Filtering with limited information

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Lars Peter Hansen (2014)

Hansen (1982) builds on a long tradition in econometrics of 'doing something without having to do everything.' This entails the study of partially specified models, that is, models in which only a subset of economic relations are formally delineated.

Motivation

- Filtering shocks from a dynamic model is a central task in macroeconomics:

    1. Path of shocks is a reality check for the model.
    2. Historical decompositions and counterfactuals.
    3. Forecasting and optimal policy recommendations when laws of motions are state-dependent.
    4. Structural estimation.

- However, filtering is hard, and there is no universal and easy-to-apply algorithm to implement it.

Alternatives

- A possibility ⇒ sequential Monte Carlos: Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016).

    1. Specification of the full model, including auxiliary assumptions.
    2. Computationally costly.
    3. Curse of dimensionality.

- Some of the previous points also hold for even the simple linear, Gaussian case where we can apply the Kalman filter.

- Can we follow Hansen's suggestion and 'do something without having to do everything"? ⇒ Yes!

- Partial information filter.

Environment

- Model:

$$f(x_t, y_t, \mathbb{E}_t[g(x_{t+1}, y_{t+1}, x_t, y_t)]) = \mathbf{0}$$

- Deterministic steady state:

$$f(\bar{x}, \bar{y}, g(\bar{x}, \bar{y}, \bar{x}, \bar{y})]) = \mathbf{0}$$

- Estimation:

$$f(\widehat{x}_t, y_t, \widehat{\mathbb{E}}_t[g(\widehat{x}_{t+1}, y_{t+1}, \widehat{x}_t, y_t)]) = \mathbf{0}$$

Factorization

- Factorization of $g(\circ)$

$$g(x_{t+1}, y_{t+1}, x_t, y_t) \equiv g_1(x_{t+1}, y_{t+1}, x_t, y_t) \times g_2(x_{t+1}, y_{t+1}, x_t, y_t)$$

- Then:

$$\mathbb{E}_t[g(x_{t+1}, y_{t+1}, x_t, y_t)_i] \equiv \mathbb{E}_t[g_1(x_{t+1}, y_{t+1}, x_t, y_t)_i] \times \mathbb{E}_t[g_2(x_{t+1}, y_{t+1}, x_t, y_t)_i] + \mathrm{Cov}_t[g_1(x_{t+1}, y_{t+1}, x_t, y_t)_i, g_2(x_{t+1}, y_{t+1}, x_t, y_t)_i]$$

- We need to approximate conditional first and second moments:

1. Equilibrium conditions of the model.
2. Observed expectations.
3. Auxiliary statistical model.

Auxiliary statistical model

- VAR(1) in $g_{1,t}, g_{2,t}$, (a subset of $n_{\widetilde{y}}$ elements of) y_t, and $\widehat{x}_t$ for $t = 1, ..., T$.
- Why?
- If we collect variables collected in ξ_t:

$$\xi_t = \mu + A\xi_{t-1} + \epsilon_t, \qquad \text{Var}[\epsilon_t] = \Sigma$$

- Ordering $g_{1,t}$ and $g_{2,t}$ as the first two variables of the VAR, we can write:

$$\mathbb{E}_t[g(x_{t+1}, y_{t+1}, x_t, y_t)]_i \equiv (\mu_i + e_i' A\xi_t)(\mu_{i+m} + e_{i+m}' A\xi_t) + \Sigma_{i,i+m}$$

where e_i is a selection vector.

- Thus:

$$f(x_t, y_t, (\mu_i + e_i' A\xi_t)(\mu_{i+m} + e_{i+m}' A\xi_t) + \Sigma_{i,i+m})_i = 0$$

Two approaches

- (Fixed point): we find $\widehat{x}^T$ (and associated parameters for the VAR) that solve for all i:

$$f(x_t, y_t, (\mu_i + e_i' A\xi_t)(\mu_{i+m} + e_{i+m}' A\xi_t) + \Sigma_{i,i+m})_i = 0$$

  In practice, initialize $\widehat{E}_t^{(0)}[g(\circ)]$ based on $\widehat{x}_t^{(0)} = \bar{x}\ \forall t$ and $A^{(0)} = \mathbf{0}$, $\mu^{(0)} = \bar{\xi}_T$ and $\Sigma^{(0)} = \mathbf{0}$ and iterate until convergence.

