Timeless Perspective Policymaking: When is Discretion Superior?*

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Abstract

In this paper I show that discretionary policymaking can be superior to timeless perspective policymaking and identify model features that make this outcome more likely. Developing a measure of conditional loss that treats the auxiliary state variables that characterize the timeless perspective equilibrium appropriately, I use a New Keynesian DSGE model to show that discretion can dominate timeless perspective policymaking when the Phillips curve is relatively flat, due, perhaps, to firm-specific capital (or labor) and/or Kimball (1995) aggregation in combination with nominal price rigidity. These results suggest that studies applying the timeless perspective might also usefully compare its performance to discretion, paying careful attention to how policy performance is evaluated.

Keywords: *Discretion, timeless perspective, policy evaluation.*

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1 Introduction

In this paper I ask whether discretionary monetary policy can dominate policy designed according to the timeless perspective and answer in the affirmative. I then examine the factors that govern this result, employing a microfounded dynamic stochastic general equilibrium (DSGE) model to ascertain the role that nominal and real rigidities play in determining whether discretion is superior. Indeed, I show that discretion is more likely to dominate timeless perspective policymaking in models where nominal and real rigidities are important. Two additional contributions of the paper are that it develops a measure of conditional loss suitable for consistently evaluating timeless perspective and discretionary policies and that it shows how timeless perspective equilibria can be obtained from the solution to an unmodified formulation of the optimal commitment problem. An important conclusion of the paper is that studies applying the timeless perspective might also usefully compare its performance to discretion.

The timeless perspective approach to policy design was first outlined in Woodford (1999a), advanced as a solution to the "initial period" problem that characterizes optimal commitment policies.[1] At that time, Woodford (1999a) argued that the initial period problem could be overcome if the central bank were to "adopt, not the pattern of behavior from now on that would be optimal to choose, taking expectations as given, but rather the pattern of behavior *to which it would have wished to commit itself to at a date far in the past*, contingent upon the random events that have occurred in the meantime." Simply put, the initial period problem ceases to be a problem when the initial period has long since passed.[2] In subsequent work, the concepts of timeless perspective policymaking and timeless perspective equilibria have been refined and made formal.[3] Because this approach overcomes the initial period problem, the literature on monetary policy has embraced it, to the point where such policies increasingly form the backbone of policy analysis, and one central bank—Norges Bank—employs the timeless perspective to construct its public interest rate forecasts.

Timeless perspective policies are closely related to optimal commitment policies. In particular, both policies involve auxiliary state variables whose role is to track the value of commit-

[1] As explained in Section 2, optimal commitment policies are subject to an "initial period" problem because such policies involve the central bank exploiting expectations held as of the optimization date while promising never to do so in the future.

[2] Notice that for this to be true the timeless perspective equilibrium must be stationary, a condition stronger than standard transversality conditions. In a linear-quadratic model, the standard transversality condition requires the economy to grow at a rate no greater than $\frac{1}{\sqrt{\beta}}$, where $\beta \in (0,1)$ is the discount factor.

[3] See Woodford (2003), Giannoni and Woodford (2002a,b), and Benigno and Woodford (2003, 2006).

ments over time. One implication of these auxiliary state variables is that timeless perspective policies involve commitments and are not time-consistent in the sense of Kydland and Prescott (1977). At the same time, timeless perspective policies are not optimal in the sense of Kydland and Prescott (1980), opening the door to the possibility that they may be inferior to other suboptimal policies, such as discretion. It is important to compare the performance of timeless perspective policies to discretion because such a comparison helps to identify and understand situations where timeless perspective policymaking may be inferior to discretion. More generally, such a comparison allows us to better understand when discretionary policies perform well and when timeless perspective policies perform less well.

Rather than employ unconditional loss to compare discretion to timeless perspective policymaking (McCallum and Nelson, 2004; Sauer, 2007), I develop a measure of conditional loss suitable for the task.[4,5] In particular, I show how the auxiliary state variables that enter the timeless perspective equilibrium can be "integrated out" to produce a measure of loss that depends upon the state variables that describe the original optimization problem and that are common to both equilibria. For linear-quadratic models, this integration lowers the performance of the timeless perspective policy relative to the optimal commitment policy by terms that quantify the conditional mean and the conditional volatility of the auxiliary states. Of course, it is far from automatic that these adjustments will permit a timeless perspective to be dominated by discretion. However, using standard New Keynesian DSGE models, I show that factors that flatten the New Keynesian Phillips curve, such as nominal price rigidity, firm-specific labor/capital, and Kimball (1995) aggregation, can raise the conditional volatility (in particular) of the auxiliary state variables to the point where discretion becomes the superior policy. Indeed, the intuition for this result is reasonably clear. As the Phillips curve becomes increasingly flat, the central bank must generate greater volatility in real marginal costs in order to stabilize inflation. To the extent that real marginal costs are correlated

[4] Indeed, some have interpreted the term "timeless perspective" to mean that timeless perspective policies should be derived as the solution to an unconditional optimization problem (Blake, 2001, Jensen and McCallum, 2002, Damjanovic, Damjanovic, and Nolan, 2008). Since Woodford's approach to timeless perspective policy design does not do this, these studies have found that timeless perspective policies are not optimal from the timeless perspective. The policy associated with the solution to the unconditional optimization problem is sometimes referred to as the Blake-Jensen-McCallum alternative.

[5] There are several good reasons why unconditional loss should not be used to compare policies. First, the loss function common to both the timeless perspective and discretionary optimization problems is (invariably) conditional. Second, because the timeless perspective policy and the optimal commitment policy share the same asymptotic equilibrium, using unconditional loss to evaluate performance amounts to comparing discretion to the optimal commitment policy. Third, by ignoring transition dynamics, the use of unconditional loss can generate spurious performance reversals (Kim, Kim, Schaumburg, and Sims, 2005).

with the central bank's other policy objectives, this volatility in real marginal costs raises the volatility of the commitments that characterize the timeless perspective policy, penalizing its performance.

The remainder of this paper is organized as follows. Section 2 introduces the timeless perspective approach to policy design, applying it to a simple New Keynesian model. Section 3 shows how standard control methods for rational expectations models can be used to construct and analyze the equilibrium of a timeless perspective policy. Section 4 turns to methods for evaluating timeless perspective policies, showing why the treatment of the auxiliary states matters importantly for performance comparisons. Subsequently, Section 4 shows how the auxiliary state variables can be conditionally integrated out to construct a measure of conditional loss that is easy to compute and that is suitable for comparing the performance of discretion and timeless perspective policies. Applying this measure of conditional loss to the simple New Keynesian model described in Section 2, Section 4 shows that discretion can be superior to timeless perspective policymaking. Section 5 extends the analysis to a small-scale DSGE model and shows that factors that flatten the slope of the Phillips curve increase the likelihood that discretion will be superior to timeless perspective policymaking. Section 6 concludes.

2 Timeless perspective policy design

Commitment policies that are optimal in the sense of Kydland and Prescott (1980) are not stationary. If a commitment mechanism is available, then the optimal policy is to exploit private sector expectations in the initial period while promising never to do so in the future. Such policies are not stationary because the initial period, the period in which the optimization just happens to occur, holds a special significance. To some, this feature of optimal commitment policies, although fundamental to their being optimal, is unattractive and undesirable. After all, when conducting policy today, why should a policymaker implement a policy that, while optimal from the perspective of some arbitrary date in the past, is suboptimal today?

To overcome this "initial-period problem" and obtain a stationary policy, it is now common to assume that policymakers approach policy design from a timeless perspective (Woodford, 1999a). Broadly speaking, a timeless perspective policymaker promises not to exploit initial conditions. Instead, the policymaker ties its hands and commits to behave in the initial period as it does in all subsequent periods. Put differently, the timeless perspective policymaker implements policy as if that policy had been formulated in the distant past, such that any advantage from exploiting initial conditions has long since evaporated.

Timeless perspective policies have a certain attraction insomuch as they are both optimal and time consistent—from the timeless perspective. However, timeless perspective policies are neither optimal nor time consistent in the sense of Kydland and Prescott (1977, 1980). As a consequence, timeless perspective policies face credibility problems, such that it is unclear whether they can feasibly be implemented, and it is not obvious whether they are necessarily superior to discretion.

