Policy Uncertainty in Brexit U.K.

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Abstract

We develop a model with heterogeneous firms and non-convex adjustment costs to study the effects economic policy uncertainty. In this model *(i)* uncertainty sparks from news about future policy reforms, which bears no direct effect on current fundamentals; *(ii)* uncertainty is about idiosyncratic and aggregate fundamentals in the long run; *(iii)* the timing of the reforms is uncertain. We use this framework to measure the effects of Brexit uncertainty.

The long run idiosyncratic and aggregate effects of Soft and Hard Brexit are identified using expectation data from the Decision Maker Panel, a novel survey of U.K. businesses, where each CFO provides probability distributions over the long-run expected outcome of Brexit on firm-level sales, for different Brexit scenarios. The long-run effects of Brexit implied by U.K business expectations are found to be large, around 8.3% and 4.8% of GDP for Hard and Soft Brexit, respectively. The transitional dynamics under policy uncertainty reveal that the referendum has produced significant economic damage, with a drop in investment leading to a three-year cumulative loss of GDP of about 3%. Most of these effects have been driven by wait-and-see effects rather than by the anticipation of a worsening in future fundamentals. The effects of policy uncertainty are larger when the expected duration of uncertainty is short, as the expected inefficiency cost of inaction is smaller.

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1 Introduction

One the most important contributions to the field of macroeconomics in recent years, is the finding that that uncertainty shocks have important consequences for business cycle fluctuations. These shocks are commonly defined as a temporary increase in the variance of aggregate and idiosyncratic TFP innovations. As shown by Bloom et al. (2018), economic recessions appear indeed to be systematically associated with a sudden and short lived increase in the variance of aggregate and firm-level shocks to fundamentals. Yet, not all episodes of uncertainty are alike, and not all are best represented by a second moment shock to fundamentals over the short run.

More recently, the world economy has been rocked by extraordinary episodes of economic policy uncertainty, including the decision of the U.K. to leave the European Union (Brexit) and the repeated threats by the U.S. president Trump to reshape international trade agreements, to name a few. These dramatic cases of policy uncertainty can hardly be interpreted as conventional uncertainty shocks. Indeed, the introduction of these policies would have permanent effects, which implies that uncertainty is primarily about aggregate and idiosyncratic fundamentals in the long run. Moreover, these policies would affect fundamentals only after they are implemented, which implies that the news does not affect fundamentals directly. Both the news, and the associated uncertainty may still affect economic activity in the short run, but only indirectly, through anticipation effects. Finally, an important aspect of this type of economic policy uncertainty is timing uncertainty, as it is not clear when these policies will be implemented, if at all. We develop a structural framework that is suitable to study economic policy uncertainty, and carry out a quantitative analysis of the case of Brexit.

The environment is a closed economy in which heterogeneous firms face fixed costs of investment and partial investment irreversibility in the spirit of Bloom (2009). We assume that the steady state of this economy is perturbed by the unexpected news that a package of policy reforms may be implemented in the future. The timing of these reforms is uncertain. Every period, one of the following three stochastic outcomes can take place: the reforms are either implemented, taken off the table once and for all, or else uncertainty drags on to the next period. Firms are also uncertain about what policy will be implemented and, for a given policy, what the idiosyncratic effects will be. Specifically, a policy is defined as a state-contingent distribution of permanent idiosyncratic productivity shocks and an aggregate shock. For simplicity, we assume that the policy can take only two possible realizations. Uncertainty about the policy will generate aggregate uncertainty. When a policy is implemented, firms have to take a draw of the productivity shock from the distribution, which adds up permanently to their current productivity. Uncertainty about the productivity draw generates idiosyncratic uncertainty. Because the model is real, these shocks should not be interpreted literally as technology shocks, but more broadly as permanent sales shocks induced by the reforms, or as the outcome of idiosyncratic policy wedges in the spirit of Restuccia and Rogerson (2008) and Buera and Shin (2013, 2018). We emphasize that the model puts no a-priori restrictions on the sign of the state-contingent aggregate and idiosyncratic shocks that characterize the policy, so the reforms can lead to either a permanent increase or a permanent decrease in aggregate production.

We calibrate the model to the U.K. economy to assess the effects of the Brexit referendum on the behaviour of key macroeconomic outcomes before Brexit uncertainty is resolved. Our empirical strategy consists of two steps. First, we calibrate the policy parameters governing the state-contingent aggregate and idiosyncratic shocks, assuming two possible policy outcomes, which we call Soft and Hard Brexit. In order to do so, we rely on the Decision Maker Panel (DMP) dataset, a representative survey of British firms, which was specifically designed to gather detailed information on the expected effect of Brexit on individual U.K. businesses. For each respondent firm in this dataset, the Chief Financial Officer is asked to provide probability distributions on the perceived long-run effects of Brexit on sales, relative to the case where the Brexit referendum had not happened. The questions asked require the CFO to provide both an unconditional distribution, and a distribution conditional on the case of a disorderly exit (Hard Brexit). Because the dataset also contains detailed information that allows us to infer the average probability assigned to a disorderly Brexit, we are able to precisely identify the policy parameters governing the long run effects of both Hard and Soft Brexit, by requiring that Brexit expectations in the model cross-section of firms match key moments of the expectation distribution of actual U.K. firms. Moreover, the survey also provides a breakdown of the probability distributions about expected sales gains and losses by productivity, which we exploit to pin down how the intensity of productivity shocks varies with firm-level productivity. Equipped with these policy parameters, we can retrieve the Hard and Soft Brexit steady states that are implied by the expectations of U.K. businesses.

In the second step, we elicit the effects of the Brexit referendum by computing transitional dynamics under policy uncertainty, and comparing the dynamics relative to the case where the referendum didn't happen, and the economy remains in the pre-Brexit steady state. We then run a number of exercises to investigate the effects of policy and timing uncertainty, as well analysing the role of idiosyncratic uncertainty.

The DMP survey reveals that on average firms believe that they are much more likely to suffer sale losses from Brexit, than to obtain gains. This is even more so in the case of a disorderly exit. As a result, the calibrated model indicates that the expected fall in GDP associated with Brexit may be large, in the order of approximately 8.3% in the case of Hard Brexit, and 4.8% in the case of Soft Brexit, which is in line with government estimates (see HM Government, 2018). We note that these steady-state calculations are implied by the individual expectations of U.K. businesses, which may well be affected by individual ideological biases, and hence may not correctly reflect the true effects of Brexit on sales. However, because we are ultimately interested in assessing the short-term effects of Brexit uncertainty, it is the expectations of U.K. businesses that matter for their investment decisions, however biased or inaccurate they may possibly be.

The transitional dynamics under policy uncertainty reveal that the effects of the Brexit referendum have been large. In the first year that follows the referendum investment falls by about 17% and employment by about 1%. Because the economy's fundamentals not change with the news, the fall in output, which is also around 1%, is mainly driven by the drop in employment. But as uncertainty drags on, the fall in investment leads to a drop in the stock of capital, which in turn reduces production and employment even further. Three years after the referendum, the fall in GDP is already close to 3%. The reason why investment falls is twofold. One the one hand, the referendum is bad news about productivity in expectation, which implies that firms anticipate the convergence to a steady-state with lower productivity and capital. Hence, the capital stock falls, just like in the standard neoclassical growth model with a representative firm, after a negative aggregate technology shock. On the other hand, the referendum also produces uncertainty over idiosyncratic outcomes, giving rise to a traditional wait and see channel, whereby the investment inaction region expands as firms freeze their investment decisions in the hope that uncertainty subsides. Eliminating investment frictions in order to shut down this channel of propagation reveals that most of the investment dynamics under policy uncertainty are driven by the increase in inaction, rather than by the effects of the news itself.

Importantly, we find that timing uncertainty plays a key role in driving investment decisions. If uncertainty is expected to be very persistent, the option value of waiting is small, so investment falls by less. Indeed, if uncertainty is expected to take long to resolve, freezing investment to wait and see the outcome of the reforms is likely to result in a sub-optimal scale of production, as the existing capital stock deteriorates. The model can therefore offer a rationale to the findings in ongoing work by Bloom et al.(2019), which show that the reduction in investment has been lower in the immediate aftermaths of the referendum, compared to the run up to the end of March 2019, when there was a large probability that uncertainty would be resolved.

The paper is closely related to the literature on uncertainty, sparked by the seminal work by Bloom (2009). See, among others, Baker and Bloom (2013), Baker, Bloom, and Davis (2015), Fernandez Villaverde et al. (2015), and Bloom et al. (2016, 2018). The focus of this literature is on uncertainty shocks, which are modelled as temporary shocks to the second moment of an exogenous process, typically TFP, be it at the aggregate or firm level or both. The nature of uncertainty, in this context, is about the realizations of technology shocks at business cycle frequencies, and is typically interpreted as uncertainty on the states of idiosyncratic and aggregate demand over the very *short run*. In our model uncertainty is about the implementation of economic policies that have implications for aggregate and idiosyncratic productivity in the *long run*. We provide a structural framework that is useful to study the short-run effects of aggregate, idiosyncratic and timing uncertainty that characterize this type of economic policy uncertainty. Our analysis reveals that the mechanism explored by Bloom (2009), whereby non-convex investment adjustment costs comprising both of a fixed investment cost and partial investment irreversibility generate option values of waiting, can produce powerful dynamics also in a set-up with long-run economic policy uncertainty. Our results suggest that investment will tend to freeze by more at times when uncertainty is expected resolve quickly, and is therefore consistent with the micro-evidence in Bloom et al. (2019) on the response of U.K. businesses to Brexit uncertainty.

