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Multiple Credit Constraints and Time-Varying Macroeconomic Dynamics*

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Please find updated versions of the paper [here](#).

Abstract

I explore the macroeconomic implications of borrowers facing both loan-to-value (LTV) and debt-service-to-income (DTI) limits, using an estimated DSGE model. I identify when each constraint dominated over the period 1984-2019: LTV constraints dominate in contractions, when house prices are relatively low – and DTI constraints dominate in expansions, when interest rates are relatively high. I also find that DTI standards were relaxed during the mid-2000s’ boom, and that lower DTI limits or higher interest rates, but not lower LTV limits, would have prevented the boom. Finally, county panel data attest to multiple credit constraints as a source of nonlinear dynamics.

JEL classification: C33, D58, E32, E44.

Keywords: Multiple credit constraints. Nonlinear estimation of DSGE models. State-dependent credit origination.

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1 Introduction

Numerous empirical and theoretical papers emphasize the role of loan-to-value (LTV) limits on loan applicants in causing financial acceleration.¹ In these contributions, the supply of collateralized credit to households moves up and down proportionally to asset prices, thereby acting as an impetus that expands and contracts the economy. In reality, however, banks *also* impose debt-service-to-income (DTI) limits on loan applicants. Given that LTV and DTI constraints generally do not allow for the same amount of debt, households effectively face the single constraint that yields the lowest amount. In turn, endogenous switching between the two constraints can occur depending on various determinants of mortgage borrowing, such as house prices, incomes, and mortgage rates. This raises some questions, all of which are fundamental to macroeconomics and finance. When and why have LTV and DTI requirements historically restricted mortgage borrowing? Did looser LTV or DTI limits cause the credit boom prior to the Great Recession? Is the credit cycle best controlled by adjusting LTV or DTI limits or monetary policy rates? How, if at all, does switching between different credit constraints affect the propagation and amplification of economic shocks? The answers to these questions have profound implications for how we model the economy and implement macroprudential policies.

In order to understand these issues better, I develop a tractable New Keynesian dynamic stochastic general equilibrium (DSGE) model with long-term fixed-rate mortgage contracts and two occasionally-binding credit constraints: an LTV constraint and a DTI constraint. With this setup, homeowners must fulfill a collateral requirement and a debt-service requirement in order to qualify for a mortgage loan.

I estimate the model by Bayesian maximum likelihood on time series covering the U.S. economy in the period 1984-2019. The solution of the model is based on a piecewise first-order perturbation method, so as to handle the occasionally-binding nature of the constraints (Guerrieri and Iacoviello, 2015, 2017). Using this framework, I present five main sets of results.

¹See, e.g., Kiyotaki and Moore (1997), Iacoviello (2005), Iacoviello and Neri (2010), Jermann and Quadrini (2012), Liu, Wang, and Zha (2013), Justiniano, Primiceri, and Tambalotti (2015), Kydland, Rupert, and Šustek (2016), Guerrieri and Iacoviello (2017), Jensen, Ravn, and Santoro (2018), and Jensen, Petrella, Ravn, and Santoro (2020).

The first set relates to the historical evolution in credit conditions. The estimation allows me to identify when the two credit constraints were binding and which shocks caused them to bind. At least one constraint binds throughout the estimation period, signifying that borrowers have generally been credit constrained. The LTV constraint often binds during and after recessions, when house prices, which largely determine housing wealth, are relatively low (i.e., 1984-1985, 1990-1997, and 2007-2012). The DTI constraint reversely mostly binds in expansions, when interest rates, which impact debt servicing, are relatively high, due to countercyclical monetary policy (i.e., 1986-1989, 1998-2006, and 2013-2019). The setup allows for heterogeneity in credit control: a binding constraint entails that a majority of borrowers is restricted by the requirement labeling the constraint, and that the complementary minority is restricted by the other requirement. According to the estimation, when the LTV constraint binds, 74 pct. of the borrowers are restricted by the LTV requirement and 26 pct. by the DTI requirement. Conversely, in a DTI regime, 88 pct. of the borrowers are DTI restricted, and 12 pct. are LTV restricted.

The second set of results relates to the evolution in DTI limits. [Corbae and Quintin \(2015\)](#) and [Greenwald \(2018\)](#) hypothesize a relaxation of DTI limits as the cause of the mid-2000s' credit boom. My estimation corroborates this hypothesis, inferring that the maximally allowed back-end DTI ratio was raised from 39 pct. in 1998 to 56 pct. in 2008, as well as tightened to 36 pct. by 2013. To my knowledge, this is the first evidence of a DTI cycle obtained within an estimated model. Using data from Fannie Mae and Freddie Mac, I show that this development is consistent with the rise and fall of the 90th and 95th percentiles of the cross-sectional distribution of DTI ratios on originated loans. The chronology is also accordant with [Justiniano, Primiceri, and Tambalotti's \(2019\)](#) conclusion that looser LTV limits cannot explain the credit boom. They instead argue that it was an increase in credit supply that caused the surge in mortgage debt. My results qualify this discovery, together suggesting that the increase in credit supply translated into a relaxation of DTI limits. The results also show that DTI standards were eased during the financial deregulation in the mid-1980s and tightened following the Savings and Loan Crisis of the late 1980s, in line with narrative accounts ([Campbell and Hercowitz, 2009](#); [Drehmann, Borio, and Tsatsaronis, 2012](#); [Mian, Sufi, and Verner, 2020](#)).

The third set of results relates to the optimal timing of macroprudential policy. Recent studies show that credit expansions predict subsequent banking and housing market crises (e.g., [Mian and Sufi, 2009](#); [Schularick and Taylor, 2012](#); [Baron and Xiong, 2017](#)). Motivated by this, I consider how mortgage debt would historically have evolved if LTV and DTI limits had responded countercyclically to deviations of credit from its long-run trend. I find that countercyclical DTI limits are effective in curbing increases in mortgage debt, since these increases typically occur in expansions, when most borrowers are DTI constrained. The flip-side of this result is that countercyclical LTV limits cannot prevent debt from rising, since only a minority of borrowers are LTV constrained in expansions. Tighter LTV limits would therefore – unlike tighter DTI limits – not have been able to prevent the mid-2000s’ boom. Countercyclical LTV limits can, however, mitigate the adverse consequences of house price slumps on credit availability by raising credit limits. In this way, the lowest credit volatility is reached by combining the LTV and DTI policies into a two-stringed policy entailing that *both* credit limits respond countercyclically. Macroprudential policy then takes into account that the effective tool changes over the business cycle, with an LTV tool in contractions and a DTI tool in expansions. Because this policy inhibits the deleveraging-induced flow of funds from borrowers to lenders in recessionary episodes, the policy efficiently redistributes consumption risk from borrowers to lenders. Such theoretical guidance on how to combine multiple credit constraints for macroprudential purposes is scarce within the existing literature, which focuses on stabilization through LTV limits, as also noted by [Jácome and Mitra \(2015\)](#).²

The fourth set of results relates to "leaning-against-the-wind" monetary policy, again aimed at limiting the deviations of credit from its trend. I show that the macroprudential potency of monetary policy increases with the share of DTI constrained households, since their borrowing ability depends directly on the interest rate. Unfortunately, however, leaning against the wind comes at the cost of redistributing consumption risk from borrowers to savers. This is because the policy entails that interest rates, c.p., rise when debt levels rise, unavoidably increasing borrowers’ interest payments when they are most

²Two exceptions are [Greenwald \(2018\)](#), who focuses on counterfactuals around the Great Recession, and [Gelain, Lansing, and Mendicino \(2013\)](#), who show that loan-to-income constraints may be effective at stabilizing mortgage borrowing in both booms and busts, using a linear model.

indebted. Thus, if policymakers have distributional concerns, they may prefer the two-stringed policy over leaning against the wind, as the borrowers are relatively poor already in the absence of leaning against the wind.

The fifth set of results relates to how endogenous switching between credit constraints transmits shocks nonlinearly through the economy. I observe a *constraint-switching effect* on *labor supply*: if borrowers become more DTI constrained, they increase their labor supply to compensate for their diminished borrowing ability, and vice versa. Turning to house price shocks, I show that these shocks exert *asymmetric* effects on the economy: Adverse shocks are amplified by an endogenous negative response of borrowers' housing demand, tightening the LTV constraint. Favorable shocks are, by contrast, dampened by countercyclical monetary policy, c.p., tightening the DTI constraint. I also show that house price shocks exert *state-dependent* effects: shocks that occur when the LTV constraint binds (typically in contractions) are amplified by this constraint and by an endogenous response of housing demand, while shocks that occur when the DTI constraint binds (typically in expansions) are curbed by countercyclical monetary policy. These predictions fit with a number of empirical studies, as well as evidence provided in this paper, documenting the presence of substantial nonlinear responses to housing market shocks.³ Models with only an occasionally-binding LTV constraint, in comparison, have difficulties in reproducing these dynamics, since nonlinearities there only arise following *large favorable* shocks that unbind the LTV constraint.⁴ Such kinds of expansionary events occur more rarely than simple switching between LTV and DTI constraints.

As a final contribution, I use a county-level panel dataset covering 1991-2017 to test two key predictions of homeowners facing both LTV and DTI requirements. The predictions are that (i) income growth, not house price growth, drives credit growth if homeowners' housing-wealth-to-income ratio is sufficiently high, as they will be DTI constrained, and that (ii) house price growth, not income growth, drives credit growth if homeowners' housing-wealth-to-income ratio is sufficiently low, as they will be LTV constrained. My

³Engelhardt (1996) and Skinner (1996) find that economic activity drops following decreases in housing wealth, but does not rise following increases in wealth, using panel surveys. Guerrieri and Iacoviello (2017) show that economic activity is more sensitive to house prices in low house price states than in high house price states.

⁴For instance, Guerrieri and Iacoviello (2017) need to apply a 20 pct. house price increase in order for borrowing demand to become saturated and their LTV constraint to unbind.

identification strategy is based on Bartik-type house price and income instruments, along with county and state-year fixed effects. The specific test involves estimating the elasticities of mortgage loan origination with respect to house prices and personal incomes, importantly after partitioning the elasticities based on the detrended house-price-to-income ratio. The exercise confirms that both elasticities depend on the state of counties' house-price-to-income ratio, in line with the predictions of the DSGE model. The elasticity with respect to house prices is 0.33 when the house-price-to-income ratio in a county is above its long-run trend and 0.65 when it is below the trend. Correspondingly, the elasticity with respect to incomes is zero when the house-price-to-income ratio is below its trend and 0.40 when it is above the trend. These estimates are among the first, in an otherwise large micro-data literature, to suggest that house prices and incomes amplify each others' effect on credit origination.

The rest of the paper is structured as follows. Section 2 discusses how the paper relates to the existing literature. Section 3 presents the theoretical model. Section 4 performs the estimation of the model. Section 5 highlights the nonlinear dynamics that the credit constraints introduce. Section 6 decomposes the historical evolution in credit conditions. Section 7 conducts the macroprudential experiments. Section 8 presents the panel evidence on state-dependent credit elasticities. Section 9 contains the concluding remarks.

2 Related Literature

The paper is, to my knowledge, the first to include both an occasionally-binding LTV constraint and an occasionally-binding DTI constraint in the same estimated general equilibrium model.⁵ A small theoretical literature already studies house price propagation through occasionally-binding LTV constraints. [Guerrieri and Iacoviello \(2017\)](#) illustrate that the macroeconomic sensitivity to house price changes is smaller during booms (when LTV constraints may unbind) than during busts (when LTV constraints bind).

⁵The heterogeneous agents models in [Gorea and Midrigan \(2017\)](#), [Chen, Michaux, and Roussanov \(2020\)](#), and [Kaplan, Mitman, and Violante \(2020\)](#) also impose both LTV and DTI constraints, but do not study their interactions over the business cycle. Moreover, while including rich descriptions of financial markets and risk, the models lack general equilibrium dynamics related to interactions between the constraints and housing demand, labor supply, output, residential investment, and stabilization policy. Focusing on firms' borrowing, [Drechsel \(2018\)](#) establishes a connection between corporations' current earnings and their access to debt, and formalizes this link through an earnings-based constraint.

Furthermore, [Jensen et al. \(2018\)](#) and [Jensen et al. \(2020\)](#) explain how higher LTV limits can negatively skew a business cycle, by damping the effects of expansionary shocks and amplifying the effects of contractionary shocks.

[Greenwald \(2018\)](#) studies complementarily the implications of LTV and DTI constraints. He relies on a calibrated model with an always-binding constraint that is an endogenously weighted average of an LTV and a DTI requirement, and considers linearized impulse responses. While this approach provides an elegant micro-to-macro mapping, it also excludes certain analyses – contained in the present paper – of the implications of multiple constraints. First, the estimation allows for a full-information identification of both when the respective constraints were dominating over the period 1984-2019 and the impact of stabilization policies.⁶ Second, the discrete switching between the constraints generates asymmetric and state-dependent impulse responses, incompatible with linear models. Third, the occasionally-binding constraints imply that borrowers may become credit unconstrained if both constraints unbind simultaneously, unlike the case with an always-binding constraint.⁷

The paper is finally, again to my knowledge, the first to examine the interacting effects of house price and income growth on equity extraction, using panel data methods. A large literature already studies the effects of house price growth on equity extraction.⁸ However, this literature mainly considers the effects of a separate variation in house prices, rather than the interacting effects of changes in house prices and other drivers of credit. A notable exception to this is [Bhutta and Keys \(2016\)](#), who interact house price and interest rate changes and find that they amplify each other considerably. This prediction fits with my theoretical model, as simultaneous expansionary shocks to house prices and monetary policy there relax both credit constraints directly.

⁶Formal identification is important, in that the relative dominance of the two constraints hinges on the magnitude and persistence of house price shocks relative to the magnitude and persistence of income and interest rate shocks. These moments, in turn, largely depend on the shock processes, which are difficult to calibrate accurately, due to their reduced-form nature and cross-model inconsistency.

⁷The borrowers' patience, which is an estimated parameter, determines whether or not both constraints unbind following a given housing wealth and income appreciation.

⁸See, e.g., [Engelhardt \(1996\)](#), [Skinner \(1996\)](#), [Campbell and Cocco \(2007\)](#), [Mian and Sufi \(2011\)](#), [Bhutta and Keys \(2016\)](#), [Guerrieri and Iacoviello \(2017\)](#), [Guren, McKay, Nakamura, and Steinsson \(2018\)](#), and [Cloyne, Huber, Ilzetzki, and Kleven \(2019\)](#).

3 Model

The model has an infinite time horizon. Time is discrete, and indexed by t . The economy is populated by two representative households: a patient and an impatient household. The households consume goods and housing services, and supply labor. Goods and housing are produced by a representative intermediate firm, by combining labor, nonresidential capital, and land. Retail firms unilaterally set prices subject to downward-sloping demand curves. The time preference heterogeneity implies that the patient household lends funds to the impatient household. The patient household also owns and operates the firms, non-residential capital, and land. The equilibrium conditions are derived in Online Appendix B-C.

3.1 Patient and Impatient Households

Variables and parameters without (with) a prime refer to the patient (impatient) household. The household types differ with respect to their pure time discount factors, $\beta \in (0, 1)$ and $\beta' \in (0, 1)$, since $\beta > \beta'$. The economic size of each household is measured by its wage share: $\alpha \in (0, 1)$ for the patient household and $1 - \alpha$ for the impatient household.

The patient and impatient households maximize their utility functions,

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t s_{I,t} \left[\chi_C \log(c_t - \eta_C c_{t-1}) + \omega_H s_{H,t} \chi_H \log(h_t - \eta_H h_{t-1}) - \frac{s_{L,t}}{1+\varphi} n_t^{1+\varphi} \right] \right\}, \quad (1)$$

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta'^t s'_{I,t} \left[\chi'_C \log(c'_t - \eta_C c'_{t-1}) + \omega_H s_{H,t} \chi'_H \log(h'_t - \eta_H h'_{t-1}) - \frac{s_{L,t}}{1+\varphi} n_t'^{1+\varphi} \right] \right\}, \quad (2)$$

where $\chi_C \equiv \frac{1-\eta_C}{1-\beta\eta_C}$, $\chi'_C \equiv \frac{1-\eta_C}{1-\beta'\eta_C}$, $\chi_H \equiv \frac{1-\eta_H}{1-\beta\eta_H}$, $\chi'_H \equiv \frac{1-\eta_H}{1-\beta'\eta_H}$,⁹ c_t and c'_t denote goods consumption, h_t and h'_t denote housing, n_t and n'_t denote labor supply and, equivalently, employment measured in hours, $s_{I,t}$ is an intertemporal preference shock, $s_{H,t}$ is a housing preference shock, and $s_{L,t}$ is a labor preference shock. Moreover, $\eta_C \in (0, 1)$ and $\eta_H \in (0, 1)$ measure habit formation in goods consumption and housing services, while $\omega_H \in \mathbb{R}_+$ weights the utility of housing services relative to that of goods consumption.¹⁰

⁹The scaling factors ensure that the marginal utility of goods consumption and housing services are $\frac{1}{c}$, $\frac{1}{c'}$, $\frac{\omega_H}{h}$, and $\frac{\omega_H}{h'}$ in the steady state.

¹⁰It is not necessary to weight the disutility of labor supply, since its steady-state level only affects the scale of the economy, as in [Guerrieri and Iacoviello \(2017\)](#).

The patient household's utility maximization is subject to a budget constraint,

$$\begin{aligned}
c_t + q_t[h_t - (1 - \delta_H)h_{t-1}] + k_t + \frac{\iota}{2} \left(\frac{k_t}{k_{t-1}} - 1 \right)^2 k_{t-1} + p_{X,t}[x_t - x_{t-1}] \\
= w_t n_t + \underbrace{div_t + b_t - \frac{1 - (1 - \rho)(1 - \sigma) + r_{t-1}}{1 + \pi_t} l_{t-1}}_{\text{Debt Expenses}} + (r_{K,t} + 1 - \delta_K)k_{t-1} + r_{X,t}x_{t-1}, \quad (3)
\end{aligned}$$

where q_t denotes the real house price, k_t denotes nonresidential capital, $r_{K,t}$ denotes the real net rental rate of nonresidential capital, x_t denotes land, $p_{X,t}$ denotes the real price of land, $r_{X,t}$ denotes the real net rental rate of land, w_t denotes the real wage, div_t denotes dividends from retail firms, b_t denotes newly issued net borrowing, l_t denotes the net level of outstanding mortgage loans, r_t denotes the average nominal net interest rate on the outstanding mortgage loans, and π_t denotes net price inflation. $\delta_H \in [0, 1]$ measures the depreciation of residential capital, $\delta_K \in [0, 1]$ measures the depreciation of nonresidential capital, and $\iota \in \mathbb{R}_+$ measures capital adjustment costs. The impatient household's utility maximization is subject to a budget constraint,

$$c'_t + q_t[h'_t - (1 - \delta_H)h'_{t-1}] = w'_t n'_t + b'_t - \underbrace{\frac{1 - (1 - \rho)(1 - \sigma) + r_{t-1}}{1 + \pi_t} l'_{t-1}}_{\text{Debt Expenses}}, \quad (4)$$

where w'_t denotes the real wage, b'_t denotes newly issued net borrowing, and l'_t denotes the net level of outstanding mortgage loans.

The net level of outstanding mortgage loans evolves in the following way:

$$l_t = (1 - \rho)(1 - \sigma) \frac{l_{t-1}}{1 + \pi_t} + b_t, \quad (5)$$

$$l'_t = (1 - \rho)(1 - \sigma) \frac{l'_{t-1}}{1 + \pi_t} + b'_t. \quad (6)$$

The structure of these laws of motion is identical to the structure imposed in [Kyndland et al. \(2016\)](#) and [Garriga, Kyndland, and Šustek \(2017\)](#), reflecting that the vast majority of mortgage debt is long-term.¹¹ In every period, a share, $1 - \rho \in [0, 1]$, of the members of the impatient household amortize their outstanding loans at the rate $\sigma \in [0, 1]$, and roll over the remaining part of their loans. At the same time, the complementary share,

¹¹[Chatterjee and Eyigungor \(2015\)](#) take a different approach to modeling long-term mortgage loans, and assume that each loan is competitively priced to reflect the probability of default on the loan, in their study of homeownership and foreclosure.

ρ , refinance their entire stock of debt. I accordingly assume that the average nominal net interest rate on outstanding loans evolves according to

$$r_t = (1 - \rho)(1 - \sigma)\frac{l'_{t-1}}{l'_t}r_{t-1} + \left[1 - (1 - \rho)(1 - \sigma)\frac{l'_{t-1}}{l'_t}\right]i_t, \quad (7)$$

where i_t denotes the current nominal net interest rate.¹²

The refinancing members of the impatient household must fulfill an LTV requirement and a DTI requirement on their new stocks of debt. This gives rise to the following two occasionally-binding credit constraints:

$$b'_t \leq \rho \left(\kappa_{LTV} \xi_{LTV} \mathbb{E}_t \{ (1 + \pi_{t+1}) q_{t+1} h'_t \} + (1 - \kappa_{LTV}) \xi_{DTI,t} \mathbb{E}_t \left\{ \frac{(1 + \pi_{t+1}) w'_{t+1} n'_t}{\sigma + r_t} \right\} \right), \quad (8)$$

$$b'_t \leq \rho \left((1 - \kappa_{DTI}) \xi_{LTV} \mathbb{E}_t \{ (1 + \pi_{t+1}) q_{t+1} h'_t \} + \kappa_{DTI} \xi_{DTI,t} \mathbb{E}_t \left\{ \frac{(1 + \pi_{t+1}) w'_{t+1} n'_t}{\sigma + r_t} \right\} \right), \quad (9)$$

where $\xi_{LTV} \in [0, 1]$ measures the LTV limit on new debt, and $\xi_{DTI,t} \equiv \frac{\tilde{\xi}_{DTI} s_{DTI,t} - \xi_O}{1 - \tau_L}$ measures the front-end DTI limit (i.e., excluding non-mortgage debt services) after taxes on new debt. Here, $\tilde{\xi}_{DTI} \in [0, 1]$ measures the back-end DTI limit (i.e., including other recurring debt services) before taxes on new debt, $\xi_O \in [0, 1]$ measures recurring non-mortgage debt services,¹³ $\tau_L \in [0, 1)$ is the implicit labor tax rate,¹⁴ and $s_{DTI,t}$ is a shock to the back-end DTI limit.¹⁵

The constraints allow for heterogeneity in credit control, in that different requirements may bind for different subsets of refinancing members at the same time. Specifically, $\kappa_{LTV} \in (0.5, 1]$ measures the share of members under (8) who are restricted by the LTV

¹²This loan type is most reminiscent of a long-term fixed-rate mortgage contract, since, in the event of a monetary policy change, the effective nominal interest rate on mortgage debt evolves sluggishly. Garriga et al. (2017) and Gelain, Lansing, and Natvik (2018) explore the nature of long-term debt and its implications for monetary policy in more depth. They show that – with a time-varying amortization rate – the model-implied repayment profile mimics that of a standard annuity loan arbitrarily well. Given the different focus of my paper, I opt for a constant amortization rate.

¹³Recurring non-mortgage debt includes credit card debt, car loans, and student loans. I do not model this debt, which effectively amounts to assuming that it is owed internally in the impatient household, so that it has no influence beyond the DTI constraint.

¹⁴The households' labor incomes should be treated as after tax incomes, since there are no taxes in the model.

¹⁵I do not model a shock to the LTV limit for two reasons. First and foremost, LTV limits on newly originated mortgage loans have historically been stable, as I document in Figure 7 of Subsection 6.2, using loan-level data from Fannie Mae and Freddie Mac. Second, adding an additional exogenous shock is unfeasible unless I also observe another variable, since equality between the number of observed variables and the number of stochastic innovations is a requisite for the inversion filter, which I use to retrieve the estimates of the innovations (Cuba-Borda, Guerrieri, Iacoviello, and Zhong, 2019).

requirement, and $\kappa_{DTI} \in (0.5, 1]$ measures the share under (9) who are restricted by the DTI requirement. Because a majority of the borrowers are restricted by the LTV requirement in the first case and by the DTI requirement in the latter case, I refer to (8) as the "LTV constraint" and to (9) as the "DTI constraint".

An expression similar to the LTV term in (8)-(9) can be derived as the solution to a debt enforcement problem, as shown by Kiyotaki and Moore (1997). Online Appendix D shows that an expression similar to the DTI term in (8)-(9) can be derived as an incentive compatibility constraint on the impatient household, and that it is a generalization of the natural borrowing limit in Aiyagari (1994). Finally, the assumption $\beta > \beta'$ implies that (8) or (9) always hold with equality in (but not necessarily around) the steady state.

