of exogenous shocks that do not generate variations in the efficient level of output. This factor is a linear combination of technology shocks and financial shocks. Intuitively, the technology shock has two separate effects in our economy. On one hand, it generates an efficient variation in the efficient level of output, as in the standard model with frictionless financial markets. This variation has no effect on the output gap and on inflation. On the other hand, however, an increase in a_t increases the revenues of indebted firms and (for a given incidence of the monitoring costs) reduces the occurrence of default. This variation affects current output but not the efficient level of output, thus creating inflationary pressures. Finally, the second and third terms in equation (30) reflect endogenous "cost-push" effects of the credit spread and the real interest rate, introduced by the existence of credit market imperfections.

3.1 Impulse responses

As a benchmark for comparison with the optimal policy case, we provide some evidence on the quantitative implications of the model through an impulse response analysis. For this purpose, we close the model with a simple monetary policy rule of the Taylor-type, including an inflation response coefficient of 1.5 and an output gap response coefficient of 0.5.

The structural parameters are set in line with the literature. Following Levin, Natalucci and Zakrajsek (2004) we set long-run monitoring costs at 15% of the firm's output (i.e. $\mu = 0.15$). We then calibrate the standard deviations of idiosyncratic shocks (σ_ω) and the subsity τ so that approximately 1% of firms go bankrupt each quarter and that the steady state spread Δ is equal to approximately 2% per year. As to monopolistic competition and retail pricing, we assume $\varepsilon = 7$, leading to a steady-state mark-up of 17%, and a probability of not being able to re-optimise prices $\theta = 0.66$, implying that prices are changed on average every 3 quarters. Finally, we set the persistence of technology and monitoring cost shocks to 0.9.

Figure 1 displays the response of a few variables to a positive 1% technology shock under the Taylor rule. As is typically the case, the shock produces downward pressure on inflation (denoted as "inf" in the figure). However, contrary to what happens in the standard new-Keynesian setting with Calvo pricing, the fall in inflation is not due to a negative output gap (y_gap). It is caused instead by a negative interest rate gap (r_gap), which has an impact on inflation because of the cost-channel. At the same time, the shock tends to increase the spread between loan rates and the policy interest rate by the same amount as in the efficient equilibrium (delta_gap).

Impulse responses to a positive 1% monitoring costs shock under the Taylor rule are presented in figure 2. The shock is contractionary, but the consumption and output gaps decrease only slightly. Nevertheless, the shock has a direct impact on marginal costs, which leads to a policy tightening and an opening of the interest rate gap.

4 Second order welfare approximation

Following Woodford (2003), we obtain a policy objective function by taking a second order approximation to the utility of the economy's representative agents. Since our economy is populated by households and entrepreneurs, the policy objective function will be a weighted average of the (approximate) utility functions of these two agents. The approximation to the objective function takes a form, which nests the one in the benchmark new-Keynesian model (see Woordford, 2003) as a special case.

The appendix shows that the present discounted value of the social welfare can be approximated by

$$W_{t_0} \simeq \varsigma c^{1-\sigma}\left[\varkappa - \frac{1}{2}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}L_t\right] + t.i.p + O\left(\left\|\zeta^3\right\|\right), \tag{34}$$

where ς is the weight assigned to households' utility

$$L_t \equiv \frac{Y-c}{c}\left(1-\frac{1-\varsigma}{\varsigma}c^{\sigma}\right)\left(\widehat{e}_t + \frac{1}{2}\widehat{e}_t^2\right) + \kappa_\pi \widehat{\pi}_t^2 + \frac{1}{2}\frac{Y}{c}\varphi\widetilde{Y}_t^2 + \frac{1}{2}\sigma\widetilde{c}_t^2 - \sigma\frac{Y}{c}\frac{Y-c}{c}\widehat{Y}_t^n\widehat{e}_t \tag{35}$$

and

$$\widehat{e}_t = \widehat{R}_t + \delta_3\widehat{\Delta}_t - \delta_4\widehat{\mu}_t$$

for parameters $\varkappa$, κ_π, δ_3, δ_4 defined in the appendix.

