The gains from delegation revisited: price-level targeting, speed-limit and interest rate smoothing policies *

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Abstract

This paper revisits the argument that discretionary equilibria and associated welfare outcomes close to those under commitment can be obtained by delegating penalty functions that involve additional terms in the price level (Svensson (1999), Vestin (2006)), the change in the output gap (Walsh (2003)), or the change in the interest rate (Woodford (2003b)). These analyses were based on the assumption that there were unique equilibria under discretion. Blake and Kirsanova (2008a,b), showed that this assumption did not generalise to monetary policy models with predetermined variables like capital. In this paper, we illustrate that the welfare benefits of these delegation schemes are much less clear cut since they generally fail to eliminate multiple equilibria.

Key Words: Time Consistency, Discretion, Multiple Equilibria, Policy Delegation

JEL References: E31, E52, E58, E61, C61

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1 Introduction

This paper revisits an old question of how to cure what Svensson (1997) called the 'stabilization bias' in monetary policy that arises when monetary policymakers cannot credibly commit to their plans. The cures involved assigning the central bank an objective that was different from the one that would best characterize the welfare of the representative agent: Woodford (2003b) suggested including a term in the change in the interest rate; Walsh (2003) suggested including a term in the change in the output gap; Svensson (1999) and Vestin (2006) argued for the central bank to be mandated to follow a price level rather than an inflation target. These cures had the feature that when this modified objective was pursued in a discretionary fashion, the outcomes obtained were seen to mimic those that would result from being able to commit to pursuing the original, unmodified, social objectives.

We revisit this question because it turns out that one of the foundations of the previous cures is less general than was probably known at the time they were devised. Previous analyses were carried out in models without capital. In those models it was guaranteed that there were unique equilibria under discretion. This uniqueness meant that to evaluate the gains from a cure to the stabilization bias, one could compare one equilibrium outcome under the original unmodified regime to another under the cure, and the gains were equal to the welfare difference between the two. It turns out, as Blake and Kirsanova (2008a) describe, that in richer sticky price models with capital or other state variables, there is no guarantee that equilibria under discretion are unique, and in fact one typically finds more than one, and in some cases many. The contribution that our paper makes is to explain that this lack of uniqueness makes it much harder to draw firm conclusions about the benefits of the proposed cures to the stabilization bias. For example, if we take one of the cures, Walsh's 'speed-limit' policy, – though our point here applies to all those offered so far – in which the central bank is given an objective that involves the change in the output gap, rather than the level, we have two equilibria under the speed limit policy, and two equilibria under the benevolent policy. We now have no unique measure for the returns from moving from one policy to another. We cannot therefore state with the degree of confidence we might have previously that these delegation policies offer a potential cure for the stabilization bias.

Before embarking on a detailed account of our work, we recap in some more detail on the strands of thought which gave rise to our paper, and then we summarize our key findings.

The commitment problem for monetary policymakers was introduced to monetary economics by Kydland and Prescott (1977). In their analysis, they considered a government that had an incentive to try to inflate the economy such that output was higher than its natural rate. From this paper we learned how a government would have an incentive to announce a sound counter inflationary policy so that low inflation expectations were embedded in price and wage contracts. However, once these contracts were signed, and in the absence of any mechanism to force it to commit to its original plan, the government would be unable to resist the temptation to renege on this low inflation announcement, because higher inflation would generate lower real wages, and that would boost employment and output. The rational private sector, however, would anticipate this, and build higher inflation expectations into nominal wage contracts. That in turn would force the government to make good on the high inflation policy in the first place to avoid a recession. This analysis generated a vast literature proposing various solutions to the problem, including Rogoff (1985) 'conservative central banker', whose inherent dislike for inflation would anchor expectations, or Walsh's 'optimal contract', or, the solution one might infer from Backus and Driffill (1985) and Barro and Gordon (1983) that a government might be able to commit if by reneging it risked losing a reputation for good behavior.

The subsequent spread of the practice of handing monetary policy to an independent central bank, perhaps mandating that central bank to pursue a quantified objective for inflation, might seem to have solved the commitment problem in monetary policymaking. However, Svensson (1997) pointed out that in dynamic sticky price models of monetary policy there still existed a commitment problem even if the central bank's objectives vis a vis output and inflation were aligned with those that were socially optimal (unlike in the original Kydland and Prescott (1977) example where governments sought to inflate output above its natural rate). Svensson termed this problem the 'stabilization bias'. Under discretion, the central bank announces a plan to stabilize a (cost push) shock that threatens to push inflation away from target. In order to minimize the costs of stabilizing this shock, to minimize the cumulative output gap required to bring it under control, the central bank announces a plan that involves interest rates staying above steady state in the future, in order that by doing so inflation expectations today will not rise, and so inflation in turn, which depends on expectations, will rise by less than otherwise. However, once this plan is factored into expectations, and once the shock to inflation has passed, there will be no incentive to follow through with the extended period of higher interest rates. It will appear more beneficial to put interest rates back to steady state rather than pay the output gap costs of keeping them high as planned. Of course the private sector anticipate this and the initial promise to keep interest rates high for an extended period is not believed; inflation expectations are therefore higher than otherwise, and interest rates need to rise by more, and the central bank pays a greater output gap cost of stabilizing the shock to inflation. As a result, in dynamic models the welfare under discretion is always lower than the welfare under commitment, see also Currie and Levine (1993).

This analysis was conducted in a closed economy sticky price model in which, as Woodford (2003a) had observed, it was true that an objective function involving the square of the output gap and the square of deviations of inflation from target was a close approximation to the welfare of the representative agent. Svensson (1999) and then Vestin (2006) pointed out that if the central bank were mandated to pursue an objective where the term in the inflation rate was replaced by a term in the price level, and the central bank were further left to pursue monetary policy under discretion as before, the result would be very close to, if not the same as the outcomes that would obtain if the central bank had had a mechanism by which to commit in the first place. A price level target forced the central bank to try to reverse the effects of past shocks to inflation on the price level and thereby generated a link between today's interest rate and historical outcomes for the economy. It was just such a link that was missing from the perspective of the central bank pursuing the original unmodified objective under discretion; it wanted to announce a plan for interest rates to be higher tomorrow, because of a shock today. Giving a central bank a term in the price level target generates this same kind of inertia. Two further solutions to the stabilization bias generate an analogous inertia into the discretionary interest rate policy of the central bank. Woodford (2003b) proposed adding a term in the change in the interest rate into the target pursued by the central bank. The inertia here stems from the central bank now having an incentive to set rates today as a function of rates yesterday. Walsh (2003) proposed replacing the term in the original central bank objective in the output gap with one in the change in the output gap. This also introduced a dependence of interest rates today on interest rates yesterday in equilibrium.

The claims for these cures, as we have stated already, were based on the observation that in these sticky price models with no capital (or other stock variables) there were unique equilibria under discretion. Those analyses abstracted from modelling capital, following a practice that is still widespread, on the grounds that nothing by way of generality was lost by so doing. However, Blake and Kirsanova (2008a) and Blake and Kirsanova (2008b) have, in two previous papers, demonstrated that in monetary policy models with endogenous state variables (like capital or debt) we should in general expect more than one equilibrium. Our contribution here is to demonstrate that the presence of multiple equilibria survives the introduction of the delegation schemes so far proposed to combat the stabilization bias. In which case we are unable to assign unique values to the welfare gains or costs associated with introducing them. We show that this is the case using two different ways of modelling capital: assuming the existence of a rental market for capital, and assuming that firms accumulate firm-specific capital. We also show that for many regimes multiplicity exists in quite wide regions of the parameter space of the model. We typically observe two equilibria under discretion under the original unmodified central bank objective, and under the various delegation schemes. We label the equilibria under the original objective 'wet' and 'dry' to describe the lack or presence of a strong counter-inflationary response of interest rates to shocks. We shall explain that using arguments of continuity there is a connection between the 'wet' and the 'dry' equilibria under the two regimes. If we make the modification to the central bank objective under delegation arbitrarily small (e.g. the weight on the term in the change in the interest rate arbitrarily small), the 'wet' equilibria under delegation often, though not always, converge on the wet equilibria without delegation. We might therefore be tempted to conclude that we can simply compare the two 'wets' or the two 'drys' to quantify the gains from delegation. But this does not necessarily follow: there is nothing to guarantee that the introduction of a delegation scheme would move an economy stuck in a wet equilibrium to a wet one under delegation. Such a scheme could shift the economy to a dry equilibrium. Or shift a dry economy into a wet equilibrium under delegation. To cut a long story short and restate our central point, the existence of multiple equilibria under discretion both with and without delegation means that we cannot come up with unique estimates of the welfare gains to these delegation schemes.

The paper is organized as follows. The next section outlines the model. We use the popular model by Sveen and Weinke (2005) and Woodford (2005), so we only explain the set of linearized equations. Section 4 demonstrates that the model has multiplicity of equilibria under the policy of benevolent discretionary policymakers. Section 5 demonstrates how the results on multiplicity change under the three delegation schemes. We discuss that these schemes not only can eliminate either of the two equilibria, but also they can create additional discretionary equilibria with undesired properties. We discuss dynamic properties and consequent advantages and disadvantages of using either of these delegation schemes. Some remarks on robustness of our results are given in Section 6. Section 7 concludes.

2 Model

We use a New Keynesian model with complete markets, as presented in Sveen and Weinke (2005) and Woodford (2005). This model is with monopolistic competition and sticky prices in goods markets. Capital accumulation is assumed to take place at the firm level and the additional capital resulting from an investment decision becomes productive with a one period delay. We assume a convex capital adjustment cost at the firm level. Since the details of the model are discussed in Sveen and Weinke (2005) we proceed directly to the equations that result form linearizing the equilibrium conditions around the zero inflation steady state.

