Estimating a DSGE model for Norway with optimal monetary policy*

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Abstract

We use Bayesian techniques to estimate a New Keynesian small open-economy model under the assumption that monetary policy is conducted optimally under commitment. To take account of the potential misspecification of the structural model, we use the DSGE-VAR methodology proposed by Del Negro and Schorfheide (2004). We compare the results to those obtained under the alternative assumption that the central bank follows a simple instrument rule. Our findings suggest that the fit of the DSGE model with optimal policy under commitment is at least as good as the model with a simple instrument rule. This result is robust to allowing for misspecification within the DSGE-VAR framework. We compare the interest rate and inflation projections to the paths published by Norges Bank after 2005.

1 Introduction

The purpose of this paper is to compare the empirical merits of different approaches to modelling monetary policy in the context of a New Keynesian small open economy model. Specifically, we use Bayesian techniques to estimate a dynamic stochastic general equilibrium (DSGE) model under two alternative representations of monetary policy: a simple instrument rule and optimal monetary policy under commitment. An important aspect of our analysis is to examine the robustness of the results with respect to model misspecification. Despite the recent progress in getting DSGE models to fit the data (see e.g., Smets & Wouters (2004), Edge et al. (2006), Adolfson et al. (2007b) and Adolfson et al. (2007c)), potential model misspecification remains a key concern. Model misspecification may distort the parameter estimates as well as the forecasts from the DSGE model.

In this paper we take account of model misspecification using the DSGE-VAR approach proposed by Del Negro & Schorfheide (2004). The DSGE-VAR approach allows us to relax the tight cross-equation restrictions implied by the DSGE model for the parameters in a VAR. An alternative interpretation is that we use the DSGE model to generate prior distributions for the VAR parameters. The DSGE-VAR approach also produces estimates of the parameters in the DSGE model that can be compared to those obtained using the traditional full-information approach. One of the questions we ask in this paper is whether

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accounting for misspecification affects the estimates of the monetary policy preferences and the optimal interest rate projections implied by the model.

In empirical DSGE models, the central bank is typically assumed to set the interest rate according to a simple instrument rule (e.g., a Taylor rule). In addition to computational simplicity, one reason behind the popularity of this approach is that simple instrument rules have been shown to give a reasonable empirical description of actual monetary policy in many countries. Moreover, simple rules are perceived to be robust in that they perform reasonably well in terms of welfare in different models. An alternative approach is to assume that monetary policy is conducted optimally. Hence, the central bank sets the interest rate to minimise an explicit intertemporal loss function. The optimal policy approach gives a more symmetric treatment of central bank and private sector behaviour and, moreover, allows the central bank to make efficient use of all relevant information. As pointed out by Svensson (2003), it seems somewhat odd to assume a priori that the central bank has a less sophisticated approach to optimization than the private agents. Finally, the optimizing framework appears to be more in line with the way monetary policy is actually conducted in most developed countries.

The main purpose of this paper is to shed light on which of these two policy assumptions provides the most plausible account of the data. From an empirical point of view, there are two opposing mechanisms at play. On the one hand, the optimal policy framework is more flexible than the simple instrument rule in the sense the implied interest rate rule contains a larger set of variables than the simple instrument rule. However, this flexibility comes at the cost of introducing a new set of restrictions on the reduced form solution of the model, restrictions that are potentially at odds with the data.

There exists a small, but increasing literature estimating New Keynesian models with optimal monetary policy. Dennis (2004) jointly estimates the parameters in the central bank's objective function and the parameters in the optimizing constraints in a New Keynesian model of the US economy, under the assumption that monetary policy is conducted optimally under discretion. In two recent papers Ilbas (2008a) and Ilbas (2008b) use a Bayesian approach to estimate the monetary policy preferences in New Keynesian closed-economy models for the euro area and the US assuming that the central bank mimimises an intertemporal loss function under commitment. Our paper is closest in spirit to the paper by Adolfson et al. (2008). In that paper the authors estimate an operational medium-scale, small open economy DSGE model for the Swedish economy and compare the in-sample fit of models with alternative assumptions about monetary policy. Their main finding, using a Bayesian approach, is that the in-sample fit of the model with a simple instrument rule is superior to the model with optimal policy under commitment. However, recognising that in-sample model comparison within a Bayesian framework can be problematic, we extend their results by also considering the out-of-sample forecasting performance of the different models. Our recursive estimation procedure has the added advantage that it allows us to investigate the stability of the parameters over time. The main contribution in our paper is, however, that we use the DSGE-VAR methodology to examine how the results are affected by acknowledging that the model is potentially misspecified.

The DSGE model is a medium-scale model estimated on data for the Norwegian economy over the period 1987Q1–2007Q4. The model is similar in size and structure to the model that has recently been adopted as the core model in the policy process in Norges Bank (Central Bank of Norway) [1] and thus constitutes a real world example of empirical interest. Moreover, Norges Bank began publishing its own interest rate forecasts in November 2005. In the final part of this paper, we compare the official Norges Bank forecasts of the interest rate and inflation to the forecasts from the DSGE model and the DSGE-VARs.

[1] See Monetary Policy Report 3/07 (available at www.norges-bank.no).

Our preliminary findings can be summarised as follows. First, the in-sample fit of the model with optimal policy is superior to the model with a simple instrument rule. This is true also when we allow for misspecification; the hyperparameter determining the weight on the DSGE model in the DSGE-VAR approach is higher for the model with optimal monetary policy than for the model with a simple instrument rule. However, in terms of forecasting accuracy, which is our favoured measure of model fit, the models perform about equally well. This result is also robust to allowing for misspecification using the DSGE-VAR framework: the forecasts performance of the two DSGE-VAR models is almost identical. Second, allowing for misspecification brings the forecasted interest path from the DSGE-VARs very much in line with Norges Bank's actual interest rate forecasts from 2005 onwards. Finally, allowing for misspecification has some interesting implications for the estimates of the DSGE model parameters and hence, the implied optimal interest rate paths computed from the model.

The remainder of the paper is organized as follows. In section 2 we give a brief description of the DSGE model used in the empirical exercise. In section 3 we discuss the DSGE-VAR approach. Section 4 presents the full-sample estimation results. Section 5 discusses the out-of-sample forecasting exercise and compares both the DSGE and DSGE-VAR forecasts to the official Norges Bank forecasts. Section 6 concludes the paper.

2 The DSGE model

The benchmark DSGE model used in the forecasting exercise is a medium-scale New Keynesian open economy model. The theoretical framework builds on the New Open Economy Macroeconomics (NOEM) literature (see e.g., Lane (2001) for a survey) as well as the closed economy models in e.g., Christiano et al. (2005) and Smets & Wouters (2003), and is similar in structure to existing open-economy models such as the Global Economy Model (GEM) model at the International Monetary Fund and the model developed in Adolfson et al. (2007a).[2]

The economy has two production sectors. Firms in the intermediate goods sector produce differentiated goods for sale in monopolistically competitive markets at home and abroad, using labour and capital as inputs. Firms in the perfectly competitive final goods sector combine domestically produced and imported intermediate goods into an aggregate good that can be used for private consumption, private investment and government spending. The household sector consists of a continuum of infinitely-lived households that consume the final good, work and save in domestic and foreign bonds. The model incorporates real rigidities in the form of habit persistence in consumption, variable capacity utilisation of capital and investment adjustment costs and nominal rigidities in the form of local currency price stickiness and nominal wage stickiness. The model is closed by assuming that domestic households pay a debt-elastic premium on the foreign interest rate when investing in foreign bonds. The model evolves around a balanced growth path as determined by a permanent technology shock. The fiscal authority runs a balanced budget each period, and we consider two alternative specifications of monetary policy. The exogenous foreign variables are assumed to follow autoregressive processes.

Final goods sector The perfectly competitive final goods sector consists of a continuum of final good producers indexed by $x \in [0,1]$ that aggregates composite domestic intermediate goods, Q, and imports, M, using a constant elasticity of substitution (CES) technology:

$$A_t(x) = \left[\eta^{\frac{1}{\mu}} Q_t(x)^{1-\frac{1}{\mu}} + (1-\eta)^{\frac{1}{\mu}} M_t(x)^{1-\frac{1}{\mu}}\right]^{\frac{\mu}{\mu-1}}, \tag{1}$$

[2] We refer to Brubakk et al. (2006) for a more thorough discussion of the model and literature references.

The degree of substitutability between the composite domestic and imported goods is determined by the parameter $\mu > 0$, whereas η $(0 \leq \eta \leq 1)$ measures the steady-state share of domestic intermediates in the final good for the case where relative prices are equal to 1.

The composite good $Q(x)$ is an index of differentiated domestic intermediate goods, produced by a continuum of firms $h \in [0, 1]$:

$$Q_t(x) = \left[\int_0^1 Q_t \left(h, x\right)^{1-\frac{1}{\theta_t}} dh \right]^{\frac{\theta_t}{\theta_t - 1}}, \tag{2}$$

where the time-varying degree of substitution between domestic intermediates is captured by θ_t and evolves according to:

$$\ln\left(\frac{\theta_t}{\theta}\right) = \lambda^{\theta} \ln\left(\frac{\theta_{t-1}}{\theta}\right) + \varepsilon_t^{\theta}, \tag{3}$$

where $\theta > 1$ is the steady-state value and $0 \leq \lambda^{\theta} < 1$. The error term ε_t^{θ} is assumed to be white noise.

Similarly, the composite imported good is a CES aggregate of differentiated import goods indexed $f \in [0, 1]$:

$$M_t(x) = \left[\frac{1}{\theta_t^*} \int_0^1 M_t \left(f, x\right)^{1-\frac{1}{\theta_t^*}} df \right]^{\frac{\theta_t^*}{\theta_t^* - 1}}, \tag{4}$$

where $\theta^* > 1$ is the degree of substitution between imported goods.