Intuitively, the central bank's utility decreases with variations of inflation around its target, and of output around its efficient level. Unlike in the benchmark new-Keynesian model, the consumption smoothing motive only applies to households' consumption, $\widetilde{c}_t^2$, rather than to total output, because entrepreneurs are indifferent to shifting consumption over time.

The main difference relative to the benchmark New-Keynesian model with frictionless financial markets is in the additional terms now appearing in the welfare approximation. Welfare is also reduced by fluctuations in the nominal interest rate $\widehat{R}_t$ and in the credit spread $\widehat{\Delta}_t$ (through the $\widehat{e}_t$ term). In spite of the fact that we have assumed the existence of a steady state subsidy to eliminate the first order effects of distortions on output, the general loss function (35) includes linear terms due to the presence of asymmetric information. The costs of this distortion are of first order and they only vanish when entrepreneurs disappears from the economy (note that the loss boils down to the usual $\kappa_\pi \widehat{\pi}_t^2 + (\varphi + \sigma)/2\widetilde{Y}_t^2$ when steady state output is equal to households' consumption $Y = c$). In general, we will therefore need a second order approximation of private agents decision rules in order to analyse welfare.

Under the special weight $\varsigma = (1 + c^{-\sigma})^{-1}$, however, first order terms disappear and the loss function simplifies to

$$L_t \equiv \kappa_\pi \widehat{\pi}_t^2 + \frac{1}{2}\frac{Y}{c}\varphi \widetilde{Y}_t^2 + \frac{1}{2}\sigma \left[\frac{Y}{c}\widetilde{Y}_t - \frac{Y-c}{c}\left(\widehat{R}_t + \delta_3 \widehat{\Delta}_t - \delta_4 \widehat{\mu}_t\right)\right]^2 - \sigma \frac{Y}{c}\frac{Y-c}{c}\widehat{Y}_t^n \left(\widehat{R}_t + \delta_3 \widehat{\Delta}_t\right) \tag{36}$$

Expression (**??**) highlights the direct influence on welfare of monitoring shocks $\widehat{\mu}_t$. It also shows that increases in the nominal interest rate and in the credit spread have also a positive effect on welfare, if they are accompanied by an increase in the efficient level of output – i.e. an increase in productivity. The reason is that households are willing to reap the benefits of the higher productivity on real wages, but wish to smooth their consumption pattern over time. A higher incidence of monitoring costs in the economy at a time of high productivity helps achieving the latter objective.

5 Optimal policy

5.1 Discretion

When the welfare function can be approximated as in (34) and (36), the problem of the central bank is to maximize that objective, subject to the system of equilibrium conditions (**??**)-(30).

Denote as $\eta_{1,t}$, $\eta_{2,t}$ and $\eta_{3,t}$ the Lagrangean multipliers associated to the constraints. Taking first-order conditions and solving for the multiplies, we can characterise the discretionary equilibrium through one equation relating the evolution of inflation to the evolution of the other variables. Such equation takes the form

$$\widehat{\pi}_t = \Theta\left(\widetilde{Y}_t, \widehat{\Delta}_t, \widehat{R}_t\right)$$

Notice that, in the frictionless case, the optimality condition amounts to

$$\widehat{\pi}_t = -\frac{\lambda}{\kappa}\widetilde{Y}_t,$$

where $\kappa > 0$ is an appropriately defined coefficient. This is the case, for instance, in Woodford (2003, chapter 7, p. 471). It implies that the central bank manages the output gap in order to achieve the constant inflation rate given by

$$\widehat{\pi}_t = \left(\frac{\kappa\lambda}{(1-\beta)\lambda + \kappa^2}\right)\widehat{Y}_t^e.$$

In our model, the target criterion (i.e. the relation between inflation and the output gap that achieves the discretionary optimum) is affected by the existence of financial frictions.

