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Expectations switching in a DSGE model of the UK*

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Abstract

Rational expectations (RE) has been dominant both in the economic literature and in the macromodels routinely used in central banks. The RE assumption has recently come under attack as one of the drawbacks of the Dynamic Stochastic General Equilibrium (DSGE modeling) paradigm. This study attempts to investigate whether other ways of modeling expectations would necessarily find a better support in the data.

We investigate the relevance of the RE assumption by introducing regime switching into the expectations formation of an otherwise standard DSGE model by Justiniano and Preston (2010). In our model, expectations switch between RE and Adaptive expectations (AE). The model is estimated on UK data using Bayesian techniques.

By introducing a switching mechanism, the model explains the data better than both the pure RE and the pure AE models. Expectation formation switches to AE during changes in monetary policy and the financial crisis.

The dynamics of the economic system is different under the two expectation regimes. Hence, should the UK economy switch to an AE regime after Brexit, or as a consequence of the COVID-19 pandemic, the dynamics of the economic system could be substantially more uncertain than under RE, given the model.

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It does require maturity to realize that models are to be used but not to be believed.

Henri Theil, *Principles of Econometrics*, p. vi, 1971.

1 Introduction

Dynamic Stochastic General Equilibrium (DSGE) models are currently enjoying a leading role in the world of macroeconomic policy and research. However, after the financial crisis of 2007-08, their usefulness has been questioned due to their difficulty both to forecast the financial crisis and explain the depth and the slow recovery.¹ Expectations are an essential part of the monetary policy transmission mechanism in DSGE models - with rational expectations as the standard assumption on how these are formed. Hence, the rational expectations assumption is one of the aspects of DSGE models that has been subject to criticism after the financial crisis.²³ Under the assumption of rational expectations, the agents — firms, households and policy-makers — have complete knowledge of the economy, including the model and its parameters. Like many other assumptions made in economic models, this is not necessarily compatible with reality in all situations. Depending upon the invariance of the economic environment and the informational resources available, agents might in some situations have limited knowledge about the economy and insufficient ability to understand it, for example due to large shocks, temporary or permanent. Several deviations from rational expectations have been put forward, predominantly from the behavioral, learning and robust control literature.⁴ In addition, the concept of rational inattention has been introduced as the case where individual capacity for processing information is limited (Sims 2003).

A possible explanation of the failure of RE-models to match the inertia of macroeconomic variables, therefore, might be that the agents follow several strategies in their formation of expectations, where the strategy chosen depends upon the information set available. Consider the situation when they have to form their expectations based on limited information about their economic environment. This is where the case of learning expectations enters the picture, where agents are modelled as forming their expectations based on an econometric learning algorithm. Assuming expectations are based on learning, exogenous shocks to the model will lead to revisions of beliefs over time - drawing out the effects of a shock.

Milani (2007) finds that learning generates a persistence in DSGE models that can replace ‘mechanical’ sources of persistence like the above-mentioned habit formation and indexation. Slobodyan and Wouters (2012) use similar learning mechanisms in a larger new Keynesian (NK) model, and finds that learning fits the data equally well or better than rational expectations. Their results indicate that the persistence introduced by the learning process does not systematically alter the estimated structural parameters of the DSGE model, contradicting the claims made by Milani (2007) that habit formation and indexation become insignificant in the presence of learning. They also find that persistence of exogenous shocks mainly remain unchanged. Both these results are based on closed-economy models, using data for the US. Jääskelä and McKibbin (2010) introduces learning in a small open-economy model, estimating it using Australian data. Comparing it with a version with rational expectations, they find that learning can replace some of the structural inertia in the model, and that the impulse response functions in the learning model exhibit more persistence than those of the RE model.

Behavioral and experimental economics also provide evidence against rational expectations. The literature survey on laboratory experiments on expectations in macroeconomics and finance, presented by Assenza et al. (2014), sum up important findings with regards to individuals’ expectations formation. With regard to New Keynesian DSGE models, evidence of individual learning taking the form of switching from one heuristics to another and subsequently heterogeneity in individual expectations are found by Assenza et. al (2012). Heterogeneity in expectations is one of the more popular alternatives to rational expectations, see for instance Galí and Gertler (1999), De Grauwe (2011) and Branch and Evans (2006). In experiments focusing on inflation expectations conducted by Pfajfar and Zakelj (2014), there is evidence of significant heterogeneity in subject’s forecasting strategies. They conclude that for about 40 % of subjects, rationality cannot be rejected, while more than 20 % of subjects are best described by adaptive learning models. However, they conclude that switching between expectations models are better at describing subjects’ behavior.

Analysing the policy implications of behavioral expectations in a multi-country NK currency model, Bertasiute, Massaro, and Weber (2020) find that the policy implications of the behavioral and rational expectations versions

1. See the collection in Vines and Wills (2018), in particular Hendry and Muellbauer (2018), and Linde, Smets, and Wouters (2016) for various degrees of criticisms of DSGE models.

2. See, for example, the critique of Hendry and Mizon (2014).

3. Another aspect is the (lack of) modelling of the financial sector in these models — see for instance Del Negro, Giannoni, and Schorfheide (2015) — which is also the case of the model used in this paper.

4. An extensive survey is given by Woodford (2013). A shorter overview to different approaches to expectations formation is given in Driscoll and Holden (2014).

of the model are similar. However, the behavioral model predicts inflation and output gap one quarter ahead more accurately. In a closed economy NK framework, Hommes, Massaro, and Weber (2019) find that contrary to the standard rational expectations model, a version with behavioral expectations predicts that inflation volatility can be lowered if the central bank reacts to the output gap in addition to inflation. This result is supported by a learning-to-forecast experiment, which also provides additional support to the use of a heuristic switching model for modelling expectations.

From a cognitive neuroscience perspective, Rolls (2019) argues that human decision processes are complex and lists several factors that need to be taken into account in macroeconomics. In particular, he emphasizes a dual system approach to human decision making, with a rational system based on information and an emotional system influenced by noise.

In this paper we use the financial crisis of 2007-08 as an example of an exogenous change in the information set, to analyse whether the expectation formation of agents in an economy shift from one expectations regime to another in the case of an exogenous shock to the economy. To investigate this hypothesis, we estimate a Markov-switching DSGE model (MS-DSGE) for the UK economy using Bayesian methods over the period from the introduction of the independence of the Bank of England (1997Q2) until after the referendum of leaving the European Union (2016Q4).

The implementation of switching expectations is done by building upon the small open economy DSGE model of Justiniano and Preston (2010), introducing Markov-switching in expectations allowing for shifts between rational expectations and simple constant-gain adaptive learning. We want to analyse whether the financial crisis may have induced a switch in expectations, as the large shock to the economy might force the economic agents of the model to alter how they form their expectations of the future⁵.

The form of learning used in this paper, is of the simplest and most well-established form, namely adaptive expectations, a hypothesis of expectations formation tracing back to Irving Fisher (1930) and formally introduced in the 1950s by Cagan (1956), Friedman (1957), and Nerlove (1958).⁶ The hypothesis states that expectations in any period are revised (linearly) in the light of past errors of expectations, i.e. revisions of beliefs. This corresponds to a simple learning rule, where agents adjust their expectations of the future values of the forward-looking variables based on past forecast errors — the simplest example of constant-gain adaptive learning. This differs from statistical learning as in Milani (2007)) and infinite-horizon learning adopted by Preston (2005a, 2005b).

The alternative to rational expectations chosen in this paper is therefore the simplest one possible and should not be interpreted to be the most realistic alternative, but rather as a simplified first attempt.

Note that the way in which the expectations shift is not obvious. A switch from rational to adaptive expectations could explain the slow recovery from the crisis — as the shock made the economy more difficult to predict, and agents then had to observe the development and learn adaptively what to expect of the future. On the other hand, before the crisis there was a period of great moderation possibly making economic agents ‘lazy’ and just expecting the economic variables to stay the way they were. A shift from adaptive to rational expectations during the crisis could then be explained with the crisis forcing economic agents to become more conscious of the effort required to form their expectations - and consequently becoming more ‘rational’.

The expectation formation of the agents might also impact the dynamic properties, and therefore also the forecasting performance of the model. When using a model to forecast, or make scenarios of, an economy subject to unknown future mechanisms, uncertain future, like the effects of Brexit or the present COVID-19 pandemic⁷, it might therefore be relevant take into consideration possible changes in the information sets available for the forecasts. One standard solution is therefore to produce forecasts conditional upon different information sets, like scenarios and structural breaks. Focusing on possible changes in expectations, the approach taken in this paper is therefore to investigate the forecasting properties of the model under different assumptions of the formation of expectations.

The Brexit vote took place on the 23rd of June 2016. We have therefore kept observations from 2017Q1-2019Q1 to use for forecasts, in order to evaluate how the Markov-switching model forecasts the domestic observables for the period after the referendum, allowing for possible changes in expectation formation schemes.

Finally, we also investigate how the two alternative assumptions of expectations formation affects the model forecasts of domestic variables over the period 2019q2-2021q1.

5. Gerdrup et al. (2017) provide a different and interesting take to incorporate the financial crisis by including a large negative, but low probability, shock to consumption demand in the same model. The probability of the shock is endogenously given, and influenced by the financial imbalances in the model. However, where we model the impact of the crisis in such a model, they look at what influences the probability of the crisis happening.

6. An extensive textbook treatment of adaptive expectations is given by Pesaran (1989) and Evans and Honkapohja (2001).

7. This research was completed in October 2019

The rest of the paper is organised as follows. The next section presents the open economy DSGE model. Section 3 introduces the Markov-switching aspects and the expectational regimes of the model in addition to the solution and estimation of the model. In section 5 the main findings are discussed, followed by an evaluation of the dynamic properties of the model, conditional upon the expectational regimes in section 6. Section 7 first evaluates the 1-step-ahead forecasts and then presents the ex-ante dynamic forecasts for the UK economy. The final section concludes.

2 Model setup

The model used as a basis for the analyses, is Justiniano and Preston's (2010) model of a small, open economy (henceforth referred to as JP10), building on the work of Monacelli (2005) and Gali and Monacelli (2005). The model comprises two economies, one small domestic economy and one large foreign economy (the rest of the world), assumed exogenous to the domestic economy. Further, the model allows for incomplete asset markets, habit formation and indexation of prices to past prices.

For completeness, a brief presentation of the log-linearized equations of the model for the aggregate economy follows below. For the derivation of the entire model, the reader is referred to the original paper.

On the demand side, households choose the optimal allocation of consumption and labor to maximize utility subject to a budget constraint. Log-linearisation of the first order conditions yields a variant of the Euler equation with habit formation:

$$c_t - h_c c_{t-1} = \mathbb{E}_t\{c_{t+1}\} - h_c c_t - \sigma^{-1}(1 - h_c)(r_t - \mathbb{E}_t\{\pi_{t+1}\}) + \sigma^{-1}(1 - h_c)(\varepsilon_{g,t} - \mathbb{E}_t\{\varepsilon_{g,t+1}\}) \quad (1)$$

where consumption c_t depends on both next period's and the preceding period's consumption, in addition to next period's real interest rate $i_t - \pi_{t+1}$. σ is the risk aversion parameter and h_c is the level of habit formation, where $h_c = 0$ will result in the usual Euler equation. The notation \mathbb{E}_t for forward-looking terms indicate that the expectations might shift, so the expression for expectations is written out following equation (13). $\varepsilon_{g,t}$ denotes a shock in preferences, and is assumed to follow an AR(1)-process.

On the supply side, there are two types of firms, producers and importers, that employ labor and maximize profits subject to costs and demand. The inflation dynamics for domestic goods results from the optimal price setting rule for producing firms resetting their price in period t and is given by:

$$\pi_{H,t} - \delta_H \pi_{H,t-1} = \frac{(1 - \theta_H)(1 - \beta\theta_H)}{\theta_H} mc_t + \beta [\mathbb{E}_t(\pi_{H,t+1}) - \delta_{H,t} \pi_{H,t}] \quad (2)$$

$$mc_t = \nu y_t - (1 + \nu)\varepsilon_{a,t} + \alpha t o t_t + \frac{\sigma}{1 - h_c}(c_t - h_c c_{t-1}) \quad (3)$$

The model assumes Calvo-prices, where a fraction θ_H of firms resets their prices in period t , while the remaining fraction $0 < \theta_H < 1$ adjust the prices according to price indexation, with δ_H being the indexation parameter. In the case of zero indexation ($\delta_H = 0$), the usual forward-looking Phillips curve applies. Domestic price inflation is consequently decided by current marginal costs mc_t , expectations about inflation in the next period and the most recent observed inflation rate. Marginal costs given in equation (3), depends on output, where ν is the inverse Frisch elasticity of labor supply, the technology disturbance $\varepsilon_{a,t}$, domestic consumption, and the terms of trade.

$$\pi_{F,t} - \delta_F \pi_{F,t-1} = \frac{(1 - \theta_F)(1 - \beta\theta_F)}{\theta_F} \psi_t + \beta [\mathbb{E}_t(\pi_{F,t+1}) - \delta_F \pi_{F,t}] + \varepsilon_{c,t} \quad (4)$$

Equation (4) results from the optimal price setting rule for importing firms resetting their prices in period t . Calvo-pricing and price indexation are assumed also here, and a cost-push shock $\varepsilon_{c,t}$ has been added, meant to capture inefficient variations in mark-ups. Retailers import foreign differentiated goods for which the law of one price holds 'at the dock'. When setting the domestic currency price of the imported goods, they are assumed to be monopolistically competitive, which leads to deviations from the law of one price in the short run. These deviations can be seen as the current marginal cost conditions for the importing firms and are defined as:

$$\psi_t = e_t + p_t^* - p_{F,t},$$

The deviation from the law of one price ψ_t is thus defined as the difference between the foreign consumer price index (p_t^*) in domestic currency (e_t is the nominal exchange rate) and the domestic currency price index of imports ($p_{F,t}$).

The consumer price inflation (π_t) is a weighted average of domestic and foreign inflation, weighted by their respective shares:

$$\pi_t = (1 - \alpha)\pi_{H,t} + \alpha\pi_{F,t}$$

where α can be understood as the openness of the domestic economy. The higher α is, the more weight on the foreign inflation. Terms of trade, tot_t , is defined as the difference between the domestic price level for foreign and domestic goods, given as $tot_t = p_{F,t} - p_{H,t}$. Time differencing the definition of terms of trade yields

$$\Delta tot_t = \pi_{F,t} - \pi_{H,t} \quad (5)$$

Using the expression for terms of trade, the relation between consumer price inflation and domestic inflation can be expressed as:

$$\pi_t = \pi_{H,t} + \alpha\Delta tot_t \quad (6)$$

Further the terms of trade, tot_t , and the real exchange rate, q_t , are related according to

$$q_t = e_t + p_t^* - p_t = \psi_t + (1 - \alpha)tot_t \quad (7)$$

where p_t is the domestic consumer price index. Hence, the real exchange rate are influenced by deviations from the law of one price and also differences in the price indices for the domestic and foreign consumption bundles. The clearing of the goods market requires that domestic production consists of domestic and foreign consumption of domestic goods. Log-linearizing the goods market clearing condition yields:

$$(1 - \alpha)c_t = y_t - \alpha\eta(2 - \alpha)tot_t - \alpha\eta\psi_t - \alpha y_t^* \quad (8)$$

Equation (8) states that equilibrium domestic consumption $(1 - \alpha)c_t$ depends on domestic output (y_t), and three sources of foreign disturbance: the terms of trade (with η being the elasticity between home and foreign goods), deviations from the law of one price and foreign output (y_t^*).