- (Gibbs sampler): From $d = 1, \ldots, D$, iterate on:
  1. Given $\{\widehat{x}_t^{(d)}\}_{t=1}^T$, $\mu^{(d)}, A^{(d)}, \Sigma^{(d)} \sim P(\mu, A, \Sigma | \xi_t)$.
  2. Given $y^T, \mu^{(d)}, A^{(d)}, \Sigma^{(d)}$, solve for $\{\widehat{x}_t^{(d+1)}\}_{t=1}^T$ for all i in:

$$f(x_t, y_t, (\mu_i + e_i' A\xi_t)(\mu_{i+m} + e_{i+m}' A\xi_t) + \Sigma_{i,i+m})_i = 0$$

  Start the Gibbs sampler from $\widehat{x}_t^{(0)} = \bar{x}$ or the fixed point above.

  The Gibbs sampler allows us to quantify estimation uncertainty.

Tobin's Q

- Representative household

$$\max_{\{c_t,n_t,i_t\}} \mathbb{E}\sum_{s=0}^{\infty}\left(\prod_{u=1}^{s}\beta_{t-1+s}\right)u(c_{t+s},n_{t+s})$$

$$\text{s.t.}$$

$$k_{t+1}=(1-\delta_t)k_t+e^{\xi_t}\left(1-\frac{\chi}{2}\left(\frac{i_t}{i_{t-1}}-(1+g)\right)^2\right)i_t$$

$$c_t+i_t=e^{z_t}k_{t-1}^{\alpha}n_t^{1-\alpha}$$

- Four shocks: $\beta_t,\delta_t,\xi_t,z_t$ follow log-linear AR(1) processes

Solution (I)

$$c_t^{-\eta}(1-\kappa(1-\eta)n_t^{1+1/\phi})^{1-\eta} = \lambda_t$$

$$c_t^{1-\eta}(1-\kappa(1-\eta)n_t^{1+1/\phi})^{-\eta}(1+1/\phi)\kappa(1-\eta)n_t^{1/\phi} = \lambda_t(1-\alpha)e^{z_t}\left(\frac{k_{t-1}}{n_t}\right)^{\alpha}$$

$$\lambda_t = \mu_t e^{\xi_t}\left(1-\frac{\chi}{2}\left(\frac{i_t}{i_{t-1}}-(1+g)\right)^2 - \chi\left(\frac{i_t}{i_{t-1}}-(1+g)\right)\frac{i_t}{i_{t-1}}\right)$$

$$+\beta_t\mathbb{E}_t\left[\mu_{t+1}\chi\left(\frac{i_{t+1}}{i_t}-(1+g)\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right]$$

$$\mu_t = \beta_t\mathbb{E}_t\left[(1-\delta_{t+1})\mu_{t+1}+\lambda_{t+1}\alpha e^{z_{t+1}}\left(\frac{n_{t+1}}{k_t}\right)^{1-\alpha}\right]$$

Solution (II)

$$c_t^{-\eta}(1-\kappa(1-\eta)n_t^{1+1/\phi})^{1-\eta} = \lambda_t$$

$$c_t^{1-\eta}(1-\kappa(1-\eta)n_t^{1+1/\phi})^{-\eta}(1+1/\phi)\kappa(1-\eta)n_t^{1/\phi} = \lambda_t(1-\alpha)e^{z_t}\left(\frac{k_{t-1}}{n_t}\right)^{\alpha}$$

$$1 = q_t e^{\xi_t}\left(1-\frac{\chi}{2}\left(\frac{i_t}{i_{t-1}}-(1+g)\right)^2 - \chi\left(\frac{i_t}{i_{t-1}}-(1+g)\right)\frac{i_t}{i_{t-1}}\right)$$

$$+\mathbb{E}_t\left[\beta_t\frac{\lambda_{t+1}}{\lambda_t}q_{t+1}\chi\left(\frac{i_{t+1}}{i_t}-(1+g)\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right]$$

$$q_t = \mathbb{E}_t\left[\beta_t\frac{\lambda_{t+1}}{\lambda_t}\left((1-\delta_{t+1})q_{t+1}+r_{t+1}^k\right)\right]$$

where $r_t^k \equiv \alpha e^{z_{t+1}}\left(\frac{n_{t+1}}{k_t}\right)^{1-\alpha}$