2.1 Linearized Equilibrium Conditions

We linearize around a zero inflation steady state. All variables are expressed in terms of log deviations from their steady state values.

From the standard household's optimization problem we obtain, respectively, an Euler equation and a labour supply equation

$$c_t = \mathcal{E}_t c_{t+1} - \frac{1}{\sigma}(i_t - \mathcal{E}_t \pi_{t+1} - \rho) \tag{1}$$

$$w_t = \phi n_t + \sigma c_t \tag{2}$$

where parameter $\rho = -\log \beta$ is the time discount rate, σ is the household's relative risk aversion or, equivalently, the inverse of the intertemporal elasticity of substitution and ϕ is the inverse of the Frisch labour supply elasticity. We denote the nominal interest rate at time t as $i_t = \log R_t$, and $\pi_t = \log\left(\frac{P_t}{P_{t-1}}\right)$ is inflation. We also denote aggregate consumption as c_t, n_t is the aggregate labour and w_t is the average real wage. $\mathcal{E}_t$ is the expectational operator conditional on information available through time t.

The low of motion of capital is obtained from averaging and aggregating optimal investment decisions on the part of firms. This implies

$$\Delta k_{t+1} = \beta \mathcal{E}_t \Delta k_{t+2} + \frac{1}{\varepsilon_\psi}\left(\left(1 - \beta\left(1-\delta\right)\right)\mathcal{E}_t ms_{t+1} - \left(i_t - \mathcal{E}_t \pi_{t+1} - \rho\right)\right) \tag{3}$$

where aggregate capital is denoted by k_t and $ms_t = w_t - k_t + n_t$ measures the average real marginal return to capital. Parameter β is the subjective discount factor, parameter δ is the rate of depreciation and parameter ε_ψ measures the capital adjustment cost at the firm level. The average real marginal return to capital is measured in terms of marginal savings in labour costs since firms are demand-constrained in this model.

Up to the first order, aggregate production function can be linearized as:

$$y_t = \alpha k_t + (1-\alpha) n_t \tag{4}$$

where parameter α denotes the capital share.

The inflation equation takes the standard form

$$\pi_t = \beta \pi_{t+1} + \kappa mc_t + \eta_t$$

where $mc_t = w_t - y_t + n_t$ denotes the average real marginal cost. If capital can be rented then parameter $\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta}$ where parameter θ gives the probability that a firm does not re-optimize its price in an given period. If the rental market does not exist, then $\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta}\frac{(1-\alpha)}{(1-\alpha+\varepsilon\alpha)}\frac{1}{\xi}$ where parameter ε denotes the elasticity of substitution between the differentiated goods, while parameter ξ is a function of the model's structural parameters which is computed numerically using the method developed in Woodford (2005). Finally, additionally to the setting in Sveen and Weinke (2005) we introduce a cost-push shock to the model. It is common to interpret this shock as a temporary discretionary change in firms' desired margins.

The goods market clearing condition can be written as:

$$y_t = \zeta c_t + \frac{1-\zeta}{\delta}\left(k_{t+1} - (1-\delta)\, k_t\right) \tag{5}$$

where ζ is the steady state consumption to output ratio, $\zeta = 1 - \frac{\delta\alpha(\varepsilon-1)}{\varepsilon(\rho+\delta)}$.

2.2 Monetary policy

The central bank instrument is the short-term interest rate i_t. We assume throughout that it acts under discretion. Throughout we will assume that the social welfare function is well captured by the following discounted quadratic loss function:

$$\frac{1}{2}\sum_{s=t}^{\infty}\beta^{s-t}\left(\pi_s^2 + \omega y_s^2\right), \tag{6}$$

where we use $\omega = \kappa/\varepsilon$.[1] This welfare function has been shown by Woodford (2003a), Ch. 6, to approximate the aggregate of individual utility functions in a model without capital, but otherwise identical to the one we work with. In our model, this approximation will not hold and so our policy objective function is to some degree *ad hoc*. However, our central claim, that we can expect widespread multiplicity under discretion is based on the general argument set out in Blake and Kirsanova (2008a), and we maintain that we can make this simplification without loss of generality. We discuss how our results are affected by the chosen form of objectives in Section 6. In what follows we simply refer to this objective as to *the* social objective. When there are no amendments to the social welfare criterion in the process of delegating monetary policy to the monetary authority we will refer to this regime as 'inflation targeting': note that we do this for convenience and not to take a stand on the optimality or the precise nature of inflation targeting regimes as practiced in real life.

Our paper is a reassessment of the gains associated with various delegation schemes for monetary policy, so we need to make these regimes concrete. They are associated with particular penalty functions assigned to the central bank that differ from the benchmark welfare function. These schemes are all nested within the following penalty function:

$$\frac{1}{2}\sum_{s=t}^{\infty}\beta^{s-t}\left(\omega_\pi\pi_s^2 + \omega_y y_s^2 + \omega_p p_s^2 + \omega_{sl}\Delta y_s^2 + \omega_i(\Delta i_s)^2\right), \tag{7}$$

[1] This relative weight is given in Woodford (2003a), Ch. 6 as a microfounded weight for the most simple model.

Using this criterion function, we can implement four regimes by the following choice of the weights ω:

Delegation Scheme	Constraints	
Strict Price level targeting	$\omega_\pi = \omega_{sl} = \omega_i = 0$	$\omega_p > 0, \omega_y = \kappa/\varepsilon$
Hybrid price level and inflation targeting	$\omega_{sl} = \omega_i = 0$	$\omega_p > 0, \omega_\pi = 1, \omega_y = \kappa/\varepsilon$
Interest rate smoothing	$\omega_{sl} = \omega_p = 0$	$\omega_y = \kappa/\varepsilon, \omega_\pi = 1, \omega_i > 0$
Speed limit policy	$\omega_i = \omega_p = \omega_y = 0$	$\omega_\pi = 1, \omega_{sl} > 0$

To put this into words, briefly: price level targeting, the focus of Svensson (1999) and Vestin (2006), is implemented by *replacing* the term in the inflation rate in the social welfare function by a term in the price level. Hybrid price level and inflation targeting, whose optimality under discretion was studied by Roisland (2008), is implemented by *adding* to the social welfare function a term in the price level. Interest rate smoothing is implemented by *adding* to the social welfare function a term in the change in the interest rate. And finally, the speed limit policy, proposed by Walsh (2003), is achieved by *replacing* the term in the level of the output gap in the social welfare function with a term in the change in the output gap.

2.3 Calibration

We use the same calibration as in Sveen and Weinke (2005). We set the capital share $\alpha = 0.36$. Our choice for the risk aversion parameter σ is 2, and a unit elasticity of labour supply is assumed: $\phi = 1$. The elasticity of substitution between goods, ε, is set to 11. The rate of capital depreciation, δ, is assumed to be 0.025 and we set $\varepsilon_\psi = 3$. Finally, our value for the Calvo price stickiness parameter, θ, is 0.75. This implies a mean duration for a particular price of 3 quarters. We use the model with a rental market for generic capital as the base line case, but we explore the robustness of our conclusions to incorporating instead a model of firm-specific capital. What we say generally goes through to the firm-specific capital model.

3 Framework of Analysis: Discretionary Policy in LQ RE Models

We assume a non-singular linear deterministic rational expectations model of the type described by Blanchard and Kahn (1980), augmented by a vector of control instruments. Specifically, the evolution of the economy is explained by the following system:[2]

$$\begin{bmatrix} y_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} y_t \\ x_t \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} [u_t] , \tag{8}$$

where y_t is an n_1-vector of predetermined variables with initial conditions y_0 given, x_t is n_2-vector of non-predetermined (or jump) variables, and u_t is a $k-$vector of policy instruments of the policymaker. For notational convenience we define the n-vector $z_t = (y_t', x_t')'$ where $n = n_1 + n_2$.

[2] We can work with the deterministic component only. This is without loss of generality because of certainty equivalence (see e.g. Anderson et al. (1996)).

At time t the policymaker has the following optimization problem ($\mathcal{E}_t$ denotes the expectations operator, conditional on information available at time t):

$$\min_{u_t} \mathcal{E}_t W_t \tag{9}$$

with the loss function

$$W_t = \frac{1}{2}\sum_{s=t}^{\infty} \beta^{s-t} g_s' \mathcal{Q} g_s = \frac{1}{2}\sum_{s=t}^{\infty} \beta^{s-t} \left(z_s' Q z_s + 2 z_s' P u_s + u_s' R u_s \right), \tag{10}$$

subject to system (8). In addition, any solution to this optimization problem should satisfy the time-consistency constraint: for any $s > t$ the policymaker will choose

$$u_s = \mathcal{E}_t u_s. \tag{11}$$

The elements of the vector g_s are the goal variables of the policymaker, $g_s = \mathcal{C}(z_s', u_s')'$. Matrix $\mathcal{Q}$ is assumed to be symmetric and positive semi-definite. In our formulation the quadratic loss function includes instrument costs, but no assumptions about the invertibility of R need be made. We are looking for solutions that ensure that the loss is finite, i.e. $W_t < \infty$.

The sequence of actions within a period is as follows. In the first stage of every period t the policymaker chooses the instrument u_t, knowing the state y_t and taking the process by which private agents behave as given. In the second stage the private sector adjusts its choice variable x_t. The optimal x_t, u_t and given y_t result in the new level of y_{t+1} by the beginning of the next period $t+1$.