A version of the Taylor (1993) monetary policy rule for the domestic economy is given by

$$r_t = \rho_r r_{t-1} + \phi_\pi \pi_t + \phi_y y_t + \phi_{\Delta e} e_t + \varepsilon_{m,t} \quad (9)$$

The nominal interest rate is determined by past interest rate, in addition to being influenced by current consumer price inflation, output, output growth and the change in the nominal exchange rate. $\varepsilon_{m,t}$ can be interpreted as a monetary policy shock. The uncovered interest parity under incomplete asset markets is given by

$$(r_t - \mathbb{E}_t \pi_{t+1}) - (r_t^* - E_t \pi_{t+1}^*) = \mathbb{E}_t \Delta q_{t+1} - \chi d_t - \varepsilon_{r,t} \quad (10)$$

where $\varepsilon_{r,t}$ is a risk-premium shock and $d_t = \log(e_t B_t / (P_t \bar{Y}))$ is the log real net foreign asset position as a fraction of steady state output. The parameter χ controls the debt sensitivity of the international markets. As foreign variables are assumed to follow a VAR(2)-process (see end of section), π_{t+1}^* will be determined by this process and not switch with the other forward-looking variables.

In addition to choose the optimal allocation of labor and consumption, households also choose to invest either in domestic bonds B_t or foreign bonds B_t^* , yielding an Euler equation for d_t :

$$d_t - \frac{d_{t-1}}{\beta} = y_t - c_t - \alpha(tot_t + \psi_t) \quad (11)$$

which can be interpreted as the evolution of the net foreign asset position.

Equations (1)-(11) are the equations for the domestic block of the open economy model in 10 unknowns $\{c_t, y_t, r_t, q_t, tot_t, \pi_t, \pi_{H,t}, \pi_{F,t}, \psi_t, d_t\}$. The exogenous disturbances

$\{\varepsilon_{a,t}, \varepsilon_{g,t}, \varepsilon_{c,t}, \varepsilon_{p,t}\}$ are assumed to be independent AR(1)-processes, while the monetary policy shock $\varepsilon_{m,t}$ is an i.i.d process.

The foreign block of the model consists of the variables y_t^*, π_t^* and r_t^* , and as in JP10 we here assume their paths to follow a vector autoregressive process of order two:

$$\begin{bmatrix} y_t^* \\ \pi_t^* \\ r_t^* \end{bmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{bmatrix} y_{t-1}^* \\ \pi_{t-1}^* \\ r_{t-1}^* \end{bmatrix} + \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{bmatrix} y_{t-2}^* \\ \pi_{t-2}^* \\ r_{t-2}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{y,t}^* \\ \varepsilon_{\pi,t}^* \\ \varepsilon_{r,t}^* \end{bmatrix} \quad (12)$$

3 The MS-DSGE model

We now extend the model presented in the preceding section. Two features of the model are assumed to be regime-dependent, namely the expectations formation and the smoothing parameter of the Taylor rule.

The original model assumes rational agents, whereas this version is endogenizing expectations formation by Markov transition probabilities, allowing the switching between rational and adaptive expectations formation. The model hence has two regimes, 1 and 2, representing rational (RE) and adaptive (AE) expectations formation respectively. Illustrating with the auxiliary variable z_t , the expectational term can be written out as:

$$\mathbb{E}_t z_{t+1} = \Upsilon(s_t) E_t^{RE} z_{t+1} + (1 - \Upsilon(s_t)) E_t^{AE} z_{t+1} \quad (13)$$

The parameter $\Upsilon(s_t)$ is governed by the 1.order Markov process s_t , and takes the value 1 in the rational expectations regime ($s_t = 1$), and 0 in the adaptive expectations regime ($s_t = 2$).

Letting $p_{ij} = Prob(s_{t+1} = j | s_t = i)$, the transition probability from state i to state j is governed by the transition matrix

$$P = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix} \quad (14)$$

The adaptive expectation formation is set up as follows in log-linear form⁸, letting f_t represent forward looking variables:

$$E_t^{AE} \{f_{t+1}\} = E_{t-1}^{AE} \{f_t\} + \mu_i (f_t - E_{t-1}^{AE} \{f_t\}) \quad (15)$$

where $\mu_i \in [0, 1]$ is the learning parameter. The rate of learning/adjustment rises as μ_i rises, with $\mu_i = 0$ being the case of no adaptive learning. Expectations regarding next period's value of a variable depends on the expected value for this period and adjustments made due to deviations in these expectations from this period's actual value. Solving iteratively we see that $E_t^{AE} \{f_{t+1}\} = \mu_i \sum_{k=0}^{\infty} (1 - \mu_i)^k x_{t-k}$, which is a distributed lag with exponentially declining weights. In JP10 there are five forward-looking variables of which expectations are formed, namely domestic consumption, c_t , domestic and imported inflation, $\pi_{H,t}$ and $\pi_{F,t}$, domestic CPI inflation, π_t , and the real exchange rate q_t . Hence, $f_t = [c_t \ \pi_t \ \pi_{H,t} \ \pi_{F,t} \ q_t]'$ and $\mu_i = [\mu_1 \ \mu_2 \ \mu_3 \ \mu_4 \ \mu_5]'$.

Also the smoothing parameter of the Taylor rule in (9) is allowed to switch following the same Markov-switching probabilities as the expectation formation processes:

$$r_t = \rho_r(s_t) r_{t-1} + \phi_\pi \pi_t + \phi_y y_t + \phi_{\Delta e} e_t + \varepsilon_{m,t}. \quad (16)$$

This is to be able to model that interest rate smoothing became less important during the financial crisis as the lowering of the official bank rate was a mean to deal with the recession.

The $\Upsilon(s_t)$ parameter is calibrated in both regimes, taking the value 1 in the rational expectations regime and 0 in the adaptive expectations regime. $\rho_r(s_t)$ on the other hand, is estimated in both regimes.

The complete model can be more compactly summarised as

$$A_0 x_t = A_1(s_t) x_{t-1} + A_2(s_t) E_t^l \{x_{t+1}\} + A_3 \varepsilon_t \quad (17)$$

for $s_t = 1, 2$, $l = RE, AE$, and where: x_t is a vector of the model variables including the exogenous variables and ε_t is a vector of exogenous shocks.

$$x_t = [c_t, y_t, r_t, q_t, tot_t, \pi_t, \pi_{H,t}, \pi_{F,t}, \psi_t, d_t, y_t^*, \pi_t^*, r_t^*, \varepsilon_{g,t}, \varepsilon_{a,t}, \varepsilon_{rp,t}, \varepsilon_{cp,t}] \quad (18)$$

$$\varepsilon_t = [e_{g,t}, e_{a,t}, e_{rp,t}, e_{cp,t}, \varepsilon_{m,t}, \varepsilon_{y,t}^*, \varepsilon_{\pi,t}^*, \varepsilon_{r,t}^*] \quad (19)$$

Due to the switching environment, standard techniques⁹ to find the solution of the model are no longer applicable. One of the solution techniques implemented in RISE is a functional iteration algorithm described in the next section, which we use to solve our model.

3.1 Solution to the Markov-switching rational expectations problem

If we denote by $p_{r_t, r_{t+1}}$ the transition probability for going from state r_t in the current period to state r_{t+1} in the next period, the generic problem to solve takes the form

8. Lower case variables are defined as $x_t = \log(X_t/X)$, where X is the non-stochastic steady state of the variable.

9. E.g. Sims (2002), Blanchard and Kahn (1980) and Klein (2000).

$$E_t \sum_{r_{t+1}=1}^h p_{r_t, r_{t+1}} [A_{r_t}^+ x_{t+1}(r_{t+1}) + A_{r_t}^0 x_t(r_t) + A_{r_t}^- x_{t-1} + B_{r_t} \varepsilon_t] = 0$$

where h is the number of regimes.

The problem can be reduced to

$$E_t \sum_{r_{t+1}=1}^h A_{r_t, r_{t+1}}^+ x_{t+1}(r_{t+1}) + A_{r_t}^0 x_t(r_t) + A_{r_t}^- x_{t-1} + B_{r_t} \varepsilon_t = 0$$

where $A_{r_t, r_{t+1}}^+ \equiv p_{r_t, r_{t+1}} A_{r_t}^+$

Under the assumption that $E_t \varepsilon_{t+s} = 0$ for $s > 0$, the exact minimum-state-variable solution to this problem takes the form

$$x_t(r_t) = \mathcal{T}_{r_t} x_{t-1} + \mathcal{R}_{r_t} \varepsilon_t \quad (20)$$

Solving the problem therefore implies finding \mathcal{T}_{r_t} and \mathcal{R}_{r_t} .

Now if the solution to the problem is given by equation (20), then it has to be the case that

$$E_t \sum_{r_{t+1}=1}^h A_{r_t, r_{t+1}}^+ [\mathcal{T}_{r_{t+1}}(\mathcal{T}_{r_t} x_{t-1} + \mathcal{R}_{r_t} \varepsilon_t) + \mathcal{R}_{r_{t+1}} \varepsilon_{t+1}] + A_{r_t}^0 [\mathcal{T}_{r_t} x_{t-1} + \mathcal{R}_{r_t} \varepsilon_t] + A_{r_t}^- x_{t-1} + B_{r_t} \varepsilon_t = 0$$

or

$$\left[\sum_{r_{t+1}=1}^h A_{r_t, r_{t+1}}^+ \mathcal{T}_{r_{t+1}} \mathcal{T}_{r_t} + A_{r_t}^0 \mathcal{T}_{r_t} + A_{r_t}^- \right] x_{t-1} + \left[\sum_{r_{t+1}=1}^h A_{r_t, r_{t+1}}^+ \mathcal{T}_{r_{t+1}} \mathcal{R}_{r_t} + A_{r_t}^0 \mathcal{R}_{r_t} + B_{r_t} \right] \varepsilon_t = 0$$

Using the method of undetermined coefficients implies that

$$\sum_{r_{t+1}=1}^h A_{r_t, r_{t+1}}^+ \mathcal{T}_{r_{t+1}} \mathcal{T}_{r_t} + A_{r_t}^0 \mathcal{T}_{r_t} + A_{r_t}^- = 0 \quad (21)$$

and

$$\left(A_{r_t}^0 + \sum_{r_{t+1}=1}^h A_{r_t, r_{t+1}}^+ \mathcal{T}_{r_{t+1}} \right) \mathcal{R}_{r_t} + B_{r_t} = 0 \quad (22)$$

This shows that the solving for \mathcal{T}_{r_t} does not depend on \mathcal{R}_{r_t} . But conditional on knowing $\{\mathcal{T}_{r_t}\}_{r_t=1}^h$, the solution for \mathcal{R}_{r_t} is readily given by

$$\mathcal{R}_{r_t} = - \left(A_{r_t}^0 + \sum_{r_{t+1}=1}^h A_{r_t, r_{t+1}}^+ \mathcal{T}_{r_{t+1}} \right)^{-1} B_{r_t}$$

Solving for $\{\mathcal{T}_{r_t}\}_{r_t=1}^h$ is more involved. A simple technique, which we use in this paper, is presented in Maih (2015) and consists of iterating over the functional equation

$$\mathcal{T}_{r_t} = - \left[A_{r_t}^0 + \sum_{r_{t+1}=1}^h A_{r_t, r_{t+1}}^+ \mathcal{T}_{r_{t+1}} \right]^{-1} A_{r_t}^- \quad (23)$$

until convergence for a given start value $\{\mathcal{T}_{r_t}^0\}_{r_t=1}^h$.

3.2 The dynamics of the expectation formations

To motivate the later results, we illustrate how the two assumptions on expectation formation used in our model might affect the dynamics of the model, using simulated data from a much simpler model as a stylised example. The model consists of three equations for output gap, inflation and interest rate respectively, and is similar to the one presented in Pfajfar and Zakelj (2014) with a few adjustments:

$$y_t = -\phi(i_t - \mathbb{E}\pi_{t+1}) + \mathbb{E}y_{t+1} + u_{y,t} \quad (24)$$

$$\pi_t = \beta \mathbb{E}\pi_{t+1} \lambda y_t + u_{\pi,t} \quad (25)$$

$$i_t = \psi i_{t-1} + \gamma(\pi_t - \bar{\pi}) + v y_t + u_{i,t} \quad (26)$$

where the u 's are exogenous shocks following AR(1) processes and $\bar{\pi}$ is the inflation target. \mathbb{E} denotes that the expectations can be formed either by rational or adaptive expectations as explained in the preceding section. The model is calibrated in two versions, where the only difference is the expectations formation. The learning parameters for the output gap and inflation under adaptive expectations are both set to 0.5. The rest of the parameters are the same in the two model versions. After calibrating and solving the model versions, data are simulated using the same "random" shocks.

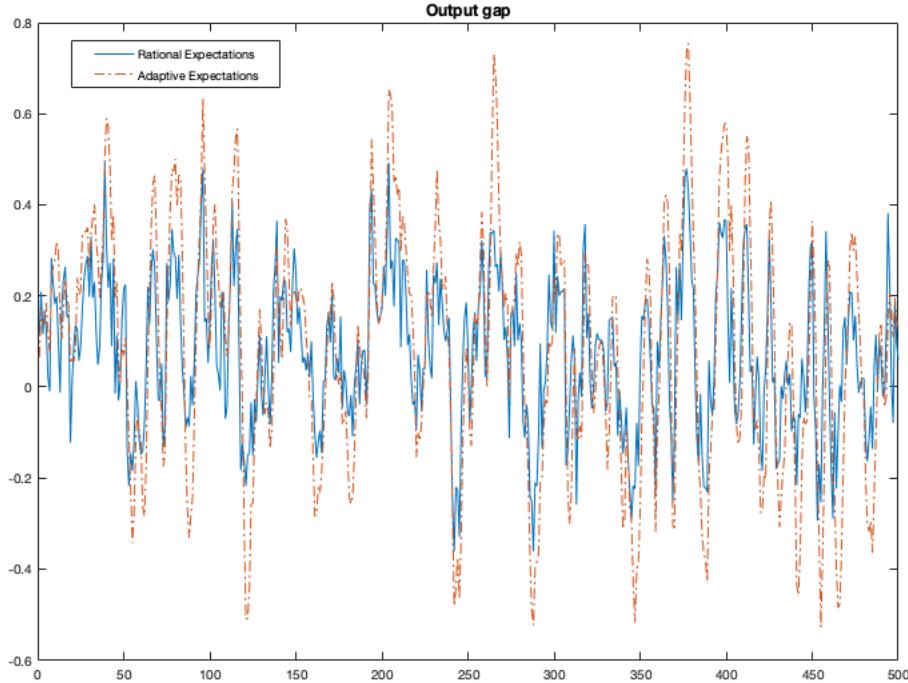


Figure 1: The output gap in the 3-equation model

For all variables, the adaptive expectations formation introduces more volatility, as can be seen from Figure 1 of the output gap. Hence, regime switching in expectations, might be able to capture uncertainty, as measured by volatility, and not only possible heteroscedasticity.