Quantitative set-up

- Calibration:
  1. $\bar{n} = \frac{1}{3}$.
  2. $\bar{g} = 0.5\%$, a 2% annual growth rate.
  3. $\eta = 1.0$, to have separable preferences for now.
  4. $\phi = 1$, a typical value for the Frisch elasticity.
  5. $\bar{\delta} = 2\%$, implying an 8% annual depreciation rate.
  6. $\beta = 0.995$, implying a annualized real rate of about $2\% + 4\eta\bar{g} = 5\%$.
  7. $\rho_z = 0.95$, $\sigma_z = 0.76\%$.
  8. $\rho_\beta = 0.8$, $\sigma_\beta = 1.0\%$.
  9. $\rho_\xi = 0.9$, $\sigma_\xi = 1.0\%$.
  10. $\rho_\delta = 0.75$, $\sigma_\delta = 1.0\%$.
- Solved the model using 3rd order perturbation methods with pruning, as in Andreasen, Fernández-Villaverde, and Rubio Ramírez (2017).
- Simulated data: We simulate the model for 2,000 periods after a burn-in of 1,000 periods.

Filter set up

- We want to filter a single variable, q_t using data on the SDF $M_{t+1} \equiv \beta_t \frac{\lambda_{t+1}}{\lambda_t}$, the rental rate on capital r_t^k, and the risk-free rate r_t^f.

- Set $x_t = q_t$, $y_t = [\widetilde{M}_t, r_t^k, r_t^f]$, and rewrite the conditional expectation of the return on investment as:

$$\begin{aligned} g(x_{t+1}, y_{t+1}, x_t, y_t) &\equiv y_{1,t+1} \times \left((1-\bar{\delta})x_{t+1} + y_{2,t+1}\right) \\ &= \widetilde{M}_{t+1} \times \left((1-\bar{\delta})q_{t+1} + r_{t+1}^k\right) \end{aligned}$$

  with $\widetilde{M}_{t+1} = \bar{\beta}\frac{c_t}{c_{t+1}}$.

- Note use of misspecified model.

- Thus

$$\begin{aligned} f(x_t, y_t, \mathbb{E}_t[g(x_{t+1}, y_{t+1}, x_t, y_t]) &\equiv -x_t + \mathbb{E}_t\Big[g(x_{t+1}, y_{t+1}, x_t, y_t)\Big] \\ &= -q_t + \mathbb{E}_t\Big[\widetilde{M}_{t+1} \times \left((1-\bar{\delta})q_{t+1} + r_{t+1}^k\right)\Big] \end{aligned}$$

Auxiliary statistical model

- Using that, in equilibrium, $1 + r_t^f \equiv \mathbb{E}_t[M_{t+1}]^{-1}$, we can re-write previous expression as an VAR approximation:

$$f(x_t^{(d)}, y_t, \widehat{\mathbb{E}}_t^{(d)}[g(x_{t+1}^{(d-1)}, y_{t+1}, x_t^{(d-1)}, y_t]) \approx$$
$$-x_t^{(d-1)} + (1-\bar{\delta})\Sigma_{1,4}^{(d)} + \Sigma_{1,2}^{(d)}$$
$$+(1+r_t^f)^{-1} \times \left((1-\bar{\delta})e_4 + e_1\right)\left(\mu^{(d)} + A^{(d)} X_t^{(d-1)}\right)$$

- We already are setting up notation for Gibbs sampler.
- For comparison purposes, we will also run:
    1. A naive guess for q computed as $q_t = \frac{(1-\bar{\delta}) + mpk_{t+1}}{1+r_t^f}$.
    2. A Kalman filter and smoother.

VAR

- VAR(1) in $\xi_t^{(d)} \equiv [M_{t+1}, r_{t+1}^k, r_{t+1}^f, \widehat{q}_t^{(d)}]$:

1. Fixed point: We solve for a fixed point in the 2,000-dimensional vector $\widehat{q}^{(fp),T}$ and the VAR parameters $\widehat{\mu}^{(fp)}, A^{(fp)}$, and $\Sigma^{(fp)}$.