Intermediate goods sector Each intermediate firm h is assumed to produce a differentiated good $T_t\left(h\right)$ for sale in domestic and foreign markets using the following CES production function:

$$T_t\left(h\right) = \left[\left(1-\alpha\right)^{\frac{1}{\xi}} \left(Z_t z_t^L l_t\left(h\right)\right)^{1-\frac{1}{\xi}} + \alpha^{\frac{1}{\xi}} \overline{K}_t\left(h\right)^{1-\frac{1}{\xi}} \right]^{\frac{\xi}{\xi-1}}, \tag{5}$$

where $\alpha \in [0, 1]$ is the capital share and ξ denotes the elasticity of substitution between labour and capital. The variables $l_t\left(h\right)$ and $\overline{K}_t\left(h\right)$ denote, respectively, hours used and effective capital of firm h in period t. There are two exogenous shocks to productivity in the model: Z_t refers to an exogenous permanent (level) technology process, which grows at the gross rate π_t^z, whereas z_t^L denotes a temporary (stationary) shock to productivity (or labour utilization). The technology processes are modelled as

$$\ln(Z_t) = \ln(Z_{t-1}) + \ln(\pi^z) + \ln\frac{\pi_t^z}{\pi^z}, \tag{6}$$

where

$$\ln\frac{\pi_t^z}{\pi^z} = \lambda^z \ln\frac{\pi_{t-1}^z}{\pi^z} + \varepsilon_t^z, \tag{7}$$

and

$$\ln\left(\frac{z_t^L}{z^L}\right) = \lambda^L \ln\left(\frac{z_{t-1}^L}{z^L}\right) + \varepsilon_t^L. \tag{8}$$

The variable $K_t(h)$ is defined as firm h's capital stock that is chosen in period t and becomes productive in period $t+1$. Firm h's *effective* capital in period t is related to the capital stock that was chosen in period $t-1$ by

$$\overline{K}_t(h) = u_t(h) K_{t-1}(h), \tag{9}$$

where $u_t(h)$ is the endogenous rate of capital utilization. When adjusting the utilization rate the firm incurs a cost of $\gamma_t^u(h)$ units of final goods per unit of capital. The cost function is

$$\gamma_t^u(h) = \phi^{u_1}\left(e^{\phi^{u_2}(u_t(h)-1)} - 1\right), \tag{10}$$

where ϕ_1^u and ϕ_2^u are parameters determining the cost of deviating from the steady state utilization rate (normalized to one).

Firm h's law of motion for physical capital reads:

$$K_t(h) = (1-\delta) K_{t-1}(h) + \kappa_t(h) K_{t-1}(h), \tag{11}$$

where $\delta \in [0,1]$ is the rate of depreciation and $\kappa_t(h)$ denotes capital adjustment costs. The latter takes the following form:

$$\begin{aligned}\kappa_t(h) \quad = \quad & \frac{I_t(h)}{K_{t-1}(h)} - \frac{\phi_1^I}{2}\left[\left(\frac{I_t(h)}{K_{t-1}(h)} - \frac{I}{K}\right)\right]^2 \\ & -\frac{\phi_2^I}{2}\left(\frac{I_t(h)}{K_{t-1}(h)} - \frac{I_{t-1}}{K_{t-2}}\right)^2,\end{aligned} \tag{12}$$

where I_t denotes investment and z_t^I is an investment shock[3] that evolves according to

$$\ln\left(\frac{z_t^I}{z^I}\right) = \lambda^I \ln\left(\frac{z_{t-1}^I}{z^I}\right) + \varepsilon_t^I, \tag{13}$$

where $0 \leq \lambda^I < 1$ and ε_t^I is white noise.

The labour input is a CES aggregate of hours supplied by the different households:

$$l_t(h) = \left[\frac{1}{n}\int_0^n l_t(h,j)^{1-\frac{1}{\psi_t}}\,dj\right]^{\frac{\psi_t}{\psi_t - 1}}, \tag{14}$$

where ψ_t denotes the elasticity of substitution between different types of labour that evolves according to:

$$\ln\left(\frac{\psi_t}{\psi}\right) = \lambda^\psi \ln\left(\frac{\psi_{t-1}}{\psi}\right) + \varepsilon_t^\psi, \tag{15}$$

where $0 \leq \lambda^\psi < 1$ and ε_t^ψ is white noise.

Firms sell their goods in markets characterised by monopolistic competition. International goods markets are segmented and firms set prices in the local currency of the buyer. An individual firm h charges $P_t^Q(h)$ in the home market and $P_t^{M*}(h)$ abroad, where the latter is denoted in foreign currency. Nominal price stickiness is modelled by assuming that firms face quadratic costs of adjusting prices,

$$\gamma_t^{PQ}(h) \equiv \frac{\phi_1^Q}{2}\left[\frac{P_t^Q(h)}{\pi P_{t-1}^Q(h)} - 1\right] \tag{16}$$

[3] This shock could e.g., represent changes in the relative price of consumption and investment.

$$\gamma_t^{P^{M^*}}(h) \equiv \frac{\phi_1^{M^*}}{2}\left[\frac{P_t^{M^*}(h)}{\pi P_{t-1}^{M^*}(h)} - 1\right] \tag{17}$$

in the domestic and foreign market, respectively. The price adjustment costs are intangible and do not affect cash flow. Rather, the costs enter the firm's maximisation problem as a form of "disutility". In every period cash-flows are paid out to the households as dividends.

Firms choose hours, capital, investment, the utilization rate and prices to maximize present discounted value of cash-flows, adjusted for the intangible cost of changing prices, taking into account the law of motion for capital, and demand both at home and abroad, $T_t^D(h)$. The latter is given by:

$$T_t^D(h) = \int_0^n Q_t(h,x)dx + \int_{1-n}^1 M_t^*(h,x^*)dx^*$$

Households The economy is inhabited by a continuum of infinitely-lived households indexed by $j \in [0,1]$. The period utility function is additively separable in consumption and leisure. The lifetime expected utility of household j is:

$$U_t(j) = E_t \sum_{i=0}^{\infty} \beta^i \left[z_{t+i}^u u\left(C_{t+i}(j)\right) - v\left(l_{t+i}(j)\right)\right], \tag{18}$$

where C denotes consumption, l is hours worked and β is the discount factor $0 < \beta < 1$. The consumption preference shock, z_{t+i}^u, evolves according to

$$\ln\left(\frac{z_t^u}{z^u}\right) = \lambda^u \ln\left(\frac{z_{t-1}^u}{z^u}\right) + \varepsilon_t^u$$

The current period utility functions for consumption and labour choices, $u(C_t(j))$ and $v(l_t(j))$, are

$$u\left(C_t(j)\right) = \left(1 - b^c/\pi^z\right)\ln\left[\frac{\left(C_t(j) - b^c C_{t-1}\right)}{1 - b^c/\pi^z}\right], \tag{19}$$

and

$$v\left(l_t(j)\right) = \frac{1}{1+\zeta} l_t(j)^{1+\zeta}. \tag{20}$$

where the degree of external habit persistence in consumption is governed by the parameter b^c $(0 < b^c < 1)$.

Each household is the monopolistic supplier of a differentiated labour input and sets the nominal wage subject to the labour demand of intermediate goods firms and subject to quadratic costs of adjustment, γ^W:

$$\gamma_t^W(j) \equiv \frac{\phi^W}{2}\left[\frac{W_t(j)/W_{t-1}(j)}{W_{t-1}/W_{t-2}} - 1\right]^2 \tag{21}$$

where W_t is the nominal wage rate.

The individual flow budget constraint for agent j is:

$$\begin{aligned} P_t C_t(j) + S_t B_{H,t}^*(j) + B_t(j) \leq {} & W_t(j)\, l_t(j)\left[1 - \gamma_t^W(j)\right] \\ & + \left[1 - \gamma_{t-1}^{B^*}\right]\left(1 + r_{t-1}^*\right) S_t B_{H,t-1}^*(j) \\ & + \left(1 + r_{t-1}\right) B_{t-1}(j) + DIV_t(j) - TAX_t(j), \end{aligned} \tag{22}$$

where S_t is the nominal exchange rate, $B_t(j)$ and $B^*_{H,t}(j)$ are household j's end of period t holdings of domestic and foreign bonds, respectively. Only the latter are traded internationally. The domestic short-term nominal interest rate is denoted by r_t, and the nominal return on foreign bonds is r^*_t. The variable DIV includes all profits from intermediate goods firms and nominal wage adjustment costs, which are rebated in a lump-sum fashion. Finally, home agents pay lump-sum (non-distortionary) net taxes, TAX_t, denominated in home currency.

A financial intermediation cost, γ^{B^*}, is introduced to guarantee that aggregate net foreign assets follow a stationary process. This cost depends on the average net foreign asset position of the domestic economy. The intermediation cost takes the following form:

$$\gamma_t^{B^*} = \phi^{B1} \frac{\exp\left(\phi^{B2}\left(\frac{S_t B_t^{H^*}}{P_t Z_t}\right)\right) - 1}{\exp\left(\phi^{B2}\left(\frac{S_t B_t^{H^*}}{P_t Z_t}\right)\right) + 1} + z_t^B, \tag{23}$$

where $0 \leq \phi^{B1} \leq 1$ and $\phi^{B2} > 0$. The exogenous 'risk premium', z_t^B, evolves according to

$$\ln\left(\frac{z_t^B}{z^B}\right) = \lambda^B \ln\left(\frac{z_{t-1}^B}{z^B}\right) + \varepsilon_t^B, \tag{24}$$

where $0 \leq \lambda^B < 1$ and ε_t^B is white noise.