3.2 Firms

3.2.1 Intermediate Firm

The intermediate firm produces intermediate goods and housing under perfect competition. It hires patient and impatient labor, rents nonresidential capital and land, and purchases its own intermediate housing inputs, in order to maximize profits.¹⁶ The profits are given by

$$\frac{Y_t}{M_{P,t}} + q_t I_{H,t} - w_t n_t - w'_t n'_t - r_{K,t} k_{t-1} - g_t - r_{X,t} x_{t-1}, \quad (10)$$

subject to the available goods production and housing transformation technologies,

$$Y_t = k_{t-1}^\mu (s_{Y,t} n_t^\alpha n'_t^{1-\alpha})^{1-\mu}, \quad (11)$$

$$I_{H,t} = g_t^\nu x_{t-1}^{1-\nu}, \quad (12)$$

where Y_t denotes goods production, $M_{P,t}$ denotes an average gross price markup over marginal costs set by the retail firms, $I_{H,t}$ denotes residential gross investment, g_t denotes intermediate housing inputs, and $s_{Y,t}$ is a labor-augmenting technology shock.¹⁷ Lastly,

¹⁶Online Appendix H shows that the main results of the paper are robust to letting the impatient workers' employment drive the aggregate variation in hours worked, leaving the patient workers' employment constant at its steady-state level.

¹⁷Nonresidential capital and labor are not used directly in the housing transformation technology, since they already enter into the production of intermediate housing inputs.

$\mu \in (0, 1)$ measures the goods production elasticity with respect to nonresidential capital, and $\nu \in (0, 1)$ measures the housing transformation elasticity with respect to intermediate housing inputs.

3.2.2 Retail Firms

Retail firms are distributed over a unit continuum by product specialization. They purchase and assemble intermediate goods into retail firm-specific final goods at no additional cost. The final goods are then sold as goods consumption, nonresidential investments, and intermediate housing inputs. The specialization allows the firms to operate under monopolistic competition. All dividends are paid out to the patient household:

$$div_t \equiv \left(1 - \frac{1}{M_{P,t}}\right) Y_t. \quad (13)$$

The solution of the retail firms' price-setting problem yields a hybrid New Keynesian Price Phillips Curve:

$$\pi_t = \gamma_P \pi_{t-1} + \beta \mathbb{E}_t \{ \pi_{t+1} - \gamma_P \pi_t \} - \lambda_P \left(\log M_{P,t} - \log \frac{\epsilon_P}{\epsilon_P - 1} \right) + \varepsilon_{P,t}, \quad (14)$$

where $\lambda_P \equiv \frac{(1-\theta_P)(1-\beta\theta_P)}{\theta_P}$ and $\varepsilon_{P,t}$ is a price markup innovation. Furthermore, $\epsilon_P > 1$ measures the price elasticity of retail firm-specific goods demand, $\gamma_P \in [0, 1)$ measures backward price indexation, and $\theta_P \in (0, 1)$ measures the Calvo probability of a firm not being able to adjust its price in a given period.

3.3 Monetary Policy

The central bank sets the current nominal net interest rate according to a Taylor-type monetary policy rule,

$$i_t = \tau_R i_{t-1} + (1 - \tau_R) i + (1 - \tau_R) \tau_P \pi_{P,t}, \quad (15)$$

where i denotes the steady-state nominal net interest rate. Moreover, $\tau_R \in (0, 1)$ measures deterministic interest rate smoothing, and $\tau_P > 1$ measures the policy response to price inflation.

3.4 Equilibrium

The model contains a goods market, a housing market, a loan market, and a land market, in addition to two redundant labor markets. The market-clearing conditions are

$$c_t + c'_t + k_t - (1 - \delta_K)k_{t-1} + \frac{\iota}{2} \left(\frac{k_t}{k_{t-1}} - 1 \right)^2 k_{t-1} + g_t = Y_t, \quad (16)$$

$$h_t + h'_t - (1 - \delta_H)(h_{t-1} + h'_{t-1}) = I_{H,t}, \quad (17)$$

$$b_t = -b'_t, \quad (18)$$

$$x_t = \mathcal{X}, \quad (19)$$

where $\mathcal{X} \in \mathbb{R}_+$ measures the fixed stock of land.

3.5 Stochastic Processes

All stochastic shocks except for the price markup innovation follow AR(1) processes. The price markup innovation is a single-period innovation, so that any persistence herein is captured by backward price indexation. All six stochastic innovations are normally independent and identically distributed, with a constant standard deviation.

4 Solution and Estimation of the Model

4.1 Methods

I solve the model with the perturbation method from [Guerrieri and Iacoviello \(2015, 2017\)](#). This allows me to account for the two occasionally-binding credit constraints and handle the associated nonlinear solution when implementing the Bayesian maximum likelihood estimation. The model economy will always be in one of four regimes, depending on whether the LTV constraint binds or not and whether the DTI constraint binds or not.¹⁸ The solution method performs a first-order approximation of each of the four regimes around the nonstochastic steady state of a reference regime (one of the four regimes). In the regime where both constraints are binding, the borrowing limits imposed by the

¹⁸Multiple solutions could, in principle, arise if a given shock vector simultaneously favors two or more regimes. However, my application of the model has not found any evidence of such multiplicity.

two constraints are, as a knife-edge case, identical. Outside this regime, the borrowing limits may naturally differ, causing discrete switching between which of the three other regimes that applies. As long as a constraint is slack, the households will expect it to bind again at some forecast horizon.¹⁹ The households therefore base their decisions on the expected duration of the current regime, which, in turn, depends on the state vector. As a result, the solution of the model is nonlinear in two dimensions. First, it is nonlinear *between* regimes, depending on which regime that applies. Second, it is nonlinear *within* each regime, depending on the expected duration of the regime.

I choose the regime where both constraints are binding as the reference regime from which the steady state is computed, in order to treat the constraints symmetrically.²⁰ Owing to this assumption, the calibration of ξ_{LTV} and $\tilde{\xi}_{DTI}$ must ensure that the right-hand sides of (8)-(9) are identical in the steady state. However, this restriction on the parameterization of the model does not entail that it is not possible to calibrate the model realistically. Instead, as will be evident in Subsection 4.3, a highly probable calibration can be reached. Because both constraints bind in the steady state, both Lagrange multipliers are positive here:

$$\lambda_{LTV} = v\lambda_{DTI} > 0, \quad (20)$$

where λ_{LTV} denotes the steady-state multiplier on (8), λ_{DTI} denotes the steady-state multiplier on (9), and $v \in \mathbb{R}_+$ measures the steady-state tightness of the LTV constraint relative to that of the DTI constraint.

The policy functions of the model depend nonlinearly on which constraint that binds, which depends on the model's innovations. Because of this, it is unfeasible to apply the Kalman filter to retrieve the estimates of the innovations when estimating the model. I instead recursively solve for the innovations, given the state of the economy and the observations, as in Fair and Taylor (1983). My implementation of the filtering algorithm

¹⁹The expectation that both constraints eventually will bind stems from the transitory nature of the shocks, implying that, as innovations decay, the economy returns to its reference regime, where both constraints are binding.

²⁰I avoid specifying a reference regime where only one constraint binds, since this could bias the model towards that regime. The regime where both constraints are slack is unattainable as a reference regime, in that the time preference heterogeneity is inconsistent with both households being credit unconstrained in the steady state.

is identical to [Guerrieri and Iacoviello’s \(2017\)](#) implementation except that I do not need to deal with stochastic singularity in zero-lower-bound episodes, on account of my model not incorporating this constraint.²¹ A methodological comment on identification when both constraints are slack is provided in Online Appendix E, along with tests evaluating the accuracy of the solution method.

4.2 Data

The estimation sample covers the U.S. economy in 1984Q1-2019Q4, at a quarterly frequency. This starting point coincides with the onset of the Great Moderation. The sample contains the following six time series: 1. Real personal consumption expenditures per capita, measuring aggregate consumption ($c_t + c'_t$). 2. Real home mortgage loan liabilities per capita, measuring the net level of outstanding mortgage loans (l'_t). 3. Real house prices, measuring real house prices (q_t). 4. Real disposable personal income per capita, measuring aggregate labor income ($w_t n_t + w'_t n'_t$). 5. Aggregate weekly hours per capita, measuring aggregate employment ($n_t + n'_t$). 6. Log change in the GDP price deflator, measuring net price inflation (π_t).

Series 1-5 are log-transformed and detrended by a one-sided HP filter (with a smoothing parameter of 100,000), in order to remove their low-frequency components, following [Guerrieri and Iacoviello \(2017\)](#).²² This filter produces plausible trend and gap estimates for the variables. For instance, the troughs of consumption and mortgage debt following the Great Recession lie 7 pct. and 23 pct. below the trend, in 2009Q2 and 2012Q4, according to the filter. Furthermore, the one-sided filter preserves the temporal ordering of the data, as the correlation of current observations with subsequent observations is not affected by the filter ([Stock and Watson, 1999](#)). Series 6 is demeaned. Data sources and time series plots are reported in Online Appendix F.

²¹[Guerrieri and Iacoviello \(2017\)](#) remove the interest rate from their vector of observed variables during zero-lower-bound periods, as their monetary policy shock is impotent in these periods. [Cuba-Borda et al. \(2019\)](#) thoroughly discuss estimation of models with occasionally-binding constraints.

²²The one-sided HP filter is initialized over the period 1975-1983, without this period being used for the maximization of the posterior kernel.

Table 1: CALIBRATED PARAMETERS

Description		Value	Source or Steady-State Target
Time discount factor, pt. hh.	β	0.985	Annual net real interest rate: 6.2 pct.
Housing utility weight	ω_H	0.69	Steady-state target*
Marginal disutility of labor supply	φ	1.00	Standard value
LTV limit	ξ_{LTV}	0.8200	See text
Steady-state back-end DTI limit	ξ_{DTI}	0.43	See text
Non-mortgage DTI limit	ξ_O	0.15	See text
Labor tax rate	τ_L	0.231	Jones (2002)
Amortization rate	σ	1/80	Loan term: 80 quarters or 20 years
Depreciation rate, res. capital	δ_H	0.01	Standard value
Depreciation rate, nonres. capital	δ_K	0.025	Standard value
Capital income share	μ	0.33	Standard value
Housing transformation elasticity	ν	0.65	Std. dev. of res. investment: 0.18 [†]
Price elasticity of goods demand	ϵ	5.00	Standard value
Stock of land	\mathcal{X}	1.00	Normalization

*The model matches the average ratio of residential fixed assets to nondurable goods consumption expenditures (27.2) over the sample period in the National Income and Product Accounts.

[†]The model matches the standard deviation of residential fixed gross investment in the National Income and Product Accounts. Online Appendix G plots the model-implied and empirical paths of residential investment. The correlation between these series is 63 pct.

4.3 Calibration and Prior Distribution

A subset of the parameters are calibrated using information complementary to the estimation sample. Table 1 reports the calibrated parameters and information on their calibration. I assume a front-end DTI limit of 28 pct. or, equivalently, a back-end DTI limit of 43 pct., in both cases before taxes: $\tilde{\xi}_{DTI} = 0.43$ and $\xi_O = 0.15$. This front-end limit is identical to the cut-off imposed in Linneman and Wachter (1989) and Greenwald (2018), and concordant with the U.S. Consumer Financial Protection Bureau’s mortgage lending rule of thumb (see Consumer Financial Protection Bureau, 2015, p. 5). The back-end limit matches the back-end limit stipulated in the Federal Housing Administration’s Single Family Housing Policy Handbook (see Federal Housing Administration, 2019, art. II.A.5.viii). I finally set the labor tax rate to $\tau_L = 0.231$, consistent with Jones (2002). The steady-state front-end DTI limit accordingly becomes $\xi_{DTI} \equiv \frac{0.43-0.15}{1-0.231} = 0.364$ for incomes after taxes. Given the calibration of the DTI limit, an LTV limit of approximately 82 pct. ensures that the borrowing limits imposed by the two constraints are identical in the steady state (cf., the discussion on the solution of the model in Subsection 4.1). This

Table 2: PRIOR AND POSTERIOR DISTRIBUTIONS

Prior Distribution				Posterior Distributions					
				Baseline			Only LTV Constraint		
Type	Mean	S.D.		Mode	5 pct.	95 pct.	Mode	5 pct.	95 pct.
Structural Parameters									
α	B	0.66	0.10	0.6932	0.6794	0.7070	0.3265	0.3136	0.3393
β'	B	0.9740	0.006	0.9806	0.9804	0.9807	0.9809	0.9807	0.9812
η_C	B	0.70	0.10	0.6266	0.6081	0.6450	0.4447	0.4090	0.4804
η_H	B	0.70	0.10	0.5490	0.5365	0.5614	0.3611	0.3376	0.3847
ρ	B	0.25	0.05	0.3925	0.3565	0.4285	0.3268	0.3028	0.3507
ι	N	10.0	10.0	60.113	47.519	72.707	23.099	19.800	26.398
γ_P	B	0.50	0.20	0.9513	0.9280	0.9745	0.3903	0.3577	0.4228
θ_P	B	0.80	0.05	0.8922	0.8855	0.8989	0.8934	0.8781	0.9087
τ_R	B	0.75	0.05	0.8814	0.8759	0.8870	0.5938	0.5582	0.6294
τ_P	N	1.50	0.25	2.0006	1.9078	2.0934	1.5833	1.4961	1.6706
κ_{LTV}	B	0.75	0.25	0.7423	0.7249	0.7597	–	–	–
κ_{DTI}	B	0.75	0.25	0.8753	0.8695	0.8811	–	–	–
v	N	1.00	0.50	0.9674	0.8680	1.0668	–	–	–
Autocorrelation of Shock Processes									
IP	B	0.50	0.20	0.9800	0.9785	0.9816	0.9055	0.8959	0.9152
HP	B	0.50	0.20	0.8928	0.8827	0.9028	0.9790	0.9744	0.9836
DTI	B	0.50	0.20	0.9824	0.9798	0.9851	0.8817	0.8580	0.9054
AY	B	0.50	0.20	0.9934	0.9919	0.9949	0.9235	0.9106	0.9365
LP	B	0.50	0.20	0.9888	0.9839	0.9938	0.9758	0.9647	0.9869
Standard Deviation of Innovations									
IP	IG	0.010	0.10	0.0351	0.0237	0.0465	0.0243	0.0183	0.0303
HP	IG	0.010	0.10	0.0649	0.0419	0.0879	0.0305	0.0224	0.0386
DTI	IG	0.010	0.10	0.0408	0.0295	0.0522	0.0102	0.0019	0.0184
AY	IG	0.010	0.10	0.0209	0.0159	0.0258	0.0169	0.0116	0.0222
LP	IG	0.010	0.10	0.0037	0.0031	0.0043	0.0029	0.0024	0.0034
PM	IG	0.010	0.10	0.0098	0.0064	0.0131	0.0127	0.0105	0.0149

Distributions: N: Normal. B: Beta. IG: Inverse-Gamma.

Shocks: IP: Intertemporal preference. HP: Housing preference. DTI: DTI limit. AY: Labor-augmenting technology. LP: Labor preference. PM: Price markup.

Note: Parameter and shock process estimates for the DSGE model. The bounds indicate the confidence intervals surrounding the posterior mode. The prior distribution of β' is truncated with an upper bound at 0.9849, and the prior distributions of κ_{LTV} and κ_{DTI} are truncated with a lower bound at 0.50. In the LTV model, the DTI shock refers to a shock to the LTV limit.

LTV limit is well within the range of typically applied limits (e.g., [Linneman and Wachter \(1989\)](#) and [Justiniano et al. \(2019\)](#) use 0.80, and [Iacoviello and Neri \(2010\)](#) use 0.85).

Table 2 reports the prior distributions of the estimated parameters. A detailed description of these distributions and comparison with the existing literature is contained

in Online Appendix E.

4.4 Posterior Distribution

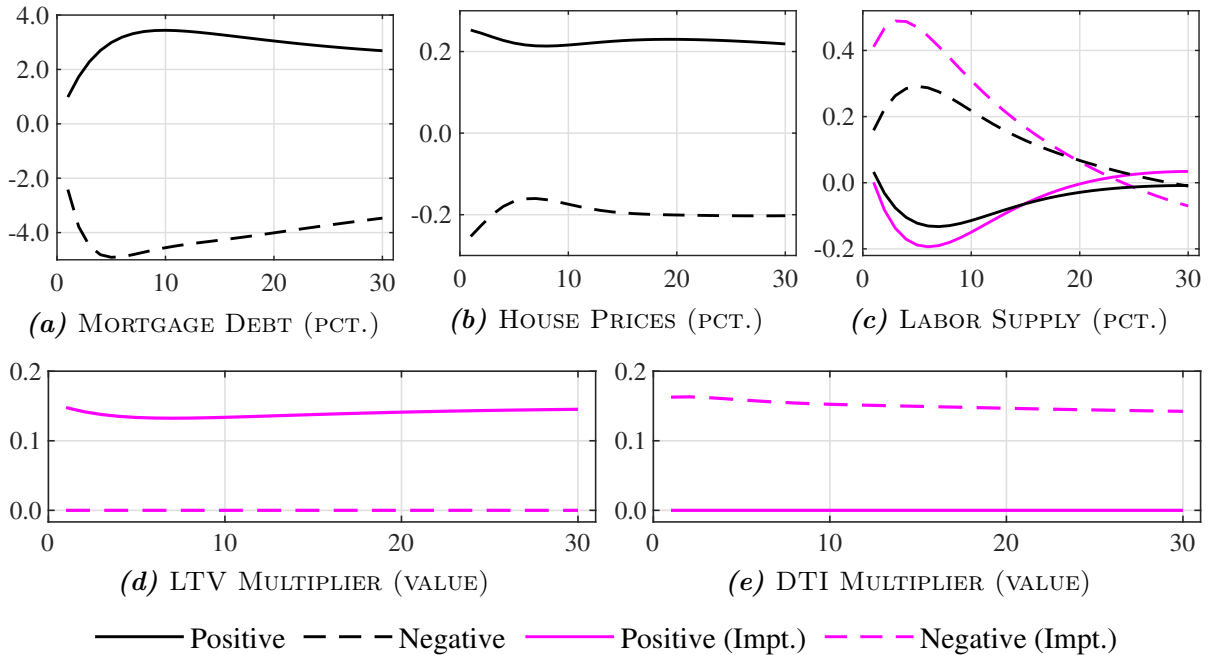
Table 2 reports two posterior distributions: One from the baseline model with two occasionally-binding credit constraints and one from a model with only an occasionally-binding LTV constraint. Apart from not featuring a DTI constraint and having a stochastic LTV shock instead of the DTI shock, this latter model is identical to the baseline model.

The parameters measuring the relative dominance of the credit requirements are not identified in any existing application. In a typical LTV regime, 74 pct. of the borrowers are restricted by the LTV requirement and 26 pct. by the DTI requirement ($\kappa_{LTV} = 0.74$). In contrast, in a DTI regime, only 12 pct. are LTV constrained, while 88 pct. are DTI constrained ($\kappa_{DTI} = 0.88$). Finally, in the steady state, the DTI constraint binds 3 pct. more strenuously than the LTV constraint ($v = 0.97$), possibly reflecting that the DTI constraint binds more frequently than the LTV constraint in the historical simulation (see Figure 4). The estimate of the refinancing rate ($\rho = 0.33$) in the LTV model is close to the estimate in Guerrieri and Iacoviello (2017). This is comforting considering that this parameter is decisive in determining the debt dynamics of the model. The confidence bounds surrounding several estimates are considerably smaller than in Guerrieri and Iacoviello (2017). A plausible explanation for this higher precision is that mortgage debt, which is intimately related to the dynamics of the model, is observed in my estimation, but not in Guerrieri and Iacoviello's (2017) estimation. Finally, note that the Taylor rule parameters are similar to what, e.g., Smets and Wouters (2007) have found, in spite of the interest rates not being an observed variable.

5 Asymmetric and State-Dependent Dynamics

This section illustrates how endogenous switching between the credit constraints generates nonlinear responses to changes in DTI limits and to housing preference shocks. The section also shows that these responses are radically different from the responses of the model with only an LTV constraint. In the LTV model, nonlinearities only arise if the LTV constraint

Figure 1: ASYMMETRIC IMPULSE RESPONSES: CHANGES IN DTI LIMITS



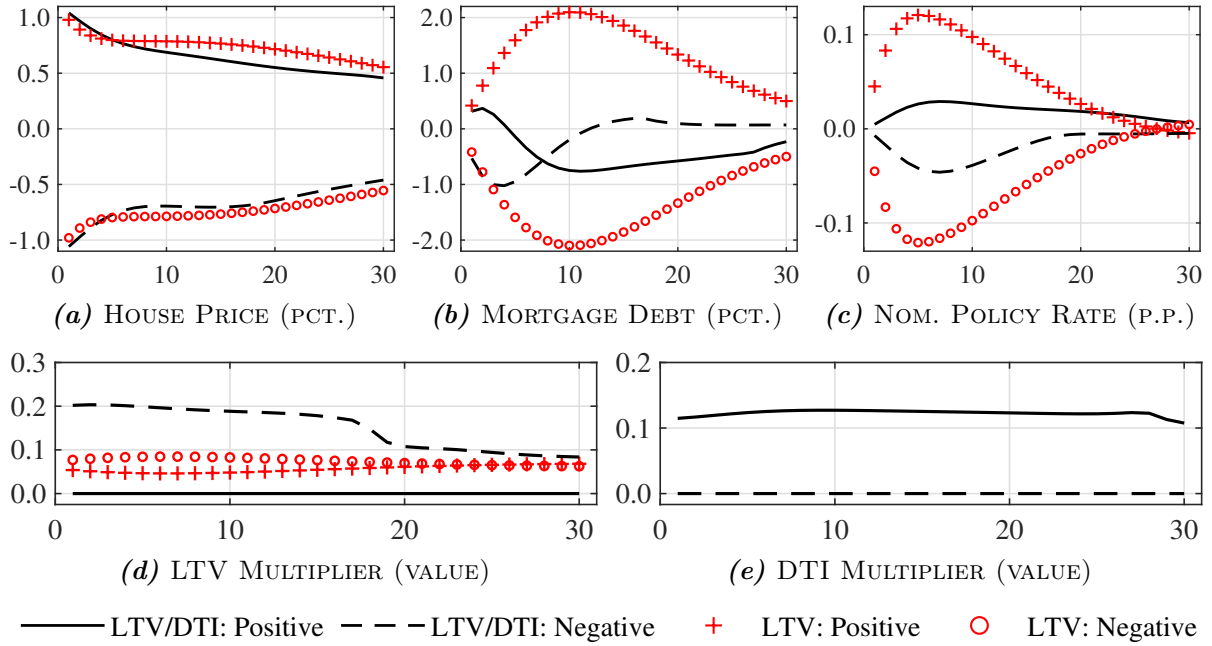
Note: The figures report the effects of unit-standard-deviation positive and negative shocks, in the baseline model. The model is parameterized to its posterior mode. Vertical axes measure deviations from the steady state (Figures 1a-1c) or utility levels (Figures 1d-1e).

unbinds, which presupposes that borrowing demand is saturated. As we will see, this type of event occurs much more rarely than simple switching between the constraints.

Responses to Changes in DTI Limits To begin, Figure 1 presents the effects of unit-standard-deviation positive and negative shocks to the DTI limit. In each case, the DTI limit is adjusted by 4.1 pct. or 1.76 p.p. away from its steady state. The positive shock causes the debt level and house prices to rise, while the negative shock causes them to fall. However, the size of the responses is asymmetric to the sign of the shock, with mortgage debt moving by around 50 pct. more after the negative shock, as compared to the positive shock. Such asymmetry is line with [Kuttner and Shim \(2016\)](#), who find significant negative effects of DTI tightenings on household credit and insignificant positive effects of relaxations, using a sample of 57 economies across 1980-2012.