Notice that both the endogenous spread $\widehat{\Delta}_t$ and the exogenous financial shock $\widehat{\mu}_t$ limit the ability of the central bank to use the output gap to achieve the desired level of inflation. Moreover, the desired inflation level is not constant. To realize this, plug the optimality condition into the aggregate supply equation. Imposing rational expectations of private agents, i.e. $\widehat{\pi}_t = E_t\widehat{\pi}_{t+1}$, we obtain the optimal inflation rate for a central bank that operates under discretion:

$$\widehat{\pi}_t = \frac{(\sigma+\varphi)\widehat{Y}_t^e + \widetilde{r}_t + (\sigma\alpha_1 + \alpha_2 - (\sigma+\varphi)\overline{\kappa}_\Delta)\widehat{\Delta}_t - (\sigma+\varphi)\overline{\kappa}_\Delta\frac{\mu\Phi}{g}\widehat{\mu}_t - \widehat{\xi}_{1,t}}{\frac{(1-\beta)(c+\sigma e)}{c\lambda} + (\sigma+\varphi)\overline{\kappa}_\pi}.$$

When financial frictions create an endogenous, time-varying spread between the loan rate and the risk-free rate, the optimal inflation rate under discretion reflects variations of the real interest rate gap, the credit spread and the exogenous shocks.

6 Optimal monetary policy in the benchmark model

We characterize numerically the optimal monetary policy under commitment in the case of the model described in section 2, where deposits pay a nominal return R_t^d. The optimal reaction of monetary policy can be derived for the general case in which linear terms are present in the quadratic approximation to the welfare function.

Figure 3 displays impulse responses to a technology shock with optimal policy under commitment (and adopting a timeless perspective, as in Woodford, 2003). Compared to the results in Figure 1, optimal policy tends to close welfare-relevant gaps much more quickly and the inflationary effects of the shock are more contained. After 2 periods, the output gap and the spread between loan and policy interest rates are closed. However, optimal policy also produces a small positive interest rate gap, which has an impact on marginal costs and therefore generates some positive inflationary pressures. This transmission channel prevents a quick and complete stabilisation of inflation in our model.

7 Conclusion

[TO BE COMPLETED...]

Appendix

A. The log-linearized system of equilibrium conditions

The equilibrium can then be characterized as the solution to the following system of log-linearized equations in the variables $\left\{\widehat{c}_t, \widehat{Y}_t, \widehat{h}_t, \widehat{q}_t, \widehat{\chi}_t, \widehat{\pi}_t, \widehat{R}_t, \widehat{\Delta}_t\right\}$,

$$\begin{aligned}
\alpha_1 \widehat{\Delta}_t &= -\widehat{Y}_t - \frac{(y-c)}{c}\widehat{\chi}_t + \widehat{c}_t - \alpha_1 \frac{g_\mu \mu}{g}\widehat{\mu}_t \\
\widehat{q}_t &= a_t - \sigma \widehat{c}_t - \varphi \widehat{h}_t - \widehat{\chi}_t \\
\widehat{h}_t &= \widehat{Y}_t - a_t \\
\widehat{R}_t &= \widehat{q}_t - \alpha_2 \widehat{\Delta}_t + \alpha_3 \widehat{\mu}_t \\
\widehat{Y}_t &= \widehat{\chi}_t + \widehat{R}_t + \alpha_4 \widehat{\Delta}_t + \alpha_5 \widehat{\mu}_t \\
\widehat{R}_t &= E_t \widehat{\pi}_{t+1} + \sigma \left(E_t \widehat{c}_{t+1} - \widehat{c}_t\right) \\
\widehat{\pi}_t &= -\lambda \widehat{\chi}_t + \beta E_t \widehat{\pi}_{t+1}
\end{aligned}$$

where $\lambda \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$, plus a monetary policy rule.