The paper also relates to the literature on news shocks sparked by the work of Beaudry and Portier (2004, 2006). A key message from this literature is that beliefs about future TFP are important drivers of business cycle fluctuations, and that economic fluctuations may arise even absent actual changes in future fundamentals.[1] The case of Brexit analysed in this paper is just a prominent example of this type of news. Our contribution to this literature is that we have introduced news shocks in an environment with heterogeneous firms, where the news itself is uncertain and firms face non-convex costs of adjustment. The quantitative analysis supports the view that the news of the Brexit referendum, and in particular the uncertainty associated with it, has been a key driver a economic fluctuations in the U.K economy over the recent years.

Our work also links to the macro development literature on resource misallocation, pioneered by Restuccia and Rogerson (2008) and further developed by Buera and Shin (2013, 2018). In these models, large scale reforms that impinge on disparate fields of regulations, including trade-policy, competition and industrial policy, labour market and migration policy and so on, are parsimoniously modelled as affecting idiosyncratic distortions at the firm level. The productivity shocks induced by the policy in the calibrated model could be interpreted as reflecting the introduction such idiosyncratic distortions, which produce resource misallocation in the long run. Indeed, a major advance in the growth literature of the last twenty years is the enhanced recognition of the role of resource allocation as a determinant of productivity in the long run (see Jones (2011) and Restuccia and Rogerson (2013)). The main lesson learnt in this literature is that by affecting the efficiency of resource allocation, policy may matter a great deal for the growth prospects of a country, for good or for bad (Buera and Shin (2013)). In this context, the Brexit referendum offers an interesting example of a developed country that faces the prospect of going down the route of economic decline; a very different case to the ones that have been typically studied in this literature.

Finally, our model relates to the recent literature that has studied the effects of Brexit. Most of this literature has focussed on the long run economic impact of Brexit, resorting to either trade gravity models or structural general equilibrium models. A common feature of this types of analysis is that they rely on assumptions on the imposition of tariff and non-tariff barriers. Our model circumvents these difficulties by aggregating firm level expectations on sales through the structure of a heterogeneous firms model. The main aim of our project though, is measuring the short term effects of Brexit uncertainty. The literature on this subject is more scant. Born et al (2019) estimate the effects of the Brexit referendum using a purely empirical model. Their quantitative findings, that by the end of 2018 the referendum has caused a cumulative decline of over two percentage points of GDP is in line with ours, although we find a stronger role for the uncertainty channel relative to the anticipation effect of the news. Steinberg (2019) measures the effects of Brexit policy uncertainty through the lenses of a standard trade models where the assumption that firms face irreversible fixed costs of export generates an option value of waiting. His findings, that the economic cost of uncertainty is negligible is very different from ours. We conclude that while fixed costs of export may fail to generate meaningful costs of uncertainty, non-convex adjustment costs of investment are a powerful mechanism, which can help make sense of the micro-data evidence discussed above.

[1] Faccini and Melosi (2018) find that beliefs that are unrelated to fundamentals have been the single most important driver of business cycle fluctuations in the U.S. economy.

The paper is organized as follows. In the next Section we discuss why Brexit is modelled as an economic policy uncertainty shock, explaining why it differs from a conventional uncertainty shock. Section 3 presents the model and Section 4 discusses how we exploit expectation data from the DMP survey to assign parameter values. Section 5 presents the quantitative analysis, discussing implications for the long run effects of Brexit under different policy scenarios and the short-term effects of policy uncertainty. Section 6 concludes.

2 Brexit as Economic Policy Uncertainty

Economic policy uncertainty arises from the announcement of a policy that may be implemented at some date in the future. Brexit, i.e. the U.K.'s decision to leave the E.U. following the referendum on 23^{rd} June 2016, can be interpreted exactly as an unexpected news of a possible future implementation of new economic arrangements with the E.U. which would have permanent implication for the U.K. economy. Economic policy uncertainty shocks are characterized by four unique features: *(i)* the news of the shock is unexpected; *(ii)* the news of the shock does not alter the economy's fundamentals in the short run; *(iii)* the policy that may or not be implemented will have permanent effects on the economy's fundamentals in the long run; and *(iv)* the timing of the policy's implementation is uncertain.

Firstly, economic policy uncertainty arises following the unexpected news that a policy will be implemented in the future. The outcome of the E.U referendum was indeed unexpected. Figure 1 shows the referendum outcome probabilities implied by bets offered on the Betfair terminal[2], where the probability of leaving the E.U. is illustrated by the green line and the probability of remaining by the blue one. The figure reveals the extent to which the result of the E.U. referendum was unexpected by the market agents. As it can be observed, the implied probability of leaving the E.U. oscillated close to 25% in the lead up to the ballot and plummeted by 10 percentage points in the very last week. Nevertheless, after the polls closed on the evening of the 23^{rd} of June 2016, the U.K. discovered that the Vote Leave campaign had won the referendum as highlighted by the reversal of the implied probabilities for Leave and Remain in Figure 1. This evidence emphasizes how U.K.'s decision to leave the E.U. was unforeseen by economic agents given the stark difference between the expectations and the result of the referendum.

Another key feature of Brexit which makes it a major canonical case of policy uncertainty, is the lack of direct short run effects on the U.K. economy following the unexpected news of the E.U. referendum. Indeed, following the Vote Leave's campaign victory, the fundamentals of the U.K. economy did not change and would not change until the U.K. modifies its economic arrangements with the E.U. after leaving. Figure 2 shows the U.K.'s Economic Policy Index (EPU) constructed by Baker, Bloom, and Davis (2015) and the U.K.'s FTSE100 Volatility Index (VIX), which substantiates this argument. The EPU Index (blue line) is a newspaper based index which quantifies the level of economic policy uncertainty in a given country at any given

[2] Source: BETdata, accessible via `https://betdata.io/historical-odds/uk-eu-referendum-2016`

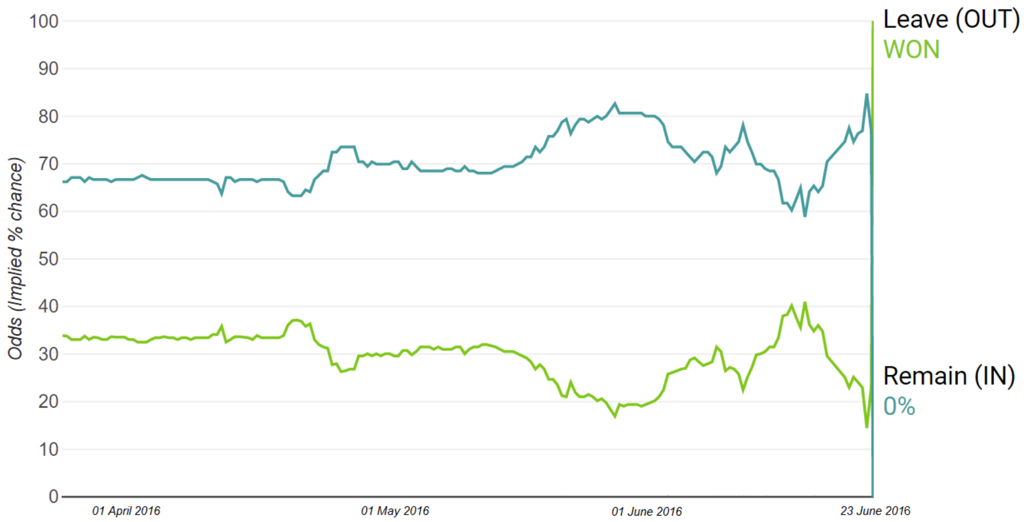

Figure 1: Implied probability of E.U. referendum outcome

time by scanning articles and counting those containing the terms uncertain or uncertainty, economic or economy, as well as policy relevant terms. On the other hand, the VIX (orange dotted line) is a common measure of the market's expectations of economic volatility calculated from the FTSE100 index 30 days options. Importantly, Figure 2 highlights how the EPU index exploded upon the news of the referendum outcome, whilst the VIX index remained virtually unchanged. The reason for this discrepancy lies in the type of uncertainty that these indexes capture. Whilst the VIX index measures uncertainty relating to the near-future, as indicated by the maturity of the stock options used in its calculation[3], the EPU index by construction is able to capture longer-term uncertainty sparking from policies that may impact the economy's fundamentals years and decades in the future.

Evidence that corroborates further the observation of a lack of direct short-run effects from the news of E.U. referendum result can be found in a Bank of England report (Bloom et al, 2019). The report exploits the Decision Maker Panel dataset to provide some in-depth analysis of the reaction of U.K. businesses to Brexit uncertainty asking firms direct questions about the effect of Brexit on their investment, sales and employment decisions. One of the most striking discoveries is that the variance of expected year-ahead sales growth between firms that are affected by Brexit uncertainty and those that are not affected has remained virtually unchanged. This implies that uncertainty surrounding Brexit is mostly about the long-run effects on fundamentals rather than the short-term prospects. The obvious reason for this finding is that even though the U.K. voted to leave the E.U., it has not yet done so at the time of writing, meaning that although firms have been plagued by uncertainty, there have been no direct effects on the U.K. economic fundamentals in the short-run. This piece of evidence, alongside the different responses of the EPU and VIX indexes, illustrate the distinction between Brexit, which we

[3]Using longer maturity VIX indexes does not change the result as the VIX only reaches a maximum maturity of a year.

view as a salient episode of economic policy uncertainty, from any ordinary uncertainty shock, which may be modelled as a second moment disturbance to the short-run fundamentals of the economy.

