Monetary Policy under Alternative Asset Market Structures: the Case of a Small Open Economy*

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Abstract

Can the structure of asset markets change the way monetary policy should be conducted? Recent literature has emphasized that welfare in an open economy can be affected by a terms of trade externality. Following a linear-quadratic approach, this paper investigates how the implications of this externality for monetary policy changes with the structure of asset markets. Our results reveal that this configuration significantly affects optimal monetary policy and the performance of standard policy rules. In particular, when comparing complete and incomplete markets, the ranking of policy rules is entirely reversed, and so are the policy prescriptions regarding the optimal level of exchange rate volatility.

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1 Introduction

How does the structure of international asset markets affect monetary policy? The debate surrounding optimal monetary policy in open economies has been extensive over the past decade. Many works have emphasized that optimal monetary policy in an open economy may be influenced the presence of a "terms of trade externality". Part of the literature highlights the fact that the presence of such an externality can affect the optimality of inward looking policies. But are these policy incentives affected by the degree of international risk sharing? The current paper characterizes a utility-based loss function for a small open economy under different asset market structures and derives the corresponding optimal monetary policy. Our analysis shows that the degree of risk sharing can significantly affect the optimal policy prescription and the performance of standard policy rules.

Early contributions on optimal monetary policymaking in an open economy, such as Clarida et al. (2001) and Gali and Monacelli (2005), show the policy problem in an open economy may be isomorphic to the one in a closed economy environment. Their results suggest that policymakers in an open economy should follow a purely inward looking policy, responding solely to movements in domestic prices (or producer prices). Hence, there is no role for exchange rate stabilization, even if movements in the exchange rate affect consumer prices.

*This work was largely carried out before the author joined the Bank of England, and the views expressed in this paper are those of the author and do not necessarily reflect the views of this institution.

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However, recent theoretical literature on policy objectives in open economies suggests that this result is not a robust one. As emphasized in Obstfeld and Rogoff (1998), welfare in an open economy can be influenced by a "terms of trade externality". This externality arises because imported goods may not be perfect substitutes to goods produced domestically. This fact implies that a social planner in an open economy may wish to exploit a certain degree of monopoly power.[1] Corsetti and Pesenti (2001) analyze welfare and monetary policy in a setting characterized by this external distortion - related to the country's monopoly power in trade - and an internal distortion – related to monopolistic supply in the domestic market. The internal distortion implies that monetary surprises can increase output towards its efficient level. But in open economies these surprises also reduce domestic consumers' purchasing power internationally. Because of the latter effect, expansionary policies can reduce welfare. As emphasized in Tille (2001), the overall impact of such shocks depends on the relative size of these two distortions.[2]

In a complete markets setting, Benigno and Benigno (2003) explore the consequences of such internal and external distortions for optimal monetary policy in a stochastic two-country framework. De Paoli (2008) and Faia and Monacelli (2008) present a similar analysis in a small open economy setting.[3] These studies show that, if policymakers in different countries act independently, they may have an incentive to affect the terms of trade in their own advantage. If domestic and foreign goods are close substitutes, an improvement in the terms of trade can induce agents to consume more imported goods (that is, terms of trade improvements have a so-called expenditure-switching effect). These consumers are better off, since they can reduce their labor effort without a corresponding fall in their consumption levels. A terms of trade improvement (or a real exchange rate appreciation) ceases to be welfare improving when these elasticities are small, and the terms of trade cannot divert consumption towards foreign goods. In this case, a more depreciated real exchange rate on average can be welfare improving. Moreover, unless countries are insular to terms of trade movements, domestic inflation targeting is no longer the policy choice of individual countries.

The present paper evaluates whether or not the above policy incentives are influenced by the degree of international risk sharing. Our analysis confirms that, under complete markets and a *high* elasticity of substitution between domestic and foreign, there is a policy incentive to engineer a terms of trade improvement (or a real exchange rate appreciation). Moreover, in this case, a exchange rate peg can outperform (that is, lead to higher welfare) a policy that focus on domestic price stabilization. This is because a fixed exchange rate regime ties policymakers hands who, for this reason, under-stabilize output relative to the flexible price allocation. When compared with price stability, this regime is associated with a lower level of output and a more appreciated real exchange rate on average.

However, the results are different in the case of imperfect risk sharing. Whereas efficient risk sharing severs the link between domestic consumption and domestic production, with incomplete markets these are more tightly related. Under financial autarky, for example, consumption has to be fully financed by domestic output. So, while in the complete markets setting, optimal risk sharing prevents home agents from suffering negative income effects if they were to reduce domestic production and engineer a terms of trade improvement, this is

[1] This externality is also discussed in the trade theory context. The literature on trade policy points out that imposing taxes on exports might be welfare improving because, due to imperfect substitutability between the domestic and foreign goods, it is in the country's interest to behave like a monopolist and restrict its supply of exports.

[2] Tille (2001) shows that the overall impact of changes in money supply depends on the degree of substitutability between goods produced within a country and the degree of substitutability between goods produced in different countries.

[3] Many other studies analyze welfare and monetary policy in different open economy settings. For alternative works investigating the case for exchange rate stabilization see, for example, Devereux and Engel (2003), Pappa (2004), Corsetti and Pesenti (2005), Sutherland (2005), Benigno and Benigno (2006).

no longer the case under incomplete markets. That is, under imperfect risk sharing it may no longer be possible to decrease agent's disutility from producing domestically without decreasing their utility from consumption. In fact, under incomplete markets, a policy of exchange rate stabilization would only be beneficial if the degree of substitutability between home and foreign good is *low*. This is because a low elasticity of substitution between imported and domestic goods reduces the negative income effect of terms of trade improvement on consumption.

Therefore, our welfare comparison highlights that while an exchange rate peg may outperform a domestic inflation targeting regime when asset markets are complete and domestic and imported goods are substitutes, the opposite holds when asset markets are incomplete. Our results suggest that optimal monetary policymaking in a small open economy crucially depend on the degree of substitutability between goods and the degree of international risk sharing.

In terms of our modelling approach, we characterize a small open economy framework as a limiting case of a two-country dynamic general equilibrium model. The baseline framework features monopolistic competition, nominal rigidities and home bias in consumption. In our analysis we consider three different asset markets specification: complete asset markets (optimal international risk sharing), incomplete asset markets (sub-optimal international risk sharing) and financial autarky (absence of international risk sharing).

Our policy evaluation methodology follows the linear quadratic approach developed by Benigno and Woodford (2003) and Sutherland (2002), and characterizes a utility-based loss function for the different asset market settings. The method delivers an analytical representation of the policy problem that is similar to the one used in the traditional literature on monetary policy evaluation (that is, policymakers minimize a quadratic loss function subject to linear constraints). But the utility-based loss function for the small open economy depends not only on the volatility of output and domestic inflation but also on the real exchange rate volatility. Moreover, the weights of these variables in the loss function depend on the form of asset markets. Finally, we derive the optimal monetary policy for the different settings and represent it in terms of a targeting rule *à la* Svensson (2003).

The remainder of the paper is structured as follows. Section 2 introduces the model. The system of log-linearized equilibrium conditions is presented in Section 3. In Section 4 we derive welfare and Subsection 4.1 presents the linear-quadratic loss function. The analysis of monetary policy under different asset market structure is illustrated in Section 5. Finally, Section 6 concludes.

2 The Model

The framework consists of a small open economy setup derived from two-country dynamic general equilibrium model. The baseline framework is fairly standard, following the work of Gali and Monacelli (2005) and De Paoli (2008). Nevertheless, in our analysis, we consider three different asset market specifications. These are presented in Subsection 2.1.

In the model deviations from purchasing power parity arise from the existence of home bias in consumption. This bias depends on the degree of openness and the relative size of the economy. The specification allows us to characterize the small open economy by taking the limit of the home economy size to zero. Prior to applying the limit, we derive the optimal equilibrium conditions for the general two-country model. After the limit is taken, the two countries, Home and Foreign, represent the small open economy and the rest of the world, respectively.

Monopolistic competition and sticky prices are introduced in order to address issues of monetary policy. We further assume that home price setting follows a Calvo-type contract, which introduces richer dynamic effects of monetary policy than in a setup where prices are set one period in advance. Moreover, we abstract from monetary frictions by considering a cashless economy as in Woodford (2003, Chapter 2).

Preferences

We consider two countries, H (Home) and F (Foreign). The world economy is populated with a continuum of agents of unit mass, where the population in the segment $[0,n)$ belongs to country H and the population in the segment $(n,1]$ belongs to country F. The utility function of a consumer j in country H is given by[4]

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s) - \frac{1}{n} \int_0^n V(y_s^j, \varepsilon_s) dj \right], \tag{1}$$

Households obtain utility from consumption $U(C^j)$ and contribute to the production of all domestic goods y^j attaining disutility $\frac{1}{n}\int_0^n V(y_s^j, \varepsilon_s)dj$. Risk is pooled internally to the extent that agents participate in the production of all goods and receive an equal share of production revenue. Productivity shocks are denoted by ε_s, and C is a Dixit-Stiglitz aggregate of home and foreign goods, defined by

$$C = \left[v^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}. \tag{2}$$

The parameter $\theta > 0$ is the intratemporal elasticity of substitution between home and foreign-produced goods, C_H and C_F. As in Sutherland (2005), the parameter determining home consumers' preferences for foreign goods, $(1-v)$, is a function of the relative size of the foreign economy, $(1-n)$, and of the degree of openness, λ; more specifically, $(1-v) = (1-n)\lambda$.

Similar preferences are specified for the rest of the world

$$C = \left[v^{*\frac{1}{\theta}} C_H^{*\frac{\theta-1}{\theta}} + (1-v^*)^{\frac{1}{\theta}} C_F^{*\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

with $v^* = n\lambda$. That is, foreign consumers' preferences for home goods depend on the relative size of the home economy and the degree of openness. Note that the specification of v and v^* generates a home bias in consumption.

The sub-indices C_H (C_H^*) and C_F (C_F^*) are Home (Foreign) consumption of the differentiated products produced in countries H and F. These are defined as follows

$$C_H = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \qquad C_F = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \tag{4}$$

$$C_H^* = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c^*(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \qquad C_F^* = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 c^*(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \tag{5}$$

where $\sigma > 1$ is the elasticity of substitution across the differentiated products. The consumption-based price indices that correspond to the above specifications of preferences are given by

$$P = \left[v P_H^{1-\theta} + (1-v)(P_F)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{6}$$

and

$$P^* = \left[v^* P_H^{*1-\theta} + (1-v^*)(P_F^*)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{7}$$

[4] In the subsequent sections, we assume the following isoelastic functional forms: $U(C_t) = \frac{C_t^{1-\rho}}{1-\rho}$ and $V(y_t, \varepsilon_{Y,t}) = \frac{\varepsilon_{Y,t}^{-\eta} y_t^{1+\eta}}{1+\eta}$, where ρ is the coefficient of relative risk aversion and η is equivalent to the inverse of the elasticity of labor supply.

where P_H (P_H^*) is the price sub-index for home-produced goods expressed in the domestic (foreign) currency and P_F (P_F^*) is the price sub-index for foreign produced goods expressed in the domestic (foreign) currency:

$$P_H = \left[\left(\frac{1}{n} \right) \int_0^n p\left(z\right)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, P_F = \left[\left(\frac{1}{1-n} \right) \int_n^1 p\left(z\right)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, \tag{8}$$

$$P_H^* = \left[\left(\frac{1}{n} \right) \int_0^n p^*\left(z\right)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, P_F^* = \left[\left(\frac{1}{1-n} \right) \int_n^1 p^*\left(z\right)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}. \tag{9}$$

We assume that the law of one price holds, so

$$p(h) = Sp^*(h) \text{ and } p(f) = Sp^*(f), \tag{10}$$

where the nominal exchange rate, S_t, denotes the price of foreign currency in terms of domestic currency. Equations (6) and (7), together with condition (10), imply that $P_H = SP_H^*$ and $P_F = SP_F^*$. However, as Equations (8) and (9) illustrate, the home bias specification leads to deviations from purchasing power parity; that is, $P \neq SP^*$ For this reason, we define the real exchange rate as $Q \equiv \frac{SP^*}{P}$.