B. The coefficients of the log-linearized equations

The coefficients of the system (25)-(**??**) are given by

$$
\begin{aligned}
\alpha_1 &= -\frac{\frac{f_{\overline{\omega}}\overline{\omega}}{\chi}\frac{y}{c}}{1-g_{\overline{\omega}}\Delta} > 0 \\
\alpha_2 &= -\frac{\mu\frac{f\overline{\omega}}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}}-\frac{\phi^2}{f_{\overline{\omega}}}\right)\frac{q}{R}}{(1-g_{\overline{\omega}}\Delta)} > 0 \\
\alpha_3 &= \left[\frac{\frac{g_\mu\mu}{g}\frac{f\overline{\omega}}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}}-\frac{\phi^2}{f_{\overline{\omega}}}\right)}{(1-g_{\overline{\omega}}\Delta)}+\frac{f\phi}{f_{\overline{\omega}}}-\Phi\right]\mu\frac{q}{R} \\
\alpha_4 &= -\frac{\mu\frac{f\overline{\omega}}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}}-\frac{\phi^2}{f_{\overline{\omega}}}\right)+\overline{\omega}\left(f_{\overline{\omega}}+\mu\phi\right)}{\left(f+\frac{\mu f\phi}{f_{\overline{\omega}}}\right)(1-g_{\overline{\omega}}\Delta)} > 0 \\
\alpha_5 &= \left(\frac{\alpha_4 g_\mu}{g}-\frac{\phi}{f_{\overline{\omega}}+\mu\phi}\right)\mu \\
\delta_1 &\equiv \left(1+\frac{\sigma e}{c}\right)\alpha_4-\sigma\alpha_1-\alpha_2 \\
\delta_2 &\equiv \alpha_3-\sigma\alpha_1\frac{g_\mu\mu}{g}.
\end{aligned}
$$

C. Welfare approximation

Our monetary policy objective is derived as the second order approximation to a weighted average of the utilities of the household and of the entrepreneur, i.e.

$$E_o\left\{\sum_0^\infty \beta^t\left[\varsigma U_t+(1-\varsigma)U_t^e\right]\right\}$$

where ς is the weight of the utility of households in the policy objective. Households' temporary utility can then be approximated as

$$U_t \simeq U+u_c c\left(\widehat{c}_t+\frac{1}{2}\left(1+\frac{u_{cc}c}{u_c}\right)\widehat{c}_t^2\right)-v_h h\left(\widehat{h}_t+\frac{1}{2}\left(1+\frac{v_{hh}h}{v_h}\right)\widehat{h}_t^2\right)$$

where hats denote log-deviations from the deterministic steady state and c and h denote steady state levels. Similarly, entrepreneurial temporary utility U_t^e can be expanded as

$$U_t^e \simeq e\left(1+\widehat{e}_t+\frac{1}{2}\widehat{e}_t^2\right)$$

where e is the steady state level of entrepreneurial consumption.

Under the functional form $U_t=\frac{c_t^{1-\sigma}}{1-\sigma}-\psi\frac{h_t^{1+\varphi}}{1+\varphi}$, households' temporary utility can be rewritten as

$$U_t \simeq \frac{c^{1-\sigma}}{1-\sigma}-\psi\frac{h^{1+\varphi}}{1+\varphi}+c^{1-\sigma}\widehat{c}_t-\psi h^{1+\varphi}\widehat{h}_t+\frac{1}{2}\left(c^{1-\sigma}(1-\sigma)\widehat{c}_t^2-\psi h^{1+\varphi}(1+\varphi)\widehat{h}_t^2\right)$$

We can express hours, households' consumption and entrepreneurial consumption as

$$h_t = \frac{s_t Y_t}{A_t}$$

$$c_t = Y_t - e_t$$

$$e_t = \tau R_t H\left(\mu_t, \overline{\omega}_t\right)$$

where $H\left(\mu_t, \overline{\omega}_t\right) \equiv \left[1 + \frac{\mu_t \phi(\overline{\omega}_t)}{f_{\overline{\omega}}(\overline{\omega}_t)}\right]^{-1}$.