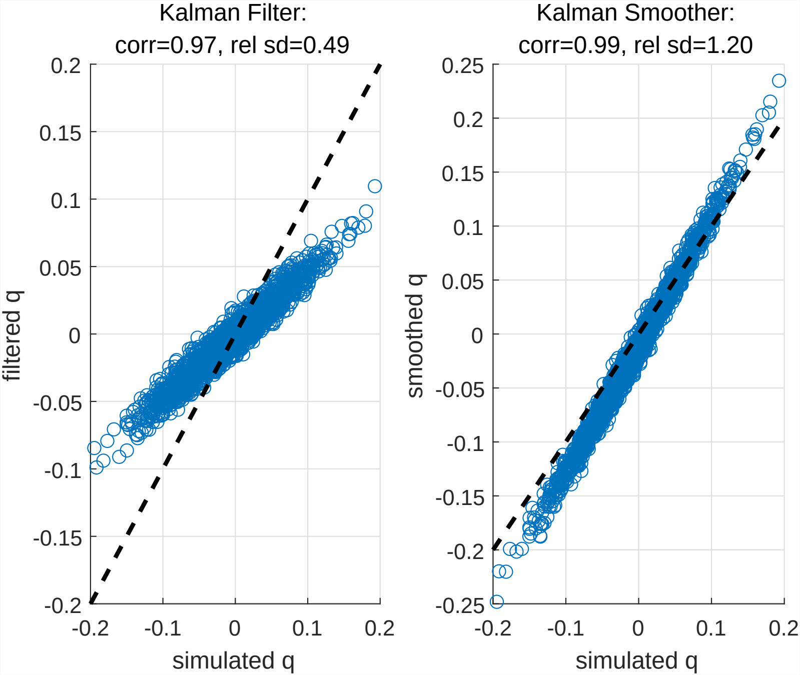
2. Gibbs sampler:

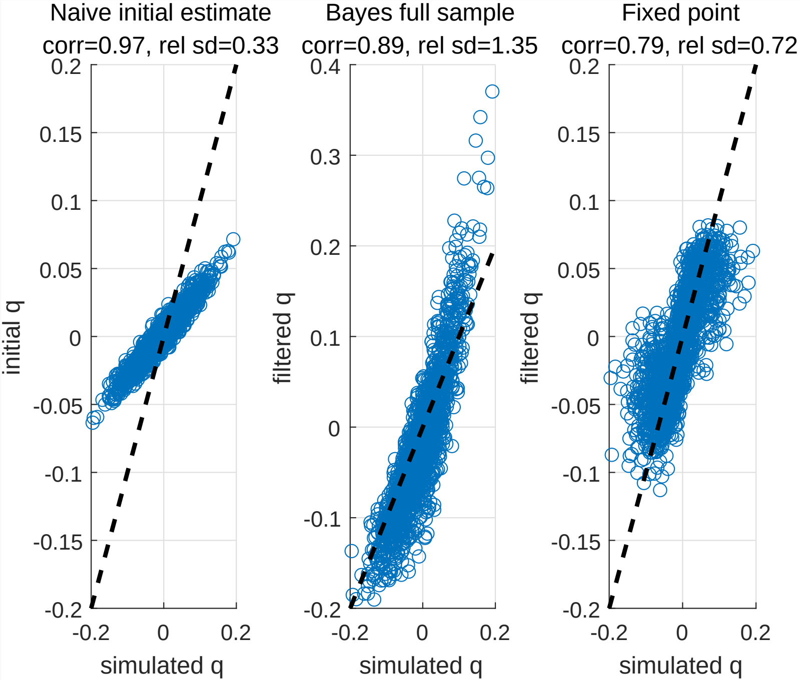
   2.1 Set $\xi_0^{(d)} = [M_0, r_0^k, r_0^f, \bar{q}]$

   2.2 We sample
   $\Sigma^{(d)}$ and $\beta^{(d)} = [\text{vec}(A^{(d)})', (\mu^{(d)})']'$
   from
   $\Sigma^{(d)}|\xi^{(d),T} \sim \mathcal{IW}(T-1, \widehat{\Sigma}_{OLS}^{(d)} \times T)$
   and
   $\beta^{(d)}|\xi^{(d),T}, \Sigma^{(d)} \sim \mathcal{N}(\widehat{\beta}_{OLS}^{(d)}, \Sigma^{(d)} \otimes ((\bar{\xi}^{(d-1)})'\bar{\xi}^{(d-1)})^{-1})$, conditioning on
   $\xi_0^{(d)} = [M_0, r_0^k, r_0^f, \bar{q}]$.