Government The government purchases final goods financed through a lump-sum tax. Real government spending (adjusted for productivity), $g_t \equiv G_t/Z_t$, is modelled as an AR(1) process

$$\ln\left(\frac{g_t}{g}\right) = \lambda^G \ln\left(\frac{g_{t-1}}{g}\right) + \varepsilon_t^G, \tag{25}$$

where G_t is real per capita government spending.

The central bank sets a short-term nominal interest rate, r^*_t. We consider two alternative specifications of monetary policy. First, we assume that the behaviour of the central bank can be represented by a simple instrument rule, which in its log-linearised version takes the form

$$r_t^* = \omega_r r_{t-1}^* + (1-\omega_r)\left[\omega_\pi \pi_t + \omega_y gdp_t + \omega_{rer} rer_t\right] \tag{26}$$

where π_t is the aggregate inflation rate, and rer_t is the real exchange rate. The parameter $\lambda^r \in [0, 1\rangle$ determines the degree of interest rate smoothing. Output (gdp_t) is measured in deviation from the stochastic productivity trend[4], the remaining variables are in deviation from their steady-state levels.

The alternative assumption about monetary policy is that the central bank sets the interest rate to minimise the intertemporal loss function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \omega_y^2 (gdp_t)^2 + \omega_{\Delta r}(r_t^* - r_{t-1}^*)^2\right] \tag{27}$$

under commitment subject to the log-linearised first-order conditions of the private sector and the exogenous shock processes. As argued by e.g., Holmsen et al. (2007) including this term is necessary in order to produce interest rate paths that do not look immediately

[4] Empirically, and under both assumptions about monetary policy, this measure of the output gap turns out to be quite similar to the output gap obtained using a standard Hodrick-Prescott filter which again resembles the preferred measure of the output gap published by Norges Bank.

unacceptable. Adolfson et al (2008) also include an interest-rate smoothing term in their loss function.

For both specifications of monetary policy we assume that the interest rate that enters into the decisions of households and firms, r_t, equals the interest rate set by the monetary policy authority, r_t^*, plus a shock, z_t^r, that is

$$r_t = r_t^* + z_t^r \tag{28}$$

where

$$\ln\left(\frac{z_t^r}{z^r}\right) = \lambda^r \ln\left(\frac{z_{t-1}^r}{z^r}\right) + \varepsilon_t^r \tag{29}$$

This shock could be interpreted e.g., as variations in the banks interest rate margins or in the spread between the key policy rate and the short-term interest rate in the money market.

Foreign variables The two foreign variables that enter the model are the real marginal cost of foreign firms, mc_t^*, (which enters in the Phillips curve for imported inflation) and the foreign output gap, y_t^* (which enters in the (reduced form) demand function for domestic exports). In the current version of the model, the foreign variables are assumed to follow simple first-order autoregressive processes, that is

$$\ln\left(\frac{mc_t^*}{mc^*}\right) = \lambda^{mc^*} \ln\left(\frac{mc_{t-1}^*}{mc^*}\right) + \varepsilon_t^{mc^*} \tag{30}$$

$$y_t^* = \lambda^{y^*} y_{t-1}^* + \varepsilon_t^{y^*} \tag{31}$$

Model solution To solve the model we first transform the model into a stationary representation by detrending the relevant real variables by the permanent technology shock. Next, we take a first-order approximation (in logs) of the equilibrium conditions around the steady-state. In the computation of the optimal policy we treat the model as exactly linear. Following the exposition in Juillard & Pelgrin (2007), the equilibrium conditions of the model can be written

$$F_+ E_t x_{t+1} + F_0 x_t + F_- x_{t-1} + G r_t + H \varepsilon_t = 0 \tag{32}$$

where x_t is a vector of endogenous variables, r_t is the policy instrument (here: the nominal interest rate) and ε_t is the vector of white noise disturbances. Letting $z_t = \begin{bmatrix} x_t' & r_t \end{bmatrix}'$ we can rewrite the intertemporal loss function (27) as

$$\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t z_t' W z_t \tag{33}$$

the Lagrangian of the optimal policy problem can be expressed as

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} \frac{1}{2}\left(x_t' W_{xx} x_t + 2 x_t' W_{xr} r_t + r_t' W_{rr} r_t\right) + \\ \lambda_t' \left(F_+ E_t x_{t+1} + F_0 x_t + F_- x_{t-1} + G r_t + H \varepsilon_t\right) \end{bmatrix} \tag{34}$$

or, alternatively, in matrix form, as

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} z_t' W z_t + \lambda_t' \left(\begin{bmatrix} F_+ & 0 \end{bmatrix} E_t z_{t+1} + \begin{bmatrix} F_0 & G \end{bmatrix} z_t + \begin{bmatrix} F_- & 0 \end{bmatrix} z_{t-1} + H \varepsilon_t\right)\right] \tag{35}$$

The first-order conditions can be written as

$$W z_t + \beta^{-1} \begin{bmatrix} F_+' \\ 0' \end{bmatrix} \lambda_{t-1} + \begin{bmatrix} F_0' \\ B' \end{bmatrix} \lambda_t + \beta \begin{bmatrix} F_-' \\ 0' \end{bmatrix} E_t \lambda_{t+1} = 0 \tag{36}$$

and

$$\begin{bmatrix} F_+ & 0 \end{bmatrix} E_t z_{t+1} + \begin{bmatrix} F_0 & G \end{bmatrix} z_t + \begin{bmatrix} F_- & 0 \end{bmatrix} z_{t-1} + H \varepsilon_t = 0 \tag{37}$$

with $\lambda_0 = 0$ and x_0 given. Inspection of equations (36) and (37) reveals that this is a linear rational expectations model expanded with difference equations for the Lagrange multipliers that can be solved using standard techniques.

Notice that the optimal commitment rule involves treating the first period differently from subsequent periods. When setting the interest rate in the first period, the policy maker takes the expectations of the private sector as given and is not constrained by any previous commitments. This is reflected in the initial value of the Lagrange multiplier being zero. The optimal commitment policy is time-inconsistent; for all periods $t > 0$ the policy maker will have an incentive to deviate from the previously announced path and exploit the private sector expectations. To overcome this 'initial value' problem Woodford (1999) proposes instead that the policy maker behaves as if the commitment to the optimal policy was made far in the past. This approach is referred to as 'timeless perspective commitment'. To compute optimal policy projections under commitment in a timeless perspective one must provide initial values for the Lagrange multipliers. See Juillard & Pelgrin (2007), Ilbas (2008a) and Adolfson et al. (2008) for alternative methods to compute these initial values.

In this paper we simply assume that monetary policy has been conducted optimally under commitment since the start of the estimation period and that the central bank never re-optimizes. The unobserved state variables, including the Lagrange multipliers, are initialized at zero which correspond to the steady-state values of the variables. When the effect of the initial conditions have died out, the optimal commitment policy will coincide with the timelessly optimal policy. Following the suggestion in Ilbas (2008a), we have also experimented with using a presample approach to initialise the multipliers. So far, our experience is that the estimation results are not much affected by how we initialize the multipliers in the estimation.

Adopting the notation in Fernández-Villaverde et al. (2007), the transition equations describing the model solution can be expressed in state-space form as

$$\begin{aligned} Z_{t+1} &= A(\theta) Z_t + B(\theta) \varepsilon_t \\ Y_t &= C(\theta) Z_t + D(\theta) \varepsilon_t \end{aligned} \tag{38}$$

where Y_t is a $k \times 1$ vector of variables observed by the econometrician. In the case of optimal commitment policies, the state vector Z_t will contain the Lagrange multipliers associated with the behavioural equations of the private sector and the structural shock processes. The matrices A, B, C and D are non-linear functions of the structural parameters in the DSGE model as represented by the vector θ. In this paper we focus on the case with an equal number of shocks and observable variables so that where D is square and invertible.

In the DSGE-VAR approach, the finite-order VAR approximation to the DSGE model plays a key role. Fernández-Villaverde et al. (2007) show that iff the eigenvalues of $A - BD^{-1}C$ are strictly less than one in modulus, Y_t has an infinite-order VAR representation given by:[5]

$$Y_t = \sum_{j=1}^{\infty} C(A - BD^{-1}C)^{j-1} BD^{-1} Y_{t-j} + D\varepsilon_t \tag{39}$$

[5] If one or more of the eigenvalues of $A - BD^{-1}C$ are exactly equal to one in modulus, Y_t does not have a VAR representation, i.e., the autoregressive coefficients do not converge to zero as the number of lags tends to infinity. Often, roots on the unit circle indicate that the observables have been overdifferenced. Fernandez-Villaverde et al (2007) refer to this as a 'benign borderline case'.

In general, a finite-order VAR is not an exact representation of the linearised DSGE model. Specifically, the finite order VAR approximation will only be exact if all the endogenous state variables are observable and included in the VAR (see e.g., Ravenna (2007)). The rate at which the autoregressive coefficients converge to zero is determined by the largest eigenvalue of $A - BD^{-1}C$. If this eigenvalue is close to unity, a low order VAR is likely to be a poor approximation to the infinite-order VAR implied by the DSGE model.