From consumers' preferences, we can derive the total demand for a generic good h, produced in country H, and the demand for a good f, produced in country F

$$y_t^d(h) = \left[\frac{p_t(h)}{P_{H,t}} \right]^{-\sigma} \left\{ \left[\frac{P_{H,t}}{P_t} \right]^{-\theta} \left[vC_t + \frac{v^*(1-n)}{n} \left(\frac{1}{Q_t} \right)^{-\theta} C_t^* \right] + G_t \right\}, \tag{11}$$

$$y_t^d(f) = \left[\frac{p_t(f)}{P_{F,t}} \right]^{-\sigma} \left\{ \left[\frac{P_{F,t}}{P_t} \right]^{-\theta} \left[\frac{(1-v)\,n}{1-n} C_t + (1-v^*) \left(\frac{1}{Q_t} \right)^{-\theta} C_t^* \right] \right\}, \tag{12}$$

where G_t and G_t^* are the country-specific government shocks. We assume that the public sector in the Home (Foreign) economy only consumes Home (Foreign) goods and has preferences for differentiated goods analogous to the ones of the private sector (given by Equations 4 and 5). The government budget constraints in the Home and Foreign economy are respectively given by

$$\tau_t \int_0^n p_t(h) y_t(h) dh = nP_{H,t}(G_t + Tr_t) \tag{13}$$

and

$$\tau_t \int_n^1 p_t^*(f) y_t^*(f) dh = (1-n)P_{F,t}(G_t^* + Tr_t^*). \tag{14}$$

Fluctuations in proportional taxes, τ_t (τ_t^*), or government spending, G_t (G_t^*), are exogenous and completely financed by lump-sum transfers, Tr_t (Tr_t^*), made in the form of domestic (foreign) goods.

Finally, to portray our small open economy, we use the definition of v and v^* and take the limit for $n \to 0$. Consequently, conditions (11) and (12) can be rewritten as

$$y^d(h) = \left[\frac{p_t(h)}{P_{H,t}} \right]^{-\sigma} \left\{ \left[\frac{P_{H,t}}{P_t} \right]^{-\theta} \left[(1-\lambda)C_t + \lambda \left(\frac{1}{Q_t} \right)^{-\theta} C_t^* \right] + G_t \right\}, \tag{15}$$

$$y^d(f) = \left[\frac{p_t^*(f)}{P_{F,t}^*} \right]^{-\sigma} \left\{ \left[\frac{P_{F,t}^*}{P_t^*} \right]^{-\theta} C_t^* + G_t^* \right\}. \tag{16}$$

Equations (15) and (16) show that external changes in consumption affect demand in the small open economy, but the opposite is not true. Moreover, movements in the real exchange rate do not affect the rest of the world's demand.

Price-setting Mechanism

Prices follow a partial adjustment rule *à la* Calvo (1983). Producers of differentiated goods know the form of their individual demand functions (given by Equations (15) and (16)), and maximize profits taking overall market prices and products as given. In each period a fraction, $\alpha \in [0, 1)$, of randomly chosen producers is not allowed to change the nominal price of the goods they produce. The remaining fraction of firms, given by $(1 - \alpha)$, chooses prices optimally by maximizing the expected discounted value of profits. The optimal choice of producers that can set their price $\tilde{p}_t(j)$ at time T is, therefore

$$E_t \left\{ \sum (\alpha\beta)^{T-t} U_c(C_T) \left(\frac{\tilde{p}_t(j)}{P_{H,T}} \right)^{-\sigma} Y_{H,T} \left[\frac{\tilde{p}_t(j)}{P_{H,T}} \frac{P_{H,T}}{P_T} - \frac{\sigma V_y \left(\tilde{y}_{t,T}(j), \varepsilon_t \right)}{(1-\tau_T)(\sigma-1) U_c(C_T)} \right] \right\} = 0. \tag{17}$$

Monopolistic competition in production leads to a wedge between marginal utility of consumption and marginal disutility of production. We allow for fluctuations in this wedge by assuming a time-varying proportional tax τ_t. Hereafter, we refer to these fluctuations as markup shocks μ_t, where $\mu_t = \frac{\sigma}{(1-\tau_t)(\sigma-1)}$.

Given the Calvo-type setup, the price index evolves according to the following law of motion,

$$(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha) \left(\tilde{p}_t(h) \right)^{1-\sigma}. \tag{18}$$

The rest of the world has an analogous price setting mechanism.

2.1 Asset Markets

The structure of financial markets can significantly alter the way idiosyncratic shocks affect consumption, output and other macroeconomic variables. As described in Obstfeld and Rogoff (1996, Chapter 5), *"The presence of international markets for risky assets weakens and may sever the link between shocks to a country's output or factor productivity and shocks to its resident's income. Sophisticated international financial markets thus force us to rethink the channels through which macroeconomic shocks impinge on the world economy."*

In this Section, we introduce three different specifications for asset market structure. First, we present the scenario in which international financial markets are incomplete, by assuming that agents can internationally trade nominal riskless bonds subject to intermediation costs. Then we describe two benchmark cases of asset market structure: at one extreme, we analyze the case of financial autarky, in which the small open economy has no access to international financial markets; at the other, we examine the most developed form of capital markets, in which households have access to a complete set of contingent claims.

Incomplete Markets

We characterize the environment of incomplete markets by assuming that agents can trade nominal riskless bonds denominated in *Home* and *Foreign* currency. We consider that home-currency denominated bonds are only traded domestically. Moreover, following Benigno (2001), the international trade of foreign currency-denominated bonds is subject to intermediation costs. This cost is proportional to the country's aggregate net foreign asset position. If the small open economy is a net debtor, its agents pay a premium on the foreign interest rates when borrowing from abroad. On the other hand, if the country is a net creditor, households lending in foreign currency receive a rate of return lower than foreign interest rates. The spread is the remuneration of international intermediaries, and is assumed to be rebated equally among foreign households.

The intermediation cost assumption is introduced for technical reasons: it solves the stationarity problem in the style of Obstfeld and Rogoff (1995).[5] By ensuring that the model is stationary, this assumption guarantees the precision of any quantitative exercises involving a log linear version of the model. In addition, it allows for the examination of the second moments of macroeconomic variables. Nevertheless, for some of our qualitative analysis, we consider the case of zero intermediation costs. This is done in order to simplify the analytical derivation of the optimal plan and improve our intuition on the policy prescriptions under incomplete markets.

We can write the household's budget constraint at Home as follows:

$$P_t C_t + \frac{B_{H,t}}{(1+i_t)} + \frac{S_t B_{F,t}}{(1+i_t^*)\psi\left(\frac{S_t B_{F,t}}{P_t}\right)} \leq B_{H,t-1} + S_t B_{F,t-1} + \frac{(1-\tau_t)\int_0^n p_t(h) y_t(h) dh}{n} + P_{H,t} Tr_t, \tag{19}$$

where $B_{H,t}$ and $B_{F,t}$ denote domestic-currency and foreign-currency denominated nominal bonds and Tr_t are government transfers, made in the form of domestic goods. The function $\psi(\cdot)$ represents the cost from international borrowings and it is increasing in the aggregate level of foreign debt: $\psi'(.) < 0$. We further assume a zero steady-state risk premium by setting $\psi\left(\overline{B}_F\right) = 1$. Moreover, in specifying the budget constraint (19), we also assume that households in a given country produce all goods and share the revenues from production in equal proportions. We also consider the case in which the initial wealth of all households within a country are equal. These two assumptions ensure that households in the same country face the same budget constraints in every period and state of the world. Consequently, we can consider a representative consumer for each economy. But even though idiosyncratic risk is pooled among households from the same country, there is imperfect risk sharing across borders.

Foreign households are assumed to trade only in foreign currency bonds. Thus, their budget constraint can be written as

$$P_t^* C_t^{*i} + \frac{B_{F,t}}{(1+i_t^*)} \leq B_{F,t-1} + \frac{(1-\tau_t^*)\int_{1-n}^1 p(f) y_t(f) df}{1-n} + P_t^* Tr_t^* + \frac{K}{1-n}. \tag{20}$$

The intermediation profits K, which are shared equally among foreign households, can be written as

$$K = \frac{B_{F,t}}{P_t^*(1+i_t^*)}\left[1 - \frac{Q_t}{\psi\left(\frac{S_t B_{F,t}}{P_t}\right)}\right]. \tag{21}$$

Given the above specification, we can write the consumer's intertemporal optimal choices as

$$U_C(C_t) = (1+i_t)\beta E_t\left[U_C(C_{t+1})\frac{P_t}{P_{t+1}}\right], \tag{22}$$

$$U_C(C_t^*) = (1+i_t^*)\beta E_t\left[U_C(C_{t+1}^*)\frac{P_t^*}{P_{t+1}^*}\right], \tag{23}$$

and

$$U_C(C_t) = (1+i_t^*)\psi\left(\frac{S_t b_t}{P_t}\right)\beta E_t\left[U_C(C_{t+1})\frac{S_{t+1}P_t}{S_t P_{t+1}}\right], \tag{24}$$

where (22) and (24) are Home and Foreign Euler equations derived from the optimal choice of foreign-currency denominated bonds. Equation (23) results from the small open economy optimal choice of home-currency denominated bonds. Moreover, Equations (22) and (24) imply that there is an interest rate differential across countries. Finally, we should note that, given that idiosyncratic risk is pooled among domestic households and foreign households

[5] See Ghironi.(2006) for a comprehensive discussion.

only trade foreign-currency denominated bonds, domestic-currency denominated bonds are in zero net supply. That is, in reality only foreign-currency denominated bonds are traded in equilibrium.[6]

Financial Autarky

In this setup, the economy does not have access to international borrowing or lending. Consequently, there is no risk sharing across borders. As in the case of incomplete markets, we assume that there is a symmetric initial distribution of wealth across domestic agents.

The household budget constraints, at Home and abroad, can be written as

$$P_t C_t \leq \frac{(1-\tau_t)\int_0^n p_t(h) y_t(h) dh}{n} + P_{H,t} Tr_t \tag{25}$$

and

$$P_t^* C_t^{*i} \leq \frac{(1-\tau_t^*)\int_{1-n}^1 p(f) y_t(f) dh}{1-n} + P_t^* Tr_t^*. \tag{26}$$

Under financial autarky, the value of domestic production has to be equal to the level of public and private consumption in nominal terms. Aggregating private and public budget constraints, we have:

$$P_H(Y_t - G_t) = P_t C_t \tag{27}$$

The inability to trade bonds with the rest of the world imposes that the value of imports should equal the value of exports:

$$(1-n) S_t P_{H,t}^* C_{H,t}^* = n P_{F,t} C_{F,t}. \tag{28}$$

Complete Markets

As in Gali and Monacelli (2005) and De Paoli (2008), we characterize the most developed form of capital markets following Chari et al. (2002). The complete market environment is introduced by assuming that agents have access to state contingent nominal claims that deliver a unit of Home currency in each state of the world. In this setup, the rate of marginal utilities is equalized across countries at all times and states of nature.

$$\frac{U_C\left(C_{t+1}^*\right)}{U_C\left(C_t^*\right)} \frac{P_t^*}{P_{t+1}^*} = \frac{U_C\left(C_{t+1}\right)}{U_C\left(C_t\right)} \frac{S_{t+1} P_t}{S_t P_{t+1}}. \tag{29}$$

3 Log-linearized equilibrium conditions

In the current Section we present a summary of the model's equilibrium conditions in log-deviations from steady state. A full description of the steady state can be found in Appendix A and Appendix B presents the derivation of the first and second order approximations of the model. In the previous section we present a general version of the model, while the log-linearized system of equilibrium conditions described below imposes some restrictions on parameter values and steady-state conditions. In particular, we assume a log-utility function (i.e. $\rho = 1$). Moreover, we consider the case of a symmetric steady state which implies a zero steady-state net foreign asset position (that is, $\overline{B}_F = 0$, where upper-bar indicates a steady-state condition). In addition, we abstract from fiscal shocks and a time-varying markup. The implication of these restrictions are discussed in Section 5.2, where such assumptions are relaxed.

[6] The present framework does not include a portfolio problem for households. For recent contributions on optimal international portfolios in incomplete markets settings, see, for example, Devereux and Sutherland (2007) and Evans and Hnatkovska (2005).

The system of equilibrium conditions for the small open economy can be described by an aggregate supply, an aggregate demand and an equilibrium condition(s) implied by the financial market structure. These can be found in Tables 1, 2 and 3, which represent, respectively, the case of complete markets, financial autarky and incomplete markets.

Table 1: Equilibrium Conditions under Complete Markets

$\pi_t = k(c_t + \eta y_t + \frac{\lambda}{1-\lambda} q_t - \eta \varepsilon_t) + \beta E_t \pi_{t+1}$	AS
$y_t = (1-\lambda)c_t + \lambda c_t^* + \gamma q_t$	AD
$c_t = c_t^* + q_t$	CM

Table 2: Equilibrium Conditions under Financial Autarky

$\pi_t = k(c_t + \eta y_t + \frac{\lambda}{1-\lambda} q_t - \eta \varepsilon_t) + \beta E_t \pi_{t+1}$	AS
$y_t = (1-\lambda)c_t + \lambda c_t^* + \gamma q_t$	AD
$y_t - \frac{\lambda}{1-\lambda} q_t = c_t$	FA

Table 3: Equilibrium Conditions under Incomplete Markets

$\pi_t = k(c_t + \eta y_t + \frac{\lambda}{1-\lambda} q_t - \eta \varepsilon_t) + \beta E_t \pi_{t+1}$	AS
$y_t = (1-\lambda)c_t + \lambda c_t^* + \gamma q_t$	AD
$E_t(c_{t+1} - c_t) = E_t(c_{t+1}^* - c_t^*) + E_t \Delta q_{t+1} - \delta b_t$	IM
$\beta b_t = b_{t-1} + y_t - c_t - \frac{\lambda}{1-\lambda} q_t$	IM′

Lower case variables are expressed in log deviations from steady-state. In summary: c_t and c_t^* denote domestic and foreign consumption, y_t denotes domestic output, q_t denotes the real exchange rate, b_t represents net foreign assets (expressed in real terms) and π_t represents domestic (or producer price) inflation. The stochastic environment is characterized by the presence of productivity shocks ε_t. The parameters of the model are described in Table 4.