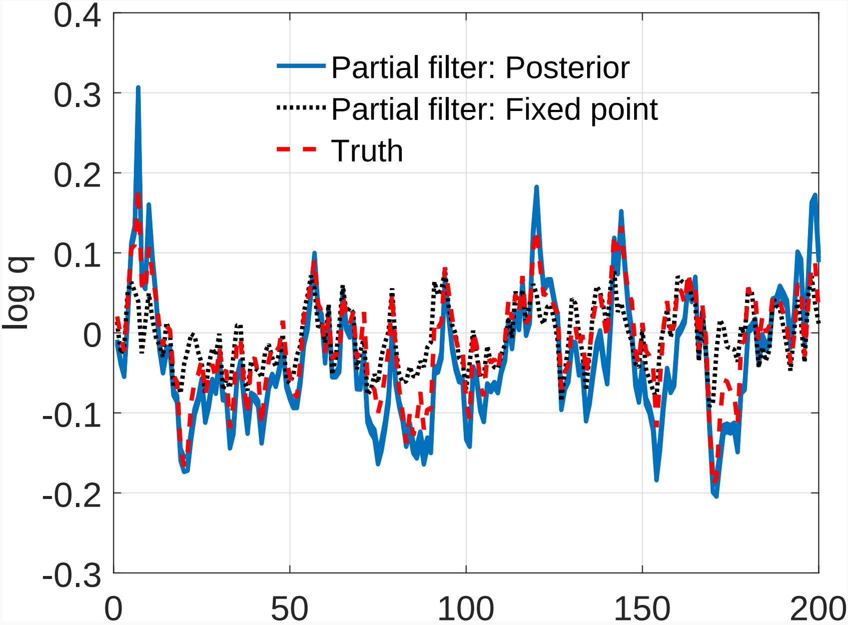
   2.3 We use a flat prior and define $\widehat{\Sigma}_{OLS}^{(d)} \times T$ as the OLS sum of squared residuals and $\widehat{\beta}_{OLS}^{(d)}$ the OLS estimator/MLE of the coefficients. $\bar{\xi}$ is a $T \times 5$ matrix with rows $\bar{\xi}_t = [\xi_{t-1}, 1]$.

   2.4 We solve optimality condition for $\widehat{q}_t^{(d)}$.


Kalman Filter:
corr=0.97, rel sd=0.49
filtered q
simulated q
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.2
-0.1
0
0.1
0.2
Kalman Smoother:
corr=0.99, rel sd=1.20
smoothed q
simulated q
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.2
-0.1
0
0.1
0.2



Naive initial estimate
corr=0.97, rel sd=0.33
initial q
simulated q
Bayes full sample
corr=0.89, rel sd=1.35
filtered q
simulated q
Fixed point
corr=0.79, rel sd=0.72
filtered q
simulated q
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.2
0
0.2



Partial filter: Posterior
Partial filter: Fixed point
Truth
log q
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
50
100
150
200

Political distribution risk and aggregate fluctuations

Budd (2012), 'Labor Relations – Striking a Balance,' 4th ed.

A popular framework for thinking about labor law is to consider a **pendulum that can range from strong bargaining power for labor** ... **to strong bargaining power for companies** ....

We stress the role that changes in

1. statutory labor law (including executive orders),
2. case law (courts and NLRB), and
3. political climate

have on business cycles, income shares, and asset prices.

Model

- RBC model with search and matching frictions.
  (Andolfatto, 1996; Merz, 1995; Shimer, 2010)
    - Household with a continuum of members. Members are either employed or unemployed.
    - Household insures members against idiosyncratic employment risk.
    - Competitive firms that choose recruiting intensity.
    - Government.
    - Complete markets.
- Bargaining power subject to persistent redistribution shocks.

Households

- Recursive problem of the head of household:

$$V(a, n_{-1}) = \max_{a', n, c} \frac{c^{1-\sigma}(1 + (\sigma - 1)\gamma n_{-1})^{\sigma} - 1}{1 - \sigma} + \beta \mathbb{E}[V(a', n)]$$

  with

$$c \equiv c_e n_{-1} + c_u(1 - n_{-1})$$

- Budget constraint:

$$c + \mathbb{E}[m' * a'] = (1 - \tau_n) w n_{-1} + T + a$$

  with stochastic discount factor m.

- Law of motion of employment:

$$n = (1 - x) n_{-1} + f(\theta)(1 - n_{-1}),$$

  with job finding rate $f(\theta) = \xi \theta^{\eta}$.

Firms

- Firm produces output y using effective capital uk_{-1} and production workers $(1-\nu)n_{-1}$:

$$y = \left(\alpha^{\frac{1}{\varepsilon}}(uk_{-1})^{1-\frac{1}{\varepsilon}} + (1-\alpha)^{\frac{1}{\varepsilon}}(z(1-\nu)n_{-1})^{1-\frac{1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

  Fraction ν workers devoted to recruiting activities.