3 The DSGE-VAR approach

As alluded to in the introduction, the basic idea of the DSGE-VAR approach is to use the DSGE model to construct prior distributions for the VAR. The starting point for the estimation is an unrestricted VAR of order p

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + u_t, \tag{40}$$

where y_t is an $n \times 1$ vector of observables, ϕ_0 is an $n \times 1$ vector of constant terms, ϕ_i are $n \times n$ matrices of autoregressive parameters $i = 1, \ldots, n$ and $u_t \sim N(0, \Sigma_u)$. If we let the vector of regressors in the VAR be denoted $x_t = [1, y_{t-1}, y_{t-2}, \ldots, y_{t-p}]$, the VAR can be written compactly as

$$Y = X\Phi + U, \tag{41}$$

where Y is $T \times n$ with rows y_t', X is $T \times (1 + np)$ with rows x_t' , U is $T \times n$ with rows u_t' and $\Phi = \left[\phi_0', \phi_1', \ldots, \phi_p'\right]$. The likelihood function for the VAR is given by

$$\begin{aligned} p(Y|\Phi, \Sigma_u) \quad &\propto \quad |\Sigma_u|^{-T/2} \\ & \times \exp\left\{-\frac{1}{2}\mathrm{tr}\left[\Sigma_u^{-1}\begin{pmatrix} Y'Y - \Phi'X'Y \\ -Y'X\Phi + \Phi'X'X\Phi \end{pmatrix}\right]\right\} \end{aligned} \tag{42}$$

The prior distribution for the VAR parameters proposed by Del Negro & Schorfheide (2004) is based on the VAR approximation to the DSGE model. Let Γ_{xx}^*, $\Gamma_{yy}^*, \Gamma_{xy}^*$ and Γ_{yx}^* be the theoretical second-order moments of the variables in Y and X implied by the DSGE model. Then

$$\begin{aligned} \Phi^*(\theta) &= \Gamma_{xx}^{*-1}(\theta)\Gamma_{xy}^*(\theta) \\ \Sigma_u^*(\theta) &= \Gamma_{yy}^*(\theta) - \Gamma_{yx}^*(\theta)\Gamma_{xx}^{*-1}(\theta)\Gamma_{xy}^*(\theta) \end{aligned} \tag{43}$$

can be interpreted as the probability limits of the coefficients in a VAR estimated on artificial observations generated by the DSGE model. Conditional on the vector of structural parameters in the DSGE model θ, the prior distribution for the VAR parameters $p(\Phi, \Sigma_u|\theta)$, is of the Inverted-Wishart ($\mathcal{IW}$) - Normal ($\mathcal{N}$) form

$$\begin{aligned} \Sigma_u|\theta &= \mathcal{IW}\left(\lambda T \Sigma_u^*(\theta), \lambda T - k, n\right) \\ \Phi|\Sigma_u, \theta &= \mathcal{N}\left(\Phi^*(\theta), \Sigma_u \otimes (\lambda T \Gamma_{xx}^*)^{-1}\right) \end{aligned} \tag{44}$$

where $k = 1+np$. The tightness of the prior distribution is governed by the hyperparameter $\lambda \in [0, \infty]$. This hyperparameter can be loosely interpreted as the size of the sample of artificial or dummy observations generated by the DSGE model relative to the size of the actual sample in the estimation.

The posterior distribution of the VAR parameters is also of the Inverted-Wishart - Normal form (see Del Negro & Schorfheide (2004))

$$\begin{aligned} \Sigma_u|Y, \theta &= \mathcal{IW}\left((\lambda + 1)\, T\widetilde{\Sigma}_u(\theta), (1 + \lambda)\, T - k, n\right) \\ \Phi|Y, \Sigma_u, \theta &= \mathcal{N}\left(\widetilde{\Phi}(\theta), \Sigma_u \otimes \left(\lambda T \Gamma_{xx}^* + X'X\right)^{-1}\right) \end{aligned} \tag{45}$$

The matrices $\widetilde{\Phi}(\theta)$ and $\widetilde{\Sigma}_u(\theta)$ have the interpretation of maximum likelihood estimates of the VAR parameters based on the combined sample of actual observations and artificial observations generated by the DSGE model, that is

$$\widetilde{\Phi}(\theta) = \left(\lambda T \Gamma^*_{xx} + X'X\right)^{-1} \left(\lambda T \Gamma^*_{xy}(\theta) + X'Y\right) \tag{46}$$

$$\begin{aligned} \widetilde{\Sigma}_u(\theta) = & \frac{1}{(\lambda+1)T} \left(\lambda T \Gamma^*_{yy}(\theta) + Y'Y\right) \\ & -\frac{1}{(\lambda+1)T} \left(\lambda T \Gamma^*_{yx}(\theta) + Y'X\right) \left(\lambda T \Gamma^{*-1}_{xx}(\theta) + X'X\right)^{-1} \left(\lambda T \Gamma^*_{xy}(\theta) + X'Y\right) \end{aligned} \tag{47}$$

From the above expressions we see that if λ is small, the prior on the DSGE model restrictions is diffuse. In particular, setting $\lambda = 0$ we would retrieve the unrestricted OLS estimates. Notice, however, that in order for the prior distribution (44) to be proper, λ has to take a value larger than $\lambda_{\min} = (k+n)/T$ (see e.g., Adolfson et al. (2007b)). The higher is λ, the more the VAR estimates will be tilted towards the parameters in the VAR approximation of the DSGE model ($\Phi^*(\theta)$ and $\Sigma^*_u(\theta)$). Del Negro et al. (2007a) choose λ by maximising the marginal data density $p_\lambda(Y)$ over a pre-specified grid for λ. In this paper we specify a uniform distribution for λ over the interval $[\lambda_{\min}, \infty)$.

The specification of the VAR prior is completed with the specification of prior distributions for the DSGE model parameters θ.The DSGE-VAR approach allows us to draw posterior inferences about the DSGE model parameters θ. As explained by Del Negro & Schorfheide (2004), the posterior estimate of θ has the interpretation of a minimum-distance estimator, where the minimand or distance function is given by the discrepancy between the unrestricted OLS estimates of the VAR parameters and the coefficients in the VAR approximation to the DSGE model, the latter being functions of θ. Obviously, then, the posterior estimates of θ will depend on the hyperparameter λ. In the limit $\lambda \to 0$, there will not be any information about θ in $p(Y|\theta)$, and hence, the posterior estimates of θ will be equal to the prior estimates.

4 Empirical results

This section documents the estimation results of the full-sample exercise. First, we describe the data and the estimation method. Then we document estimation results for the DSGE and DSGE-VAR models based on the full sample 1987Q1-2007Q4.

4.1 The data

The model is estimated on quarterly, seasonally adjusted data for the Norwegian economy covering the period from 1987Q1 to 2007Q4. The sample period available for presample estimation is 1981Q4-1986Q4. The estimation is based on the following eleven variables: GDP, private consumption, business investment, exports, the real wage, the real exchange rate, overall inflation, imported inflation, the 3-month nominal money market rate, the overnight deposit rate (the policy rate) and hours worked. Since the model predicts that domestic GDP, consumption, investment, exports and the real wage are non-stationary, these variables are included in first differences. We take the log of the real exchange rate and hours worked.

The data series relate to the mainland economy, that is, the total economy excluding the petroleum sector. The series for GDP, exports, consumption, business investment and hours worked are measured relative to the size of the working age population (16-74 yrs.). The real wage is measured as total wage income per hour divided by the private consumption deflator. The quarterly series for growth in wage income per hour is obtained by a taking a linear interpolation of the annual series from the national accounts. The nominal exchange rate is an effective import-weighted exchange rate based on the bilateral exchange rates of the Norwegian krone versus 44 countries. Consumer price inflation is measured as the total CPI adjusted for taxes and energy (CPI-ATE), and imported inflation is measured as the inflation rate for imported goods in the CPI-ATE. The money market rate is the 3 months effective nominal money market rate (NIBOR). All the series are demeaned prior to estimation.

The choice of information set is based on data availability and on the perceived quality of the data series as well as a desire to obtain good estimates of the structural parameters in the DSGE model. In general, the issue of parameter identification points to including a large number of variables in the information set.[6] Within the context of a DSGE-VAR, however, the price of working with a large set of variables is that the size of the VAR becomes large relative to the sample size, resulting in imprecise estimates of the VAR parameters and wide forecast error bands. In particular, the VAR becomes much larger than what is typically used in standard forecasting applications.[7]

4.2 Estimation method

We estimate the DSGE and DSGE-VAR models using Bayesian techniques. The estimation of the DSGE model is based on the state-space representation (38). The likelihood function is evaluated using the Kalman filter and we use a Metropolis-Hastings (MH) algorithm to draw from the posterior distribution of the structural parameters starting from the posterior mode of the parameters computed in a first step. The estimation of the DSGE-VAR is based on the MCMC algorithm to draw from the joint posterior distribution of ϕ, Σ_u, θ described in Del Negro & Schorfheide (2004). The full-sample results reported below are based on 3 million draws from the posterior distribution. In the forecasting experiment, the number of draws in each recursion is 100000.[8]

Some of the parameters are fixed prior to estimation. The steady-state per capita growth rate π_z is calibrated to equal 2.25 per cent on an annualised basis. Based on current estimates,[9] we assume a long-run annual real interest rate of 2.5 per cent. Consistent with this, we set the discount factor β to 0.9994. The quarterly depreciation rate of capital is set to 1.8 per cent, which is in line with the recent estimates from the national accounts. The steady-state elasticity of substitution between differentiated intermediate goods, θ and θ^* is set to 6 corresponding to a price mark-up on marginal cost of 20 per cent. The home bias parameter,[10] ν, is set to 0.644 to ensure a steady state import share of roughly 30 per cent, and the elasticity of substitution between capital and labour, ξ, is set to 0.7, which yields a steady state wage income share of 0.6.