Table 4: Model parameters

θ	Intratemporal elasticity of substitution
η^{-1}	Elasticity of labor production
λ	Degree of openness
β	Subjective discount factor
σ	Elasticity of substitution across the differentiated products
δ	Sensitivity of intermediation costs to the level of foreign debt
k	$(1-\alpha\beta)(1-\alpha)/\alpha(1+\sigma\eta)$
γ	$\theta\lambda(2-\lambda)/(1-\lambda)$

Equation (AS) represents the small open economy's Phillips curve. Note that the flexible price allocation is identical to the equilibrium allocation that would prevail were policymakers to target domestic inflation. That is, the case in which $\alpha \to 0$ and, therefore, $k \to \infty$, is equivalent to the case in which $\pi_t = 0$, ∇t. Equation (AD) illustrates how the demand for the small open economy's products depends on foreign and domestic consumption. Equation (CM) in Table 1 is derived from the complete market assumption, and represents the optimal risk sharing agreement between agents in the small economy and agents in the rest of the world. In Table 2, which summarizes the equilibrium conditions under financial autarky, Equation (FA) represents the aggregate resource constraint. This Equation illustrates that under financial autarky, domestic consumption has to be fully financed by domestic production. Finally, in the case of market incompleteness (Table 3), combining domestic and foreign Euler equations, we derive Equation (IM). Moreover, in this setup, the aggregate budget constraint of the small open economy can be written as (IM$'$).

Given domestic exogenous shocks ε_t and external conditions, c_t^*, the small open economy system of equilibrium conditions is closed by specifying the monetary policy rule. In the next sections we examine different specifications for this rule. Apart from analyzing the optimal monetary policy regime, we evaluate the performance of alternative policy rules such as an exchange rate peg, and both consumer price index (CPI) and producer price index (PPI) inflation targeting regimes.

Foreign dynamics are governed by foreign supply and demand conditions (AS* and AD*):

Table 5: Foreign Equilibrium Conditions

$\pi_t^* = k(c_t^* + \eta y_t^* - \eta\varepsilon_t) + \beta E_t \pi_{t+1}^*$	AS*
$y_t^* = c_t^*$	AD*

The specification of the foreign policy rule completes the system of equilibrium conditions which determine the evolution of y_t^*, c^* and π_t^*. For simplicity, and without loss of generality, we assume that the foreign economy targets domestic inflation (i.e. sets $\pi_t^* = 0$, ∇t).[7] We should note that the dynamics of the rest of the world are not affected by *Home* variables. Therefore, the small open economy can treat c_t^* as an exogenous shock.

[7] Since the policy choice in the rest of the world determines how the endogenous variables respond to structural shocks, it may also affect the correlation between c_t^* and π_t^*. But in the case of a symmetric steady state, the system of equilibrium conditions in the small open economy is only affected by c_t^*. Thus, in this case, the policy choice of the rest of the world is irrelevant for the dynamics of the small open economy.

4 Welfare

In this Section we present the objective function and the optimal monetary policy plan for the small open economy, under the different asset market structures. A full derivation of these expressions can be found in Appendix C. We should note that, while the appendix allows for a general level of steady-state monopolistic markup, for clarity of the exposition, in the sections to follow we assume a specific level for this markup (in particular, we set $\overline{\mu} = (1-\lambda)^{-1}$, as in Gali and Monacelli (2005)). This parameterization guarantees that the steady state is efficient when the elasticity of intratemporal and intertemporal substitution are unitary, or when the economy is closed.

In a microfounded model, welfare can be precisely derived from households' utility. Therefore, we obtain the monetary authority's objective function, which should reflect the economy's level of welfare, from a second-order Taylor expansion of this utility:

$$L_{to} = (1-\lambda)U_c\bar{C}E_{t_0}\sum\beta^t\left[d_t + \frac{1}{2}(\eta+1)(y_t - y_t')^2 + \frac{1}{2}\frac{\sigma}{k}(\pi_t)^2\right] + t.i.p + \mathcal{O}(||\xi||^3). \quad (30)$$

The term $t.i.p.$ stands for terms independent of policy (in particular, these refer to exogenous shocks that are not affected by the policy choice) and $\mathcal{O}(||\xi||^3)$ refers to terms of order strictly higher than two. In addition, we define $d_t \equiv y_t - \frac{1}{(1-\lambda)}c_t$ and $y_t' \equiv \frac{\eta}{(\eta+1)}\varepsilon_t$. Note that, in the case of a closed economy, in which $\lambda = 0$, the term d_t is eliminated from the above expression. Moreover, in this case y_t' coincides with the flexible price allocation, or equivalently, the equilibrium allocation that would prevail if a policy of price stability is implemented. In other words, in this specification of a closed economy, there is no trade-off between stabilizing inflation and the output gap. As demonstrated in Benigno and Woodford (2003) this results relies on the fact that the steady-state output is efficient and there are only productivity shocks. As replicated in our Appendix, in the presence of non-efficient steady state or markup shocks, this result no longer holds.

But as described in Obstfeld and Rogoff (1998) and Corsetti and Pesenti (2001), in an open economy welfare is influenced by an external distortion that gives rise to a terms of trade externality. Such a distortion arises because imported goods are not perfect substitutes to goods produced domestically, and, as a result, a social planner in an open economy may wish to exploit a certain degree of monopoly power. Thus, apart from nominal rigidities, welfare in our small open economy is affected an internal monopolistic distortion as well as an external distortion. The presence of these distortions can be illustrated by the term d_t in the loss function.

Importantly, the implication of these distortions depends on the asset market structure. We can illustrate this fact by comparing the complete markets specification with the case of financial autarky. By inspection of Table 1 and Table 2 we can see that while under complete markets the term d_t can be written as a function of $E[(1-\theta)q_t]$, under financial autarky, this term depends on $E[(\theta-1)q_t]$.[8] Therefore, while a more appreciated real exchange rate on average is welfare improving under complete markets and substitute goods ($\theta > 1$), the opposite holds under financial autarky.

Under complete markets, when domestic and foreign goods are substitutes in the utility an real exchange rate appreciation can improve welfare by decreasing the disutility of producing at home without an equivalent reduction in the utility of consumption. But while the complete markets assumption prevents agents in the small economy from experiencing negative income effects (were they to reduce their production levels), under financial autarky this would no longer be the case. When domestic agents have no access to international asset markets, domestic agents borrowing constraints imply a tight link between

[8] Using a first-order approximation of demand equation and the risk sharing condition, we can write d_t as a function of θ and q_t. However, the full welfare implications of the linear term can only be accessed when this term is approximated to second order. This is derived in Appendix C and explained in the next Section.

domestic consumption and income. In this case, if goods are substitutes an appreciated real exchange rate is actually welfare inferior, as it induces a lower demand for domestic goods and, thus, lower domestic income. Lower domestic income, in turn, decreases consumption and welfare.

On the other hand, when goods are complements in the utility (i.e. when $\theta < 1$), under complete markets a more depreciated unconditional mean of the real exchange rate increases welfare by creating a rise in consumption utility larger than the rise in labor disutility. And, in this case, an appreciated exchange rate could improve welfare under financial autarky. For small values of θ, output would fall little relative the movement in real exchange rate, and the effect of the appreciation on agents' purchasing power would outweigh the reduction in output.

In the knife-edge case in which the marginal utility of consuming one good does not depend on the consumption of the other good (i.e. when $\theta = 1$), welfare would not depend on the level of the real exchange rate. Under this specification, the economy never experiences trade imbalances and the dynamics of the current account and the asset market structure are irrelevant.[9] As a result, the welfare characterization is also independent of the degree of risk sharing. Furthermore, the real exchange rate externality is eliminated and the utility-based loss function becomes isomorphic to the one in a closed economy.

4.1 The Linear-quadratic Loss Function

In order to obtain an approximation of the optimal plan that is fully accurate to second-order, we follow the linear-quadratic approach of Benigno and Woodford (2003) and Sutherland (2002).[10] We eliminate the linear term d_t in the Taylor expansion, using a second-order approximation of the model's equilibrium conditions. Because alternative asset market characterizations imply different equilibrium conditions, the final expression for welfare varies according to the structure of the asset market. As shown in Appendix C, we obtain the following expression for the objective function:

$$L_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\begin{array}{c} \frac{1}{2} l_{yy}^m y_t^2 + l_{yq}^m y_t q_t + \frac{1}{2} l_{qq}^m q_t^2 + \frac{1}{2} l_\pi^m \pi_t^2 \\ + L_{ey}^{m'} e_t y_t + L_{eq}^{m'} e_t q_t \end{array} \right] + t.i.p + \mathcal{O}(||\xi||^3). \qquad (31)$$

The weights $l_{yy}^m, l_{qq}^m, l_{yq}^m, l_\pi^m$, and the vectors L_{ey}^m and L_{eq}^m depend on the structural parameters of the model and on the asset market configuration. In what follows we let the superscript $m = c$ represent the case of complete markets, while $m = fa$ is the financial autarky setup and the incomplete market case is denoted by $m = i$. The vector of exogenous variables, e_t, is defined as:

$$e_t = \left[\begin{array}{cc} \varepsilon_t & c_t^* \end{array} \right].^{11}$$

Even though the weights in the loss function are a complex function of structural parameters, we can show that when domestic and foreign goods are substitutes (complements) in the utility function, inflation variability is less (more) costly if asset markets are complete. In particular, the weight of inflation in the loss function, l_π, can be expressed as:

[9] The irrelevance of the asset market structure under this specification has been extensively discussed in the literature (e.g. Cole and Obstfeld (1991), Obstfeld and Rogoff (1995) and Benigno (2001), among others).

[10] Chari et al. (1994, 1995) suggest that the linear-quadratic approach may lead to an inaccurate approximation of the optimal policy problem. However, as explained in Benigno and Woodford (2006a, 2006b), their analysis is based on a "naïve" linear-quadratic approximation of the policy problem. As emphasized by Judd (1996, 1999), in order to obtain an approximation of the optimal plan that is fully accurate to second-order, the effect of second moments on the mean of the variables should be taken into account. The linear-quadratic approach adopted in this paper incorporates these effects by obtaining a purely quadratic approximation for the policy objective. Indeed, Benigno and Woodford (2006a, 2006b) demonstrate that a purely quadratic representation of the loss function leads to the correct local approximation of the problem for small enough disturbances.

[11] In the appendix we also allow for fiscal and markup shocks. These are discussed in Section 5.2.

$$l_\pi^i = l_\pi^{fa} = \frac{\sigma(1-\lambda)}{k}\left(1 + l\frac{\lambda}{(l+1)(1-\lambda)}\right)$$

and

$$l_\pi^c = \frac{\sigma(1-\lambda)}{k}\left(1 - l\frac{\lambda(\eta+1)}{(1+\eta)+\eta\lambda l}\right)$$

where $l = (\theta - 1)(2 - \lambda)$. Therefore, $l_\pi^i = l_\pi^{fa} > l_\pi^c$ when $\theta > 1$ and $l_\pi^i = l_\pi^{fa} < l_\pi^c$ when $\theta < 1$. The implication of these different weights for monetary policy will be explored in the subsequent sections.

5 Monetary Policy under Alternative Asset Market Structures

We proceed by characterizing the optimal plan under the assumption that policymakers can commit to maximizing the small open economy's welfare. The policy problem consists of minimizing the loss function given the equilibrium conditions and the initial conditions π_{t_0} and y_{t_0}.[12] In the case of complete markets and financial autarky, the policymakers choose the path of $\{\pi_t, y_t, c_t, q_t\}$ in order to minimize (31), subject to the equilibrium conditions given by Tables 1 and 2, respectively. The first order conditions to the minimization problem can be written in the form of the following targeting rules:

$$Q_y^c \Delta(y_t - y_t^{T,c}) + Q_q^c \Delta(q_t - q_t^{T,c}) + Q_\pi^c \pi_t = 0 \tag{32}$$

and

$$Q_y^{fa} \Delta(y_t - y_t^{T,fa}) + Q_q^{fa} \Delta(q_t - q_t^{T,fa}) + Q_\pi^{fa} \pi_t = 0, \tag{33}$$

where Δ denotes first difference operator, the superscript c denotes the complete market case and fa refers to the financial autarky setting. The above targeting rules set the objectives for monetary policy. This is done by specifying the targets y_t^T and q_t^T as functions of the different shocks.