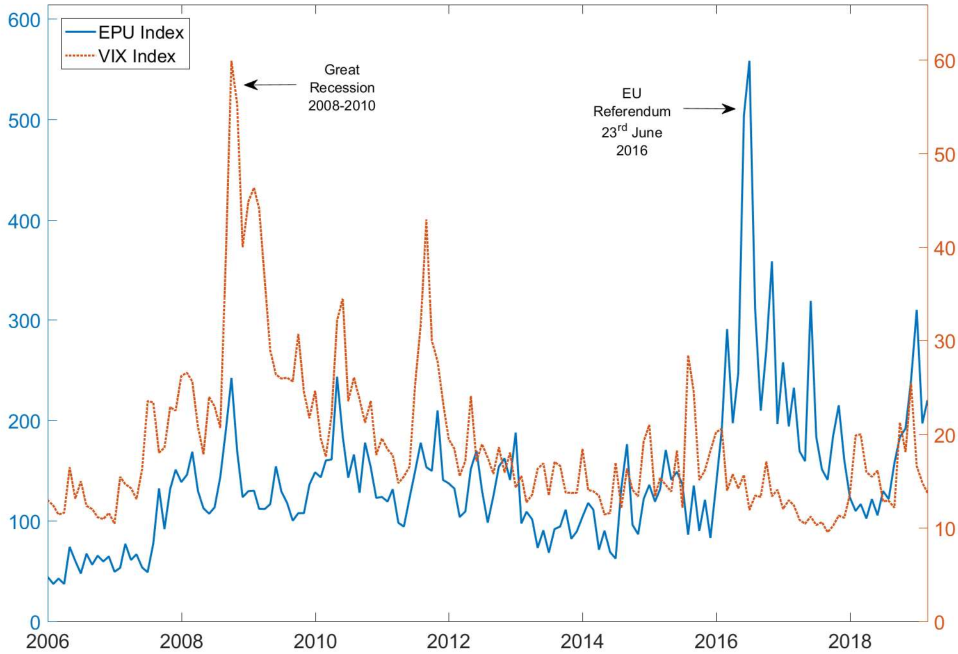

Figure 2: Uncertainty indices in U.K.

Another characteristic of policy uncertainty is that economic policies typically carry permanent direct effects on the economy's fundamentals in the long-run. More specifically, under policy uncertainty, firms face uncertainty about their future aggregate and idiosyncratic fundamentals. Brexit is an ideal example of a policy which has permanent consequences for U.K. economy because it entails a negotiation between the British government and the European Union about the long-term economic, political, and legal arrangements. For the purposes of this paper, such arrangements include but do not limit themselves to: trading rules, regulations on goods and services, immigration controls, transportation, security, contributions to the E.U. budget, and industrial strategy. All of these arrangements will impact the U.K. economy through an aggregate shock to fundamentals, to the extent that the arrangements equally affects all of the firms in the economy, and/or though idiosyncratic shocks to fundamentals, so long as the arrangements only affect a subset of economic agents or produce heterogeneous effects.

Finally, a distinguishing feature of Brexit is timing uncertainty. From the news of the E.U. referendum result in June 2016 up to the time of writing, the U.K. has yet to leave the E.U. and the date of the U.K.'s departure is still unknown. Throughout the Brexit process deadlines for the U.K.'s exit from the E.U. have been set, missed, and extended. Following the referendum, on 29^{th} March 2017, the U.K.'s Prime Minister, with the consent of parliament, sent the Article 50 letter to the President of the E.U. council notifying the U.K.'s wish to start the Brexit process. As stated by Article 50, this sets a maximum deadline of two-years for the U.K. to leave the E.U. with or without reaching an agreement. Alternatively, the U.K. could have left earlier either with a Withdrawal Agreement providing for a transitional period to define the details of the departure, or or without any agreement. Therefore, the very nature of exiting the E.U. introduces uncertainty over the timing of the implementation of the Brexit policy.

Furthermore, the failure to have reached an agreement, meant that the U.K. applied for an Article 50 extension twice on the 20^{th} of March and on the 5^{th} of April 2019. Both extensions were accepted by the E.U. leaders subject to certain conditions. Under the first extension the Brexit *'cliff-edge'* shifted from 29^{th} March 2019 to 22^{th} May 2019, conditional on the Withdrawal Agreement being accepted by the U.K. parliament, or the 12^{th} April 2019 otherwise. With the second extension the Brexit *'cliff-edge'* was moved to the 30^{th} of June 2019 conditional on the U.K. parliament passing the Withdrawal Agreement by the 22^{th} May 2019, and to the 30^{th} of October 2019 otherwise. Moreover, the U.K. and the E.U. in the midst of the Brexit process also introduced a transitional implementation period lasting until the 31^{st} of December 2020, which allows some time after the UK leaves for both economies to make the necessary preparations for the new future relationship. The Article 50 process, in conjunction with the constant moving of deadlines, added with the transitional implementation period, and the persistent threat of U.K. unilaterally exiting the E.U. at any time without an agreement, has made Brexit a unique case study for the effects of timing uncertainty.

3 Model

We present a simple, discrete-time economy with heterogeneous firms facing non-convex costs of adjusting the capital stock comprising of fixed costs in investment as well as partial investment irreversibility. These assumptions are required to allow for potentially meaningful effects of policy uncertainty (Bloom, 2009). We further assume that the stationary equilibrium of this economy is perturbed by the news that a policy may be implemented at some future, stochastic point in time. The details of the policy, described below, imply that firms face a combination of permanent state-contingent productivity shocks both at the aggregate and firm level. As a result, timing uncertainty about the implementation of the policy induces aggregate and idiosyncratic state uncertainty. We deliberately take a partial equilibrium approach and abstract from the explicit modelling of the household sector. Indeed, because our quantitative exercise focuses on the U.K. economy, we do not want to impose the equality between savings and investment.

3.1 The States Variables

The economy is populated by a unit mass of heterogeneous firms. Each period, firms produce by renting labour services at the wage rate w and using their capital stock, which evolves according to $k' = (1-\delta)k + i$, where $\delta \in \{0,1\}$ is a rate of depreciation, i denotes investment and $k \in \mathbb{R}_+$. Firms are characterized by a four-tuple of idiosyncratic state variables: (i) idiosyncratic productivity; (ii) an idiosyncratic policy state; (iii) the capital stock; (iv) a fixed cost associated with investment. Let's examine each element in turn.

The evolution of individual productivity, denoted by $a \in \mathcal{A} = \{a_1, ..., a_{N_a}\}$, is governed by an exogenous Markov stochastic AR(1) process: $log(a') = \psi log(a) + \epsilon$, where $\epsilon \sim \mathcal{N}(0, \sigma)$ and $\psi \in [0,1)$. Each firm is also characterized by an idiosyncratic policy state $z \in \mathcal{Z} = \{z_+, z_-, z_0\}$, which captures whether the implementation of the new policy contributes a positive, negative or neutral permanent component to firm level productivity, respectively. This state becomes relevant only after the policy is implemented, at which time firms take an independent draw of this idiosyncratic policy state. Specifically, firms will draw z_+ and z_- with associated probability q_+ and q_-, respectively, and z_0 with the complement probability $1 - q_+ - q_-$. These assumptions generate idiosyncratic uncertainty over the outcome of the policy. Such stochastic process allows us to separate idiosyncratic uncertainty from other sources of policy uncertainty. Finally, we assume that changing the capital stock is subject to fixed costs of adjustment, where ξ is stochastically drawn, each period, from the distribution $G(\xi) \sim \mathcal{U}\left[0, \overline{\xi}\right]$.

In addition to these idiosyncratic states, we introduce a fifth state variable, which captures the aggregate policy state, and is denoted by $\zeta \in \{\zeta^R, \zeta^U, \zeta^S, \zeta^H\}$. We let ζ^R denote the initial aggregate state of the economy, prior to the arrival of the news, and ζ^U denote the state after the news is announced but before the policy is actually implemented. The latter state is therefore characterized by uncertainty about the future outcome of the policy. Specifically, we assume that while the economy is in state ζ^U, a new policy is implemented at Poisson rate θ. In this case the economy transitions either to the absorbing state ζ^H, with probability γ^H, or to the other absorbing state ζ^S, with probability $\gamma^S = 1 - \gamma^H$, depending on which policy is implemented. We refer to these states as "Soft" and "Hard", respectively, in relation to our quantitative exercise on Brexit, but at this stage they can be thought of as two generic, unrestricted outcomes of the policy. If the new policy is not implemented, the economy can revert back to the initial absorbing state ζ^R with Poisson rate γ^R. If the policy is neither implemented nor taken off the table once and for all, uncertainty drags on to the next period with period probability $1 - \theta - \gamma^R$. The transition matrix for the stochastic policy state of the economy, $\Gamma_\zeta(\zeta_{t+1} = \zeta^i | \zeta_t = \zeta^q)$, can therefore be described by the following transition matrix:

$$
\begin{array}{ll}
\downarrow \zeta_t, \zeta_{t+1} \rightarrow & \begin{array}{cccc} \zeta^U & \zeta^S & \zeta^H & \zeta^R \end{array} \\
\begin{array}{l} \zeta^U \\ \zeta^S \\ \zeta^H \\ \zeta^R \end{array} &
\begin{pmatrix}
1 - \gamma^N - \theta & \theta(1-\gamma^H) & \theta\gamma^H & \gamma^R \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}.
\end{array}
\tag{1}
$$

3.2 The Production Function and the Effects of the Policy

The production function is given by

$$\tilde{f} = (1 + \tilde{\tau})a(k^{\alpha}l^{1-\alpha})^{(1-\nu)}, \tag{2}$$

where $1 - \nu$ is a span of control parameter, which represents the share of output that remunerates the variable factors and governs the degree of decreasing returns to scale. The term $\tilde{\tau}$ captures instead the contribution of the policy to firm level productivity. We assume that the economy is in an initial steady state,(ζ^R), where all firms face $\tilde{\tau} = 0$, when it is unexpectedly hit by the news that a policy package may be be implemented at some future stochastic point in time. Conditional on the new policy being implemented, firms will be subject to a distribution of non-zero values for $\tilde{\tau}$.