Some of the parameters turned out to be difficult to identify, like for example one of the parameters related to investment costs, ϕ_1^I. Furthermore, it is not possible to identify both intermediation cost parameters ϕ_1^B and ϕ_2^B, using a first order approximation of the model. We therefore set $\phi_1^B = \phi_1^I = 1$. The shape, the mean and the standard deviation of the prior distributions for the estimated parameters are given in tables 3 and 4.

[6] See e.g., the discussion in Adolfson et al. (2007a).

[7] For example, a typical VAR analysis of a small open economy contains a measure of real activity, inflation, the exchange rate and the interest rate in addition to foreign variables.

[8] The results are obtained using Dynare (see http://www.cepremap.cnrs.fr/dynare/) and our own Matlab codes for estimating of DSGE models with optimal policy under commitment and forecasting with a DSGE-VAR.

[9] See Norges Bank's Inflation Report 2/06.

[10] This parameter represents the share of domestic intermediates in the final goods aggregate that would prevail in the hypothetical case where the prices on domestic and imported goods were equal.

4.3 Full-sample estimation results

An important modelling choice for the DSGE-VAR is the choice of lag length. As argued by Del Negro et al. (2007b) there are essentially two dimensions to the choice of lag length for a DSGE-VAR. The first dimension is related to the accuracy of the VAR approximation to the DSGE model. This suggests we choose the lag-length to minimise the approximation error, that is, to minimise the discrepancy between the dynamics of the DSGE-VAR(∞) and the dynamics of the DSGE model. Since, in general, the accuracy of the approximation increases with lag length, this criterion points to having a fairly large number of lags. In the previous literature (see e.g., Adolfson et al. (2007b) and Del Negro et al. (2007a)), the lag-length has commonly been set to four based on this criterion. The second dimension to the choice of lag length is the empirical fit of the DSGE-VAR with the optimal value of λ,that is the DSGE-VAR($\widehat{\lambda}$). This suggests the we choose the lag length to maximise the marginal data density associated with the DSGE-VAR($\widehat{\lambda}$). As emphasized by Del Negro et al. (2007a), the DSGE-VAR exercise is meaningful even if the VAR approximation to the DSGE model is not exact. For our model(s) we find that the marginal data density is maximised for the model with two lags. The optimal value of the hyperparameter lambda is smaller in the model with two lags compared with the model with four lags, however. This reflects that the benefit gained from shrinking towards the theoretical model are smaller in the former case, since there are fewer free parameters in the VAR. Similar findings were reported by Del Negro & Schorfheide (2008).

Table 1 reports measures of the in-sample fit of the DSGE and DSGE-VAR models for alternative assumptions about the conduct of monetary policy. The marginal data density is measured using the modified harmonic mean estimator proposed by Geweke .A key result is that the model with a simple instrument rule is clearly dominated by the model with optimal policy in terms of in-sample fit. Hence, the implicit rule following from the assumption of optimal monetary policy, appears to be give a more accurate description of the way monetary policy was conducted over the sample period than does a simple instrument rule. This contrasts with the findings in Adolfson et al. (2008) on Swedish data, who report better in-sample fit for the model with a simple instrument rule. However, their instrument rule is somewhat more flexible than the instrument rule considered in this paper; it includes additional terms in both the change in inflation and in the growth rate of GDP, and moreover, includes a stochastic monetary policy shock.[11] The empirical merits of the optimal policy assumption is also reflected in the higher value of the hyperparameter λ in the DSGE-VAR with optimal policy (the posterior mean is 1.14 in the case of optimal policy and 0.89 in the model with a simple rule). We also see that the fit of the model is improved if we shrink the VAR parameters towards the restrictions implied by the DSGE model, or, alternatively, if we relax the DSGE model restrictions in the direction of the unrestricted VAR estimates. That is, the marginal data density is higher for the DSGE-VAR than for the DSGE model. This is true under both assumptions about monetary policy. In the next subsection we examine whether this holds true also in terms of out-of-sample forecasting performance.

Turning to the parameter estimates, table 2 reports the estimates of the monetary policy preferences from the DSGE model and the DSGE-VAR. In both cases, the estimates imply a high relative weight on interest rate changes in the loss function. The relative weight on inflation is slightly higher once we allow for model misspecification: the DSGE-

[11] The reason why we decided not to include a monetary policy shock in the instrument rule is that we wanted to make the model comparison exercise more meaningful. Assume that you allow for a random disturbance term in the simple instrument rule, but not in the model with optimal monetary policy, and that the monetary policy shock turns out to explain a large fraction of the variation in the policy rate. It is not clear which model explains the systematic part of monetary policy better, even if the model with a simple instrument rule achieves a higher marginal data density.

VAR estimates of the coefficients on the output gap and interest rate changes in the loss function are smaller than the DSGE estimates. Regarding the simple instrument rule, both the DSGE and the DSGE-VAR estimates point to a high weight on inflation and a substantial degree of interest rate smoothing.

The posterior estimates of the remaining parameters are reported in tables 3 and 4. Comparing the DSGE models, the parameter estimates do not seem to be significantly influenced by the choice of monetary policy. This conclusion is supported by the impulse responses of the estimated shocks, which appear fairly similar. However, there is one notable exception to this conclusion. The price stickiness on domestic goods is estimated to be significantly higher in the model employing a simple instrument rule. As we shall see in the next section, this has important implications for the forecasting properties of the two models, in particular for inflation and the interest rate. Turning to the DSGE-VAR models, we note that the parameter estimates differ even less than for the DSGE models. This reflects the fact that part of the differences in the estimated DSGE parameters are due to misspecification. One way to think about this is that misspecification adds an extra source of variation to the estimated parameters. Another robust finding is that the degree of external persistence is reduced significantly once misspecification is taken into account. It is clear from 4 that both the correlation coefficient and the standard deviation of the shock processes are in general lower in the DSGE-VAR models than the DSGE models. Hence, taking into account misspecification reduces the need for exogenous persistence.[12]

5 Forecast comparison

The forecast experiment is constructed as follows. We estimate each model on a sample period ending 1998Q4 and compute forecasts for horizons of one up to twelve quarters. We then extend the sample by one quarter, demean the data, re-estimate the models and compute new forecasts. The implicit steady-states of the variables are allowed to vary over time; we demean the data prior to estimation in each recursion. This exercise is repeated until the end of the sample. Notice that all the parameters in the DSGE model and the DSGE-VAR, including the hyperparameter λ are re-estimated in each recursion. The DSGE and DSGE-VAR forecasts are based on 100000 MH draws starting from the posterior mean of the previous recursion. In addition to the DSGE and DSGE-VAR we compute forecasts from an unrestricted VAR and a Bayesian VAR (BVAR) with a Minnesota-type prior. The prior in the BVAR will tilt the VAR towards univariate random walks of the variables in levels. The lag-length in all the VAR models is set to two.

We measure forecasting accuracy by univariate root mean squared forecast error (RMSE). The point forecasts used to calculate the RMSEs are the posterior means of the forecast draws. Following Adolfson et al. (2007c) we also a compute a measure of multivariate forecast accuracy, namely the trace of the mean squared forecast error (MSE) matrix for horizon h. The MSE matrix is denoted $\Omega_M(h)$ and is defined as

$$\Omega_M(h) = \frac{1}{N_h} \sum_{t=T}^{T+N_h-1} \left(y_{t+h} - \widehat{y}_{t+h|t}\right) M^{-1} \left(y_{t+h} - \widehat{y}_{t+h|t}\right)', \qquad (48)$$

where N_h is the number of forecasts and M is a diagonal matrix with the sample variances of the variables as diagonal elements. For the variables that enter the model in growth rates, we follow Del Negro et al. (2007a) and report the RMSE for the cumulative changes in the variables.

Figure 1 plots the univariate RMSEs from the DSGE model under the different assumptions about monetary policy. The ranking of the models is less clear than was the

[12] Similar findings are again reported by Del Negro & Schorfheide (2008).

case when using measures of in-sample fit based on the full sample. In terms of forecasting accuracy the models perform about equally well. The model with optimal policy produces more accurate forecasts of the growth rates of GDP, consumption and investment, whereas the model with a simple instrument rule produces more accurate forecasts of the inflation and interest rates. We conjecture that one reason why the model with a simple instrument rule produces more accurate forecasts of inflation and interest rates is that price stickiness parameter is estimated to be higher in this version of the model, giving rise to weaker equilibrium-correction. Figure 2 compares the univariate RMSEs from the DSGE model with optimal policy to those obtained using a DSGE-VAR approach, an unrestricted VAR and the BVAR. For most variables, relaxing the cross-equation restrictions in the DSGE model towards an unrestricted VAR improves the forecasting performance. However, in terms of forecasting performance the DSGE-VAR models are inferior to the BVAR with a Minnesota prior. This findings is confirmed in figure 3 which reports a multivariate measure of forecast accuracy.

The fact that the BVAR outperforms the DSGEs and the DSGE-VARs is perhaps not surprising. The BVAR prior tilts the unrestricted VAR towards univariate random walks. Given that inflation and interest rates are only borderline stationary in our sample, this seems like a very reasonable prior. The DSGE prior, on the other hand, tilts the VAR towards a multivariate mean reverting process.

5.1 Comparison with Norges Bank's official forecasts

In this section, we compare the official Norges Bank projections of interest rates and inflation with the corresponding model projections. The exercise is somewhat restricted by the fact that official forecasts are only available from 2005 onwards, however, we still believe that it provides some interesting insights.