- Laws of motion for employment and capital:

$$n = n_{-1}(\nu\mu(\theta) + 1 - x)$$
$$k = (1-\delta(u))k_{-1} + I\left(1 - \frac{1}{2}\kappa\left(\frac{I}{k_{-1}} - \tilde{\delta}\right)^2\right)$$

  where $\mu(\theta) = f(\theta)/\theta$ is hiring probability per recruiter.

- Firm value:

$$J(n_{-1}, k_{-1}) = \max_{n,k,\nu}(1-\tau_k)(y - wn_{-1}) - I + \tau_k\delta(\bar{u})k_{-1} + \mathbb{E}\left[m' * J(n,k)\right]$$

Wage determination

- Generalized Nash bargaining between firms and households.
  - Workers have bargaining power ϕ.
  - Exogenous shifts in ϕ capture political shocks to bargaining process (Binmore *et al.*, 1986).
  - Other bargaining protocols? (Hall and Milgrom, 2008).
- Equilibrium wage solves

$$w = \arg\max_{\tilde{w}} \tilde{V}_n(\tilde{w})^{\phi} \tilde{J}_n(\tilde{w})^{1-\phi},$$

where $\tilde{V}_n$ and $\tilde{J}_n$ are marginal values of employment for households and firms given an arbitrary wage $\tilde{w}$.

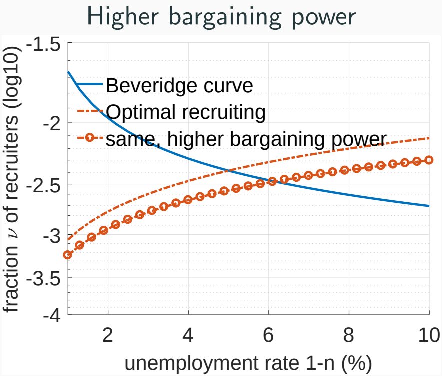
- Equilibrium wage along the balanced growth path:

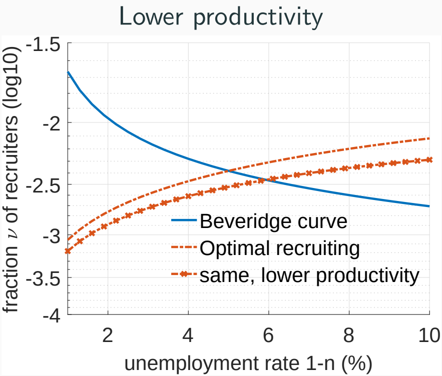
$$\bar{w} = \bar{\phi} \times (1 + \bar{\theta})\overline{mpl} + (1 - \bar{\phi}) \times \frac{\sigma}{1 - \tau_n} \left(\frac{\gamma \bar{c}}{1 + (\sigma - 1)\gamma \bar{n}} \right).$$

Equilibrium

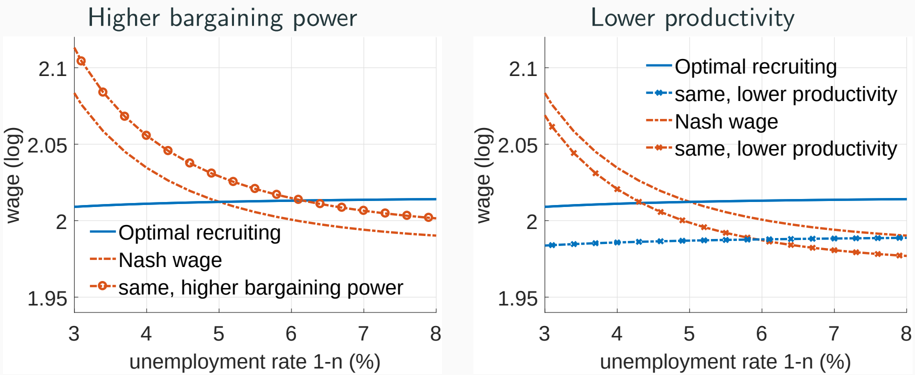
- Government.
- Standard competitive equilibrium definition.
- Market clearing $y = c + I$.
- Aggregate capital and employment follow their law of motion.
- Two exogenous AR(1) shocks:
    1. Labor productivity z_t.
    2. Bargaining power $\ln \frac{\phi_t}{1-\phi_t}$:
        2.1 Baseline: half-life shocks of 8.5 years $\approx$ average control of presidency/house/senate after WWII.
        2.2 Middle-run: half-life shocks of 20 years $\approx$ medium-term in Comín and Gertler (2006).
        2.3 Short run: half-life of 3.5 months.
        2.4 Long-run: new steady state.