Specifically, it is assumed that:

$$\tilde{\tau} \equiv \tau(z_i, \zeta^j, a) = \begin{cases} 0 & \text{for} \quad j = \{R, U\} \\ (1 + X^j)(1 + x_i^j)a^{\omega} & \text{for} \quad j = \{S, H\} \text{ and } \quad i \in \{z_+, z_-, z_0\} \end{cases} \tag{3}$$

where $X^j \in \mathbb{R}$ is the parameter capturing the permanent aggregate productivity shock associated with the aggregate policy state $j \in \{\zeta^S, \zeta^H\}$ and $x_i^j \in \mathbb{R}$ is the parameter associated with the permanent idiosyncratic shock that is contingent on the aggregate policy state $j \in \{\zeta^S, \zeta^H\}$ and the idiosyncratic policy state $i \in \{z_+, z_-, z_0\}$. Further, $\omega > 0$ is a parameter governing how the intensity of the policy shocks varies with firm-level productivity. We assume that $x_i^j = 0$ for $i = z_0$. In this case, the implementation of the new policy affects the firms with idiosyncratic state z_0 only through aggregate effects X^j.

3.3 The Firms' Problem

Every period, a firm can invest to any future level of capital k' only upon payment of its fixed adjustment cost ξ. Because the firm takes each period an i.i.d. draw of ξ from the uniform distribution G, for a given end-of-period stock of capital, a firm's current adjustment cost has no implication for its future adjustment. As a result, it is sufficient to describe differences across firms by their productivity a, individual policy state z and capital k. We therefore summarize the joint distribution of firms over (a, z, k) by a probability measure $\mu(a, z, k)$ defined on the Borel algebra $\mathbf{S}$ for the product space $\mathbb{S} = \mathcal{A} \times \mathcal{Z} \times \mathbb{R}_+$.

After observing the aggregate policy state of the economy ζ, and having inherited its capital stock k from the previous period, the firm will first take a new productivity draw, and a new draw of the idiosyncratic policy state z, conditional on a new aggregate policy having been implemented. It then chooses its current level of employment, produces and pays its workers. Next, it draws its fixed adjustment cost ξ. The current period value of a firm before drawing the fixed adjustment cost, can therefore be represented as the expected value of the firm over all possible realizations of ξ:

$$
\begin{aligned}
v(a,z,k;\zeta) &= \mathbb{E}_{\xi}\tilde{v}(a,z,k,\xi;\zeta) \\
&= \int_0^{\bar{\xi}} \tilde{v}(a,z,k,\xi;\zeta)\ G(d\xi).
\end{aligned} \tag{4}
$$

Once the firm observes the draw of the fixed adjustment cost ξ, it decides whether to pay the cost and adjust its capital to the desired level k', or avoid the cost and retain its previous capital stock k. The optimal choice solves:

$$
\tilde{v}(a,z,k,\xi;\zeta) = \quad \max\ \{-\xi + v^A(a,z,k;\zeta)\ ,\ v^{NA}(a,z,k;\zeta)\}, \tag{5}
$$

where v^A and v^{NA} represent the value functions of adjusting and non-adjusting capital, respectively. In turn, given the current aggregate state ζ, the value upon adjustment of capital solves the following dynamic problem:

$$
\begin{aligned}
v^A(a,z,k;\zeta) = \max_{k',l}\ &\Big[\tilde{f}(a,z,k,l;\zeta) - wl - k' + (1-\delta)k - \mathbb{I}(i<0)\,|i|\,\chi \\
&+ \mathbb{E}_{\Gamma_a,\Gamma_z,\Gamma_\zeta}\beta v(a',z',k';\zeta'|a,z,\zeta)\Big]\,,
\end{aligned} \tag{6}
$$

subject to the law of motion for the aggregate policy state Γ_ζ, for idiosyncratic productivity Γ_a, and the individual policy state Γ_z. In the above maximization problem β is the discount factor, $\mathbb{I}$ is an indicator function that takes the value of one if the firm disinvests, and χ is a parameter governing the partial irreversibility of investment (i.e. the resale of capital occurs at a price that is only a share $(1-\chi)$ of its purchase price). The expression in the first line represents the period flow of profits, after substituting investment using the law of motion for capital.

The value function $v^{NA}(a,z,k;\zeta,\mu)$ is instead the solution to the following equation:

$$
\begin{aligned}
v^{NA}(a,z,k;\zeta) = \max_{l}\ &\Big[\tilde{f}(a,z,k,l;\zeta) - wl \\
&+\mathbb{E}_{\Gamma_a,\Gamma_z,\Gamma_\zeta}\beta v(a',z',k',(1-\delta)k;\zeta'|a,z,\zeta)\Big]\,,
\end{aligned} \tag{7}
$$

subject to the laws of motion for the states a, z, and ζ, which implies that the firm neither invests nor disinvests, bringing into the next period its current stock of capital k, after depreciation.

It is then possible to define for every firm a threshold adjustment cost $\xi^T(a,z,k;\zeta)$ such that, for the current period, the value of adjusting its capital stock is equal to the value of not adjusting:

$$
\xi^T(a,z,k;\zeta) + v^A(a,z,k;\zeta) = v^{NA}(a,z,k;\zeta), \tag{8}
$$

or equivalently:

$$
\xi^T(e,z,k;\zeta) = v^{NA}(e,z,k;\zeta) - v^A(e,z,k;\zeta). \tag{9}
$$

In this set-up, the policy function for the next period choice of capital can be represented by the following piece-wise function:

$$K(a,z,k,\xi;\zeta) = \begin{cases} k'(a,z,k;\zeta) & \text{if} \quad \xi < \xi^T(a,z,k;\zeta) \\ (1-\delta)k & \text{otherwise,} \end{cases}$$

which implies that $K(a,z,k,\xi;\zeta)$ is equal to the optimal level of capital if the draw of the adjustment cost is below the threshold, and equal to a fraction $1-\delta$ of the current capital stock otherwise. As a result, the optimal investment decision will exhibit an inaction region for any value of $\xi > \xi^T$. We expect aggregate uncertainty about the future policy state to widen this region along the lines discussed by Bloom (2009), with firms postponing investment, or disinvestment, as they wait for uncertainty to resolve. Finally, we denote by $L(a,z,k;\zeta)$ the policy function for employment[4].

4 Calibration

The model is calibrated at a quarterly frequency. The model comprises three main sets of parameters: those that affect the pre-Brexit steady state of the U.K. economy, those that affect the stochastic process for Brexit and those governing the policy shocks in the post-Brexit steady states. The first set comprises relatively conventional parameter values, which are assigned following the literature. The other two sets of parameters are instead less conventional. In order to assign values we make use of expectations data regarding the timing and the impact of Brexit, which are obtained from the survey of U.K. firms in the Decision Makers Panel. The parameters governing the stochastic process of Brexit are set to the values directly inferred from the survey. The parameters governing the permanent shocks associated with Brexit are instead calibrated to minimize deviations from key moments in this survey. In what follows, we present the DMP data and discuss separately how we tackle the calibration of the three sets of parameter values.

4.1 The Decision Maker Panel

The Decision Maker Panel (DMP) is a new, large and representative survey of U.K. businesses that was launched in August 2016 by the Bank of England, together with Stanford University and the University of Nottingham. This survey was specifically designed to investigate how the uncertainty sparked by the Brexit referendum would affect U.K. businesses. More specifically, the questions ask about: *(i)* self-reported views on the importance of Brexit as a source of uncertainty to each business; *(ii)* uncertainty about the eventual, long term impact on the sales of each business; *(iii)* uncertainty about one-year ahead growth in sales and investment; *(iv)* uncertainty about the timing of Brexit. A particularly appealing feature of this survey is that individual businesses are asked to provide probability distributions about their expectations,

[4]Note the independence of the labour policy function from the fixed costs of adjustment as optimization of labour is a static choice.

and not just the first moment. As a result, the survey incorporates valuable information on the impact that Brexit is expected to have, for each business, across various outcomes of the negotiations.

The sampling pool of firms for the DMP was selected from the registry of all active companies in the U.K., and comprises all those businesses who were not a subsidiary of a U.K. parent company, who had a complete set of company accounts and at least ten employees. For firms that were randomly sampled, the Chief Financial Officer (CFO) was contacted and asked to participate. Those who agreed were asked to fill in a survey on a monthly basis. The survey has a rotating panel structure with each member being asked one-third of the quarterly questionnaire each month, with random ordering of the three monthly questionnaires in each quarter. The average monthly response rate was 53% and has ranged between 40% and 65%.

Descriptive analysis by Bloom et al. (2019) shows that the survey provides a good coverage of the different industries and firm sizes in the U.K. economy. The analysis also reveals that the amount of uncertainty sparked by Brexit has been particularly important for about 40% of firms and not important only for around 15% of firms. Overall, and so far, there have not been large changes in the uncertainty that business face as the Brexit negotiations have evolved, so the high aggregate level of uncertainty has not subsided.

In what follows we discuss the specific questions of the DMP that are key, in the context of our model, to identify the parameters.