It can be proved the solution in any time t gives a value function which is quadratic in the state variables,

$$W_t = \frac{1}{2} y_t' S y_t$$

and a pair of linear rules

$$u_t = -F y_t, \tag{12}$$

$$x_t = -J y_t - K u_t = -\left(J - KF\right) y_t = -N y_t \tag{13}$$

where $K = -\partial x_t / \partial u_t$: in a leadership equilibrium the follower treats the leader's policy instrument parametrically. Matrix F describes the policy reaction of the policymaker, i.e. $i_t = -F k_t$ in our benchmark case. Matrix N defines the reduced form reaction function of the private sector, i.e. $(k_{t+1}, \pi_t, c_t)' = -N k_t$.

Note that introducing additional targets into the objective function, like interest rate smoothing $\left(i_s - i_{s-1}\right)^2$, would introduce additional predetermined variables into the system, i_{s-1} in this case, so in system (8) vector y_t would become $(k_t, i_{t-1})'$.

Definition 1 *The system of first order conditions to optimization problem (8)-(11) for matrices* $\{N, S, F\}$ *can be written in the following form:*

$$
\begin{aligned}
S &= Q^* + \beta A^{*\prime} S A^* - \left(P^{*\prime} + \beta B^{*\prime} S A^*\right)' F, && (14)\\
F &= \left(R^* + \beta' B^* S B^*\right)^{-1} \left(P^{*\prime} + \beta B^{*\prime} S A^*\right), && (15)\\
N &= (A_{22} + N A_{12})^{-1}((A_{21} - B_2 F) + N(A_{11} - B_1 F)), && (16)\\
Q^* &= Q_{11} - Q_{12} J - J' Q_{21} + J' Q_{22} J, \ P^* = J' Q_{22} K - Q_{12} K + P_1 - J' P_2, && (17)\\
R^* &= K' Q_{22} K + R - K' P_2 - P_2' K, \ A^* = A_{11} - A_{12} J, \ B^* = B_1 - A_{12} K, && (18)\\
J &= (A_{22} + N A_{12})^{-1}(A_{21} + N A_{11}), \ K = (A_{22} + N A_{12})^{-1}(B_2 + N B_1). && (19)
\end{aligned}
$$

The proof can be found in e.g. Blake and Kirsanova (2008a). There is a one-to-one mapping between equilibrium trajectories and $\{y_s, x_s, u_s\}_{s=t}^{\infty}$ and the triplet $\mathcal{T} = \{N, S, F\}$, so it is convenient to continue with definition of policy equilibrium in terms of $\mathcal{T}$, not trajectories. Hence, in what follows it is convenient to use the following definition.

Definition 2 *A triplet* $\mathcal{T} = \{N, S, F\}$ *is a discretionary equilibrium if it satisfies the system of FOCs (14)-(19).*

It is apparent from system (14)-(19) that matrices N, S and F satisfy three quadratic algebraic matrix equations (Riccati equations) (14)-(16), where the coefficients in these equations are also non-linear functions of model matrices and matrix N. This makes the whole system (14)-(19) very non-linear and it is not surprising that it may have many solution triplets $\mathcal{T}^J = \{N^J, S^J, F^J\}$, $J = 1, ..M$ where M is the total number of solutions. We investigate properties of these discretionary solutions in Blake and Kirsanova (2008a).

Oudiz and Sachs (1985) and Backus and Driffill (1986) iterative procedures search for solutions to the system of first order conditions (14)-(19) but can only find those that are asymptotically stable fixed points of corresponding recursions. Blake and Kirsanova (2008a) call them R-stable. These equilibria can be deduced by rational agents if they use backward induction. In what follows we shall only consider R-stable equilibria.

Notation 3 *Suppose system matrices A, B and* $\mathcal{Q}$ *depend on parameter* ζ. *Then solution triplet will also be a function of this parameter and we denote it as* $\mathcal{T}(\zeta) = \{N(\zeta), S(\zeta), F(\zeta)\}$.

4 Inflation Targeting: The Two Policy Regimes

This case was the centerpiece of Blake and Kirsanova (2008b), and we briefly repeat it here as a necessary benchmark against which to judge our delegation regimes. The baseline calibration produces two stable discretionary equilibria: following a shock, the economy can follow one of the two transition paths, both of which satisfy the conditions for optimality and time-consistency. (Formally, we say that we have two different triplets $T^D = \{N^D, S^D, F^D\}$ and $T^W = \{N^W, S^W, F^W\}$, see Definition 1.)

Figure 1 shows these two transition paths: two impulse responses of endogenous variables to a cost push shock; the solid and the dashed lines denote responses in the two equilibria. In both equilibria the interest rate rises in response to a positive cost-push shock, but the amount by which interest rate rises is substantially greater in one than the other. As a consequence, in our first equilibrium we see a larger fall in the output gap and a smaller rise in inflation than in the second equilibrium. We call the first equilibrium 'seemingly dry' and the second 'seemingly wet', a reference to the greater determination of the central bank in the first equilibrium to raise interest rates to combat the cost-push shock. Figure 1 suggests that the 'seemingly dry' policymaker implements 'nearly optimal' solution as its actions are more similar to the ones under commitment than the actions of the 'seemingly wet' policymaker. For brevity we shall refer to these equilibria as to just 'dry' and 'wet' correspondingly, but bear in mind that these two equilibria are generated by the same policy objective.

Such difference in reactions can be explained by the multiplicity of policy-induced private sector equilibria, see Blake and Kirsanova (2008a). Essentially, for every policy there is more than one locally optimal response of the private sector, which is of course, conditional on the forecast of future policy. In order to understand how they arise we can look at the role which capital plays in the determination of the laws of motion for marginal costs. After some algebra, the marginal cost can be written out as:

$$mc_t = \left(\frac{(\alpha + \phi)}{(1-\alpha)}\zeta + \sigma \right) c_t + \frac{(\alpha+\phi)}{(1-\alpha)} \frac{1-\zeta}{\delta} \left(k_{t+1} - (1-\delta) k_t \right) - \frac{(\phi+1)\alpha}{(1-\alpha)} k_t$$

On the one hand, investment demand increases marginal cost and marginal return to capital, on the other hand the resulting additional capital tends to decrease them when it becomes productive. When a positive cost-push shock hits the system and interest rates rise the resulting investment slump reduces the capital stock. This reduction in capital increases marginal costs and is inflationary, so the private sector may expect that the economy will be brought back to the equilibrium *slowly.* The policymaker will find it optimal to validate the beliefs of the private sector and to keep interest rate high for longer to reduce inflationary pressure in all consequent periods while also keeping capital stock below the steady state. This is what the 'reputational' and the 'seemingly dry' policymakers do, as is shown by a negative feedback coefficient on capital in their policy reactions. Alternatively, the private sector may expect that the economy will be brought back to the steady state *quickly.* The policymaker will find it optimal to validate these beliefs and will choose not to allow the large fall in capital stock in the first period, and then to bring capital back to the steady state within several periods only. This is what the 'seemingly wet' policymaker does.

More generally, the presence of capital accumulation in the model necessary implies sluggish adjustment of the economy back to the steady state. In contrast, if we did not include the capital accumulation process into the model, all adjustment would happen within a single time period. No different beliefs about the different speed of adjustment would be possible and no multiple equilibria would arise.[3] If there are multiple equilibria then coordination failure happens: the agents are not able to coordinate on the best equilibrium and any random event, a sunspot, can decide which equilibrium will realize.

In order to see how relevant the two regimes, Blake and Kirsanova (2008b) investigate how regions of multiplicity depend on the calibration of parameters that govern the capital accumulation process: the share of capital α, the rate of capital depreciation δ, and the adjustment cost

[3] This is a rather general fact, see Proposition 1 in Blake and Kirsanova (2008a), which demonstrates that a model without endogenous predetermined state variables can only have one discretionary equilibrium.

of capital, ε_{ψ}.We repeat their results in Figure 3. The the first row of subplots in the top panel presents the results for the model with rental market for capital, and the first row of subplots in the bottom panel presents the results for the model with firm-specific capital. We also plot outcomes for either the model with rental market for capital, or for the model with firm-specific capital in upper and lower panels correspondingly. (To explain what happens in Figure 3: we vary two out of the three of the following parameters: δ, the capital share α and the capital adjustment costs ε_{ψ}. The third parameter in each case is held fixed at the baseline value.) It is apparent that the base line case of the model with rental market for capital produces two equilibria for the wide range of realistic calibrations of the model, while in the case of firm-specific capital there is a wide range of plausible parameters where only bad, 'wet' equilibrium survives.

Out of these results arises the central question of this paper: how do delegation schemes – like speed-limit policies, interest rate penalties, or price level targeting – affect the likelihood of obtaining multiple equilibria? If multiplicity survives – and, as we have already stated, it does – how are we to quantify the welfare implications of choosing one of these delegation schemes? To which of potentially multiple equilibria under inflation targeting under discretion do we compare to which other equilibria under our delegation scheme?

5 Multiple equilibria under monetary policy delegation

In this section, we study the equilibria that arise under the three delegation schemes. We begin with interest rate smoothing, discussed by Woodford (2003b). We shall use the case of benevolent policymaker as the benchmark case.

5.1 Interest Rate Smoothing

We implement interest rate smoothing by setting an additional penalty on the change in interest rate, Δi_s, so our flow objective becomes $\pi_s^2 + \omega_y y_s^2 + \omega_i (\Delta i_s)^2$.

The first thing to note is that for a wide range of parameter values, we still get two equilibria under interest rate smoothing. These equilibria we label wet and dry, since they are related to the corresponding equilibria under inflation targeting in the following sense. If the 'dry' equilibrium under the interest rate smoothing ω_i is described by a triplet $T_D(\omega_i)$ and the 'wet' equilibrium is $T_W(\omega_i)$, see Definition 1, then for any $J \in \{D, W\} : \lim_{\omega_i \to 0} T_J(\omega_i) = T_J(0) = T^J$. In other words, if we were to compute a continuum of 'dry' equilibria that result from a continuum of weights ω_i that represent different delegation policies, then as we moved along the continuum towards $\omega_i = 0$ we would eventually observe the 'dry' equilibrium under the benevolent policy of inflation targeting. Similarly for the 'wet' equilibria.