The period aggregate utility can be approximated as

$$\begin{aligned}
\varsigma U_t + (1-\varsigma) U_t^e \quad \simeq \quad & \varsigma c^{1-\sigma}\left(\frac{1}{1-\sigma} - \frac{\psi}{1+\varphi}\frac{h^{1+\varphi}}{c^{1-\sigma}}\right) + (1-\varsigma) e \\
& + \varsigma c^{1-\sigma}\widehat{c}_t + (1-\varsigma) e\widehat{e}_t - \varsigma c^{1-\sigma}\frac{\psi h^{1+\varphi}}{c^{1-\sigma}}\widehat{h}_t \\
& + \frac{1}{2}\varsigma c^{1-\sigma}(1-\sigma)\widehat{c}_t^2 - \frac{1}{2}\varsigma c^{1-\sigma}\frac{\psi h^{1+\varphi}}{c^{1-\sigma}}(1+\varphi)\widehat{h}_t^2 + \frac{1}{2}(1-\varsigma) e\widehat{e}_t^2
\end{aligned}$$

Now note that the resource constraint $c_t = Y_t - e_t$ can be approximated to second order as

$$\widehat{c}_t = \frac{Y}{c}\widehat{Y}_t - \frac{e}{c}\widehat{e}_t + \frac{1}{2}\frac{Y}{c}\widehat{Y}_t^2 - \frac{1}{2}\frac{e}{c}\widehat{e}_t^2 - \frac{1}{2}\widehat{c}_t^2$$

while the production function implies simply

$$\widehat{h}_t = -a_t + \widehat{s}_t + \widehat{Y}_t.$$

It follows that utility can be rewritten as

$$\begin{aligned}
\frac{\varsigma U_t + (1-\varsigma) U_t^e - \varkappa}{c^{1-\sigma}} \quad \simeq \quad & -\varsigma\frac{\psi h^{1+\varphi}}{c^{1-\sigma}}\widehat{s}_t - \varsigma\left(\frac{\psi h^{1+\varphi}}{c^{1-\sigma}} - \frac{Y}{c}\right)\widehat{Y}_t \\
& - \frac{e}{c}\left(\varsigma - (1-\varsigma) c^{\sigma}\right)\left(\widehat{e}_t + \frac{1}{2}\widehat{e}_t^2\right) \\
& - \frac{1}{2}\varsigma\sigma\widehat{c}_t^2 - \frac{1}{2}\varsigma\left(\frac{\psi h^{1+\varphi}}{c^{1-\sigma}}(1+\varphi) - \frac{Y}{c}\right)\widehat{Y}_t^2 \\
& + \varsigma\frac{\psi h^{1+\varphi}}{c^{1-\sigma}}(1+\varphi) a_t\widehat{Y}_t + tips
\end{aligned}$$

where

$$\varkappa \equiv \varsigma c^{1-\sigma}\left(\frac{1}{1-\sigma} - \frac{1}{1+\varphi}\frac{\psi h^{1+\varphi}}{c^{1-\sigma}}\right) + (1-\varsigma) e$$