2.1 A simple example

To make timeless perspective policymaking concrete, consider the following simple example. The central bank's optimization problem is to choose the sequence of nominal interest rates $\{i_t\}_0^\infty$ to minimize

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \mu y_t^2 + \nu i_t^2\right), \tag{1}$$

where π_t denotes inflation, y_t denotes the output gap, $\beta \in (0,1)$ denotes the subjective discount factor, and $\mu \in [0,\infty)$ and $\nu \in [0,\infty)$ denote the relative weights on output and interest rate stabilization relative to inflation stabilization, respectively, and E_0 is the mathematical expectations operator conditional on period 0 information. Under certain circumstances, equation (1) can be viewed as a second-order accurate approximation to household welfare (Benigno and Woodford, 2006). For the purposes of this section, however, I will simply take equation (1) to be primal.

Constraining the central bank's optimization problem are

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa y_t + u_t, \tag{2}$$

$$y_t = \mathrm{E}_t y_{t+1} - \sigma \left(i_t - \mathrm{E}_t \pi_{t+1}\right) + r_t^n, \tag{3}$$

$$u_{t+1} = \rho_u u_t + \epsilon_{ut+1}, \tag{4}$$

$$r_{t+1}^n = \rho_r r_t^n + \epsilon_{rt+1} \tag{5}$$

where u_t denotes a markup shock, r_t^n denotes a neutral rate shock, and the initial conditions u_0 and r_0^n are known. The innovations ϵ_{ut} and ϵ_{rt} are assumed to be *i.i.d.* with zero mean and finite variance. Equation (2) is the New Keynesian Phillips curve associated with Calvo-pricing (Calvo, 1983). Equation (3) is the standard consumption-Euler equation that, abstracting from government spending, investment, and trade, is written in terms of the output gap. Equations (4) and (5) describe the laws of motion for the markup shock and the neutral rate shock. The parameters $\{\kappa, \sigma\} \in (0, \infty)$ describe the price rigidity and the elasticity of intertemporal substitution, respectively, while $\{\rho_u, \rho_r\} \in (-1, 1)$ summarize the persistence properties of the two shocks.

In addition to equations (2) through (5), the first-order conditions for the optimal commitment policy are

$$\beta\pi_t + \lambda_{\pi t+1} = 0, \quad t = 0, \tag{6}$$

$$\mu\beta y_t - \kappa\lambda_{\pi t+1} + \beta\lambda_{yt+1} = 0, \quad t = 0, \tag{7}$$

$$\beta\pi_t + \lambda_{\pi t+1} - \lambda_{\pi t} - \sigma\lambda_{yt} = 0, \quad t > 0, \tag{8}$$

$$\mu\beta y_t - \kappa\lambda_{\pi t+1} + \beta\lambda_{yt+1} - \lambda_{yt} = 0, \quad t > 0, \tag{9}$$

$$\nu i_t + \sigma\lambda_{yt+1} = 0, \quad t \geq 0, \tag{10}$$

where $\lambda_{\pi t+1}$ and λ_{yt+1} are the Lagrange multipliers associated with equations (2) and (3), respectively. The time inconsistency of the optimal commitment policy is reflected in the difference's between equations (6) through (7) and equations (8) through (9), which imply a different policy when $t = 0$ than when $t > 0$. Notice, however, that these differences disappear when $\lambda_{\pi 0} = \lambda_{y0} = 0$. As a consequence, the optimal commitment policy can be obtained by applying standard saddle-point solution methods to equations (2) through (5) and (8) through (10), with the initial conditions $\lambda_{\pi 0} = \lambda_{y0} = 0$ and u_0, r_0^n, known.

2.2 Timeless perspective policymaking

To obtain the Woodford (1999a) timeless perspective policy for this model the approach is to proceed as follows. First, to introduce the timeless perspective, assume that equations (8) and (9) also apply when $t = 0$, effectively discarding equations (6) and (7). Then, to obtain a policy that is implementable, use equation (10) to solve for λ_{yt+1} and equation (9) to solve for $\lambda_{\pi t+1}$ and substitute these expressions into equation (8) to eliminate the two Lagrange multipliers. With these substitutions, the timeless perspective policy is

$$\pi_t + \frac{\mu}{\kappa}(y_t - y_{t-1}) - \frac{\nu}{\sigma\kappa\beta}\left[(\beta + \sigma\kappa)(i_t - i_{t-1}) - (i_{t-1} - i_{t-2})\right] + \frac{\nu}{\beta}i_t = 0, \quad t \geq 0. \tag{11}$$

Provided $\nu > 0$, equation (11) can be solved for i_t, giving rise to what is known as an explicit targeting rule. The timeless perspective equilibrium is now obtained by solving for the rational expectations equilibrium of equations (2) through (5) and (11), with u_0, i_{-1}, and i_{-2} known. Notice that the timeless perspective policy depends on the *change* in the output gap, a point emphasized by Walsh (2003) in his discussion of "speed limit" policies, and on lags of the interest rate, a point Woodford (1999b) highlights in his analysis of optimal interest rate inertia. Further, as Giannoni and Woodford (2002b) stress, because the shocks are not directly present, the timeless perspective targeting rule, as represented by equation (11), is robust to misspecification of the shock processes.

3 A general approach

Researchers are often less interested in the timeless perspective targeting rule itself than they are in the timeless perspective equilibrium. For example, if they are interested in impulse responses, assessing the importance of particular shocks for business cycle volatility, or understanding the economic effects of particular nominal/real rigidities, then the object of interest is the timeless perspective equilibrium. Similarly, if they are interested in forecasting or in system estimation of the model, then the object of interest is also the timeless perspective equilibrium. This is certainly not to say that the timeless perspective targeting rule is never of interest,[6] and, to the extent that insight can be gained from both, the techniques described in this section complement those in the previous section and those described in Giannoni and Woodford (2002a) and Benigno and Woodford (2007).

For reasons that I explain below, timeless perspective equilibrium can be obtained from the following three-step procedure.

1. Formulate and solve the period 0 optimal commitment problem.

2. Use the equilibrium relationships to derive expressions for the Lagrange multipliers (shadow prices).

3. Assume that the expressions derived in step 2 hold in the initial period, thereby introducing the timeless perspective, and use these expressions to eliminate the shadow prices from the system.

Notice that this three-step approach obtains the timeless perspective equilibrium directly from the optimal commitment problem, without requiring modifications to either its objective function or its constraints (c.f. Giannoni and Woodford, 2002a, and Benigno and Woodford, 2007).

[6] Indeed, timeless perspective policies are often analyzed for the purpose of establishing policy representations that can implement uniquely the timeless perspective equilibrium.

To understand how this three-step procedure works, consider the following general linear-quadratic control problem. Let the economic environment be one in which an $n \times 1$ vector of endogenous variables, $\mathbf{z}_t$, consisting of n_1 predetermined variables, $\mathbf{x}_t$, and n_2 $(n_2 = n - n_1)$ nonpredetermined variables, $\mathbf{y}_t$, evolves over time according to

$$\mathbf{x}_{t+1} = \mathbf{A_{xx}}\mathbf{x}_t + \mathbf{A_{xy}}\mathbf{y}_t + \mathbf{B_{xu}}\mathbf{u}_t + \boldsymbol{\varepsilon}_{t+1}, \tag{12}$$

$$\mathrm{E}_t\mathbf{y}_{t+1} = \mathbf{A_{yx}}\mathbf{x}_t + \mathbf{A_{yy}}\mathbf{y}_t + \mathbf{B_{yu}}\mathbf{u}_t, \tag{13}$$

where $\mathbf{u}_t$ is a $p \times 1$ vector of policy control variables, $\boldsymbol{\varepsilon}_t \sim i.i.d.\,[\mathbf{0}, \boldsymbol{\Sigma}]$ is an $s \times 1$ $(s \leq n_1)$ vector of white-noise innovations, and E_t is the private sector's mathematical expectations operator conditional upon period t information. The matrices $\mathbf{A_{xx}}$, $\mathbf{A_{xy}}$, $\mathbf{A_{yx}}$, $\mathbf{A_{yy}}$, $\mathbf{B_{xu}}$, and $\mathbf{B_{yu}}$ contain the structural parameters that govern preferences and technology and are conformable with $\mathbf{x}_t$, $\mathbf{y}_t$, and $\mathbf{u}_t$ as necessary. The matrix $\mathbf{A_{yy}}$ is assumed to have full rank.