4.2 Calibration of the Pre-Brexit Steady State Economy

Technology & Prices			
β	0.995	Annual interest rate of 1.8%	ONS
w	1.621	Labour hours to 33.7%	ONS
α	0.333	Capital share of 1/3	
ν	0.250	DRTS parameter	Bloom et al., 2018
Idiosyncratic Shocks & Adjustment Costs			
ψ	0.950	Persistence of idiosyncratic productivity	Khan and Thomas (2008)
σ	0.022	St. dev. of idiosyncratic productivity	Khan and Thomas (2008)
δ	0.020	Annual depreciation rate of 8.2%	ONS
$\overline{\xi}$	$1.4e^{-4}$	Annual inaction rate of investment of 8%	Khan and Thomas (2008)
χ	0.339	Resale loss of capital	Bloom et al., 2018

Table 1: Calibration of the No Brexit Steady State

In this section we calibrate the steady state of the model prior to the Brexit referendum under the assumption that one period of time equals one quarter. The calibration of the pre-Brexit economy follows the literature on models with heterogeneous firms (see Kahn and Thomas 2008, and Bloom et al. 2018), and Table 1 displays the parameters, their values, as well as the targets of the calibration.

We start by discussing the first set of parameters that affect the technology and prices common to all firms. Using data from the Office of National Statistics (ONS), the prices in the model are calibarted such that, the discount factor β matches an average annual real interest rate of 1.8% for the U.K. between 1992-2015, and the wage is calibrated to obtain labour hours to be 33.7% the time available between 1992-2015 . The parameters of the production function, namely, α and ν are set in accordance with Bloom et al. (2018) and such that we achieve a capital share of a third.

The second set of parameters relates to the the idiosyncratic shocks and capital adjustment costs. As for the parameters governing the persistence and the variance of the stochastic idiosyncratic productivity process, we rely on estimates by Khan and Thomas (2008). The capital depreciation rate δ is fixed to achieve an annual capital depreciation of 8.2% as calculated by the ONS. The upper support of the uniform distribution from which the fixed costs is drawn, $\bar{\xi}$, is set in line with the findings of Cooper and Haltiwanger (2006) and Khan and Thomas (2008), which observe that annually 8% of firms remain inactive with respect to their investment[5]. Finally, the capital partial irreversibility parameter, χ, is calibrated to match a resale loss of capital of 33.9% as found by Bloom (2009) and Bloom et al. (2018).

4.3 Calibration of the Brexit Stochastic Transition Parameters

The stochastic transition set in motion by the Brexit referendum result is governed by the following three parameters: *(i)* the probability that in a given quarter Brexit takes place, θ; *(ii)* the probability that, conditional on Brexit occurring, Brexit is Hard, γ^H; *(iii)* the probability that the decision of the Brexit referendum is reversed, γ^R. We calibrate these parameters using expectation data from the DMP on the timing of Brexit and on the probability that no successful deal is reached (Hard Brexit).

"U.K.'s expected withdrawal date from the E.U., after any transition period, average probability (%)"					
2019	2020	2021	2022	2023 or later	Never
19	18	29	15	9	9

Table 2: Question S.18 from the Bank of England's Decision Maker Panel, Nov. 2017-Jan. 2018

We first calibrate θ and γ^R using question S.18, which asks: *"What do you think is the percentage likelihood (probability) of the U.K. leaving the E.U. (after the end of any transitional arrangements) in each of the following years?:* i*) 2019,* ii) *2020,* iii) *2021,* iv) *2022,* v) *2023 or later,* vi) *Never"*. Table 2 shows the average probability attached to each date. The answers to this question indicate that the probability of the U.K. not having left the E.U. by the end of 2022 is 18%. In turn, this probability comprises a 9% chance of Brexit happening any time after 2022, and a 9% chance of Brexit never taking place, thus capturing the possibility that a second referendum eventually reverses the Brexit decision.

In the model, the probability of the U.K. not having left the E.U. by the end of 2022 can be expressed as the probability of remaining in the announcement state for 26 consecutive quarters

[5]Inactivity in investment is defined as a firm having an annual invetsment rate between -1% and 1% analogously to Cooper and Haltiwanger (2006) and Khan and Thomas (2008).

(from 2016Q3 up to and including 2022Q4):

$$Pr(\text{U.K. remaining in E.U. until end 2022}) = (1-\theta-\gamma^R)^{26}. \tag{10}$$

Moreover, the probability that the U.K. never leaves the E.U. can be represented in the model by the summation of the probabilities that the decision of the Brexit referendum is reversed at any future point in time:

$$Pr(\text{U.K. never leaving the in E.U.}) = \sum_{i=1}^{\infty}(1-\theta-\gamma^R)^{i-1}\gamma^R = \frac{\gamma^N}{1-(1-\theta-\gamma^R)}. \tag{11}$$

Equations (10) and (11) solve for θ and γ^N. Specifically, from eq.(10) we get that $(1-\theta-\gamma^N) = 0.18^{\frac{1}{26}} = 0.9362$. Plugging this result into eq.(11), we can work out $\gamma^R = 0.09 \times (1-0.9362) = 0.0057$. Hence, $\theta = 1 - 0.9362 - 0.0057 = 0.0581$. This means that the average expected probability of Brexit happening over any quarter is 5.81%, whilst the Poisson rate of reversing the outcome of the Brexit referendum is 0.57%.

"What probability, do you attach to a disorderly Brexit, whereby no deal is reached by the end of March 2019?"				
$< 20\%$	$\geq 20-40\%$	$\geq 40-60\%$	$\geq 60-80\%$	$\geq 80\%$
17	31	25	17	10

Table 3: Question S.25 from the Bank of England's Decision Maker Panel, Feb. 2017-Apr.2018

To calibrate the conditional probability of Hard Brexit, γ^H, and the implied probability of Soft Brexit, γ^S, we make use of Question S.25, reported in Table 3. The question asks *"What probability, in percent, do you attach to a disorderly Brexit, whereby no deal is reached by the end of March 2019?"*. Attributing mid-points of $10, 30, 50, 70$ and 90 to the five bins in Table 3, we get that the average perceived probability of Hard Brexit is 44%.

In the model, Hard Brexit is not restricted to take place in a particular date, but follows a Poisson process. The unconditional probability of Hard Brexit happening at any future point in time is given by:

$$Pr(\text{Hard Brexit}) = \sum_{i=1}^{\infty}(1-\theta-\gamma^N)^{i-1}\theta\gamma^H = \frac{\theta\gamma^H}{1-(1-\theta-\gamma^N)}. \tag{12}$$

Given the values for γ^N and θ inferred from Table 2, and making use of $Pr(\text{Hard Brexit}) = 0.44$ from Table Table 3, we can solve eq.(12) for the conditional probability of Hard Brexit, and get $\gamma^H = 0.4835$, which implies $\gamma^S = 0.5165$.

4.4 Calibration of the Brexit Policy Parameters

In this Section we discuss how we assign values to the following nine Brexit policy parameters: *(i)* the aggregate shocks in Soft and Hard Brexit, X^j for $j = S, H$; the positive and negative idiosyncratic shocks x^j_+ and x^j_- for $j = S, H$, the probabilities associated with drawing a positive and aggregate idiosyncratic shock, q_+ and q_-, and the parameter ω, which governs how the intensity of the policy shocks varies with firm-level productivity. To calibrate these parameters we make use of two questions from the DMP survey. The first question asks: *"The Prime Minister has said that the U.K. government does 'not seek membership of the Single Market. Instead we seek the greatest possible access to it through a new, comprehensive, bold and ambitious Free Trade Agreement.' How likely do you think it is that the eventual agreement will have the following effects, compared to what would have been the case had the U.K. remained a member of the E.U., with five scenarios provided about the effect on sales at home and abroad: i) a large positive effect adding* 10% *or more, ii) modest positive effect adding less than* 10%*, iii) make little difference, iv) modest negative effect subtracting less than* 10%*, v) large negative effect subtracting* 10% *or more."* [6]

"Expected impact of Brexit on sales, average probability"			*"Expected impact of disorderly Brexit on sales, average probability"*		
	Data	*Model*		*Data*	*Model*
$< -10\%$	17.5	17.6	$< -10\%$	19.7	19.9
$> -10\%$	27.6	28.9	$> -10\%$	26.9	27.6
$\simeq 0\%$	37.2	36.3	$\simeq 0\%$	43.5	42.4
$< 10\%$	12.3	12.1	$< 10\%$	6.9	7.1
$> 10\%$	5.4	5.1	$> 10\%$	2.9	3.0
	Data	*Model*		*Data*	*Model*
Within firm st. dev. of sales expectations	5.9	6.6	*Elasticity of expected sales to productivity*	-0.37	-0.37

Table 4: Policy Parameters Calibration Moments

The second question asks about the expected effects of Brexit on sales, conditional on the specific case where the U.K. economy exits the E.U. without a deal (Hard Brexit). The question asks: *"The Prime Minister has said that Brexit negotiations will be tough and 'no deal is better than a bad deal'. If the U.K. leaves the E.U. without a deal then there could be an increase in non-tariff barriers to trade with the E.U. (for example from a higher cost of meeting required standards and regulation in E.U. markets, or an inability to acquire the necessary permissions). How likely do you think it is that this outcome will have the following effect on the sales of your business, compared to what would have been the case had the U.K. remained a member of the E.U.: i) a large positive effect adding* 10% *or more, ii) modest positive effect adding less than* 10%*, iii) make little difference, iv) modest negative effect subtracting less than* 10%*, v) large negative effect subtracting* 10% *or more."* In both questions every respondent is therefore asked to attach a probability to each of these five outcomes, thereby providing an individual probability distribution on the expected effect of Brexit on their sales. The first question elicits information on the unconditional distribution, while the second is conditional on Hard Brexit.

[6] Note that the question asks about the expected effects of "the eventual agreement" and not about the effects of a possible Free Trade Agreement, and is therefore intended to capture average individual expectations over all possible Brexit outcomes (cf. Bloom et al. (2018)).