4 Data and estimation procedure

The model is estimated using Bayesian methods on quarterly UK (domestic economy) and US (foreign economy) time series spanning 1997Q2 : 2016Q4, while the observations 2017Q1 : 2019Q1 are retained for forecasting evaluation. The aggregate variables used as observables include output, inflation, interest rate, terms of trade, real exchange rate, foreign output, foreign inflation, foreign interest rate, shown in Figure 2, with the start of the forecasting period indicated by the vertical line. The data have mainly been collected from Macrobond, while the exchange rate data and the data for the foreign economy were taken from the Database at the Federal Reserve Bank of St. Louis. More information on the observables used is found in the appendix.

In May 1997 the Monetary Policy Committee was formed under the independent Bank of England (BoE) and given the responsibility of setting the interest rate. This move of monetary policy from the Treasury to the BoE started a period of increased interest rates in the UK, and is the main reason for starting our sample period in 1997. Throughout most of the sample period, the UK interest rate is higher than the US interest rate as displayed in the lower left panel of figure 2. Both the BoE and the Federal Reserve (Fed) reacted to the financial crisis by lowering their rates. The Fed lowered its base rate to 0.25%, which is effectively zero - and kept it there until 2015. During the same period, BoE did not go lower than 0.5%. However, this changed in August 2016, when BoE set its base rate to 0.25% to boost quantitative easing after the Brexit vote two months earlier.

The top left panel of figure 2 shows the growth rates of the two countries' real GDP per capita. The series are rarely corresponding, except for the financial crisis when both series show a negative growth rate of approximately -2.4% in the last quarter of 2008. The growth rates are mainly positive, with a couple of other exceptions with negative growth. For instance is there a couple of quarters in the early 2000s with negative growth in the US, presumably corresponding to the dot-com crash. Regarding the consumer price (CPI) inflation rates

displayed in the top right panel of figure 2, they are mainly moving around a mean of approximately 0.5% corresponding well to an annual inflation target of 2%. The financial crisis led to a much steeper drop in the CPI of the US than of the UK (-3.48% to -0.70%).

The lower right panel of figure 2 displays the real exchange rate and the terms of trade variables. The real exchange rate shows some large fluctuations around the financial crisis. The British pound is first appreciated relatively to the U.S. dollar, presumably due to the fact that the Fed started to lower its base rate already in late 2007 while the BoE first cut its interest rate in late 2008. This is reflected in the real exchange rate growth which increases drastically before becoming negative for a short period during the financial crisis. Another curiosity of the real exchange rate growth rate plot, is the sharp decrease immediately following the Brexit vote in 2016. The terms of trade depicted in the same panel is relatively stable throughout the series, with the exception of the beginning of the sample period. The large fluctuations in the beginning of the sample of the terms of trade are caused by changes in the export price deflator - possibly reflecting the uncertainty following the Asian financial crisis of 97.



Figure 2: The observables of the model, seasonally adjusted quarterly data

The 8 observable variables are represented by the following vector

$$z_t = \begin{bmatrix} \Delta \log(rGDP)_t \\ \Delta \log(CPI)_t \\ \Delta \log(\text{Real exch. rate})_t \\ \Delta \log(\text{Terms of trade})_t \\ LIBOR_t/4 \\ \Delta \log(rGDP)_t^* \\ \Delta \log(CPI)_t^* \\ FEDFUNDS/4 \end{bmatrix}$$

which is connected to the model state variables through a measurement equation of the form

$$z_t = Hv_t \quad (27)$$

where no measurement errors are needed, as the eight stochastic shocks of the model help avoid the potential problem of stochastic singularity.

The solution of the model combined with the measurement equation (27) form the regime-switching state-space form. We apply a modification of the Kim and Nelson filter¹⁰ to this state-space form to compute the (approximate) likelihood of the data given a parameter vector θ , which includes both the structural model parameters and the parameters pertaining to the transition matrix¹¹.

The likelihood is then combined with a prior distribution on the parameters to form the posterior kernel that is maximized to find the mode of the posterior distribution. For this optimization process we use a trust-region reflective algorithm based on the method described in Coleman and Liu (1994,1996).

We take a sample from the posterior distribution using a standard Markov Chain Monte Carlo (MCMC) technique, namely the Metropolis-Hastings algorithm.

To evaluate the model fit to the data and to compare it to rival models in a Bayesian context, one needs the marginal data density defined as

$$p(Y_T) \equiv \int p(Y_T|\theta) p(\theta) d\theta \quad (28)$$

All those algorithms mentioned are coded in the RISE toolbox, which we use for implementation.

4.1 Prior information

The priors are mainly the same as those used by Justiniano and Preston (2010), and described in the four first columns in Table 1. The discount rate and the debt sensitivity parameter β and χ are calibrated and set equal to 0.99 and 0.01 respectively. For the inverse Frisch elasticity and the elasticity of substitution between home and foreign goods, loose Gamma priors with large tails are adopted. The risk aversion parameter is also given a Gamma prior, accommodating values from 0.5 to well above 1, as all are plausible estimates in a model of this type. Parameters that are restricted to lie between 0 and 1 are mainly given Beta priors. For the learning parameters, the Beta priors are in accordance with the work of Levine et al. (2012), while the other Beta priors are the same as in Justiniano and Preston (2010). The switching probabilities are given a uniform distribution, restricting the values to be between 0 and 1. The standard deviations of the shocks are given an inverse gamma distribution with a mean of 0.5 and infinite variance (by fixing the degrees of freedom to 2), reflecting that we have little prior information on the shocks. Regarding the VAR(2)-priors which are not included in table 1, we also here follow Justiniano and Preston (2010), as they also use US data for the foreign economy.

5 Results

The switching model is estimated as described earlier in RISE, finding a posterior mode. Following the estimation, a MCMC procedure is initiated with the inverse Hessian estimated at the posterior mode. Two chains of 200,000 draws are simulated with 10,000 initial draws discarded and every 20th draw afterwards saved for a total volume of 10,000 draws for each block. The two chains converged towards the same mean, which together with standard deviation and the 90% credibility intervals are reported in Tables 1 and 2. Convergence diagnostics are found in the appendix, with plots of prior and posterior distributions, trace plots, recursive mean plots in addition to Gelman-Rubin plots.

Figure 3 shows the data together with shadowed fields for the smoothed regime probabilities of the AE-regime. The model is in the adaptive expectations regime in the start of the sample (from 1997Q2 to 1998Q2) and during the financial crisis in the last quarter of 2008 and the first quarter of 2009. The shift corresponds well to a sharp decrease in the UK interest rates. From October 2008 to January 2009 the 3-month LIBOR decreased from 4.82% to 1.17%. In figure 3, it is also shown that the growth of both UK and US real output are at their lowest at the time of the shift to the AE-regime. The real exchange rate growth is also unusually high, indicating that the British pound weakened against the US dollar due to the crisis.

The transition probabilities p_{12} and p_{21} are estimated to be 0.02 and 0.30 respectively. This indicates that the probability of staying in the RE-regime is 0.98, and the probability of leaving the RE-regime cannot be said to be significantly different from zero. As can be seen from Figure 3 however, the model is in the AE-regime two

10. See the appendices.

11. The difference between the Kim and Nelson filter and the filtering algorithm used in this paper resides in the collapsing rule. The latter algorithm is computationally more efficient but yields the same results as the Kim and Nelson algorithm

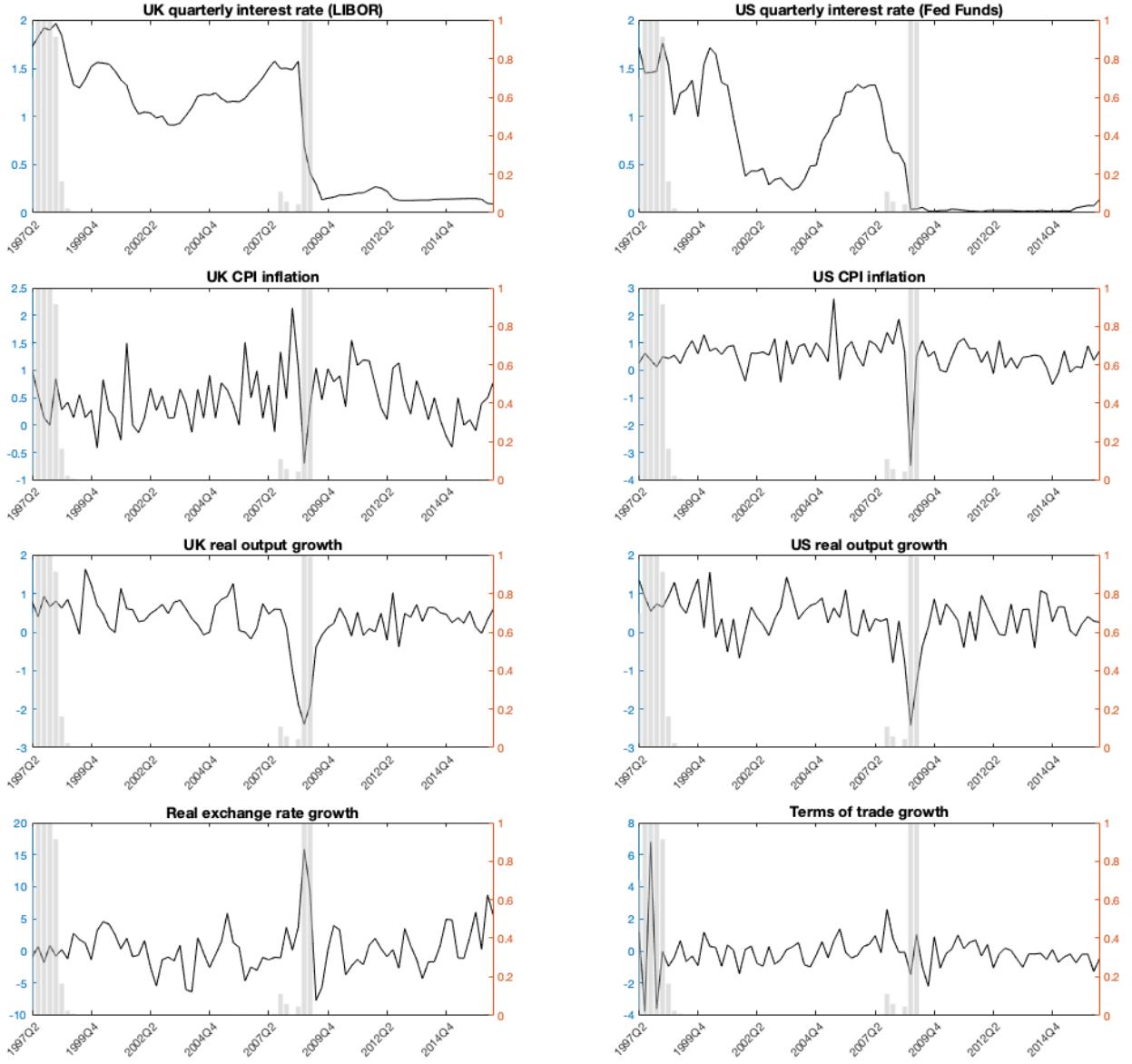


Figure 3: Data (left-hand scale) and regime probability (right-hand scale) for the AE-regime.

times during the sample period. The regime-dependent smoothing parameter in the Taylor rule is estimated to be relatively higher in the RE-regime than in the AE-regime with mean values of 0.93 and 0.64 respectively. This reflects that the Bank of England lowered its interest rate to deal with the recession following the financial crisis.

Some of the (non-switching) parameters are unusually low. The openness parameter α is estimated to be 0.05.¹² The risk aversion parameter σ is estimated to 0.28, which is somewhat lower than what is usual (0.5-1.0). The inverse Frisch elasticity of labor supply ν is 3.19 with a rather large credible interval. The elasticity between home and foreign goods, η is 1.98. The estimates for the Calvo-parameters are 0.30 and 0.76 for domestic and import prices respectively. This indicates price contract durations of approximately 1.5 and 4 quarters.

The habit parameter h_c is not significantly different from zero, estimated to be 0.05, with a standard error of 0.04. As can be seen from the posterior distribution of h_c in figure 10, most of the distribution is concentrated close to zero. The two other potential sources of persistence in the RE model, the price indexation parameters δ_H and δ_F have mean estimates 0.22 and 0.03, respectively. As with the habit-parameter, the price indexation parameters are not significantly different from zero with their large standard errors. The Taylor-parameters for output and exchange rate are estimated to be around 0.10 and 0.06, respectively. The learning parameters range between 0.30-0.90, introducing varying degrees of persistence to the model when it shifts to the AE-regime. The

12. This parameter was calibrated in Justiniano and Preston (2010). We found that calibrating α led to the risk aversion parameter σ to be close to zero, and hence chose to estimate both.