Also, although we introduced a new endogenous predetermined state variable into the system, past interest rate i_{t-1}, we did not find any other policy equilibria, except the two we have just described.

Impulse responses of the economy to a positive cost-push shock are shown on Figure 2. The smoothing policy works as follows. The biggest social loss in the welfare objective is the cost of inflation variability, so it is not surprising that the interest rate smoothing policy should try to replicate *the path of inflation* under commitment (that is plotted as the 'reputational' solution, where the authorities are assumed to maximize the social welfare function (6)). The distinctive feature of the reputational solution is that inflation overshoots the steady state level before converging back to it. The social gain under commitment arises because the overlapping-contracts price setters perceive negative future inflation and price their goods less in preceding periods. Similarly, optimal interest rate smoothing policy under discretion does generate *some* overshooting, but it fails to generate as much of it as the commitment policy does. This is because a penalty on interest rate smoothing requires monetary policymaker to keep interest rate high in periods after the shock, and also requires a smaller increase of interest rate in the first period than under commitment. Such policy leads to lower *future* inflation, but the first-period inflation remains sufficiently high, and the first-period loss still outweighs all future-periods gain in the intertemporal social welfare loss. The overall loss remains noticeably higher than under commitment. In other words, an interest-rate smoothing policy under discretion remains less flexible than a policy under commitment, as it cannot fully manipulate expectations of the private sector to achieve desired impulse responses. At the same time the 'dry' equilibrium delivers responses *closer* to the 'reputational' solution, that is consistent with claims from the original work based on single-equilibrium settings. There is an optimal weight ω_i^* that maximizes the gain of delegation for the 'dry' equilibrium.

Interest rate smoothing policy monotonically improves the 'wet' equilibrium. The 'wet' equilibrium is characterized by a weak response of interest rate to cost-push shock and by smooth behavior of investment and capital. The requirement of keeping interest rate smooth leads to higher future interest rate and lower future inflation that determines the costs. As a result, the loss of the 'wet' equilibrium falls monotonically with ω_i until the 'wet' equilibrium disappears at $\bar{\omega}_i$. However, the 'wet' equilibrium does not produce inflation overshooting.

These impulse responses translate into fairly intuitive outcomes for welfare (defined according to social welfare metric (6)), see Table 1.

The figures in this Table express social losses as a ratio to the figures obtained under the optimal commitment solution with social loss (6). As we can see, the 'dry' equilibrium under interest rate smoothing delivers outcomes close to the optimum (1.106), and closer than both discretionary equilibria that we get in the absence of the delegation of the interest-rate smoothing objective. By contrast, the 'wet' equilibrium under interest rate smoothing delivers losses some way off the optimum.

We might well ask whether, despite the presence of multiple equilibria under discretion in the absence or presence of an interest rate smoothing policy, we could still establish that delegation would improve welfare. Actually we cannot guarantee this. Suppose that if we began in the dry equilibrium under the benevolent policy, and we were to impose a delegated interest rate smoothing policy with weight ω_i^*, we would anticipate moving to the 'corresponding' dry equilibrium under the delegation policy. And that in this case we would record a small welfare gain by cutting losses from 1.126 to 1.106, relative to the optimum of 1. Analogously, we might be tempted to reason that if we began in the wet equilibrium under the benevolent policy, we would shift to the corresponding wet equilibrium under the interest rate smoothing policy, cutting losses from 7.523 to 7.514.

However, this reasoning is hazardous. First, although by arguments of continuity these pairs of equilibria are related, when we announce a new delegation policy it is a *regime change*, so we cannot rule out that we might start from a wet equilibrium and move to a dry under delegation,

	rental capital			firm-specific capital		
Interest Rate Smoothing	$\omega_i = 0$	$\omega_i^* = 0.01$	$\bar{\omega}_i = 99.8$	$\omega_i = 0$	$\omega_i^* = 0.01$	$\bar{\omega}_i = 26.1$
'Seemingly Dry'	1.126	1.106	3.151	n/a	1.2515	1.7961
'Seemingly Wet'	7.523	7.514	6.863	2.3961	2.3880	2.2640
Speed-Limit Policy	$\omega_{sl} = 0$	$\omega_{sl}^* = 0.005$	$\bar{\omega}_{sl} = 6.1$	$\omega_{sl} = 0$	$\omega_{sl}^* = 0.001$	$\bar{\omega}_{sl} = 0.243$
'Seemingly Dry'	1.126	1.020	6.809	n/a	1.045	2.170
'Seemingly Wet'	7.523	7.517	7.331	2.3961	2.392	2.336
Hybrid Price Level Targeting	$\omega_\pi = 1$ $\omega_p = 0$	$\omega_\pi = 1$ $\omega_p^* = 0.5$	$\omega_\pi = 1$ $\underline{\omega}_p = 0.0013$	$\omega_\pi = 1$ $\omega_p = 0$	$\omega_\pi = 1$ $\omega_p^* = 0.5$	$\omega_\pi = 1$ $\underline{\omega}_p = 10^{-14}$
'Seemingly Dry'	1.126	1.052	1.116	n/a	1.0917	n/a
'Seemingly Wet'	7.523	–	–	2.3961	–	–
'Passive'	–	11.95	1053.7	–	3.984	152.7
Strict Price Level Targeting	$\omega_\pi = 1$ $\omega_p = 0$	$\omega_\pi = 0$ $\omega_p^* = 1.7$	$\omega_\pi = 0$ $\underline{\omega}_p = 0.023$	$\omega_\pi = 1$ $\omega_p = 0$	$\omega_\pi = 0$ $\omega_p^* = 1.2$	$\omega_\pi = 0$ $\underline{\omega}_p = 10^{-14}$
'Seemingly Dry'	1.126	1.00007	4.079	n/a	1.0002	n/a
'Seemingly Wet'	7.523	–	–	2.3961	–	–
'Passive'	–	16.789	10.90	–	5.0613	4.424
Reputational	1.00	–	–	1.00	–	–

Table 1: Social loss of Interest Rate Smoothing regime under discretion relative to the loss under commitment with social objectives

or vice versa. And we cannot therefore guarantee that any gains would arise from the delegation. The wet equilibrium under benevolent policy gives lower welfare than the wet equilibrium for the interest rate smoothing policy; similarly for the dry equilibrium. But it does not follow that we can assert that interest rate smoothing improves welfare.

Second, by imposing a penalty ω_i on interest rate smoothing, we can get to the area where one of the two equilibria does not exist any more. For example, we might be in the area of multiplicity and in the 'dry' equilibrium but the delegation of the policy makes this equilibrium non-existent with this choice of ω_i: only the 'wet' equilibrium might survive. Here, a 'jump' from one equilibrium to another and a change in regime seems to be granted (we shall consider this situation in the section on price-level targeting). Differently, we might be in an area of uniqueness and, having delegated the policy, get into the area of multiplicity. Again, a move to a new equilibrium might occur. Figure 3 demonstrates regions for which we can find multiple equilibria if we vary values of parameters that affect capital accumulation. Each panel contains three top subplots that show regions of multiplicity (white) and uniqueness (filled) under *benevolent* discretionary policy. The second row of three subplots in each panel demonstrates these regions for penalty ω_i^*. In each plot we mark our base case with a '$\times$'.

It is apparent that with higher penalty on interest rate smoothing the 'dry' policymaker acts in a more smooth way that is also required if the labour share is sufficiently high. So the area of existence of the 'dry' equilibrium widens and, correspondingly, the area where only 'wet' equilibrium survives becomes smaller. The both equilibria also become closer to each other. The delegation policy also widens the area where only good equilibrium survives. However, this area exists for unrealistic values of parameters for example, for unrealistically high depreciation rate and/or for unrealistically high capital share. The interest rate smoothing policy, on its own, is not able to select the 'dry' equilibrium in the area with realistic parameter values.

5.2 Speed-limit policy

Walsh (2003) suggested that delegating the policy to a central bank that minimizes $(\Delta y_t)^2$ *instead* of y_t^2 can substantially increase welfare. The reason for the welfare improvement is the same as above: we make the discretionary policy to be 'inertial', that would guide the expectations of the private sector in the right direction and so this might bring us closer to the first best commitment solution. Indeed, we can find rather similar result to the interest rate smoothing policy.

We find two equilibria for the base line calibration of the model and for $\omega_y = 0$, $\omega_{sl} > 0$, $T_D(\omega_y = 0, \omega_{sl})$ and $T_W(\omega_y = 0, \omega_{sl})$. By continuity, these equilibria are the 'dry' and 'wet' equilibria discussed above, because for any $J \in \{D, W\}$ and for $0 \leq \xi \leq 1$: $\lim_{\omega_{sl} \to 0, \xi \to 1} T_J(\xi\kappa/\varepsilon, \omega_{sl}) = T_J(\omega_y, 0) = T^J$.

Despite an additional endogenous predetermined state variable in the system, past output gap y_{t-1}, we cannot find any additional equilibria besides these 'dry' and 'wet' equilibria.