The asymmetries arise from differences in the constraint that binds. Following the positive shock, the DTI constraint unbinds, causing a majority of borrowers to be LTV constrained. The increased value of housing as collateral boosts borrowers' housing demand, leading house prices to rise. In addition, because fewer households find themselves

Figure 2: ASYMMETRIC IMPULSE RESPONSES: HOUSING PREFERENCE SHOCKS

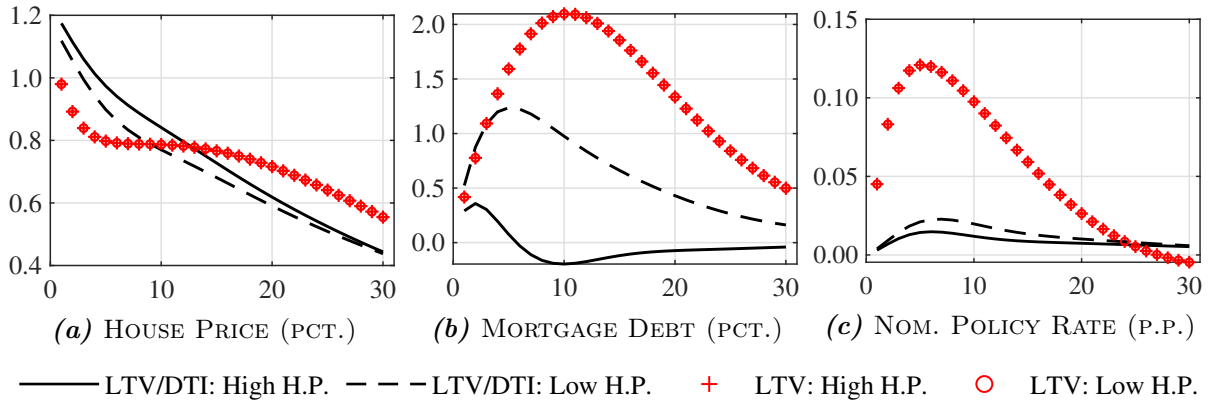


Note: The figures report the effects of unit-standard-deviation positive and negative shocks, in the baseline model and the LTV model. The models are parameterized to their respective posterior modes. Vertical axes measure deviations from the steady state (Figures 2a-2c) or utility levels (Figures 2d-2e).

constrained by the DTI requirement, labor supply shrinks. Following the negative shock, the converse qualitative effects apply. However, since a majority of borrowers are now DTI constrained, the effects on the economy of the pared DTI limit are accentuated relative to the case of a positive shock, where most borrowers were LTV constrained. The effect of DTI changes on housing prices resembles the constraint-switching effect, highlighted by [Greenwald \(2018\)](#), which also works through the collateral motive and amplifies the transmission of monetary policy onto house prices. Moreover, as discussed, an equivalent constraint-switching effect of the income-based requirement onto labor supply is present in the model.

Responses to Housing Preference Shocks Figure 2 next plots the effects of unit-standard-deviation positive and negative housing preference shocks, in the baseline model and the LTV model. The responses of mortgage debt and monetary policy are asymmetric in the baseline model and completely symmetric in the LTV model. The asymmetries of the baseline model again result from differences in the constraint that binds. Following a positive shock, house prices and residential investment increase. The central bank raises

Figure 3: STATE-DEPENDENT IMPULSE RESPONSES: HOUSING PREFERENCE SHOCKS



Note: The figures report the effects of positive unit-standard-deviation housing preference shocks, which occur in low and high house price states of the baseline model and the LTV model. The house price states are simulated by permanently shifting the housing preference of both households up or down by two standard deviations. The models are parameterized to their respective posterior modes. Vertical axes measure deviations from the house price states.

the interest rate to combat the associated rise in inflation, which, as the borrowers predominantly are DTI constrained, suppresses the initial increase in credit. Following the negative shock, instead, house prices fall, and the LTV constraint is tightened. The borrowers are thereupon forced to delever. However, this process is amplified by the borrowers lowering their housing demand further, to finance the repayment of loans. Moreover, because most borrowers are now LTV constrained, the central bank is less able to stimulate the economy, causing the bank to change the policy rate by more.

Finally, Figure 3 charts the effects of positive unit-standard-deviation housing preference shocks, which occur in low and high house price states of the baseline model and the LTV model. In the baseline model, the housing preference shock only significantly expands borrowing in the low house price state. This contrasts with the LTV model, where the housing preference shock expands borrowing in both states. The responses of the baseline model are caused by differences across the state of the economy in the constraint that binds. When house prices are relatively low and the LTV constraint binds, this constraint forcefully propagates the house price appreciation onto borrowing. The now more liquid impatient household increases its housing demand, thereby enlarging its borrowing ability further. When house prices are already high and the DTI constraint binds, these amplification channels are attenuated, significantly muting the effects of the housing preference shock. Instead, DTI constrained households are eventually forced to

delever, because the central bank raises its interest rate, as particularly residential investment expand the economy. The relative difference in debt responses fits well with the panel results of Subsection 8: this data show that the loan origination elasticity with respect to house prices is about twice as large when the house-price-to-income ratio is above its long-run trend and as when the ratio is below its trend.

The symmetric and state-invariant responses in the LTV model, shown in Figures 2-3, arise, since its constraint does not unbind following the impulses. As a result, debt always moves in tandem with housing wealth, leaving the model completely linear. If the constraint were to unbind, nonlinearities would arise, but they would, in general, be smaller than in the baseline model. The differences between the two models suggest that frameworks with only an LTV constraint misidentify the propagation from lone housing preference shocks.

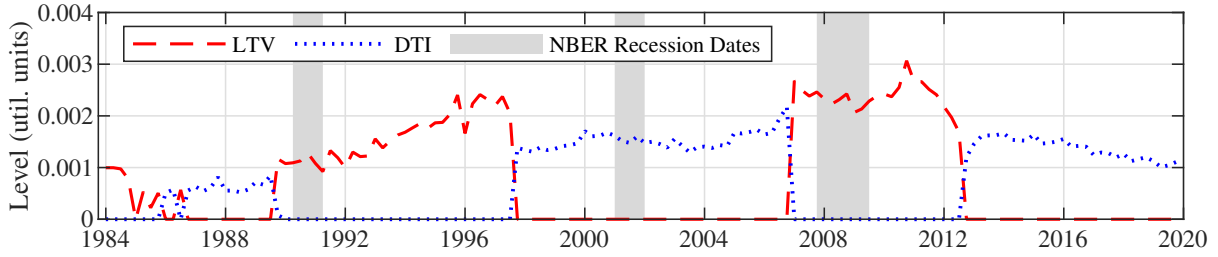
6 The Historical Evolution in Credit Conditions

This section gives a historical account of the evolution in credit conditions. The first subsection focuses on when the respective constraints restricted borrowing, and the circumstances that led them to do so. The second subsection zooms in on the estimated path of DTI limits. A decomposition of the drivers of house prices and mortgage debt is delegated to Online Appendix G.

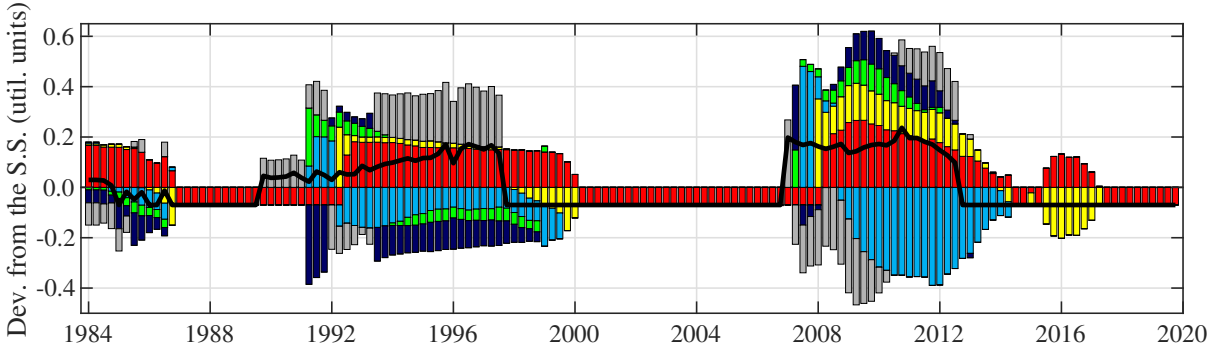
6.1 Historical Credit Regimes

Figure 4a superimposes the smoothed posterior Lagrange multipliers of the two credit constraints onto shaded NBER recession date areas. The LTV constraint binds when $\lambda_{LTV,t} > 0$, while the DTI constraint binds when $\lambda_{DTI,t} > 0$. Figures 4b-4c plot the historical shock decomposition of the Lagrange multipliers, in deviation from the steady state. At least one constraint binds throughout the period 1984-2019, signifying that borrowers have generally been credit constrained. However, the source of this control changed appreciably over time. Above all, we observe a consecutive pattern: the LTV constraint usually binds during and after recessions, while the DTI constraint binds in

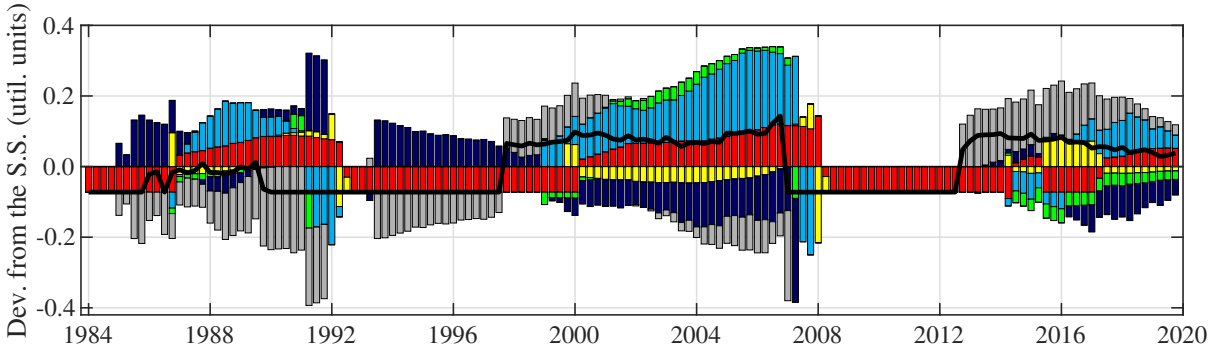
Figure 4: SMOOTHED POSTERIOR VARIABLES



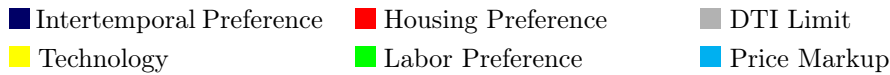
(a) LAGRANGE MULTIPLIERS



(b) SHOCK DECOMPOSITION: LTV LAGRANGE MULTIPLIER



(c) SHOCK DECOMPOSITION: DTI LAGRANGE MULTIPLIER

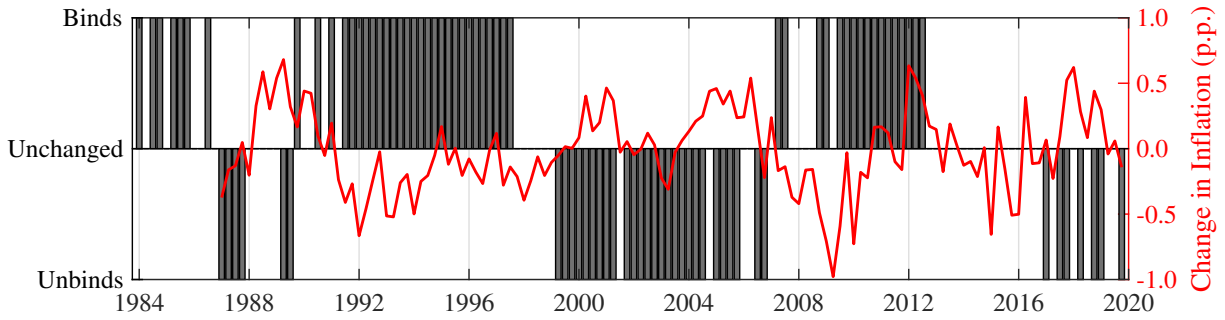


Note: The decomposition is performed at the baseline posterior mode. Figures 4b-4c illustrate the shock decomposition of the Lagrange multipliers. The steady-state values are positive, since both constraints bind in the steady state. Each bar indicates the contribution of a given shock to a certain variable. The shocks were marginalized in the following order: (1) housing preference, (2) labor-augmenting technology, (3) price markup, (4) labor preference, (5) intertemporal preference, and (6) DTI limit. This ordering is identical to the one applied by Guerrieri and Iacoviello (2017), with the novel DTI shock ordered last.

expansions.

The switching pattern is, to a large extent, caused by housing market sentiments (housing preference shocks) being more volatile than technology and labor preference shocks. House prices thereby materialize as more volatile than personal incomes, implying that the LTV constraint is tightened more than the DTI constraint in recessions and vice

Figure 5: SUBDUED MONETARY POLICY: EFFECT ON DTI CONSTRAINT



Note: The figure reports the effect on the DTI constraint of setting the monetary policy response to price inflation to $\tau_P = 1.01$, so that the Taylor principle is just barely fulfilled. The figure superimposes the change in inflation over the past 12 quarters. The simulations are performed at the baseline posterior mode.

versa in expansions.²³ The switching pattern is also a result of countercyclical monetary policy, which, c.p., relaxes the DTI constraint in recessions and tightens it in expansions. Figure 5 illustrates this point, by superimposing the change in inflation over the past three years on indicators for the periods when the two constraints would have bound differently from their historical paths if the Taylor principle was just barely fulfilled (i.e., $\tau_P = 1.01$). It emerges that, with a pruned inflation reaction, the DTI constraint becomes less likely to bind when inflation has risen recently and more likely to bind when inflation has fallen.

For this variety of reasons, the LTV constraint was binding in 1984-1985, after the early-1980s' double-dip recession. LTV control at that time aligns well with [Linneman and Wachter's \(1989\)](#) finding that down-payment requirements had a larger impact on households' homeownership decision than income-based requirements in the early 1980s. Eventually, however, the DTI constraint would start binding, as the mid-1980s boom caused consumer and house price inflation, along with interest rates, to surge. This continued being the case until just before the early-1990s' recession, when eased DTI standards and other factors caused the LTV constraint to start binding. Later on, accelerating house price growth loosened this constraint up again, prompting it to unbind by 1997.²⁴ With the onset of the Great Recession, the LTV constraint started to bind again, as housing market conditions deteriorated. Recently, from around 2013, the DTI constraint has been

²³The standard deviation of the detrended house price and personal income series is 0.099 and 0.020, respectively.

²⁴The decomposition echoes [Guerrieri and Iacoviello's \(2017\)](#) finding that the LTV constraint was slack in the early 2000s, due to soaring house prices. However, the decomposition also shows that this did not imply that homeowners could borrow freely, because of DTI requirements.

binding, in particular, due to a renewed surge in house prices and inflation, in addition to stricter DTI standards.

Finally, at odds with the prediction of cyclical switching, we observe that LTV control failed to dominate during the mild early-2000s' recession, as a result of positive housing market sentiments lingering, thereby preventing house prices from adjusting downward.

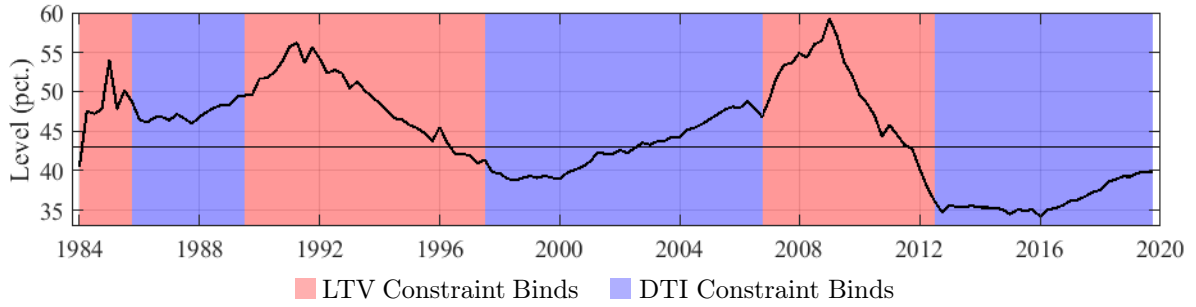
6.2 Debt-Service-to-Income Cycles

[Drehmann et al. \(2012\)](#) and [Borio \(2014\)](#) suggest the existence of a slowly moving financial cycle, which can be parsimoniously described in terms of credit and property prices and is disjunct from the regular business cycle. In this subsection, I ask how the financial cycle has shifted DTI limits historically? To shed light on this, [Figure 6](#) superimposes the smoothed posterior back-end DTI limit onto shaded areas indicating when each credit constraint was binding. Broadly unaffected by the switching between LTV and DTI constraints, DTI limits have undergone two boom-busts in the past 36 years.

The first cycle started in the 1980s. Here, the DTI limit was raised from around 46 pct. in 1984 to 55 pct. by 1991. The relaxation likely resulted from the first major financial deregulation since the Great Depression. The Depository Institutions Deregulation and Monetary Control Act of 1980 and the Garn-St. Germain Depository Institutions Act of 1982 deregulated and increased the competition between banks and thrift institutions, according to [Campbell and Hercowitz \(2009\)](#). In addition, state deregulation allowed banks to expand their branch networks within and between states, further increasing bank competition, as emphasized by [Mian et al. \(2020\)](#). Due to these changes in legislation, greater access to alternative borrowing instruments (e.g., adjustable-rate loans) reduced effective down payments and allowed households to delay repayment through cash-out refinancing. This process continued until the Savings and Loan Crisis, after which the DTI limit gradually returned to its steady state.

The second cycle occurred in the 2000s. This time, the DTI limit was lifted from 39 pct. in 1998 to 56 pct. in 2008. This chronology aligns with [Justiniano et al.'s \(2019\)](#) conclusion that looser LTV limits cannot explain the recent surge in mortgage credit. They instead argue that it was an increase in credit supply that caused the boom. They

Figure 6: BACK-END DTI LIMIT



Note: The figure plots the smoothed back-end DTI limit ($\tilde{\xi}_{DTI|S_{DTI,t}}$), identified at the baseline posterior mode. The horizontal line indicates its steady-state value ($\tilde{\xi}_{DTI}$).

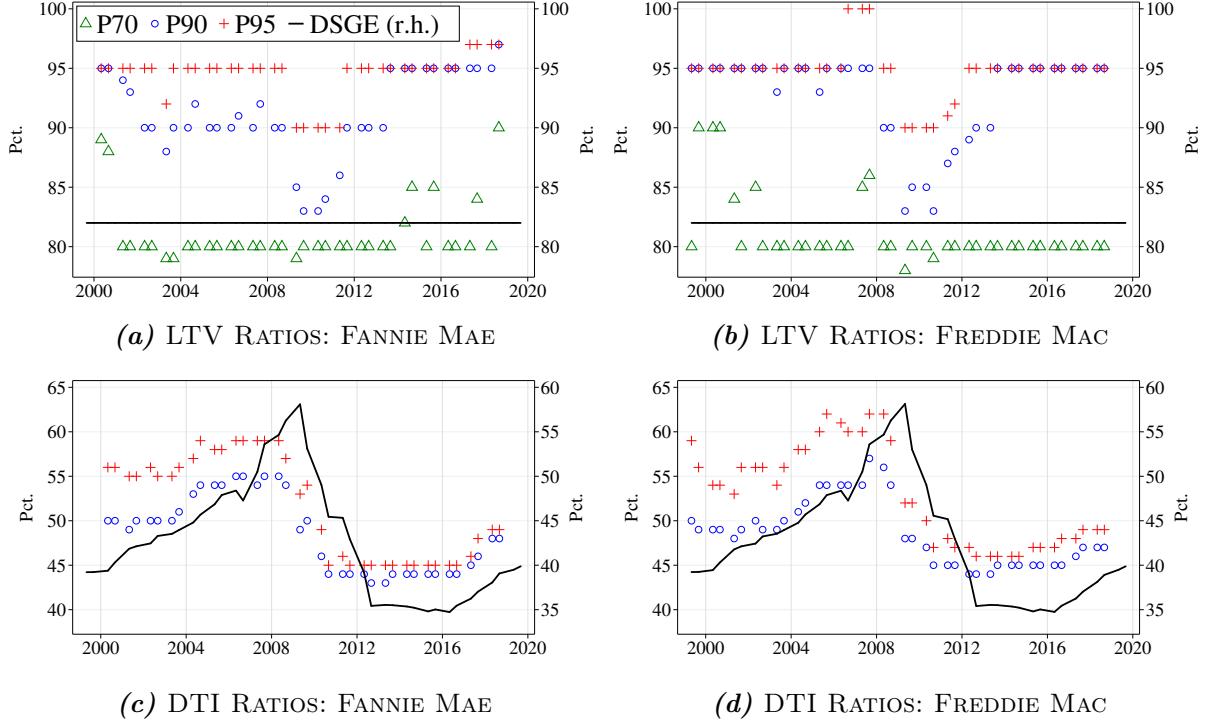
mention the pooling and tranching of mortgage bonds into mortgage-backed securities and the global savings influx into the U.S. mortgage market following the late-1990s Asian financial crisis. My finding that the DTI limit was relaxed, in turn, suggests that the increase in credit supply translated into lax credit limits.²⁵ Later on, from the eruption of the Financial Crisis and into the ensuing recession, the DTI limit was gradually tightened, and fell to 36 pct. in 2013 and 40 pct. in 2019. These developments presumably reflect a smaller post-crisis risk appetite on behalf of lenders, in addition to enhanced financial regulation. Accordingly, the estimated values are within the range of the 36 pct. limit specified by large U.S. retail banks on their websites (see Online Appendix A) and the 43 pct. Ability-to-Repay rule of the Dodd-Frank Act.

Mapping the Results to Loan-Level Data To assess the realism of the model, I now compare the estimated credit limits to leverage ratios found in loan-level data. Specifically, Figure 7 charts the upper percentiles of the cross-sectional distribution of combined LTV ratios and back-end DTI ratios on newly issued conventional fixed-rate mortgages, securitized by Fannie Mae since 2000 and Freddie Mac since 1999.²⁶ Figure 7 also charts the constant LTV limit and time-varying back-end DTI limit from the DSGE estimation.

²⁵An increase in saving cannot be captured by just the DTI shock, since the credit constraints are frictions on the demand-side of the loan market. However, for an increase in saving to be intermediated onward to the impatient household, the DTI shock must increase in the absence of any other shock that relaxes the credit constraints.

²⁶The combined LTV ratio is the ratio of total mortgage debt to the home value, if applicable, summing over multiple mortgages collateralized against the same property. Greenwald (2018) uses the same data to document bunching around institutional LTV and DTI limits.

Figure 7: LTV AND DTI RATIOS: LOAN-LEVEL DATA AND DSGE ESTIMATION



Note: The data are from the acquisitions files in Fannie Mae’s Single-Family Fixed Rate Mortgage Dataset, covering 2000Q1-2018Q4, and the origination files in Freddie Mac’s Single Family Loan-Level Dataset, covering 1999Q1-2018Q3. The DSGE values refer to the LTV limit (ξ_{LTV}) and to the smoothed back-end DTI limit ($\xi_{DTI,DTI,t}$), identified at the baseline posterior mode.

On the whole, there is a remarkable similarity, transversely to the datasets, in how the upper parts of the LTV and DTI distributions appear over time. Moreover, across the sample periods, the upper parts of both distributions lie slightly above the LTV and DTI limits in the model, something that should be seen in the light of the model not incorporating losses on lending. Focusing on the LTV ratios, the cross-sectional distributions changed little across time. For instance, the 95th percentile is constant at 95 pct., primarily except for a brief period around the Great Recession, when it descended to 90 pct. It is, in part, this near constancy that motivates my assumption of a time-constant LTV limit in the model. We also see that the 70th percentile has largely remained constant at 80 pct., the point where borrowers must acquire private mortgage insurance, throughout most of the periods considered.

Turning to the DTI plots, we observe a completely different configuration. The 90th and 95th percentiles grew by 5-10 p.p. from the turn of the millennium until 2008, after

which they fell until 2013 by around 15 p.p., hence overshooting their reference points. There is a reasonably close correspondence between this development and the DSGE path. In the latter case, the DTI limit rises by approximately 10 p.p. until 2007, and falls by approximately 20 p.p. after the crisis. The only point in time where the DTI measures diverge is in 2009, where the DSGE limit spikes, presumably because the model, with its time-constant refinancing rate, underestimates the degree of debt overhang in the data. Finally, in both the loan-level and DSGE data, we observe a recent surge in DTI limits by about 5 p.p.

7 Macprudential Implications

Recent studies show that credit expansions predict subsequent banking and housing market crises (e.g., [Mian and Sufi, 2009](#); [Schularick and Taylor, 2012](#); [Baron and Xiong, 2017](#)). Motivated by this, I now examine how mortgage debt would historically have evolved if credit limits or monetary policy had responded countercyclically to deviations of credit from its long-run trend.