Table 1: Prior densities and posterior estimates of structural parameters for the RS-DSGE model

Parameters		Prior			Posterior			
		Prior density	Mean	SD	Mean	SD	5%	95%
Openness parameter	α	Beta	0.50	0.20	0.05	0.02	0.02	0.10
Risk aversion	σ	Gamma	1.20	0.40	0.28	0.10	0.14	0.46
Inverse Frisch	ν	Gamma	1.50	0.75	3.19	0.95	1.79	4.87
Calvo dom. prices	θ_H	Beta	0.50	0.10	0.30	0.06	0.20	0.40
Calvo import prices	θ_F	Beta	0.50	0.10	0.76	0.04	0.70	0.82
Elasticity H-F goods	η	Gamma	1.50	0.75	1.98	0.65	1.02	3.15
Habit	h_c	Beta	0.50	0.25	0.05	0.04	0.006	0.12
Indexation domestic	δ_H	Beta	0.50	0.25	0.22	0.17	0.03	0.57
Indexation foreign	δ_F	Beta	0.50	0.25	0.03	0.02	0.003	0.08
Taylor smoothing RE regime	ϕ_{r1}	Beta	0.50	0.25	0.93	0.05	0.84	0.99
Taylor smoothing AE regime	ϕ_{r2}	Beta	0.50	0.25	0.64	0.20	0.27	0.91
Taylor inflation	ϕ_π	Gamma	1.50	0.10	1.51	0.10	1.36	1.68
Taylor output	ϕ_y	Gamma	0.25	0.13	0.13	0.06	0.05	0.24
Taylor exch.rate	$\phi_{\Delta e}$	Gamma	0.25	0.13	0.06	0.02	0.03	0.09
Learning consumption	μ_1	Beta	0.50	0.20	0.81	0.11	0.61	0.95
Learning CPI inflation	μ_2	Beta	0.50	0.20	0.30	0.13	0.13	0.52
Learning dom. inflation	μ_3	Beta	0.50	0.20	0.51	0.19	0.20	0.83
Learning import inflation	μ_4	Beta	0.50	0.20	0.90	0.05	0.81	0.96
Learning real exch. rate	μ_5	Beta	0.50	0.20	0.70	0.08	0.56	0.81

^a Calibrated $\beta = 0.99$ and $\chi = 0.01$.

mean estimates of learning for CPI inflation and domestic inflation μ_2 and μ_3 are both very imprecise, with the 90% credible interval ranging from 0.13 to 0.52 for the former and from 0.20 to 0.83 for the latter. Figure 10) also show that the posterior distribution for μ_2 has its mean to the right of its mode which is as low as 0.2.

Regarding the estimates of the foreign block, the priors were based on estimating pre-sample individual autoregressions for the three variables y_t^* , π_t^* and r_t^* (see Justiniano and Preston (2010)). The posteriors for the first lags of the VAR(2)-estimates in figure 10 are identical to the priors, indicating that the dataset did not bring additional information to the estimation. The other exogenous shocks are mostly highly persistent, with estimates ranging from 0.63-0.72. This excludes the import cost-push shock parameter, which is estimated to as low as 0.05, not significantly different from zero. The data otherwise looks informative about the volatility of all the exogenous shocks, judging from Figure 10 in the Appendix.

In addition to the suggested model with switching in expectations, we have also estimated the rational expectations model version as well as a model version with adaptive expectations. The filtered outcomes from the various versions, together with the observables, are shown in Figure 4. With the exception of UK inflation, the filtered model seems to be tracking the observables quite well. This lack of fit of the inflation specification corresponds well with the the insignificance of the indexation parameter δ_H and the very low estimate of the openness parameter α and could therefore be a source of uncertainty when using the model for forecasting.

Table 3 reports the log marginal data densities (MDD) using different algorithms, including Modified Harmonic Mean, Chib-Jeliazkov, Reciprocal Importance Sampling, Laplace approximation using the normal distribution and Laplace approximation using the covariance matrix of the MCMC draws. For all the different measures of log-MDDs used here, the switching model is preferred to the two alternatives.

6 Impulse response functions

Figures 5 and 6 shows the impulse responses following a negative monetary policy shock and a negative technology shock under the two regimes. Variables respond similarly in the different regimes, but in the adaptive expectations regime the responses are much stronger. For the monetary policy shock in figure 5, the interest rate is also responding differently in the two regimes. The shock is negative, but in the AE regime, the domestic interest rate is responding positively. Due to the equations for adaptive expectations, there is more persistence in the AE-regime. This makes it necessary for variables to respond stronger than in RE-regime to the same

Table 2: Prior densities and posterior estimates of structural parameters for the RS-DSGE model (continued).

Parameters		Prior			Posterior			
		Prior density	Mean	SD	Mean	SD	5%	95%
Technology	ρ_a	Beta	0.80	0.10	0.72	0.08	0.58	0.85
Preferences	ρ_g	Beta	0.80	0.10	0.92	0.02	0.89	0.94
Risk premium	ρ_{rp}	Beta	0.80	0.10	0.63	0.08	0.49	0.76
Import cost-push shock	ρ_{cp}	Beta	0.50	0.25	0.05	0.04	0.01	0.13
1st lag y^*	a_{11}	Normal	0.90	0.01	0.90	0.01	0.88	0.91
1st lag π^*	a_{22}	Normal	0.59	0.04	0.53	0.04	0.46	0.59
1st lag r^*	a_{33}	Normal	0.90	0.01	0.90	0.01	0.88	0.92
sd foreign output	σ_y^*	Inv. Gamma	0.50	inf.	0.65	0.05	0.57	0.75
sd foreign inflation	σ_π^*	Inv. Gamma	0.50	inf.	0.77	0.07	0.67	0.89
sd foreign interest rate	σ_r^*	Inv. Gamma	0.50	inf.	0.16	0.01	0.14	0.19
sd technology	σ_a	Inv. Gamma	0.50	inf.	0.43	0.04	0.36	0.50
sd preferences	σ_g	Inv. Gamma	0.50	inf.	1.68	0.30	1.28	2.25
sd Taylor rule	σ_m	Inv. Gamma	0.50	inf.	0.75	0.08	0.63	0.89
sd risk premium	σ_{rp}	Inv. Gamma	0.50	inf.	1.05	0.22	0.72	1.44
sd import cost push	σ_{cp}	Inv. Gamma	0.50	inf.	1.41	0.20	1.12	1.76
Transition probability, from RE to AE	p_{12}	Uniform	0.00	1.00	0.02	0.01	0.005	0.04
Transition probability, from AE to RE	p_{21}	Uniform	0.00	1.00	0.30	0.17	0.08	0.62

^a For the uniform distribution min and max value for the prior density is reported.

Table 3: Comparing model fit

Algorithm	AE model	RE model	Switching model
Chib-Jeliazkov	-848.73	-727.42	-696.07
Laplace	-826.56	-722.76	-685.26
Laplace MCMC	-829.4	-723.74	-685.66
Modified Harmonic Mean	-821.76	-714.51	-677.11
Reciprocal Importance Sampling	-839.97	-714.4	-677.01

shocks. In the UIP condition given in equation (10), there is learning in the domestic CPI inflation and the real exchange rate variable. A monetary policy shock and a lower interest rate can normally be ‘neutralised’ in the UIP condition by reducing the expected CPI inflation or expected real exchange rate for the next period. Due to the learning mechanisms in the adaptive expectations regime, the expectations cannot simply be lowered as easily, as they depend (rather strongly) on the expectation past period. A negative monetary policy shock ceteris paribus in the AE-regime will therefore force the interest rate up to keep the UIP condition stable. As can be seen from the impulse responses, also the CPI inflation and real exchange rate react positively and stronger than in the RE regime, allowing for the interest rate to become positive the next period (before moving back to its steady state).

Another difference between the two regimes is found in the Taylor rule, where the parameter for the lagged interest rate is higher in the RE-regime. The combination of the stronger response in the domestic CPI inflation in the AE-regime, and the higher degree of interest rate smoothing in the RE-regime is also likely contributing to the opposite reactions in the interest rate in the two regimes.

In many cases the dynamics of the system, following a shock in the adaptive expectations regime are more complex. For instance, the terms of trade variable initially responds negatively to the negative monetary policy shock in both regimes. A negative monetary policy shock normally leads to a lower interest rate, higher inflation and hence the terms of trade also responds negatively. The lower interest rate leads to a depreciation of the domestic currency, so the interest rate will be raised again to prevent this and the terms of trade will thus move back to its steady state in the RE-regime. In the AE-regime however, where the initial response in the interest rate was positive, the terms of trade variable is oscillating towards its steady state.

Regarding the negative technology shock, the response of the domestic GDP per capita is similar for both regimes. The technology shock is included in the equation for marginal costs (see equation (3)), which are increased following a negative shock to $\epsilon_{a,t}$. This also leads to an increase in the domestic goods inflation and consequently the domestic CPI inflation, through equations (2) and (6). As with the monetary policy shock, the response in domestic CPI inflation is stronger in the AE-regime than in the RE-regime. This also affects the response in the domestic nominal interest rate. As mentioned above, the smoothing parameter is higher in the RE-regime. This makes the already smaller response in π in the RE-regime also affecting the interest rate

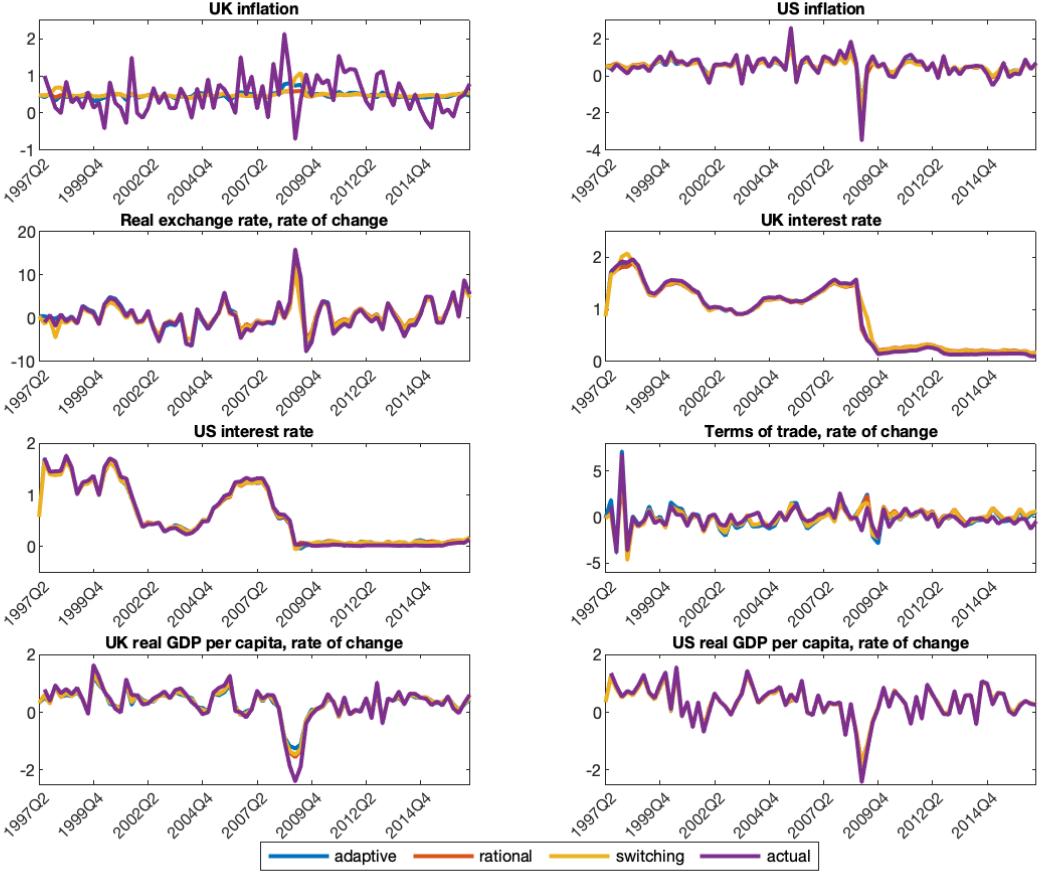


Figure 4: Filtered variables from different model versions with the actual observables

to a lesser extent than in the AE-regime.

As with the monetary policy shock and the domestic interest rate, the technology shock leads to opposite responses in the imported inflation. In the AE-regime, the domestic goods inflation reacts more on a technology shock due to the adaptive expectations equation for π_H . This requires stronger responses also in the real exchange rate, terms of trade and the law of one price gap through the equations (5) and (7). Through the price equation for imported inflation this initially decreases the imported inflation in the AE-regime. In the RE-regime however, domestic goods inflation does not react as strongly, and therefore the responses of the real exchange rate, terms of trade and the law of one price gap are also relatively modest. Therefore, the response of the imported inflation is mainly derived from the equation for terms of trade (5), and hence mostly positive.

The response of imported inflation π_F to a technology shock is another example of a variable oscillating towards its steady state in the AE-regime. The variable has an initial negative response to a technology shock, before becoming positive. As can be seen from the price-setting mechanism of the imported price inflation, it is forward-looking and hence following a learning mechanism in the AE-regime. The lower the learning parameter is, the longer time the variable will use to get back to its steady state. If the learning parameter is low enough, it might never get back to its steady state after a shock.

In Figure 7, we include an exercise with a combination of negative shocks to technology, preferences, foreign output, foreign inflation, and foreign interest rate, qualitatively trying to capture some elements of the current economic situation. The responses of output growth are surprisingly similar in the two regimes, while the effects on inflation and interest rate are stronger in the adaptive regime. As described earlier, the main differences of responses due to expectations formation is to be found in the exchange rate, as represented in the behaviour of imported inflation, the real exchange rate, and the terms of trade. Even though there is a negative impulse to foreign inflation, imported inflation increases initially due to a large depreciation of the exchange rate, leading to oscillations in both real exchange rate growth and terms of trade growth rates, which do not seem to go to zero, and thus indicating an unstable regime.

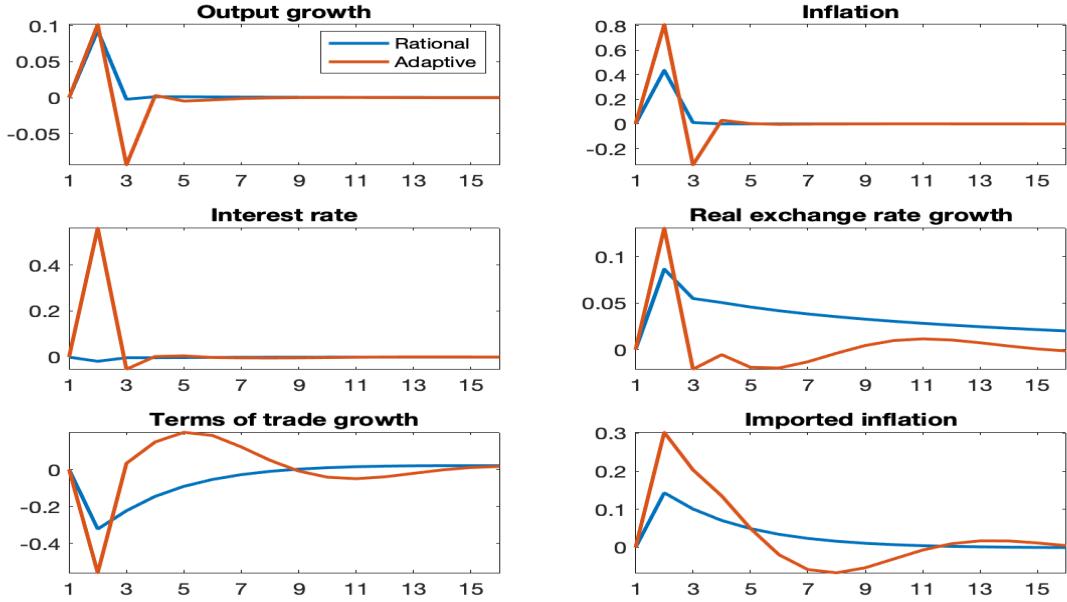


Figure 5: Impulse responses of a negative monetary policy shock.