Subject to equations (12) and (13) and $\mathbf{x}_0$ known, the control problem is for the policymaker to choose the sequence of control variables $\{\mathbf{u}_t\}_0^\infty$ to minimize

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\mathbf{z}_t^{'}\mathbf{W}\mathbf{z}_t + 2\mathbf{z}_t^{'}\mathbf{U}\mathbf{u}_t + \mathbf{u}_t^{'}\mathbf{R}\mathbf{u}_t\right], \tag{14}$$

where $\mathbf{z}_t \equiv \begin{bmatrix} \mathbf{x}_t^{'} & \mathbf{y}_t^{'} \end{bmatrix}^{'}$. Methods to solve this optimal commitment problem are by now well known (see Oudiz and Sachs (1985) and Backus and Driffill (1986)). For the purposes of this section, however, what is important is that the equilibrium has the form

$$\mathbf{x}_{t+1} = \mathbf{M_{xx}}\mathbf{x}_t + \mathbf{M_{xp}}\mathbf{p}_t + \boldsymbol{\varepsilon}_{t+1}, \tag{15}$$

$$\mathbf{p}_{t+1} = \mathbf{M_{px}}\mathbf{x}_t + \mathbf{M_{pp}}\mathbf{p}_t, \tag{16}$$

$$\mathbf{y}_t = \mathbf{H_x}\mathbf{x}_t + \mathbf{H_p}\mathbf{p}_t, \tag{17}$$

$$\mathbf{u}_t = \mathbf{F_x}\mathbf{x}_t + \mathbf{F_p}\mathbf{p}_t, \tag{18}$$

where $\mathbf{p}_t$ is the $n_2 \times 1$ vector of shadow prices associated with the nonpredetermined variables and the system is initialized with $\mathbf{x}_0$ known and $\mathbf{p}_0 = \mathbf{0}$. These shadow prices are the direct analog to the Lagrange multipliers employed earlier, and they serve as state variables, keeping track of the current value of past promises, in the equilibrium (Kydland and Prescott, 1980).

With the solution to the optimal commitment problem in hand, the second step is to use these equilibrium relationships to derive an expression for the shadow prices. Define $\mathbf{d}_t \equiv \begin{bmatrix} \mathbf{y}_t \\ \mathbf{u}_t \end{bmatrix}$ and rewrite equations (17) and (18) as

$$\mathbf{d}_t = \mathbf{G_x}\mathbf{x}_t + \mathbf{G_p}\mathbf{p}_t, \tag{19}$$

where the construction of $\mathbf{G_x}$ and $\mathbf{G_p}$ is obvious and straightforward. Importantly, since $\mathbf{d}_t$ contains all of the nonpredetermined variables and $\mathbf{A_{yy}}$ has full rank, $\mathbf{G_p}$ is a $(n_2 + p) \times n_2$ matrix with $\text{rank}(\mathbf{G_p}) \geq n_2$. Rewriting equation (19) to make $\mathbf{p}_t$ the subject leads to

$$\mathbf{p}_t = \mathbf{G_p}^{-1} \left(\mathbf{d}_t - \mathbf{G_x} \mathbf{x}_t \right), \tag{20}$$

where $\mathbf{G_p}^{-1}$ represents the generalized (left) inverse of $\mathbf{G_p}$.

In the final step, I substitute equation (20) into equation (15) and the lag of equation (16), thereby dispensing with the initial condition $\mathbf{p}_0 = \mathbf{0}$. After some reorganization, the timeless perspective equilibrium can be written as

$$\begin{bmatrix} \mathbf{x}_{t+1} \\ \mathbf{x}_t \\ \mathbf{d}_t \end{bmatrix} = \begin{bmatrix} \mathbf{M_{xx}} & \mathbf{M_{xp}} \left(\mathbf{M_{px}} - \mathbf{M_{pp}} \mathbf{G_p}^{-1} \mathbf{G_x} \right) & \mathbf{M_{xp}} \mathbf{M_{pp}} \mathbf{G_p}^{-1} \\ \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{G_x} & \mathbf{G_p} \left(\mathbf{M_{px}} - \mathbf{M_{pp}} \mathbf{G_p}^{-1} \mathbf{G_x} \right) & \mathbf{G_p} \mathbf{M_{pp}} \mathbf{G_p}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{x}_{t-1} \\ \mathbf{d}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} [\boldsymbol{\varepsilon}_{t+1}]. \tag{21}$$

To understand why this procedure correctly recovers the timeless perspective equilibrium, consider the relationship between the optimal commitment problem and the timeless perspective problem. In both problems the policymaker has access to a mechanism that it uses to commit to its policy. The value of the central bank's policy commitments is encapsulated in shadow prices. Critically, aside from the initial period, the timeless perspective does not change either the constraints or the objectives in the optimization problem. As a consequence, the timeless perspective does not change the system's stability properties, nor does it change the system's eigenvalues or whether the shadow prices are predetermined, which is why the optimal commitment policy and the timeless perspective policy share the same asymptotic equilibrium. What the timeless perspective does change, however, is the system's initial conditions, which is why the optimal commitment policy and the timeless perspective policy have different period-0 transition dynamics and, with discounting, yield different losses.

But, although saddle-point solution methods require the partitioning between stable and unstable eigenvalues to conform to the partitioning between predetermined and nonpredetermined variables (both unaffected by the timeless perspective), they do not require an explicit declaration of the initial conditions. Thus, the timeless perspective equilibrium can be found by first applying standard rational expectations control methods. Then, once the equilibrium has been obtained for arbitrary initial conditions, the timeless perspective can be introduced by using the equilibrium relationships to make stationary, and subsequently eliminate, the shadow prices.

Note the role of the rank condition on $\mathbf{G_p}$. This rank condition ensures that the shadow prices obtained from equation (20) fully satisfy the model's equilibrium relationships. It follows that a valid solution for $\mathbf{p}_t$ can be obtained from any subset of the variables in $\mathbf{y}_t$ and $\mathbf{u}_t$ provided that the resulting $\mathbf{G_p}$ matrix has $\text{rank}(\mathbf{G_p}) \geq n_2$. Although the particular state variables that enter the timeless perspective equilibrium will depend on which equilibrium relationships are used when solving for $\mathbf{p}_t$, by construction, they all imply the same welfare and equilibrium behavior. That timeless perspective equilibria have multiple representations is also reflected in the fact that although the procedure described above yields an equilibrium in which $\mathbf{u}_t$ is a function of $\mathbf{x}_t$, $\mathbf{x}_{t-1}$, $\mathbf{y}_{t-1}$, and $\mathbf{u}_{t-1}$, the approach described in Section 2 would yield an equilibrium in which $\mathbf{u}_t$ is a function of $\mathbf{x}_t$, $\mathbf{y}_{t-1}$, $\mathbf{u}_{t-1}$, and $\mathbf{u}_{t-2}$.[7]

One situation where the rank condition, $\text{rank}(\mathbf{G_p}) \geq n_2$, will often fail is when equation (18) is used in isolation to solve for the shadow prices, as per Tetlow and von zur Muehlen (2001). They solve for $\mathbf{p}_t$ according to

$$\mathbf{p}_t = \mathbf{F}_\mathbf{p}^{-1} \left(\mathbf{u}_t - \mathbf{F_x x}_t \right), \tag{22}$$

where $\mathbf{F}_\mathbf{p}^{-1}$ denotes the generalized (left) inverse of $\mathbf{F_p}$, and use this expression to eliminate the shadow prices from the system. The problem with using equation (22) is that, unless $p \geq n_2$, the solution for $\mathbf{p}_t$ will be generated from a system that is underdetermined. Consequently, the shadow prices will no longer necessarily obey the model's equilibrium relationships. In the Tetlow and von zur Muehlen (2001) application there was one nonpredetermined variable and one control variable, so $p = n_2$ and no problem arose. But if $p < n_2$, as is invariably the case, then employing equation (22) will lead to erroneous results.

Two further points are worth emphasizing before leaving this section. First, provided $\mathbf{d}_t \neq \mathbf{u}_t$, and therefore that $\mathbf{G_p} \neq \mathbf{F_p}$, a representation for the timeless perspective targeting rule can be recovered by substituting equation (20) back into equation (18). This substitution leads to the expression

$$\mathbf{u}_t = \left(\mathbf{F_x} - \mathbf{F_p G_p^{-1} G_x} \right) \mathbf{x}_t + \mathbf{F_p G_p^{-1} d}_t, \tag{23}$$

which describes a relationship that $\mathbf{u}_t$, $\mathbf{x}_t$, and $\mathbf{y}_t$ must satisfy along the equilibrium path. Second, the timeless perspective equilibrium described by equation (21) essentially has two components. The first component is a transition equation for the predetermined variables,

$$\mathbf{x}_{t+1} = \mathbf{N_{xx1} x}_t + \mathbf{N_{xx2} x}_{t-1} + \mathbf{N_{xd} d}_{t-1} + \varepsilon_{t+1}. \tag{24}$$

[7] Importantly, this well-known multiplicity of representations makes no material difference for the analysis or conclusions that follow, since I assume—for consistency—that the conditioning variables satisfy the timeless perspective equilibrium relationships.