Now consider the FOC

$$\frac{\psi h^{\varphi}}{c_t^{-\sigma}} = \frac{A_t}{q_t \chi_t}$$

Under perfect competition and frictionless credit markets, firms set the real wage at the marginal product of labor, $w_t = A_t$. In our model, equation (11) implies that $w_t = \frac{A_t}{q_t \chi_t}$. We provide households with a subsidy Ω_1 such that $\frac{\psi h^{\varphi}}{c^{-\sigma}} = w_t \Omega_1$, and

$$\Omega_1 = q\chi.$$

Moreover, we assume that Ω_1 is small. It follows that in such a steady state

$$\frac{\psi h^{1+\varphi}}{c^{1-\sigma}} = \frac{Ah}{c} = \frac{Y}{c}.$$

In addition, we focus on the case of a special Pareto weight $\varsigma = \frac{c^{\sigma}}{1+c^{\sigma}}$, which allows us to ignore first order terms in entrepreneurial consumption. It follows that the loss can be written as

$$\frac{\varsigma U_t + (1-\varsigma) U_t^e - \varkappa}{\frac{c^{1-\sigma}}{1+c^{-\sigma}}} \simeq -\frac{Y}{c}\widehat{s}_t - \frac{1}{2}\frac{Y}{c}\varphi \widehat{Y}_t^2 - \frac{1}{2}\sigma\left(\frac{Y}{c}\widehat{Y}_t - \frac{e}{c}\widehat{e}_t\right)^2 + \frac{Y}{c}(1+\varphi) a_t \widehat{Y}_t + t.i.s.p. + O\left(\left\|\zeta^3\right\|\right)$$

Now note that in the fully-efficient steady state we would have

$$(1+\varphi) a_t = \sigma c_t^e + \varphi Y_t^e$$

which can be used to substitute out the technology shock a_t from the loss function.

In addition, a first order approximation to the equation for price dispersion, of first-order in $\widehat{s}_t$ and second-order in $\widehat{\pi}_t$ takes the form

$$\widehat{s}_t \simeq \frac{\theta}{1-\theta}\varepsilon \frac{\widehat{\pi}_t^2}{2} + \theta \widehat{s}_{t-1}.$$

This latter can be integrated forward to obtain

$$\widehat{s}_t \simeq \frac{\theta}{1-\theta}\varepsilon \sum_{s=t_0}^{t} \theta^{t-s}\frac{\widehat{\pi}_s^2}{2} + \theta^{t-t_0+1}\widehat{s}_{t_0-1} + O\left(\left\|\zeta^3\right\|\right).$$

Multiplying this by β^{t-t_0} and realizing that multiples of $\widehat{s}_{t_0-1}$ are t.i.p., we get

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0}\widehat{s}_t \simeq \frac{\theta}{(1-\theta)(1-\beta\theta)}\varepsilon \sum_{t=t_0}^{\infty} \beta^{t-t_0}\frac{\widehat{\pi}_t^2}{2} + t.i.p. + O\left(\left\|\zeta^3\right\|\right)$$

as long as $\widehat{s}_{t_{0-1}} = O\left(\left\|\zeta^2\right\|\right)$.

Finally, note that entrepreneurial consumption can be written as

$$\widehat{e}_t = \widehat{R}_t + \delta_3 \widehat{\Delta}_t - \delta_4 \widehat{\mu}_t$$

where

$$\begin{aligned}
\delta_3 &= -\frac{\mu\overline{\omega}\left(\phi_{\overline{\omega}} - \frac{\phi^2}{f_{\overline{\omega}}}\right)}{\left(f_{\overline{\omega}} + \mu\phi\right)\left(1 - \Delta g_{\overline{\omega}}\right)} > 0 \\
\delta_4 &= \left(\frac{\phi}{\mu\phi + f_{\overline{\omega}}} + \frac{\delta_3 g_\mu}{g}\right)\mu
\end{aligned}$$

It follows that the approximated welfare function can be written as in the main text, for

$$\kappa_\pi \equiv \frac{Y}{c}\frac{\varepsilon\theta}{(1-\theta)(1-\beta\theta)} > 0$$