In the case of incomplete markets, the policy problem consists of choosing the path of $\{\pi_t, y_t, c_t, q_t, b_t\}$ in order to minimize (31) subject to the equations specified in Table 3. The resulting first order conditions are shown in the appendix. The characterization of the optimal policy under incomplete markets is more complicated because of the intertemporal representation of the constraints (IM) and (IM′). The presence of intermediation costs also adds to the complexity of the problem. Nevertheless, in the special case in which there are no intermediation costs involved in the international trade of bonds (i.e. $\delta = 0$), the above first order conditions imply

$$Q_y^i E_t \Delta(y_{t+1} - y_{t+1}^{T,i}) + Q_q^i E_t \Delta(q_{t+1} - q_{t+1}^{T,i}) + Q_\pi^i E_t \pi_{t+1} = 0.^{13} \tag{34}$$

The general formulation of the optimal rule is similar under the different asset market structures. According to these rules, policymakers should respond to real exchange rate and output movements, as well as inflation. Nevertheless, the coefficients Q_y, Q_q and Q_π vary with the structure of the asset market. Such coefficients depend on the weights of

[12] In effect, the constraints on the initial conditions impose that the first order conditions to the problem are time invariant. This method follows Woodford's (1999) timeless perspective approach and ensures that the policy prescription does not constitute a time inconsistent problem.

[13] We should note that Equation (34) is not a targeting rule, since economic dynamics are not *determined* under this rule. The monetary policy plan which determines the optimal evolution of variables under incomplete markets is shown by equations (C.43) to (C.46) in the appendix. Equation (34) can be obtained by combining these equations, but it does not represent a monetary policy rule.

output, the real exchange rate and inflation in the loss function, which, in turn depend on the asset market structure.

Nevertheless, as previously stated, when the elasticity of intratemporal substitution is unitary, the dynamics of the small open economy are independent of the asset market structure. Under this specification, the first order conditions of the policy problem - for every asset market structure - can be expressed as:

$$0 = \Delta(y_t - y_t') + \sigma\pi_t \tag{35}$$

where y_t' coincides with the flexible price allocation for output. Therefore, under this parameterization, a policy of complete domestic price stabilization closes the welfare relevant output gap. In other words, it is optimal to target producer price inflation regardless of the asset market structure.

5.1 Evaluating different policy rules

In the previous Section, we presented the optimal monetary policy in the form of targeting rules. But the implementation of such rules may not be straightforward, either because the targets are difficult to monitor (y_t^T and q_t^T depend on unobservable shocks) or because the weights are complex functions of structural parameters. For this reason, in this section we numerically evaluate the optimal policy and compare the performances of simple policy rules. In particular, we compute a ranking, based on our welfare measure, of producer price inflation (PPI) targeting, consumer price inflation (CPI) targeting and fixed exchange rate regime (or PEG). This is done for different degrees of substitutability between domestic and foreign goods (i.e., different values of θ), under the different asset market structures. Table 6 presents the benchmark specification for the parameter values used in our numerical exercises.

Table 6: Parameter values used in the quantitative analysis

Parameter	Value	Notes:
β	0.99	Specifying a quarterly model with 4% steady-state real interest rate
η	0.47	Following Rotemberg and Woodford (1997)
λ	0.25	(unless specified otherwise)
α	0.66	Characterizing an average length of price contract of 3 quarters
σ	10	Following Benigno and Woodford (2005)
θ	1.5	(unless specified otherwise)
δ	0.01	Benigno (2001)

Tables 9 to 10 illustrate the results. Let's consider the case in which domestic and foreign goods are substitutes in the utility. In this case, while under sub-optimal risk sharing (financial autarky or incomplete markets), producer price inflation targeting is always the best policy available, under complete markets an exchange rate PEG or a CPI inflation targeting might outperform such policy. In particular, a domestic inflation targeting is welfare inferior under complete markets when foreign and domestic goods are close substitutes. This is true for domestic and foreign disturbances.

On the other hand, when domestic and foreign goods are complementary to one another, a policy that is more flexible with respect to movements in inflation and more restrictive towards exchange rate fluctuations can be welfare improving when asset markets are incomplete, but not when these are complete. And, when a country cannot perfectly share risk with the rest of the world, the larger the degree of complementarity between goods, the stronger is the case for targeting the exchange rate.

Productivity shocks				
		Optimal risk sharing	Sub-optimal risk sharing	
		Complete Markets	Incomplete Markets	Financial Autarky
Substitute	$\theta = 6$	PEG	PPI	PPI
Goods	$\theta = 4$	CPI	PPI	PPI
	$\theta = 2$	PPI	PPI	PPI
	$\theta = 1$	PPI	PPI	PPI
Complementary	$\theta = 0.8$	PPI	PPI	PPI
Goods	$\theta = 0.7$	PPI	PPI	PPI
	$\theta = 0.6$	PPI	PEG	PEG
	$\theta = 0.5$	PPI	PEG	PEG

Table 7: Preferred Policy Rule following a Productivity Shock

Foreign shocks				
		Optimal risk sharing	Sub-optimal risk sharing	
		Optimal Markets	Incomplete Markets	Financial Autarky
Substitute	$\theta = 6$	PEG	PPI	PPI
Goods	$\theta = 4$	CPI	PPI	PPI
	$\theta = 2$	PPI	PPI	PPI
	$\theta = 1$	PPI	PPI	PPI
Complementary	$\theta = 0.8$	PPI	PPI	PPI
Goods	$\theta = 0.7$	PPI	PPI	PPI
	$\theta = 0.6$	PPI	PEG	PEG
	$\theta = 0.5$	PPI	PEG	PEG

Table 8: Preferred Policy Rule following a Foreign Shock

These findings are entirely consistent with the results shown in Section 4.1. In this section we demonstrate that, when home and foreign goods are substitutes in utility, the coefficient of inflation variability in the loss function is smaller under perfect risk sharing than it is under incomplete markets, while the opposite holds when the goods are complementary to one another.

Moreover, a fixed exchange rate regime or a policy that targets the CPI inflation leads to less volatile equilibrium real exchange rates relative to a policy that only targets domestic prices. And, as shown in De Paoli (2008), restricting movements in the exchange rate can lead to a more appreciated real exchange rate on average.[14] An exchange rate peg, for example, ties policymakers hands who will, in turn, under-stabilize output relative to its flexible price allocation. It follows that a policy of fixed exchange rates will lead to a lower level of output and a more appreciated real exchange rate on average. But when is a more appreciated real exchange rate welfare improving? As illustrated in Section 4, this depends crucially on the degree os substitutability between goods and the degree of risk sharing. The economic mechanism behind this result is emphasized below.

When asset **markets are complete** and goods are substitutes, attaining a more appreciated exchange rate on average might be beneficial for the small open economy. As shown in De Paoli (2008) and Benigno and Benigno (2003), such appreciation would divert some

[14] This can be seen in our model by using the second order approximation of the equilibrium conditions and expressing the unconditional mean of the exchange rate in terms of its variance.

output production to the foreign economy and therefore reduces the disutility of producing at home. At the same time, the complete market specification ensures that consumption at home does not suffer significantly with the policy of diverting production. Moreover, when domestic and foreign goods are close substitutes to each other, the decrease in output and production disutility will be larger, and in this case, such policy can outperform an inflation targeting regime. When goods are complements, however, it is no longer possible to shift consumption towards foreign goods by inducing a greater appreciation in the exchange rate. In this case, domestic inflation targeting is the preferred policy.

In the case of **incomplete markets**, there is a greater link between consumption and output. In the extreme case of financial autarky, for example, consumption has to be fully financed by domestic production. Consequently, a policy that tries to reduce the disutility of production will inevitably reduce consumption utility. When the elasticity of substitution between the goods is high, restricting the exchange rate movements has a strong impact on output and consequently on consumption. Therefore, it does not lead to welfare gains. In this case, the authorities should focus on stabilizing inflation and on minimizing the distortions that price dispersion brings. On the other hand, lowering the degree of substitutability between the goods reduces output sensitivity to exchange rate movements. Hence, the income effect on consumption of the appreciation is smaller. In addition, a relatively appreciated exchange rate can improve the small open economy's purchasing power under market incompleteness (see equations (FA) and (IM)). When θ is sufficiently low, the income effect in consumption is small and therefore its negative impact on welfare is smaller than the positive welfare effect from an improvement in purchasing power. Hence, in this case, an exchange rate peg outperforms a domestic inflation target.

These findings are also in line with the results shown in Table 9. This Table shows that under complete markets the volatility of the real exchange rate under the optimal rule is lower (higher) than under a policy of domestic price stability when goods are substitutes (complements) in the utility. Under sub-optimal risk sharing the results are completely reversed.

Productivity shocks			
	Optimal risk sharing	Sub-optimal risk sharing	
	Complete Markets	Incomplete Markets	Financial Autarky
$\theta = 1.5$	$\text{var}^{opt}(Q_t) < \text{var}^{ppi}(Q_t)$	$\text{var}^{opt}(Q_t) > \text{var}^{ppi}(Q_t)$	$\text{var}^{opt}(Q_t) > \text{var}^{ppi}(Q_t)$
$\theta = 0.7$	$\text{var}^{opt}(Q_t) > \text{var}^{ppi}(Q_t)$	$\text{var}^{opt}(Q_t) < \text{var}^{ppi}(Q_t)$	$\text{var}^{opt}(Q_t) < \text{var}^{ppi}(Q_t)$

Table 9: Volatility of the Real Exchange Rate under the Optimal Rule vs. Producer Price Inflation Targeting

5.2 Further sensitivity analysis

Our baseline model assumes a logarithmic utility function, a zero steady-state net foreign asset position, a specific degree of intermediation costs in the incomplete market setting, and abstracts from fiscal or markup shocks. Under this specification and the benchmark calibration of Table 6, a producer price inflation targeting regime is the preferred policy rule regardless of the asset market structure. In the current Section we assess the implications of relaxing the aforementioned assumptions.

Table 10 shows that the conclusions presented in the previous sections are not altered if we consider fiscal shocks. As before, the performance of different policy rules is completely reverted when comparing optimal or sub-optimal risk sharing.

Fiscal shocks				
		Optimal risk sharing	Sub-optimal risk sharing	
		Complete Markets	Incomplete Markets	Financial Autarky
Substitute	$\theta = 6$	PEG	PPI	PPI
Goods	$\theta = 4$	PEG	PPI	PPI
	$\theta = 2$	PPI	PPI	PPI
	$\theta = 1$	PPI	PPI	PPI
Complementary	$\theta = 0.8$	PPI	PPI	PPI
Goods	$\theta = 0.7$	PPI	PPI	PPI
	$\theta = 0.6$	PPI	CPI	CPI
	$\theta = 0.5$	PPI	PEG	PEG

Table 10: Preferred Policy Rule following a Fiscal Shock

Given that our welfare derivations assume a general constant relative risk aversion (CRRA) utility function, we can conduct further sensitivity analysis for different values of this risk aversion coefficient, ρ. As for the case of the intratemporal elasticity of substitution θ, increasing the values of ρ increases the degree of substitutability between domestic and foreign goods in agents' utility. Therefore, comparing Table 9 and Table 11, we can see that when ρ is larger, under complete markets, an exchange rate PEG outperforms a PPI regime for lower levels of θ.

Productivity, Foreign or Fiscal shocks			
	Optimal risk sharing	Sub-optimal risk sharing	
ρ	Complete Markets	Incomplete Markets	Financial Autarky
6	PEG	PPI	PPI
4	PEG	PPI	PPI
2	PEG	PPI	PPI

Table 11: Preferred Policy Rule following a Productivity Shock: varying the Elasticity of Intertemporal Substitution

For the case of incomplete markets, we also analyze whether the assumption of zero steady-state net foreign asset and the level of intermediation costs affect our results by computing the policy rankings under different values of $\bar{B}_F/\bar{Y}$ and δ. As shown in the table bellow, the result that PPI is the preferred policy rule when goods are substitutes and markets are incomplete hold regardless of the value of δ and $\bar{B}_F/\bar{Y}$.

Incomplete Markets: Productivity, Foreign or Fiscal shocks			
$\bar{B}_F/\bar{Y} \setminus \delta$	0.001%	0.5%	1%
0	PPI	PPI	PPI
0.3	PPI	PPI	PPI
0.5	PPI	PPI	PPI

Table 12: Sensitivity Analysis: varying the Steady-state Debt to GDP Ratio and Risk Premium

Finally, we analyze the case of markup shocks and found that, in the presence of such disturbances, domestic price stability ceases to be optimal for the majority of parameter values, regardless of the asset market structure. But this result is not surprising, as markup shocks introduce inefficiencies in the flexible price allocation, even in a closed economy setting.

