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How New Keynesian is the US Phillips Curve?

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Abstract

I provide a generalization of Calvo price setting, to include non-overlapping contracts as a special case and embed this in a small DSGE model. The resulting Generalized Phillips Curve (GPC) nests New-Keynesian and Neoclassical versions. I linearize the model around a potentially non-zero trend inflation rate, and estimate it on US data using Bayesian methods, allowing for Markov switching in the variances of structural shocks. I find that the Phillips curve is 100% New Keynesian. There is no evidence of either forward or backward indexation. I illustrate that trend inflation affects the estimation of the Phillips curve.

JEL-codes:

E13, E31.

Keywords:

Phillips curve, neoclassical, indexation, trend inflation, regime switch.

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Introduction

I provide a generalization of the price setting described in Calvo (1983) that includes non-overlapping contracts as a special case.¹ The resulting Generalized Phillips Curve (GPC) nests New Keynesian and Neoclassical versions. Models with one-period price contracts are applied in the theoretical literature, and have policy implications that differ from the overlapping contracts case.² The empirical relevance of the non-overlapping contract model is therefore of interest. I estimate this generalized Phillips curve³ as part of a small DSGE model on US data using Bayesian methods. I allow for Markov switching in structural shocks. I find that the GPC may be described as 100% New Keynesian.

I obtain the GPC model by assuming that a fraction of the agents in the Calvo model who do not reset their price optimally in a given period, instead index their price to the expected next period price index. Full forward indexation reduces to a model with prices set one period in advance⁴. That produces a Neoclassical Phillips curve. On the other hand, zero indexation produces the standard New Keynesian Phillips curve, with overlapping contracts. The GPC model captures both cases, as well as intermediate ones.

As emphasized by Hornstein (2007), Cogley and Sbordone (2008), Cogley, Primiceri, and Sargent (2010) and Ascari and Sbordone (2013), the reduced form parameters of the Phillips curve are not policy invariant: They

¹Since Fischer (1977), Gray (1978), Taylor (1979) and Calvo (1983), staggered (overlapping) contracts as opposed to one-period contracts, have been the standard for nominal rigidities in applied macromodels. See Taylor (1999) and Fuhrer (2010).

²A discussion of optimal policy in a one-period non-overlapping contract model is provided in Mankiw and Weinzierl (2011). The model is a simple version of a sticky-information model. One period contracts are also discussed in Woodford (2003), chapter 3, section 1. As a minimalist way of introducing real effects of monetary policy, the non-overlapping contracts model has been used in the new open economy macro literature, see Obstfeld and Rogoff (1996) chapter 10, and also Corsetti and Pesenti (2005). The framework of Krugman (1998) has an interpretation as one with a one-period price contract.

³In accordance with common practice, I use the term "Phillips curve" to describe a relationship between inflation and output. Some would argue that the term "aggregate supply curve" would be more appropriate for this purpose, and reserve the term "Phillips curve" for the relationship between unemployment and inflation. Fuhrer (2010) uses the term "inflation Euler equation".

⁴That model is also equal to the information lag model of Ball, Mankiw, and Reis (2005), in the special case of an information lag equal to one.

depend on the trend price inflation rate. Hence, I linearize the model around a potentially non-zero trend inflation rate, and I also recognize the effect of trend growth on the parameters of the GPC.

In the empirical section of this paper, I ask whether the US Phillips curve has been a mix between Neoclassical and New Keynesian versions, or whether the pure version of either fits the data better. I answer by estimating the degree of forward indexation in the GPC. Using US data for price inflation, output and interest rates, I follow Liu, Waggoner, and Zha (2011), and allow for Markov switching in structural shocks to the economy. For comparison, I also estimate a non-nested version of the model with a standard hybrid New Keynesian Phillips curve (HNKPC), which is based on potential backward indexation. The two models are identical and equal to the purely forward-looking NKPC when forward- and backward indexation, respectively, is equal to zero. That special case is preferred by the data. The model with Markov switching in variances outperforms a version with constant variances. I also estimate the models on demeaned data, where I counterfactually calibrate trend inflation and output growth to zero. Based on those estimations, one would mistakenly find evidence of indexation.

The next section presents the model. In section 2, I present the equilibrium conditions and the steady state, in section 3 monetary and fiscal policy is discussed, and in section 4 the full set of log-linearized equilibrium conditions are presented. In section 5, I describe empirical results.

1 The model

A representative yeoman farmer⁵ maximizes her objective with respect to consumption \tilde{C} , her output price X , money m and bonds B , subject to a period budget constraint,⁶

$$E_n \left\{ \sum_{t=n}^{\infty} \beta^{t-n} u(\tilde{C}_t) + v(Y_t; \tilde{k}_t) + f\left(\frac{m_t}{P_t}\right) \right\}, \quad (1)$$

$$(1 + \omega_t)X_t\tilde{Y}_t + m_{t-1} + (1 + i_{t-1})B_{t-1}^g + B_{t-1} = t_t + P_t\tilde{C}_t + m_t + B_t^g + \delta_{t,t+1}\mathbf{B}_t. \quad (2)$$

⁵See Woodford (2003), p. 149 for a discussion of the yeoman farmer model.

⁶To save notation, capital letters are used for individual as well as aggregate variables. With a total mass of identical agents equal to one, and perfect risk sharing, aggregate consumption and output will equal individual consumption in equilibrium.

The constraint says that nominal income from production \tilde{Y}_t , sold at price X_t including taxes or subsidies, $(1+\omega_t)$, plus financial assets and their return brought over from last period (money m_{t-1} , state contingent claims B_{t-1} and government bonds $(1+i_{t-1})B_{t-1}^g$) must equal (lump sum) nominal taxes t_t , nominal consumption expenditure $P_t\tilde{C}_t$ and new holdings of financial assets⁷.

A no-Ponzi game constraint rules out unbounded borrowing;

$$E_t \left\{ \lim_{s \rightarrow \infty} \frac{m_{t+s} + B_{t+s}}{P_{t+s}} \prod_{k=t}^{k=s} (1+i_k)^{-1} \geq 0 \right\}. \quad (3)$$

Period utility from the composite consumption good \tilde{C} is

$$u(\tilde{C}_t) = \frac{\tilde{C}_t^{1-\rho} - 1}{1-\rho}, \quad (4)$$

where the composite consumption good is

$$\tilde{C}_t \equiv \left[\int_{j=0}^1 (\tilde{C}(j)_t)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}. \quad (5)$$

Producers of different period t goods $\tilde{C}(j)_t$ are indexed by j , and θ describes the demand elasticity of substitution between goods. As described in B, demand for consumption good j is given by

$$\tilde{C}(j)_t = \left(\frac{X(j)}{P} \right)^{-\theta} \cdot \tilde{C}_t. \quad (6)$$

The period disutility from producing output \tilde{Y}_t for each agent is

$$v(\tilde{Y}_t; \tilde{\kappa}_t) = -\frac{1}{2} \tilde{\kappa}_t \tilde{Y}_t^2. \quad (7)$$

⁷ B_t^g is the nominal value of risk free government bonds, while B_t is a vector of quantities of state contingent claims, and $\delta_{t,t+1}$ is the vector of the prices of those claims. Each state contingent claim pays one unit of currency in the subsequent period given a particular realization of the state in that period. The gross risk free nominal interest rate, $1+i_t$ (I will also use I_t for this variable) is therefore equal to $[\delta_{t,t+1} \cdot \mathbf{1}]^{-1}$, where $\mathbf{1}$ is a vector of ones.

$\tilde{\kappa}_t$ is an exogenous aggregate supply shock, or "laziness" shock⁸. Utility from real money balances is additively separable and given by some function $f(\frac{m}{P})$, $f' \geq 0$, $f'' \leq 0$.

1.1 The flexible price model

First order conditions for utility maximization with respect to consumption and asset holdings give the consumption Euler equation,

$$\tilde{C}_t^{-\rho} = E_t \left\{ \frac{\beta(1+i_t)}{P_{t+1}/P_t} \tilde{C}_{t+1}^{-\rho} \right\}. \quad (8)$$

The condition for optimal price setting of price $X(i)$ by agent i , if prices are perfectly flexible is (see appendix A)

$$\frac{X(i)_t}{P_t} = \frac{\frac{\theta}{\theta-1} \tilde{\kappa}_t \tilde{Y}_t}{1 + \omega_t \tilde{C}_t^{-\rho}}. \quad (9)$$

This says that the relative price $\frac{X(i)_t}{P_t}$ should equal the marginal rate of substitution between production and consumption, corrected for any markup net of subsidies,

$$\mu_t = \frac{\frac{\theta}{\theta-1}}{1 + \omega_t}.$$

I will use the notation $\widetilde{MC} = v'_y = \tilde{\kappa}_t \tilde{Y}_t$ and $\widetilde{MU} = u'_c = \tilde{C}_t^{-\rho}$.

There is no government consumption. Equilibrium output and consumption under flexible prices is determined by productivity ($\tilde{\kappa}_t$) and the distortion from monopolistic competition and fiscal policy (μ_t);

$$\tilde{Y}_t = (\mu_t \tilde{\kappa}_t)^{-\frac{1}{1+\rho}}. \quad (10)$$

With output given by exogenous shocks and fiscal policy, monetary policy and the consumption Euler equation are left to pin down price inflation and interest rates in the flexible price model.