Identification (I)



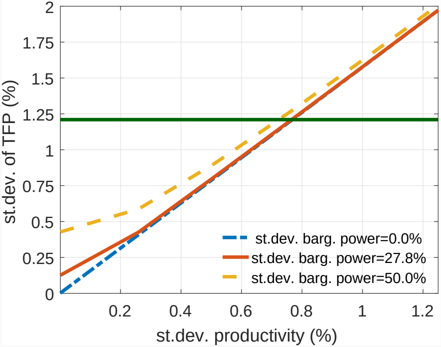


Identification (II)

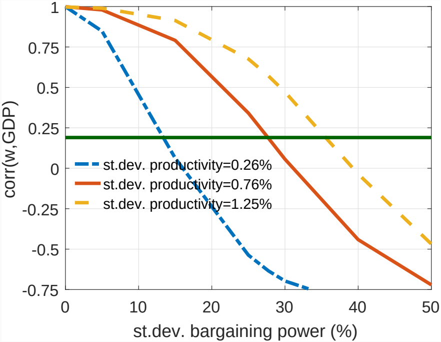


Identification (III)

Volatility of measured ftp

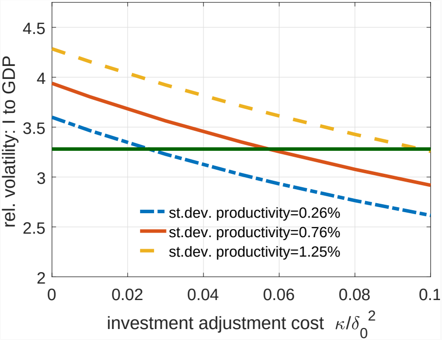


Cyclicality of wages

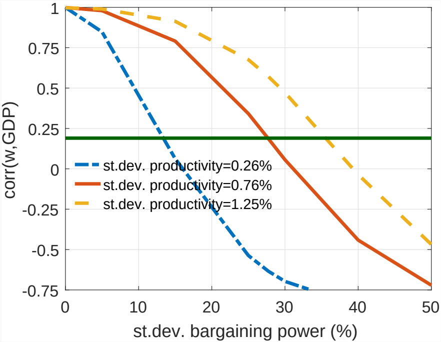


Identification (IV)

Relative volatility of investment



Cyclicality of wages



Moment matching (I)

- Solve using pruned 3rd-order approximation (Andreasen *et al.*, 2017).
- Select β, δ_0, α, and τ_k to match moments from corporate non-financial business sector:
  1. 31.2% gross capital share.
  2. 12.7% gross depreciation share.
  3. 29.9% share of taxes in net surplus.
  4. 2.3 annual K/Y ratio.
- Match labor market statistics following Shimer (2010).
- Parametrized productivity and bargaining power process to match:
  1. 1.6% annual labor productivity growth.
  2. Volatility of measured z given persistence $0.95^{1/3}$.
  3. Cyclicality of wages.
  4. Relative standard deviation of investment I relative to Y.

Moment matching (II)