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Figure 1

Impulse responses to a positive technology shock under the Taylor rule

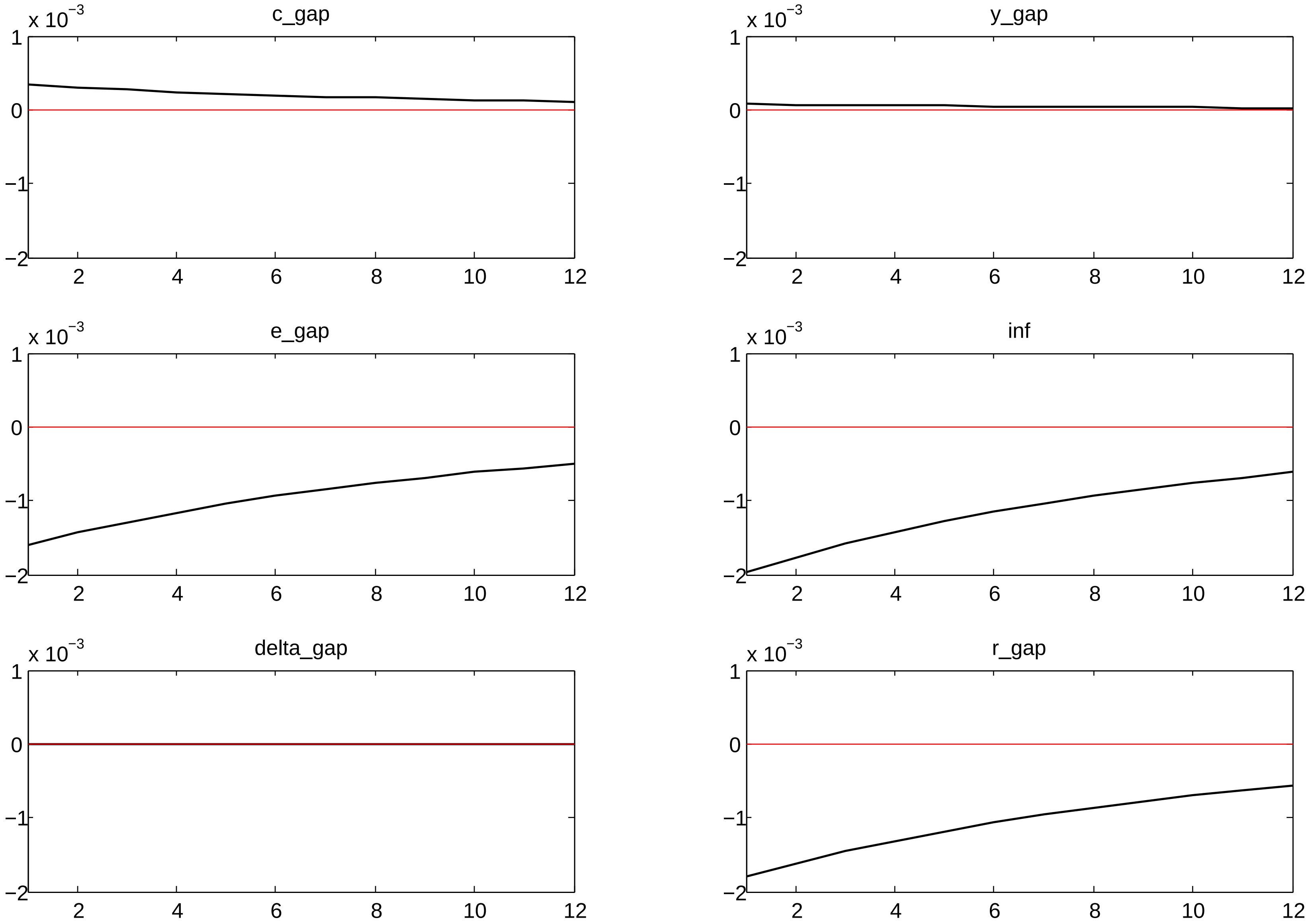

Figure 2