The second component is a measurement equation for the nonpredetermined variables and the decision variables

$$\mathbf{d}_t = \mathbf{S}_{\mathbf{dx1}}\mathbf{x}_t + \mathbf{S}_{\mathbf{dx2}}\mathbf{x}_{t-1} + \mathbf{S}_{\mathbf{dd}}\mathbf{d}_{t-1}. \tag{25}$$

It should be clear from equations (24) and (25) that system estimation of timeless perspective models involves the very same techniques that are used to estimate rational expectations models. Thus, the timeless perspective raises no hurdles for estimation other than those already encountered for rational expectations models (c.f. Juillard and Pelgrin, 2007). Specifically, after introducing any necessary measurement error terms, the likelihood function can be evaluated directly (Fuhrer and Moore, 1995; Dennis, 2004; Schmitt-Grohé and Uribe, 2007) or be built up recursively using the Kalman filter (Hansen and Sargent, 1980), depending on the model, and then either maximized or combined with a prior for Bayesian estimation.

4 Evaluating timeless perspective policies

An obvious alternative to timeless perspective policymaking is for the policymaker to conduct policy with discretion. Discretion is an obvious alternative because discretionary policies are rule-based, time-consistent, and, critically, they do not require a commitment mechanism. In the absence of a commitment mechanism, the discretionary policymaker simply reoptimizes period by period. Since neither policy is optimal, an essential question is whether timeless perspective policymaking dominates discretion. This is the question I now address.

However, in order to address this question I need a method for evaluating the performance of timeless perspective policies, an exercise that is complicated by the presence of the auxiliary state variables. I begin by considering two possible methods. The first method is to evaluate loss conditional on the entire (including the auxiliary) initial state vector; the second method is to evaluate performance using unconditional loss. Using the simple new Keynesian model to illustrate, I show that neither of these approaches is entirely satisfactory and use their deficiencies to motivate and derive an alternative measure of policy performance.

4.1 The simple example continued

To illustrate the central issues, it is useful to return to the simple model introduced earlier. Accordingly, I state without proof that the explicit targeting rule associated with discretionary policymaking in that model is given by

$$\pi_t + \frac{\mu}{\kappa} y_t - \frac{\nu}{\sigma\kappa} i_t = 0. \tag{26}$$

Simplifying, in the special case that $\nu = 0$, equations (11) and (26) collapse to

$$\pi_t + \frac{\mu}{\kappa}(y_t - y_{t-1}) = 0, \quad (27)$$

$$\pi_t + \frac{\mu}{\kappa} y_t = 0, \quad (28)$$

respectively. It follows that the state is described by u_t for the discretionary policy, by u_t and $\lambda_{\pi t}$ for the optimal commitment policy, and by u_t and y_{t-1} for the timeless perspective policy. As I now illustrate numerically,[8] the performances associated with each of these policies depends importantly on how these differences among the state variables is treated.

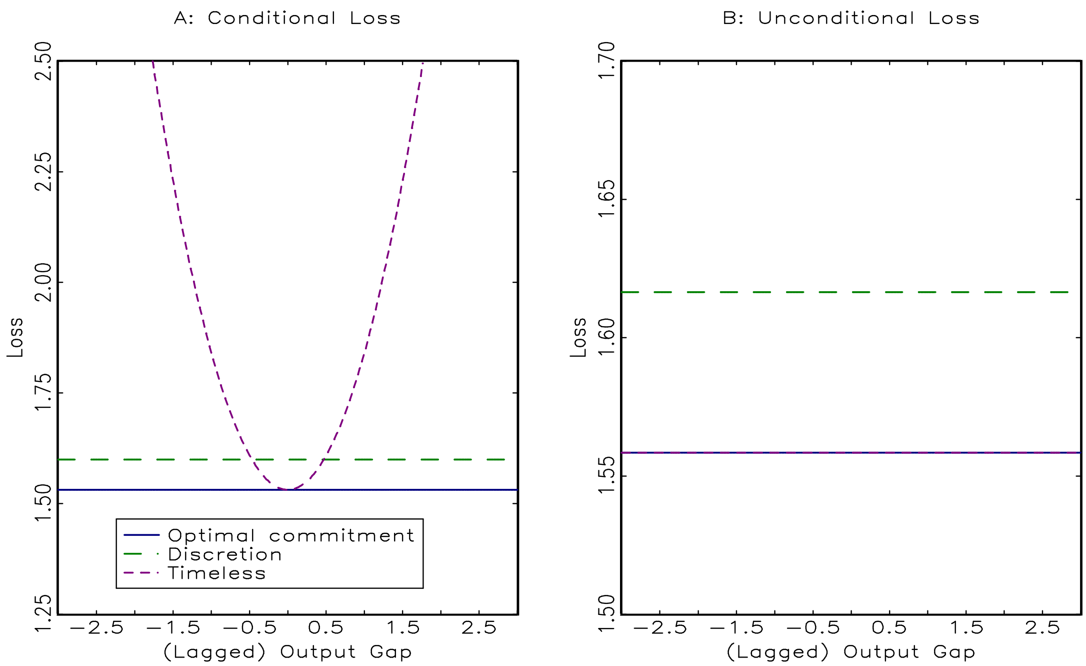

Fig. 1

One way to measure the performance of each policy is to simply evaluate equation (14) conditional on the relevant initial states. For the optimal commitment policy and the discretionary policy, it is straightforward to evaluate equation (14), since both policies assume a given known value for u_0 and since for the optimal commitment policy it is known that $\lambda_{\pi 0} = 0$. It is slightly more complicated for the timeless perspective policy, since that policy requires an initial value for y_{-1}, the lagged output gap.

Consider Figure 1A, which displays performances for $u_0 = 0$ and for an array of different initial values for the lagged output gap. By construction, the optimal commitment policy

[8] I parameterize the model according to $\kappa = 0.025$, $\rho_u = 0.20$, $\beta = 0.99$, $\sigma_{\epsilon_u} = 1$, and $\mu = 0.50$.

generates the best performance, with the optimal commitment policy delivering a 4.3 percent improvement in performance relative to discretion. Also by construction, since y_{-1} is not a state variable in either the discretionary equilibrium or the optimal commitment equilibrium, the performances associated with these policies are invariant to this variable.

Now consider the performances associated with the timeless perspective policy. For the timeless perspective, performance is maximized when $y_{-1} = 0$ and rises symmetrically for absolute deviations in y_{-1} about 0. In fact, when $y_{-1} = 0$, the timeless perspective policy performs identically to the optimal commitment policy. Most obviously, Figure 1A reveals that as y_{-1} becomes larger in magnitude, loss for the timeless perspective policy increases to become larger than the loss for discretion. Clearly there exist states (here a lagged output gap greater than about 0.5 percent) for which discretion is superior, delivering a better performance than the timeless perspective policy. This is an issue for a central bank pursuing a timeless perspective policy because in states where discretion is superior it is not clear that the central bank would continue to implement the inferior policy, highlighting the time inconsistency of the timeless perspective policy. Timeless perspective policies perform poorly when the output gap is large because the timeless perspective assumes that it is the stationary asymptotic equilibrium—and not initial expectations or transition dynamics—that govern outcomes.

With policies evaluated according to equation (14), it is not difficult to see that it will always be possible to find states where discretion is superior to timeless perspective policymaking for any model in which there is a time-consistency problem.[9] An alternative to evaluating policy according to equation (14), hinted at in the discussion above, is to use unconditional loss. By using the unconditional expectation of equation (14), the equation's dependence on the initial state is eliminated. Figure 1B displays unconditional loss for each policy, where the initial state has been integrated out using the (unconditional) probability density implied by the model.[10] Since policies are now being evaluated according to the characteristics of their asymptotic equilibrium, and the optimal commitment policy and the timeless perspective policy share the same asymptotic equilibrium, these policies deliver the same unconditional loss. Clearly, if unconditional loss is the appropriate measure of performance, then discretion is the inferior policy.

[9] To the extent that timeless perspective commitments are untenable in such states, this consideration provides motivation for the "quasi-commitment" equilibrium analyzed by Schaumburg and Tambalotti (2007) and the "loose commitment" equilibrium studied by Debartoli and Nunes (2006).