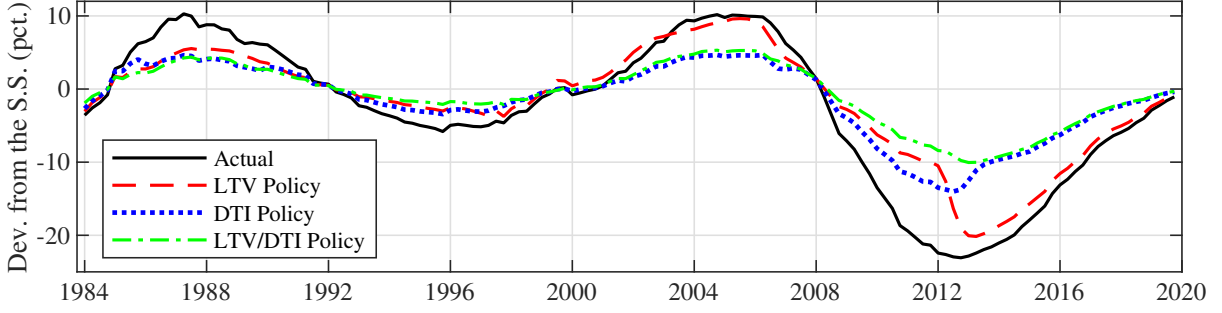
7.1 Countercyclical Credit Limits

Figure [8a](#) plots the reaction of mortgage debt to the estimated sequence of shocks under four different credit regimes. In the first regime, there is no active macroprudential policy, so the LTV limit is constant and the DTI limit is shifted by the DTI shock, as in the estimated model. Thus, the observed variables in the model, by construction, match the data. In the three other regimes, the following policies apply: a countercyclical LTV limit, a countercyclical DTI limit, and countercyclical LTV and DTI limits. Figures [8b-8c](#) plot the credit limits implied by the policies. I introduce the countercyclical credit limits by augmenting the credit constraints in [\(8\)-\(9\)](#) with two macroprudential stabilizers:

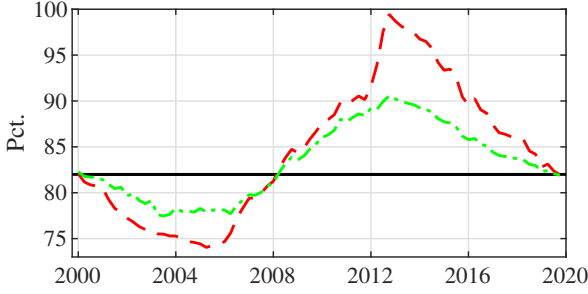
$$b'_t \leq \rho \left(\kappa_{LTV} \xi_{LTV} \hat{s}_{LTV,t} \mathbb{E}_t \{ (1 + \pi_{t+1}) q_{t+1} h'_t \} + (1 - \kappa_{LTV}) \xi_{DTI,t} \mathbb{E}_t \left\{ \frac{(1 + \pi_{t+1}) w'_{t+1} n'_t}{\sigma + r_t} \right\} \right),$$

$$b'_t \leq \rho \left((1 - \kappa_{DTI}) \xi_{LTV} \hat{s}_{LTV,t} \mathbb{E}_t \{ (1 + \pi_{t+1}) q_{t+1} h'_t \} + \kappa_{DTI} \xi_{DTI,t} \mathbb{E}_t \left\{ \frac{(1 + \pi_{t+1}) w'_{t+1} n'_t}{\sigma + r_t} \right\} \right),$$

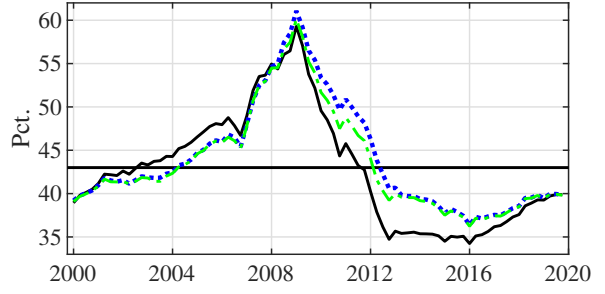
Figure 8: COUNTERCYCLICAL CREDIT LIMITS



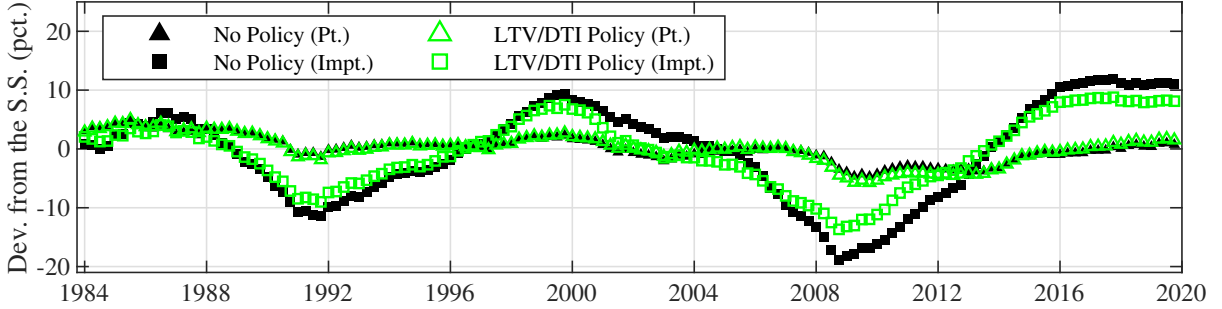
(a) MORTGAGE DEBT



(b) LTV LIMIT



(c) BACK-END DTI LIMIT



(d) HOUSEHOLD CONSUMPTION

Note: The simulations are performed at the baseline posterior mode. Figures 8b-8c plot the LTV limit ($\xi_{LTV} \hat{s}_{LTV,t}$) and the back-end DTI limit ($\tilde{\xi}_{DTI} s_{DTI,t} \hat{s}_{DTI,t}$), with horizontal lines indicating the steady-state values (ξ_{LTV} and $\tilde{\xi}_{DTI}$).

where $\xi_{DTI,t} \equiv \frac{\tilde{\xi}_{DTI} s_{DTI,t} \hat{s}_{DTI,t} - \xi_0}{1 - \tau_L}$, $\hat{s}_{LTV,t}$ is an LTV stabilizer, and $\hat{s}_{DTI,t}$ is a back-end DTI stabilizer. As the simplest imaginable policy rule to stabilize mortgage debt, the stabilizers respond negatively with a unit elasticity to the expected deviation of debt from its steady-state level:

$$\log \hat{s}_{LTV,t} = -(\mathbb{E}_t \log l'_{t+1} - \log l') \quad \text{and} \quad \log \hat{s}_{DTI,t} = -(\mathbb{E}_t \log l'_{t+1} - \log l'), \quad (21)$$

where l' denotes the steady-state net level of outstanding mortgage loans.

The LTV policy reduces the standard deviation of mortgage debt by 25 pct. relative to

the historical benchmark. It does so principally by mitigating the adverse effects of house price slumps on credit availability. For instance, across 2009-2012, following the Financial Crisis, the LTV limit is, on average, 7.9 p.p. higher under (21) than in the benchmark simulation, which considerably limits the credit bust. The flip-side of this result is that the LTV policy often cannot curb credit expansions during house price booms, since most borrowers are constrained by the DTI requirement in these situations. Thus, even though the LTV limit is 6.7 p.p. lower in 2003-2006 with the LTV policy, as compared to the benchmark level, macroprudential policy does not prevent the mid-2000s' boom in credit. The DTI policy is, by contrast, able to curb credit growth during house price booms, by enforcing stricter DTI norms. In the above simulations, this policy reduces the standard deviation of mortgage debt by 48 pct. relative to the benchmark. Zooming in on the mid-2000s' credit boom, the DTI policy dictates that this limit should have been 1.4 p.p. lower, again across 2003-2006. This would roughly have halved the expansion in credit around this time. The lowest volatility in mortgage debt is reached by combining the LTV and DTI policies. This reduces the standard deviation by 57 pct. relative to the benchmark. In this case, macroprudential policy takes into account that the effective tool changes over the business cycle, mostly with a DTI tool in expansions and an LTV tool in contractions. The implementation of such a policy does not require that the policymaker in real time knows when either constraint binds. Rather, it merely presupposes that the policymaker conducts a two-stringed policy entailing that *both* requirements respond countercyclically to the credit gap.

The underlying objective of a macroprudential policy that stabilizes credit fluctuations is arguably to minimize the probability of large drops in consumption. For this reason, I now compute a measure of consumption-at-risk in the no-policy scenario and under the two-stringed policy. I define consumption-at-risk as the maximum negative deviation of consumption from its steady state occurring within the top 95 pct. of the distribution of consumption observations.²⁷ Historical consumption-at-risk is 3.7 pct. of steady-state consumption for the patient household and 15.7 pct. for the impatient household. The two-stringed policy increases this metric to 4.5 pct. for the patient household, and decreases

²⁷This definition of consumption-at-risk is congruous with the value-at-risk measure commonly used within finance and the output-at-risk measure of Nicolò and Lucchetta (2013) and Jensen et al. (2018).

it to 11.0 pct. for the impatient household. Figure 8d sheds some light on these changes by plotting the paths of household consumption in the two scenarios. With an active policy, deleveraging in busts is significantly curtailed, as shown in Figure 8a. This limits the redistribution of funds from the impatient to the patient household in these episodes, leaving borrowers able to consume more and lenders necessitated to consume less. As a result, the left tail of the consumption distribution is lower in the patient household and higher in the impatient household. The two-stringed policy thus redistributes consumption *risk* from the impatient household to the patient household, while roughly maintaining average household consumption levels.²⁸

7.2 Leaning Against the Wind

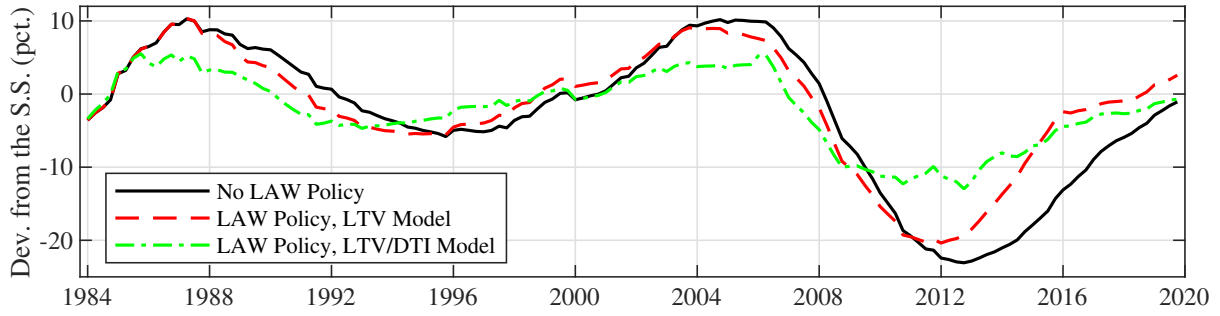
Figure 9a plots the reaction of mortgage debt to the estimated sequence of shocks under two different monetary policy regimes, in the baseline model and the LTV model. In the first regime, monetary policy follows the estimated Taylor rule in (15), implying that the central bank is devoid of any interest in debt stabilization. In this case, the observed variables in the model again match the data. In the alternative regime, the central bank adjusts its interest rate with the aim of stabilizing mortgage debt. I introduce this "leaning-against-the-wind" policy by augmenting the monetary policy rule with a term reacting positively to the expected deviation of debt from its steady-state level:

$$i_t = \tau_R i_{t-1} + (1 - \tau_R) i + (1 - \tau_R) \tau_P \pi_{P,t} + 0.0075 \cdot (\mathbb{E}_t \log l'_{t+1} - \log l'). \quad (22)$$

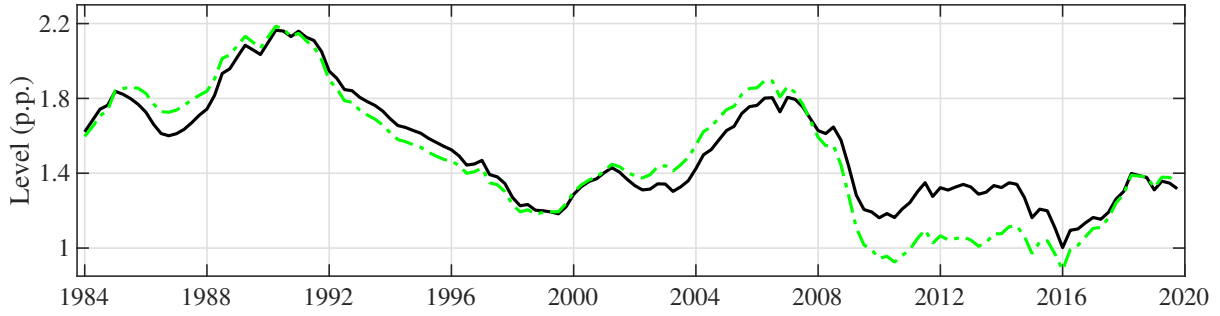
Leaning against the wind stabilizes mortgage debt, thereby reducing its standard deviation by 48 pct. relative to the benchmark. This is, to a large extent, a result of a significant fraction of borrowers being DTI constrained. Thus, in the LTV model, monetary policy only weakly influences the credit cycle. This small effect of monetary policy on mortgage debt in LTV-only environments has two causes, as already shown in Gelain et al. (2018). First, when borrowers are LTV constrained, monetary policy only affects

²⁸Consumption is 0.02 pct. lower in the patient household and 0.02 pct. higher in the impatient household, on average across 1984-2019, under the two-stringed policy. Aggregate consumption and output are largely unaffected by the policy, because the responses of borrowers and lenders "wash out in the aggregate", as coined by Justiniano et al. (2015).

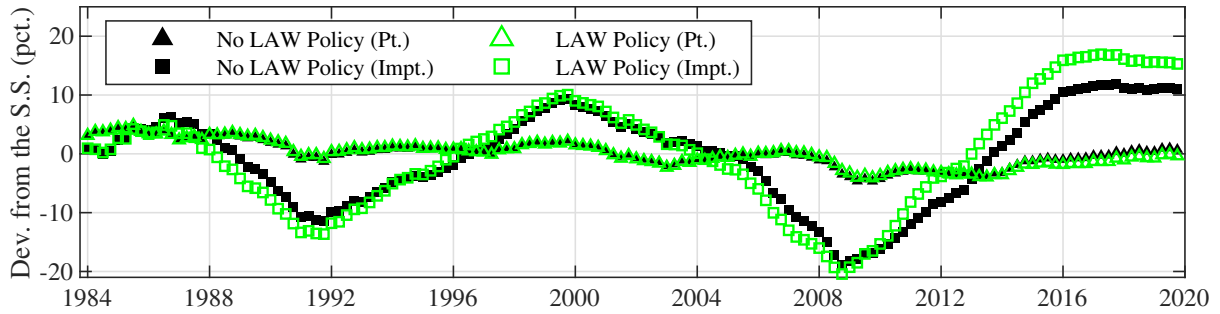
Figure 9: LEANING AGAINST THE WIND



(a) MORTGAGE DEBT



(b) NOMINAL POLICY RATE



(c) HOUSEHOLD CONSUMPTION: BASELINE MODEL

Note: The simulations are performed at the respective posterior modes.

borrowing indirectly, by changing housing demand and house prices. Second, an interest rate hike, for instance, reduces inflation, leading to higher – not lower – real levels of debt. These effects are compounded when debt is long-term.

Finally, Figure 9c reports the paths of household consumption in the two scenarios, from the baseline model. Consumption-at-risk decreases to 3.6 pct. for the patient household, and increases to 16.2 pct. for the impatient household. Thus, unlike with the two-stringed policy, leaning against the wind redistributes consumption risk from the patient to the impatient household. This is because the policy, c.p., implies an increase in interest rates when the debt level rises, unavoidably increasing the impatient household's

interest payments when they are most indebted. Thus, if policymakers have distributional concerns, they may prefer the two-stringed policy over leaning against the wind, since the impatient household is relatively poor already in the absence of leaning against the wind.

8 Evidence on State-Dependent Credit Origination

Given that homeowners face both LTV and DTI requirements, we should expect income (house price) growth – not house price (income) growth – to be the main driver of credit growth when the homeowners’ housing-wealth-to-income ratio is relatively high (low). In this section, I test this prediction by estimating the elasticities of mortgage loan origination with respect to house prices and personal incomes after having partitioned the elasticities based on a proxy for the aforementioned ratio.

8.1 Data

The dataset contains data on the dollar amount of originated mortgage loans, house prices, personal incomes, and population size, across U.S. counties in all 50 states and the District of Columbia at an annual longitudinal frequency. Data on originated mortgage loans are from the Loan Application Register of the Home Mortgage Disclosure Act (HMDA).²⁹ I consider originated mortgage loans that are secured by a first or subordinate lien in an owner-occupied principal dwelling, consistent with the theoretical measure of credit in the DSGE model. The results are robust to broader credit measures, such as total originated mortgage loans. A limitation of the HMDA data is its inability to exactly identify equity extraction. However, as shown by [Mondragon \(2018\)](#), the behavior of aggregate mortgage origination is similar to that of aggregate equity extraction. Coverage of the HMDA dataset starts in 1990. I collect the data from two sources: the U.S. Library of Congress (1990-2006) and the U.S. Consumer Financial Protection Bureau (2007-2017).³⁰

²⁹HMDA was enacted in 1975, and obligates most U.S. financial institutions to disclose information about home mortgages. With the implementation of the Dodd-Frank Act, HMDA rule-writing authority was transferred from the U.S. Federal Reserve Board to the U.S. Consumer Financial Protection Bureau. HMDA data are also used by [Mondragon \(2018\)](#) and [Gilchrist, Siemer, and Zakrajšek \(2018\)](#), to study the effects of credit supply shocks.

³⁰The National Archives Identifier is 2456161. Coverage technically goes back to 1981, but some variables of interest (e.g., the type of action taken) are unavailable before 1990.

The house price data are from the All-Transactions House Price Index of the U.S. Federal Housing Finance Agency, and is available from 1975. The income and population data are from the Personal Income, Population, Per Capita Personal Income (CAINC1) table in the Regional Economic Accounts of the U.S. Bureau of Economic Analysis, and are available from 1966. The merged sample effectively covers the years 1991-2017, as I lose the first year of observations, because I am regressing log-differences. The dataset is unbalanced, since observations on loan origination and house prices are sporadically missing if the transaction volume in a given county and year was insufficient.

Online Appendix I reports summary statistics of the data. The dataset contains 62,424 unique county-year observations on population size and the growth rates of mortgage loan origination, house prices, and incomes. Unconditionally, loan origination growth has a small positive correlation with house price growth (15 pct.), and is uncorrelated with income growth (2 pct.), while house price and income growth are themselves positively correlated (36 pct.).

8.2 Identification Strategy

The goal of the analysis is to identify the causal effect of house prices, incomes, and interactions between house prices and incomes on loan origination. A challenge to doing this is that house prices and incomes are endogenously determined by each other, along with forces determining home credit. For instance, a favorable credit or productivity shock may increase loan origination, house prices, and incomes without any causal relationship between these variables. In that case, not only would the house price and income elasticities be positively biased, but the interacting effect of house price and income growth would also be positively biased.

In order to overcome the described identification challenge, I rely on an instrumental variable strategy, in combination with a rich set of fixed effects. The instrumental variable strategy uses systematic differences in the sensitivity of local house prices (incomes) to the national house price (income) cycle to instrument house price (income) variation. This approach builds on work by [Sinai \(2013\)](#), showing continual differences in how sensitive local house prices are to the national house price cycle. The strategy is also inspired by

the commonly used "Bartik instrument", which in labor economics involves using national employment to instrument local labor demand (e.g., [Blanchard and Katz, 1992](#)).³¹ [Palmer \(2015\)](#) and [Guren et al. \(2018\)](#) similarly use aggregate house prices to instrument local house prices, in their studies of the effects of house prices on, respectively, mortgage defaults and retail employment.

I construct the instrument by, for each county i , estimating the following first-stage time series relations:

$$\Delta \log hp_{i,t} = \gamma_{i, hp} + \hat{\beta}_{i, hp} \Delta \log hp_{-i,t} + v_{i,t, hp}, \quad (23)$$

$$\Delta \log inc_{i,t} = \gamma_{i, inc} + \hat{\beta}_{i, inc} \Delta \log inc_{-i,t} + v_{i,t, inc}, \quad (24)$$

where $\mathbb{E}\{v_{i,t, hp}\} = \mathbb{E}\{v_{i,t, inc}\} = 0$. $\Delta \log hp_{i,t}$ and $\Delta \log inc_{i,t}$ denote the log change in house prices and personal incomes in county i in year t . Moreover, $\Delta \log hp_{-i,t}$ and $\Delta \log inc_{-i,t}$ denote the log change in national house prices and personal incomes in year t after weighing out the contribution of county i to the national indices.³² Finally, $\gamma_{i, hp}$ and $\gamma_{i, inc}$ are county fixed effects. I use the predicted values from (23)-(24) as instruments for the growth rates of house prices and personal incomes across counties. Note that (23)-(24) are not first-stage regressions in a traditional two-stage least squares sense, in that the loading factors, $\hat{\beta}_{i, hp}$ and $\hat{\beta}_{i, inc}$, vary across counties. Rather, the predicted values from (23)-(24) proxy the magnitude by which house prices and incomes move at a given point in time, abstracting from local shocks that do not affect the aggregate economy. The difference across counties in how much the national cycles load on local conditions, in turn, plays the same role in my empirical strategy as [Saiz's \(2010\)](#) estimates of housing supply elasticities play in, e.g., [Mian and Sufi \(2011\)](#), namely to determine by how much house prices are expected to change at a given point in time.³³

³¹[Bartik \(1991\)](#) used local industry shares to proxy how much the national change in employment within each industry loaded on local labor demand. In a similar way, my approach uses the estimated loading of the national house price (income) cycle on local house prices (incomes) as a source of exogenous variation in local conditions.

³²This weighing-out is meant to remove the mechanical contribution of county i to the national indices. I use the county population shares as weights. For all practical purposes, the transformed indices are identical to the national indices, as the population shares of even large counties are tiny. The results are thereupon robust to simply using the national indices as instruments.

³³It is not possible to use [Saiz's \(2010\)](#) housing supply elasticities as a house price instrument with the current setup, since these elasticities do not vary over time.

In addition to instrumenting house price and income growth, I rely on county and state-year fixed effects, in order to control for potential confounders, as in [Cloyne et al. \(2019\)](#). County fixed effects control for fixed differences in the propensity to originate loans, while state-year fixed effects control for time-varying state shocks to loan origination. Identification hence arises from time-varying differences in credit origination across counties that cannot be explained by the average origination within a county’s state. With these controls, e.g., state fiscal or credit shocks will not threaten identification, as they will be captured by the state-year effects.

Under the following two conditions, a regression of credit origination on the house price and income instruments identifies the causal effect of local house price and income growth on local origination. First, the national house price (income) cycle must yield predictive power over local house prices (incomes), so that the instruments are relevant.³⁴ Second, conditional on the fixed effects, the loading of national house prices (incomes) on local house prices (incomes) must not be influenced by local shocks to credit origination, implying that the instruments are exogenous. Thus, importantly, the approach does not assume that the nationwide variation in house prices and incomes is exogenous. Rather, it presupposes that there is no systematic time-varying divergence in the uptake of the national variables on local variables, conditional on the fixed effects.

8.3 Results

The second-stage regression specification is given by

$$\begin{aligned} \Delta \log d_{i,t} = & \delta_i + \zeta_{j,t} + \beta_{hp} \Delta \log \widehat{hp}_{i,t-1} + \beta_{inc} \Delta \log \widehat{inc}_{i,t-1} \\ & + \tilde{\beta}_{hp} \mathcal{I}_{i,t}^{LTV} \Delta \log \widehat{hp}_{i,t-1} + \tilde{\beta}_{inc} \mathcal{I}_{i,t}^{DTI} \Delta \log \widehat{inc}_{i,t-1} + u_{i,t}, \end{aligned} \quad (25)$$

where $\mathbb{E}\{u_{i,t}\} = 0$. $\Delta \log d_{i,t}$ denotes the log change in the amount of originated mortgage loans in county i in year t . Moreover, δ_i denotes the county fixed effect in county i , and $\zeta_{j,t}$ denotes the state-year fixed effect in state j in year t . Finally, $\Delta \log \widehat{hp}_{i,t}$ and $\Delta \log \widehat{inc}_{i,t}$ denote the predicted values from (23)-(24). (25) uses lagged house price and income vari-

³⁴In (23)-(24), the restrictions $\hat{\beta}_{i, hp} = 0$ or $\hat{\beta}_{i, inc} = 0$ are rejected at a one-percent confidence level in 84 pct. of the counties for house prices and 97 pct. for incomes, indicating that the instruments are broadly relevant. The average t-statistic is 5.28 for house prices and 9.65 for incomes across the counties.

ables, to prevent any confounding shocks that have not already been instrumented out or are captured by the fixed effects from biasing the results, as in [Guerrieri and Iacoviello \(2017\)](#). The results below are qualitatively robust to a number of alternative econometric assumptions, such as not using the Bartik-instruments, as well as using current house price and income variables. They are also robust to omitting the county fixed effects or replacing the state-year fixed effects with year fixed effects.