Markup Shocks			
	Optimal risk sharing	Sub-optimal risk sharing	
	Complete Markets	Incomplete Markets	Financial Autarky
$\theta = 6$	PEG	CPI	CPI
$\theta = 2$	CPI	CPI	CPI
$\theta = 0.5$	PPI	PEG	PEG

Table 13: Sensitivity Analysis: varying the Steady-state Debt to GDP Ratio and Risk Premium

6 Concluding Remarks

In this work, we formalize the dynamics of the small open economy under different degrees of international risk sharing and show that these have significant implications for monetary policy. When a country can optimally share risk with the rest of the world, and home and foreign goods are substitutes, restricting real exchange rate volatility may improve its welfare. Conversely, if goods are complements in the utility, an inflation targeting is the preferred policy rule. But under imperfect risk sharing, these results are entirely reversed.

Optimal monetary policy is independent of the financial market structure only when the latter is entirely irrelevant for the economy's dynamics. This is the case when trade imbalances are ruled out and the steady-state level of net foreign assets is zero. Under this specification, and provided there are no markup shocks or steady-state inefficiencies in output, price stability coincides with the optimal plan, regardless of the degree of risk sharing.

In light of the results presented, an interesting exercise would be analyze empirically the conduct of monetary policy in countries with different import profiles and different asset market characteristics.

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A Appendix: The Steady State

In this Section, we derive the steady-state conditions. We allow for an *asymmetric* steady state in the analysis of the incomplete market case. All variables in steady state are denoted with a bar. We assume that in steady state $1+i_t = 1+i_t^* = 1/\beta$ and $P_t^H/P_{t-1}^H = P_t^F/P_{t-1}^F = 1$. We normalize the price indexed such that $\overline{P}_H = \overline{P}_F$.

The steady-state versions of the demand equation at Home and in the rest of the world are

$$\overline{Y} = (1-\lambda)\overline{C} + \lambda\overline{C^*} + \overline{G} \tag{A.1}$$

and

$$\overline{Y}^* = \overline{C}^* + \overline{G}^*. \tag{A.2}$$

From the household and government budget constraints we have

$$(1-\beta)\overline{B}_F = \overline{C} - \overline{Y}(1-\overline{\tau}) + \overline{Tr} \tag{A.3}$$

and

$$\overline{G} = \overline{\tau}\overline{Y} - \overline{Tr} \tag{A.4}$$

where $\overline{B}_H = 0$, given the assumption that domestic bonds are in zero net supply.

We can therefore write the following relationship between the steady-state foreign asset position and the consumption differential:

$$(1-\beta)\overline{B}_F = \lambda(\overline{C} - \overline{C}^*). \tag{A.5}$$

Finally, applying our normalization to the price setting equations we have

$$U_C(\overline{C}) = \overline{\mu}V_y\left(\lambda\overline{C^*} + (1-\lambda)\overline{C} + \overline{G}\right) \tag{A.6}$$

and

$$U_C(\overline{C}^*) = \overline{\mu}^* V_y\left(\overline{C}^* + \overline{G}^*\right), \tag{A.7}$$

where

$$\overline{\mu} = \frac{\sigma}{(1-\overline{\tau})(\sigma-1)};\ \overline{\mu} > 1$$

Equations (A.5), (A.6) and (A.7) determine the relationship between $\overline{B}_F$, $\overline{G}$, $\overline{G}^*$ and $\overline{\mu}^*$. In particular, when $\overline{G} = \overline{G}^* = \overline{B}_F = 0$, $\overline{\mu}^* = \overline{\mu}$.

B Appendix: Approximating the Model

In this Appendix, we derive first and second order approximations to the equilibrium conditions of the model. Lowercase variables denote log deviations from steady state. Moreover, we show the second order approximation to the utility function in order to conduct our welfare analysis. We assume $\overline{G} = 0$ and use the following isoelastic functional forms for the utility functions:

$$U(C_t) = \frac{C_t^{1-\rho}}{1-\rho} \tag{B.8}$$

$$V\left(y_t(h), \varepsilon_T\right) = \frac{\varepsilon_{y,t}^{-\eta} y_t(h)^{\eta+1}}{\eta+1} \tag{B.9}$$

B.1 Demand

The first order approximation to the small open economy demand is

$$y_H = -\theta p_H + d_b c + (1-d_b)c^* + \theta(1-d_b)q + g, \tag{B.10}$$

where $d_b = (1-\lambda)(1+a)$ and $a = \frac{\lambda(\overline{C}-\overline{C}^*)}{\overline{Y}}$. Moreover, Home relative prices are denoted by $p_H = P_H/P$ and the fiscal shock g_t is defined as $\frac{Gt-\overline{G}}{\overline{Y}}$, allowing for the analysis of this shock even when steady-state government consumption is non-zero. In the symmetric steady state, in which $d_b = 1-\lambda$, Equation (B.10) becomes

$$y_H = -\theta p_H + (1-\lambda)c + \lambda\hat{C}^* + \theta\lambda q + g. \tag{B.11}$$

The second order approximation to the demand function is

$$\sum \beta^t \left[d_y' y_t + \frac{1}{2} y_t' D_y y_t + y_t' D_e e_t \right] + t.i.p + \mathcal{O}(||\xi||^3) = 0, \tag{B.12}$$

where

$$y_t = \begin{bmatrix} y_t & c_t & p_{Ht} & q_t \end{bmatrix},$$

$$e_t = \begin{bmatrix} \varepsilon_{yt} & \mu_t & g_t & c_t^* \end{bmatrix},$$

$$d_y' = \begin{bmatrix} -1 & d_b & -\theta & \theta(1-d_b) \end{bmatrix},$$

$$D_y' = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & (1-d_b)d_b & 0 & -\theta(1-d_b-d_g)d_b \\ 0 & 0 & 0 & 0 \\ 0 & -\theta(1-d_b)d_b & 0 & \theta^2(1-d_b)d_b \end{bmatrix},$$

and

$$D_e' = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -d_b & -(1-d_b)d_b \\ 0 & 0 & 0 & \theta & 0 \\ 0 & 0 & 0 & -\theta(1-d_b) & \theta(1-d_b)d_b \end{bmatrix}.$$

B.2 The Real Exchange Rate

Given that, in the rest of the world, $P_F = SP^*$, Equation (6) can be expressed as:

$$\left(\frac{P_t}{P_{H,t}}\right)^{1-\theta} = (1-\lambda) + \lambda\left(Q_t \frac{P_t}{P_{H,t}}\right)^{1-\theta}. \tag{B.13}$$

The first order approximation to the above expression is:

$$p_{H,t} = -\frac{\lambda q_t}{1-\lambda}. \tag{B.14}$$

The second order approximation to Equation (B.13) is:

$$\sum E_t \beta^t \left[f_y' y_t + \frac{1}{2} y_t' F_y y_t + y_t' F_e e_t \right] + t.i.p + \mathcal{O}(||\xi||^3) = 0, \tag{B.15}$$

where

$$f_y' = \begin{bmatrix} 0 & 0 & -(1-\lambda) & -\lambda \end{bmatrix},$$

$$f_e' = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix},$$

$$F_y' = \lambda(\theta - 1)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & (1-\lambda/(1-\lambda)) \end{bmatrix},$$

and

$$F_e' = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

B.3 Price Setting

The first and second-order approximations to the price setting equation follow Benigno and Benigno (2001) and Benigno and Benigno (2003). These conditions are derived from the following first order condition of sellers that can reset their prices:

$$E_t\left\{\sum(\alpha\beta)^{T-t}U_c(C_T)\left(\frac{\tilde{p}_t(h)}{P_{H,t}}\right)^{-\sigma}Y_T\left[\frac{\tilde{p}_t(h)}{P_{H,T}}\frac{P_{H,T}}{P_T} - \mu_T\frac{V_y\left(\tilde{y}_{t,T}(h),\varepsilon_t\right)}{U_c(C_T)}\right]\right\} = 0, \quad \text{(B.16)}$$

where

$$\tilde{y}_t(h) = \left(\frac{\tilde{p}_t(h)}{P_{H,t}}\right)^{-\sigma} Y_t, \quad \text{(B.17)}$$

and

$$(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha)\left(\tilde{p}_t(h)\right)^{1-\sigma}. \quad \text{(B.18)}$$

With markup shocks, μ_t, defined as $\frac{\sigma}{(\sigma-1)(1-\tau_t^H)}$, the first order approximation to the price setting equation can be written in the following way:

$$\pi_t = k\left(\rho c_t + \eta y_t - p_{H,t} + \mu_t - \eta\varepsilon_t\right) + \beta E_t\pi_{t+1}, \quad \text{(B.19)}$$

where $k = (1-\alpha\beta)(1-\alpha)/\alpha(1+\sigma\eta)$.

The second order approximation to Equation (B.16) can be written as follows:

$$\begin{aligned} Q_{to} = {} & \phi\sum E_t\beta^t\left[a_y' y_t + \frac{1}{2}y_t' A_y y_t + y_t' A_e e_t + \frac{1}{2}a_\pi\pi_t^2\right] \\ & + t.i.p + \mathcal{O}(||\xi||^3), \end{aligned} \quad \text{(B.20)}$$

$$a_y' = \begin{bmatrix} \eta & \rho & -1 & 0 \end{bmatrix},$$

$$a_e' = \begin{bmatrix} -\eta & 1 & 0 & 0 \end{bmatrix},$$

$$A_y' = \begin{bmatrix} \eta(2+\eta) & \rho & -1 & 0 \\ \rho & -\rho^2 & \rho & 0 \\ -1 & \rho & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A'_e = \begin{bmatrix} -\eta(1+\eta) & 1+\eta & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$a_\pi = (\eta+1)\frac{\sigma}{k}.$$

B.4 Incomplete Markets: Approximating the Current Account equation

We assume that home-currency denominated bonds are in zero net supply. The net foreign asset position is therefore fully denominated in foreign currency. Aggregating private and public budget constraints, the law of motion for b_t can be written as

$$P_t C_t + \frac{S_t B_{F,t}}{(1+i_t^*)\,\psi\left(\frac{S_t B_{F,t}}{P_t}\right)} = S_t B_{F,t-1} + P_{H,t}(Y_t - G_t) \tag{B.21}$$

Defining $\frac{B_{F,t}S_t}{P_t} \equiv B_t$ we can rewrite the government budget constraint as

$$B_t = B_{t-1}\frac{S_t P_{t-1}}{S_{t-1}P_t}(1+i_t^*)\,\psi(B_t) + \frac{P_{H,t}}{P_t}(Y_t - G_t)(1+i_t^*)\,\psi(B_t) - C_t(1+i_t^*)\,\psi(B_t). \tag{B.22}$$

From agents' intertemporal choice,

$$U_C(C_t) = (1+i_t^*)\psi(B_t)\,\beta E_t\left[U_C(C_{t+1})\frac{S_{t+1}P_t}{S_t P_{t+1}}\right]. \tag{B.23}$$

We can therefore write (B.22) as

$$B_t\beta E_t\left[U_C(C_{t+1})\frac{S_{t+1}P_t}{S_t P_{t+1}}\right] = B_{t-1}\frac{S_t P_{t-1}}{S_{t-1}P_t}U_C(C_t) + \frac{P_{H,t}}{P_t}(Y_t - G_t)U_C(C_t) - C_t U_C(C_t). \tag{B.24}$$

And the log linear representation of the above equation, defining $a_\beta = \frac{a}{1-\beta}$, is

$$\begin{aligned} &-\rho a_\beta c_t + b_{t-1} + a_\beta(\Delta Q_t - \pi_t^*) \\ &= -y_t + (1+a)c_t - \rho a c_t - p_{H,t} + g_t \\ &+ \beta E_t\left[-\rho a_\beta c_{t+1} + b_t + a_\beta(\Delta Q_{t+1} - \pi_{t+1}^*)\right]. \end{aligned} \tag{B.25}$$

Furthermore, if we allow $B_{W,t} = B_{F,t-1}\frac{P_{t-1}}{P_t}\frac{S_t}{S_{t-1}}U_C(C_t)$ and $s_t = -\frac{P_{H,t}}{P_t}(Y_t - G_t) + C_t$, the intertemporal government solvency condition (**??**) can be written as

$$b_{W,t} = U_C(C_t)\,s_t + E_t\beta b_{W,t+1} = E_t\sum_{T=t}^{\infty}\beta^{T-t}U_C(C_T)\,s_t, \tag{B.26}$$

and the term $U_C(C_T, \xi_{C,T})\,s_t$ can be approximated, up to the second order, by

$$U_C\overline{Y}\left\{\begin{array}{c} a - \widehat{Y}_t + (1+a(1-\rho))c_t - p_{H,t} - \frac{1}{2}y_t^2 + \rho y_t c_t - y_t p_{H,t} \\ +\frac{1}{2}(a\rho^2 + (1+a)(1-2\rho))c_t^2 + \rho c_t p_{H,t} - \frac{1}{2}p_{H,t}^2 \end{array}\right\}.$$

Thus, defining $b_{W,t} = \frac{B_{W,t} - \overline{B}_W}{\overline{B}'_W}$ and $\overline{B}'_W = \frac{U_c\overline{Y}}{(1-\beta)}$, we have,

$$
\begin{aligned}
b_{W,t} = (1-\beta)\left[b_y^{i\prime} y_t + \frac{1}{2} y_t' B_y^i y_t + y_t' B_e^i e_t\right] + \beta E_t b_{W,t+1} \\
+ t.i.p + \mathcal{O}(||\xi||^3)
\end{aligned} \tag{B.27}
$$

where

$$
b_y^{i\prime} = \begin{bmatrix} -1 & 1 + a(1-\rho) & -1 & 0 \end{bmatrix},
$$

$$
B_y^{i\prime} = \begin{bmatrix} -1 & \rho & -1 & 0 \\ \rho & a(1-\rho)^2 + (1-2\rho) & \rho & 0 \\ -1 & \rho & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},
$$

and

$$
B_e^{i\prime} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\rho & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.
$$

Special Case: Note that if $\rho = \theta = 1$, $a = 0$ and $b_{-1} = 0$, the second order current account approximation becomes

$$
c_t = y_t + p_{H,t} - g_t - g_t p_{H,t} + \widehat{g_t C}_t, \tag{B.28}
$$

which combining with the demand equation implies

$$
c_t = q_t + c_t^*. \tag{B.29}
$$

This is identical to the perfect risk sharing condition.