[10] Importantly, to the extent that observed data are not well explained by the model, very different results might be obtained if the integration used the observed frequency distribution for the state variables rather than using the model-implied density function.

However, while it is common to use unconditional loss when assessing timeless perspective policy performance (Jensen and McCallum, 2002), there are good reasons for not doing so. Aside from the most obvious point, which is that the discretionary problem, the optimal commitment problem, and the timeless perspective problem are all explicitly conditioned on an observed known initial state, $\mathbf{x}_t$, it is well known that ignoring transition dynamics and evaluating policies according to their asymptotic behavior can lead to spurious welfare reversals (Kim, Kim, Schaumburg, and Sims, 2005).

Although Figure 1A shows that discretion can be superior to timeless perspective policy-making, neither equation (14) nor its unconditional expectation seems entirely satisfactory for quantifying timeless perspective policy performance: the former depends on auxiliary state variables, here y_{-1}, while the latter ignores initial conditions and transition dynamics. To address this issue, in the next section I develop a measure of performance suitable for evaluating timeless perspective policies.

4.2 Evaluating policy performance

For the general linear-quadratic control problem described by equations (12) through (14), the three policy approaches examined above have equilibria that can be written in the form

$$\mathbf{s}_{t+1} = \mathbf{M}_{\mathbf{ss}}\mathbf{s}_t + \mathbf{N}\boldsymbol{\varepsilon}_{t+1}, \tag{29}$$

$$\mathbf{y}_t = \mathbf{H}_{\mathbf{s}}\mathbf{s}_t, \tag{30}$$

$$\mathbf{u}_t = \mathbf{F}_{\mathbf{s}}\mathbf{s}_t, \tag{31}$$

where $\mathbf{s}_t \equiv \begin{bmatrix} \mathbf{x}_t' & \mathbf{q}_t' \end{bmatrix}'$. For the discretionary policy $\mathbf{q}_t$ is the null vector, for the optimal commitment policy $\mathbf{q}_t = \mathbf{p}_t$, and for the timeless perspective policy $\mathbf{q}_t = \begin{bmatrix} \mathbf{x}_{t-1}' & \mathbf{d}_{t-1}' \end{bmatrix}'$. Now, for arbitrary period t, equations (29) through (31) allow the loss function, conditional on $\mathbf{s}_t$, to be expressed as

$$L_t = \mathbf{s}_t'\mathbf{P}\mathbf{s}_t + \frac{\beta}{1-\beta}\mathrm{tr}\left[\mathbf{N}'\mathbf{P}\mathbf{N}\boldsymbol{\Sigma}\right], \tag{32}$$

where

$$\mathbf{P} = \widehat{\mathbf{W}} + \beta\mathbf{M}_{\mathbf{ss}}'\mathbf{P}\mathbf{M}_{\mathbf{ss}}, \tag{33}$$

$$\widehat{\mathbf{W}} \equiv \mathbf{H}_{\mathbf{s}}'\mathbf{W}\mathbf{H}_{\mathbf{s}} + \mathbf{H}_{\mathbf{s}}'\mathbf{U}\mathbf{F}_{\mathbf{s}} + \mathbf{F}_{\mathbf{s}}'\mathbf{U}'\mathbf{H}_{\mathbf{s}} + \mathbf{F}_{\mathbf{s}}'\mathbf{R}\mathbf{F}_{\mathbf{s}}. \tag{34}$$

In light of equation (32), unconditional loss is given by

$$\begin{aligned}\overline{L} &= \int_{\mathbf{s}} \left[\mathbf{s}_t' \mathbf{P} \mathbf{s}_t + \frac{\beta}{1-\beta} \mathrm{tr} \left[\mathbf{N}' \mathbf{P} \mathbf{N} \mathbf{\Sigma} \right] \right] p(\mathbf{s}_t)\, d\mathbf{s}_t, \\ &= \mathrm{tr}\left[\mathbf{P}\mathbf{\Omega} \right] + \frac{\beta}{1-\beta} \mathrm{tr} \left[\mathbf{N}' \mathbf{P} \mathbf{N} \mathbf{\Sigma} \right], \end{aligned} \tag{35}$$

where $p(\mathbf{s}_t)$ denotes the density function for $\mathbf{s}_t$ and $\mathbf{\Omega}$ represents the unconditional variance of $\mathbf{s}_t$. The performances shown in Figure 1A were calculated using $(1-\beta)L_t$, while those in Figure 1B were calculated using $(1-\beta)\overline{L}$.

Rather than integrate with respect to the entire state, $\mathbf{s}_t$, as equation (35) does, I propose to integrate with respect to $\mathbf{q}_t$ conditional on $\mathbf{x}_t$, and to evaluate timeless perspective policies according to

$$\widehat{L}_t = \int_{\mathbf{q}} \left[\mathbf{s}_t' \mathbf{P} \mathbf{s}_t + \frac{\beta}{1-\beta} tr \left[\mathbf{N}' \mathbf{P} \mathbf{N} \mathbf{\Sigma} \right] \right] p(\mathbf{q}_t | \mathbf{x}_t)\, d\mathbf{q}_t, \tag{36}$$

where $p(\mathbf{q}_t|\mathbf{x}_t)$ denotes the density function for $\mathbf{q}_t$ conditional on $\mathbf{x}_t$. To evaluate this integral, partition $\mathbf{\Omega}$ (and subsequently $\mathbf{M_{ss}}$ and $\mathbf{P}$) conformably with $\mathbf{x}_t$ and $\mathbf{q}_t$, then the mean and variance of $\mathbf{q}_t$ conditional on $\mathbf{x}_t$, are given by

$$\overline{\mathbf{q}}_t = \mathbf{\Omega_{qx}} \mathbf{\Omega_{xx}^{-1}} \mathbf{x}_t, \tag{37}$$

$$\mathbf{\Omega}_{\mathbf{q}_t|\mathbf{x}_t} = \mathbf{\Omega_{qq}} - \mathbf{\Omega_{qx}} \mathbf{\Omega_{xx}^{-1}} \mathbf{\Omega_{xq}}, \tag{38}$$

and equation (36) is equivalent to

$$\widehat{L}_t = \mathbf{x}_t' \left(\mathbf{P_{xx}} + \mathbf{P_{xq}} \mathbf{\Omega_{qx}} \mathbf{\Omega_{xx}^{-1}} + \mathbf{\Omega_{xx}^{-1\prime}} \mathbf{\Omega_{qx}'} \mathbf{P_{qx}} \right) \mathbf{x}_t + \mathrm{tr} \left[\mathbf{P_{qq}} \mathbf{\Omega}_{\mathbf{q}_t|\mathbf{x}_t} \right] + \frac{\beta}{1-\beta} \mathrm{tr} \left[\mathbf{N}' \mathbf{P} \mathbf{N} \mathbf{\Sigma} \right]. \tag{39}$$

Equation (39) contains three terms. The first and third terms represent the penalties attributable to the known initial state and to the stochastic shocks, respectively. The second term represents the penalty associated with the conditional variance of the auxiliary states that are introduced by timeless perspective policymaking. By integrating out the auxiliary state variables, equation (39) measures average loss for a given state, $\mathbf{x}_t$. In the absence of any auxiliary states, equation (39) is equivalent to equation (32). Further, in the limit as $\beta \uparrow 1$, equation (39) converges to equation (35).

Before leaving this section, it is worth noting that policy performance, as assessed by equation (39), is invariant to how the timeless perspective equilibrium is represented, unaffected by the particular choice of $\mathbf{d}_t$ or by the fact that $\mathbf{G_p^{-1}}$ may not be unique. To understand why, note that the substantive difference between the optimal commitment equilibrium and the timeless perspective equilibrium is that the shadow prices, $\mathbf{p}_t$, are not initialized to $\mathbf{0}$, but rather behave in the initial period as they do in all subsequent periods. It follows that there is actually no need to eliminate the shadow prices from the system (the step that leads to multiple representations) when evaluating equation (39). Instead, one can simply integrate with respect to the shadow prices conditional on $\mathbf{x}_t$. Because the optimal commitment policy has a unique representation in terms of $\mathbf{x}_t$ and $\mathbf{p}_t$ (under standard and quite general conditions), so too does the timeless perspective equilibrium, and this unique representation yields unique values for the mean and variance of $\mathbf{p}_t$ conditional on $\mathbf{x}_t$.[11]

4.3 The simple example again

Returning to the simple model, I now evaluate performance using $(1-\beta)\widehat{L}_t$ while varying κ, the slope of the Phillips curve, and μ, the weight on gap stabilization.[12] Figure 2A displays the performances for the optimal commitment policy, the timeless perspective policy, and the time-consistent policy as κ is varied between $(0, 0.1]$, holding ρ_u and μ constant at their benchmark values. Complementing Figure 2A, Figure 2C displays the performances associated with varying μ between $(0, 10]$ while holding ρ_u and κ constant at their benchmark values. In contrast, Figures 2B and 2D are generated allowing both κ and μ to vary between $(0, 0.1]$ and $(0, 10]$, respectively, displaying as a percent the fraction of occasions for which the discretionary policy performs better than the timeless perspective policy against particular values of κ and μ, respectively.