In general, the volatility of the estimated AE-regimes indicate that the model cannot stay in that regime for too long, which is also reflected in the estimated probability of staying in that regime ($p_{22} = 0.70$ corresponding to 9 quarters).

7 Forecasts

7.1 1-step ahead forecasts

The Brexit vote took place on the 23rd of June 2016. We have therefore kept observations from 2017Q1-2019Q1 to use for forecasts, in order to evaluate how the Markov-switching model forecasts the domestic observables for the period after the referendum. The forecasts are 1-step ahead, so the model is re-estimated for each quarter and a forecast for the next period is made.

The distributions of the forecasts are generated by drawing parameters from the posterior distribution, using 1000 replications for each time period. Of particular interest in the present setting is to allow for changes in expectation formation as new information accrues when forming the 1-step-ahead forecasts. We therefore use forecasts with uncertainty in regimes and shocks, dependent upon transition probabilities and shock distributions, in addition to allowing for parameter estimation uncertainty.

Not surprisingly, therefore, the 1-step ahead forecasts, shown in Figure 8 are characterised by substantial uncertainty, as documented by the prediction intervals. However, using this model, we find that the Brexit referendum result in 2016 has not lead to a new shift from rational to adaptive expectations as of 2019q1, as shown in Figure 14. Also, the smoothed regime probabilities are invariant to the extension of the sample, indicating that the earlier result of shifts from rational expectations to adaptive expectations during the change in monetary policy and the financial crisis is robust.

This is perhaps not so surprising, since only the real exchange rate seem to have been heavily influenced by the outcome of the Brexit-vote, as shown in Figure 8.

7.2 Ex-ante dynamic forecasts

The actual consequences of the Brexit vote were still not clear by the end of 2019. Given the uncertainty of the outcome and the resulting economic environment, the model was used to forecast the period post Brexit. We therefore computed two sets of dynamic forecasts for the period 2019q2-2020q4, conditional upon the two regimes of rational and adaptive expectations formation. The main results are reported in Figure 9. Regardless of regime,

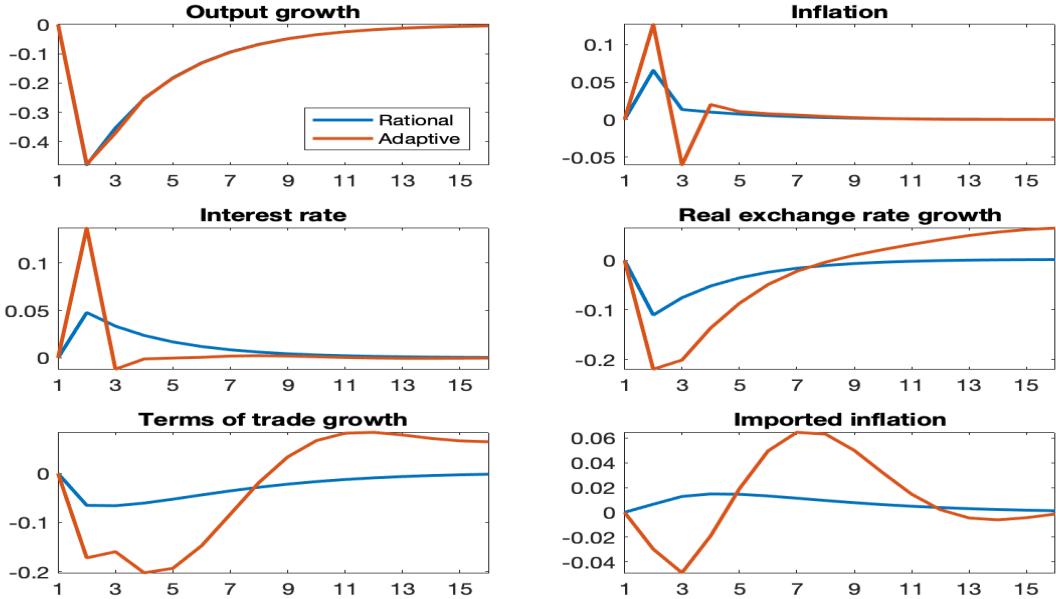


Figure 6: Impulse responses of a negative technology shock

the model forecasts suffer from considerable uncertainty. The main difference between the two regimes, however, is that the adaptive expectations regime induces higher uncertainty with often substantially wider prediction bands for the forecasted outcomes, corresponding to less information about the economic environment. This is in particular evident for variables including the exchange rate, with a substantial higher prediction uncertainty affecting both real exchange rate and terms of trade growth under an adaptive expectations regime.

8 Conclusions

We have used the past financial crisis to investigate the possibility of allowing for switching in expectation formation in a DSGE model. The model we used is based on the work of Justiniano and Preston (2010), where we introduced Markov switching between rational expectations and adaptive expectations. The model has then been estimated using U.K. and U.S. data to analyse whether an economic shock in the scale of the past financial crisis or the possible consequences of Brexit could induce a shift in the expectation formation.

When estimating the model on a sample from 1997Q2-2016Q4, we find evidence of switching, in particular during the financial crisis. Based on the log marginal data densities, using different algorithms, our model also was an improved fit compared to non-switching model alternatives. The model is mainly in the rational expectations regime, but switches to the adaptive expectations regime during the financial crisis possibly reflecting less information about the economic environment.

The switching of expectations is also robust to an expanded estimation sample, where we have estimated the model adding more observations at the end of the sample from 2016Q4-2019Q1.

We have also investigated the properties of the model in terms of impulse responses and forecasting in the different expectations regimes. The impulse response functions indicated that the adaptive expectations regime is more volatile and that the learning mechanism might prevent variables from returning to their steady states after a shock. In the dynamic forecasting exercise, there were considerably higher uncertainty associated with the forecasts in the adaptive expectations regime, being in line with an interpretation of lack of information.

The alternative hypothesis to rational expectations chosen in this paper is the simplest one possible and should therefore not be interpreted to be the most realistic alternative, but rather as a simplified first attempt. It would therefore be of interest to investigate both more realistic alternatives to expectation formation in a switching environment, as well as alternative proxies for information switching mechanisms.

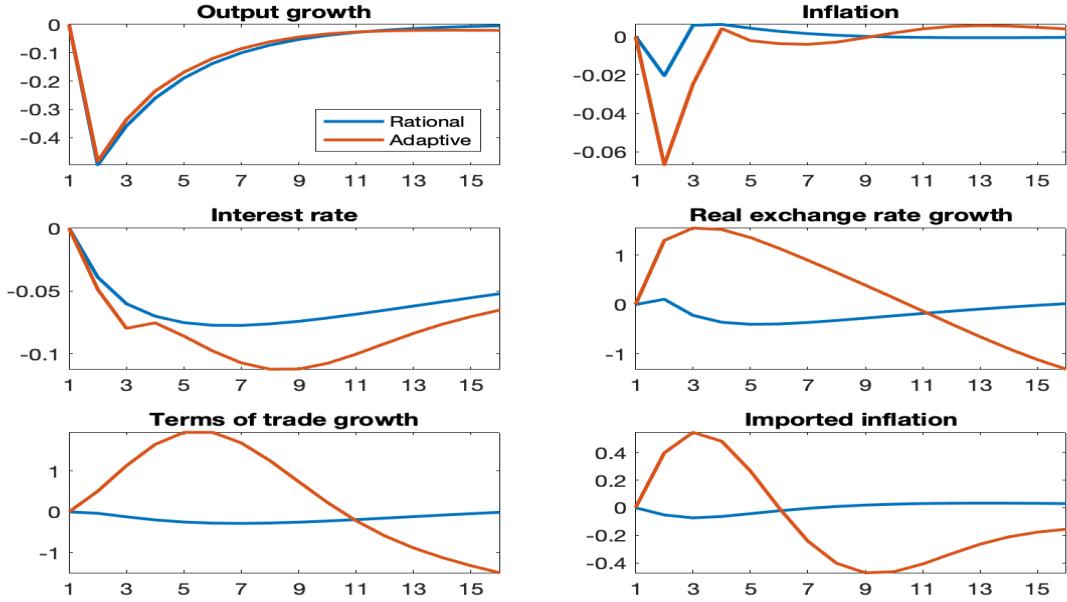


Figure 7: Impulse responses of negative shocks to technology, preferences, foreign output, foreign inflation, foreign interest rate.

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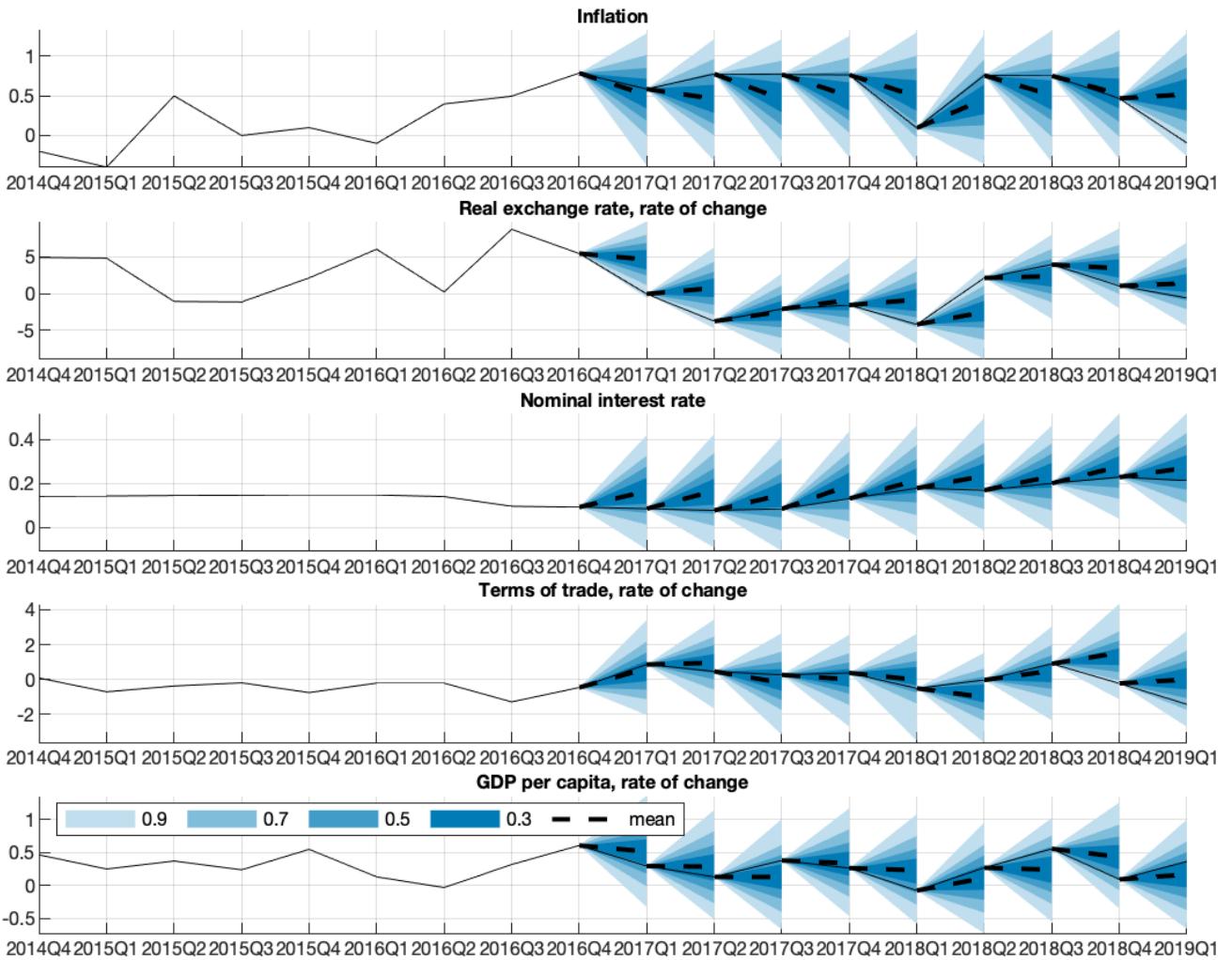


Figure 8: 1-step-ahead forecasts for the domestic variables

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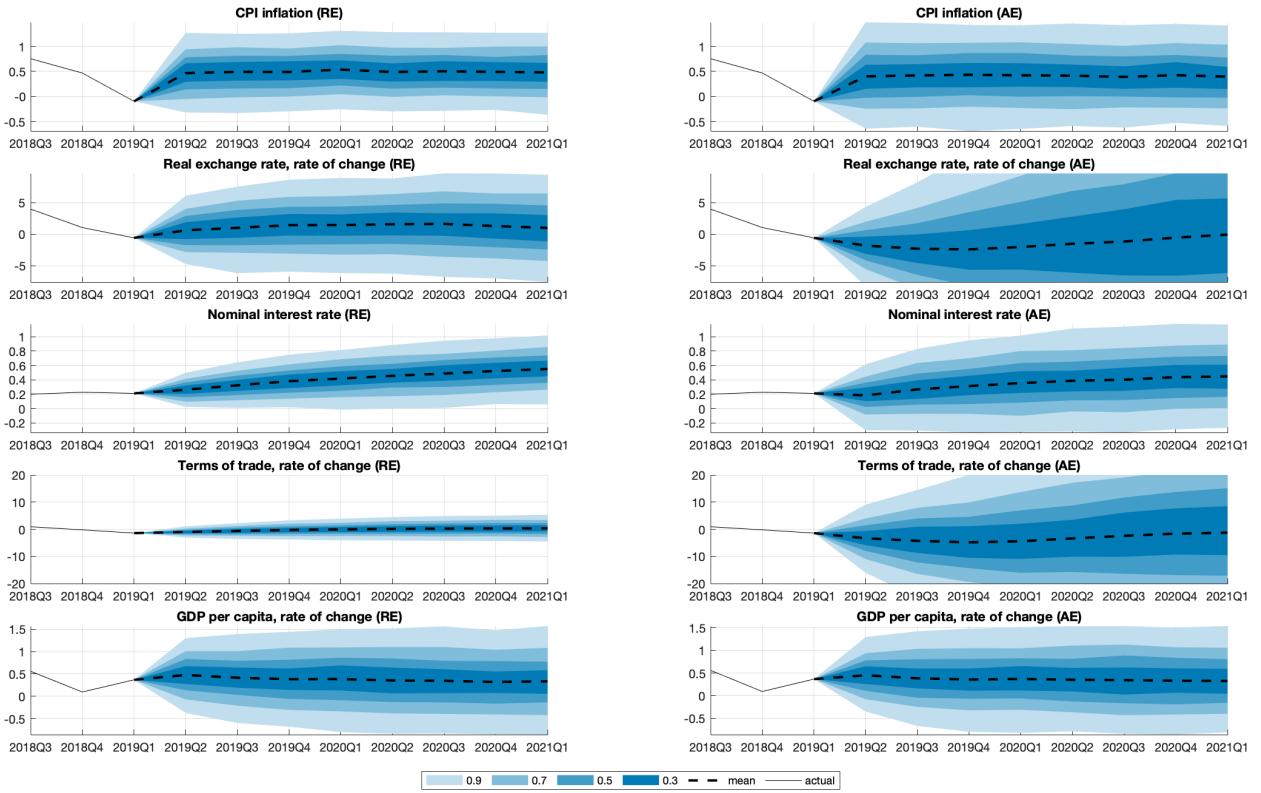


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Appendices

A The modified Kim and Nelson filter

Consider a state space form in which the transition equation is given by

$$x_t(r_t) = b_{r_t} + \mathcal{T}_{r_t} x_{t-1} + \mathcal{R}_{r_t} \varepsilon_t \quad (\text{A.1})$$

and the measurement equation by

$$y_t = a_{r_t} + \mathcal{Z}_{r_t} x_t + H_{r_t} \eta_t \quad (\text{A.2})$$

where $\varepsilon_t \sim N(0, I)$ is the vector of structural shocks and $\eta_t \sim N(0, I)$ the vector of measurement errors.