In my baseline specification, I let $\mathcal{I}_{i,t}^{LTV}$ and $\mathcal{I}_{i,t}^{DTI}$ denote *level* indicators for house prices and personal incomes in county i in year t . The LTV (DTI) indicator takes the value "0" if the ratio of house prices to incomes is above (below) its long-run county-specific ratio in a given year and the value "1" if it is below (above):

$$\mathcal{I}_{i,t}^{LTV} \equiv 1 - \mathcal{I}_{i,t}^{DTI} \equiv \begin{cases} 0 & \text{if } \log\left(\frac{hp_{i,t}}{inc_{i,t}}\right) \geq \overline{\log\left(\frac{hp_{i,t}}{inc_{i,t}}\right)} \\ 1 & \text{else,} \end{cases} \quad (26)$$

where $\overline{\log\left(\frac{hp_{i,t}}{inc_{i,t}}\right)}$ denotes a separately estimated county-specific quadratic or cubic time trend.³⁵ With this specification, the indicators partition the house price and income elasticities in (25) based on the prevailing detrended ratio of house prices to incomes.³⁶ More forces than just multiple credit constraints could, in principle, cause house price and income growth to amplify each other. Nonetheless, this partitioning does facilitate a test of whether the state-dependent credit dynamics implied by the LTV and DTI requirements are present in the data. If home values are sufficiently below incomes, then the house price elasticity should likely be high ($\beta_{hp} + \tilde{\beta}_{hp}$) and the income elasticity low (β_{inc}), since households will primarily be LTV constrained. Symmetrically, if incomes are below home values, then the income elasticity should likely be high ($\beta_{inc} + \tilde{\beta}_{inc}$) and the house price elasticity low (β_{hp}), because households will predominantly be DTI constrained.³⁷

³⁵I avoid using linear trends, as the trend growth rate is unlikely to have been constant over the entire estimation period. For instance, shifts in total factor productivity growth, relative sectoral productivity levels, or migration patterns could affect the trend.

³⁶The value $\log\left(\frac{hp_{i,t}}{inc_{i,t}}\right)$ does not have a meaningful interpretation by itself, as $hp_{i,t}$ is an index. Subtracting from $\log\left(\frac{hp_{i,t}}{inc_{i,t}}\right)$ its county-specific time trend serves to create a balanced mix of high and low price-income observations within each county.

³⁷Whether LTV or DTI constraints dominate should ideally depend on the housing-wealth-to-income ratio, rather than on the house-price-to-income ratio relative to its trend. However, estimating such a specification is not possible with the current data, as it requires information on both the size of the housing stock and the actual house price level (not an index) within each county.

Table 3: CATALYSTS FOR CREDIT ORIGINATION: LEVEL SHIFTERS (1991-2017)

Detrending Method	$\Delta \log b_t$					
	N/A	Quadratic		Cubic		N/A
	(1)	(2)	(3)	(4)	(5)	(6)
$\widehat{\Delta \log hp_{i,t-1}}$	0.523*** (0.0926)	0.331*** (0.115)	0.330*** (0.116)	0.207 (0.130)		0.130 (0.114)
$\widehat{\Delta \log inc_{i,t-1}}$	0.0906 (0.193)	-0.0610 (0.203)		-0.0778 (0.198)		0.151 (0.184)
$\mathcal{I}_{i,t}^{LTV} \widehat{\Delta \log hp_{i,t-1}}$		0.317** (0.127)	0.315** (0.125)	0.483*** (0.148)	0.553*** (0.117)	
$\mathcal{I}_{i,t}^{DTI} \widehat{\Delta \log inc_{i,t-1}}$		0.400*** (0.112)	0.396*** (0.108)	0.509*** (0.116)	0.547*** (0.0999)	
$\widehat{\Delta \log hp_{i,t-1}} \widehat{\Delta \log inc_{i,t-1}}$						9.795*** (1.692)
Observations	62424	62424	62424	62424	62424	62424
Adjusted R^2	0.674	0.674	0.674	0.674	0.674	0.674

Note: County and state-year fixed effects are always included. Observations are weighted by the county population in a given year. Standard errors are clustered at the county level, and reported in parentheses. ***, **, and * indicate statistical significance at the 1 pct., 5 pct., and 10 pct. confidence levels.

Table 3 reports the ordinary least squares estimates of the second-stage regression equation in (25) under (26). In specification 1, I do not allow for state-dependent elasticities, in which case only the house price elasticity is significantly positive. In specification 2, I partition the elasticities as explained above, based on quadratic trends. The point estimates of the unconditional elasticities dwindle. More interestingly, however, the estimates of the newly introduced conditional elasticities are significantly positive and, as compared to the unconditional elasticities, sizable. In particular, in the parsimonious specification 3, the house price elasticity is about twice as large when the house-price-to-income ratio is low (0.65) than when it is high (0.33), while the income elasticity (0.40) is only positive when the house-price-to-income ratio is high. This shift in the house price elasticity aligns well with the DSGE impulse responses, illustrated in Figure 3, showing that debt responds by about twice as much to a given house price change if the LTV constraint dominates, as compared to if the DTI constraint dominates. In specifications 4-5, as a robustness test, I rerun the estimation with cubic trends. The previous results on state-dependent elasticities now appear more distinctly: in the parsimonious specification 5, the house price

elasticity is 0.55 if incomes are relatively high, and, coincidentally, the income elasticity is 0.55 if house prices are relatively high.³⁸ Otherwise, the elasticities are zero. Lastly, in specification 6, I add a continuous interaction term. If house price and income growth amplify each other, then this might also show up as a continuous interaction, something that I find to be the case.

Online Appendix I contains some robustness checks, including one where I estimate (25) on just the period 2009-2017, as well as one where I partition the indicators based on growth rates instead of house-price-to-income ratios. The aforementioned results carry through in these cases. All in all, it emerges that the process through which growth in house prices and incomes leads to growth in mortgage credit is not linear. Instead, variation in house-price-to-income ratios across counties shifts the effect of house prices and incomes on credit origination, in conformance with the presence of multiple credit constraints.

9 Concluding Remarks

Banks impose both LTV and DTI constraints on loan applicants. Yet, because house prices and interest rates are low in recessions and high in expansions, LTV constraints tend to dominate in recessions, and DTI constraints tend to dominate in expansions. This – until now, unexplored – systematic switching between credit constraints has fundamental implications for the workings of the economy. The switching causes an asymmetric and state-dependent variation in the transmission of economic shocks onto credit uptake: adverse shocks have larger effects than favorable shocks, and a given shock has the largest effect in contractions. The switching also implies that the effective macroprudential tool changes over the business cycle, with an LTV tool in contractions and a DTI tool in expansions. The potency of monetary policy increases with the share of DTI constrained households, but using this policy as a macroprudential tool may not be desirable for distributional reasons. Finally, county panel data on mortgage loan origination, house prices, and incomes attest to multiple credit constraints as a source of nonlinear dynamics.

³⁸I arrive at the parsimonious specification after sequentially having restricted the most insignificant term out and reestimated the model.

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Multiple Credit Constraints and Time-Varying Macroeconomic Dynamics

Online Appendix

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A Qualitative Evidence on the DTI Limits of Banks

Table A.1 reports the DTI limits that the ten largest U.S. retail banks post on their websites. The specific statements about DTI limits follow below the table. All banks that issue mortgage loans instruct loan applicants to fulfill a front-end requirement of 28 pct. or a back-end requirement of 36 pct.

Table A.1: DTI LIMITS OF THE TEN LARGEST U.S. RETAIL BANKS

Rank	Name	Domestic Assets (million \$)	DTI Limit	
			Front-end	Back-end
1	JPMorgan Chase Bank	1,676,806	28 pct.	36 pct.
2	Wells Fargo Bank	1,662,311	–	36 pct.
3	Bank of America	1,661,832	–	36 pct.
4	Citibank	821,805	–	36 pct.
5	U.S. Bank	442,844	28 pct.	–
6	PNC Bank	364,084	28 pct.	36 pct.
7	TD Bank	294,830	28 pct.	36 pct.
8	Capital One	289,808	–	–
9	Branch Banking and Trust Company	214,817	28 pct.	–
10	SunTrust Bank	199,970	28 pct.	36 pct.

Note: No DTI limit is available from Capital One, since this bank stopped issuing mortgage loans in 2017. All websites were accessed on September 23, 2018. The banks are ranked by the size of their domestic assets as of March 31, 2018, see [Federal Reserve Statistical Release \(2018\)](#).

JPMorgan Chase Bank

"Some lending institutions sometimes ascribe to a "28/36" guideline in assessing appropriate debt loads for individuals, meaning housing costs should not exceed 28 percent of gross monthly income, and back end costs should be limited to an additional 8 points for a total of 36 percent."

Website: [chase.com/news/121115-amount-of-debt](https://www.chase.com/news/121115-amount-of-debt)

Wells Fargo Bank

*"Calculating your debt-to-income ratio
(Rule of thumb: At or below 36%)"*

"Is your ratio above 36%?"

There are loan programs that allow for higher debt-to-income ratios. Consult with a home mortgage consultant to discuss your options. You can also try to reduce your existing monthly debt by paying off one or more obligations. And you may want to think about consolidating existing loan balances at a lower interest rate and payment."

Website: [wellsfargo.com/mortgage/learning/calculate-ratios/](https://www.wellsfargo.com/mortgage/learning/calculate-ratios/)

Bank of America

"Why is my debt-to-income ratio important?"

Banks and other lenders study how much debt their customers can take on before those customers are likely to start having financial difficulties, and they use this knowledge to set lending amounts. While the preferred maximum DTI varies from lender to lender, it's often around 36 percent."

"How to lower your debt-to-income ratio"

If your debt-to-income ratio is close to or higher than 36 percent, you may want to take steps to reduce it."

Website: bettermoneyhabits.bankofamerica.com/en/credit/what-is-debt-to-income-ratio

Citibank

"Your debt-to-income (DTI) ratio is the percentage of your monthly gross income that goes toward paying debts. The lower your DTI ratio, the more likely you are to qualify for a mortgage. Lenders include your monthly debt expenses and future mortgage payments when they consider your DTI."

"The preferred DTI ratio is generally around 36%. You can reduce your DTI ratio by limiting your credit card usage and paying down your existing debt."

Website: online.citi.com/US/JRS/portal/template.do?ID=mortgage_what_affects_my_rates

U.S. Bank

"A standard rule for lenders is that your monthly housing payment (principal, interest, taxes and insurance) should not take up more than 28 percent of your income."

"Mortgage payments should not exceed more than 28% of your income before taxes (a standard rule for lenders)"

Website: usbank.com/home-loans/mortgage/first-time-home-buyers/how-much-house-can-i-afford.html

PNC Bank

"Know How Much You Can Afford"

Depending on the amount you have saved for a down payment, your mortgage payment should typically be no more than 28% of your monthly income, and your total debt should be no more than 36%, although debt ratios have some flexibility, depending on mortgage type you choose."

Website: <https://www.pnc.com/en/personal-banking/borrowing/home-lending/understanding-home-lending-center/home-buyers-basics.html>

"Start by assessing your income. Then consider liabilities like student loans, credit card balances and auto loans. Ideally, the amount of your monthly debt payments, including your proposed mortgage payment, should be equal to or less than 36% of your gross monthly income."

Website: <https://www.pnc.com/en/personal-banking/borrowing/home-lending/understanding-home-lending-center/learn-mortgage.html>

TD Bank

"Monthly housing payment (PITI)

This is your total principal, interest, taxes and insurance (PITI) payment per month. This includes your principal, interest, real estate taxes, hazard insurance, association dues or fees and principal mortgage insurance (PMI). Maximum monthly payment (PITI) is calculated by taking the lower of these two calculations:

- 1. Monthly Income X 28% = monthly PITI*
- 2. Monthly Income X 36% - Other loan payments = monthly PITI*

Maximum principal and interest (PI)

This is your maximum monthly principal and interest payment. It is calculated by subtracting your monthly taxes and insurance from your monthly PITI payment. This calculator uses your maximum PI payment to determine the mortgage amount that you could qualify for."

Website: <https://tdbank.mortgagewebcenter.com/Resources/Resources/MortgageMax>

Branch Banking and Trust Company

"Gross annual income

Providing this enables us to estimate how much you will be able to borrow assuming a 28% debt-to-income ratio. Include the total of your gross annual wages and other income that can be used to qualify for this home equity loan or line of credit."

Website: <https://www.bbt.com/iwov-resources/calculators/BBLoanLine.html>

SunTrust Bank

"28. The maximum percentage of your gross monthly income that should go to housing expenses, including your mortgage, taxes and insurance."

Website: <https://www.suntrust.com/content/dam/suntrust/us/en/resource-center/documents/2018/avoid-these-common-budget-mistakes.pdf>

Your DTI ratio is all of your monthly debt payments divided by your gross monthly income (the amount earned before taxes and other deductions). It's typically an important part of the home buying process since some lenders require your debt (including your new potential mortgage payments) to make up less than 36% percent of your income.

Website: <https://www.suntrust.com/resource-center/homeownership/article/ways-to-manage-your-debt-and-still-buy-a-home>

B Dynamic Equilibrium Conditions

The appendix describes the derivation of the first-order conditions, which constitute the model together with the laws of motion. All variables, with the exception of inflation and interest rates and the Lagrange multipliers, are log-transformed prior to entering the equations into the solution code. The equations are linearized as a part of the solution procedure.

Patient Household

The patient household maximizes its utility function,

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t s_{I,t} \left[\chi_C \log(c_t - \eta_C c_{t-1}) + \omega_H s_{H,t} \chi_H \log(h_t - \eta_H h_{t-1}) - \frac{s_{L,t}}{1 + \varphi} n_t^{1+\varphi} \right] \right\}, \quad (\text{B.1})$$

subject to a budget constraint,

$$\begin{aligned} c_t + q_t [h_t - (1 - \delta_H) h_{t-1}] + k_t + \frac{\iota}{2} \left(\frac{k_t}{k_{t-1}} - 1 \right)^2 k_{t-1} + p_{X,t} [x_t - x_{t-1}] \\ = w_t n_t + div_t + b_t - \frac{1 - (1 - \rho)(1 - \sigma) + r_{t-1}}{1 + \pi_t} l_{t-1} + (r_{K,t} + 1 - \delta_K) k_{t-1} + r_{X,t} x_{t-1}, \end{aligned} \quad (\text{B.2})$$

and to the laws of motion for the net level of outstanding mortgage loans and the average nominal net interest rate on outstanding mortgage loans,

$$l_t = (1 - \rho)(1 - \sigma) \frac{l_{t-1}}{1 + \pi_t} + b_t, \quad (\text{B.3})$$

$$r_t = (1 - \rho)(1 - \sigma) \frac{l_{t-1}}{l_t} r_{t-1} + \left[1 - (1 - \rho)(1 - \sigma) \frac{l_{t-1}}{l_t} \right] i_t, \quad (\text{B.4})$$

where $\chi_C \equiv \frac{1 - \eta_C}{1 - \beta \eta_C}$ and $\chi_H \equiv \frac{1 - \eta_H}{1 - \beta \eta_H}$.

The budget constraint in (B.2) can be rewritten by substituting (B.3) into it:

$$\begin{aligned} c_t + q_t [h_t - (1 - \delta_H) h_{t-1}] + k_t + \frac{\iota}{2} \left(\frac{k_t}{k_{t-1}} - 1 \right)^2 k_{t-1} + p_{X,t} [x_t - x_{t-1}] \\ = w_t n_t + div_t + l_t - \frac{1 + r_{t-1}}{1 + \pi_t} l_{t-1} + (r_{K,t} + 1 - \delta_K) k_{t-1} + r_{X,t} x_{t-1}. \end{aligned} \quad (\text{B.5})$$

The marginal determinant of the patient household's consumption-saving decision is the current interest rate (i_t) and not the average interest rate (r_t).¹ On account of this, the

¹This implies that it is the current interest rate which enters into the household's first-order condition with respect to net mortgage debt, rather than the average interest rate. Results assuming that the average interest rate is the marginal rate are nearly identical.

budget constraint of the marginal patient lender j is

$$\begin{aligned} c_t + q_t[h_t - (1 - \delta_H)h_{t-1}] + k_t + \frac{\iota}{2} \left(\frac{k_t}{k_{t-1}} - 1 \right)^2 k_{t-1} + p_{X,t}[x_t - x_{t-1}] \\ = w_t n_t + div_t + l_t(j) - \frac{1 + i_{t-1}}{1 + \pi_t} l_{t-1}(j) + (r_{K,t} + 1 - \delta_K)k_{t-1} + r_{X,t}x_{t-1}. \end{aligned} \quad (\text{B.6})$$

The marginal utility of goods consumption ($u_{c,t}$) and housing services ($u_{h,t}$) is

$$u_{c,t} \equiv \frac{1 - \eta_C}{1 - \beta\eta_C} \left(\frac{s_{I,t}}{c_t - \eta_C c_{t-1}} - \beta\eta_C \mathbb{E}_t \left\{ \frac{s_{I,t+1}}{c_{t+1} - \eta_C c_t} \right\} \right), \quad (\text{B.7})$$

$$u_{h,t} \equiv \omega_H \frac{1 - \eta_H}{1 - \beta\eta_H} \left(\frac{s_{I,t} s_{H,t}}{h_t - \eta_H h_{t-1}} - \beta\eta_H \mathbb{E}_t \left\{ \frac{s_{I,t+1} s_{H,t+1}}{h_{t+1} - \eta_H h_t} \right\} \right). \quad (\text{B.8})$$

The patient household maximizes its utility function with respect to housing, labor supply, net mortgage debt, nonresidential capital, and land. The resulting first-order conditions are

$$u_{c,t} q_t = u_{h,t} + \beta(1 - \delta_H) \mathbb{E}_t \{ u_{c,t+1} q_{t+1} \}, \quad (\text{B.9})$$

$$u_{c,t} w_t = s_{I,t} s_{L,t} n_t^\varphi, \quad (\text{B.10})$$

$$u_{c,t} = \beta \mathbb{E}_t \left\{ u_{c,t+1} \frac{1 + i_t}{1 + \pi_{t+1}} \right\}, \quad (\text{B.11})$$

$$u_{c,t} \left[1 + \iota \left(\frac{k_t}{k_{t-1}} - 1 \right) \right] = \beta \mathbb{E}_t \left\{ u_{c,t+1} \left[r_{K,t+1} + 1 - \delta_K + \frac{\iota}{2} \left(\frac{k_{t+1}^2}{k_t^2} - 1 \right) \right] \right\}, \quad (\text{B.12})$$

$$u_{c,t} p_{X,t} = \beta \mathbb{E}_t \{ u_{c,t+1} (r_{X,t+1} + p_{X,t+1}) \}. \quad (\text{B.13})$$

Impatient Household

The impatient household maximizes its utility function,

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t s_{I,t} \left[\chi'_C \log(c'_t - \eta_C c'_{t-1}) + \omega_H s_{H,t} \chi'_H \log(h'_t - \eta_H h'_{t-1}) - \frac{s_{L,t}}{1+\varphi} n_t'^{1+\varphi} \right] \right\}, \quad (\text{B.14})$$

subject to a budget constraint,

$$c'_t + q_t[h'_t - (1 - \delta_H)h'_{t-1}] = w'_t n'_t + b'_t - \frac{1 - (1 - \rho)(1 - \sigma) + r_{t-1}}{1 + \pi_t} l'_{t-1}, \quad (\text{B.15})$$

to the laws of motion for the net level of outstanding mortgage loans and the average nominal net interest rate on outstanding mortgage loans,

$$l'_t = (1 - \rho)(1 - \sigma) \frac{l'_{t-1}}{1 + \pi_t} + b'_t, \quad (\text{B.16})$$

$$r_t = (1 - \rho)(1 - \sigma) \frac{l'_{t-1}}{l'_t} r_{t-1} + \left[1 - (1 - \rho)(1 - \sigma) \frac{l'_{t-1}}{l'_t} \right] i_t, \quad (\text{B.17})$$

and to the two occasionally binding credit constraints,

$$b'_t \leq \rho \left(\kappa_{LTV} \xi_{LTV} \mathbb{E}_t \{ (1 + \pi_{t+1}) q_{t+1} h'_t \} + (1 - \kappa_{LTV}) \xi_{DTI,t} \mathbb{E}_t \left\{ \frac{(1 + \pi_{t+1}) w'_{t+1} n'_t}{\sigma + r_t} \right\} \right), \quad (\text{B.18})$$

$$b'_t \leq \rho \left((1 - \kappa_{DTI}) \xi_{LTV} \mathbb{E}_t \{ (1 + \pi_{t+1}) q_{t+1} h'_t \} + \kappa_{DTI} \xi_{DTI,t} \mathbb{E}_t \left\{ \frac{(1 + \pi_{t+1}) w'_{t+1} n'_t}{\sigma + r_t} \right\} \right), \quad (\text{B.19})$$

where $\chi'_C \equiv \frac{1-\eta_C}{1-\beta'\eta_C}$ and $\chi'_H \equiv \frac{1-\eta_H}{1-\beta'\eta_H}$.

The budget constraint in (B.15) can be rewritten by substituting (B.16) into it:

$$c'_t + q_t[h'_t - (1 - \delta_H)h'_{t-1}] = w'_t n'_t + l'_t - \frac{1 + r_{t-1}}{1 + \pi_t} l'_{t-1}. \quad (\text{B.20})$$

As with the patient household, the marginal determinant of the impatient household's consumption-saving decision is the current interest rate (i_t) and not the average interest rate (r_t). Therefore, the budget constraint of the marginal impatient borrower j is

$$c'_t + q_t[h'_t - (1 - \delta_H)h'_{t-1}] = w'_t n'_t + l'_t(j) - \frac{1 + i_{t-1}}{1 + \pi_t} l'_{t-1}(j). \quad (\text{B.21})$$

I solve the utility maximization problem through the method of Lagrange multipliers. The Lagrange function before substitution of (B.21) is

$$\begin{aligned} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t s_{I,t} \left[\chi'_C \log(c'_t - \eta_C c'_{t-1}) + \omega_H s_{H,t} \chi'_H \log(h'_t - \eta_H h'_{t-1}) - \frac{s_{L,t}}{1+\varphi} n_t^{1+\varphi} \right. \right. \\ \left. \left. + \lambda_{LTV,t} \left((1-\rho)(1-\sigma) \frac{l'_{t-1}}{1+\pi_t} + \rho \left(\kappa_{LTV} \xi_{LTV,t} \mathbb{E}_t \{ (1+\pi_{t+1}) q_{t+1} h'_t \} \right. \right. \right. \\ \left. \left. \left. + (1-\kappa_{LTV}) \xi_{DTI,t} \mathbb{E}_t \left\{ \frac{(1+\pi_{t+1}) w'_{t+1} n'_t}{\sigma+r_t} \right\} \right) - l'_t \right) \right. \\ \left. \left. + \lambda_{DTI,t} \left((1-\rho)(1-\sigma) \frac{l'_{t-1}}{1+\pi_t} + \rho \left((1-\kappa_{DTI}) \xi_{LTV,t} \mathbb{E}_t \{ (1+\pi_{t+1}) q_{t+1} h'_t \} \right. \right. \right. \right. \\ \left. \left. \left. + \kappa_{DTI} \xi_{DTI,t} \mathbb{E}_t \left\{ \frac{(1+\pi_{t+1}) w'_{t+1} n'_t}{\sigma+r_t} \right\} \right) - l'_t \right) \right] \right\}, \end{aligned}$$

where $\lambda_{LTV,t}$ denotes the multiplier on (B.18), and $\lambda_{DTI,t}$ denotes the multiplier on (B.19).