[11] Without wishing to labor the point, this invariance property is also a feature of unconditional loss, and for the same reason. Unconditional loss is invariant to the multiplicity of representations because it integrates out the entire state vector, which includes the auxillary states.

[12] Consistent with Figure 1, I set $\kappa = 0.025$, $\mu = 0.50$, $\rho_u = 0.20$, and $\sigma_{\epsilon_u} = 1$, and the initial state is given by $u_0 = 0$.

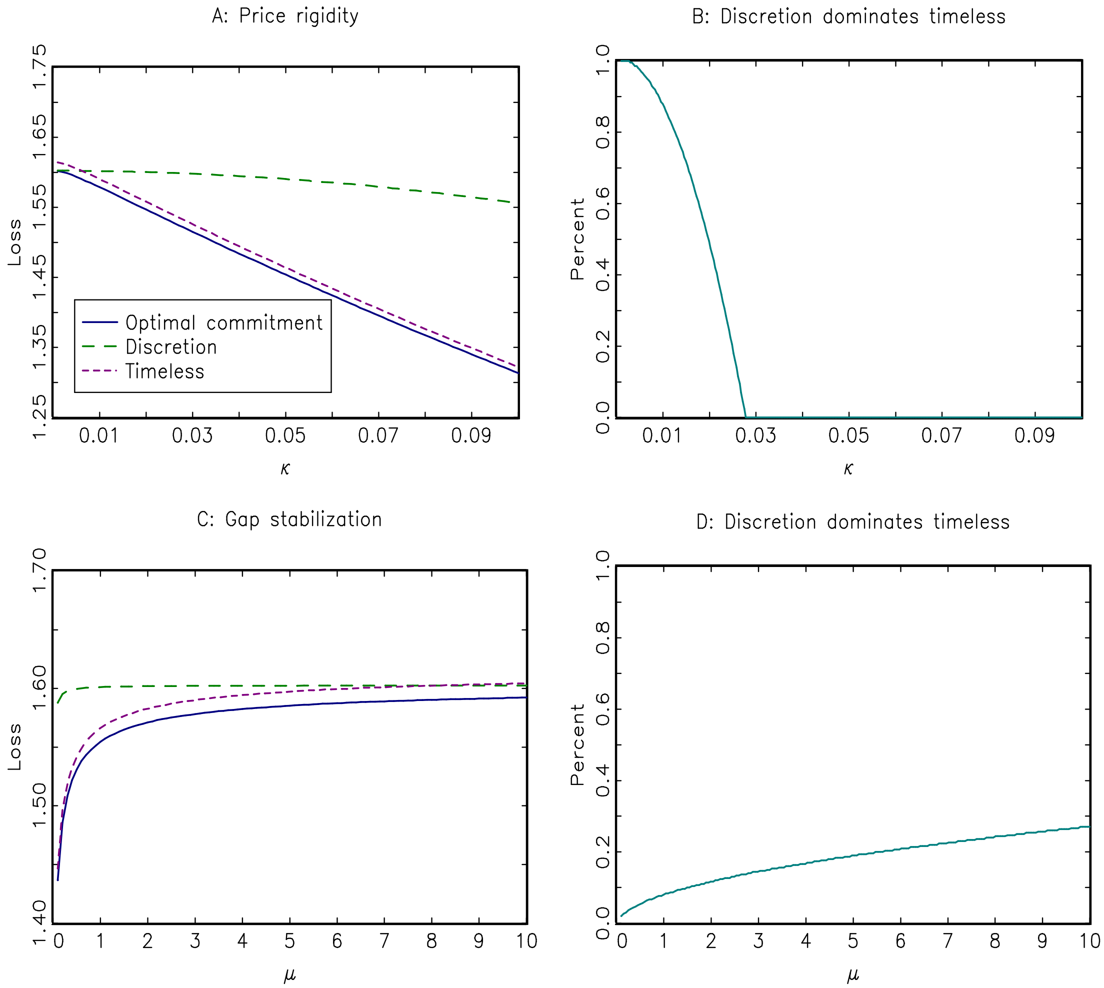

Fig. 2

Figure 2A reveals that, ceteris paribus, discretion does better than timeless perspective policymaking when κ is small and the Phillips curve is relatively flat. Low values for κ can arise when prices adjust infrequently and/or when strategic complementarities are important. When the Phillips curve is relatively flat, monetary policy must generate large movements in the output gap to stabilize inflation, and, relative to discretion, these large movements in the output gap undermine the performance of the timeless perspective policy. Similarly, ceteris paribus, Figure 2C shows that discretion does better than timeless perspective policymaking when μ is large. Complementing these findings, Figure 2B shows that, while the share of the parameter space for which discretion dominates the timeless perspective is decreasing in κ, there appears to be a threshold value for κ above which timeless perspective policymaking always dominates. Figure 2D reveals that the share of the parameter space for which discretion dominates timeless perspective policymaking increases monotonically with the weight on output gap stabilization. The main conclusions to take away from Figure 2 are that, although the improvement in loss may be small, discretion is more likely to perform better than timeless perspective policymaking when the Phillips curve is relatively flat and when the weight on output gap stabilization is relatively large.

5 A small-scale DSGE model

In the previous section, I analyzed a simple New Keynesian model and found that discretion could be superior to timeless perspective policymaking. In this section, I undertake a broader analysis using a more sophisticated business cycle model that contains a wider array of shocks and propagation mechanisms. Importantly, the model contains margins for substitution and mechanisms for propagating shocks that are present in most modern New Keynesian business cycle models. As with the simple model, I find that discretion can be superior to timeless perspective policymaking. Next, I extend this DSGE model to include features, such as firm-specific labor markets and Kimball (1995) aggregation, that are increasingly employed in DSGE models. I show that these modeling features and others like them raise the likelihood that discretion will dominate timeless perspective policymaking.

5.1 The model

Let the economy be populated by households, intermediate-good producing firms, final-good producing firms, and a central bank. Households are identical and infinitely lived, choosing consumption, c_t, investment, i_t, labor, l_t, and nominal holdings of next period bonds, b_{t+1}, to maximize

$$\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left[e^{g_{t+j}} \frac{c_{t+j}^{1-\sigma}}{1-\sigma} - \frac{l_{t+j}^{1+\chi}}{1+\chi} \right], \tag{40}$$

where $\{\sigma, \chi\} \in (0, \infty)$ represent the (inverse) elasticity of intertemporal substitution and the Frisch labor supply elasticity, respectively, subject to the budget constraint

$$c_t + i_t + \frac{b_{t+1}}{P_t} = w_t l_t + R_t k_t + (1 + r_t) \frac{b_t}{P_t} + D_t, \tag{41}$$

and the accumulation equation

$$k_{t+1} = (1 - \delta) k_t + i_t, \tag{42}$$

where $\delta \in (0, 1)$ represents the depreciation rate. In equations (41) and (42), w_t denotes the consumption real wage, R_t denotes the real rental rate of capital, k_t, P_t denotes the price of the final good, r_t denotes the net real return on the one-period nominal bond, b_t, and D_t denotes (in units of the final good) the lump-sum dividend that households receive from the intermediate-good producing firms.