The modified Kim and Nelson filter we use proceeds through the following steps

for $t = 1, \dots, n$

- $v_t(r_t) = y_t - a_{r_t} - \mathcal{Z}_{r_t} x_{t|t-1}(r_t)$
- $F(r_t) = \mathcal{Z}_{r_t} P_{t|t-1}(r_t) \mathcal{Z}'_{r_t} + H_{r_t} H'_{r_t}$
- $p(r_t|Y_{t-1}) = \sum_{r_{t-1}=1}^h p_{r_{t-1}, r_t}(Y_{t-1}) p(r_{t-1}|Y_{t-1})$
- $p(r_t|Y_t) = \frac{p(y_t|Y_{t-1}, r_t) p(r_t|Y_{t-1})}{\sum_{r_t=1}^h p(y_t|Y_{t-1}, r_t) p(r_t|Y_{t-1})}$
- $p(r_t|Y_t, r_{t+1}) = p(r_t|Y_t)$
- $K_{t+1,t}(r_{t+1}, r_t) = \mathcal{T}_{r_{t+1}} P_{t|t-1}(r_t) \mathcal{Z}'_{r_t} F(r_t)^{-1}$
- $x_{t+1|t}(r_{t+1}) = \sum_{r_t=1}^h p(r_t|Y_t, r_{t+1}) [b_{r_{t+1}} + \mathcal{T}_{r_{t+1}} x_{t|t-1}(r_t) + K_{t+1,t}(r_{t+1}, r_t) v_t(r_t)]$
- $P_{t+1|t}(r_{t+1}) = \sum_{r_t=1}^h p(r_t|Y_t, r_{t+1}) [\mathcal{T}_{r_{t+1}} P_{t|t-1}(r_t) \mathcal{Z}'_{r_t} - K_{t+1,t}(r_{t+1}, r_t) F_t(r_t) K_{t+1,t}(r_{t+1}, r_t)' + \mathcal{R}_{r_{t+1}} \mathcal{R}'_{r_{t+1}}]$
- $p(y_t|Y_{t-1}, r_t) = (2\pi)^{-\frac{n_y}{2}} |F(r_t)|^{-\frac{1}{2}} \exp\left(-\frac{1}{2} v_t(r_t)' F(r_t)^{-1} v_t(r_t)\right)$
- $p(y_t|Y_{t-1}) = \sum_{r_t=1}^h p(y_t|Y_{t-1}, r_t) p(r_t|Y_{t-1})$

and the likelihood is given by

$$\log Lik = \sum_{t=1}^n \log p(y_t|Y_{t-1})$$

B Data description

Output (y_t): Real seasonally adjusted GDP per capita for the UK. Log differenced.

Nominal interest rate (r_t): The average three-month LIBOR expressed as net quarterly interest rate.

Real exchange rate (q_t): The real exchange rate is calculated using the nominal exchange rate between the UK and the U.S., and the consumer price indices from the two countries. Log differenced.

Terms of trade (s_t): Calculated using the import and export price deflators from the UK. Log differenced.

CPI Inflation (π_t): The seasonally adjusted consumer price index for all goods in the UK. Log differenced.

Foreign output (y_t^*): Seasonally adjusted real US GDP per capita. Log differenced.

Foreign inflation (π_t^*): The seasonally adjusted consumer price index for all goods in the U.S. Log differenced.

Foreign interest rate (r_t^*): The Federal Funds interest rate expressed as net quarterly interest rate.

All variables are in logs and demeaned unless otherwise specified. The data were collected from Macrobond and the Federal Reserve Bank of St. Louis.

C Convergence diagnostics for 2016

See figures 10, 11, 12 and 13.

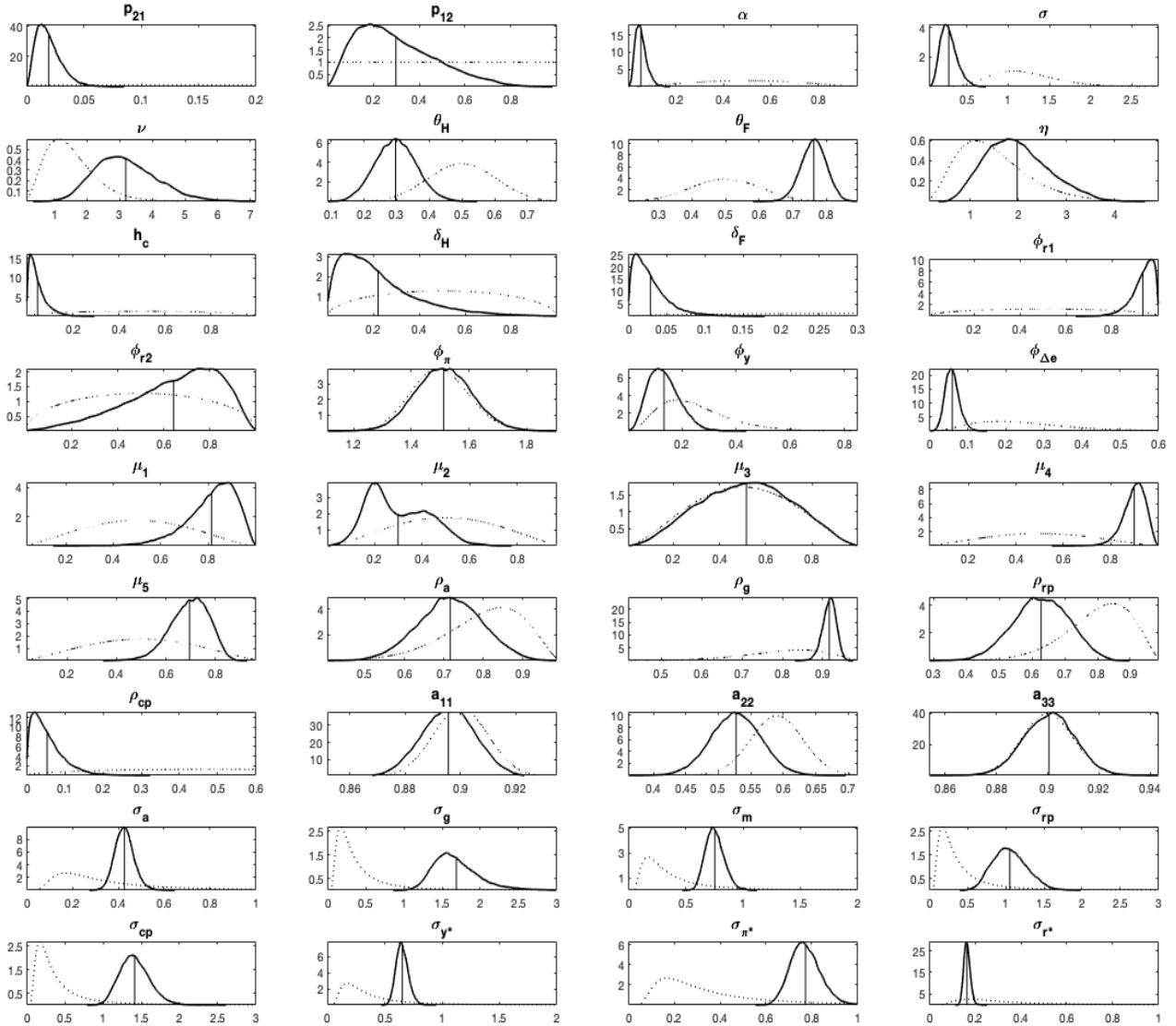


Figure 10: Prior and posterior distributions of parameters from UK model (2016 sample). Dashed curves are the prior distributions, while the solid curves are the posterior distributions of the respective parameters.

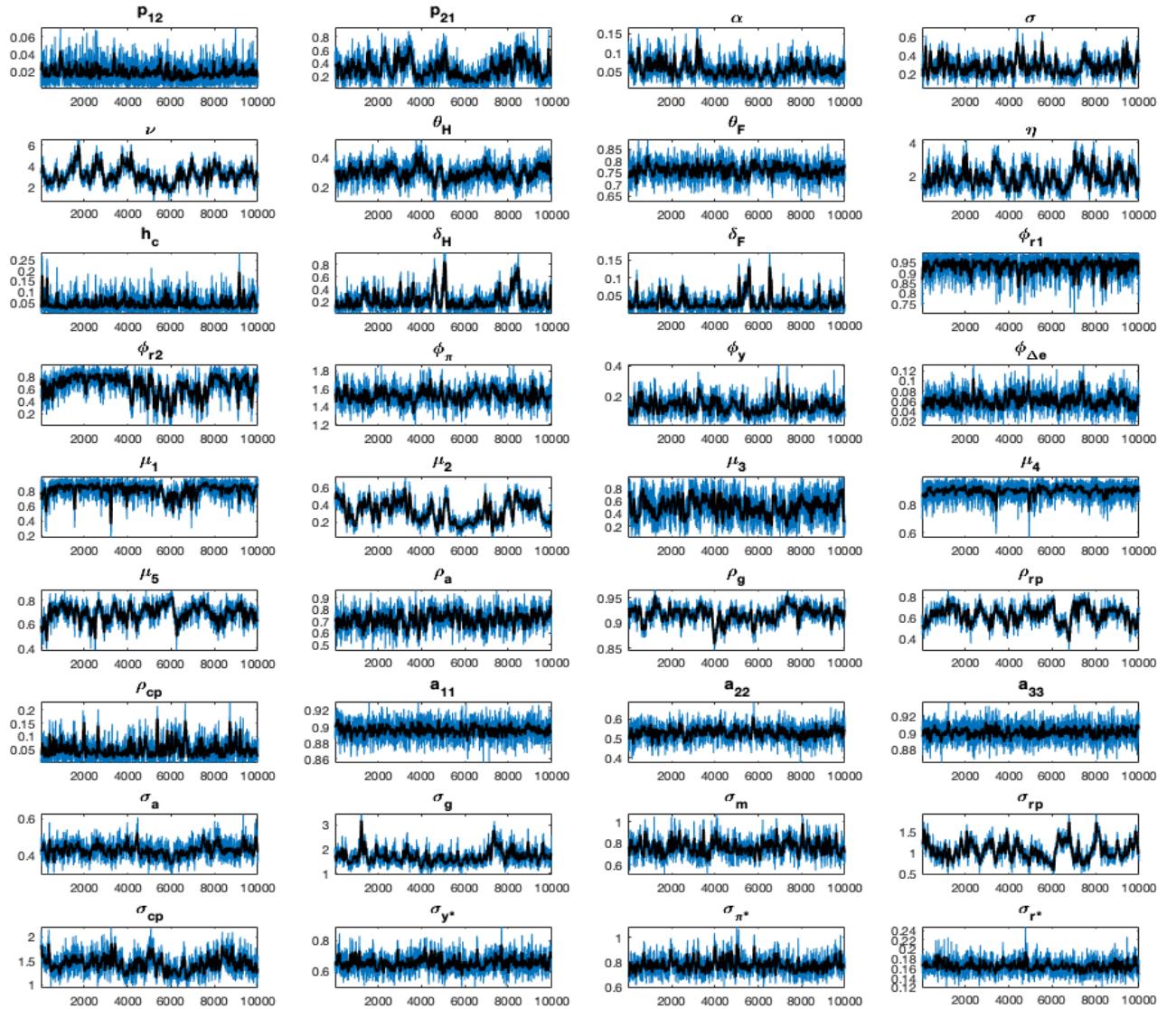


Figure 11: Traceplot UK model (2016 sample)