For the base line calibration both equilibria survive for a wide range of penalty ω_{sl}. Table 1 demonstrates that the Walsh's results on welfare improvement remain valid in the model with capital accumulation. In terms of welfare, the 'dry' equilibrium performs very well and generates the social loss which is only marginally higher than the one under commitment. The social loss of the 'wet' equilibrium falls with ω_{sl} monotonically until the point $\bar{\omega}_{sl}$ at which this equilibrium disappears. At $\bar{\omega}_{sl}$ both equilibria are very close to each other, that in this case means that the loss of the 'dry' equilibrium has risen very high. Moreover, the 'dry' speed-limit policy equilibrium outperforms the 'dry' interest rate smoothing policy. To understand this result it is instructive to look at impulse responses in Figure 4. As before, it is beneficial to try to repeat the commitment adjustment path for inflation, as this will determine the most of social losses. And therefore, following a cost-push shock, a optimal speed-limit policy keeps output below the steady state long enough to generate desired inflation overshooting *and* reduce initial inflation by making the private sector to believe that the future inflation can be sufficiently negative. The 'wet' policy, again, fails to produce inflation overshooting.

Figure 5 demonstrates that, similar to the interest rate smoothing case, the speed limit policy widens the region where only the best equilibrium survives and shrinks the area where the worst equilibrium survives. The delegating the policy with optimal ω_{sl} affects the area of multiplicity by more than the delegating the policy with optimal ω_i. However, for the realistic values of parameters we still have multiplicity of equilibria, though the region of a unique bad equilibrium is nearly completely eliminated and certainly much further from the base line calibration. Here again, delegating policy to a speed-limit-concerned policymaker is not able to solve the problem of equilibrium selection.

5.3 Price-level targeting schemes

Price-level targeting has been extensively discussed in the literature. The most close setting to our can be found in Svensson (1999), Vestin (2006), Batini and Yates (2003) and Roisland (2008). The intuition behind the superiority of price-level targeting is slightly different from the one behind interest rate smoothing and speed-limit policy proposals. Although we also introduce inertia in the model by introducing past price p_{t-1} into the system and so we also can guide expectations of the private sector in a desired way, the price level targeting also *explicitly* exploits dynamic properties of commitment solution. By introducing the price level target into the policymaker's objectives we force the price level to be stationary, *as under commitment*, so inflation behavior should also be very similar. In a model with forward-looking behavior the short run inflation volatility falls indeed. Following a cost-push shock the price-setters perceive the policymaker would raise interest rate aggressively to achieve *lower than average* inflation so they do not raise inflation high at the first moment. As the cost of inflation variability dominates social loss, the price level targeting reduces the loss. However, we target the *integral* of inflation and this might be a very strong additional constraint on optimal discretionary solution. As we shall see the results of such delegation can be very different from 'just inertial' regimes, that impose much weaker dynamic constraints. In particular, the results on multiplicity of equilibria *are* affected: we shall see that one of the equilibria can be ruled out, but one other, very different, equilibrium arises.

In what follows we modify our flow objective to $\omega_\pi \pi_s^2 + \omega_y y_s^2 + \omega_p p_s^2$. If $\omega_\pi = 1$ and $\omega_y = \kappa/\varepsilon$ are social weights in loss (6) then any $\omega_p > 0$ gives us the 'hybrid price level targeting' scheme. If we tend ω_π to zero, but keep $\omega_p > 0$ we end up with the 'strict price level targeting' scheme.[4] We start with hybrid price level targeting scheme, as it is easier to contrast with the benchmark

[4]This set-up is slightly different from the one in Batini and Yates (2003) and Roisland (2008), who modify inflation term π_t^2 into $(p_t - \phi p_{t-1})^2$. Relative to their approach, we ignore cross-terms $\pi_t p_t$, but this makes easier to convey our results.

case, and then move to the case of strict price level targeting.

5.3.1 Hybrid Price Level Targeting

It is convenient to start by assuming some value of $\omega_p > 0$. For $\omega_p = 0.5$, for example, we can find two solutions, see optimal responses to a unit cost-push shock in Figure 6, the first column of subplots. One solution is the familiar 'seemingly dry' solution discussed above, in the sense that if ω_p tends to zero, then this solution tends to the 'dry' solution: $\lim_{\omega_p \to 0} T_D(\omega_p) = T_D(0) \equiv T^D$. In a response to a cost-push shock interest rate rises and remains positive for several consequent periods, inflation overshoots the steady state level and then converges back to the steady state. The price level is stationary. With $\omega_p > 0$ this solution becomes very close to the reputational solution, as it is also apparent from Table 1, and the minimum of losses is achieved when $\omega_p \simeq 0.5$.[5]

The second solution is far from familiar: here, in response to a positive cost-push shock interest rate *falls*. We can call it 'passive' solution and denote it $T_P(\omega_p)$. It is not the 'wet' solution i.e. $\lim_{\omega_p \to 0} T_P(\omega_p) \neq T^W$. In other words, if we set the weight on the price level target ω_p to values that tend to zero, then the 'passive' solution does not converge on the 'wet' solution under the benevolent policy of inflation targeting.

In order to understand this result recall again that the stationarity of the price level requires inflation overshooting. If inflation stays positive along the whole convergence paths then its integral, the price level, is a strictly positive number so the price level does not return to its steady state, once disturbed.

It is possible to achieve inflation overshooting in two ways. A reputational policymaker raises interest rate and *keeps it high* for longer to ensure negative marginal cost, i.e. below its steady state level. Negative marginal cost means inflation should *rise* while converging to the steady state zero level. Following a cost push shock and an interest rate rise inflation falls in the first period, overshoots the zero level and then converges to the steady state from below, rising. The 'seemingly dry' policymaker tries to repeat this policy, but under a time-consistency constraint. Similar to the commitment case, marginal cost is kept below zero for most of the periods, inflation overshoots the zero steady state level and price level is stationary.

However, the policymaker can keep marginal cost below zero by keeping sufficiently high capital stock *for some time*. This is achieved by lowering interest rate sharply in the response to a cost-push shock, see the dashed line scenario in the first column of plots in Figure 6. Following a sharp fall in interest rate and a negative real interest rate, future consumption falls below its steady state level. An initial rise in investment leads to higher stock of capital one period later. As all its components fall, the marginal cost also fall below the steady state level one period after the shock; it also stays below for several consequent periods and this ensures inflation overshooting and stationarity of the price level. Stabilization of the economy and, thus, the capital stock back to the steady state level requires small disinvestment within long period of time. Consumption also stays below the steady state level but rises as interest rates remain low. All this ensures

[5] If we put ω_p but add an additional equation $p_t = p_{t-1} + \pi_t$ into system ()-(), then such system under the optimal discretionary control that minimises loss () will have an eigenvalue $\lambda_k = 1$. For any $\omega_p > 0$ this eigenvalue (and any orthers) will be less than one, but $\lim_{\omega_p \to 0} \lambda_k(\omega_p) = 1$.

inflation remains negative for a long period of time while the economy adjusts towards the steady state.

If ω_p becomes smaller then there is a smaller need to bring the price level back to the steady state. Additionally, controlling inflation variability gains the priority as we consider the hybrid price level targeting. Therefore, monetary policy wants to bring price level down *slowly.* This, however, is impossible to do *smoothly*, under a time-consistent policy. For any given $\omega_p > 0$ monetary policy still has to ensure inflation overshooting. But if inflation stays below zero *for a long time,* as we have seen for $\omega_p = 0.5$, then the price level falls *too* quickly for a small penalty ω_p and inflation cost dominates the loss. So, inflation might need to rise quicker and even to return to the positive area again. To achieve this, capital cannot *stay* high, it should go down to increase marginal cost and this would allow inflation to rise. Interest rate has to go *down* to allow this increase in marginal cost. The second column of plots in Figure 6 suggests that when ω_p becomes smaller ($\omega_p = 0.002$) all variables have to change direction of movements three times (up-down-up) before they monotonically convergence to the steady state. The welfare loss of the 'passive' solution is 16.77, substantially higher than 11.95 for $\omega_p = 0.5$. Further reduction in penalty ω_p requires a 'zig-zag' dynamics for all economic variables, this ensures slow convergence of price level and (relatively) small inflation cost.[6]

For the base line calibration the 'passive' solution does not exist if $\omega_p \lesssim \underline{\omega}_p = 0.0013$. With near unit-root dynamic process for the price level, inflation 'zig-zags' should be nearly symmetric with respect to zero inflation line, but this cannot be achieved in an economy with investment and positive depreciation rate. (If we reduce the depreciation rate then this solution survives for smaller penalties ω_p.) Smaller effect of marginal cost on inflation also reduces the problem: in the model with firm-specific capital for our base line calibration the 'passive' equilibrium survives for $\omega_p > 0$, as we checked numerically. When $\omega_p < 10^{-14}$ then the 'passive' equilibrium disappears but the 'wet' equilibrium is reinstalled.

This explains the existence of the 'dry' and 'passive' equilibria. The 'seemingly wet' equilibrium, however, cannot exist for any $\omega_p > 0$ (numerically the threshold is 10^{-14}). The reason for this is again the need of inflation overshooting. If $\omega_p = 0$ then the seemingly wet policymaker initially rises interest rate in order to lower it sharply in the consequent period so that the resulting higher investment corrects capital to the steady state quickly. Moreover, it lowers the interest rate by more than the 'seemingly dry' policymaker does. Under this regime there is no additional requirement of controlling an additional 'stock' – the price level. With an additional requirement to stabilize the price level, the second-period reduction in interest rate is not helpful: it does not generate inflation overshooting and so does not lead to the stationarity of the price level. That is why any small $\omega_p > 0$ that would, by continuity, lead to smaller *fall* in interest rate in the second and consequent periods, would not corresponds to any *price-stationary* equilibrium.

Finally, regions of multiplicity for the model with either rental market for capital or with firm specific capital are plotted in Figure 7, in the second row of the corresponding panel. For comparison, we also reproduce the area of multiplicity for $\omega_p = 0$ in the first row of the corresponding panel.