The marginal utility of goods consumption ($u'_{c,t}$) and housing services ($u'_{h,t}$) is

$$u'_{c,t} \equiv \frac{1-\eta_C}{1-\beta'\eta_C} \left(\frac{s_{I,t}}{c'_t - \eta_C c'_{t-1}} - \beta'\eta_C \mathbb{E}_t \left\{ \frac{s_{I,t+1}}{c'_{t+1} - \eta_C c'_t} \right\} \right), \quad (\text{B.22})$$

$$u'_{h,t} \equiv \omega_H \frac{1-\eta_H}{1-\beta'\eta_H} \left(\frac{s_{I,t} s_{H,t}}{h'_t - \eta_H h'_{t-1}} - \beta'\eta_H \mathbb{E}_t \left\{ \frac{s_{I,t+1} s_{H,t+1}}{h'_{t+1} - \eta_H h'_t} \right\} \right). \quad (\text{B.23})$$

The impatient household maximizes its utility function with respect to housing, labor supply, and net mortgage debt. The resulting first-order conditions are

$$\begin{aligned} u'_{c,t} q_t = u'_{h,t} + \beta'(1-\delta_H) \mathbb{E}_t \{ u'_{c,t+1} q_{t+1} \} \\ + s_{I,t} \rho [\kappa_{LTV} \lambda_{LTV,t} + (1-\kappa_{DTI}) \lambda_{DTI,t}] \xi_{LTV,t} \mathbb{E}_t \{ (1+\pi_{t+1}) q_{t+1} \}, \end{aligned} \quad (\text{B.24})$$

$$u'_{c,t} w'_t + s_{I,t} \rho [(1-\kappa_{LTV}) \lambda_{LTV,t} + \kappa_{DTI} \lambda_{DTI,t}] \xi_{DTI,t} \mathbb{E}_t \left\{ \frac{(1+\pi_{t+1}) w'_{t+1}}{\sigma+r_t} \right\} = s_{I,t} s_{L,t} n_t^{1+\varphi}, \quad (\text{B.25})$$

$$\begin{aligned} u'_{c,t} + \beta'(1-\rho)(1-\sigma) \mathbb{E}_t \left\{ s_{I,t+1} \frac{\lambda_{LTV,t+1} + \lambda_{DTI,t+1}}{1+\pi_{t+1}} \right\} \\ = \beta' \mathbb{E}_t \left\{ u'_{c,t+1} \frac{1+i_t}{1+\pi_{t+1}} \right\} + s_{I,t} (\lambda_{LTV,t} + \lambda_{DTI,t}). \end{aligned} \quad (\text{B.26})$$

Intermediate Firm

The intermediate firm maximizes its profits,

$$\frac{Y_t}{M_{P,t}} + q_t I_{H,t} - w_t n_t - w'_t n'_t - r_{K,t} k_{t-1} - g_t - r_{X,t} x_{t-1}, \quad (\text{B.27})$$

subject to the goods production and housing transformation technologies,

$$Y_t = k_{t-1}^\mu (s_{Y,t} n_t^\alpha n'_t)^{1-\alpha} \quad (\text{B.28})$$

$$I_{H,t} = g'_t x_{t-1}^{1-\nu}. \quad (\text{B.29})$$

The firm's profit maximization occurs with respect to nonresidential capital, patient and impatient labor, intermediate housing inputs, and land. The resulting first-order conditions are

$$\mu \frac{Y_t}{M_{P,t} k_{t-1}} = r_{K,t}, \quad (\text{B.30})$$

$$(1 - \mu) \alpha \frac{Y_t}{M_{P,t} n_t} = w_t, \quad (\text{B.31})$$

$$(1 - \mu)(1 - \alpha) \frac{Y_t}{M_{P,t} n'_t} = w'_t, \quad (\text{B.32})$$

$$\nu q_t I_{H,t} = g_t, \quad (\text{B.33})$$

$$(1 - \nu) \frac{q_t I_{H,t}}{x_{t-1}} = r_{X,t}. \quad (\text{B.34})$$

Household Constraints and Market-Clearing Conditions

The goods, housing, loan, and land market-clearing conditions are

$$c_t + c'_t + k_t - (1 - \delta_K) k_{t-1} + \frac{\iota}{2} \left(\frac{k_t}{k_{t-1}} - 1 \right)^2 k_{t-1} + g_t = Y_t, \quad (\text{B.35})$$

$$h_t + h'_t - (1 - \delta_H)(h_{t-1} + h'_{t-1}) = I_{H,t}, \quad (\text{B.36})$$

$$b_t = -b'_t, \quad (\text{B.37})$$

$$x_t = \mathcal{X}. \quad (\text{B.38})$$

C Steady-State Computation

The appendix describes the derivation of the solution to the model's nonstochastic steady state. An exact numerical solution can be reached by combining the resulting relations as it is done in the solution code for the steady state.

Simple Expressions

The marginal utility of goods consumption is from (B.7) and (B.22) given by

$$\begin{aligned} u_c &= \frac{1 - \eta_C}{1 - \beta\eta_C} \left(\frac{1}{c - \eta_C c} - \beta \frac{\eta_C}{c - \eta_C c} \right) & u'_c &= \frac{1 - \eta_C}{1 - \beta'\eta_C} \left(\frac{1}{c' - \eta_C c'} - \beta' \frac{\eta_C}{c' - \eta_C c'} \right) \\ &= \frac{1 - \eta_C}{1 - \beta\eta_C} \frac{1 - \beta\eta_C}{1 - \eta_C} \frac{1}{c} & &= \frac{1 - \eta_C}{1 - \beta'\eta_C} \frac{1 - \beta'\eta_C}{1 - \eta_C} \frac{1}{c'} \\ &= \frac{1}{c}, & &= \frac{1}{c'}. \end{aligned}$$

The marginal utility of housing services is from (B.8) and (B.23) given by

$$\begin{aligned} u_h &= \omega_H \frac{1 - \eta_H}{1 - \beta\eta_H} \left(\frac{1}{h - \eta_H h} - \beta \frac{\eta_H}{h - \eta_H h} \right) & u'_h &= \omega_H \frac{1 - \eta_H}{1 - \beta'\eta_H} \left(\frac{1}{h' - \eta_H h'} - \beta' \frac{\eta_H}{h' - \eta_H h'} \right) \\ &= \omega_H \frac{1 - \eta_H}{1 - \beta\eta_H} \frac{1 - \beta\eta_H}{1 - \eta_H} \frac{1}{h} & &= \omega_H \frac{1 - \eta_H}{1 - \beta'\eta_H} \frac{1 - \beta'\eta_H}{1 - \eta_H} \frac{1}{h'} \\ &= \frac{\omega_H}{h}, & &= \frac{\omega_H}{h'}. \end{aligned}$$

Net price inflation is

$$\pi = 0. \tag{C.1}$$

The current nominal net interest rate is from the first-order condition of the patient household with respect to net mortgage debt in (B.11) given by

$$\begin{aligned} u_c &= \beta u_c \frac{1 + i}{1 + \pi} \\ i &= \frac{1}{\beta} - 1. \end{aligned} \tag{C.2}$$

The real net rental rate of nonresidential capital is from the first-order condition of the patient household with respect to nonresidential capital in (B.12) given by

$$\begin{aligned} u_c \left[1 + \iota \left(\frac{k}{k} - 1 \right) \right] &= \beta u_c \left[r_K + 1 - \delta_K - \frac{\iota}{2} \left(\frac{k^2}{k^2} - 1 \right) \right] \\ 1 &= \beta [r_K + 1 - \delta_K] \\ r_K &= i + \delta_K. \end{aligned} \tag{C.3}$$

The average nominal net interest rate on outstanding loans is from its law of motion in

(B.17) given by

$$\begin{aligned} r &= (1 - \rho)(1 - \sigma) \frac{l'}{l} r + \left[1 - (1 - \rho)(1 - \sigma) \frac{l'}{l} \right] i \\ &= i. \end{aligned} \tag{C.4}$$

Analytical Steady-State Ratios

The first-order condition of the intermediate firm with respect to nonresidential capital is from (B.30) given by

$$\mu \frac{Y}{M_P k} = r_K. \tag{C.5}$$

Combining (C.3) and (C.5), one gets an expression for the $\frac{k}{Y}$ ratio:

$$\begin{aligned} \mu \frac{Y}{M_P k} &= \frac{1}{\beta} - (1 - \delta_K) \\ \frac{Y}{k} &= \frac{1 - \beta(1 - \delta_K)}{\beta \mu} M_P \\ \frac{k}{Y} &= \frac{\beta \mu}{1 - \beta(1 - \delta_K)} \frac{1}{M_P} \equiv \aleph_1. \end{aligned} \tag{C.6}$$

The first-order condition of the patient household with respect to housing is from (B.9) given by

$$\begin{aligned} u_c q &= u_h + \beta(1 - \delta_H) u_c q \\ \frac{1}{c} q &= \frac{\omega_H}{h} + \beta(1 - \delta_H) \frac{1}{c} q \\ \frac{qh}{c} &= \frac{\omega_H}{1 - \beta(1 - \delta_H)} \equiv \aleph_2. \end{aligned} \tag{C.7}$$

The first-order condition of the impatient household with respect to net mortgage debt is from (B.26) given by

$$\begin{aligned} u'_c + \beta'(1 - \rho)(1 - \sigma) \frac{\lambda_{LTV} + \lambda_{DTI}}{1 + \pi} &= \beta' u'_c \frac{1 + i}{1 + \pi} + \lambda_{LTV} + \lambda_{DTI} \\ \frac{1}{c'} + \beta'(1 - \rho)(1 - \sigma)(\lambda_{LTV} + \lambda_{DTI}) &= \frac{\beta'}{\beta} \frac{1}{c'} + \lambda_{LTV} + \lambda_{DTI} \\ (\lambda_{LTV} + \lambda_{DTI})[\beta'(1 - \rho)(1 - \sigma) - 1] &= \frac{1}{c'} \left[\frac{\beta'}{\beta} - 1 \right] \\ \lambda_{LTV} + \lambda_{DTI} &= \frac{1 - \frac{\beta'}{\beta}}{c' [1 - \beta'(1 - \rho)(1 - \sigma)]}. \end{aligned}$$

Both credit constraints are, by assumption, binding in the steady state, implying that

$$\lambda_{LTV} = \nu \lambda_{DTI} > 0.$$

Using this condition, one gets the following expressions for the Lagrange multipliers:

$$\lambda_{LTV} = \frac{1 - \frac{\beta'}{\beta}}{(1 + \frac{1}{v})c'[1 - \beta'(1 - \rho)(1 - \sigma)]} > 0, \quad (C.8)$$

$$\lambda_{DTI} = \frac{1 - \frac{\beta'}{\beta}}{(1 + v)c'[1 - \beta'(1 - \rho)(1 - \sigma)]} > 0. \quad (C.9)$$

Numerical Solution: Household Variables

The first-order condition of the impatient household with respect to housing is from (B.24) given by

$$\begin{aligned} u'_c q &= u'_h + \beta'(1 - \delta_H)u'_c q + \rho[\kappa_{LTV}\lambda_{LTV} + (1 - \kappa_{DTI})\lambda_{DTI}]\xi_{LTV}(1 + \pi)q \\ \frac{1}{c'}q &= \frac{\omega_H}{h'} + \beta'(1 - \delta_H)\frac{1}{c'}q + \rho\left[\kappa_{LTV}\frac{1}{1 + \frac{1}{v}} + (1 - \kappa_{DTI})\frac{1}{1 + v}\right]\frac{1 - \frac{\beta'}{\beta}}{c'[1 - \beta'(1 - \rho)(1 - \sigma)]}\xi_{LTV}q \\ \frac{1}{c'}qh' &= \omega_H + \beta'(1 - \delta_H)\frac{1}{c'}qh' + \rho\left[\kappa_{LTV}\frac{1}{1 + \frac{1}{v}} + (1 - \kappa_{DTI})\frac{1}{1 + v}\right]\frac{1 - \frac{\beta'}{\beta}}{c'[1 - \beta'(1 - \rho)(1 - \sigma)]}\xi_{LTV}qh' \\ \frac{qh'}{c'} &= \frac{\omega_H}{1 - \beta'(1 - \delta_H) - \rho\left[\kappa_{LTV}\frac{1}{1 + \frac{1}{v}} + (1 - \kappa_{DTI})\frac{1}{1 + v}\right]\frac{1 - \frac{\beta'}{\beta}}{1 - \beta'(1 - \rho)(1 - \sigma)}\xi_{LTV}} \equiv \aleph_3. \end{aligned} \quad (C.10)$$

The net level of outstanding mortgage loans is from the law of motion for this variable in (B.16) and the LTV constraint in (B.18) given by

$$\begin{aligned} l' &= (1 - \rho)(1 - \sigma)\frac{l'}{1 + \pi} + \rho\left(\kappa_{LTV}\xi_{LTV}(1 + \pi)qh' + (1 - \kappa_{LTV})\xi_{DTI}\frac{(1 + \pi)w'n'}{\sigma + r}\right) \\ l'[1 - (1 - \rho)(1 - \sigma)] &= \rho\left(\kappa_{LTV}\xi_{LTV}qh' + (1 - \kappa_{LTV})\xi_{DTI}\frac{w'n'}{\sigma + r}\right) \\ l' &= \frac{\rho}{\rho + (1 - \rho)\sigma}\left(\kappa_{LTV}\xi_{LTV}qh' + (1 - \kappa_{LTV})\xi_{DTI}\frac{w'n'}{\sigma + r}\right). \end{aligned} \quad (C.11)$$

The net level of outstanding mortgage loans is from the law of motion for this variable in (B.16) and the DTI constraint in (B.19) given by

$$\begin{aligned} l' &= (1 - \rho)(1 - \sigma)\frac{l'}{1 + \pi} + \rho\left((1 - \kappa_{DTI})\xi_{LTV}(1 + \pi)qh' + \kappa_{DTI}\xi_{DTI}\frac{(1 + \pi)w'n'}{\sigma + r}\right) \\ l'[1 - (1 - \rho)(1 - \sigma)] &= \rho\left((1 - \kappa_{DTI})\xi_{LTV}qh' + \kappa_{DTI}\xi_{DTI}\frac{w'n'}{\sigma + r}\right) \\ l' &= \frac{\rho}{\rho + (1 - \rho)\sigma}\left((1 - \kappa_{DTI})\xi_{LTV}qh' + \kappa_{DTI}\xi_{DTI}\frac{w'n'}{\sigma + r}\right). \end{aligned} \quad (C.12)$$

The model automatically chooses the LTV limit,

$$\xi_{LTV} = \frac{\xi_{DTI}\frac{w'n'}{\sigma + r}}{qh'}, \quad (C.13)$$

that ensures that both constraints are binding in the steady state, i.e., that

$$-l = l' = \frac{\rho}{\rho + (1 - \rho)\sigma} \xi_{LTV} q h' = \frac{\rho}{\rho + (1 - \rho)\sigma} \xi_{DTI} \frac{w'n'}{\sigma + r}, \quad (\text{C.14})$$

with the first equality following from the loan market-clearing condition in (B.37).

The dividends that the retail firms pay to the patient household are

$$div = \left(1 - \frac{1}{M_P}\right) Y. \quad (\text{C.15})$$

The $\frac{c}{Y}$ ratio is from the budget constraint of the patient household in (B.5) given by

$$\begin{aligned} c + q[h - (1 - \delta_H)h] + k + \frac{t}{2} \left(\frac{k}{k} - 1\right)^2 k + p_X[x - x] \\ &= wn + div + l - \frac{1+r}{1+\pi} l + (r_K + 1 - \delta_K)k + r_X x \\ c + \delta_H q h - r_X x &= wn + div - rl + (r_K - \delta_K)k \\ c + \delta_H q h - (1 - \nu)qI_H \cdot 1 &= wn + div + r \frac{\rho}{\rho + (1 - \rho)\sigma} \xi_{DTI} \frac{w'n'}{\sigma + r} + rk \\ c + \delta_H q h - (1 - \nu)q\delta_H(h + h') \\ &= (1 - \mu)\alpha \frac{Y}{M_P n} n + \left(1 - \frac{1}{M_P}\right) Y + r \frac{\rho}{\rho + (1 - \rho)\sigma} \xi_{DTI} \frac{1}{\sigma + r} (1 - \mu)(1 - \alpha) \frac{Y}{M_P n'} n' + r \aleph_1 Y \\ [1 + \nu\delta_H \aleph_2]c - (1 - \nu)\delta_H \aleph_3 c' &= \left[(1 - \mu)[\alpha + \aleph_4(1 - \alpha)] \frac{1}{M_P} + 1 - \frac{1}{M_P} + r \aleph_1 \right] Y \\ \beth_1 c - \beth_2 c' &= \beth_3 Y, \end{aligned} \quad (\text{C.16})$$

where $\aleph_4 \equiv r \frac{\rho}{\rho + (1 - \rho)\sigma} \xi_{DTI} \frac{1}{\sigma + r}$, $\beth_1 \equiv 1 + \nu\delta_H \aleph_2$, $\beth_2 \equiv (1 - \nu)\delta_H \aleph_3$, $\beth_3 \equiv (1 - \mu)[\alpha + \aleph_4(1 - \alpha)] \frac{1}{M_P} + 1 - \frac{1}{M_P} + r \aleph_1$, and $x = \mathcal{X} = 1$ by normalization.

The $\frac{c'}{Y}$ ratio is from the budget constraint of the impatient household in (B.20) given by

$$\begin{aligned} c' + q[h' - (1 - \delta_H)h'] &= w'n' + l' - \frac{1+r}{1+\pi} l' \\ c' + \delta_H q h' &= w'n' - rl' \\ c' + \delta_H q h' &= w'n' - r \frac{\rho}{\rho + (1 - \rho)\sigma} \xi_{DTI} \frac{w'n'}{\sigma + r} \\ c' + \delta_H \aleph_3 c' &= w'n' \left(1 - r \frac{\rho}{\rho + (1 - \rho)\sigma} \xi_{DTI} \frac{1}{\sigma + r}\right) \\ [1 + \delta_H \aleph_3]c' &= (1 - \mu)(1 - \alpha) \frac{Y}{M_P n'} n' (1 - \aleph_4) \\ \frac{c'}{Y} &= \frac{\beth_5}{\beth_4}, \end{aligned} \quad (\text{C.17})$$

where $\beth_4 \equiv 1 + \delta_H \aleph_3$ and $\beth_5 \equiv (1 - \mu)(1 - \alpha) \frac{1}{M_P} (1 - \aleph_4)$.

Numerical Solution: Labor Market Variables

The real wages are from (B.31) and (B.32) given by

$$w = (1 - \mu)\alpha \frac{Y}{M_P n}, \quad (\text{C.18})$$

$$w' = (1 - \mu)(1 - \alpha) \frac{Y}{M_P n'}. \quad (\text{C.19})$$

The first-order condition of the patient household with respect to labor supply is from (B.10) given by

$$u_c w = n^\varphi.$$

Combining this expression with (C.18) gives the following expression for the employment of the patient household:

$$\begin{aligned} \frac{1}{u_c} n^\varphi &= (1 - \mu)\alpha \frac{Y}{M_P n} \\ c n^\varphi &= (1 - \mu)\alpha \frac{1}{M_P n^{\frac{1}{Y}}} \\ n &= \left[(1 - \mu)\alpha \frac{1}{M_P \frac{c}{Y}} \right]^{\frac{1}{1+\varphi}}. \end{aligned} \quad (\text{C.20})$$

The first-order condition of the impatient household with respect to labor supply is from (B.25) given by

$$\begin{aligned} u'_c w' + \rho [(1 - \kappa_{LTV})\lambda_{LTV} + \kappa_{DTI}\lambda_{DTI}] \xi_{DTI} \frac{(1 + \pi)w'}{\sigma + r} &= n'^\varphi \\ u'_c w' + \rho \left[(1 - \kappa_{LTV}) \frac{1}{1 + \frac{1}{v}} + \kappa_{DTI} \frac{1}{1 + v} \right] \frac{1 - \frac{\beta'}{\beta}}{c'[1 - \beta'(1 - \rho)(1 - \sigma)]} \xi_{DTI} \frac{w'}{\sigma + r} &= n'^\varphi \\ w' &= \frac{1}{u'_c + \rho \left[(1 - \kappa_{LTV}) \frac{1}{1 + \frac{1}{v}} + \kappa_{DTI} \frac{1}{1 + v} \right] \frac{1 - \frac{\beta'}{\beta}}{c'[1 - \beta'(1 - \rho)(1 - \sigma)]} \xi_{DTI} \frac{1}{\sigma + r}} n'^\varphi. \end{aligned}$$

Combining this expression with (C.19) gives the following expression for the employment of the impatient household:

$$\begin{aligned} \frac{1}{\frac{1}{c'} + \rho \left[(1 - \kappa_{LTV}) \frac{1}{1 + \frac{1}{v}} + \kappa_{DTI} \frac{1}{1 + v} \right] \frac{1 - \frac{\beta'}{\beta}}{c'[1 - \beta'(1 - \rho)(1 - \sigma)]} \xi_{DTI} \frac{1}{\sigma + r}} n'^\varphi &= (1 - \mu)(1 - \alpha) \frac{Y}{M_P n'} \\ n' &= \left[(1 - \mu)(1 - \alpha) \frac{1}{M_P \frac{c'}{Y}} \cdot \left(1 + \rho \left[(1 - \kappa_{LTV}) \frac{1}{1 + \frac{1}{v}} + \kappa_{DTI} \frac{1}{1 + v} \right] \frac{1 - \frac{\beta'}{\beta}}{1 - \beta'(1 - \rho)(1 - \sigma)} \xi_{DTI} \frac{1}{\sigma + r} \right) \right]^{\frac{1}{1+\varphi}}. \end{aligned} \quad (\text{C.21})$$

Numerical Solution: Production and Housing Market Variables

Goods production is from (B.28) given by

$$\begin{aligned}
 Y &= k^\mu (n^\alpha n'^{1-\alpha})^{1-\mu} \\
 Y^{\frac{1}{1-\mu}} &= k^{\frac{\mu}{1-\mu}} n^\alpha n'^{1-\alpha} \\
 Y &= \left(\frac{k}{Y}\right)^{\frac{\mu}{1-\mu}} n^\alpha n'^{1-\alpha} \\
 Y &= \aleph_1^{\frac{\mu}{1-\mu}} n^\alpha n'^{1-\alpha}.
 \end{aligned} \tag{C.22}$$

The $\frac{qI_H}{Y}$ ratio is determined by the housing market-clearing condition in (B.36), as

$$\begin{aligned}
 I_H &= h + h' - (1 - \delta_H)(h + h') \\
 qI_H &= \delta_H q(h + h') \\
 qI_H &= \delta_H (\aleph_2 c + \aleph_3 c') \\
 \frac{qI_H}{Y} &= \delta_H \left(\aleph_2 \frac{c}{Y} + \aleph_3 \frac{c'}{Y} \right).
 \end{aligned} \tag{C.23}$$

Residential gross investment is from (B.29) and (B.33) given by

$$\begin{aligned}
 I_H &= g^\nu x^{1-\nu} \\
 &= (\nu q I_H \cdot 1)^\nu \\
 &= \left(\nu Y \frac{q I_H}{Y} \right)^\nu,
 \end{aligned} \tag{C.24}$$

where $x = \mathcal{X} = 1$ by normalization.

The real house price is determined by the following identity:

$$q = \frac{qI_H}{Y} \frac{Y}{I_H}. \tag{C.25}$$

The stocks of housing are determined by the following identities:

$$h = \frac{qh}{c} \frac{c}{q} = \aleph_2 \frac{c}{q} = \aleph_2 \frac{\frac{c}{Y} Y}{q}, \tag{C.26}$$

$$h' = \frac{qh' c'}{c' q} = \aleph_3 \frac{c'}{q} = \aleph_3 \frac{\frac{c'}{Y} Y}{q}. \tag{C.27}$$

Analytical Solutions

Goods consumption is determined by the following identities:

$$c = \frac{c}{Y}Y, \quad (\text{C.28})$$

$$c' = \frac{c'}{Y}Y. \quad (\text{C.29})$$

Nonresidential capital is determined by the following identity:

$$k = \frac{k}{Y}Y = \aleph_1 Y. \quad (\text{C.30})$$

Intermediate housing inputs are from (B.33) given by

$$g = \nu q I_H. \quad (\text{C.31})$$

D Derivation of the DTI Requirement

The appendix derives the DTI requirement as an incentive compatibility constraint imposed by the patient household on the impatient household, and shows that it is a generalization of the natural borrowing limit in [Aiyagari \(1994\)](#). The derivation is separate from the LTV requirement in the sense that the patient household does not internalize the LTV requirement when imposing the DTI requirement.