⁸In a yeoman farmer model, the labor market is internalized. κ may be interpreted as a labor supply shock or a productivity shock. In particular, following Obstfeld and Rogoff (1996), the productivity variable κ may be understood as follows: Let disutility from work effort l be given by $-\phi l$ and the production function be Al^α , $\alpha < 1$. Inverting the production function gives $l = (\frac{y}{A})^{1/\alpha}$. Given $\alpha = \frac{1}{2}$ and $\kappa = \frac{2\phi}{A^{1/\alpha}}$, we get $-\phi(\frac{y}{A})^{1/\alpha} = -\frac{1}{2}\kappa y^2$.

1.2 Price setting that nests Calvo Price setting and one-period contracts

In order to introduce nominal rigidities, I assume that in any period, a fraction α of arbitrarily chosen price setters are not free to adjust their price, as in Calvo (1983). Price setters sell whatever volume is demanded at the price they set. There is indexation of some or all of the sticky prices to expected next period price inflation⁹;

$$X_t(j) = [E_{t-1}(\Pi_t)]^\sigma X_{t-1}(j), \Pi_t = P_t/P_{t-1}. \quad (11)$$

One interpretation of this price indexation scheme is that the non-optimizing price setters have access to one period lagged information. A fraction σ of agents (arbitrarily chosen), are allowed to act on it. With $\sigma = 1$, this price setting corresponds to the lagged information model of Mankiw and Reis (2002) and Ball, Mankiw, and Reis (2005), in the special case of a one period information lag.

Thus, the model allows for full updating of non-optimal prices with one-period delayed information ($\sigma = 1$), or only partial updating ($\sigma < 1$). The case of full indexation ($\sigma = 1$), implies that the overlapping contracts of the Calvo model in effect are replaced by a fraction $(1 - \alpha)$ of flexible prices and a fraction α of one-period contracts.

In appendix C, I derive optimal price setting. The relative price set by flexible-price agents today, $X_t/P_t = x_t$, depends on the relationship between current and expected future costs from producing on the one hand, and current and future marginal utility from consuming on the other, where the future is weighted by the likelihood α that the price will stay effective going forward. This is captured in equation (1.2) in table 1.

The optimal relative price also depends on competition among producers as captured by θ , and the production subsidy ω_t , and a possible need to front-load price increases (decreases) due to trend inflation (deflation), in case of less than full indexation ($\sigma < 1$). The equilibrium conditions are summarized in table 1. Equation 1.1 is the consumption Euler equation with equilibrium output substituted in for consumption. Equation 1.2 is the price setting equation, and 1.3 is the price index. There is no explicit production sector in the yeoman farmer model. Output and inflation are

⁹I thank an anonymous referee for suggesting that this indexation scheme could capture the Neoclassical one period contract case.

determined jointly in combination with some monetary policy that remains to be specified.

Table 1: Equilibrium conditions	
$\tilde{Y}_t^{-\rho} = E_t\left\{\frac{\beta(1+i_t)}{P_{t+1}/P_t}\tilde{Y}_{t+1}^{-\rho}\right\}.$	(1.1)
$x_t^{1+\theta} = \frac{\theta}{(\theta-1)}\frac{\tilde{K}_t}{\tilde{D}_t}, \quad x_t = X_t/P_t$	(1.2a)
$\tilde{K}_t \equiv \widetilde{MC}_t + \alpha\beta E_t[G_{t,1}\psi_{t+1}^{-2\theta}\tilde{K}_{t+1}], \quad G_{t,1} \equiv \frac{\tilde{Y}_{t+1}}{\tilde{Y}_t}$	(1.2b)
$\tilde{D}_t \equiv \widetilde{MU}_t(1 + \omega_t) + \alpha\beta E_t[G_{t,1}\psi_{t+1}^{1-\theta}\tilde{D}_{t+1}],$	(1.2c)
$1 = (1 - \alpha)x_t^{1-\theta} + \alpha\{\psi_t\}^{1-\theta}, \quad \psi_t \equiv [E_{t-1}(\Pi_t)]^\sigma (\Pi_t)^{-1}$	(1.3)

2 Normalized equilibrium conditions and the steady state

I redefine the real variables in order to establish a model in terms of stationary real variables. Non-detrended variables have decorations like \tilde{Z} , while the corresponding detrended variables do not. Let the detrended level of consumption C_t be equal to $\frac{\tilde{C}_t}{\tilde{Z}_t}$, and detrended output $Y_t = \frac{\tilde{Y}_t}{\tilde{Z}_t}$. The trend growth factor is $\frac{\tilde{Y}_t}{\tilde{Y}_{t-1}} = \frac{\tilde{Z}_t}{\tilde{Z}_{t-1}} \equiv \Gamma$.

The flexible-price level of output Y_t^{flex} in detrended form is a function of the detrended productivity shock defined as $\kappa_t \equiv \tilde{\kappa}_t Z_t^{1+\rho}$;

$$\begin{aligned} \tilde{Y}_t^{flex} &= (\mu_t \tilde{\kappa}_t)^{-\frac{1}{1+\rho}}, \\ \tilde{Y}_t^{flex}/Z_t &= (\mu_t \tilde{\kappa}_t Z_t^{-(1+\rho)})^{-\frac{1}{1+\rho}} \frac{1}{Z_t} \Rightarrow \\ Y_t^{flex} &\equiv (\mu_t \kappa_t)^{-\frac{1}{1+\rho}}. \end{aligned} \tag{12}$$

Growth in actual (non-detrended) output is equal to Γ times growth in detrended potential output;

$$\tilde{Y}_t^{flex}/\tilde{Y}_{t-1}^{flex} = \Gamma \cdot \left(\frac{\mu_t \tilde{\kappa}_t}{\mu_{t-1} \tilde{\kappa}_{t-1}}\right)^{-\frac{1}{1+\rho}} = \Gamma \cdot Y_t^{flex}/Y_{t-1}^{flex} \tag{13}$$

Detrended marginal cost is derived:

$$\widetilde{MC}_t = \tilde{\kappa}_t \tilde{Y}_t = \kappa_t Z_t^{-(1+\rho)} Y_t Z_t = \kappa_t Y_t Z_t^{-\rho} \Rightarrow MC_t = \widetilde{MC}_t \cdot Z_t^\rho, \tag{14}$$

while detrended marginal utility follows from

$$\widetilde{MU}_t = \tilde{C}_t^{-\rho} = C_t^{-\rho} Z_t^{-\rho} \Rightarrow MU_t = \widetilde{MU}_t \cdot Z_t^\rho. \tag{15}$$

Table 2 repeats the equilibrium conditions in terms of detrended real variables.

Table 2: Normalized equilibrium conditions	
$Y_t^{-\rho} = E_t \left\{ \frac{\beta(1+i_t)}{P_{t+1}/P_t} Y_{t+1}^{-\rho} \right\} \Gamma^{-\rho}.$	(2.1)
$x_t^{1+\theta} = \frac{\theta}{(\theta-1)} \frac{K_t}{D_t}.$	(2.2a)
$K_t \equiv \kappa_t Y_t + \alpha \beta E_t \left[\frac{Y_{t+1}}{Y_t} \psi_{t+1}^{-2\theta} K_{t+1} \Gamma^{1-\rho} \right].$	(2.2b)
$D_t \equiv (1 + \omega_t) C_t^{-\rho} + \alpha \beta E_t \left[\frac{Y_{t+1}}{Y_t} \psi_{t+1}^{1-\theta} D_{t+1} \Gamma^{1-\rho} \right].$	(2.2c)
$1 = (1 - \alpha) x_t^{1-\theta} + \alpha \{ \psi_t \}^{1-\theta}, \quad \psi_t = [E_{t-1}(\Pi_t)]^\sigma \Pi_t^{-1}$	(2.3)

The steady state versions of the above equations associated with some nominal steady state $\bar{\Pi}$, \bar{x} , and real growth rate Γ are given below. I impose the normalization $\bar{\kappa} = 1$, and I assume an elimination of steady state effects of monopolistic competition by fiscal policy, so that $\bar{\mu} = 1 \Rightarrow \bar{y} = \bar{c} = \bar{\mu} \bar{\kappa}^{-\frac{1}{1+\rho}} = 1$.