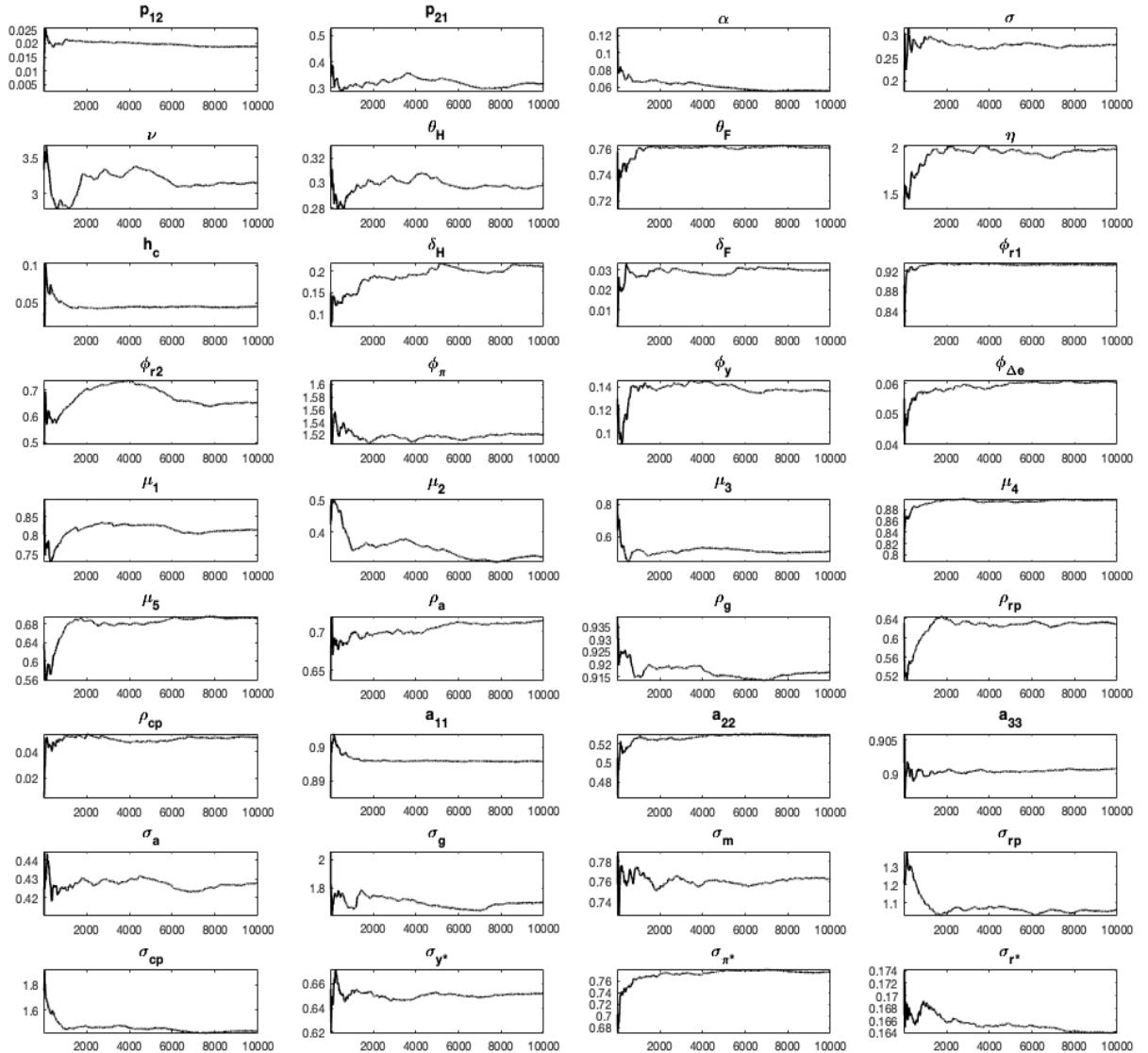
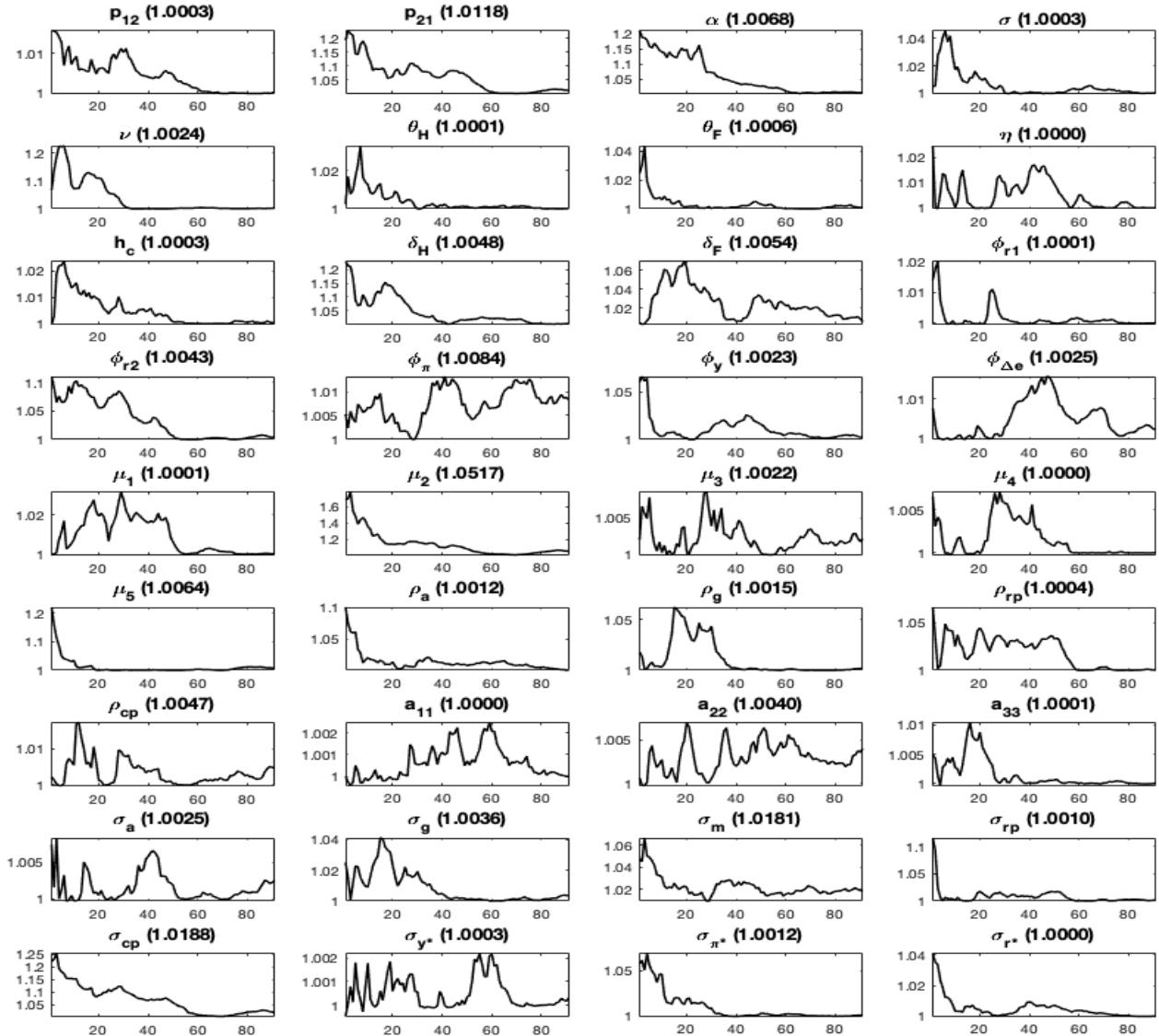


Figure 12: Recursive means of the parameters in the UK model (2016 sample)

Figure 13: Gelman-Rubin plots for selected parameters of the UK model (2016 sample)



D Results and convergence diagnostics for 2019

Table 4: Prior densities and posterior estimates of structural parameters for the RS-DSGE model

Parameters		Prior			Posterior			
		Prior density	Mean	SD	Mean	SD	5%	95%
Openness parameter	α	Beta	0.50	0.20	0.06	0.02	0.03	0.10
Risk aversion	σ	Gamma	1.20	0.40	0.26	0.10	0.13	0.44
Inverse Frisch	ν	Gamma	1.50	0.75	3.75	1.15	2.06	5.77
Calvo dom. prices	θ_H	Beta	0.50	0.10	0.30	0.06	0.20	0.40
Calvo import prices	θ_F	Beta	0.50	0.10	0.76	0.04	0.70	0.82
Elasticity H-F goods	η	Gamma	1.50	0.75	2.39	0.77	1.28	3.79
Habit	h_c	Beta	0.50	0.25	0.04	0.04	0.005	0.11
Indexation domestic	δ_H	Beta	0.50	0.25	0.27	0.22	0.03	0.73
Indexation foreign	δ_F	Beta	0.50	0.25	0.03	0.02	0.003	0.07
Taylor smoothing RE regime	ϕ_{r1}	Beta	0.50	0.25	0.94	0.04	0.86	0.99
Taylor smoothing AE regime	ϕ_{r2}	Beta	0.50	0.25	0.70	0.18	0.35	0.93
Taylor inflation	ϕ_π	Gamma	1.50	0.10	1.51	0.10	1.36	1.68
Taylor output	ϕ_y	Gamma	0.25	0.13	0.13	0.06	0.05	0.23
Taylor exch.rate	$\phi_{\Delta e}$	Gamma	0.25	0.13	0.05	0.02	0.03	0.08
Learning consumption	μ_1	Beta	0.50	0.20	0.84	0.09	0.65	0.96
Learning CPI inflation	μ_2	Beta	0.50	0.20	0.28	0.13	0.10	0.50
Learning dom. inflation	μ_3	Beta	0.50	0.20	0.49	0.20	0.18	0.82
Learning import inflation	μ_4	Beta	0.50	0.20	0.91	0.04	0.83	0.97
Learning real exch. rate	μ_5	Beta	0.50	0.20	0.71	0.07	0.58	0.81

^a Sample 1997Q2-2019Q1

^b Log-MDD (Laplace): -705.16

^c Calibrate $\beta = 0.99$ and $\chi = 0.01$.

For convergence diagnostics, see figures 15, 16, 17 and 18.

Table 5: Prior densities and posterior estimates of structural parameters for the RS-DSGE model (continued).

Parameters		Prior			Posterior			
		Prior density	Mean	SD	Mean	SD	5%	95%
Technology	ρ_a	Beta	0.80	0.10	0.72	0.08	0.58	0.85
Preferences	ρ_g	Beta	0.80	0.10	0.92	0.02	0.89	0.95
Risk premium	ρ_{rp}	Beta	0.80	0.10	0.67	0.08	0.53	0.79
Import cost-push shock	ρ_{cp}	Beta	0.50	0.25	0.06	0.05	0.01	0.15
1st lag y^*	a_{11}	Normal	0.90	0.01	0.90	0.01	0.88	0.91
1st lag π^*	a_{22}	Normal	0.59	0.04	0.52	0.04	0.46	0.58
1st lag r^*	a_{33}	Normal	0.90	0.01	0.90	0.01	0.89	0.92
sd foreign output	σ_{y^*}	Inv. Gamma	0.50	inf.	0.62	0.05	0.55	0.70
sd foreign inflation	σ_{π^*}	Inv. Gamma	0.50	inf.	0.74	0.06	0.65	0.84
sd foreign interest rate	σ_{r^*}	Inv. Gamma	0.50	inf.	0.16	0.01	0.14	0.18
sd technology	σ_a	Inv. Gamma	0.50	inf.	0.41	0.04	0.35	0.48
sd preferences	σ_g	Inv. Gamma	0.50	inf.	1.68	0.32	1.25	2.27
sd Taylor rule	σ_m	Inv. Gamma	0.50	inf.	0.73	0.08	0.61	0.86
sd risk premium	σ_{rp}	Inv. Gamma	0.50	inf.	1.00	0.21	0.69	1.37
sd import cost push	σ_{cp}	Inv. Gamma	0.50	inf.	1.38	0.18	1.10	1.69
Transition probability, from RE to AE	p_{12}	Uniform	0.00	1.00	0.02	0.01	0.005	0.04
Transition probability, from AE to RE	p_{21}	Uniform	0.00	1.00	0.29	0.17	0.07	0.61

^a Sample 1997Q2-2019Q1

^b Log-MDD (Laplace): -705.16

^c For the uniform distribution min and max value for the prior density is reported.

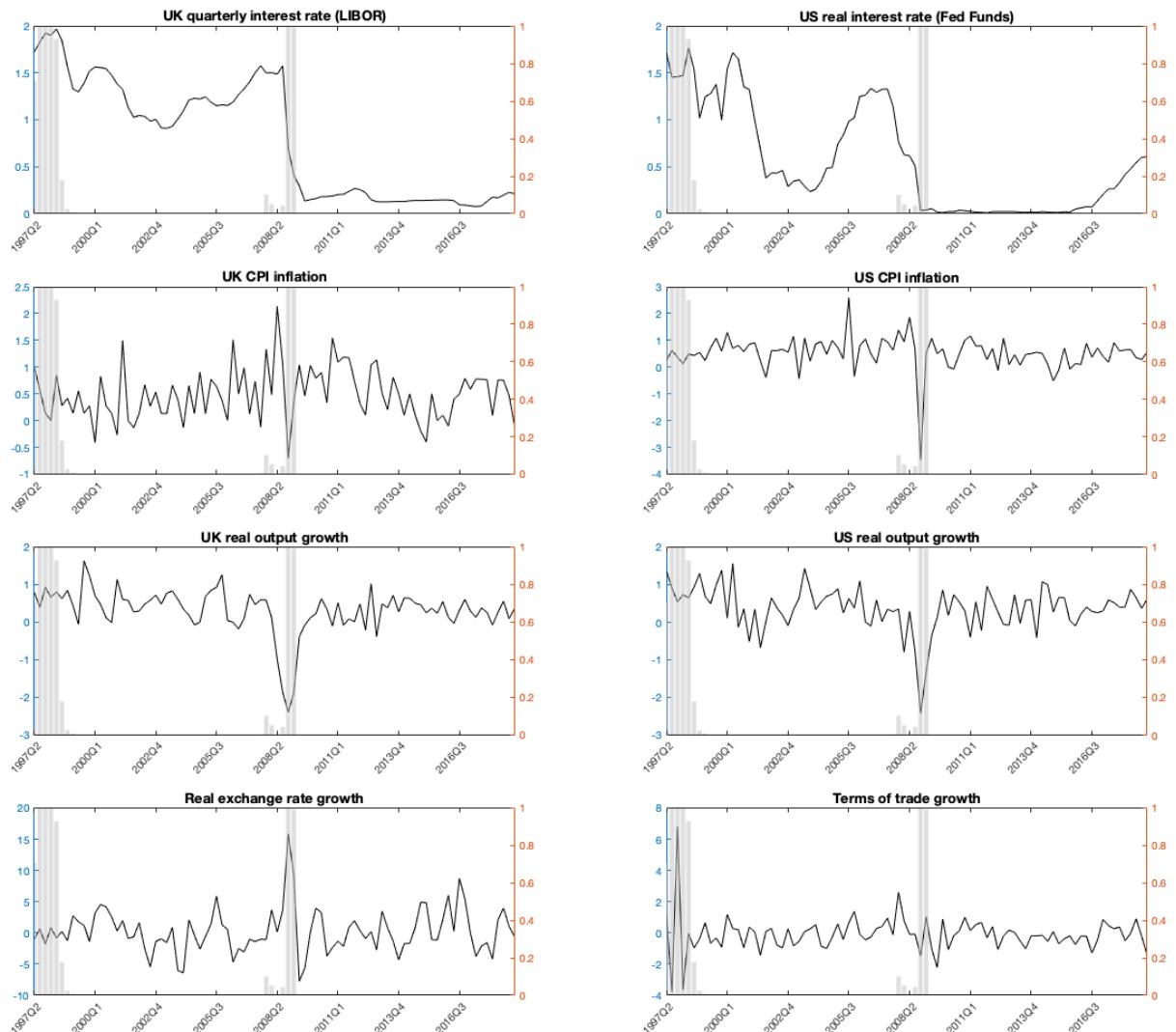
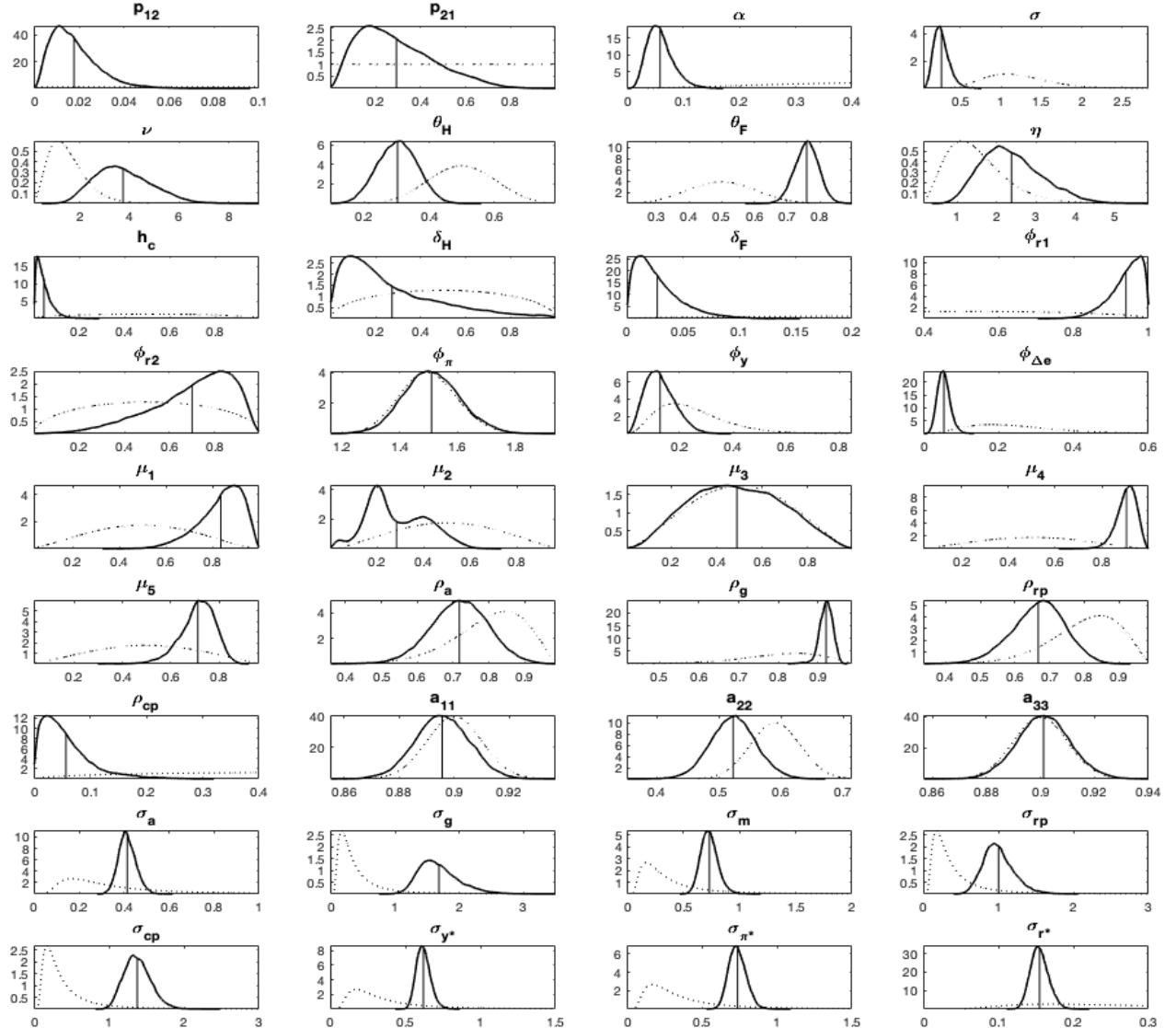