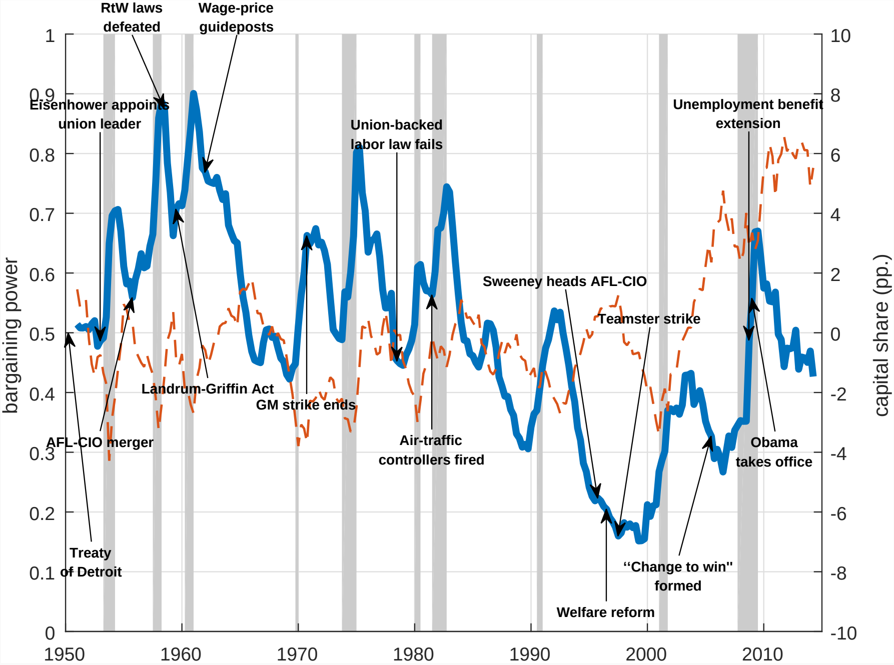
Parameter	Value	
Risk aversion σ	2	Consumption of unemployed
Discount factor β	$0.976^{1/12}$	Corp. non-financial sector
Disutility of working γ	such that $\bar{n} = 0.95$	5% unemployment rate
Capital share α	0.31	Corp. non-financial sector
Elasticity of substitution ε	1	Cobb-Douglas
Depreciation δ_0	$5.5\%/12$	Corp. non-financial sector
Trend productivity growth g_z	$1.016^{1/12}$	Cooley and Prescott '95
Inv. adj. cost κ	$0.0575 \times (\delta_0)^{-2}$	Rel. volatility of I
Capacity util. cost δ_1	such that $\bar{u} = 1$	Normalization
Capacity util. cost δ_2	$2\delta_1$	BGP ela. w.r.t. $\frac{mpk_t}{u_t}$ of $\frac{1}{2}$
Separation rate x	3.3%	Shimer '05
Bargaining power $\bar{\phi}$	0.5	
Matching elasticity η	0.5	
Matching efficiency $\bar{\mu}$	2.3 ($\mu(\bar{\theta}) = 8.4$)	Recruiting efficiency
Income tax rate τ_n	0.4	Prescott '04
Corporate tax rate τ_k	0.3	Corp. non-financial sector
Productivity persistence ρ_z	$0.95^{1/3}$	Cooley and Prescott '95
Productivity s.d. ω_z	0.76%	z volatility
Barg. power persistence ρ_ϕ	$0.98^{1/3}$	8 year half-life
Bargaining power s.d. ω_ϕ	27.75%	Wage cyclicality

Implementation of the partial filter

- Bargaining power enters only wage-setting.
- Wage-setting equation implies:

$$
\begin{aligned}
& e^{\ln \frac{\phi_t}{1-\phi_t}} \left(\left(mpl_t \left(1 + \frac{1-x}{\mu(\theta_t)}\right) - w_t \right) \right. \\
& \left. - (1 - x - f_t(\theta_t)) e^{\kappa_\phi + (\rho_\phi - 1)\ln \frac{\phi_t}{1-\phi_t} + \frac{1}{2}\omega_\phi^2} \left(\mathrm{Cov}_t[\circ] + \frac{mpl_t}{\mu(\theta_t)} \right) \right) \\
= & w_t - \frac{1}{1-\tau_n} \left(\frac{c_t}{1 + (\sigma - 1)\gamma n_{t-1}} \right) \gamma\sigma, \qquad \text{where} \\
\mathrm{Cov}_t[\circ] = & \mathrm{Cov}_t \left[\ln \frac{\phi_t}{1-\phi_t}, m_{t+1} \left(mpl_{t+1} \left(1 + \frac{1-x}{\mu(\theta_{t+1})} \right) - w_{t+1} \right) \right].
\end{aligned}
$$

- Given Cov_t, solve for $\ln \frac{\phi_t}{1-\phi_t}$. Iterate in Gibbs-Sampler.



RtW laws defeated
Wage-price guideposts
Eisenhower appoints union leader
Union-backed labor law fails
Unemployment benefit extension
Sweeney heads AFL-CIO
Teamster strike
Landrum-Griffin Act
GM strike ends
AFL-CIO merger
Air-traffic controllers fired
Obama takes office
Treaty of Detroit
"Change to win" formed
Welfare reform
bargaining power
capital share (pp.)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10
8
6
4
2
0
-2
-4
-6
-8
-10
1950
1960
1970
1980
1990
2000
2010

Concluding remarks

- We have other implementations.
- For example: sticky leverage of Gomes, Jermann, and Schmid (2016).
- However, many things to do:
    1. Embedding the model in an RBC model could aid in the calibration.
    2. Can we use machine-learning tools to improve the covariance/expectations computation?
    3. Heterogeneous agent model.
    4. Small sample results.
    5. Role for state smoothing due to estimation uncertainty/approximation error.