Table 3: Steady state equilibrium conditions	
$\bar{\Pi} = I \beta \Gamma^{-\rho}.$	(3.1)
$\bar{x}^{1+\theta} = \frac{\theta}{1-\theta} \frac{\bar{K}}{\bar{D}}, \quad \bar{\mu} \equiv \frac{\theta}{(1+\bar{\omega})(\theta-1)} = 1.$	(3.2a)
$\bar{K} = \frac{1}{1 - \alpha \beta \Gamma^{1-\rho} \bar{\Pi}^{2\theta(1-\sigma)}}.$	(3.2b)
$\bar{D} = \frac{(1+\bar{\omega})}{1 - \alpha \beta \Gamma^{1-\rho} \bar{\Pi}^{(1-\sigma)(\theta-1)}}.$	(3.2c)
$1 = (1 - \alpha) \bar{x}^{1-\theta} + \alpha \bar{\Pi}^{(\sigma-1)(1-\theta)}, \quad \bar{\psi} = \bar{\Pi}^{\sigma-1}$	(3.3)

3 Monetary and Fiscal Policy

The instrument of monetary policy is the nominal interest rate. Authorities respond to deviations in the price inflation rate from the target, which also determines trend inflation, and deviation of output from some benchmark¹⁰, when they set the gross nominal interest rate I_t :

$$\frac{I_t}{I} = \left(\frac{I_{t-1}}{I} \right)^{\rho_i} \left[\left(\frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^{bench}} \right)^{\phi_Y} \right]^{(1-\rho_i)} \cdot e^{m_{i,t}}, \quad (16)$$

$$I \equiv \bar{\Pi} \beta \Gamma^{-\rho}, \quad \Pi^* = \bar{\Pi}.$$

¹⁰The definition of the output gap is discussed in section 4.

Π_t is gross period inflation and Π^* is the gross inflation target. I_t is the gross nominal interest rate. The steady state nominal rate is pinned down by the inflation target and the consumption Euler equation.

Fiscal authorities collect nominal lump sum taxes and hand out subsidies, so that the steady state effect of monopolistic competition is eliminated. Fiscal policy is noisy, however, implying that there will be a difference between the flexible price output level and the first best output level.¹¹ I do not consider fiscal policy in the following, other than its effect on the markup. I justify that by assuming that fiscal policy is always Ricardian. This means that fiscal policy makes sure that the public sector transversality condition, or debt sustainability condition, holds in nominal (as well as real) terms, given any path for nominal interest rates and price inflation that is being considered by monetary authorities. For example, implementing a balanced budget rule,

$$t_t - \omega_t X_t \tilde{Y}_t = 0,$$

will make the value of public nominal debt stay constant, and that will be sufficient for the transversality condition to hold in nominal terms under most forms of monetary policy.¹²

4 A log-linear approximation around steady state

I use small letters in place of capital letters to denote normalized variables' log deviations from their steady state values. Where necessary, I will use a hat \hat{x} to state that a variable x is expressed in terms of log deviation from its steady state. The disturbance κ_t in the log-linearized model in this section, is the normalized version of the original shock, in terms of deviation from its steady state. For simplicity, I still use notation κ_t in the rest of the paper.

4.1 Two definitions of the output gap

With $\bar{\kappa}$ normalized at $\bar{\kappa} = 1$, we have normalized $\bar{Y} = 1$, and the definition of the flexible-price output gap expressed in terms of log deviations from

¹¹This way of including markup shocks has precedence in Ball, Mankiw, and Reis (2005).

¹²One exception is that a balanced budget policy will not be compatible with a strictly zero nominal interest rate with probability one at all times. In order to allow for a strictly zero nominal interest rate at all times, fiscal policy has to make the path of nominal public debt fall over time. See Schmitt-Grohe and Uribe (2000).

trend is

$$y_t^{fgap} \equiv y_t - y_t^{flex} = y_t + \frac{1}{1+\rho}(\hat{\mu}_t + \kappa_t). \quad (17)$$

Since

$$\mu_t \equiv \frac{\frac{\theta}{\theta-1}}{1+\omega_t} \quad \text{and} \quad \frac{\frac{\theta}{\theta-1}}{1+\bar{\omega}} = 1, \quad (18)$$

the log deviation from steady state of $\hat{\mu}_t \approx -\omega_t$, and hence

$$y_t^{fgap} \equiv y_t - \frac{1}{1+\rho}(\omega_t - \kappa_t). \quad (19)$$

The above expression says that flexible-price output gap will be zero if above (below) trend output y_t is explained with high (low) productivity or a negative (positive) markup distortion. The deviation from first best trend output is given by

$$y_t^{gap} \equiv y_t - \left(-\frac{1}{1+\rho}\kappa_t\right). \quad (20)$$

4.2 The consumption Euler equation

The linearized consumption Euler equation (2.1) is

$$-\rho y_t = E_t\{i_t - \pi_{t+1} - \rho y_{t+1}\}. \quad (21)$$

4.3 Price setting and the Phillips curve

The Price index (2.3) expressed on log-linear form is

$$0 = (1-\alpha)\bar{x}^{1-\theta}\hat{x}_t + \alpha\bar{\Pi}^{(\sigma-1)(1-\theta)}\hat{\psi}_t, \quad (22)$$

where

$$\hat{\psi}_t \equiv -(\pi_t - \sigma E_{t-1}\pi_t). \quad (23)$$

In appendix D, I use the price setting equations (2.a-c) and the price index equations (22) and (23) to derive the log-linearized price setting equation:

$$\begin{aligned} & -\hat{\psi}_t(\mu_1 + \mu_2 L^{-1} + \mu_3 L^{-2}) = \\ & E_t(\mu_4 + \mu_5 L^{-1})m c_t - E_t(\mu_6 + \mu_7 L^{-1})(m u_t + \omega_t) + \mu_8 E_t(y_{t+1} - y_t), \end{aligned} \quad (24)$$

where the μ - coefficients are functions of structural parameters and L is the lag operator.

4.4 The complete model with a Generalized Phillips Curve (GPC)

In appendix G, I substitute in the output gap for $(mu_t + \omega_t)$ and mc_t in equation (24), and rearrange to show that the generalized Phillips curve is given by (25). The generalized Phillips curve (GPC), along with the consumption Euler equation (26) and the interest rate rule (27), now together determine price inflation and output deviations from their trends and the deviation of the interest rate from its steady state:

$$\begin{aligned} (\mu_1 + \mu_2 L^{-1} + \mu_3 L^{-2})(\pi_t - \sigma E_{t-1} \pi_t) = \\ E_t(\mu_6 + \mu_7 L^{-1})(1 + \rho)y_t^{fgap} \\ + E_t \mu_8 (1 - L^{-1}) \left[\left(\frac{1 - \rho}{1 + \rho} \right) \kappa_t - 2y_t^{gap} \right], \end{aligned} \quad (25)$$

$$y_t = -\frac{1}{\rho} E_t(i_t - \pi_{t+1}) + E_t y_{t+1}, \quad (26)$$

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) [\phi_\pi \cdot \pi_t + \phi_Y \cdot y_t^{gap}] + m_{i,t}. \quad (27)$$

There are six structural parameters in the model; $\Gamma, \sigma, \alpha, \beta, \rho,$ and $\theta,$ in addition to the four policy parameters: ρ_i, ϕ_π, ϕ_y and $\bar{\Pi}$.

4.4.1 Some special cases for the GPC

In appendix H, I show that in the special case of a trend inflation rate equal to zero; $\bar{\Pi} = 1,$ or full indexation $\sigma = 1,$ the Phillips curve equation (25) reduces to:

$$E_t(1 - \beta \Gamma^{(1-\rho)} L^{-1})(\pi_t - \sigma E_{t-1} \pi_t) = \kappa_0 \cdot y_t^{fgap}, \quad (28)$$

where

$$\kappa_0 = \frac{(1 - \alpha \beta \Gamma^{1-\rho})(1 - \alpha)(1 + \rho)}{(1 + \theta)\alpha}, \quad y_t^{fgap} \equiv y_t + \frac{1}{1 + \rho}(\kappa_t - \omega_t). \quad (29)$$

With $\alpha = 1,$ κ_0 approaches zero, and there is then no link between the output gap and deviations of the inflation rate from its trend. With $\alpha = 0$ (fully flexible prices), κ_0 is infinite, but the output gap is always zero, and the expression on the right hand side of (28) is not well defined. The model then reduces to the one presented in section 1.1 on page 5, where monetary policy and the Euler equation together determine the paths of nominal variables, and the real and nominal dichotomy applies.

4.4.2 The pure New Keynesian Phillips curve

In the case with $\bar{\Pi} = 1$, no indexation ($\sigma = 0$), and $\Gamma = 1$ or $\rho = 1$ (log utility), equation (25) reduces to the familiar New Keynesian Phillips curve:

$$\pi_t = E_t \beta \cdot \pi_{t+1} + \kappa_0 \cdot y_t^{fgap}.$$

4.4.3 The pure Neoclassical Phillips curve

With full indexation, ($\sigma = 1$), and given *any* $\bar{\Pi}$ and Γ , equation (25) reduces to what we may call a Neoclassical Phillips curve, or aggregate supply curve:

$$\pi_t - E_{t-1} \pi_t = \kappa_0 \cdot y_t^{fgap}.$$

4.5 The hybrid New Keynesian Phillips curve (HNKPC)

As discussed in appendix F, the New Keynesian version of this model, with lagged indexation instead of forward indexation, is obtained by replacing equation (25) with the following Phillips curve.

$$\begin{aligned} (\mu_1 + \mu_2 L^{-1} + \mu_3 L^{-2})(\pi_t - \sigma \pi_{t-1}) = \\ E_t(\mu_6 + \mu_7 L^{-1})(1 + \rho)y_t^{fgap} \\ + E_t \mu_8 (1 - L^{-1}) \left[\left(\frac{1 - \rho}{1 + \rho} \right) \kappa_t - 2y_t^{fgap} \right], \end{aligned} \tag{30}$$

The difference between this Phillips curve and the one with some forward indexation, is on the left hand side of the equality sign only: Lagged inflation π_{t-1} replaces $E_{t-1} \pi_t$. Both the hybrid New Keynesian Phillips curve (HNKPC) (30) and the Generalized Phillips curve (GPC) (25) encompass the pure forward-looking Phillips curve as a special case, when $\sigma = 0$ in each model.