Suppose first that the impatient household faces the choice of whether or not to default in period $t + 1$ on the borrowing issued to it in period t . Suppose next that if the impatient household defaults, the patient household obtains the right to repayment through a perpetual income stream commencing at period $t + 1$. The payments in the income stream are based on the amount $\mathbb{E}_t\{(1 + \pi_{t+1})w'_{t+1}n'_t\}$, and decrease by the amortization rate, reflecting a gradual repayment of the loan. Hence, from a period t perspective and assuming that the patient household discounts the future at the rate r_t , the net present value of the perpetual income stream is

$$\begin{aligned} S_t &= \mathbb{E}_t \left\{ \frac{(1 + \pi_{t+1})w'_{t+1}n'_t}{1 + r_t} + (1 - \sigma) \frac{(1 + \pi_{t+1})w'_{t+1}n'_t}{(1 + r_t)^2} + (1 - \sigma)^2 \frac{(1 + \pi_{t+1})w'_{t+1}n'_t}{(1 + r_t)^3} + \dots \right\} \\ &= \mathbb{E}_t \left\{ \frac{(1 + \pi_{t+1})w'_{t+1}n'_t}{1 + r_t} \left[1 + \frac{1 - \sigma}{1 + r_t} + \left(\frac{1 - \sigma}{1 + r_t} \right)^2 + \dots \right] \right\}. \end{aligned}$$

Since the income stream is a converging infinite geometric series ($\frac{1 - \sigma}{1 + r_t} < 1$ applies), its net present value can be expressed as

$$\begin{aligned} S_t &= \mathbb{E}_t \left\{ \frac{(1 + \pi_{t+1})w'_{t+1}n'_t}{1 + r_t} \frac{1}{1 - \frac{1 - \sigma}{1 + r_t}} \right\} \\ &= \mathbb{E}_t \left\{ \frac{(1 + \pi_{t+1})w'_{t+1}n'_t}{\sigma + r_t} \right\}. \end{aligned}$$

Suppose finally that it is uncertain whether or not the patient household will receive the income stream to which it is entitled in the case of default. With probability $\xi_{DTI,t}$, the household will receive the full stream, and with complementary probability $1 - \xi_{DTI,t}$, the household will not receive anything. The DTI requirement now arises as an incentive compatibility constraint that the patient household imposes on the impatient household in period t . Incentive compatibility requires that the value of the loan about to be lent is not greater than the expected income stream in the event of default:

$$b'_t \leq \xi_{DTI,t} \mathbb{E}_t \left\{ \frac{(1 + \pi_{t+1})w'_{t+1}n'_t}{\sigma + r_t} \right\} + (1 - \xi_{DTI,t}) \cdot 0.$$

This requirement is a generalization of the natural borrowing limit in [Aiyagari \(1994\)](#). In his seminal paper, he assumed that households may borrow up to the discounted sum of all their future minimum labor incomes, giving him the following constraint: $b'_t \leq \frac{wn_{min}}{r}$. Thus, in the phrasing of the present paper, [Aiyagari \(1994\)](#) assumed that stream payments are certain ($\xi_{DTI,t} = 1$) and not amortized ($\sigma = 0$).

E DSGE Estimation: Methodological Comments

Identification When Both Constraints Are Slack

The net level of outstanding mortgage loans is an observed variable in the estimation. It is mainly the DTI shock which ensures that this theoretical variable matches its empirical measure. When a credit constraint binds, the DTI shock has an immediate effect on the debt level via the binding constraint, leading to a direct econometric identification of the shock. If both constraints are slack, this direct channel is switched off, due to the constraints no longer contemporaneously predicting borrowing. Even in this case, however, the model is not stochastically singular, since the DTI shock also has an effect on the debt level when both constraints are slack. Only now, this effect works through the impatient household's first-order condition with respect to net mortgage debt in (B.26):

$$\begin{aligned} u'_{c,t} + \beta'(1-\rho)(1-\sigma)\mathbb{E}_t \left\{ s_{I,t+1} \frac{\lambda_{LTV,t+1} + \lambda_{DTI,t+1}}{1 + \pi_{t+1}} \right\} \\ = \beta' \mathbb{E}_t \left\{ u'_{c,t+1} \frac{1 + i_t}{1 + \pi_{t+1}} \right\} + s_{I,t}(\lambda_{LTV,t} + \lambda_{DTI,t}). \end{aligned}$$

Through recursive substitution v periods ahead, this condition can be restated as

$$\begin{aligned} u'_{c,t} = & \beta'^v \mathbb{E}_t \left\{ u'_{c,t+v} \prod_{j=0}^{v-1} \frac{1 + i_{t+j}}{1 + \pi_{t+j+1}} \right\} \\ & + \sum_{i=1}^{v-1} \beta'^i \mathbb{E}_t \left\{ s_{I,t+i} (\lambda_{LTV,t+i} + \lambda_{DTI,t+i}) \prod_{j=0}^{i-1} \frac{1 + i_{t+j}}{1 + \pi_{t+j+1}} \right\} \\ & - \sum_{i=1}^{v-1} \beta'^{i+1} (1-\rho)(1-\sigma) \mathbb{E}_t \left\{ s_{I,t+i+1} \frac{\lambda_{LTV,t+i+1} + \lambda_{DTI,t+i+1}}{1 + \pi_{t+i+1}} \prod_{j=0}^{i-1} \frac{1 + i_{t+j}}{1 + \pi_{t+j+1}} \right\} \\ & + s_{I,t}(\lambda_{LTV,t} + \lambda_{DTI,t}) - \beta'(1-\rho)(1-\sigma) \mathbb{E}_t \left\{ s_{I,t+1} \frac{\lambda_{LTV,t+1} + \lambda_{DTI,t+1}}{1 + \pi_{t+1}} \right\}, \end{aligned}$$

for $v \in \{v \in \mathbb{N} | v > 1\}$. According to this expression, the current levels of consumption and (via the budget constraint) borrowing are pinned down by the current and expected future Lagrange multipliers for $v \rightarrow \infty$. The current multipliers are zero ($\lambda_{LTV,t} = \lambda_{DTI,t} = 0$) when both constraints are slack. The expected future multipliers will, however, be positive at some forecast horizon, due to the model being stable with zero-mean stochastic innovations. As a result, if a constraint (or both) is slack, the constraint(s) will continue to impact the economy, via its (their) expected future limits and consequently the expected future Lagrange multiplier(s). A corollary of this is that, in the case where both constraints are slack, the current DTI shock (along with any other shock) may still – through its persistent effects on future credit limits – affect the contemporaneous economy.¹

¹For the case where one constraint binds, in experiments not reported here, I found the indirect effects of future Lagrange multipliers to be minuscule when compared to the direct effects coming through the binding constraint and contemporaneously positive Lagrange multiplier.

Prior Distribution

I now motivate the prior distribution. The distributions of the individual parameters are reported in Table 2 of the main text. The prior means of the wage share parameter ($\alpha = 0.66$), the habit formation parameters ($\eta_C = \eta_H = 0.70$), and the refinancing rate ($\rho = 0.25$) follow the prior means in [Guerrieri and Iacoviello \(2017\)](#). Moreover, the prior mean of the impatient time discount factor ($\beta' = 0.974$) ensures that the gap between this value and calibrated value of the patient time discount factor ($\beta = 0.985$) is identical to the corresponding gap ($0.984 - 0.995$) in [Guerrieri and Iacoviello \(2017\)](#). Next, the prior means of the price setting parameters ($\theta_P = 0.80$ and $\gamma_P = 0.50$) are broadly in line with the estimates in [Galí and Gertler \(1999\)](#) and [Sbordone \(2002\)](#).

Three parameters – all governing the relative dominance of the credit requirements – are specific to my model. I remain *a priori* agnostic about this relative dominance, by assigning the parameters with broad prior distributions. To the parameters measuring the distribution of LTV and DTI constrained borrowers, I assign truncated beta distributions centered at the median value in the interval over which the parameters are defined ($\kappa_{LTV} = \kappa_{DTI} = 0.75$). Furthermore, to the parameter controlling the relative steady-state tightness of the constraints, I assign a normal distribution centered around unity ($v = 1$).

The prior means of the remaining estimated parameters follow the prior means of the corresponding parameters in [Iacoviello and Neri \(2010\)](#).

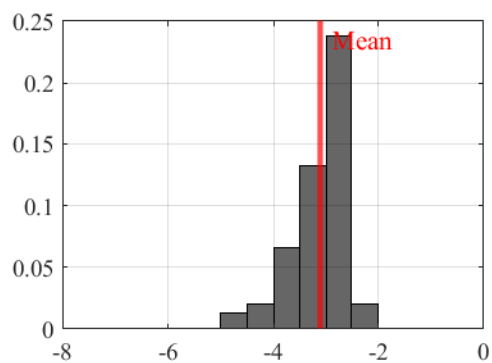
Accuracy Test

The model is solved by means of a piecewise first-order perturbation method. I verify the accuracy of this method by numerically computing the intertemporal errors in the Euler equations of the model, as proposed by [Judd \(1992\)](#). The errors arise both because of the linearization of the originally nonlinear regimes of the model, and because the solution method does not fully internalize the precautionary motives stemming from the possibility of future regime switches.² I compute the expectation terms in the Euler equations by standard monomial integration (see [Judd, Maliar, and Maliar \(2011\)](#) for a description of this method), following [Guerrieri and Iacoviello \(2017\)](#).

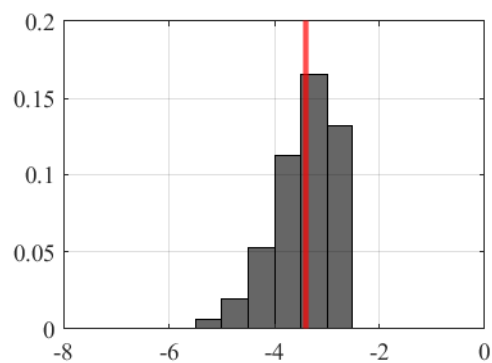
Figure E.1 reports histograms of the intertemporal errors for the first-order conditions of both households with respect to net mortgage debt and housing, stated in (B.9), (B.11), (B.24), and (B.26). The errors are expressed on an absolute log 10 scale. The mean values of the errors are -3.12 and -3.41 for the patient household and -2.98 and -2.87 for the impatient household. These values imply that, on average, the patient household loses about \$1 for every \$1,900 spent on goods consumption and housing services, while the impatient household loses \$1 for every \$900 spent.

²The method does partly internalize the possibility of future regimes switches, in that, if a constraint is slack, the households will expect it to bind again at some forecast horizon. However, once a constraint starts binding, the households will not expect it to unbind at any forecast horizon.

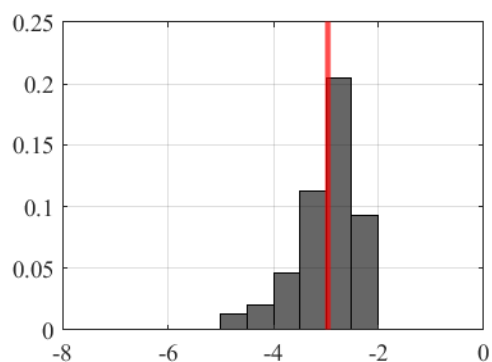
Figure E.1: INTERTEMPORAL ERRORS FOR THE DSGE MODEL



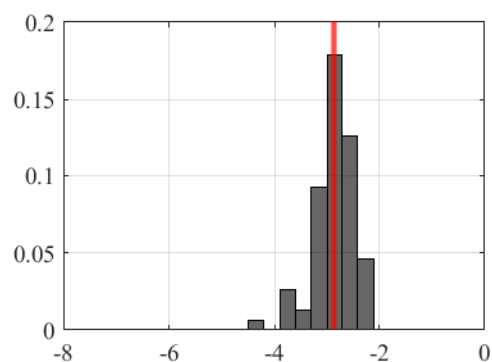
(a) CONDITION W.R.T. MORTGAGE DEBT (PT.)



(b) CONDITION W.R.T. HOUSING (PT.)



(c) CONDITION W.R.T. MORTGAGE DEBT (IMPT.)



(d) CONDITION W.R.T. HOUSING (IMPT.)

Note: The histograms report the intertemporal errors for the first-order conditions on an absolute log 10 scale. The model is parameterized to the baseline posterior mode.

F DSGE Estimation: Data

The sample covers the U.S. economy in 1984Q1-2019Q4, at a quarterly frequency. The time series are retrieved from the database of the U.S. Federal Reserve Bank of St. Louis and transformed as described below.

$$\text{Real personal consumption expenditures p.c.: } \frac{\text{PCEC}_t}{\text{PCECTPI}_t \cdot \text{CNP16OV}_t}. \quad (\text{F.1})$$

$$\text{Real home mortgage loan liabilities p.c.: } \frac{\text{HHMSDODNS}_t}{\text{GDPDEF}_t \cdot \text{CNP16OV}_t}. \quad (\text{F.2})$$

$$\text{Real house prices: } \frac{\text{CSUSHPISA}_t}{\text{GDPDEF}_t}. \quad (\text{F.3})$$

$$\text{Real disposable personal income p.c.: } \frac{\text{HNODPI}_t}{\text{GDPDEF}_t \cdot \text{CNP16OV}_t}. \quad (\text{F.4})$$

$$\text{Aggregate weekly hours p.c.: } \frac{\text{AWHI}_t}{\text{CNP16OV}_t}. \quad (\text{F.5})$$

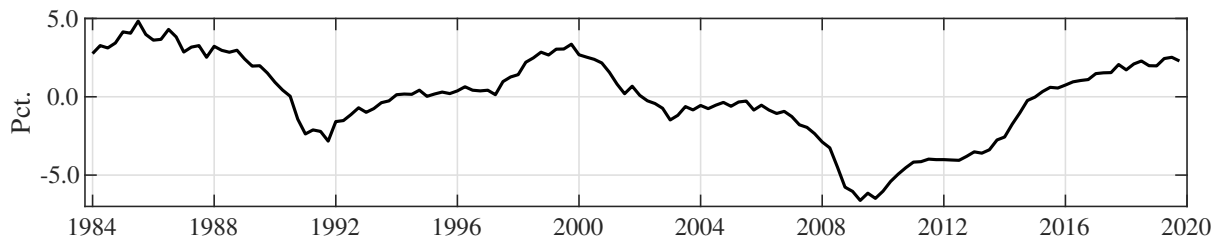
$$\text{Log change in the GDP price deflator: } \log \left(\frac{\text{GDPDEF}_t}{\text{GDPDEF}_{t-1}} \right). \quad (\text{F.6})$$

(F.1)-(F.5) are normalized relative to 1975Q1, then log-transformed, and lastly detrended by series-specific one-sided HP filters, with the smoothing parameter set to 100,000. (F.6) is demeaned across 1984Q1-2019Q4. Figure F.1 plots the resulting time series across this period.

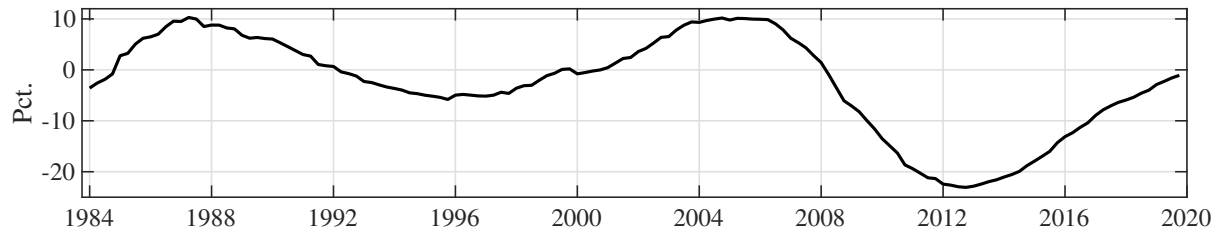
The text codes in (F.1)-(F.6) are the identifiers used by the U.S. Federal Reserve Bank of St. Louis. They abbreviate:

- PCEC: Personal Consumption Expenditures (billions of dollars, SA annual rate).
- HHMSDODNS: Households and Nonprofit Organizations; Home Mortgages; Liability, Level (billions of dollars, SA).
- CSUSHPISA: S&P Case-Shiller U.S. National Home Price Index (index, SA).
- HNODPI: Households and Nonprofit Organizations; Disposable Personal Income (billions of dollars, SA annual rate).
- AWHI: Index of Aggregate Weekly Hours: Production and Nonsupervisory Employees: Total Private Industries (index, SA).
- PCECTPI: Personal Consumption Expenditures: Chain-type Price Index (index, SA).
- GDPDEF: Gross Domestic Product: Implicit Price Deflator (index, SA).
- CNP16OV: Civilian Noninstitutional Population (thousands of persons, NSA).

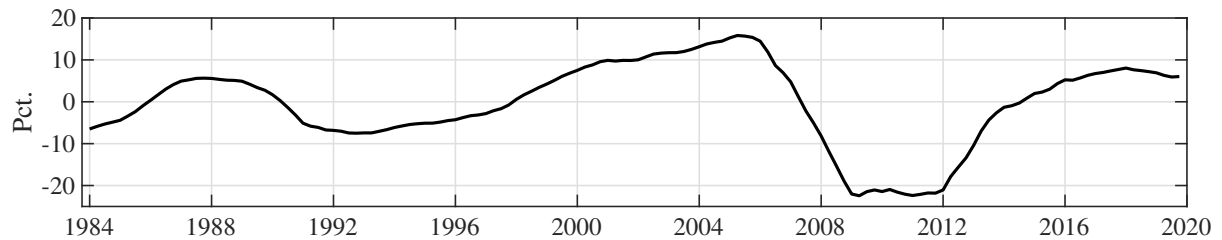
Figure F.1: DATA PLOTS (DEVIATION FROM MEAN OR TREND)



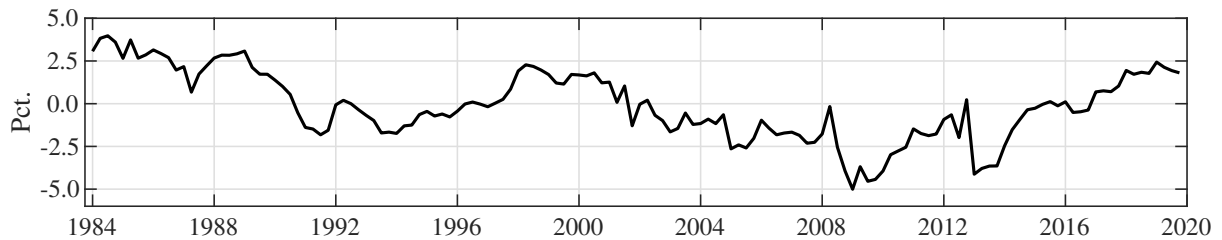
(a) REAL PERSONAL CONSUMPTION EXPENDITURES P.C.



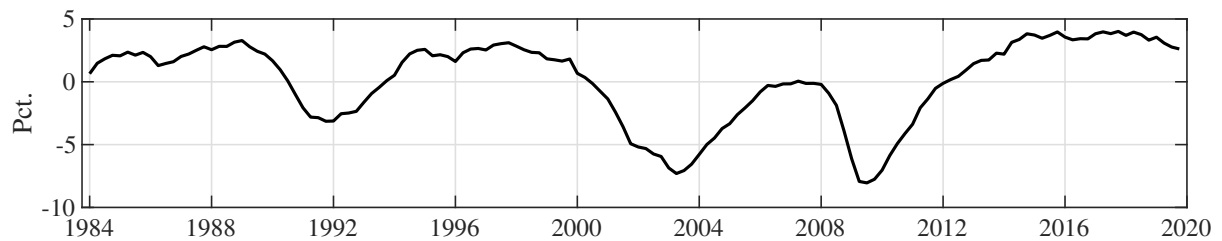
(b) REAL HOME MORTGAGE LOAN LIABILITIES P.C.



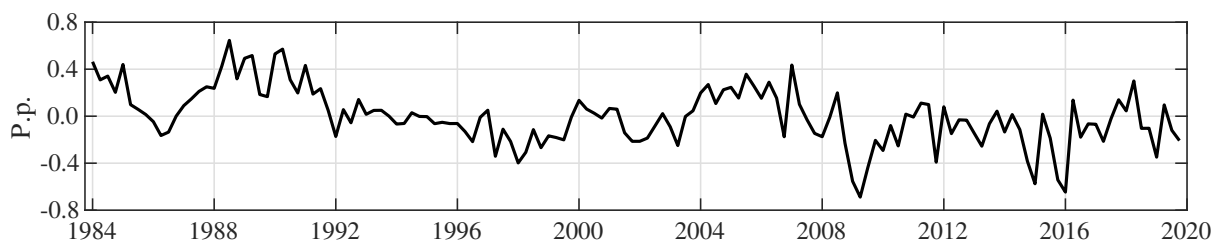
(c) REAL HOUSE PRICES



(d) REAL DISPOSABLE PERSONAL INCOME P.C.



(e) AGGREGATE WEEKLY HOURS P.C.

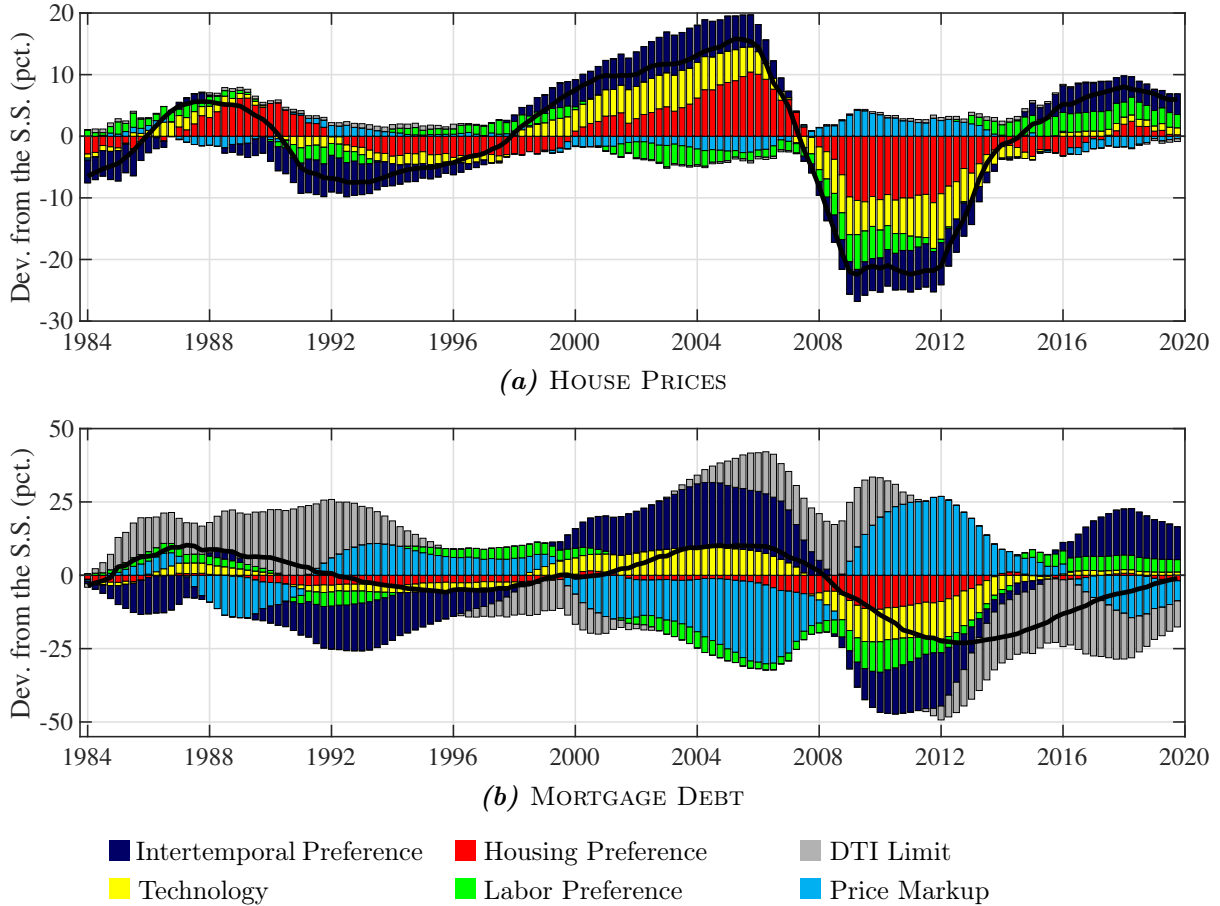


(f) LOG CHANGE IN THE GDP PRICE DEFLATOR

G DSGE Estimation: Additional Results

Section 6 in the main text has already examined how the shocks in the DSGE model affected when the respective credit constraints were binding. This appendix explores the historical impact of the shocks on house prices and mortgage debt. To do so, Figure G.1 plots the shock decomposition of these two variables, in deviation from the steady state.