Each question provides us with four independent moments, which can be used as calibration targets. Indeed, we have five aggregate probability bins for each question, but because probabilities must add to one, we can only rely on four independent targets. These four moments in the aggregate distribution about the sales impact of Hard Brexit elicit information on the Hard Brexit policy parameters X^H, x^H_-, x^H_+. Given this information, the probability bins of the unconditional distribution, together with our assumption that there can be only two types of Brexit, Hard or Soft, identify the policy parameters of the Soft Brexit policy, X^S, x^S_-, x^S_+. Together, these two distributions also pin down the probability parameters q_- and q_+.

To elicit information that is useful to identify how the policy shocks correlate with firm-level productivity, as captured by the parameter ω, we make use of estimates by Bloom et al. (2019) on the elasticity of expected sales to firm-level productivity in the DMP panel. Their finding of a negative correlation implies that more productive firms expect heavier losses from Brexit.[7] Finally, we also target the average dispersion of the bins measured in the answers to the first question, the one on the unconditional post-Brexit-sales expectations. Because we have nine parameters for ten calibration targets, our model is over-identified. How well the model is able to capture this measure of idiosyncratic dispersion in firms' expectation is therefore a useful validation test for the assumptions on the policy in our model.

In order to take the model to the data, we compute artificial probability distributions for the firms in our model in the same way as in the DMP survey, that is, we produce five bin histograms, both unconditional and conditional on Hard Brexit. Specifically, we generate a large number of artificial panels of $2,500$ firms, the same size as the number of average respondents in the DMP survey, where every firm (i) is a productivity draw (a_i) from the time-invariant distribution of ability $p(a)$. When computing the unconditional probability distribution, we consider that every firm may end up in different aggregate policy states with probabilities implied by the transition matrix in eq. 3.1. Moreover, conditional on the states of Soft and Hard Brexit occurring, each firm may end up in a different idiosyncratic policy state z_i for $i \in \{z_+, z_-, z_0\}$. We compute the steady state sales changes associated with each of these aggregate and idiosyncratic states, and then add up each probability to the corresponding bin in the histogram. We compute Hard Brexit distribution in a similar way, just noting that the only uncertainty over the states is idiosyncratic, i.e. related to the draw of z_i for $i \in \{z_+, z_-, z_0\}$.

With the individual probability distributions at hand, we then average every bin of the distribution across all firms in every panel, and then across all panels, to produce aggregate moment that correspond to the data reported in Table 4. We calibrate the parameters of the policy using the method of simulated moments (MSM), that is, we minimize the squared deviation of model moments from the targets (see Appendix A for details on the numerical algorithm).

Table 4 reports the moments targeted in our calibration exercise, and their corresponding value in our model. Looking at the data on the expected effect of Brexit on sales, we see that companies placed more weight on Brexit reducing sales than on it increasing them. This is even more so in the case of Hard Brexit. The average unconditional probability attached to Brexit increasing sales was 17.7%, while the average probability of a negative impact was 45.1%; the

[7] We thank the authors for sharing their results.

Aggregate Policy Parameters		*Idiosyncratic Policy Parameters*					*Idiosyncratic Probabilities*	
X^S	X^H	x_+^S	x_-^S	x_+^H	x_-^H	ω	q_+	q_-
0.009	-0.005	0.037	-0.054	0.044	-0.042	-0.003	0.101	0.457

Table 5: Policy Parameters Calibration

average probability of it having no effect was 37.2%. The average probability of experiencing sales losses conditional on Hard Brexit is only slightly larger, 46.6%, but within that we observe a relative increase in the probability of experiencing sales losses above 10%. The probability of experiencing gains from Hard Brexit, 9.8% is nearly half than the unconditional probability. A good chunk of the mass in the unconditional probability distribution assigned to positive effects on sales shifts to the left in the case of Hard Brexit, raising the probability of receiving no material impact, to 43.5%. When mapping the probability distribution of the data to those constructed in the model, we assume that expected changes in sales that are lower than 2% in absolute value correspond to firms reporting "no material impact" in the survey.

The results of the calibration can be observed in Table 4. The model does remarkably well at matching all targeted moments. Even the average standard deviation of the individual histograms is very close to the data.

The calibrated parameter values are reported in Table 5.

5 Quantitative analysis

Equipped with the calibrated parameter values we first work-out the implications for the long-run effects of Hard and Soft Brexit and then analyse the short term effects of the referendum, disentangling anticipation effects from those of economic policy uncertainty.

5.1 The Long-Run Effects of Brexit

	No-Brexit	*Soft Brexit*	*Hard Brexit*
Output	1.091	1.038 *(-4.8%)*	1.000 *(-8.3%)*
labour	0.337	0.320 *(-4.8%)*	0.309 *(-8.3%)*
Investment	0.244	0.232 *(-5.2%)*	0.222 *(-9.3%)*
TFP	1.005	0.994 *(-1.1%)*	0.987 *(-1.9%)*

Table 6: Brexit Steady States Comparison

Table 6 shows the long-run effects of Soft and Hard Brexit implied by the calibrated model. The results reveal that these effects are large, in the order of 8.3% in the case of Hard Brexit and 4.8% in the case of Soft. We note that the result that both Soft and Hard Brexit imply a media/IRFS/PE_RemainIRF.pdf

Figure 3: Economy's Response to Policy Uncertainty Shock

fall in aggregate production, employment, consumption and investment is not implicit in the assumption of the model, rather it is implied by the sales expectations of U.K. businesses.

The large effects associated with Brexit are due to three main reasons reflected in the data from the DMP. Firstly, the skewness of the distribution of sales expectations means that on average firms expect losses with a much higher probability than gains in sales after any Brexit scenario. Secondly, the non-linearity of average expected sales post-Brexit with respect to firm productivity indicates that more productive firms lower sales with a higher probability than less productive firms. Finally, the long-run effects of Brexit worsen in the Hard Brexit case since all firms tend to expect losses with a higher probability should the U.K. fail to reach a deal with the E.U..

The DMP data only allows the model to identify two Brexit scenarios: Hard and Soft. This is a potential limitation as we know that in reality there is a spectrum of possible outcomes of the U.K.-E.U. negotiations that range from a close EEA single market arrangement akin to the Norway-E.U. deal, to a no deal Brexit with WTO arrangements, passing by a Free Trade Agreement similar to the E.U.-Canadian deal. The Hard Brexit scenario neatly corresponds to the no deal Brexit case in the DMP dataset, and the model estimates a fall of GDP very close to the HM Government estimates of a no deal Brexit which range from -9.0% to -6.3% (see HM Government, 2018). However, Soft Brexit is not identified as neatly. Indeed, Soft Brexit in economic and political commentaries has come to represent different Brexit outcomes. Nevertheless for the purposes of this study, Soft Brexit represents the average of the scenarios whereby the U.K. and E.U. reach any deal, ranging from a single market arrangement to a Free Trade Agreement. Indeed, the the long-run estimates of the Soft Brexit scenario fall within the range of estimates from the HM Government assessment, which span from -6.4% (Free Trade Agreement) to -0.9% (EEA arrangement)(see HM Government, 2018).

5.2 The Effects of Brexit-Policy Uncertainty

In order to assess the short-term impact of the Brexit referendum, we trace the response of the model economy to the news under policy uncertainty. Figure 3 shows the responses of key macroeconomic aggregates in percentage deviations from the steady state, for the case in which after 17 quarters, the aggregate policy shock hits the economy bringing it back to steady state. We interpret this case as the result of a second referendum, which reverses the outcome of the first. We choose a 17 quarters time span for the duration of uncertainty as it corresponds to its expected duration, based on the quarterly estimated value of $\theta = 0.0581$. We therefore assume that the Brexit policy, or the second referendum, occur on the 18^{th} quarter. The impulse responses can be interpreted as those of a noise shock, which is defined as a shock to beliefs media/IRFS/PE_RemainIRFtheta.pdf

Figure 4: Economy's Response to Timing Uncertainty

about future TFP that is orthogonal to future fundamentals.

Even if the referendum does not affect fundamentals directly, investment falls on impact. As capital gradually falls, the demand for labour also falls, and so does GDP. The fall in investment in the first year that follows the referendum is about 17%. Interestingly, our model predicts that the impact of the referendum on the economy builds up over time. By the third year, the fall in GDP is around 2% and the cumulated fall over a three year period is around 3%. This exercise suggests that the Brexit referendum has already produced substantial damage to the U.K. economy. These results are in line with the empirical findings by Born et al (2019), who conclude that by the end of 2018 the referendum has caused a cumulative decline of over two percentage points of GDP.

The response of the economy to the news is driven both by an anticipation effect and by policy uncertainty. As a result of the news, firms anticipate that future reforms are likely to lead to a new steady state with lower capital. Hence, they immediately respond lowering their investment. On the other hand, firms also face aggregate and idiosyncratic uncertainty, which implies that they might halt their investment and avoid paying the fixed cost of investment so as to wait for uncertainty to resolve. Both effects work in the same direction, contributing to lowering investment and the capital stock. In order to disentangle the importance of these two forces, we shut down the wait and see channel of propagation by restricting the fixed cost of capital adjustment to equal zero.

Figure 4 shows transitional dynamics with and without capital adjustment costs, under the assumption of policy uncertainty. These are represented by the red line and the blue line, respectively. In the absence of investment frictions investment still falls, but less than in the benchmark case where both propagation channels are active (the red line). The difference between these two paths widens over time. So after a few quarters the effects of uncertainty already dominate those produced by the anticipation channel.