5 Estimation

I use Junior Maih's RISE toolbox for the estimations. See Alstadheim, Bjornland, and Maih (2013) for description and references.

5.1 Data and calibration

I use quarterly US data for GDP growth and PCE inflation (both SA) and 3-month interest rates from the St. Louis FRED database, for q1 1960 to q2 2013¹³. I read in the data series without demeaning or detrending, as they appear in figure 1.

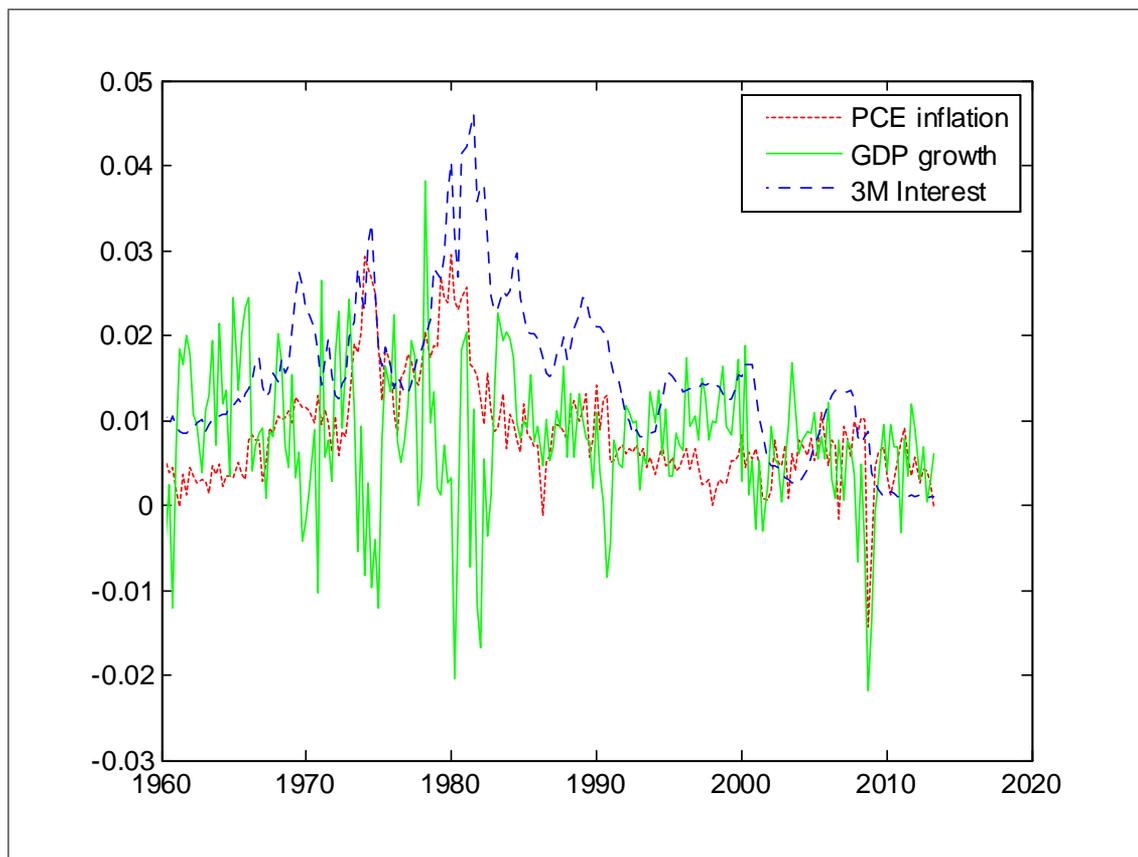


Figure 1: US Quarterly data; 3m interest rate divided by 400, log growth rates for PCE and GDP

¹³The FRED series ID for the inflation and GDP series are PCECTPI (inflation) and GDPC1 (GDP). The source for both is the US Dep. of Commerce: Bureau of Economic Analysis. For the interest rate, the FRED series ID is IR3TED01USQ156N (source: OECD MEI). I use the log first difference of the PCE series and the GDP series. I divide the interest rate series by 400.

The shock processes are specified as follows,

$$\begin{aligned}\kappa_t &= \rho_\kappa \cdot \kappa_{t-1} + \sigma_\kappa \cdot \varepsilon_{\kappa,t} \\ \omega_t &= \rho_\omega \cdot \omega_{t-1} + \sigma_\omega \cdot \varepsilon_{\omega,t} \\ m_t &= \rho_m \cdot m_{t-1} + \sigma_m \cdot \varepsilon_{m,t}.\end{aligned}$$

In addition, I allow for a measurement error in output, ε_y . The observation equations are, with observed log change in the price index given by π_t^{obs} , observed log change in real GDP given by dy_t^{obs} and observed nominal interest rate divided by 400 given by i_t^{obs} :

$$\begin{aligned}\pi_t^{obs} &= \pi_t + \log(\bar{\Pi}), \\ dy_t^{obs} &= y_t - y_{t-1} + \log(\Gamma) + \varepsilon_{y,t}, \\ i_t^{obs} &= i_t + \log(I).\end{aligned}$$

I is implicitly defined by the steady state condition $\bar{\Pi} = I\beta\Gamma^{-\rho}$, and therefore depends on the estimation of $\bar{\Pi}$ and structural parameters.

The structural parameters of the model are $\Gamma, \sigma, \alpha, \beta, \rho$, and θ , the policy parameters are ρ_i, ϕ_π, ϕ_y and $\bar{\Pi}$, and the parameters for exogenous processes are their variances $\sigma_m, \sigma_\kappa, \sigma_\omega$, and their autocorrelation coefficients $\rho_\kappa, \rho_m, \rho_\omega$. The standard deviation of the measurement error ε_y , given by σ_y , is also estimated. In the Markov Switching environment, the parameters to estimate will also include the transition probabilities.

I impose a tight prior on the trend inflation rate $\bar{\Pi}$ in order to make the model implication for the steady state nominal interest rate I be equal to the sample mean for the nominal interest rate. Had I used sample means to calibrate (priors for) the inflation rate and the real growth rate, as well as the inflation rate, there would have been dynamic inconsistency (the Euler equation would not hold for any $\beta < 1$).

I set the prior on the trend inflation rate to $\bar{\Pi} \sim \mathbf{N}$, with 99,9% of the distribution between 1.004 and 1.006. I calibrate $\rho = 1.2$, $\beta = 0.999$ and $\log(\Gamma) = 0.0076$ (equal to the sample mean growth rate). With an estimated trend inflation rate around $\log(\bar{\Pi}) = 0.005$, (annual inflation of 2%), the implied steady state nominal rate will be about equal to the sample mean, which is 0.0152 (corresponding to a 6% annual interest rate). Inspecting the data in the figure, we see that this imposes the assumption that deviations

from the trend inflation rate were large in the 1970s and 1980s. A version of the model with Markov switching in the trend inflation rate was estimated as a robustness check. In terms of fit to the data as measured by the MDD, that model was dominated by the fixed steady-state inflation-rate version. This is consistent with results in Liu, Waggoner, and Zha (2011) and Sims and Zha (2006).

5.2 Results

I estimate two versions of the model with the GPC; The Constant parameter model (C), and the Switching Variance model (SV). In the latter version, the standard deviations of the four disturbances (technology shock ε_κ , markup shock ε_ω , the monetary policy shock ε_m and the measurement error in the observation equation for output ε_y) are allowed to switch - in a synchronized fashion - between two states. I estimate the corresponding model versions with the HNKPC as well. As can be seen from the tables below, which report the indexing parameter σ along with Marginal Data Densities (MDD) for the different specifications, a version with Markov switching in the variances of structural shocks fits the data best.

5.2.1 Main models:

Model	MDD*	GPC ¹ : σ	MDD	HNKPC ¹ : σ
Constant	2483	0.23	2469	0.11
Switch Variance	2533	0.00	2533	0.00

*)Marginal Data Density (LaPlace approximation)

1) Posterior mode

The table shows that the special case where the GPC and the HNKPC models are identical, when there is zero indexation in both, is preferred by the data. The result is the same when the models are estimated on a sample that ends in 2008 q2, indicating that the result is robust to not including the great recession period.

5.2.2 Robustness checks

Above, I used PCE inflation in the dataset. Estimation results with the CPI instead are given below, confirming the case of no indexation.