Generally, the reasons for survival of only one equilibrium are similar to those for $\omega_p = 0$, for both models of capital.

[6] Neither Batini and Yates (2003) nor Roisland (2008) study this conflict of targets.

(i) When δ increases only the good equilibrium survives. As capital depreciates at a higher rate, then monetary policy becomes relatively more effective in its immediate effects. So, it becomes welfare-improving to change interest rate by more in the response to a positive cost-push shock. The 'passive' solution starts to exhibit bigger zig-zags and disappears if the rate of depreciation is too high. However the depreciation rate needed to eliminate the 'passive' solution is twice bigger than the one needed to eliminate the 'seemingly wet' solution under $\omega_p = 0$.

(ii) If the capital share α is very large then it becomes difficult to control the economy as even slight movements in interest rates can have large effects on the volatility of macroeconomic variables. To avoid excessive volatility the policymaker chooses to change interest rates smoothly. Additionally, the 'passive' policymaker chooses to keep the interest rate persistently *low* in order to keep capital high and control high inflation. The policymaker cannot achieve negative future inflation and negative real interest rate. It becomes difficult to keep marginal costs below zero and so the 'passive' regime disappears at some threshold α – close to unity – where the incentive to lower the interest rate further becomes inconsistent with the incentive to keep the interest rate smooth.

(iii) In the model with firm-specific capital there is a substantial area where only 'passive' equilibrium survives. With larger price stickiness it becomes difficult to control the economy in a 'dry' way if α is large enough. If α is not close to one, but still large, then an efficient control of inflation in the 'seemingly dry' equilibrium requires more aggressive monetary policy. Higher capital share requires less aggressive interest rate movements as they cause bigger investment and consumption movements. These two requirements conflict with each other. Although a penalty on price stabilization lessens the conflict slightly, at some large value of α the 'seemingly dry' solution does not exist. It is reinstated if α becomes even bigger, but still not very close to one. This is because the effect of α on κ is non-linear and the need to raise interest rate to control inflation in a 'dry' way becomes smaller with very large α. The conflict lessens and both equilibria are reappear.

5.3.2 Strict Price Level Targeting

The price level targeting can be taken to its extreme case, the strict price level targeting, where we reduce the relative weight on inflation, ω_π, to zero as was originally proposed by Vestin (2006). We present the results on multiplicity in Figure 7 and all welfare numbers are given in Table 1. The following three results are apparent.

First, this delegation scheme does not affect the results on multiplicity: relative to the hybrid price level targeting case, we still have regions where only one of the two equilibria survive, see Figure 7, the third and the sixth row of subplots. The area where only dry equilibrium exists noticeably shrinks even further, while the region of the only 'passive' equilibrium remains hardly affected.

Second, the dry equilibrium nearly replicates the commitment equilibrium the difference in welfare is less than 0.001%.

Third, the 'passive' equilibrium requires less 'zig-zag' behavior. With no need to keep inflation on target, the adjustment of the economy to shocks can be made more smooth. The strict price level targeting, however, does not change the nature of the 'passive equilibrium.

6 Remarks on Robustness

We have chosen the policymaker's objective function nearly arbitrary. The methodology to derive a quadratic approximation to the social welfare metric is only available in assuming that the policymaker acts under either commitment policy or timeless-perspective policy (Benigno and Woodford (2007)). This methodology is unsuitable to use under discretion. However, the *existence* of multiple equilibria under discretion cannot be guaranteed in models with social objectives (see results in Blake and Kirsanova (2008a) for the model with social objectives). Moreover, the main aim of this paper to study different delegation schemes that, by definition, assume changing policy objectives *away* from 'the reference objective function', where the social function may or may not play its role. In this paper we have chosen the 'social' welfare objective that is widely considered as 'reasonable' and is likely to be used in 'applied' policy analysis.

Having said this, we run several experiments with different policy objectives. We kept the order of magnitude ω the same as in the base line case, but modified ωy_t^2 term into $\omega_y y_t^2 + \omega_c \zeta^2 c_t^2 + \omega_i I_t^2$, as either one might expect the social welfare metric might look like, or if one just wants to pin down consumption and investment separately. We then varied ω_y, ω_c and ω_i between zero and some numbers of order κ/ε. There were only negligibly small quantitative changes to our simulations. The fact of existence of multiple equilibria and the relative performance of different delegation schemes remained the same.

7 Summary of Results and Conclusions

This paper revisits three policy delegation schemes for the discretionary policy. We are particularly interested in how these schemes perform in a model that is known to have multiplicity of discretionary equilibria if the policymaker is benevolent. We demonstrate that the speed-limit and interest-rate-smoothing policies modify the existing 'dry' and 'wet' equilibria but they do not create any new equilibria. Generally speaking, these two delegation policies improve both equilibria in welfare terms. They also reduce the region where only the 'wet' equilibrium survives. The price-level targeting scheme eliminates the worst equilibrium of those existing under the benevolent policy. However, unless the weight on price stabilization objective is extremely small the price level targeting policy creates another equilibrium which is even worse in welfare terms. Moreover, the worst equilibrium can be a realistic outcome in the model with firm-specific capital, as the corresponding region of uniqueness was found for realistic values of parameters for this model.

Our analysis suggests that because of possible switches between different equilibria, in particular as a result of sudden implementation of policy delegation, the previous welfare analysis should be taken with care. Any slight modification to the plain vanilla New Keynesian model that brings endogenous predetermined state variables in (one can think about capital, debt, habit persistence etc.) is likely to generate multiple equilibria under discretionary policy and create problems with equilibrium selection.

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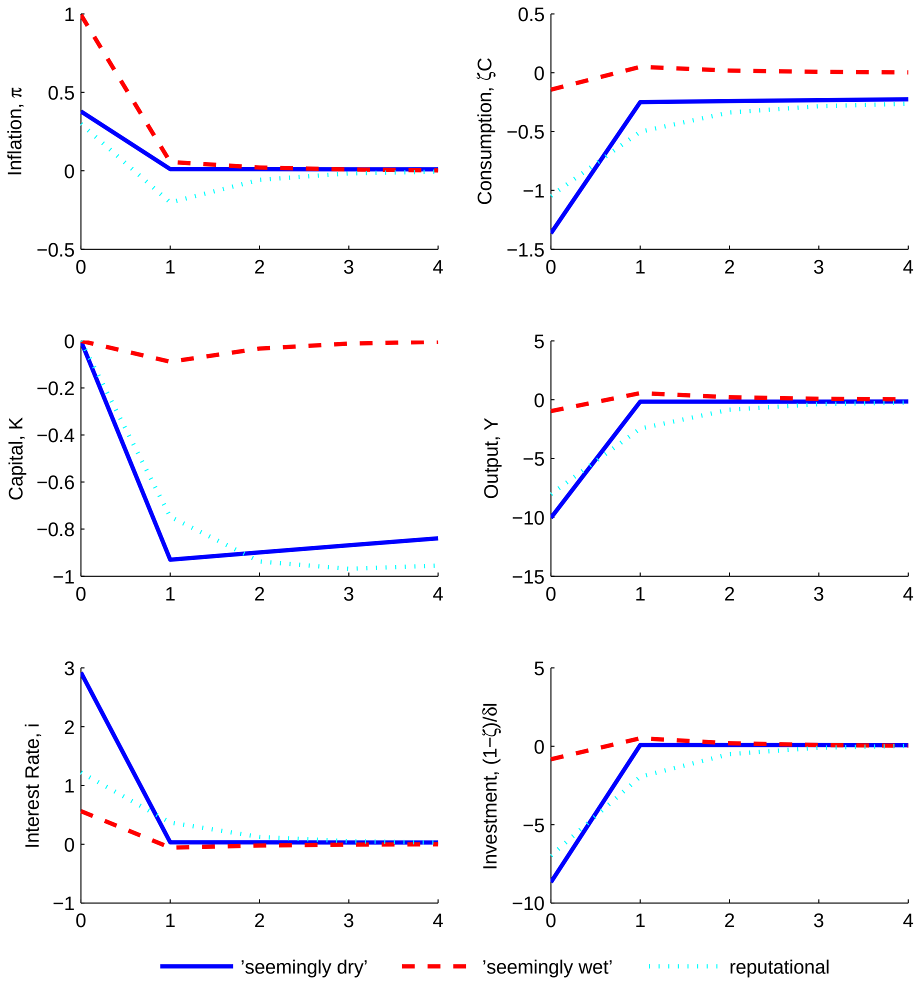

Figure 1: Impulse responses to a unit cost-push shock under three different regimes.