Impulse responses to a positive monitoring cost shock under the Taylor rule

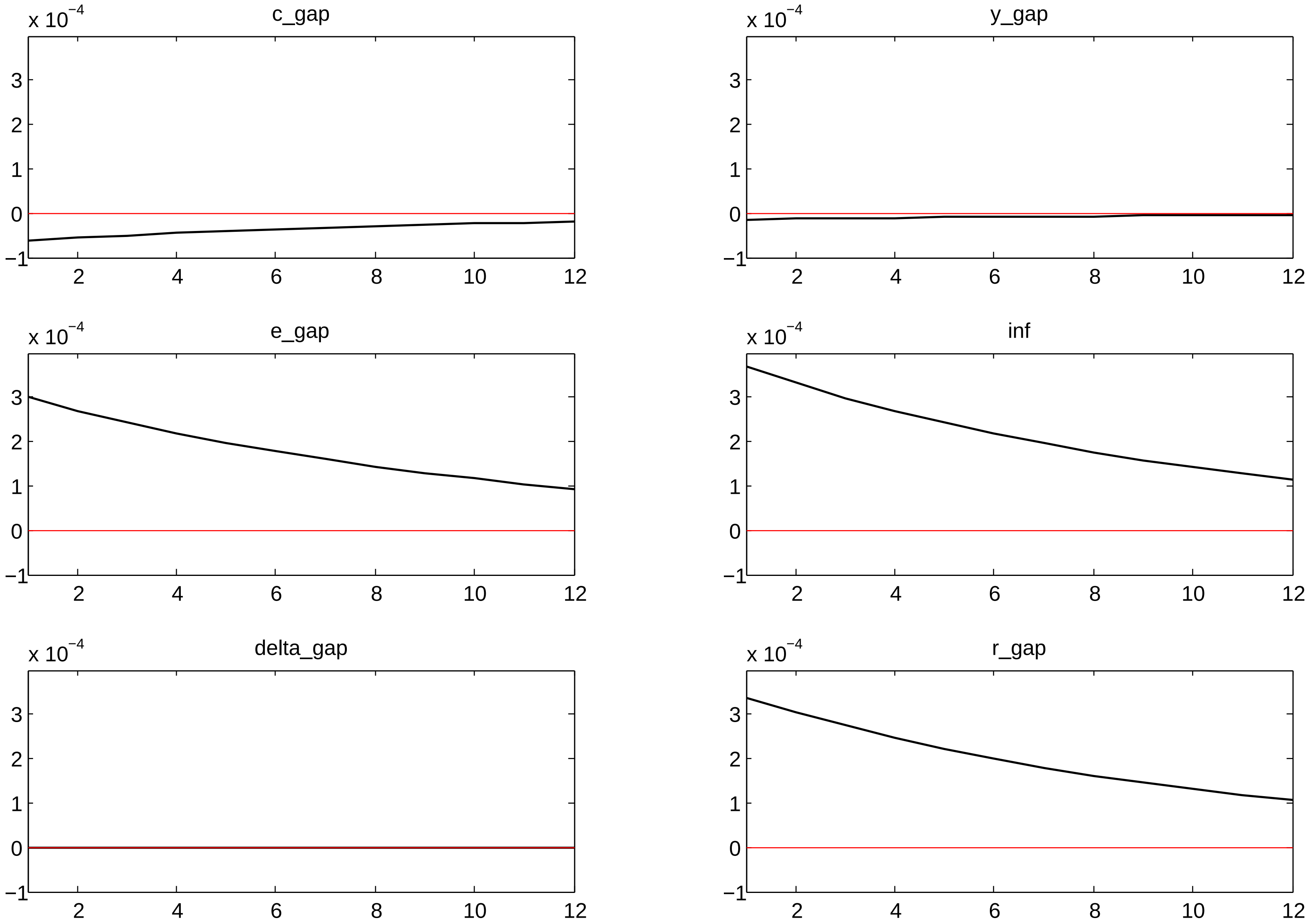

Figure 3

Impulse responses to a positive technology shock under optimal policy