Results with the CPI instead of PCE inflation:

Model	MDD*	GPC ¹⁾ : σ	MDD	HNKPC: σ
Constant	2435	0.25	2435	0.04
Switch Variance	2479	0.00	2478	0.00

*)Marginal Data Density (LaPlace approximation)

1) Posterior mode

I also estimate the model on demeaned data series. In that case, I calibrate $\bar{\Pi} = 1.00$ and $\Gamma = 1.00$, and also impose a steady state nominal interest rate equal to zero on the observation equation for the nominal interest rate. Results show that parameter estimates are sensitive to both demeaning the data and to allowing variances of disturbances to switch:

Demeaned data, PCE inflation:

Model	MDD*	GPC ¹⁾ : σ	MDD*)	HNKPC ¹⁾ : σ
Constant	2506	0.16	2507	0.47
Switch Variance	2562	0.27	2508	0.04

*)Marginal Data Density (LaPlace approximation)

1) Posterior mode

5.2.3 Estimated model with Markov switching in the variance of shocks

The parameter estimates of the SV models for the GPC case and the HNKPC case are given below. The estimates reflect that the two models are the same in the special case that the data prefer.

Structural parameters of SV, GPC model and HNKPC model:

Parameter	Prior dist ¹⁾	GPC ²⁾	HNKPC ²⁾
σ	U, [0,1], (100)	0.00 (0.0000)	0.00 (0.0000)
α	U, [0,1], (100)	0.32 (0.0015)	0.32 (0.0015)
ϕ_π	N, [1,3], (95)	1.06 (0.0053)	1.19 (0.0051)
ϕ_y	N,[0,3], (95)	2.39 (0.0082)	2.31 (0.0084)
ρ_i	N, [0,0,9], (95)	1.49 (0.0032)	1.47(0.0033)
$\bar{\Pi}$	N, [1.004,1.006], (99,9)	1.0048 (0.0003)	1.0048 (0.0003)
MDD		2533	2533

1) Distribution, range and percent of distribution within range

2) Mode and standard deviation of posterior.

Parameters, exogenous shock processes:

Par	Prior ¹⁾	GPC ²⁾	HNKPC ²⁾
ρ_κ	beta, [0.1,0.6], (90)	0.37 (0.0050)	0.40 (0.0048)
ρ_ω	beta, [0.1,0.6], (90)	0.58 (0.0038)	0.55 (0.0039)
ρ_m	beta, [0.1,0.6], (90)	0.61 (0.0018)	0.61 (0.0018)
σ_κ	inv. gam., [0.005 ,1.0], (90)	.0028 (0.0007)/.0145 (0.0012)	.0028 (0.0006)/.0145 (0.0012)
σ_ω	inv. gam., [0.005 ,1.0], (90)	.0067 (0.0007)/.0249 (0.0014)	.0067 (0.0007)/.0253 (0.0014)
σ_m	inv. gam., [0.005 ,1.0], (90)	.0074 (0.0005)/.0077 (0.0010)	.0074 (0.0005)/.0077 (0.0009)
σ_y	inv. gam., [0.005 ,1.0], (90)	.0073 (0.0005)/.0123 (0.0011)	.0073 (0.0005)/.0123 (0.0010)

1) Distribution, range and percent of distribution within range

2) Mode and standard deviation of posterior

Parameters, switching probabilities:

Switch prob.	Prior ¹⁾	GPC ²⁾	HNKPC ²⁾
Lo to hi var.	N [0.001, 0.2] (95)	0.0496 (0.0039)	0.0487 (0.0040)
Hi to lo var.	N [0.001, 0.2] (95)	0.0861 (0.0077)	0.0853 (0.0082)

1) Distribution, range and percent of distribution within range

2) Mode and standard deviation of posterior.

Figure 2 illustrates the smoothed probability of being in the high volatility regime, along with the graph for price inflation. This picture is based on the GPC model, but the graph for the HNKPC model is almost exactly the same.

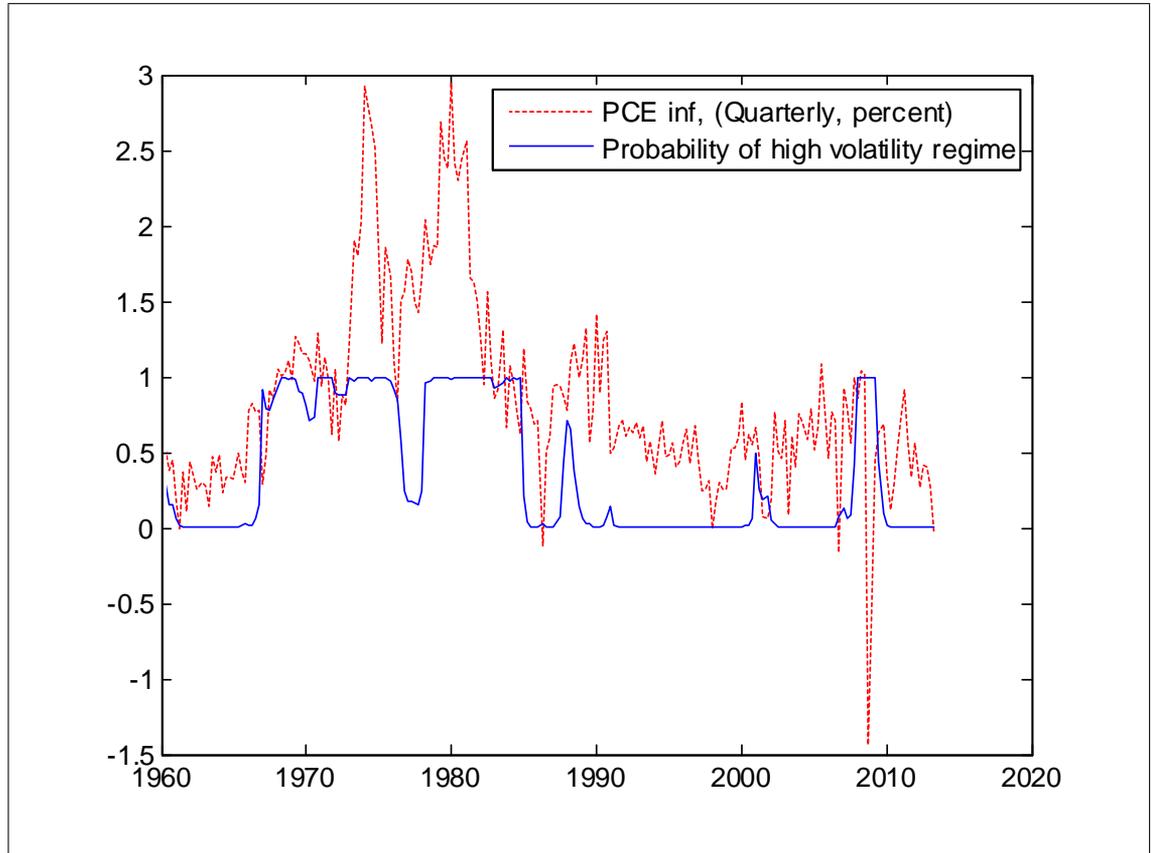


Figure 2: Probability of high volatility regime, GPC model with switching variances.

6 Concluding remarks

I find that a version of the generalized Phillips curve (GPC) with zero forward indexation fits US data better than either a more Neoclassical Phillips curve or a hybrid New Keynesian Phillips curve (HNKPC).

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Appendix

A The flexible price model

The price $X(i)_t$ is set by each representative agent i in period t , in order to maximize the utility value of revenue minus the utility loss associated with production:

$$\text{Max}_{X(i)_t} E_n \left\{ \sum_{t=n}^{\infty} \beta^{t-n} \left(\lambda_t(1 + \omega_t)X(i)_t\tilde{Y}_t - \frac{1}{2}\tilde{\kappa}_t\tilde{Y}(i)_t^2 \right) \right\},$$

or, with demand $\tilde{Y}(i)_t = (\frac{X(i)_t}{P_t})^{-\theta}\tilde{Y}_t$:

$$\text{Max}_{X(i)_t} E_n \left\{ \sum_{t=n}^{\infty} \beta^{t-n} \left(\lambda_t(1 + \omega_t)X(i)_t \left[\left(\frac{X(i)_t}{P_t} \right)^{-\theta}\tilde{Y}_t \right] - \frac{1}{2}\tilde{\kappa}_t \left[\left(\frac{X(i)_t}{P_t} \right)^{-\theta}\tilde{Y}_t \right]^2 \right) \right\}.$$

The first order condition for optimal price setting if prices are flexible is then given by equation (9) in the main text.