Figure G.1: SHOCK DECOMPOSITION

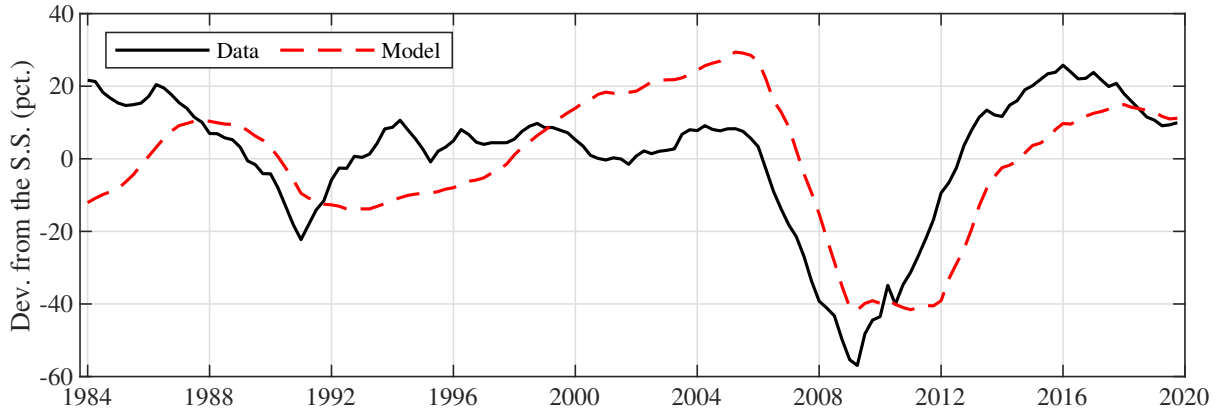


Note: The decomposition is performed at the baseline posterior mode. Each bar indicates the contribution of a given shock to a certain variable. The shocks were marginalized in the following order: (1) housing preference, (2) labor-augmenting technology, (3) price markup, (4) labor preference, (5) intertemporal preference, and (6) DTI limit. This ordering is identical to the one applied by [Guerrieri and Iacoviello \(2017\)](#), with the novel DTI shock ordered last. The results are robust to alternative orderings.

Figure G.1a shows that housing preference shocks explain the largest portion of the movements in house prices.¹ These preference shifts could represent genuine changes in consumers' taste for housing. For instance, [Shiller \(2015, ch. 6\)](#) report that media outlets have increased their coverage of interior design and home improvement since the end-1990s, which could be both reflective of and conducive to an increased taste for housing. Alternatively, the preference shifts could, in a reduced form, represent changes in consumers' expectations about future house prices that are not captured by the expectations

¹This finding is consistent with DSGE results in [Liu, Wang, and Zha \(2013\)](#) and [Guerrieri and Iacoviello \(2017\)](#), as well as with the approach taken by [Berger, Guerrieri, Lorenzoni, and Vavra \(2018\)](#) to explain the 2000s' housing boom-bust in an OLG housing model.

Figure G.2: RESIDENTIAL INVESTMENT: DATA AND MODEL



Note: The model series is identified at the baseline posterior mode. The data series is real residential fixed gross investment from the National Income and Product Accounts of the U.S. Bureau of Economic Analysis. The data series has been log-transformed and detrended by a one-sided HP filter, with the smoothing parameter equal to 100,000.

formation in the model. Such a line of reasoning has been proposed by, e.g., [Kaplan, Mitman, and Violante \(2020\)](#), who find changes in beliefs to be a main source of movements in house prices. These authors likewise find that shifts in credit conditions do not move house prices but are important for leverage dynamics, in line with the above results. Figure G.1a also documents that other shocks influence house prices. For instance, procyclical technology and labor preference shocks spur housing demand and house prices in expansions and contract them in recessions.

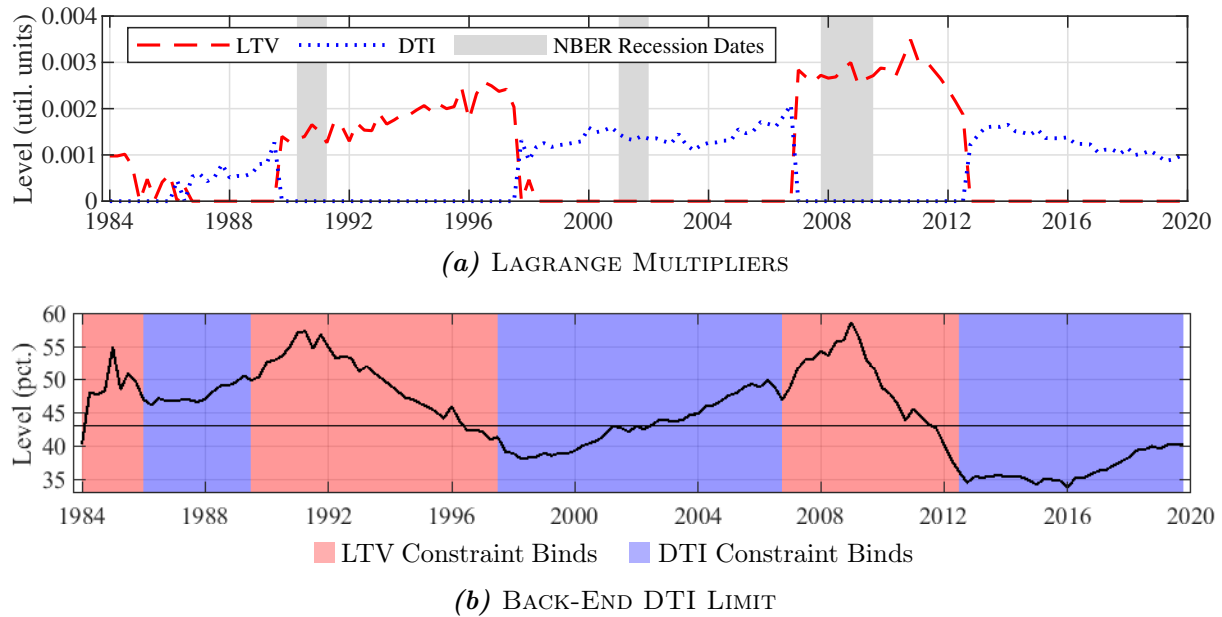
The decomposition of mortgage debt in Figure G.1b next illustrates how the adjustments in DTI limits, discussed in the main text, caused the debt level to fluctuate. It is evident that DTI relaxations in both the 1980s and the 2000s contributed to raising mortgage debt. Moreover, low DTI limits after the Great Recession have been paramount in keeping debt levels low. House prices, conversely, only impacted debt levels considerably following the Great Recession, when falling house prices forced LTV constrained homeowners to delever. Lastly, we observe that technology shocks impact debt procyclically, by causing both house prices and incomes to rise in expansions, relaxing both constraints, and fall in recessions, tightening both constraints.

I finally evaluate the fit of the model with respect to residential investment. Figure G.2 plots the smoothed path of residential investment from the baseline estimation of the DSGE model, along with the evolution in residential fixed gross investment from the National Income and Product Accounts. The model reasonably precisely predicts residential investment. The correlation between the two series is 63 pct. The calibration of the housing transformation elasticity ($\nu = 0.65$) ensures that the standard deviation of the two series is approximately identical (0.18 in both cases).

H DSGE Estimation: Sensitivity Analysis

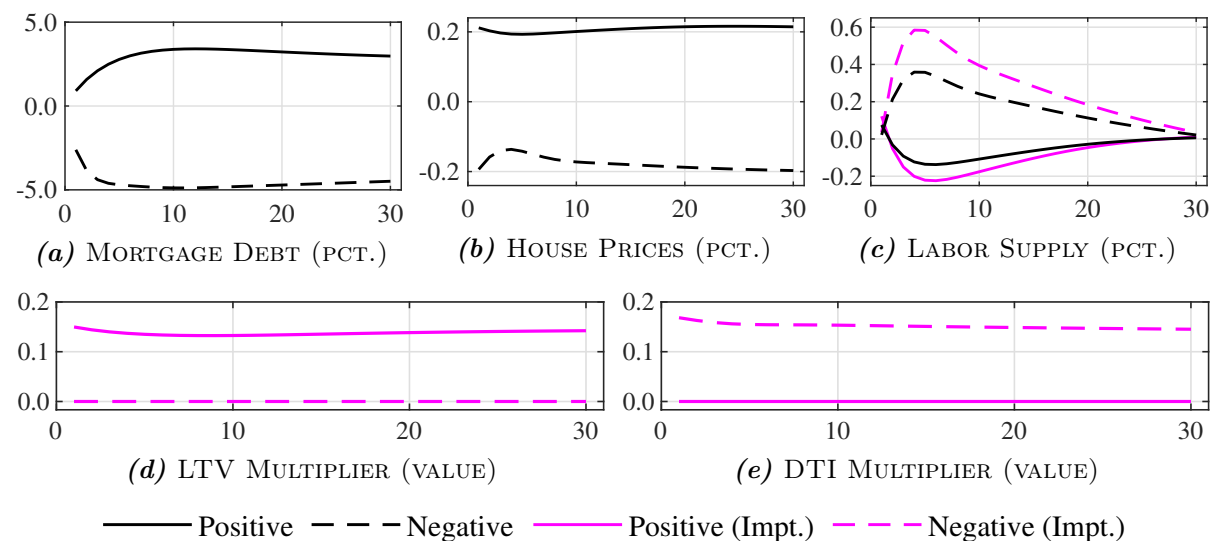
The model presented in the main text assumes that the aggregate variation in hours worked is driven by variation within both households. Figures H.1-H.2 show that the results are robust to assuming heterogeneity in labor market attachment. In this latter case, it is the impatient workers' employment that drives the aggregate variation in hours worked, leaving patient workers' employment constant at its steady-state level.

Figure H.1: HETEROGENEITY IN LABOR MARKET ATTACHMENT: ESTIMATION RESULTS



Note: Figure H.1a plots the smoothed Lagrange multipliers. Figure H.1b plots the smoothed back-end DTI limit ($\tilde{\xi}_{DTI,DTI,t}$), with the horizontal line indicating the steady-state DTI limit ($\tilde{\xi}_{DTI}$). All variables are identified at the posterior mode.

Figure H.2: HETEROGENEITY IN LABOR MARKET ATTACHMENT: CHANGES IN DTI LIMITS



Note: The figures report the effects of unit-standard-deviation positive and negative shocks. The model is parameterized to its posterior mode. Vertical axes measure deviations from the steady state (Figures H.2a-H.2c) or utility levels (Figures H.2d-H.2e).

I Evidence on State-Dependent Credit Origination

Table I.1 reports summary statistics of the county-level panel data. Across the years, there is a substantial variation in both the central tendency and the dispersion of the growth rates of mortgage loan origination, house prices, and incomes. Unconditionally, loan origination growth has a small positive correlation with house price growth, and is uncorrelated with income growth, while house price and income growth are themselves positively correlated.

Table I.1: SUMMARY STATISTICS: GROWTH RATES ACROSS COUNTIES (1991-2017)

Counts, Means, and Standard Deviations by Year							
	Obs.	Loan Origination		House Price		Disp. Personal Income	
		Mean	S.D.	Mean	S.D.	Mean	S.D.
1991	1337	0.198	0.576	0.013	0.038	0.037	0.023
1992	1383	0.538	0.482	0.018	0.025	0.064	0.022
1993	1505	0.430	0.406	0.014	0.036	0.044	0.023
1994	1772	-0.372	0.473	0.020	0.039	0.051	0.024
1995	1924	-0.050	0.415	0.029	0.041	0.056	0.023
1996	1932	0.383	0.349	0.028	0.024	0.058	0.022
1997	1964	0.135	0.186	0.032	0.026	0.059	0.022
1998	2038	0.603	0.133	0.043	0.024	0.068	0.026
1999	2084	-0.159	0.140	0.044	0.025	0.049	0.023
2000	2319	-0.208	0.113	0.072	0.048	0.074	0.028
2001	2343	0.657	0.172	0.065	0.029	0.041	0.037
2002	2353	0.277	0.219	0.056	0.038	0.020	0.023
2003	2501	0.307	0.176	0.046	0.034	0.037	0.023
2004	2557	-0.317	0.183	0.087	0.062	0.056	0.030
2005	2624	0.077	0.146	0.112	0.077	0.054	0.032
2006	2627	-0.074	0.119	0.075	0.056	0.069	0.031
2007	2636	-0.018	0.196	0.012	0.044	0.053	0.029
2008	2643	-0.339	0.258	-0.054	0.086	0.038	0.037
2009	2656	0.193	0.216	-0.077	0.080	-0.028	0.037
2010	2657	-0.118	0.128	-0.045	0.041	0.039	0.026
2011	2667	-0.092	0.108	-0.039	0.033	0.058	0.028
2012	2666	0.345	0.139	-0.011	0.027	0.046	0.032
2013	2663	-0.085	0.120	0.038	0.046	0.014	0.025
2014	2664	-0.297	0.124	0.059	0.051	0.054	0.026
2015	2649	0.253	0.103	0.043	0.029	0.047	0.028
2016	2631	0.152	0.086	0.050	0.033	0.024	0.028
2017	2629	-0.084	0.130	0.056	0.033	0.041	0.016
All years	62424	0.077	0.373	0.029	0.064	0.045	0.034

Correlations across all Years			
	Loan Origination	House Price	Disp. Personal Income
Loan Origination	1.00		
House Price	0.15	1.00	
Disp. Personal Income	0.02	0.36	1.00

Note: The observations are weighted by the county population in a given year.

I next check the robustness of the panel data results along three dimensions. In each case, I rely on the second-stage regression model from the main text, restated here:

$$\begin{aligned} \Delta \log d_{i,t} = & \delta_i + \zeta_{j,t} + \beta_{hp} \Delta \log \widehat{hp}_{i,t-1} + \beta_{inc} \Delta \log \widehat{inc}_{i,t-1} \\ & + \tilde{\beta}_{hp} \mathcal{I}_{i,t}^{LTV} \Delta \log \widehat{hp}_{i,t-1} + \tilde{\beta}_{inc} \mathcal{I}_{i,t}^{DTI} \Delta \log \widehat{inc}_{i,t-1} + u_{i,t}. \end{aligned} \quad (\text{I.1})$$

Post-Financial Crisis Sample A concern is that credit standards have changed over time, entailing that the house price and income elasticities have adjusted. To test this, I reestimate the model in (I.1) under the baseline definition of the indicators in (I.2) but relying on data covering only 2009-2017. The results follow in Table I.2.

$$\mathcal{I}_{i,t}^{LTV} \equiv 1 - \mathcal{I}_{i,t}^{DTI} \equiv \begin{cases} 0 & \text{if } \log \left(\frac{hp_{i,t}}{inc_{i,t}} \right) \geq \overline{\log \left(\frac{hp_{i,t}}{inc_{i,t}} \right)} \\ 1 & \text{else.} \end{cases} \quad (\text{I.2})$$

Overall, the estimated elasticities do not differ much from the baseline. However, one result is noteworthy: the unconditional house price elasticities in specifications 2-3 are now always insignificant. Thus, in counties that were predominantly DTI constrained, the effect of house prices on mortgage origination was smaller in 2009-2017 than under the historical norm. This is likely an effect of the tightening in DTI limits around the Great Recession, documented in the DSGE estimation.

Table I.2: CATALYSTS FOR CREDIT ORIGINATION: LEVEL SHIFTERS (2009-2017)

Detrending Method	$\Delta \log b_t$					
	N/A	Quadratic		Cubic	N/A	
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \log \widehat{hp}_{i,t-1}$	0.561*** (0.136)	0.0561 (0.172)		-0.100 (0.233)		0.413*** (0.150)
$\Delta \log \widehat{inc}_{i,t-1}$	-0.0142 (0.205)	-0.137 (0.221)		-0.0762 (0.209)		0.249 (0.221)
$\mathcal{I}_{i,t}^{LTV} \Delta \log \widehat{hp}_{i,t-1}$		0.687*** (0.199)	0.705*** (0.163)	0.860*** (0.253)	0.821*** (0.184)	
$\mathcal{I}_{i,t}^{DTI} \Delta \log \widehat{inc}_{i,t-1}$		0.556*** (0.182)	0.553*** (0.158)	0.608*** (0.205)	0.581*** (0.176)	
$\Delta \log \widehat{hp}_{i,t-1} \Delta \log \widehat{inc}_{i,t-1}$						5.612*** (2.113)
Observations	23882	23882	23882	23882	23882	23882
Adjusted R^2	0.807	0.809	0.809	0.809	0.809	0.807

Note: County and state-year fixed effects are always included. Observations are weighted by the county population in a given year. Standard errors are clustered at the county level, and reported in parentheses. ***, **, and * indicate statistical significance at the 1 pct., 5 pct., and 10 pct. confidence levels.

Separate Level Shifters The LTV and DTI indicators in the main text partition the house price and income elasticities on the basis of the house-price-to-income ratio. I now instead partition these elasticities solely based on the prevailing detrended levels of incomes and house prices:

$$\mathcal{I}_{i,t}^{LTV} \equiv \begin{cases} 0 & \text{if } \log inc_{i,t} \leq \overline{\log inc_{i,t}} \\ 1 & \text{else,} \end{cases} \quad \mathcal{I}_{i,t}^{DTI} \equiv \begin{cases} 0 & \text{if } \log hp_{i,t} \leq \overline{\log hp_{i,t}} \\ 1 & \text{else,} \end{cases} \quad (\text{I.3})$$

where $\overline{\log hp_{i,t}}$ and $\overline{\log inc_{i,t}}$ denote separately estimated county-specific time trends. The intuition behind this partitioning is the following. If homeowners must fulfill a DTI requirement and incomes are currently low, then the house price elasticity should likely be lower than if incomes were high. Likewise, if homeowners must fulfill an LTV requirement and house prices are currently low, then the income elasticity should likely be lower than if house prices were high.

Table I.3 reports the ordinary least squares estimates of the model in (I.1) under (I.3). Specifications 2-3 are based on quadratic estimates of $\overline{\log hp_{i,t}}$ and $\overline{\log inc_{i,t}}$, while specifications 4-5 are based on cubic estimates. With both detrending procedures, the estimates of both newly introduced conditional elasticities are significantly positive and, as compared to the unconditional elasticities, sizable. In particular, in the parsimonious specification 5, the house price elasticity is twice as large when incomes are high (0.64) as when they are low (0.32), while the income elasticity (0.38) is only positive when house prices are high.

Table I.3: CATALYSTS FOR CREDIT ORIGINATION: SEPARATE LEVEL SHIFTERS (1991-2017)

Detrending Method	$\Delta \log b_t$				
	N/A	Quadratic		Cubic	
	(1)	(2)	(3)	(4)	(5)
$\Delta \log \widehat{hp}_{i,t-1}$	0.523*** (0.0926)	0.376*** (0.109)	0.372*** (0.110)	0.322*** (0.107)	0.321*** (0.109)
$\Delta \log \widehat{inc}_{i,t-1}$	0.0906 (0.193)	-0.0731 (0.194)		-0.00924 (0.192)	
$\mathcal{I}_{i,t}^{LTV} \Delta \log \widehat{hp}_{i,t-1}$		0.244*** (0.0786)	0.244*** (0.0786)	0.317*** (0.0815)	0.317*** (0.0820)
$\mathcal{I}_{i,t}^{DTI} \Delta \log \widehat{inc}_{i,t-1}$		0.534*** (0.0799)	0.531*** (0.0782)	0.379*** (0.0756)	0.378*** (0.0746)
Observations	62424	62424	62424	62424	62424
Adjusted R^2	0.674	0.675	0.675	0.675	0.675

Note: County and state-year fixed effects are always included. Observations are weighted by the county population in a given year. Standard errors are clustered at the county level, and reported in parentheses. ***, **, and * indicate statistical significance at the 1 pct., 5 pct., and 10 pct. confidence levels.

Growth Shifters LTV and DTI requirements tie homeowners' borrowing ability to the relative *level* of their housing wealth and incomes. Nevertheless, under such requirements, we should also expect that strong growth in incomes (house prices) tend to make homeowners LTV (DTI) constrained. In that case, the house price (income) elasticity should increase in the ensuing years. I test this prediction by letting $\mathcal{I}_{i,t}^{LTV}$ and $\mathcal{I}_{i,t}^{DTI}$ denote *growth* indicators for personal incomes and house prices in county i in year t . The indicators take the value "0" if the growth rate of their input variable fell below a certain threshold in the previous year and the value "1" if it was above:

$$\mathcal{I}_{i,t}^{LTV} \equiv \begin{cases} 0 & \text{if } \Delta \log inc_{i,t-1} \leq \kappa_{inc} \\ 1 & \text{else,} \end{cases} \quad \mathcal{I}_{i,t}^{DTI} \equiv \begin{cases} 0 & \text{if } \Delta \log hp_{i,t-1} \leq \kappa_{hp} \\ 1 & \text{else,} \end{cases} \quad (\text{I.4})$$

where $\kappa_{inc} \in \mathbb{R}$ and $\kappa_{hp} \in \mathbb{R}$ measure the growth thresholds. Under this specification, the indicators partition the house price and income elasticities based on past income and house price growth.

There are two advantages of this partitioning over the baseline partitioning in the main text. First, the partitioning in (I.4) does not hinge on a specific method of detrending, unlike with the baseline partitioning. Second, the indicators are less autocorrelated than with the baseline partitioning. If the indicators are highly autocorrelated, then shifts in them may also capture low-frequency events, such as changing lending conditions, that are economically disjunct from the switching between LTV and DTI constraints.¹

Table I.4: CATALYSTS FOR CREDIT ORIGINATION: GROWTH SHIFTERS (1991-2017)

Thresholds	$\Delta \log b_t$				
	N/A	$(\kappa_{inc}, \kappa_{hp}) = (0.0597, 0.0707)$		$(\kappa_{inc}, \kappa_{hp}) = (0, 0)$	
	(1)	(2)	(3)	(4)	(5)
$\Delta \log \widehat{hp}_{i,t-1}$	0.523*** (0.0926)	0.243** (0.105)	0.238** (0.106)	-0.0626 (0.116)	
$\Delta \log \widehat{inc}_{i,t-1}$	0.0906 (0.193)	-0.158 (0.201)		0.111 (0.208)	
$\mathcal{I}_{i,t}^{LTV} \Delta \log \widehat{hp}_{i,t-1}$		0.270*** (0.0616)	0.267*** (0.0612)	0.671*** (0.113)	0.641*** (0.0903)
$\mathcal{I}_{i,t}^{DTI} \Delta \log \widehat{inc}_{i,t-1}$		0.845*** (0.102)	0.841*** (0.102)	0.242** (0.107)	0.249** (0.103)
Observations	62424	62424	62424	62424	62424
Adjusted R^2	0.674	0.675	0.675	0.675	0.675

Note: County and state-year fixed effects are always included. Observations are weighted by the county population in a given year. Standard errors are clustered at the county level, and reported in parentheses. ***, **, and * indicate statistical significance at the 1 pct., 5 pct., and 10 pct. confidence levels.

It is not *a priori* obvious which values the growth thresholds should take, i.e., what

¹The autocorrelation of $\mathcal{I}_{i,t}^{LTV}$ and $\mathcal{I}_{i,t}^{DTI}$ under the baseline partitioning is 0.70 with the quadratic detrending and 0.64 with the cubic detrending. By contrast, the autocorrelation under (I.4) is 0.24 of $\mathcal{I}_{i,t}^{LTV}$ and 0.55 of $\mathcal{I}_{i,t}^{DTI}$.

constitutes "low" and "high" growth rates of house prices and incomes. I therefore allow the data to choose the thresholds by simulating these in the following way. First, I divide the observations of house price and income growth rates, respectively, into ten percentiles, thus obtaining nine quantiles as potential thresholds for each variable. I then estimate (I.1) under (I.4), tentatively trying each of the $9 \cdot 9 = 81$ possible quantile-pair combinations. As the final threshold, I choose the quantile-pair that minimizes the root mean square error of the regression. This combination is $(\kappa_{inc}, \kappa_{hp}) = (0.0597, 0.0707)$, which is the 70 pct. income growth quantile and the 80 pct. house price growth quantile.

Specification 2-3 in Table I.4 reports the ordinary least squares estimates of the second-stage regression equation in (I.1) under (I.4), with $(\kappa_{inc}, \kappa_{hp}) = (0.0597, 0.0707)$. The results align well with the baseline results on state-dependent elasticities. In the parsimonious specification 3, the house price elasticity (0.51) is roughly twice as large when income growth was above 6.0 pct. in the previous year, as when it fell below this threshold (0.24). Moreover, the income elasticity is only positive (0.84) when house prices grew by more than 7.1 pct. in the previous year. Finally, as a robustness test in specification 4-5, I use the alternative threshold, $(\kappa_{inc}, \kappa_{hp}) = (0, 0)$, where the estimates are partitioned based on whether house prices and incomes fell or grew in the previous year. In the parsimonious specification 5, only the conditional estimates are significantly positive. In this way, only house price growth conditional on past positive income growth and income growth conditional on past positive house price growth increase loan origination.

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