Figure 4 shows the DSGE forecasts and the official forecasts for each quarter in the period 2005q3-2008q2.[13] As is evident from the figures, both versions of the DSGE model consistently predict a sharper increase in interest rates than the official forecasts. This is especially true for the model assuming optimal policy. Furthermore, we observe that Norges Banks forecasts are more in line with the actual interest path. However, in contrast to the DSGE models, there seems to be a slight tendency for the Norges Bank forecast to under-predict the actual interest path. Turning to the DSGE-VAR forecasts we note that, in this case, the differences between the model forecasts and the official forecasts are quite small. Hence, accounting for misspecification brings the model interest rate projections much more in line with the published forecasts. Similar conclusions are reached when it comes to inflation (see figure 5): the DSGE models tend to over-predict underlying inflation for the whole period, both relative to actual inflation and the official forecasts, whereas the DSGE-VAR forecasts are closer to both actual inflation and the official projections.

A notable feature of the interest rate and inflation projections from the DSGE model with optimal policy is that they 'overshoot' the long-run level in the medium run. This feature of optimal policy under commitment is less pronounced in Norges Bank's projections since 2005, and is not a feature of the DSGE-VAR forecasts. This is a sign that the model with optimal monetary policy is misspecified. One interpretation is that the Norges Bank does not fully exploit the expectations channel when setting policy, or alternatively, that it perceives the gains from commitment in the current specification of the DSGE model to be too large (e.g., that the price-setters in the model are in a sense too forward-looking).

[13] Norges Bank publishes forecasts three times a year. To compare these forecasts to the forecasts from our quarterly model we have added a "synthetic" forecast round with forecasts equal to the previously published path. In general, the forecasts made by Norges Bank are made later in time and in that sense incorporate more information than the model forecasts.

One tentative conclusion one could draw from this exercise is that the DSGE-VAR models mimics the actual 'mental' model or modelling process used by Norges Bank in its projection exercises. As noted above, the DSGE model employed in this paper is broadly similar to the core model used for policy projections at the Norges Bank. However, arriving at the final official projections is a complicated process, involving input from other forecasting models, add factors and off-model considerations. Our results indicate that the 'mental' model, or the iterative forecasting process, used by the Norges Bank can be well represented by a DSGE-VAR model, where the restrictions from the core DSGE model can be interpreted as a prior on the VAR parameters.

6 Concluding remarks

The preliminary results in this paper suggest that traditional measures of in-sample fit favour the DSGE model estimated under the assumption of optimal policy, whereas the out-of-sample forecasting exercise provides no clear ranking of the two alternative model specifications. This conclusion also holds when taking account of misspecification. One way of interpreting the DSGE-VAR results is that introducing optimal monetary policy reduces the degree of misspecification. However, as is evidenced both by the optimal value of the DSGE-VAR hyperparameter and the forecast performance of the different models, model misspecification remains a serious concern for the use of DSGE models in practical policy analysis. Ongoing work attempts to estimate the model under the assumption that monetary policy is conducted optimally under discretion rather than commitment. Another interesting extension will be to allow for regime-switching in the monetary policy parameters.

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	Marginal data density	Weight on DSGE λ
DSGE optimal policy	3223.0	–
DSGE simple rule	3157.3	–
DSGE-VAR optimal policy	3247.8	1.1381
DSGE-VAR simple rule	3223.1	0.8929

Table 1: The fit of the DSGE and DSGE-VAR models under different assumptions about monetary policy

	Prior mean	Posterior mean	
		DSGE	DSGE-VAR
Optimal policy			
Weight on output gap ω_y	0.5	0.2508	0.2285
Weight on interest rate $\omega_{\Delta r}$	0.2	0.4400	0.4688
Simple instrument rule			
Weight on inflation ω_π	2.0	1.5031	1.7986
Weight on output gap ω_y	0.2	0.4552	0.3830
Weight on interest rate ω_r	0.8	0.6720	0.7021
Weight on real exchange rate ω_{rer}	0.0	0.0202	0.0033

Table 2: Estimates of monetary policy preferences in DSGE and DSGE-VAR with optimal monetary policy. The weight on the inflation term in the loss function is normalised to unity.

Parameter	Prior		DSGE opt	DSGE-VAR opt	DSGE simp	DSGE-VAR simp
	Type	Mean (std)	Mean (std)	Mean (std)	Mean (std)	Mean (std)
α	Beta	0.30 (0.020)	0.2973 (0.0196)	0.2908 (0.0217)	0.2968 (0.0201)	0.2844 (0.0202)
ψ	Inv gam	5.50 (0.500)	4.9056 (0.5624)	5.1585 (0.5095)	4.8298 (0.4838)	5.2486 (0.5409)
ζ	Inv gam	3.00 (0.200)	2.7867 (0.2807)	2.8721 (0.2940)	2.8622 (0.2771)	2.8984 (0.2878)
μ	Inv gam	1.10 (0.200)	1.1472 (0.0494)	1.2116 (0.0621)	1.2497 (0.0557)	1.3252 (0.0571)
μ^*	Inv gam	1.10 (0.200)	1.1864 (0.1387)	1.0665 (0.2220)	1.3298 (0.2263)	1.0856 (0.2988)
b_C	Beta	0.75 (0.050)	0.7879 (0.0450)	0.7202 (0.0395)	0.7989 (0.0348)	0.7411 (0.0333)
ϕ^M	Inv gam	1.00 (1.000)	1.7113 (0.1740)	1.8087 (0.3929)	1.7180 (0.2703)	1.8344 (0.3232)
ϕ^Q	Inv gam	1.00 (1.000)	1.9145 (0.1229)	1.8168 (0.0.4621)	2.4521 (0.4844)	2.1027 (0.9027)
ϕ^W	Inv gam	1.00 (1.000)	2.7469 (0.4183)	2.2420 (0.5950)	2.8860 (0.4940)	2.1393 (0.7644)
ϕ_2^I	Gam	10.00 (5.000)	17.7661 (2.7254)	12.2380 (3.5670)	19.180 (3.9843)	14.0437 (4.0315)
ϕ_2^B	Inv gam	0.02 (0.005)	0.0173 (0.0037)	0.0167 (0.0033)	0.0166 (0.0027)	0.0199 (0.0031)

Table 3: Posterior mean of DSGE model parameters I

Parameter	Prior		DSGE opt	DSGE-VAR opt	DSGE simp	DSGE-VAR simp
	Type	Mean (std)	Mean (std)	Mean (std)	Mean (std)	Mean (std)
σ_z	Inv gam	0.0050 (Inf)	0.0115 (0.0013)	0.0060 (0.0011)	0.0140 (0.0011)	0.0062 (0.0013)
σ_r	Inv gam	0.0025 (Inf)	0.0015 (0.0002)	0.0011 (0.0001)	0.0017 (0.0001)	0.0010 (0.0001)
σ_ψ	Inv gam	1.0000 (Inf)	0.9140 (0.2168)	0.7140 (0.1640)	1.2070 (0.2393)	0.6606 (0.2587)
σ_θ	Inv gam	1.0000 (Inf)	0.5644 (0.0617)	0.5677 (0.1292)	0.5083 (0.0967)	0.5316 (0.1316)
σ_I	Inv gam	1.0000 (Inf)	1.5296 (0.2212)	0.7492 (0.2687)	1.5836 (0.2767)	0.7876 (0.3004)
σ_L	Inv gam	0.0050 (Inf)	0.0112 (0.0013)	0.0068 (0.0010)	0.0147 (0.0012)	0.0071(0.0013)
σ_B	Inv gam	0.0100 (Inf)	0.0028 (0.0001)	0.0039 (0.0005)	0.0031 (0.0004)	0.0038 (0.0005)
σ_{mc^*}	Inv gam	0.0100 (Inf)	0.2689 (0.0209)	0.2465 (0.0722)	0.2903 (0.0629)	0.2083 (0.0710)
σ_{y^*}	Inv gam	0.0100 (Inf)	0.0348 (0.0036)	0.0219 (0.0028)	0.0342 (0.0026)	0.0214 (0.0031)
σ_g	Inv gam	0.0100 (Inf)	0.0541 (0.0058)	0.0315 (0.0046)	0.0615 (0.0046)	0.0322 (0.0048)
σ_u	Inv gam	0.0100 (Inf)	0.0527 (0.0068)	0.0276(0.009)	0.0593 (0.0090)	0.0293 (0.0119)
λ^z	Beta	0.8500 (0.1)	0.1316 (0.0730)	0.1224 (0.0712)	0.0762 (0.0389)	0.0957 (0.0469)
λ^ψ	Beta	0.8500 (0.1)	0.7396 (0.0757)	0.4981 (0.0405)	0.6681 (0.0495)	0.4945 (0.0426)
λ^θ	Beta	0.8500 (0.1)	0.6767 (0.1232)	0.3310 (0.0725)	0.7495 (0.0605)	0.3830 (0.0521)
λ^I	Beta	0.8500 (0.1)	0.1538 (0.0760)	0.1585 (0.0711)	0.1166 (0.0589)	0.1591 (0.0631)
λ^L	Beta	0.8500 (0.1)	0.8538 (0.0400)	0.7429 (0.0310)	0.8632 (0.0284)	0.8215 (0.0353)
λ^B	Beta	0.8500 (0.1)	0.9386 (0.0486)	0.8194 (0.0213)	0.9252 (0.0196)	0.7862 (0.0195)
λ^{mc^*}	Beta	0.8500 (0.1)	0.5617 (0.1283)	0.4181 (0.1009)	0.5060 (0.1007)	0.4479 (0.0987)
λ^{y^*}	Beta	0.8500 (0.1)	0.6125 (0.0872)	0.2548 (0.0544)	0.6773 (0.0522)	0.3879 (0.0508)
λ^g	Beta	0.8500 (0.1)	0.8840 (0.0777)	0.6830 (0.0223)	0.8497 (0.0331)	0.6489 (0.0330)
λ^r	Beta	0.8500 (0.1)	0.7844 (0.0451)	0.5317(0.0533)	0.5251 (0.0195)	0.5017 (0.0200)
λ^u	Beta	0.8500 (0.1)	0.3143 (0.0678)	0.3902(0.0935)	0.2913 (0.0663)	0.3296(0.0664)