B The intratemporal problem

The agents' intratemporal cost minimization problem is:

$$\text{Min} [PC - \lambda[C - 1]], \tag{B.1}$$

where the agent minimizes with respect to C . P is the price index.

$$\lambda C^{-1} = \lambda \equiv P. \tag{B.2}$$

From

$$C_t \equiv \left[\int_{j=0}^1 (C_{j,t})^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \tag{B.3}$$

demand for good j in terms of the relative price $\frac{X(j)}{P}$ is;

$$\left(\frac{C_j}{C} \right) = \left(\frac{X(j)}{P} \right)^{-\theta}.$$

This means that demand for an individual firms' goods is

$$C_j = \left(\frac{X(j)}{P}\right)^{-\theta} C, \quad (\text{B.4})$$

and the price index is

$$P = \left[\int_{j=0}^1 X(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}.$$

C Sticky price setting and forward indexation

There is potential indexation of prices by the fraction α of price setters who do not optimize their price in period t :

$$X_t(j) = [E_{t-1}(\Pi_t)]^\sigma X_{t-1}(j), \Pi_t = P_t/P_{t-1}. \quad (31)$$

Inserting prices of firms $(1 - \alpha)$ that optimize their price X (they are all equal and hence set the same price, so we can disregard indexing of individual firms), and prices of sticky-price firms (α) who potentially index to expected inflation into (32), noting that the distribution of initial prices for non-optimizing firms (j) equals the lagged price index:

$$\begin{aligned} P_t &= \left\{ (1 - \alpha) X_t^{1-\theta} + \alpha \int_{j=0}^1 [[E_{t-1}(\Pi_t)]^\sigma P_{t-1}(j)]^{1-\theta} dj \right\}^{\frac{1}{1-\theta}} \\ &= \left\{ (1 - \alpha) X_t^{(1-\theta)} + \alpha [E_{t-1}(\Pi_t)]^{\sigma(1-\theta)} P_{t-1}^{1-\theta} \right\}^{\frac{1}{1-\theta}} \end{aligned}$$

Dividing through by the price index P_t , and using $x_t \equiv X_t/P_t$, gives:

$$1 = (1 - \alpha) x_t^{1-\theta} + \alpha \{ [E_{t-1}(\Pi_t)]^\sigma (\Pi_t)^{-1} \}^{1-\theta} \quad (32)$$

Analogous to Hornstein (2007), but with forward indexation instead of lagged indexation, define

$$\psi_t \equiv [E_{t-1}(\Pi_t)]^\sigma (\Pi_t)^{-1},$$

and the price index may be expressed as

$$1 = (1 - \alpha) x_t^{1-\theta} + \alpha \{ \psi_t \}^{1-\theta}. \quad (33)$$

Given indexation according to (31), the producer's relative price x_n evolves according to

$$x_{n+\tau}(j) = (\Pi_{n+\tau})^{-1} [E_{n+\tau-1}(\Pi_{n+\tau})]^\sigma x_{n+\tau-1}(j) = \psi_{n+\tau} x_{n+\tau-1}(j),$$

$$\tau \geq 1.$$

The τ -period ahead relative price is, with repeated substitution,

$$x_{n+\tau}(j) = \prod_{k=1}^{\tau} [\Pi_{n+k}^{-1} [E_{n+k-1}(\Pi_{n+k})]^\sigma] x_n(j) = \prod_{k=1}^{\tau} \psi_{n+k} x_n(j) = \Psi_{n,\tau} x_n(j),$$

$$\Psi_{n,0} \equiv 1.$$

The level of the price $X_{n+\tau}$, set at period n , develops according to

$$X_{n+\tau} = x_{n+\tau} P_{n+\tau} = x_{n+\tau} P_n \prod_{r=1}^{\tau} \Pi_{n+r} = \Psi_{n,\tau} x_n P_n \prod_{r=1}^{\tau} \Pi_{n+r}, \quad (34)$$

so that demand in period τ for producer i 's production, who is setting price in period n , is

$$\tilde{Y}_\tau(i) = (\Psi_{n,\tau} x_n)^{-\theta} \tilde{Y}_\tau \quad (35)$$

The optimal price $X_n(j)$ is chosen in period n to maximize expected utility for consumer/producer j , given that the price will stay effective (but potentially subject to forward indexation) with probability α in each period ahead:

$$\text{Max}_{X(j)^n} E_n \left\{ \sum_{\tau=n}^{\infty} (\alpha\beta)^{\tau-n} \left(\lambda_\tau (1 + \omega_\tau) X(j)_\tau \tilde{Y}(X(j))_\tau - \frac{1}{2} \tilde{\kappa}_\tau \tilde{Y}(X(j))_\tau^2 \right) \right\}.$$

Noting that the producer supplies whatever volume is demanded, given the price she sets, and disregarding indexing of agents j , the agent's max-problem when setting her price is:

$$\text{Max}_{x_n} E_n \left\{ \sum_{\tau=n}^{\infty} (\alpha\beta)^{\tau-n} \left(\lambda_\tau [(1 + \omega_\tau) [X_\tau (\Psi_{n,\tau} x_n)^{-\theta} \tilde{Y}_\tau] - \frac{1}{2} \tilde{\kappa}_\tau [(\Psi_{n,\tau} x_n)^{-\theta} \tilde{Y}_\tau]^2] \right) \right\},$$

or, using (34) and (35),

$$\text{Max}_{x_n} E_n \left\{ \sum_{\tau=n}^{\infty} (\alpha\beta)^{\tau-n} \left(P_n \left(\prod_{r=1}^{\tau} \Pi_{n+r} \right) \frac{1}{P_\tau \tilde{C}_\tau^\rho} [(1 + \omega_\tau) [\Psi_{n,\tau}^{1-\theta} x_n^{1-\theta} \tilde{Y}_\tau] - \frac{1}{2} \tilde{\kappa}_\tau [\Psi_{n,\tau}^{-\theta} x_n^{-\theta} \tilde{Y}_\tau]^2] \right) \right\}.$$

Differentiating with respect to the relative price x_n gives the first order condition

$$x_n^{1+\theta} = \frac{\theta}{(\theta-1)} \frac{E_n \sum_{\tau=n}^{\infty} (\alpha\beta)^{\tau-n} \tilde{\kappa}_{\tau} \Psi_{n,\tau}^{-2\theta} \tilde{Y}_{\tau}^2}{E_n \sum_{\tau=n}^{\infty} (\alpha\beta)^{\tau-n} \left(\prod_{r=1}^{\tau} \Pi_{n+r} \right) P_n \frac{(1+\omega_{\tau})}{P_{\tau} \tilde{C}_{\tau}^{\rho}} [\Psi_{n,\tau}^{1-\theta} \tilde{Y}_{\tau}]}. \quad (36)$$

Dividing through by \tilde{Y}_n in the numerator and denominator and defining the growth rate $G_{n,\tau} = \frac{\tilde{Y}_{\tau}}{\tilde{Y}_n}$, and marginal cost $MC_{\tau} = \tilde{\kappa}_{\tau} \tilde{Y}_{\tau}$, and marginal utility $MU_{\tau} = \tilde{C}_{\tau}^{-\rho}$:

$$x_n^{1+\theta} = \frac{\theta}{(\theta-1)} \frac{E_n \sum_{\tau=n}^{\infty} (\alpha\beta)^{\tau-n} \tilde{\kappa}_{\tau} \tilde{Y}_{\tau} \Psi_{n,\tau}^{-2\theta} G_{n,\tau}}{E_n \sum_{\tau=n}^{\infty} (\alpha\beta)^{\tau-n} \frac{(\prod_{r=1}^{\tau} \Pi_{n+r}) P_n (1+\omega_{\tau})}{P_{\tau} \tilde{C}_{\tau}^{\rho}} [\Psi_{n,\tau}^{1-\theta} G_{n,\tau}]}, \quad (37)$$

or, re-indexing, replacing n by t , and using that $P_{\tau}/P_n = \prod_{r=1}^{\tau} (\Pi_{n+r})$;

$$x_t^{1+\theta} = \frac{\theta}{(\theta-1)} \frac{E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^{\tau} \widetilde{MC}_{t+\tau} \Psi_{t,\tau}^{-2\theta} G_{t,\tau}}{E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^{\tau} \widetilde{MU}_{t+\tau} (1 + \omega_{t+\tau}) [\Psi_{t,\tau}^{1-\theta} G_{t,\tau}]}. \quad (38)$$

Define

$$\tilde{K}_t \equiv E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^{\tau} \widetilde{MC}_{t+\tau} \Psi_{t,\tau}^{-2\theta} G_{t,\tau}$$

and

$$\tilde{D}_t \equiv E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^{\tau} \widetilde{MU}_{t+\tau} (1 + \omega_{t+\tau}) [\Psi_{t,\tau}^{1-\theta} G_{t,\tau}],$$

and the first order conditions becomes:

$$x_t^{1+\theta} = \frac{\theta}{(\theta-1)} \frac{\tilde{K}_t}{\tilde{D}_t}, \quad (39)$$

where the following recursive definitions following from the definitions above will be useful:

$$\tilde{K}_t = \widetilde{MC}_t + \alpha\beta E_t [G_{t,1} \psi_{t+1}^{-2\theta} \tilde{K}_{t+1}], \quad (40)$$

$$\tilde{D}_t = \widetilde{MU}_{t+\tau} (1 + \omega_{t+\tau}) + \alpha\beta E_t [G_{t,1} \psi_{t+1}^{1-\theta} \tilde{D}_{t+1}]. \quad (41)$$

D Linearizing the price setting equation

From $\bar{\psi} = \bar{\Pi}^{\sigma-1}$ and (22) we have

$$\hat{x}_t = \frac{-\alpha\bar{\Pi}^{(\sigma-1)(1-\theta)}}{[1 - \alpha\bar{\Pi}^{(\sigma-1)(1-\theta)}]}\hat{\psi}_t. \quad (42)$$