On the production side, a unit-continuum of monopolistically competitive intermediate-good producing firms rent capital and hire labor in perfectly competitive markets. Indexing firms by $\tau \in [0,1]$, the τ'th firm chooses $l_t(\tau)$, $k_t(\tau)$, and its price, $p_t(\tau)$, to maximize its value, subject to three things: the production technology

$$y_t(\tau) = e^{u_t} k_t(\tau)^{\alpha} l_t(\tau)^{1-\alpha}, \tag{43}$$

where $\alpha \in (0,1)$; the demand schedule,

$$y_t(\tau) = \left(\frac{p_t(\tau)}{P_t}\right)^{-\varepsilon_t} Y_t, \tag{44}$$

where Y_t denotes aggregate output and $\varepsilon_t \in (1,\infty)$ represents the (stochastic) elasticity of substitution between goods; and a Calvo (1983) price rigidity. In the Calvo (1983) model, randomly chosen firms of share $1-\xi$ are able to make capital, labor, and pricing decisions while the remaining share are only able to make capital and labor decisions. All firms make their capital and labor decisions to minimize the real cost of producing a marginal unit of output, mc_t, then those firms that can choose their prices do so to maximize

$$\mathrm{E}_t \sum_{j=0}^{\infty} (\xi\beta)^j \frac{\Lambda_{t+j}}{\Lambda_t} \left(p_t(\tau) - mc_t\right) \left(\frac{p_t(\tau)}{P_{t+j}}\right)^{-\varepsilon_{t+j}} Y_{t+j}, \tag{45}$$

where Λ_t denotes the marginal utility of consumption, while the remaining firms keep their prices unchanged. Profits are aggregated and returned to households (shareholders) in the form of a lump-sum dividend.

The final-good producing firms also seek to maximize profits. They purchase intermediate goods, aggregate them into a final good according to the technology

$$Y_t = \left(\int_0^1 y_t(\tau)^{\frac{\varepsilon_t - 1}{\varepsilon_t}} d\tau\right)^{\frac{\varepsilon_t}{\varepsilon_t - 1}}, \tag{46}$$

and sell these final goods to households in a perfectly competitive market. As is well known, profit maximization by final-good producers gives rise to equation (44) while their zero-profit condition implies

$$P_t = \left(\int_0^1 p_t(\tau)^{1-\varepsilon_t} d\tau\right)^{\frac{1}{1-\varepsilon_t}}. \tag{47}$$

The model contains three stochastic elements: these translate into an aggregate markup shock, $v_t \equiv \ln\left(\frac{\varepsilon_t}{\varepsilon_t - 1}\right)$, an aggregate consumption-preference shock, g_t, and an aggregate technology shock, u_t. These three shocks evolve over time according to

$$\widehat{v}_{t+1} = \rho_{\widehat{v}}\widehat{v}_t + \epsilon_{\widehat{v}t+1}, \tag{48}$$

$$g_{t+1} = \rho_g g_t + \epsilon_{gt+1}, \tag{49}$$

$$u_{t+1} = \rho_u u_t + \epsilon_{ut+1}, \tag{50}$$

where $\widehat{v}_t$ is the percent deviation in v_t from steady state, where $\left\{\rho_{\widehat{v}}, \rho_g, \rho_u\right\} \in (-1, 1)$, and where the innovations $\left\{\epsilon_{\widehat{v}_{t+1}}, \epsilon_{g_{t+1}}, \epsilon_{u_{t+1}}\right\}$ are *i.i.d.* with zero mean and finite variance.

5.1.1 The log-linear model

When log-linearized about a zero-inflation nonstochastic steady state, the constraints and first-order conditions for this model are

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \frac{(1-\xi)(1-\beta\xi)}{\xi}\widehat{mc}_t + \widehat{v}_t, \tag{51}$$

$$\widehat{mc}_t = \alpha \widehat{R}_t + (1-\alpha)\widehat{w}_t - u_t, \tag{52}$$

$$\widehat{l}_t = \widehat{R}_t - \widehat{w}_t + \widehat{k}_t, \tag{53}$$

$$\widehat{w}_t = \chi \widehat{l}_t + \sigma \widehat{c}_t - g_t, \tag{54}$$

$$\mathrm{E}_t \widehat{R}_{t+1} = \frac{1+\rho}{\rho+\delta}\left(r_{t+1} - \mathrm{E}_t \pi_{t+1}\right), \tag{55}$$

$$\widehat{c}_t = \mathrm{E}_t \widehat{c}_{t+1} - \frac{1}{\sigma}\left(r_{t+1} - \mathrm{E}_t \pi_{t+1} - g_t + \mathrm{E}_t g_{t+1}\right), \tag{56}$$

$$\widehat{k}_{t+1} = (1-\delta)\widehat{k}_t + \delta \widehat{i}_t, \tag{57}$$

$$\widehat{y}_t = (1-\gamma)\widehat{c}_t + \gamma \widehat{i}_t, \tag{58}$$

$$\widehat{y}_t = u_t + \alpha \widehat{k}_t + (1-\alpha)\widehat{l}_t, \tag{59}$$

where $\rho \equiv \frac{1-\beta}{\beta}$ is the discount rate, $\gamma \equiv \frac{\alpha\delta}{\rho+\delta}\frac{\varepsilon-1}{\varepsilon}$ is the steady-state share of investment in output, and $\varepsilon > 1$ is the steady-state elasticity of substitution between intermediate goods.

Equation (51) is the New Keynesian Phillips curve linking inflation to movements in real marginal costs. Equation (52) documents the relationship between real marginal costs and the costs of the production factors, capital and labor. Equations (53) and (54) describe labor demand and supply, respectively. Equation (55) summarizes the connection between the rental rate of capital and the return on the one-period nominal bond that arises from the household's portfolio decision. Equation (56) is the standard consumption-Euler equation for time-separable isoelastic preferences. Equations (57) through (59) represent to first-order accuracy the capital accumulation equation, the resource constraint, and the aggregate production technology.

Turning now to monetary policy, I assume, for simplicity, that the central bank's decision problem is to choose $\{r_{t+1}\}_0^\infty$ to optimize the primal loss function

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \mu y_t^2\right), \tag{60}$$

subject to equations (48) through (59) and the known initial conditions $\widehat{v}_0$, g_0, u_0, and k_0.

5.1.2 Results

As earlier, I now analyze policy performance on a parameter grid. Specifically, I condition on the depreciation rate and on the parameters in the shock processes,[13] since preliminary investigations indicated these "persistence" parameters were largely unimportant for the results, and analyze the performance of discretion relative to timeless perspective policymaking on a grid of values for α, ξ, σ, ε, χ, and μ. The results are shown in Figure 3, which displays, for each parameter, the share of the parameter space for which discretion is the superior policy.[14]

[13] For this exercise, I set $\rho_g = \rho_v = 0.3$, $\rho_u = 0.95$, and $\delta = 0.025$. Further, the initial state is described by $\widehat{v}_0 = g_0 = u_0 = k_0 = 0$, such that the economy initially resides at its nonstochastic steady state.

[14] Blake and Kirsanova (2008) have shown recently that this model can exhibit multiple discretionary equilibria. In my simulations, I addressed this possibility by using multiple starting points to search for the worst-performing discretionary equilibrium.