Table 4: Posterior mean of DSGE model parameter II

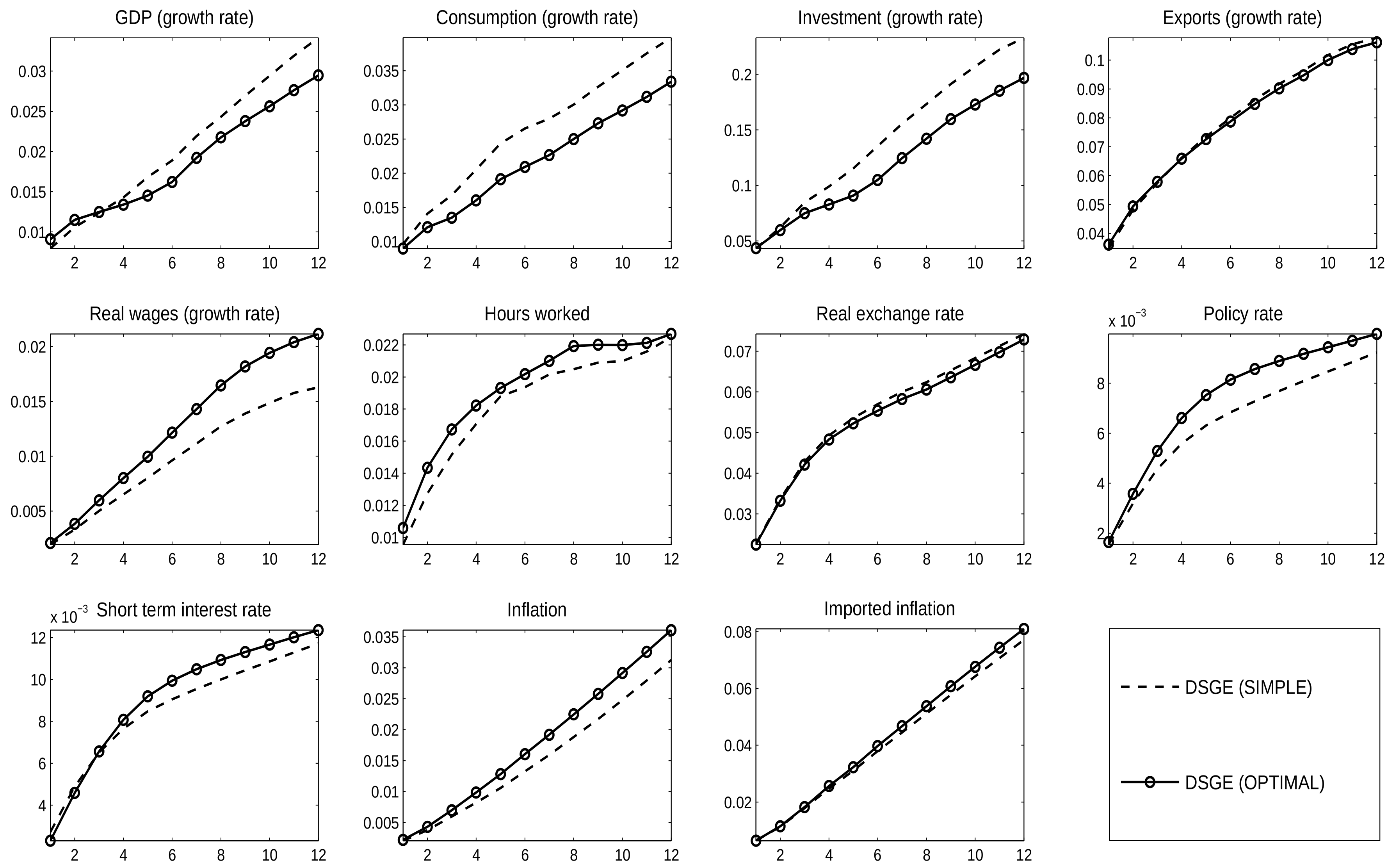

Figure 1: Univariate RMSEs for DSGE model with optimal policy and DSGE with simple instrument rule

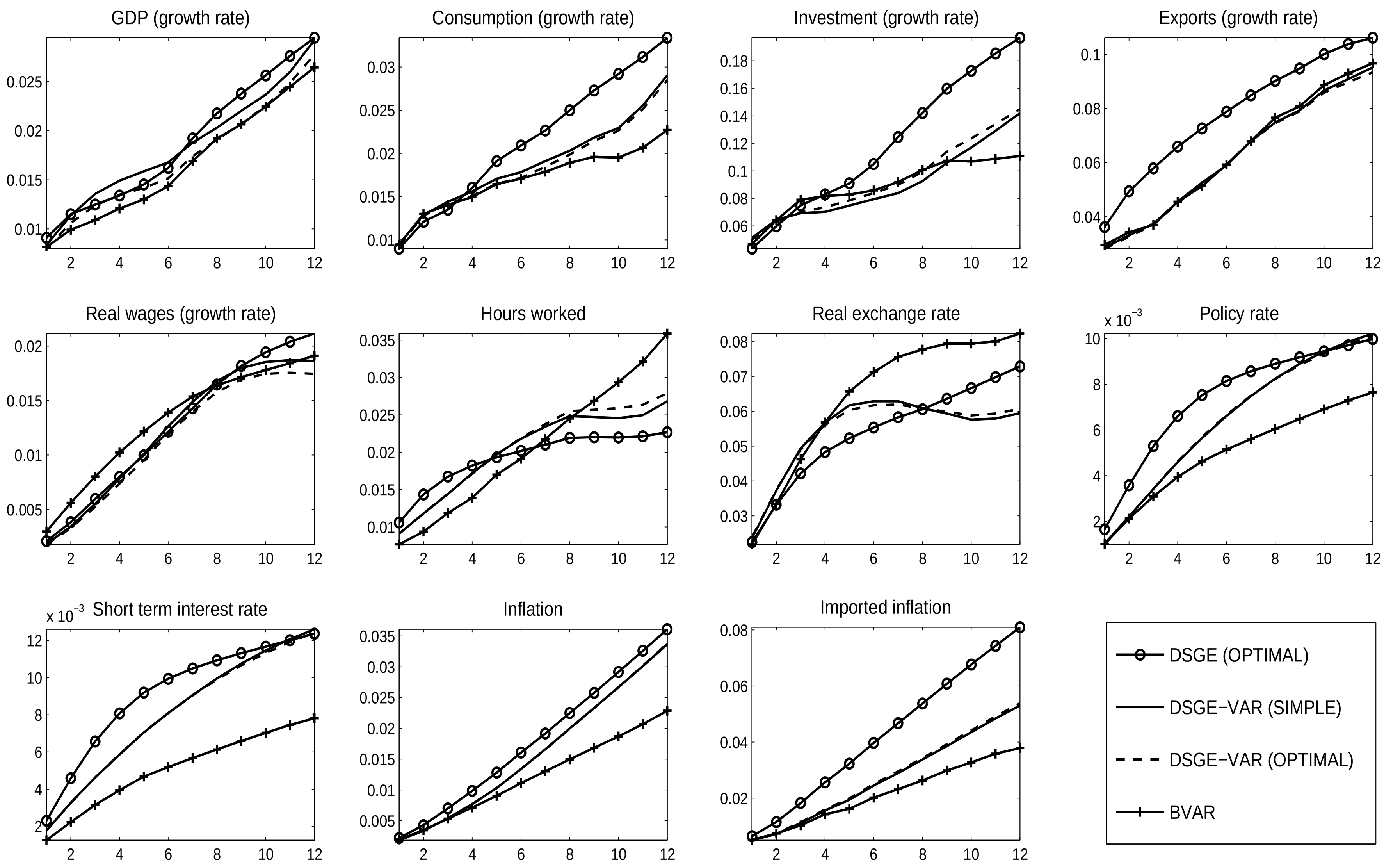

Figure 2: Univariate RMSEs for DSGE model with optimal policy, DSGE-VAR with simple instrument rule, DSGE-VAR with optimal policy and BVAR