B.5 Financial Autarky: the Extreme Case of Market Incompleteness

In this case we assume that there is no risk sharing between countries. The inability to trade bonds across borders impose that the value of imports equates the value of exports:

$$
(1-n) S P_{H,t}^* C_{H,t}^* = n P_{F,t} C_{F,t}, \tag{B.30}
$$

given the preference specification, we can write:

$$
C_{H,t} = v \left[\frac{P_{H,t}}{P_t}\right]^{-\theta} C_t, \; C_{F,t} = (1-v)\left[\frac{P_{F,t}}{P_t}\right]^{-\theta} C_t, \tag{B.31}
$$

$$
C_{H,t}^* = v^* \left[\frac{P_{H,t}^*}{P_t^*}\right]^{-\theta} C_t^*, C_{F,t}^* = (1-v^*)\left[\frac{P_{F,t}^*}{P_t^*}\right]^{-\theta} C_t^*. \tag{B.32}
$$

Substituting in Equation (B.30):

$$
C_t = \left[\frac{p_{H,t}}{Q_t}\right]^{1-\theta} [Q_t]^\theta C_t^*. \tag{B.33}
$$

And using the definition of the consumption indexes and market clearing, condition (B.33) implies

$$
P_{H,t}(Y_t - G_t) = P_t C_t. \tag{B.34}
$$

Assuming $\overline{C} = \overline{C}^*$, the second order approximation is

$$p_{H,t} + y_t - g_t + y_t g_t = c_t, \tag{B.35}$$

and can be represented as follows:

$$\sum E_t \beta^t \left[b_y^{fa\prime} y_t + \frac{1}{2} y_t' B_y^{fa} y_t + y_t' B_e^{fa} e_t \right] + t.i.p + \mathcal{O}(||\xi||^3) = 0,$$

$$b_y^{fa\prime} = \begin{bmatrix} -1 & 1 & -1 & 0 \end{bmatrix},$$

$$B_y^{fa\prime} = 0,$$

and

$$B_e^{fa\prime} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Special Case: when $\theta = 1$, Equation (B.35) combined with the demand equation becomes

$$c_t^* = c_t + q_t. \tag{B.36}$$

B.6 Complete markets: the Risk Sharing Equation

Assuming a symmetric steady-state equilibrium, the log linear approximation to the risk sharing Equation (29) is

$$c_t^* = c_t + \frac{1}{\rho} q_t. \tag{B.37}$$

Given our utility function specification, Equation (29) gives rise to an exact log linear expression and therefore the first and second order approximations are identical. In matrix notation, we have

$$\sum E_t \beta^t \left[b_y^{c\prime} y_t + \frac{1}{2} y_t' B_y^{c} y_t + y_t' B_e^{c} e_t \right] = 0,$$

where

$$b_y^{c\prime} = \begin{bmatrix} 0 & -1 & 0 & \frac{1}{\rho} \end{bmatrix},$$

$$b_y^{c\prime} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B_y^{c\prime} = 0,$$

and

$$B_e^{c\prime} = 0.$$

C Appendix: Welfare and Optimal Monetary Policy

C.1 Welfare with Incomplete Asset Markets:

Following Benigno and Benigno (2003), the second order approximation to the utility function, U_t, can be written as:

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s) - \frac{1}{n} \int_0^n V(y_s^j, \varepsilon_{Y,s}) dj \right], \tag{C.38}$$

$$W_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[w_y' y_t - \frac{1}{2} y_t' W_y y_t - y_t' W_e e_t - \frac{1}{2} w_\pi \pi_t^2 \right] + t.i.p + \mathcal{O}(||\xi||^3), \tag{C.39}$$

where

$$w_\pi' = \frac{\sigma}{\mu k},$$

$$w_y' = \left[\begin{array}{cccc} -1/\mu(1+a) & 1 & 0 & 0 \end{array} \right],$$

$$W_y' = \left[\begin{array}{cccc} \frac{(1+\eta)}{(1+a)\mu} & 0 & 0 & 0 \\ 0 & -(1-\rho) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right],$$

and

$$W_e' = \left[\begin{array}{cccc} -\frac{\eta}{\mu(1+a)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right].$$

Using the second order approximation to the equilibrium condition, we can eliminate the term $w_y' y_t$. We derive the vector Lx, such that

$$\left[\begin{array}{cccc} a_y & d_y & f_y & b_y^i \end{array} \right] Lx = w_y.$$

Thus, the loss function can be written as

$$L_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} y_t' L_y^i y_t + y_t' L_e^i e_t + \frac{1}{2} l_\pi^i \pi_t^2 \right] + t.i.p + \mathcal{O}(||\xi||^3), \tag{C.40}$$

where

$$L_y^i = W_y + Lx_1^i A_y + Lx_2^i D_y + Lx_3^i F_y + Lx_4^i B_y^i,$$

$$L_e^i = W_e + Lx_1^i A_e + Lx_2^i D_e + Lx_4^i B_e,$$

and

$$l_\pi^i = w_\pi + Lx_1^i a_\pi.$$

Given the values of a_y, d_y, b_y^i, f_y, defined in this Appendix, we have:

$$Lx_1^i = \frac{(1+a)\lambda(l_1(2-\lambda) + (\phi - \lambda)) - a(l_1 + \phi)}{(1+a)\lambda(l_2(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho-1)) - al_2},$$

$$Lx_2^i = \frac{-\lambda((\rho+\eta) - \phi(\rho-1))}{(1+a)\lambda(l_2(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho-1)) - al_2},$$

$$Lx_3^i = \frac{-(\lambda + (1-\lambda)a)\theta((\rho+\eta) - (\rho-1)(\phi - (\phi-1)a))}{(1+a)\lambda(l_2(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho-1)) - al_2},$$

and

$$Lx_4^i = \frac{(1+a)\lambda(l_3(2-\lambda) - (\phi-\lambda)) - a(l_3-\phi)}{(1+a)\lambda(l_2(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho-1)) - al_2},$$

where $l_1 = \theta a(\rho - \phi(\rho-1)) + \phi(\theta-1)$,$l_2 = \theta((\rho+\eta) - \eta(\rho-1)a)$, $l_3 = \theta((\rho+\eta(1+a) - (\rho-1)\phi)$ and $(1-\phi) = \frac{1}{\mu}$.

We write the model just in terms of the output, real exchange rate and inflation, using the matrixes N and N_e, as follows:

$$y_t' = N\left[Y_t, T_t\right] + N_e e_t,$$

$$N = \begin{bmatrix} 1 & 0 \\ \frac{1}{d_b} & -\frac{d_q}{d_b} \\ 0 & -\frac{\lambda}{(1-\lambda)} \\ 0 & 1 \end{bmatrix},$$

and

$$N_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{d_b} & -\frac{(1-d_b)}{d_b} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

where $d_q = \frac{\theta(\lambda + (1-d_b)(1-\lambda))}{(1-\lambda)}$.

Equation (C.40) can therefore be expressed as

$$L_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \begin{bmatrix} \frac{1}{2}\left[y_t, q_t\right]' L_y^{i'} \left[y_t, q_t\right] \\ + \left[y_t, q_t\right]' L_e^{i'} e_t + \frac{1}{2} l_\pi^i \pi_t^2 \end{bmatrix} + t.i.p + \mathcal{O}(||\xi||^3), \tag{C.41}$$

where:

$$L_y^{i\prime} = N' L_y^i N,$$

$$L_e^{i\prime} = N' L_y^i N_e + N' L_e,$$

$$\begin{aligned} \left[y_t, q_t\right]' L_y^{i'} \left[y_t, q_t\right] &= \left[y_t, q_t\right]' \begin{bmatrix} l_{yy}^i & l_{yq}^i \\ l_{yq}^i & l_{qq}^i \end{bmatrix} \left[y_t, q_t\right] \\ \left[y_t, q_t\right]' L_e^{i'} e_t &= \left[y_t, q_t\right]' \begin{bmatrix} l_{ye}^i \\ l_{qe}^i \end{bmatrix} L_e' e_t, \end{aligned} \tag{C.42}$$

$$l_{ye}^i = \begin{bmatrix} l_{y\varepsilon}^i & l_{y\mu}^i & l_{yg}^i & l_{yc^*}^i \end{bmatrix},$$

$$l_{qe}^i = \begin{bmatrix} l_{q\varepsilon}^i & l_{q\mu}^i & l_{qg}^i & l_{qc^*}^i \end{bmatrix},$$

$$\begin{aligned} l_{yy}^i = {} & \frac{(\eta+1)(1-\phi)}{(1+a)} + \frac{\rho-1}{d_b^2} + \left(\frac{\rho(\rho-2d_b)}{d_b^2} + \eta(2+\eta)\right) Lx_1 \\ & + \frac{(1-d_b)}{d_b^2} Lx_2 + \frac{\rho\left(a\rho + 2(a(1-\lambda)-\lambda) + (1+a)^{-1}\rho^{-1}\right)}{d_b^2} Lx_4, \end{aligned}$$

$$l^i_{yq} = l^i_{qy} = \frac{\rho(1-\lambda)^{-1} + (1+a)}{d_b^2}\left((r_1-\lambda)(1+a) + a\rho\theta\right) Lx_1 + \frac{\theta(a-\lambda\left(1-\lambda\right)^{-1})}{d_b^2} Lx_2$$
$$+\left((1-\lambda)^{-1}\lambda + \frac{\rho(1+a)\lambda}{d_b^2}\right) Lx_4 + \left(\left(\rho d_b^{-1} + r_3\right) Lx_4 - 1\right)\frac{r_2}{d_b},$$

$$l^i_{qq} = r_2^2(\rho-1) + (\lambda(1-\lambda)^{-1} - r_2\rho)^2 Lx_1 + \theta d_b(1-d_b)(1+r_2)Lx_2$$
$$+ (1-\lambda)^{-1}\lambda(\theta-1)Lx_3$$
$$+ \left((\lambda(1-\lambda)^{-1} + r_2 r_3)r_2 + \lambda(1-\lambda)^{-1}(-\lambda(1-\lambda)^{-1} + r_2\rho)\right) Lx_4,$$

$$l^i_{\pi} = \frac{\sigma}{k}\left(\frac{(1-\phi)}{(1+a)} + (\eta+1)Lx_1\right),$$

$$l^i_{y\varepsilon} = \frac{-\eta(1-\phi)}{(1+a)} - \eta(\eta+1)Lx_1,$$

$$l^i_{y\mu} = (\eta+1)Lx_1,$$

$$l^i_{yg} = \frac{-\left(\rho-1\right)}{d_b^2} + \frac{\rho\left(d_b-\rho\right)}{d_b^2}Lx_1 - \frac{Lx_2}{d_b} - \frac{(r_3+\rho)Lx_4}{d_b},$$

$$l^i_{yc^*} = \frac{-\left(\rho-1\right)\left(1-d_b\right)}{d_b^2} + -\frac{\rho\left(d_b-\rho\right)}{d_b^2}\frac{\left(1-d_b\right)}{d_b}Lx_1 - \frac{(\lambda(1+a)-a)Lx_2}{d_b} - \frac{\left(1-d_b\right)}{d_b}\left(\rho + \frac{r_3}{d_b}\right)Lx_4,$$

$$l^i_{q\varepsilon} = 0,$$

$$l^i_{q\mu} = 0,$$

$$l^i_{qg} = \frac{\theta\left(\rho-1\right)\left(a-\lambda(2-\lambda)(1+a)\right)}{(1-\lambda)d_b^2} + \frac{\left((r_1+\lambda)(1+a) - a\rho\theta\right)\rho}{(1-\lambda)d_b^2}(Lx_1 + Lx_4)$$
$$+ \frac{(\lambda(1+a)-a)Lx_2}{(1-\lambda)d_b} - \frac{\left((r_3+\rho)(2-\lambda) + \lambda)(1+a) + a\theta r_3\right)\rho}{(1-\lambda)d_b^2}\frac{Lx_4}{d_b},$$

and

$$l^i_{qc^*} = \frac{\theta\left(\rho-1\right)\left(\lambda(1+a)-a\right)((1-\lambda)^{-1} + d_b)}{d_b^2} - \frac{\left((r_1+\lambda)(1+a) - a\rho\theta\right)\left(\lambda(1+a)-a\right)Lx_1}{(1-\lambda)d_b^2}$$
$$+ \frac{\theta(\lambda(1+a)-a)Lx_2}{(1-\lambda)d_b} + \frac{(\lambda(1+a)-a)}{d_b}\left(\rho\lambda(1-\lambda)^{-1} + r_2 r_3\right) Lx_4,$$

where $r_1 = \lambda(2-\lambda)(\rho\theta-1)$, $r_2 = \frac{\theta((1-d_b)+(1-\lambda)^{-1}\lambda)}{d_b}$ and $r_3 = (1+\rho^2)a + 1 - 2\rho$