Finally, we investigate the role of timing uncertainty within the model by changing the parameter θ, which captures the period probability of Brexit. Figure 4 also reports transitional dynamics for the case where we halve the value of θ, thereby doubling the expected duration of uncertainty. The results indicate that as it takes longer for uncertainty to resolve, its effects are substantially smaller. Indeed, the cost to wait and see is that firms must produce at an inefficient scale of production as they do not adjust investment to the desired level. This inefficiency cost rises in expectation with the average duration of uncertainty. Hence, everything else equal, the longer the uncertainty, the smaller the effects. This result implies that the effects of uncertainty on investment should increase at times when uncertainty is expected to resolve quickly, i.e. around the cliff edges of negotiations.

media/IRFS/PE_NoIdioIRF.pdf

Figure 5: Effect of Idiosyncratic Uncertainty

media/IRFS/PE_SoftBrexitIRF.pdf

Figure 6: Soft Brexit Simulation

In Figure 5 we analyse the effects of idiosyncratic uncertainty brought about policy uncertainty on the economy's dynamics. In this scenario, the red line represents the same baseline response with policy uncertainty, whilst the blue line now illustrates the economy's response to a policy uncertainty shock without idiosyncratic uncertainty. We can shut off idiosyncratic uncertainty by converting the idiosyncratic policy stochastic process represented by Γ_z into a deterministic one, where firms learn about the firms specific effects of the policy as soon as the announcement is made instead of when the policy is implemented. Again, comparing the dynamics with a scenario where the referendum never occurred, we can see that without idiosyncratic policy uncertainty the losses in GDP are smaller. In fact, the impact on investment is less acute due to the reduced uncertainty brought about by the policy uncertainty shock which leads to a reduction in *'wait and see'* behaviour by firms, highlighted by the slightly muted response of inactive firms.

5.3 Brexit Simulations

In this section we elucidate the transitional dynamics whereby the U.K. decides to implement either Soft or Hard Brexit. In Figures 6 and 7 we illustrate the transitional dynamics under policy uncertainty and under perfect foresight for Soft and Hard Brexit respectively. Looking at the differences between these dynamics, we can analyze the impact of policy uncertainty relative to a case where the U.K. eventually decides to exit the E.U..

The transitional dynamics under perfect foresight show the response of the economy when firms face no policy uncertainty (black dotted line), meaning that firms know what type of policy will be implemented, when it will be implemented, and how it will directly affect them. More specifically, when the E.U. referendum result is announced, firms are communicated whether Brexit will be Soft or Hard, that it will occur on the 18^{th} quarter, and what type of idiosyncratic policy shock (z) they will receive. From Figures 6 and 7, in both the Soft and Hard Brexit case, it is noticeable how upon the announcement of the E.U. referendum result, the economy begins a path of gradual convergence to new respective steady states. As firms learn about the new policy to be implemented, some choose to remain inactive and let capital depreciate and others will actively disinvest. Employment and output slowly start to decrease as the capital stock in the economy falls. When the policy is implemented, the direct effects of media/IRFS/PE_HardBrexitIRF.pdf

Figure 7: Hard Brexit Simulation

the reform kick in and produce a downturn in all the economic aggregates, which will be more severe in the case of Hard Brexit. After the 50^{th} quarter, we can see the economy reach its new steady state.

In contrast, the transitional dynamics under policy uncertainty (red line) detail the reaction of the economy when firms face uncertainty about the type of policy to be implemented, how it will affect each specific firm, and when the policy will be implemented. In this context, the economy upon the announcement of the E.U. referendum result reacts in the same manner as explained in Section 5.2, by severely reducing investment as more firms *'wait and see'* to learn about the outcome of the reform. In comparison to the case without policy uncertainty, we see inaction increasing by 2 percentage points and investment falling by an additional 13 percentage points, which in turn results in a more severe downturn in output and employment, at least in the short-run. However, when firms learn the outcome of the policy in the 18^{th} quarter, the direct effects of the policy cause a sharp drop in labour and output. Notice that there is sudden increase in investment as firms learn about the new steady state, unlike the counter-factual scenario where firms gradually decrease their investments. This occurs as a result of the resolution of uncertainty which spurs some of the pent-up investment by firm which learn that they will benefit from Brexit. The sharp increase in inactivity as uncertainty lifts is due to firm learning they will either be negatively or not affected by Brexit and will want to reduce their capital stock, but due to the capital adjustment costs, just let it depreciate to the new optimal level. Nonetheless, after the a small upturn, the economy begins to converge to the its steady state.

6 Conclusion

Recently the world economy has been rocked by extraordinary episodes of economic policy uncertainty, from the U.K.'s decision to leave the E.U., to the U.S. president's hostility to existing international trade arrangements. This paper provides a structural framework able to study the effects of policy uncertainty in a setting whereby firms face non-convex adjustment costs in investment. Policy uncertainty is modelled as an announcement of potential future implementation of reforms which entail permanent aggregate and idiosyncratic shock to the firms' fundamentals. Using the novel Decision Maker Panel dataset, which provides U.K. firms expectation data surrounding the effects of Brexit on domestic businesses, the structural framework is then utilized to study the long-run effects of Brexit and the short-run effects of Brexit uncertainty.

The structural framework developed in this paper elucidates the importance of policy uncertainty on business cycle dynamics by highlighting the effect of policy uncertainty on firms' investment decisions. As results show, an announcement of the potential implementation of a policy at some date in the future introduces policy uncertainty into the economy which interacts with the non-convex capital adjustment costs and causes firms to adopt a *'wait and see'* attitude, leading to a downturn in investment, labour and output. We show that the effects of policy uncertainty become larger the closer the economy gets to the implementation date because firms continue their *'wait and see'* behaviour until policy uncertainty is resolved. Moreover, we find that timing uncertainty, a key component of policy uncertainty, plays a pivotal role in the dynamics of investment. Indeed, if firms expect policy uncertainty to persist, the fall in investment is attenuated. This is due to the fact that higher policy uncertainty persistence causes the option value of freezing investment to diminish, as firm must produce with a sub-optimal level of capital increases for longer, thus disincentivizing the *'wait and see'* behaviour.

With respect to the quantitative analysis of Brexit, by feeding in firm-level expectation data regarding post-Brexit sales and Brexit uncertainty from the DMP, we estimate that GDP losses associated with Brexit may be large. In fact, GDP is expected to fall by magnitude of 4.8% in the case of Soft Brexit, that is, in the scenario where the U.K. and E.U. reach a deal. More importantly, should the U.K. and the E.U. fail to reach such an agreement, that is, in the case of Hard Brexit, GDP looses are exacerbated as it is predicted to fall by 8.3%.

Furthermore, we also find that the costs relating to the policy uncertainty generated by the outcome of the E.U. referendum have been significant and have caused a downturn in economic aggregates. Investment has fallen by 17% in the year following the referendum, whereas employment and output have decreased by 1%. However, as uncertainty has prolonged, the effects of policy uncertainty have increased and the cumulative losses of output risen to about 3% in the three years after the referendum.

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Appendix A Method of Simulated Moments

The Method of Simulated Moments (MSM) obtains the parameters which minimize the sum of squared residuals between the data moments and the models' moments, which can be represented as:

$$\Theta = \arg\min_{\Theta} \mathbf{d}' W \mathbf{d}, \tag{13}$$

where Θ is a $N \times 1$ vector of parameters, $\mathbf{d}$ is a $M \times 1$ vector of residuals, and W is a $M \times M$ weighting matrix. There is requirement that there are as many parameters (N) and moments (M), that is $N \geq M$. In the case that $N = M$ then the model is just-identified, whereas if $N > M$ the model is over-identified. Note that setting W as a matrix with the reciprocal of the squared data moments on the diagonal and zero elsewhere means that solving Equation 13 is equivalent to minimizing the sum of squared residuals between the data moments and the models' moments. In order to solve the MSM we rely on the root-finding method of Nelder and Mead (1965). Since we use a local root-finding method, we conduct robust checks by altering both the initial starting values and the step factor and we find that results do not change.

Appendix B Computational Strategy

This section describes the numerical methods used to compute the model's steady state, as well as the transitional dynamics under perfect foresight and aggregate uncertainty. However before proceeding, we must describe how the the state space is discretized in order to solve numerically the model.

B.1 State Space Discretization

The model contains a total of four states: idiosyncratic productivity (a), idiosyncratic policy state (z), idiosyncratic capital (k), aggregate states (ζ). The discretization of the four states is as follows:

- The idiosyncratic productivity (a) is discretized into a grid $\mathbf{a} \in \{\overline{a}_1, ..., \overline{a}_{N_a}\}$ containing of $N_a = 15$ log-linearly spaced points.
- The idiosyncratic policy states (z) can be represented in to a grid containing $N_z = 3$ idiosyncratic states (z_+, z_-, z_0) represented by $\mathbf{z} \in \{\overline{z}_+, \overline{z}_-, \overline{z}_0\}$.
- The idiosyncratic capital (k) is discretized into a grid $\mathbf{k} \in \{\overline{k}_1, ..., \overline{k}_{N_k}\}$ containing of $N_k = 25$ log-linearly points between 1×10^{-5} and 5×20^2.
- The aggregate states (ζ) are four ($N_\zeta = 4$): the uncertainty state (ζ^U), the remain state (ζ^R), the Soft Brexit state (ζ^S), and the Hard Brexit state (ζ^H). These aggregate states can be represented into the following grid $\mathbf{Z} \in \{\overline{\zeta^U}, \overline{\zeta^R}, \overline{\zeta^S}, \overline{\zeta^H}\}$.

Overall, the state space used for the numerical method used for the computational purposes of the model is $N_a \times N_k \times N_z \times N_\zeta$, or more specifically $5 \times 25 \times 3 \times 4$.