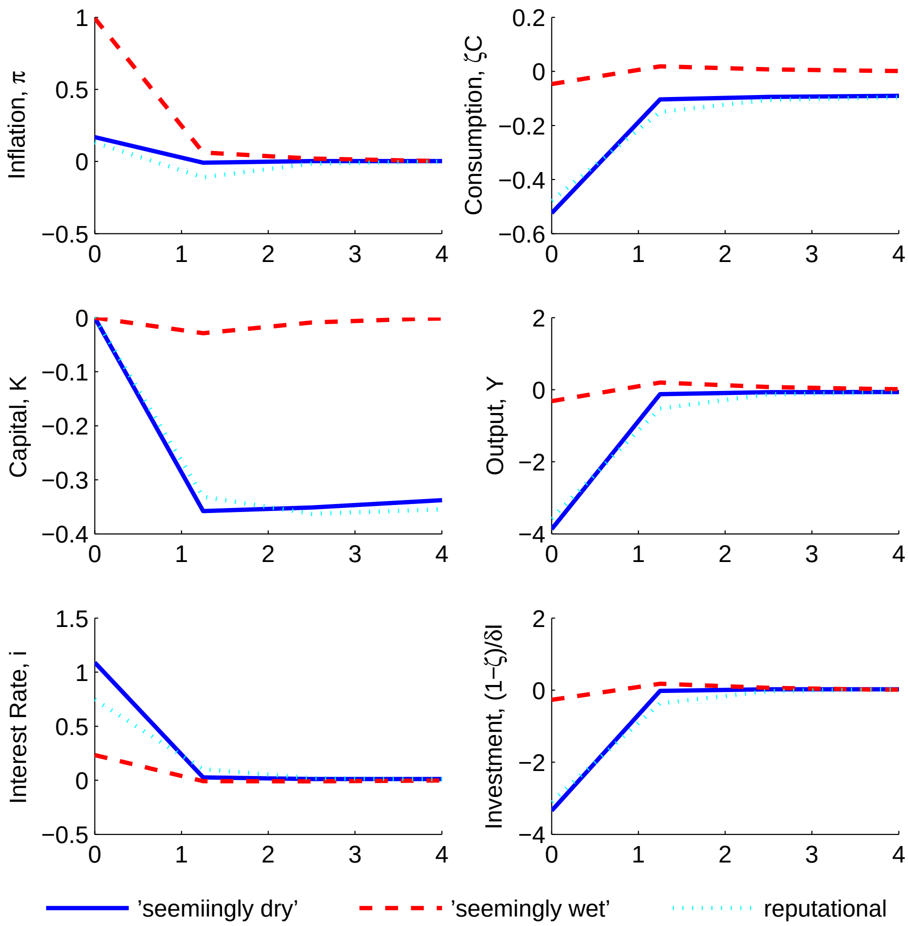

Figure 2: Impulse responses to a unit cost-push shock under interest rate smoothing policy

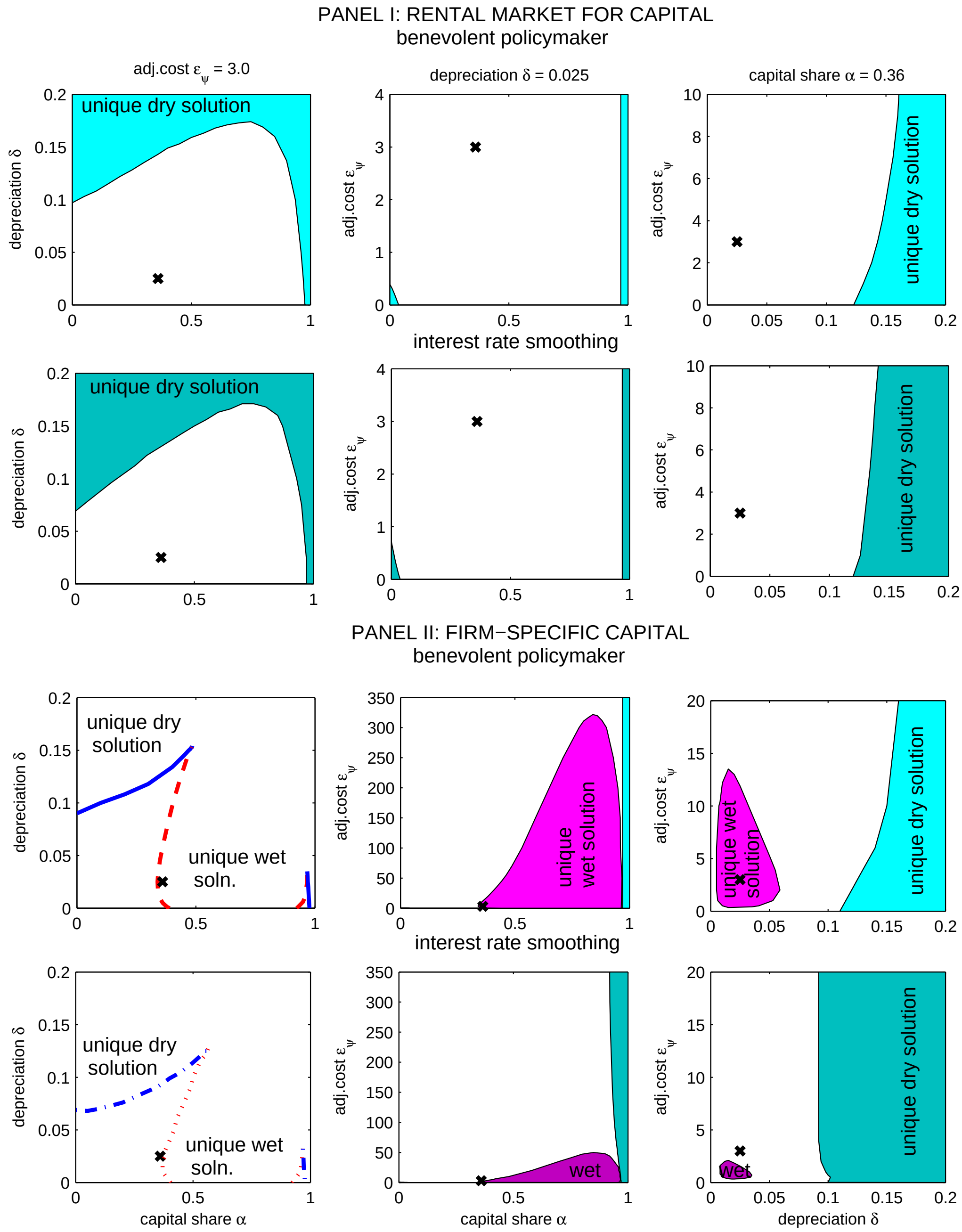

Figure 3: Regions of multiplicity for Interest Rate Smoothing policy.