Figure 14: Data and regime probability for the AE-regime, sample 1997Q2-2019Q1.

Figure 15: Prior and posterior distributions of parameters from UK model (2019 sample)



Dashed curves are the prior distributions, while the solid curves are the posterior distributions of the respective parameters.

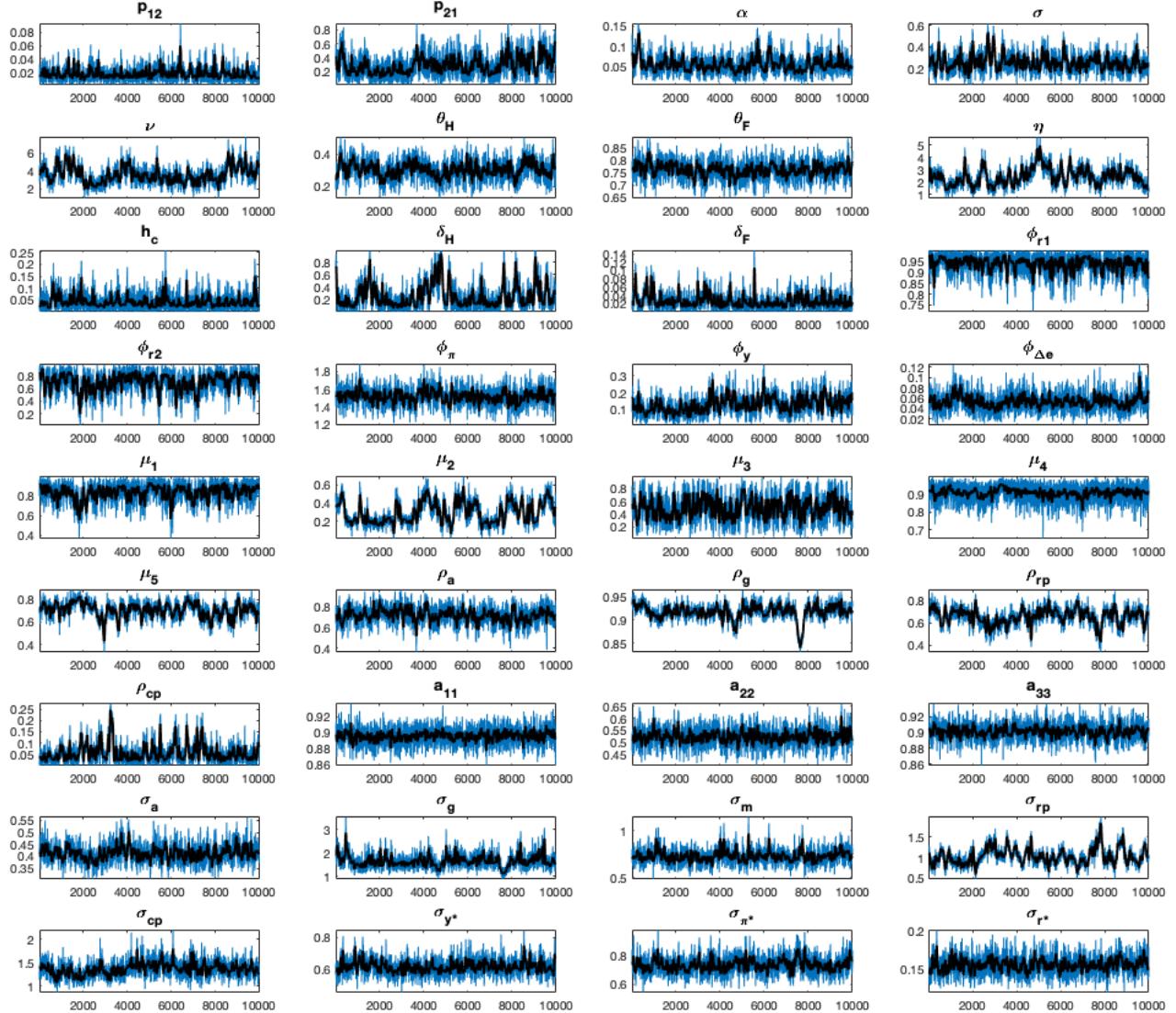


Figure 16: Traceplot UK model (2019 sample)

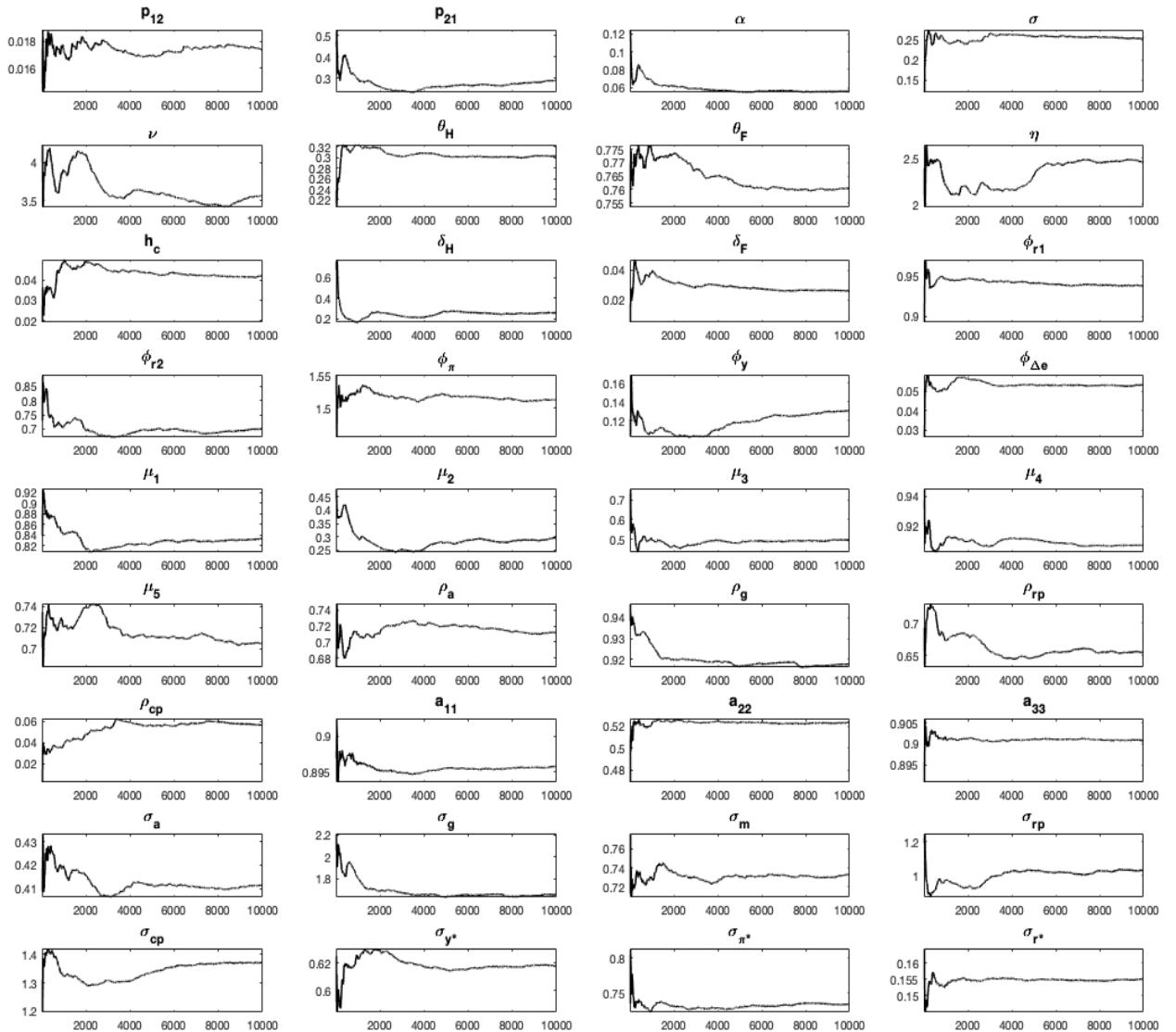


Figure 17: Recursive means of the parameters in the UK model (2019 sample)

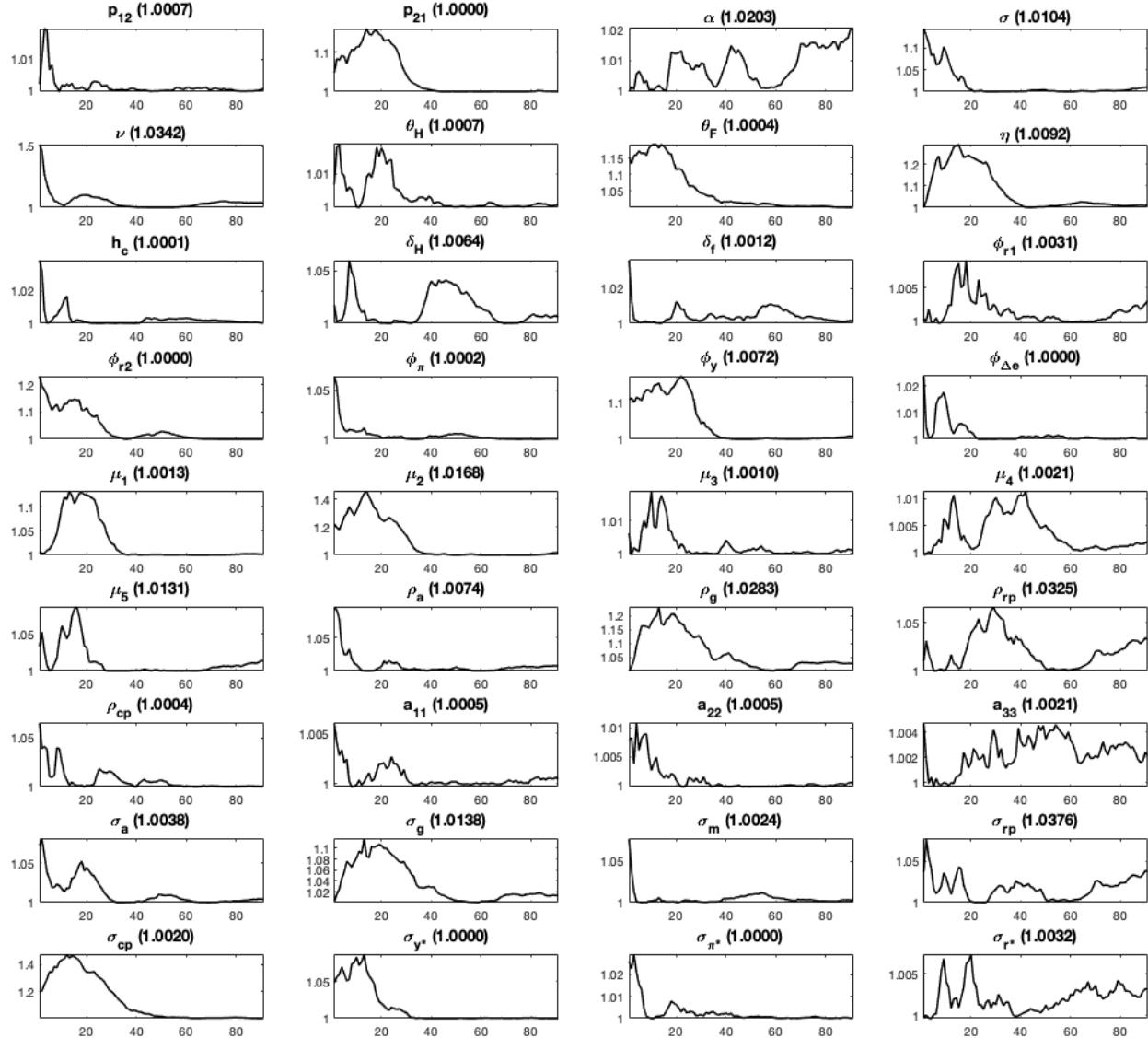


Figure 18: Gelman-Rubin plots for selected parameters of the UK model (2019 sample)