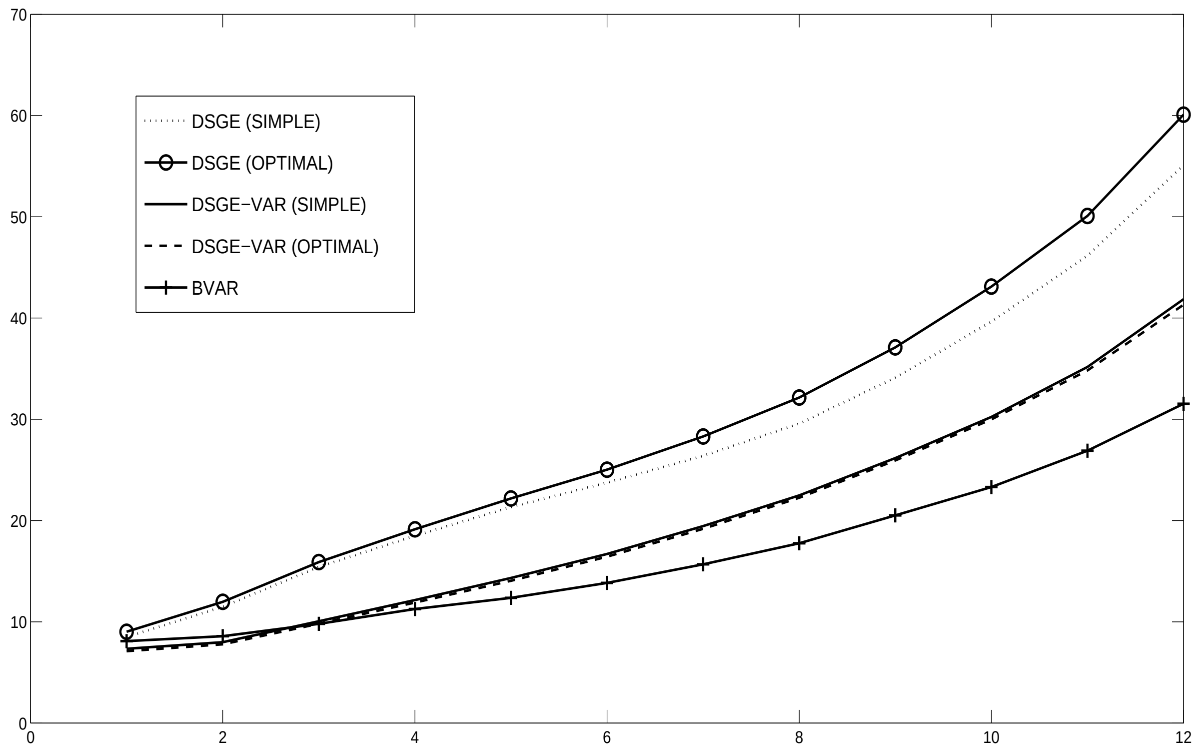

Figure 3: Multivariate trace statistic for DSGE model with optimal policy, DSGE model with simple instrument rule, DSGE-VAR with optimal policy, DSGE-VAR with simple instrument rule and BVAR

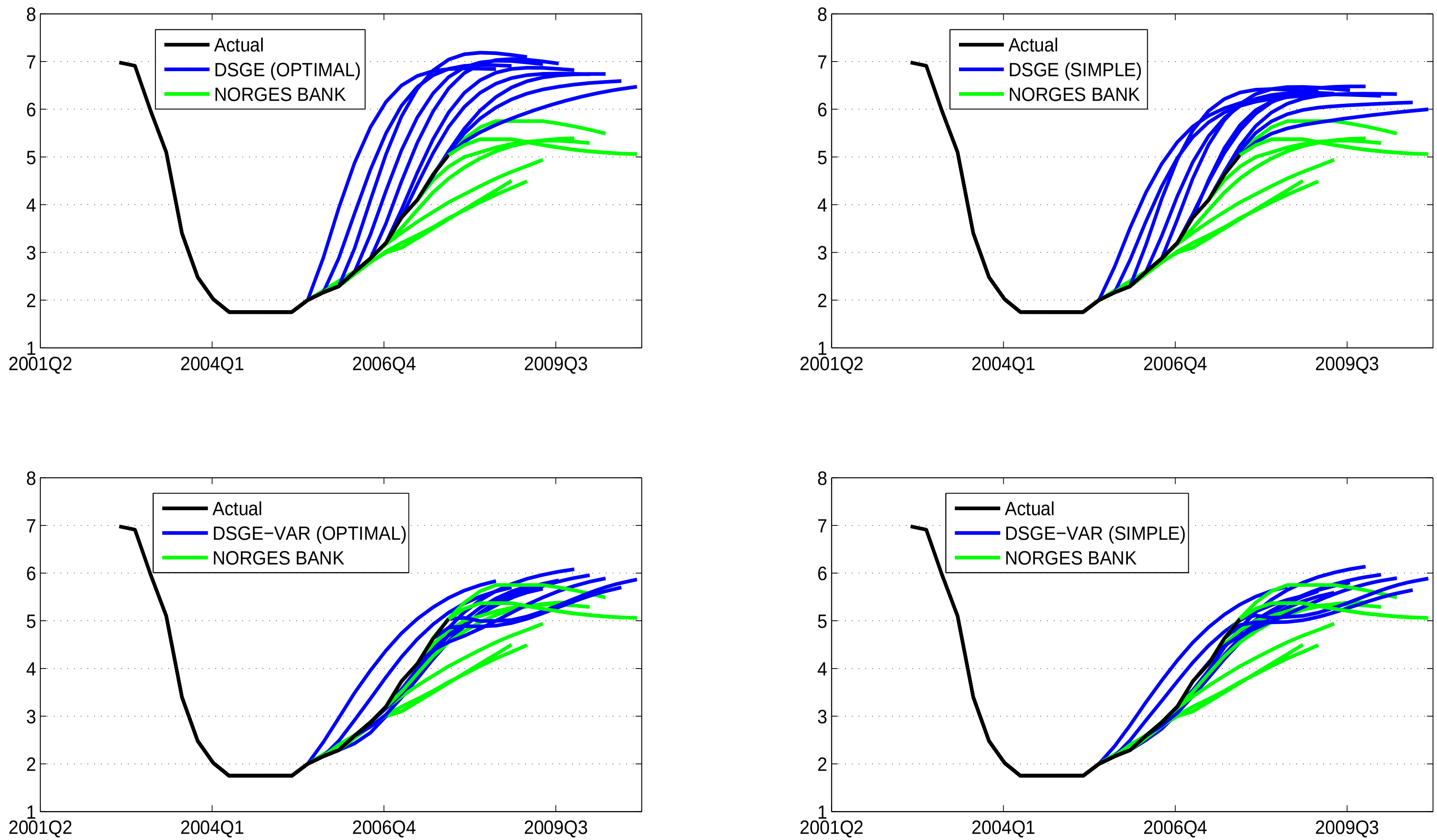

Figure 4: Actual policy rate, Norges Bank's official forecasts and model forecasts

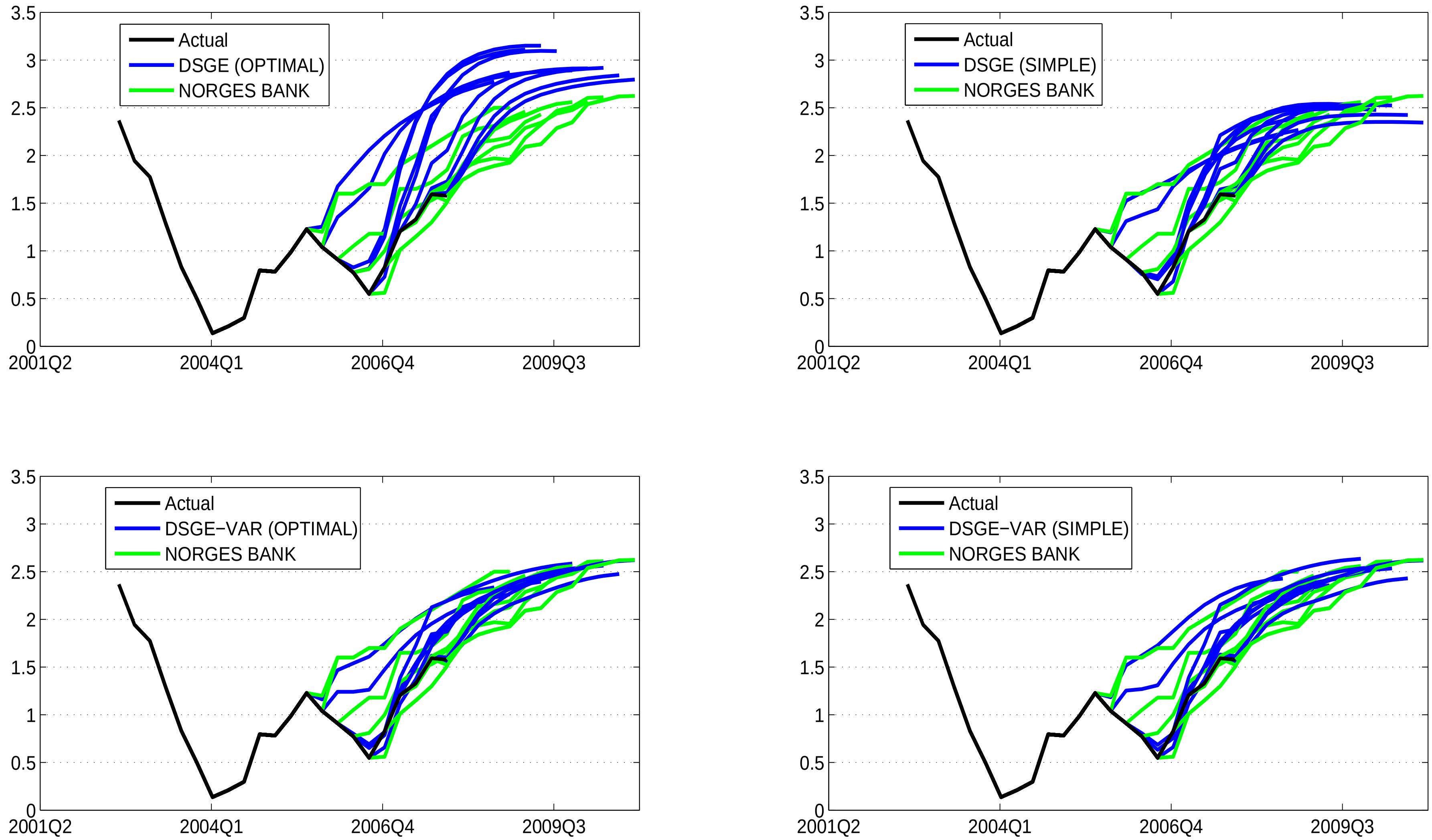

Figure 5: Actual four quarter inflation, Norges Bank's official forecasts and model forecasts