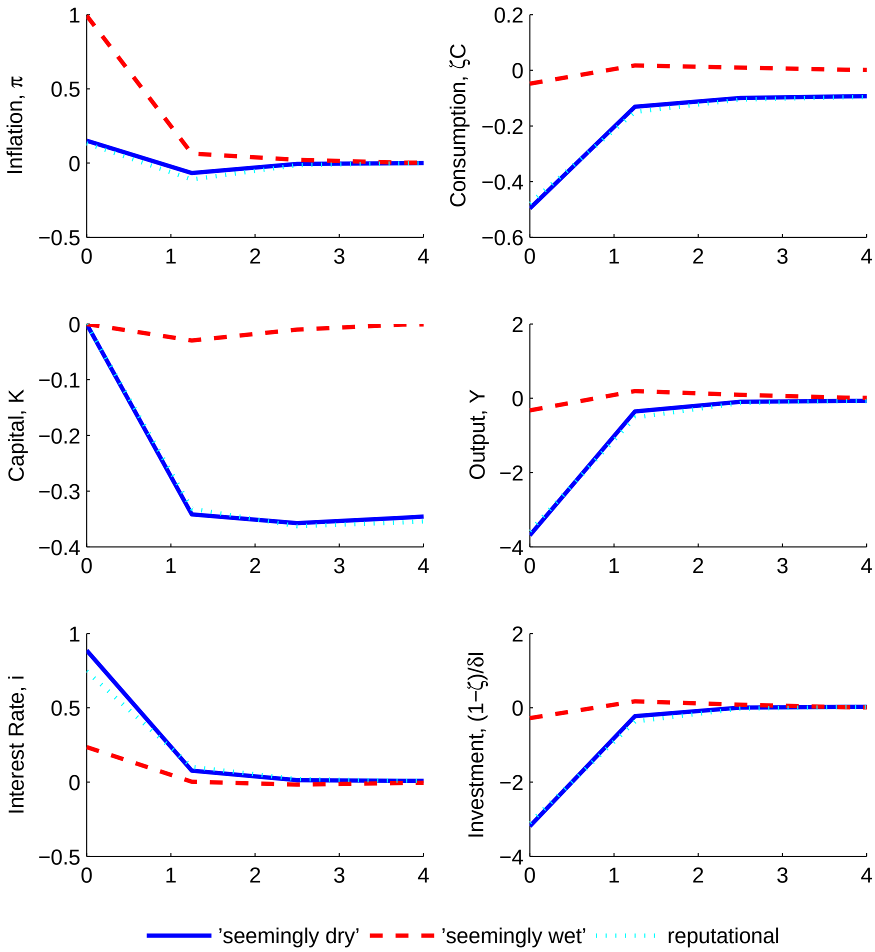

Figure 4: Impulse responses to a unit cost-push shock under speed-limit policy

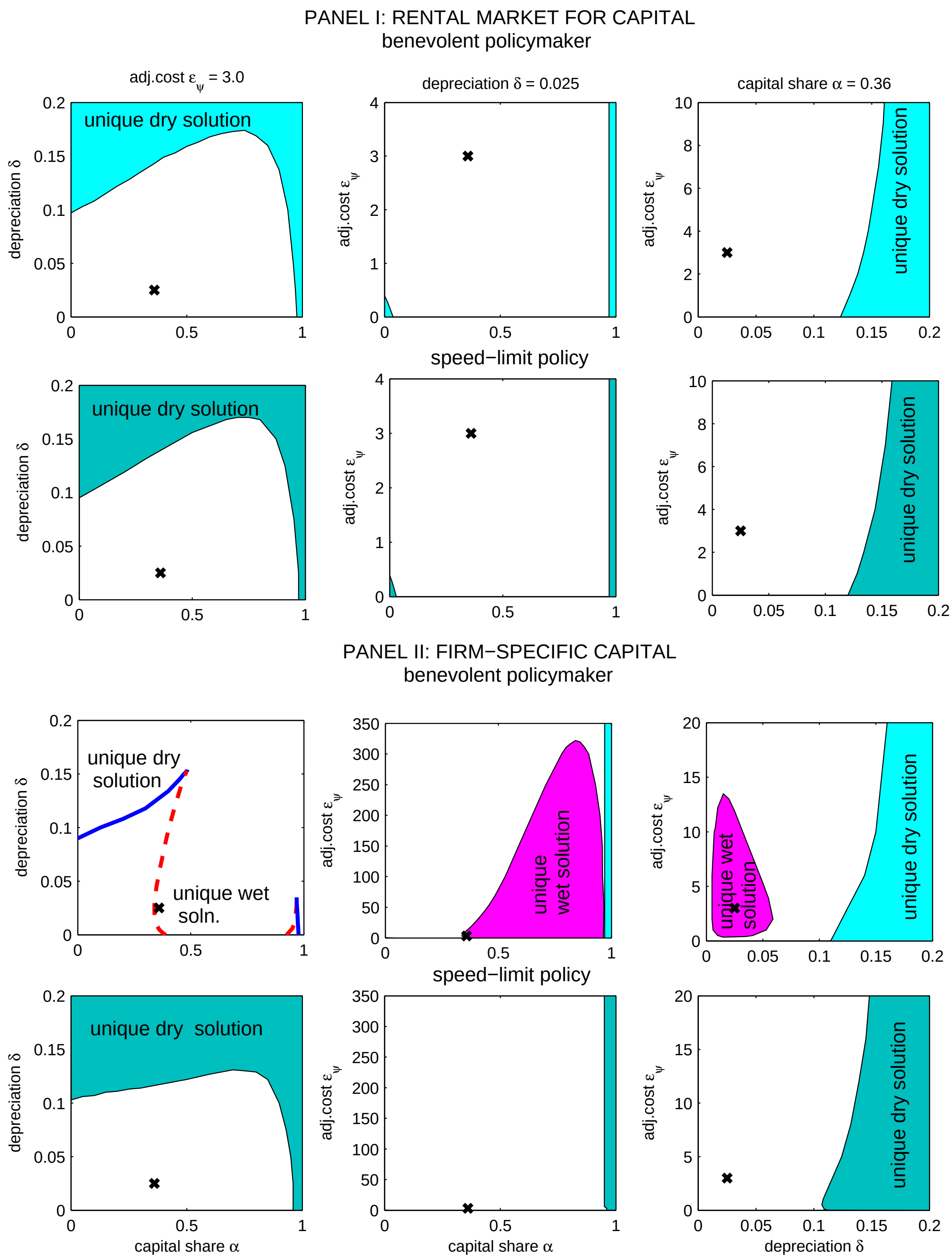

Figure 5: Regions of multiplicity for speed-limit policy

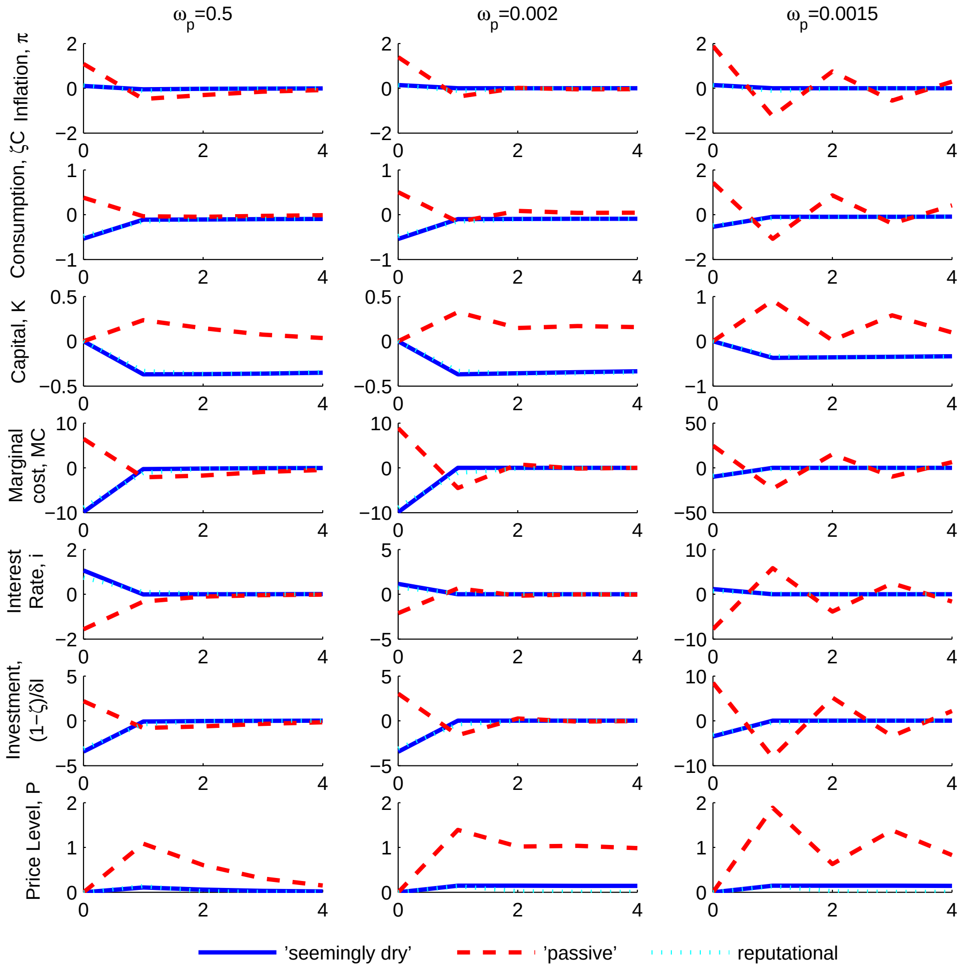

Figure 6: Impulse responses to a unit cost-push shock under price level targeting for three different penalties ω_p. Rental market for capital.

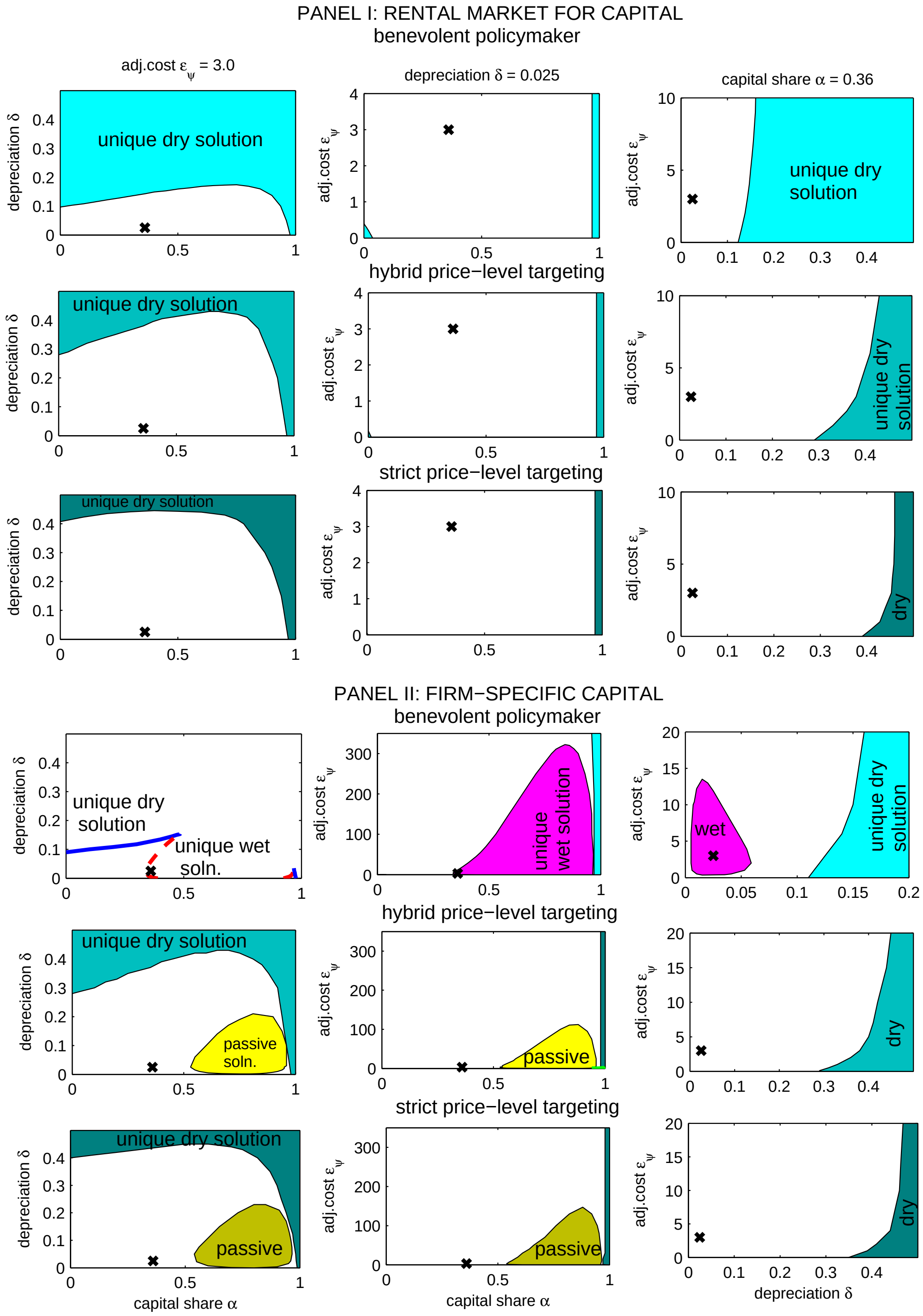

Figure 7: Regions of multiplicity for the price-level targeting