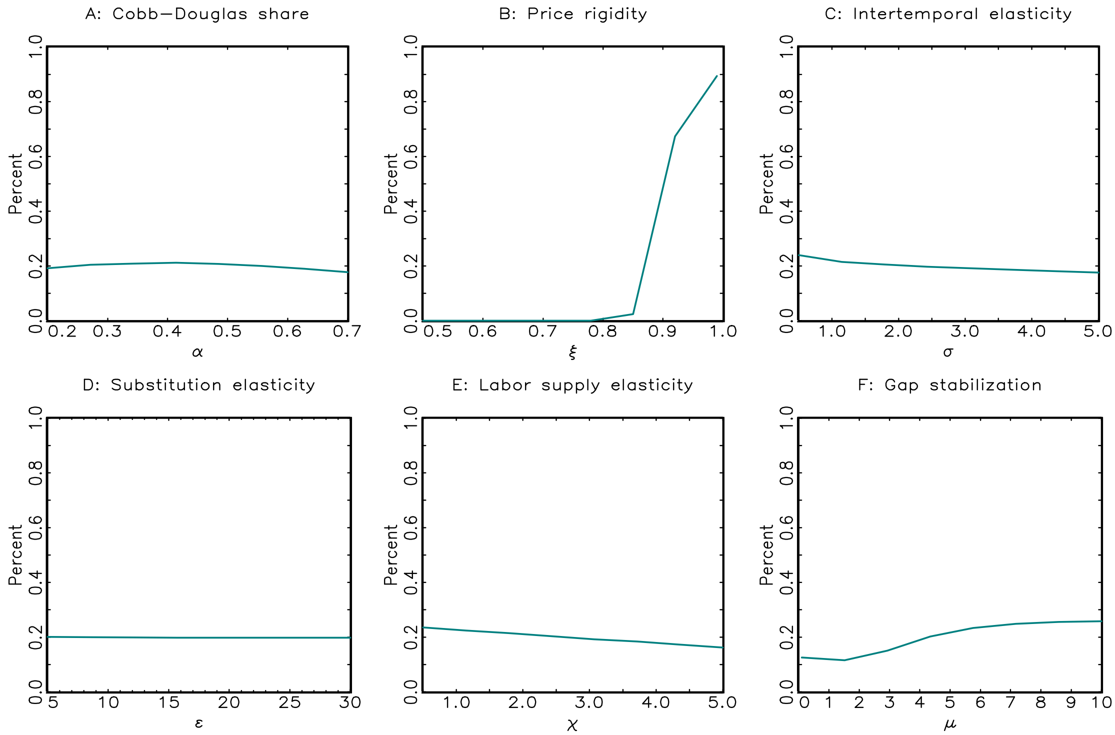

Fig. 3

Figure 3 reveals that the performance of discretion relative to timeless perspective policy-making declines as α and σ increase and increases as ξ, ε, χ, and μ increase. Since higher values of ξ imply a flatter Phillips curve, the results in Figure 3B are consistent with those in Figure 2A. With respect to α, σ, ε, and χ, whether changes in these parameters help or hinder discretion turns primarily on how they alter the trade-off the central bank faces between stabilizing inflation and stabilizing the gap, with factors that worsen the trade-off helping discretion. Thus, the relative performance of discretion improves for higher values of χ and μ and for lower values of σ. With respect to ε, higher values of ε help discretion because they lower the steady-state consumption share of output, weakening the policy channel operating on real marginal costs through consumption and wages. The effects of changes in α are more complicated. Higher values for α help discretion because they lower the steady-state share of consumption in output and because they weaken the policy channel operating on real marginal costs through consumption and wages. However, higher values of α also strengthen the policy channel operating on real marginal costs through the real rental rate, and, since interest rates have a large effect on the rental rate of capital, this effect tends to dominate. Figure 3 further reveals that the parameter space for which discretion dominates timeless perspective policymaking is relatively small, and the reason for this is clear. In this model, only when the Calvo parameter is large, with ξ greater than about 0.90, can discretion be superior. Thus, analogous to the simple model analyzed in Section 4, there appears to be a threshold value for ξ below which discretion cannot be superior.

5.2 Adding firm-specific labor and Kimball aggregation

I now make two modifications to the DSGE model. First, following Woodford (2003, 2005), I introduce firm-specific labor and dispense with the assumption that there is a single aggregate market in which to hire labor. When labor is firm-specific, a firm making its pricing decision will take into account the effect its price has on its real marginal costs, which are no longer independent of the pricing decision or identical across firms. Second, drawing on Eichenbaum and Fisher (2007) and Woodford (2005), to whom interested readers are referred, I introduce the Kimball (1995) aggregator in place of the Dixit and Stiglitz (1977) aggregator in the production of the final good. Although some variables, such as $\widehat{w}_t$ and $\widehat{mc}_t$ require new interpretation, in terms of the aggregate log-linear relationships, these two modifications manifest themselves in the inflation Phillips curve, which is now given by

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \frac{(1-\xi)(1-\beta\xi)}{\xi} \theta_l \theta_k \widehat{mc}_t + \widehat{v}_t, \tag{61}$$

where $\theta_l \equiv \left[1 + \varepsilon \frac{(1-\alpha)\chi}{(1+\alpha\chi)}\right]^{-1}$ and $\theta_k \equiv \left[1 + \frac{\omega}{(\varepsilon-1)}\right]^{-1}$ and where $\omega \in [0, \infty)$ is the Kimball curvature parameter, the price elasticity of ε. When $\omega = 0$, the Kimball (1995) aggregator is equivalent to the Dixit and Stiglitz (1997) aggregator.[15]

Since $\{\theta_l, \theta_k\} \in (0, 1)$, it is clear that the effect of these modifications is to lower the coefficient on real marginal costs in the Phillips curve, thereby reducing its slope for any given value of ξ. Of course, it is the very fact that mechanisms like Kimball aggregation and firm-specific labor markets flatten the Phillips curve that underlies their rising popularity in the New Keynesian DSGE literature. These mechanisms can rationalize with reasonable estimates of nominal price rigidity the small coefficient on real marginal costs found in estimated Phillips curves.

[15] Although I introduce both firm-specific labor and Kimball aggregation, since both appear to be plausible, I recognize, indeed exploit, the fact that they have equivalent effects on the slope of the Phillips curve (Eichenbaum and Fisher, 2007; Levin, Lopez-Salido, and Yun, 2007).

5.2.1 Results

To analyze this modified DSGE model I again condition on the depreciation rate and the parameters in the shock processes and evaluate loss on a grid for the other parameters. Figure 4 displays the percent of the parameter space for which discretion is superior to timeless perspective policymaking, for given values of α, ξ, σ, ε, χ, ω, and μ.

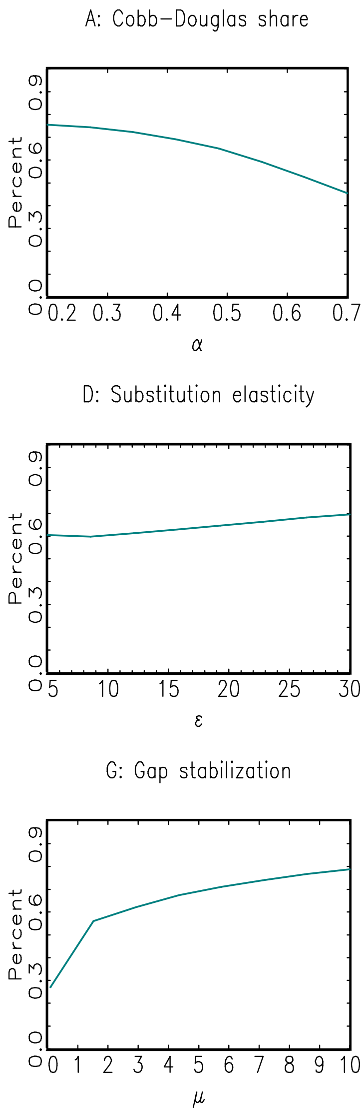

Fig. 4

Qualitatively, the results in Figure 4 are in keeping with those in Figure 3. Discretion is superior to timeless perspective policymaking for a larger share of the parameter space when α and σ are small or when ξ, χ, and μ are large. In addition, since higher values for ω, the

Kimball curvature parameter, serve to flatten the Phillips curve, discretion is more likely to dominate timeless perspective policymaking when ω is large. Unlike in Figure 3, however, the frequencies with which discretion dominates the timeless perspective are much larger at all points of the parameter space and the ability for discretion to be better than the timeless perspective occurs at much lower values of ξ, i.e., at much smaller levels of nominal price rigidity.

6 Conclusion

In this paper I have shown that discretion can be superior to timeless perspective policymaking and I have identified factors that contribute to this. Broadly speaking, discretion is more likely to be superior to timeless perspective policymaking when the Phillips curve is relatively flat, i.e., in models where nominal price rigidity is important or where factors such as Kimball aggregation or firm-specific labor/capital are present. These findings are important because these very factors are becoming widely employed in the New Keynesian DSGE models used to analyze monetary policy. Although a timeless perspective approach to policymaking may have its attractions, one should not simply assume that timeless perspective policymaking is superior to discretion.

One difficulty with comparing discretion to timeless perspective policymaking has been finding a suitable metric for assessing performance. This difficulty arises because the timeless perspective introduces auxiliary state variables that are absent from the time-consistent equilibrium. Rather than simply assigning initial values to these auxiliary state variables or using unconditional loss to evaluate policies, I propose evaluating policies using a measure of conditional loss that integrates out the auxiliary state variables conditional upon the known predetermined state variables. The measure of performance that I develop is easy to compute, provides a consistent treatment of the initial conditions in the discretion and the timeless perspective equilibria, and is consistent with the conditioning assumptions that describe the associated optimization problems.

The goal of this paper has not been to criticize the timeless perspective as an approach to policy design. Rather, because timeless perspective policies are suboptimal, the goal has been to highlight that timeless perspective policies are not necessarily superior to other suboptimal policies, of which discretion is a leading example. It is certainly unclear why a central bank should commit to implementing a timeless perspective policy when that policy is inferior to a time consistent alternative. The results in this paper suggest that studies analyzing timeless perspective policies might usefully consider their performance alongside that of discretion, evaluating the policies using the measure of conditional loss developed in this paper.

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