C.2 Optimal Plan with Incomplete Asset Markets:

The optimal plan consists of minimizing (31) subject to equations in Table 4. Therefore, the first order conditions with respect to π_t, y_t, q_t, c_t, and b_t are:

$$l_\pi^i \pi_t + \Delta\varphi_{1,t} = 0, \tag{C.43}$$

$$0 = l_{yy}^i y_t + l_{yq}^i q_t + l_{ye}^i \widehat{e}_t - k\eta\varphi_{1,t} + \varphi_{2,t} - \varphi_{4,t},$$

$$0 = l_{yq}^i y_t + l_{qq}^i q_t + l_{qe}^i \widehat{e}_t - k\frac{\lambda}{(1-\lambda)}\varphi_{1,t} - d_q\varphi_{2,t} + \varphi_{3,t} - \beta^{-1}\varphi_{3,t-1}$$
$$+ \frac{\lambda}{(1-\lambda)}\varphi_{4,t} - a_\beta\Delta\varphi_{4,t} + a_\beta\beta\Delta E_t\varphi_{4,t+1}, \tag{C.44}$$

$$0 = -\rho k(1+a)\varphi_{1,t} - (1+a)(1-\lambda)\varphi_{2,t} - \rho\varphi_{3,t} + \rho\beta^{-1}\varphi_{3,t-1} + (1+a)\varphi_{4,t} + \rho a_\beta\Delta\varphi_{4,t}, \tag{C.45}$$

and

$$E_t\Delta\varphi_{4,t+1} = \beta^{-1}\delta\varphi_{3,t}. \tag{C.46}$$

The case of no intermediation costs:
When $\delta = 0$, the first order conditions can be written as

$$Q_y^i E_t\Delta(y_{t+1} - y_{t+1}^{T,i}) + Q_q^i E_t\Delta(q_{t+1} - q_{t+1}^{T,i}) + Q_\pi^i E_t\pi_{t+1} = 0, \tag{C.47}$$

with

$$Q_y^i = l_{yq}^i + (d_q + (1+a)(1-\lambda)\rho^{-1})l_{yy}^i,$$

$$Q_q^i = l_{qq}^i + (d_q + (1+a)(1-\lambda)\rho^{-1})l_{yq}^i,$$

$$Q_\pi^i = k\left[(1+a)(\rho + \eta(1-\lambda)\rho^{-1}) + \eta d_q + \lambda(1-\lambda)^{-1}\right] l_\pi^i,$$

$$q_t^{T,i} = \frac{-l_{qe}}{Q_q^i}\widehat{e}_t,$$

and

$$y_t^{T,i} = \frac{-(d_q + (1+a)(1-\lambda)\rho^{-1})l_{ye}^i}{Q_y^i}\widehat{e}_t.$$

Special Case: Incomplete markets, symmetric steady state, no trade imbalances and specific level of steady-state output
In the case we have

1. $\mu = 1/(1-\lambda)$
2. $\rho = \theta = 1$
3. $a = 0$

In this case, the first order conditions can be written as:

$$0 = (l_{yy}^i + l_{yq}^i(1-\lambda))\Delta y_t + ((1-\lambda)l_{qq}^i + l_{yq}^i)\Delta q_t + (l_{ye}^i + l_{qe}^i(1-\lambda))\Delta\widehat{e}_t + k(\eta+1)l_\pi^i\pi_t \tag{C.48}$$

Moreover:

$$Lx_1 = 0;\ Lx_2 = -1;\ Lx_3 = -1;\ \text{and } Lx_4 = 2-\lambda.$$

And therefore:
$l_{yy} + l_{yq}(1-\lambda) = (\eta+1)(1-\lambda)$
$(1-\lambda)l_{qq} + l_{yq} = 0$
$l_{\pi} = (1-\lambda)\sigma/k$
$(l_{ye} + l_{qe}(1-\lambda)) = \begin{bmatrix} -\eta(1-\lambda) & 0 & -(1-\lambda) & 0 \end{bmatrix}$
Hence, the targeting rule can be written as

$$0 = \Delta\left(y_t - \frac{\eta}{(\eta+1)}\varepsilon_t - \frac{1}{(\eta+1)}g_t\right) + \sigma\pi_t^H. \tag{C.49}$$

In addition, using Equation (B.28), we can write the Phillips Curve as follows:

$$\pi_t = k\left((\eta+1)y_t - \eta\varepsilon_{Yt} - g_t + \mu_t\right) + \beta E_t\pi_{t+1}. \tag{C.50}$$

By inspection of Equation (C.49) and (C.50), we can see that domestic inflation target is the optimal plan if there are no markup shocks, μ_t.

C.3 Welfare under Financial Autarky

Using an analogous derivation for welfare, but substituting the matrices b_y^i, B_y^i and B_e^i for b_y^{fa}, B_y^{fa} and B_e^{fa}, the loss function under financial autarky has the following weights[15]:

$$\begin{aligned} l_{yy}^{fa} &= (\eta+1)(1-\phi) + \frac{\rho-1}{d_b^2} \\ &+ \left(\frac{\rho\left(\rho-2d_b\right)}{d_b^2} + \eta(2+\eta)\right)Lx_1^{fa} + \frac{(1-d_b)}{d_b^2}Lx_2^{fa}, \end{aligned}$$

$$\begin{aligned} l_{yq}^{fa} = l_{qy}^{fa} &= \frac{\rho(1-\lambda)^{-1}}{d_b^2}\left((r_1-\lambda)(1+a) + a\rho\theta\right)Lx_1^{fa} \\ &+ \frac{\theta(a-\lambda\left(1-\lambda\right)^{-1})}{d_b^2}Lx_2^{fa} - \frac{r_2}{d_b}, \end{aligned}$$

$$\begin{aligned} l_{qq}^{fa} &= r_2^2(\rho-1) + (\lambda(1-\lambda)^{-1} - r_2\rho)^2Lx_1^{fa} + \theta d_b(1-d_b)(1+r_2)Lx_2^{fa} \\ &+ (1-\lambda)^{-1}\lambda(\theta-1)Lx_3^{fa}, \end{aligned}$$

$$l_{\pi}^{fa} = \frac{\sigma}{k}\left((1-\phi) + (\eta+1)Lx_1^{fa}\right),$$

$$l_{y\varepsilon}^{fa} = -\eta(1-\phi) - \eta(\eta+1)Lx_1^{fa},$$

$$l_{y\mu}^{fa} = (\eta+1)Lx_1^{fa},$$

$$l_{yg}^{fa} = -\frac{(Lx_2(1-\lambda) + \rho Lx_4^{fa})}{d_b},$$

$$l_{yc^*}^{fa} = \frac{-\left(\rho-1\right)\left(1-d_b\right)}{d_b^2} - \frac{\rho\left(d_b-\rho\right)}{d_b^2}\frac{\left(1-d_b\right)}{d_b}Lx_1^{fa} - \frac{(\lambda(1+a)-a)Lx_2^{fa}}{d_b},$$

$$l_{q\varepsilon}^{fa} = 0,$$

[15] Note that for the derivation of wefare under Complete Market and Financial autarky, we assume $a = 0$.

$$l_{q\mu}^{fa} = 0,$$

$$l_{qg}^{fa} = \frac{(r_2 + \lambda) Lx_4^{fa}}{d_b^2},$$

$$l_{yc^*}^{fa} = \frac{\theta\,(\rho - 1)\,(\lambda(1+a) - a)((1-\lambda)^{-1} + d_b)}{d_b^2} - \frac{((r_1 + \lambda)(1+a) - a\rho\theta)\,(\lambda(1+a) - a)Lx_1^{fa}}{(1-\lambda)d_b^2}$$
$$+ \frac{\theta(\lambda(1+a) - a)Lx_2^{fa}}{(1-\lambda)d_b},$$

and

$$l_{yc^*}^{fa} = -\frac{(Lx_2^{fa}(1-\lambda) + (-r_2 - \lambda + \rho)Lx_4^{fa})}{d_b},$$

with

$$Lx_1^{fa} = \frac{\lambda(l_1^{fa}(2-\lambda) + (\phi - \lambda))}{\lambda(l_2^{fa}(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho - 1))},$$
$$Lx_2^{fa} = \frac{-\lambda((\rho+\eta) - \phi(\rho - 1))}{\lambda(l_2^{fa}(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho - 1))},$$
$$Lx_3^{fa} = \frac{-\lambda\theta((\rho+\eta) - (\rho - 1)\phi)}{\lambda(l_2^{fa}(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho - 1))},$$

and

$$Lx_4^{fa} = \frac{\lambda(l_3^{fa}(2-\lambda) - (\phi - \lambda))}{\lambda(l_2^{fa}(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho - 1))},$$

where $l_1^{fa} = \phi(\theta - 1)$,$l_2^{fa} = \theta((\rho + \eta)$ and $l_3^{fa} = \theta((\rho + \eta - (\rho - 1)\phi)$.

C.4 Optimal Plan under Financial Autarky

We can write the system of equations given in Table 2 in terms of y_t and q_t as follows:

$$\pi_t = \phi\left((\eta + \rho)y_t - (\rho - 1)\lambda(1-\lambda)^{-1}q_t + \mu_t - \eta\varepsilon_t\right) + \beta E_t \pi_{t+1}, \tag{C.51}$$

and

$$y_t = q_t \frac{(1 + l_i)}{(1-\lambda)} + c_t^* + \lambda^{-1} g_t. \tag{C.52}$$

The policymaker minimizes the loss function subject to the problem (C.51) and (C.52). Given that the multipliers associated with (C.51) and (C.52) are, respectively, φ_1 and φ_2, the first order conditions with respect to π_t, y_t and q_t are given by:

$$(\varphi_{1,t} - \varphi_{1,t-1}) = k l_\pi^{fa} \pi_t, \tag{C.53}$$

$$\varphi_{2,t} - (\eta + \rho)\varphi_{1,t} = l_{yy}^{fa} y_t + l_{yq}^{fa} q_t + l_{ye}^{fa} \widehat{e}_t, \tag{C.54}$$

and

$$-(1 + l_i)(1-\lambda)^{-1}\varphi_{2,t} + (\rho - 1)\lambda(1-\lambda)^{-1}\varphi_{1,t} = l_{qy}^{fa} y_t + l_{qq}^{fa} q_t + l_{qe}^{fa} \widehat{e}_t. \tag{C.55}$$

The last 3 equations can be combined, giving rise to the following targeting rule

$$Q_y^{fa}\Delta(y_t - y_t^{T,fa}) + Q_q^c\Delta(q_t - q_t^{T,fa}) + Q_\pi^{fa}\pi_t = 0, \tag{C.56}$$

where

$$Q_y^{fa} = (l_{yy}^{fa} + l_{qy}^{fa}(1-\lambda)(1+l_i)^{-1}),$$

$$Q_q^{fa} = ((1-\lambda)(1+l_i)^{-1}l_{qq}^{fa} + l_{qy}^{fa}),$$

$$Q_\pi^{fa} = k((\eta+\rho) - (\rho-1)\lambda(1-\lambda)(1+l_i)^{-1})l_\pi^{fa},$$

$$y_t^{T,fa} = (Q_y^{fa})^{-1}l_{ye}^{fa}\widehat{e}_t,$$

and

$$q_t^{T,fa} = (Q_q^{fa})^{-1}l_{qe}^{fa}(1-\lambda)(1+l_i)^{-1}\widehat{e}_t.$$

Special Case: when $\mu = 1/(1-\lambda)$ and $\rho = \theta = 1$, the targeting rule is identical to (C.49). Also, in the less restrictive case that only $\theta = 1$, the targeting rule would be given by

$$0 = \Delta\left(y_t - y_t^{Flex}\right) + \sigma\pi_t^H, \tag{C.57}$$

where $y_t^{Flex} = \frac{\eta}{(\eta+\rho)}\varepsilon_t + \frac{\rho}{(\eta+\rho)}g_t$. In other words, producer price stability consists the optimal plan under the assumptions of $\mu = 1/(1-\lambda)$ and $\theta = 1$, regardless of the value of ρ.