I log linearize optimal price setting, equation (2.2a) in table 2, to get

$$\hat{x}_t = \frac{1}{(1 + \theta)}(k_t - d_t). \quad (43)$$

(42) and (43), and the definitions in table 4 below, imply

$$-(1 + \theta)\eta_0\hat{\psi}_t = k_t - d_t. \quad (44)$$

In log-linearized form, (2.2b) becomes

$$\begin{aligned} k_t &= [1 - \alpha\beta\Gamma^{1-\rho}\bar{\Pi}^{2\theta(1-\sigma)}]mc_t + \\ \alpha\beta\Gamma^{1-\rho}\bar{\Pi}^{2\theta(1-\sigma)}E_t[(y_{t+1} - y_t) + k_{t+1} - 2\theta\hat{\psi}_{t+1}], \\ mc_t &= \kappa_t + y_t, \end{aligned} \quad (45)$$

while (2.2c) becomes

$$\begin{aligned} d_t &= [1 - \alpha\beta\Gamma^{1-\rho}\bar{\Pi}^{(1-\sigma)(\theta-1)}](\omega_t + mu_t) + \\ \alpha\beta\Gamma^{1-\rho}\bar{\Pi}^{(1-\sigma)(\theta-1)}E_t[(y_{t+1} - y_t) + d_{t+1} + (1 - \theta)\hat{\psi}_{t+1}], \\ mu_t &= -\rho y_t. \end{aligned} \quad (46)$$

It is useful to rewrite (45), and let $g_t \equiv y_{t+1} - y_t$:

$$E_t \left\{ (1 - \eta_2 L^{-1})k_t + \eta_2 2\theta\hat{\psi}_{t+1} - \eta_2 g_t \right\} = \eta_3 mc_t, \quad (47)$$

and (46):

$$E_t(1 - \eta_1 L^{-1})d_t + \eta_1 E_t[(\theta - 1)\hat{\psi}_{t+1} - g_t] = [1 - \eta_1](\omega_t + mu_t), \quad (48)$$

where:

Table 4: η parameters
$\eta_0 \equiv \frac{\alpha\bar{\Pi}^{(\sigma-1)(1-\theta)}}{[1 - \alpha\bar{\Pi}^{(\sigma-1)(1-\theta)}]}$
$\eta_1 \equiv \alpha\beta\Gamma^{(1-\rho)}\bar{\Pi}^{(1-\sigma)(\theta-1)},$
$\eta_2 \equiv \alpha\beta\Gamma^{(1-\rho)}\bar{\Pi}^{2\theta(1-\sigma)},$
$\eta_3 \equiv [1 - \alpha\beta\Gamma^{(1-\rho)}\bar{\Pi}^{2\theta(1-\sigma)}].$

Now, expand equation (48), and reorganize, to define B :

$$E_t(1 - \eta_1 L^{-1})(1 - \eta_2 L^{-1})d_t = \\ E_t(1 - \eta_2 L^{-1})\{[1 - \eta_1](\omega_t + mu_t) - \eta_1 E_t[(\theta - 1)\widehat{\psi}_{t+1} - g_t]\} = B,$$

and the same with equation (47), to define A :

$$E_t(1 - \eta_1 L^{-1})(1 - \eta_2 L^{-1})k_t = \\ E_t(1 - \eta_1 L^{-1})\{\eta_3 mc_t + [\eta_2 g_t - \eta_2 2\theta\widehat{\psi}_{t+1}]\} = A.$$

The above implies

$$E_t(1 - \eta_1 L^{-1})(1 - \eta_2 L^{-1})[k_t - d_t] = A - B = \quad (49) \\ E_t(1 - \eta_1 L^{-1})\{\eta_3 mc_t - \eta_2 2\theta\widehat{\psi}_{t+1}\} \\ + E_t(1 - \eta_2 L^{-1})\{\eta_1(\theta - 1)\widehat{\psi}_{t+1} - [1 - \eta_1](\omega_t + mu_t)\} + (\eta_2 - \eta_1)g_t.$$

And now (44) can be written in an expanded fashion as

$$E_t(1 - \eta_1 L^{-1})(1 - \eta_2 L^{-1})[-(1 + \theta)\eta_0\widehat{\psi}_t] = A - B. \quad (50)$$

I plug in for $A - B$ from the definition in (49), and collect ψ_t -terms on the left hand side, to get

$$- E_t(1 - \eta_1 L^{-1})(1 - \eta_2 L^{-1})[(1 + \theta)\eta_0\widehat{\psi}_t] + \\ E_t(1 - \eta_1 L^{-1})\{\eta_2 2\theta\widehat{\psi}_{t+1}\} - E_t(1 - \eta_2 L^{-1})\eta_1(\theta - 1)\widehat{\psi}_{t+1} \quad (51) \\ = E_t(1 - \eta_1 L^{-1})\{\eta_3 mc_t\} + \\ E_t(1 - \eta_2 L^{-1})\{-[1 - \eta_1](\omega_t + mu_t)\} + (\eta_2 - \eta_1)E_t g_t.$$

This gives

$$-\widehat{\psi}_t(\mu_1 + \mu_2 L^{-1} + \mu_3 L^{-2}) = \quad (52) \\ E_t(\mu_4 + \mu_5 L^{-1})mc_t - E_t(\mu_6 + \mu_7 L^{-1})(\omega_t + mu_t) + \mu_8 E_t g_t,$$

where

$$-\widehat{\psi}_t = (\pi_t - \sigma E_{t-1}\pi_t),$$

with μ - parameters defined in table 5.

Table 5: μ-parameters.
$\mu_1 \equiv (1 + \theta)\eta_0,$
$\mu_2 \equiv -[(1 + \theta)\eta_0(\eta_2 + \eta_1) + \eta_2 2\theta + \eta_1(1 - \theta)],$
$\mu_3 \equiv (1 + \theta)(\eta_0 + 1)\eta_1\eta_2,$
$\mu_4 \equiv \eta_3,$
$\mu_5 \equiv -\eta_3\eta_1,$
$\mu_6 \equiv [1 - \eta_1],$
$\mu_7 \equiv -\eta_2[1 - \eta_1],$
$\mu_8 \equiv (\eta_2 - \eta_1)$

E An alternative representation in terms of a factorized polynomial

Defining λ_1 and λ_2 implicitly by:

$$(\mu_1 + \mu_2 L^{-1} + \mu_3 L^{-2}) = \mu_3 \left(\frac{\mu_1}{\mu_2} + \frac{\mu_2}{\mu_3} L^{-1} + L^{-2} \right) = \frac{\mu_3}{\lambda_1 \lambda_2} (1 - \lambda_1 L^{-1})(1 - \lambda_2 L^{-1}),$$

lets us write equation (52) as

$$\begin{aligned} & \frac{\mu_3}{\lambda_1 \lambda_2} (1 - \lambda_1 L^{-1})(1 - \lambda_2 L^{-1})(\pi_t - \sigma E_{t-1} \pi_t) \\ & = E_t(\mu_4 + \mu_5 L^{-1})m c_t - E_t(\mu_6 + \mu_7 L^{-1})(\omega_t + m u_t) + \mu_8 E_t g_t, \end{aligned}$$

or

$$\begin{aligned} & (1 - \lambda_1 L^{-1})(1 - \lambda_2 L^{-1})(\pi_t - \sigma E_{t-1} \pi_t) = \\ & \kappa_p \left[E_t \left(1 + \frac{\mu_5}{\mu_4} L^{-1} \right) m c_t - E_t \left(\frac{\mu_6}{\mu_4} + \frac{\mu_7}{\mu_4} L^{-1} \right) (\omega_t + m u_t) + \frac{\mu_8}{\mu_4} E_t g_t \right], \end{aligned}$$

where

$$\kappa_p = \lambda_1 \lambda_2 \frac{\mu_4}{\mu_3},$$

and hence

$$\begin{aligned} & (\pi_t - \sigma E_{t-1} \pi_t) = \tag{53} \\ & \kappa_p \left[(1 - \lambda_2 L^{-1})(1 - \lambda_2 L^{-1}) \right]^{-1} \cdot \\ & \left[E_t \left(1 + \frac{\mu_5}{\mu_4} L^{-1} \right) m c_t - E_t \left(\frac{\mu_6}{\mu_4} + \frac{\mu_7}{\mu_4} L^{-1} \right) (\omega_t + m u_t) + \frac{\mu_8}{\mu_4} E_t g_t \right]. \end{aligned}$$

F The price setting equation with standard indexation to lagged inflation

The derivation of the price setting equation is exactly as in D, but with

$$\widehat{\psi}_t = -(1 - \sigma L)\pi_t, \quad (54)$$

and the price setting equation then becomes

$$(1 - \sigma L)\pi_t = \kappa_p \left[(1 - \lambda_2 L^{-1})(1 - \lambda_2 L^{-1}) \right]^{-1} \cdot \left[E_t \left(1 + \frac{\mu_5}{\mu_4} L^{-1} \right) m c_t - E_t \left(\frac{\mu_6}{\mu_4} + \frac{\mu_7}{\mu_4} L^{-1} \right) (\omega_t + m u_t) + \frac{\mu_8}{\mu_4} E_t g_t \right]. \quad (55)$$