Furthermore, along with discretizing all the states in the model, one must also discretize the exogenous stochastic processes in the model. There are three exogenous stochastic processes and are discretized as follows:

- The stochastic process of idiosyncratic productivity can be represented by the transition matrix Γ_a of size $N_a \times N_a$ discretized using the Tauchen's method.
- The stochastic process of idiosyncratic policy states can be represented by the transition matrix Γ_z of size $N_a \times N_a \times N_z \times N_z$. Where $\Gamma_z(\zeta' = \zeta^j, z'|\zeta = \zeta^i, z) = \mathbb{I}$ if $i = \{R, S, H\}$ and $j = \{R, U, S, H\}$, as these states do not entail the draw of z. Moreover, if $i = \{U\}$, $j = \{U, R\}$, and $z_n = \{z_+, z_0, z_-\}$: $\Gamma_z(\zeta' = \zeta^j, z'|\zeta = \zeta^i, z) = \mathbb{I}$. However, if $i = \{U\}$, $j = \{S, H\}$, and $z_n = \{z_+, z_0, z_-\}$:

$$\Gamma_z(\zeta' = \zeta^j, z'|\zeta = \zeta^i, z) =$$

$$\begin{array}{cc} \downarrow z, z' \rightarrow & \begin{array}{ccc} z_+ & \quad z_0 \quad & z_- \end{array} \\ \begin{array}{c} z_+ \\ z_0 \\ z_- \end{array} & \begin{pmatrix} q_+ & (1 - q_+ - q_-) & q_- \\ q_+ & (1 - q_+ - q_-) & q_- \\ q_+ & (1 - q_+ - q_-) & q_- \end{pmatrix} \end{array}. \tag{14}$$

- The stochastic process of the aggregate states can be represented by the transition matrix Γ_ζ of size $N_\zeta \times N_\zeta$ where $\sum_{l=1}^{N_\zeta} \pi_{j,l}^{\zeta} = 1$ for all $j \in \{1, ..., N_\zeta\}$. The transition matrix probabilities are displayed in Equation 3.1.

B.2 Steady State

It is usually the case that the steady state of a model is computed abstracting from policy uncertainty. When policy uncertainty is present in the model, one could solve for the *'risky'* steady state where the agents take into account the aggergate uncertainty (see Coeurdacier, Rey, and Winant (2011) for example). However, for the purposes of the exercises undertaken in this study, that is to calibrate the model to the economy pre-policy announcement and to calculate the short-run effects of policy uncertainty and the long-term effects of the policies, it is not necessary to take into account policy uncertainty because in the pre-announcement economy it is assumed that the agents do not expect the aggregate state to change, as well as the fact that terminal aggregate policy state are absorbing Markov states which by definition do not entail any aggregate uncertainty. As such, it is possible to describe the solution algorithm based on value function iteration for the steady state in the following manner, bearing in mind that the notional will reflect that the steady state abstracts from policy uncertainty.

B.3 Steady State Model's Solution Algorithm:

1. Solve the firms' problem using value function iteration given the prices β and w:

   (a) guess an initial value function $V_g(a,z,k)$, typically $V_g(e,z,k) = 0$ is chosen;

   (b) solve for $V^{NA}(a,z,k)$ and $V^A(a,z,k)$ by taking expectations over the exogenous processes of a and z and using $V'(a,z,k) = V_g(a,z,k)$, and obtain the policy functions $K(a,z,k)$ and $L(a,z,k)$;

   (c) using $V^{NA}(a,z,k)$ and $V^A(a,z,k)$ find $\tilde{V}(a,z,k,\xi)$;

   (d) then find the policy function for the fixed capital adjustment cost threshold $\xi^T(a,z,k)$;

   (e) calculate $V(a,z,k)$ by taking expectations of $\tilde{V}(a,z,k\xi)$ over ξ using the threshold $\xi^T(a,z,k)$;

   (f) check whether the absolute deviation in percentage between the guessed value function $V_g(a,z,k)$ and the obtained value function $V(a,z,k)$ is within a preset tolerance, typically $1e^{-6}$. If the absolute deviation smaller than the tolerance then exit the algorithm and save the optimal policy functions ($K(a,z,k)$,$L(a,z,k)$,$\xi^T(a,z,k)$), otherwise update the guess $V_g(a,z,k) = V(a,z,k)$ and repeat (a)- (e) until convergence.

2. Using the policy functions $K(a,z,k)$ and $\xi^T(a,z,k)$ solve for the stationary distribution as a fixed point, defined as $\mu'(a',z',k') = \mu(a,z,k)$, by iterating on distribution of firms over idiosyncratic productivity, idiosyncratic policy, and idiosyncratic capital holdings. In doing so, the transitional probability matrices, Γ_a and Γ_z, for the exogenous processes for a and z respectively are used for the evolution of the distribution:

$$\mu'(a',z',k') = \sum_{a\in\mathbf{a}}^{N_a}\sum_{z\in\mathbf{z}}^{N_z}\mu(a,z,k)\Gamma_a(a'=a_l|a=a_q)\Gamma_z(z'=z_i|z=z_j)\mathbb{I}(k',a,z,k), \tag{15}$$

   where $\mathbb{I}(k',a,z,k) = 1$ if $k' = K(a,z,k)$ and 0 otherwise.

3. Once obtained the stationary distribution it is possible to multiply it by the relevant policy decision to obtain the aggregates K, L, Y, I.

B.4 Transitional Dynamics

In the endeavour of unravelling the short-term effects of policy uncertainty generated by the policy announcement from the long-run effect of the policy, it is required first to compute the transitional dynamics under policy uncertainty. Secondly, compute the transitional dynamics under prefect foresight, that is, without policy uncertainty. Thirdly and finally, it is possible to extract the short-term effects of policy uncertainty for each of the different policy scenarios by comparing the transitional dynamics under policy uncertainty with the counter-factual transitional dynamics under perfect foresight.

B.4.1 Transitional Dynamics Under Policy Uncertainty

We solve for the transitional dynamics under policy uncertainty. The model at hand features policy uncertainty, in the form of a stochastic process of the policy represented by the transition matrices Γ_ζ and Γ_z. In this scenario, the firms face aggregate uncertainty as they do not know what type of policy will be implemented, they will face timing uncertainty as the timing of the policy implementation is unknown, and they will also face idiosyncratic uncertainty as they are unaware of how policies will affect specific firms.

We fix $T = 100$ and $N = 19$. For the transition from ζ^R to ζ^j where $j \in \{S, H, R\}$:

1. Solve the model for the initial steady state (ζ^R) using value function iteration and obtain the initial distribution $\mu_0(a, z, k)$ by solving the fixed-point of the stationary distribution.

2. Solve the model for the all aggregate states with the aggregate policy stochastic process (Γ_ζ) and the idiosyncratic policy stochastic process (Γ_z) and obtain the optimal policy functions $K_t(a, z, k; \zeta^i)$, $Ł_t(a, z, k; \zeta^i)$, and $\xi^T_t(a, z, k; \zeta^i)$ where $i \in \{U, S, H, R\}$ using value function iteration.

3. Using the optimal policy functions and $\mu_{t-1}(a, z, k)$, obtain aggregates and solve for next period distribution $\mu_t(a, z, k)$ for $t = 1, ..., N$ under the aggregate state ζ^U.

4. Again, using the optimal policy functions and $\mu_{t-1}(a, z, k)$, obtain aggregates and solve for next period distribution $\mu_t(a, z, k)$ for $t = N + 1, ..., T$ under the aggregate state ζ^j.

We have use alternative maximum time periods for the algorithm, namely, $T = 200, 300$ and results do not change.

B.4.2 Transitional Dynamics Under Perfect Foresight

The transitional dynamics under perfect information are computed using backward induction algorithm. In this setting, there is no aggregate uncertainty, indeed, in this counter-factual the stochastic aggregate transition matrix is replaced with a deterministic one. We obtain the transitional dynamics for a policy where firms know when the policy will occur (no timing uncertainty), what type of policy will be implemented (no aggregate uncertainty), and how its will affect the specific firm (no idiosyncratic uncertainty). In order to compute these dynamics, we assume that for the first $N - 1$ periods (where N is the period when the policy is implemented) we will solve model imposing the announcement state ζ^R, whist for $t = N, ...T$ (where T is the terminal period) we will solve the model using the policy states ζ^j where $j \in \{S, H, R\}$.

We fix $T = 100$ and $N = 19$. For the transition from ζ^R to ζ^j where $j \in \{S, H, R\}$:

1. Solve the model for the initial steady state (ζ^R) using value function iteration and obtain the initial distribution $\mu_0(a, z, k)$ by solving the fixed-point of the stationary distribution.

2. Solve the model for the terminal steady state (ζ^j) and obtain the terminal value function $V_T(a, z, k)$ using the value function iteration.

3. By backward induction, compute the value function $V_t(a,z,k)$ for $t = T-1, ..., N$ and the optimal decision rules $K_t(a,z,k)$, $L_t(a,z,k)$ and $\xi_t^T(a,z,k)$ for $t = T-1, ..., N$ under the aggregate state ζ^j.

4. Again, by backward induction, compute the value function $V_t(a,z,k)$ for $t = N-1, ..., 1$ and the optimal decision rules $K_t(a,z,k)$ and $\xi_t^T(a,z,k)$ for $t = N-1, ..., 1$ under the aggregate state ζ^R.

5. Using the initial stationary distribution $\mu_0(a,z,k)$, the optimal policy rules, and the transition matrices for the idiosyncratic productivity process (Γ_a), update the distribution $\mu_{t+1}(a,z,k)$ for every $t = 0, ..., T$.

We have use alternative maximum time periods for the backward induction, namely, $T = 200, 300$ and results do not change.