C.5 Welfare with Complete Markets

Following the derivation in De Paoli (2008), the loss function with complete markets can be written as

$$\begin{aligned} L_{to}^i = U_c\bar{C}E_{t_0}\sum\beta^t\left[\frac{1}{2}l_{yy}^c(y_t - y_t^{T,c})^2 + \frac{1}{2}l_{qq}^c(q_t - q_t^{T,c})^2 + \frac{1}{2}l_\pi^c(\pi_t)^2\right] \\ + t.i.p + \mathcal{O}(||\xi||^3) \end{aligned} \tag{C.58}$$

where:

$$\begin{aligned} l_{yy}^c = (\eta+\rho)(1-\phi) &+ \frac{(\rho-1)\left[-l^c(1-\phi) - (\lambda-\phi)\right]}{(1+l^c)} \\ &+ Lx_1^c\left[(\eta+\rho) + \eta(\eta+1) - \frac{\rho(\rho-1)}{(1+l^c)}\right] \\ &- \frac{Lx_2^c(1-\lambda)^2\lambda(\rho\theta-1)}{(1+l^c)}, \end{aligned}$$

$$\begin{aligned} l_{qq}^c = &-\frac{(\lambda+l^c)(\rho-1)}{(1-\lambda)\rho^2} \\ &+ \frac{Lx_1^c l^c(\rho-1-l^c)}{(1-\lambda)^2\rho} \\ &+ \frac{Lx_2^c\lambda(\rho\theta-1)\left[\rho\theta(1-\lambda)+\lambda+l^c\right]}{\rho^2} \\ &+ \frac{Lx_3^c\left[1+\lambda^2(2-\lambda)\right]\lambda(\theta-1)}{1-\lambda}, \end{aligned}$$

$$l_\pi^c = \frac{\sigma}{\mu k} + (1+\eta)\frac{\sigma}{k} Lx_1^c,$$

$$y_t^{T,c} = q_y^e e_t,$$

and

$$q_t^{T,c} = q_q^e e_t,$$

where

$$q_y^e = \frac{1}{\Phi_Y} \left[\begin{array}{cccc} \frac{\eta}{\mu} + Lx_1^c(1+\eta)\eta & -Lx_1^c(1+\eta) & \frac{(\rho-1)(1-\lambda)+Lx_2^c}{1+l^c} & 0 \end{array} \right],$$

$$q_Q^e = \frac{1}{\Phi_Q} \left[\begin{array}{cccc} 0 & 0 & \frac{(\rho-1-l^c)Lx_1^c}{(1-\lambda)} + \frac{Lx_2^c\lambda(\lambda(1-\lambda)+1)(\rho\theta-1)}{\rho(1-\lambda)} & \frac{-Lx_2^c\lambda(1-\lambda)(\rho\theta-1)}{\rho} \end{array} \right],$$

$$Lx_1^c = \frac{1}{(\rho+\eta)+l^c\eta}\left[l\mu^{-1} + (1-\lambda) - \mu^{-1}\right],$$

$$Lx_2^c = \frac{1}{(\rho+\eta)+l^c\eta}\left[\rho(\mu^{-1} - (1-\lambda)) + (1-\lambda)(\eta+\rho)\right],$$

$$Lx_3^c = \frac{1}{(\rho+\eta)+l^c\eta}\left[(\rho\theta-1)(1-\lambda)\mu^{-1} - (\eta\theta+1)\right], \tag{C.59}$$

and $l^c = (\rho\theta - 1)\lambda(2-\lambda)$.

C.6 Optimal Plan with Complete Markets

The optimal plan consists of minimizing the loss function subject to

$$\pi_t = k\left(\eta\hat{Y}_t + (1-\lambda)^{-1}q_t + \mu_t - \eta\varepsilon_t + \rho c_t^*\right) + \beta E_t\pi_{t+1}, \tag{C.60}$$

and

$$y_t = q_t\frac{(1+l^c)}{\rho(1-\lambda)} + g_t + c_t^*. \tag{C.61}$$

The multipliers associated with (C.60) and (C.61) are, respectively, φ_1 and φ_2. The first order conditions with respect to π_t, y_t and q_t are, therefore, given by

$$(\varphi_{1,t} - \varphi_{1,t-1}) = kl_\pi^c\pi_t, \tag{C.62}$$

$$\varphi_{2,t} - \eta\varphi_{1,t} = l_{yy}^c(y_t - y_t^T), \tag{C.63}$$

and

$$-\varphi_{2,t} - \frac{\rho}{(1+l)}\varphi_{1,t} = \frac{\rho(1-\lambda)}{(1+l)}l_{qq}^c(q_t - q_t^T). \tag{C.64}$$

To obtain a targeting rule for the small open economy, we combine equations (C.62), (C.63), and (C.64):

$$Q_y^c\Delta(y_t - y_t^{T,c}) + Q_q^c\Delta(q_t - q_t^{T,c}) + Q_\pi^c\pi_t = 0, \tag{C.65}$$

where

$$Q_y^c = (1+l^c)l_{yy}^c,$$

$$Q_q^c = \rho(1-\lambda)l_{qq}^c,$$

and

$$Q^c_\pi = (\rho + \eta(1+l))kl^c_\pi.$$

Special Case: when $\mu = 1/(1-\lambda)$ and $\rho = \theta = 1$, the targeting rule is identical to (C.49). This confirms that, under these circumstances, the asset market structure is irrelevant for monetary policy.

C.7 The welfare cost of inflation

Under some simplifying assumptions, the weight of inflation in the loss function for, l_π, can be expressed as:[16]

$$l^i_\pi = l^{fa}_\pi = \frac{\sigma(1-\lambda)}{k}\left(1 + \frac{l_i\lambda(1-\lambda)^{-1}(\eta+1)}{l_i(\rho+\eta)+\rho(1-\lambda)+\eta+\lambda}\right)$$

and

$$l^c_\pi = \frac{\sigma(1-\lambda)}{k}\left(1 - \frac{l_c(\eta+1)}{(\rho+\eta)+\eta l}\right)$$

with $l_i = (\theta-1)(2-\lambda)$ and $l_c = (\rho\theta-1)\lambda(2-\lambda)$.

These expressions demonstrate that when domestic and foreign goods are substitutes in the utility function, inflation variability is less costly when asset markets are complete. More specifically, when $\rho\theta > 1$ and $\theta > 1$ then $l^i_\pi = l^{fa}_\pi > q_\pi$ and $l^c_\pi < q_\pi$. When welfare is expressed as a purely quadratic expression, the weight on inflation under incomplete markets and financial autarky increases, while in the case of complete markets it decreases. It follows that, with complete markets, the linear term $c_t - \frac{1}{\overline{\mu}(1+a)}y_t$ in Equation (**??**) can be written as an increasing function of inflation variability. On the other hand, with imperfect risk sharing, either in the case of financial autarky or market incompleteness, this term is a decreasing function of $(\pi_t)^2$. This conclusion if reversed if domestic and foreign goods are complements in the utility. Now, if $\rho\theta < 1$ and $\theta < 1$, the conclusion is reversed: $l^i_\pi = l^{fa}_\pi < q_\pi$ and $l^c_\pi > q_\pi$.

D Appendix: Randomization Problem - the Financial Autarky Case

To ensure that the policy obtained from the minimization of the loss function is indeed the best available policy, we should confirm that no other random policy plan can be welfare improving. De Paoli (2008) analyses the case of complete markets. We present the conditions under which no random policy can enhance welfare. As shown in Woodford and Benigno (2003), these conditions coincide with the second order condition for the linear quadratic optimization problem. In the present Section, we study the case of financial autarky.

Following the same steps as De Paoli (2008), we characterize the relationship between inflation and output and inflation and the real exchange rate. Equation (AS) combined with Equation (FA) leads to the following expression:

$$\pi_t = k\left(\frac{(\eta+\rho)d_1 - (\rho-1)\lambda}{d_1}y_t + \mu_t + \eta\varepsilon_t\right) + \beta E_t\pi_{t+1}, \tag{D.66}$$

where $d_1 = (\theta-1)(1-\lambda) - \lambda\theta$. Alternatively,

$$\pi_t = k\left(\frac{(\eta+\rho)d_1 - (\rho-1)\lambda}{(1-\lambda)}q_t + \mu_t + \eta\varepsilon_t\right) + \beta E_t\pi_{t+1}. \tag{D.67}$$

[16] Here we assume that the level of output is efficient in the steady state (for the small open economy) and that the net foreign asset position is zero. In particular, we set $\overline{\mu} = 1/(1-\lambda)$ and $\overline{B} = 0$.

A random sunspot realization that adds $\varphi_j v_j$ to π_{t+j}, will, therefore, add a contribution of $\alpha_y k^{-1}(\varphi_j - \beta\varphi_{j+1})v_j$ to y_t and $\alpha_q k^{-1}(\varphi_j - \beta\varphi_{j+1})v_j$ to q_t, where

$$\alpha_q^{fa} = \frac{(\eta+\rho)d_1 - (\rho-1)\lambda}{(1-\lambda)}, \tag{D.68}$$

and

$$\alpha_y^{fa} = \frac{(\eta+\rho)d_1 - (\rho-1)\lambda}{d_1}. \tag{D.69}$$

To obtain what is the contribution to the loss function of the realization $\varphi_j v_j$ to π_{t+j}, we rewrite the loss function as follows. Noticing that $(d_q\lambda^{-1}-1)q_t = y_t + t.i.p$, the loss function under financial autarky can be written as

$$L_{to} = U_c\bar{C}E_{t_0}\sum\beta^t \left[\begin{array}{c} \frac{1}{2}(l_{yy}^{fa} + (d_q\lambda^{-1}-1)^{-1}l_{yq}^{fa})(y_t - y_t^T)^2 \\ +\frac{1}{2}(l_{qq}^{fa} + (d_q\lambda^{-1}-1)l_{yq}^{fa})(q_t - q_t^T)^2 + \frac{1}{2}l_\pi^{fa}\pi_t^2 \end{array} \right] + t.i.p, \tag{D.70}$$

where

$$y_t^T = l_y^e e_t,$$

$$l_y^e = \frac{1}{(l_{yy}^{fa} + (d_q\lambda^{-1}-1)^{-1}l_{yq}^{fa})}\left[\begin{array}{cccc} l_{ye}^{fa} & l_{y\mu}^{fa} & l_{yg}^{fa} & l_{yc^*}^{fa} \end{array}\right],$$

$$q_t^T = l_q^e e_t,$$

and

$$l_y^e = \frac{1}{(l_{yy}^{fa} + (d_q\lambda^{-1}-1)l_{yq}^{fa})}\left[\begin{array}{cccc} l_{qe}^{fa} & l_{q\mu}^{fa} & l_{qg}^{fa} & l_{qc^*}^{fa} \end{array}\right].$$

Consequently, the contribution to the loss function of a random realization in $\varphi_j v_j$ is

$$U_c\bar{C}\beta^t\sigma_v^2 E_{t_0}\sum\beta^t\left[\Phi^{fa}k^{-1}(\varphi_j - \beta\varphi_{j+1})^2 + l_\pi^{fa}(\varphi_j)^2\right], \tag{D.71}$$

where

$$\Phi^{fa} = \Phi_Y^{fa}\alpha_y^2 + \Phi_Q^{fa}\alpha_q^2,$$

$$\Phi_Y^{fa} = (l_{yy}^{fa} + (d_q\lambda^{-1}-1)^{-1}l_{yq}^{fa}),$$

and

$$\Phi_Q^{fa} = (l_{qq}^{fa} + (d_q\lambda^{-1}-1)l_{yq}^{fa}).$$

It follows that policy randomization cannot improve welfare if the expression given by Equation (D.71) is positive definite. Hence, the first order conditions to the minimization problem are indeed a policy optimal if Φ^{fa} and l_π^{fa} are not both equal to zero and either: (a) $\Phi^{fa} \geq 0$ and $\Phi^{fa} + (1-\beta^{1/2})^2k^{-2}l_\pi^{fa} \geq 0$, or (b) $\Phi^{fa} \leq 0$ and $\Phi^{fa} + (1-\beta^{1/2})^2k^2l_\pi^{fa} \geq 0$ holds.