G The Phillips curve

It is useful to define the parameter T :

$$1 - \frac{\mu_6}{\mu_4} = -\alpha \beta \Gamma^{(1-\rho)} [\Pi^{2\theta(1-\sigma)} - \Pi^{(1-\sigma)(\theta-1)}] / [1 - \alpha \beta \Gamma^{(1-\rho)} \pi^{2\theta(1-\sigma)}] = T. \quad (56)$$

Other parameter combinations equal $-T$:

$$\frac{\mu_5}{\mu_4} - \frac{\mu_7}{\mu_4} = -\alpha \beta \Gamma^{(1-\rho)} [\Pi^{(1-\sigma)(\theta-1)} - \Pi^{2\theta(1-\sigma)}] / [1 - \alpha \beta \Gamma^{(1-\rho)} \pi^{s2\theta(1-\sigma)}] = -T, \quad (57)$$

$$\frac{\mu_8}{\mu_4} = -T.$$

Parts of the last line in (53) can be expressed as

$$E_t \left(1 + \frac{\mu_5}{\mu_4} L^{-1} \right) m c_t - E_t \left(\frac{\mu_6}{\mu_4} + \frac{\mu_7}{\mu_4} L^{-1} \right) (m u_t - m c_t + m c_t) = T m c_t - T L^{-1} m c_t + E_t \left(\frac{\mu_6}{\mu_4} + \frac{\mu_7}{\mu_4} L^{-1} \right) (m c_t - m u_t). \quad (58)$$

Using this in equation (53) gives

$$(\pi_t - \sigma E_{t-1} \pi_t) = \kappa_p \left[(1 - \lambda_2 L^{-1})(1 - \lambda_2 L^{-1}) \right]^{-1} \cdot \left[E_t T [m c_t - m c_{t+1}] + E_t \left(\frac{\mu_6}{\mu_4} + \frac{\mu_7}{\mu_4} L^{-1} \right) (m c_t - (\omega_t + m u_t)) - T g_t \right]. \quad (59)$$

Or, with $g_t = y_{t+1} - y_t$,

$$\begin{aligned} & (\pi_t - \sigma E_{t-1}\pi_t) = \\ & \kappa_p [(1 - \lambda_2 L^{-1})(1 - \lambda_2 L^{-1})]^{-1} \cdot \\ & \left[E_t T[\kappa_t + y_t - (\kappa_{t+1} + y_{t+1})] + E_t \left(\frac{\mu_6}{\mu_4} + \frac{\mu_7}{\mu_4} L^{-1} \right) (m c_t - (\omega_t + m u_t)) - T(y_{t+1} - y_t) \right]. \end{aligned}$$

Furthermore,

$$y_t^{fgap} \equiv y_t + \frac{1}{1 + \rho} (\hat{\kappa}_t - \omega_t) \Rightarrow \quad (60)$$

$$\begin{aligned} (1 + \rho) y_t^{fgap} &= (1 + \rho) y_t + (\kappa_t - \omega_t) \\ &= m c_t - (\omega_t + m u_t), \end{aligned} \quad (61)$$

and hence

$$\begin{aligned} & (\pi_t - \sigma E_{t-1}\pi_t) = \\ & \kappa_p [(1 - \lambda_2 L^{-1})(1 - \lambda_2 L^{-1})]^{-1} \cdot \\ & \left[E_t T[\kappa_t + 2y_t - (\kappa_{t+1} + 2y_{t+1})] + E_t \left(\frac{\mu_6}{\mu_4} + \frac{\mu_7}{\mu_4} L^{-1} \right) ((1 + \rho) y_t^{fgap}) \right]. \end{aligned}$$

Inserting $\kappa_p = \lambda_1 \lambda_2 \frac{\mu_4}{\mu_3}$, we get

$$\begin{aligned} & (\lambda_1 \lambda_2)^{-1} \frac{\mu_3}{\mu_4} [(1 - \lambda_2 L^{-1})(1 - \lambda_2 L^{-1})] (\pi_t - \sigma E_{t-1}\pi_t) = \\ & \left[E_t T[\kappa_t + 2y_t - (\kappa_{t+1} + 2y_{t+1})] + E_t \left(\frac{\mu_6}{\mu_4} + \frac{\mu_7}{\mu_4} L^{-1} \right) ((1 + \rho) y_t^{fgap}) \right]. \end{aligned}$$

And the above is equal to

$$\begin{aligned} & \frac{1}{\mu_4} (\mu_1 + \mu_2 L^{-1} + \mu_3 L^{-2}) (\pi_t - \sigma E_{t-1}\pi_t) = \\ & \left[E_t T[\kappa_t + 2y_t - (\kappa_{t+1} + 2y_{t+1})] + E_t \left(\frac{\mu_6}{\mu_4} + \frac{\mu_7}{\mu_4} L^{-1} \right) ((1 + \rho) y_t^{fgap}) \right]. \end{aligned}$$

Use the fact that $\frac{\mu_8}{\mu_4} = -T$, and multiply by μ_4 ;

$$\begin{aligned} & (\mu_1 + \mu_2 L^{-1} + \mu_3 L^{-2}) (\pi_t - \sigma E_{t-1}\pi_t) = \\ & \left[E_t \mu_8 [(\kappa_{t+1} + 2y_{t+1}) - (\kappa_t + 2y_t)] + E_t (\mu_6 + \mu_7 L^{-1}) ((1 + \rho) y_t^{fgap}) \right]. \end{aligned}$$

And finally note that

$$\begin{aligned}
& (\kappa_{t+1} + 2y_{t+1}) - (\kappa_t + 2y_t) = \\
& (\kappa_{t+1} + 2(y_{t+1}^{fgap} - \frac{1}{1+\rho}(\widehat{\kappa}_{t+1} - \omega_{t+1}))) - (\kappa_t + 2(y_t^{fgap} - \frac{1}{1+\rho}(\widehat{\kappa}_t - \omega_t))) = \\
& 2(y_{t+1}^{gap} - y_t^{gap}) + (\frac{\rho-1}{\rho+1})(\kappa_{t+1} - \kappa_t)
\end{aligned}$$

Which implies

$$\begin{aligned}
& (\mu_1 + \mu_2 L^{-1} + \mu_3 L^{-2})(\pi_t - \sigma E_{t-1} \pi_t) = \\
& E_t(\mu_6 + \mu_7 L^{-1})(1 + \rho)y_t^{fgap} \\
& + E_t \mu_8 (1 - L^{-1})[(\frac{1-\rho}{\rho+1})\kappa_t - 2y_t^{gap}].
\end{aligned} \tag{62}$$

Note the appearance of both y_t^{gap} and y_t^{fgap} in the above equation.

H A special case: approximation around a zero trend inflation rate

In case of $\bar{\Pi} = 1$ or $\sigma = 1$; the parameters in table 4 become

Table 4b: η parameters when $\bar{\Pi} = 1$ or $\sigma = 1$
$\eta_0 = \frac{\alpha}{1-\alpha}$
$\eta_1 = \alpha\beta\Gamma^{(1-\rho)} \equiv \eta,$
$\eta_2 = \alpha\beta\Gamma^{(1-\rho)} \equiv \eta,$
$\eta_3 = [1 - \alpha\beta\Gamma^{(1-\rho)}] \equiv 1 - \eta.$

Now, with $\eta_1 = \eta_2$, (49) simplifies to

$$\begin{aligned}
& E_t(1 - \eta_1 L^{-1})\{-(1 + \theta)\eta_0 \widehat{\psi}_t\} = \\
& E_t\{\eta_3 m c_t - \eta_2 2\theta \widehat{\psi}_{t+1}\} + \{\eta_1(\theta - 1)\widehat{\psi}_{t+1} - [1 - \eta_1](\omega_t + m u_t)\} + (\eta_2 - \eta_1)g_t, \\
& \Rightarrow
\end{aligned}$$

$$\begin{aligned}
& E_t(1 - \eta L^{-1})[-(1 + \theta)\eta_0 \widehat{\psi}_t] = \\
& E_t\{[1 - \eta]m c_t - \eta 2\theta \widehat{\psi}_{t+1}\} + E_t\{\eta(\theta - 1)\widehat{\psi}_{t+1} - [1 - \eta](\omega_t + m u_t)\}.
\end{aligned} \tag{63}$$

And this simplifies to

$$\begin{aligned}
E_t(1 - (\eta + \frac{\eta\theta + \eta}{(1 + \theta)\eta_0})L^{-1})[-\widehat{\psi}_t] = & \quad (64) \\
E_t\{\frac{[1 - \eta]}{(1 + \theta)\eta_0}((1 + \rho)y_t^{fgap}), &
\end{aligned}$$

and further to

$$\begin{aligned}
E_t(1 - \beta\Gamma^{(1-\rho)}L^{-1})(\pi_t - \sigma E_{t-1}\pi_t) = & \\
\frac{(1 - \alpha\beta\Gamma^{(1-\rho)})(1 - \alpha)}{(1 + \theta)\alpha}(1 + \rho)y_t^{fgap}. & \quad (65)
\end{aligned}$$