

# **Estimation of Operational Macromodels at the Zero Lower Bound**

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*The views expressed are solely those of the authors and should not be interpreted as reflecting the views of the National Bank of Belgium, Norges Bank or Sveriges Riksbank*

# Challenges Posed by the Great Recession

- **The Global Financial Crisis and the resulting Great Recession triggered leading CBs as the Fed, ECB, and the BoE to cut policy rates to zero or near zero.**
- **Estimation sample contains period with binding ZLB constraint:**
  - **compare alternative estimation methods that allow for occasionally binding ZLB in medium-scale macro-model;**
  - **discuss impact on estimated parameters, shocks, forecasts and irfs;**
  - **discuss estimates of the cost of the ZLB.**

## **Challenges Posed by the Great Recession (Cont.)**

- **Not surprisingly, there is a rapidly expanding literature on assessing the empirical gains of the explicit treatment of the ZLB.**
  - **Fratto and Uhlig (2014), on the one hand, argue that the ZLB is seemingly unimportant to understand the behaviour of the U.S. economy in the workhorse Smets and Wouters model.**
  - **On the other hand, Kulish, Morley and Robinson (2014), Binning and Maih (2016), Guerrieri and Iacoviello (2013), Gust, Herbst, Lopez-Salido and Smith (2016), and Richter and Throckmorton (2016) suggest that ZLB is key to understand the dynamics of prices and quantities during the Great Recession.**
  - **Wu and Zhang (2016) use Shadow Rate as observable for estimating their DSGE model over the recent sample.**

# What We Do

- **We impose the ZLB through two alternative approaches during estimation of the DSGE model:**
  - **With anticipated monetary policy shocks: the expected ZLB duration is endogenously determined by the model forecast given the state of the economy and the policy rule (~endogenous ZLB duration).**
  - **With a regime switching setup: the ZLB regime is characterized by a fixed interest rate rule. In the baseline RS-version, the regime switching is treated as exogenous (~exogenous ZLB duration) but alternatives with endogenous regime switching probabilities (and breaks in the natural real rate) are also considered.**
  - **Compare with models estimated without any treatment for the ZLB.**
  - **Estimation implemented in RISE.**

# What We Do

- **Relative to the current literature, we differ by doing this in a large scale model with many shocks and observables.**
  - **Full nonlinear solution methods cannot easily be extended to setups with larger set of shocks (state variables) or observables.**
- **We pursue our analysis in an altogether linearized model, apart from the ZLB constraint.**
  - **To the extent the linearized model behaves very differently far off the steady state, the downside of this procedure can be considerable.**
  - **The benefit of this approach is that we can parse out the partial derivative of imposing the ZLB in a workhorse linearized macro model. Other studies have often mixed several mechanisms: non-linearities and ZLB.**
- **We estimate two models (SW-2007 & GSW-2011) with different views on output gap and recovery to test the robustness.**

## What We Do (Cont.)

- **We assess the empirical implications of accounting for the ZLB in estimation on several key dimensions:**
  - **Parameter estimates: no systematic changes related to ZLB;**
  - **Estimates of the shocks: confusion between MP and RP shocks;**
  - **Bayesian ML: models with ZLB treatment fit much better;**
  - **Forecasts: point forecasts and prediction densities are very sensitive;**
  - **Impulse responses: important time variation for risk premium and wage markup shocks; less so for TFP and government spending shocks;**
  - **Macroeconomic costs of the ZLB: large with ZLB constraint explaining an important share of the output gap.**

# Remainder of Talk

- **models and extensions**
- **estimation methodologies**
- **estimation results**
- **evaluation of the cost of the ZLB constraint**

**(Caveat: this is work in progress!)**

# Augmented SW and GSW Model

- **original SW 2007: 7 US-time series & 7 exogenous shocks**
- **original GSW 2011: observe UR, two wage concepts and endogenous labor supply**
- **add 2 year Treasury yield to the list of observables**
- **include risk premium and term premium in the policy rule**



## Augmented SW and GSW Model (Cont.)

- When **unconstrained**, monetary policy rule is

$$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + (1 - \rho_R) \left[ r_\pi \widehat{\pi}_t + r_y (\widehat{ygap}_t) + r_{\Delta y} \Delta(\widehat{ygap}_t) + r_{tp} \widehat{\varepsilon}_t^{tp} + r_{rp} \widehat{\eta}_t^{rp} \right] + \widehat{\varepsilon}_t^r$$

- When **constrained** by the **ZLB**:

- The **Endogenous ZLB duration** depends on a shadow interest rate concept: lower for longer policy (RW, 2000, EW, 2003).

$$\begin{aligned} \widehat{R}_t^* &= \rho_R \widehat{R}_{t-1}^* + (1 - \rho_R) \left[ r_\pi \widehat{\pi}_t + r_y (\widehat{ygap}_t) + r_{\Delta y} \Delta(\widehat{ygap}_t) + r_{tp} \widehat{\varepsilon}_t^{tp} + r_{rp} \widehat{\eta}_t^{rp} \right] \\ \widehat{R}_t &= \max \left( -\bar{r}, \widehat{R}_t^* \right) \quad \text{with max-operator implemented via } E_t(\widehat{\eta}_{t+h}^r) \end{aligned}$$

- Model with **Exogenous ZLB Duration** (Regime-Switching):

$$\widehat{R}_t = -\bar{r} + \sigma(zlb) \cdot \widehat{\eta}_t^r$$

## Augmented SW and GSW Model (Cont.)

- To enhance the consistency between the policy rule based expectation of the ZLB duration and market expectations, we include the 2-year Treasury yield as observable:

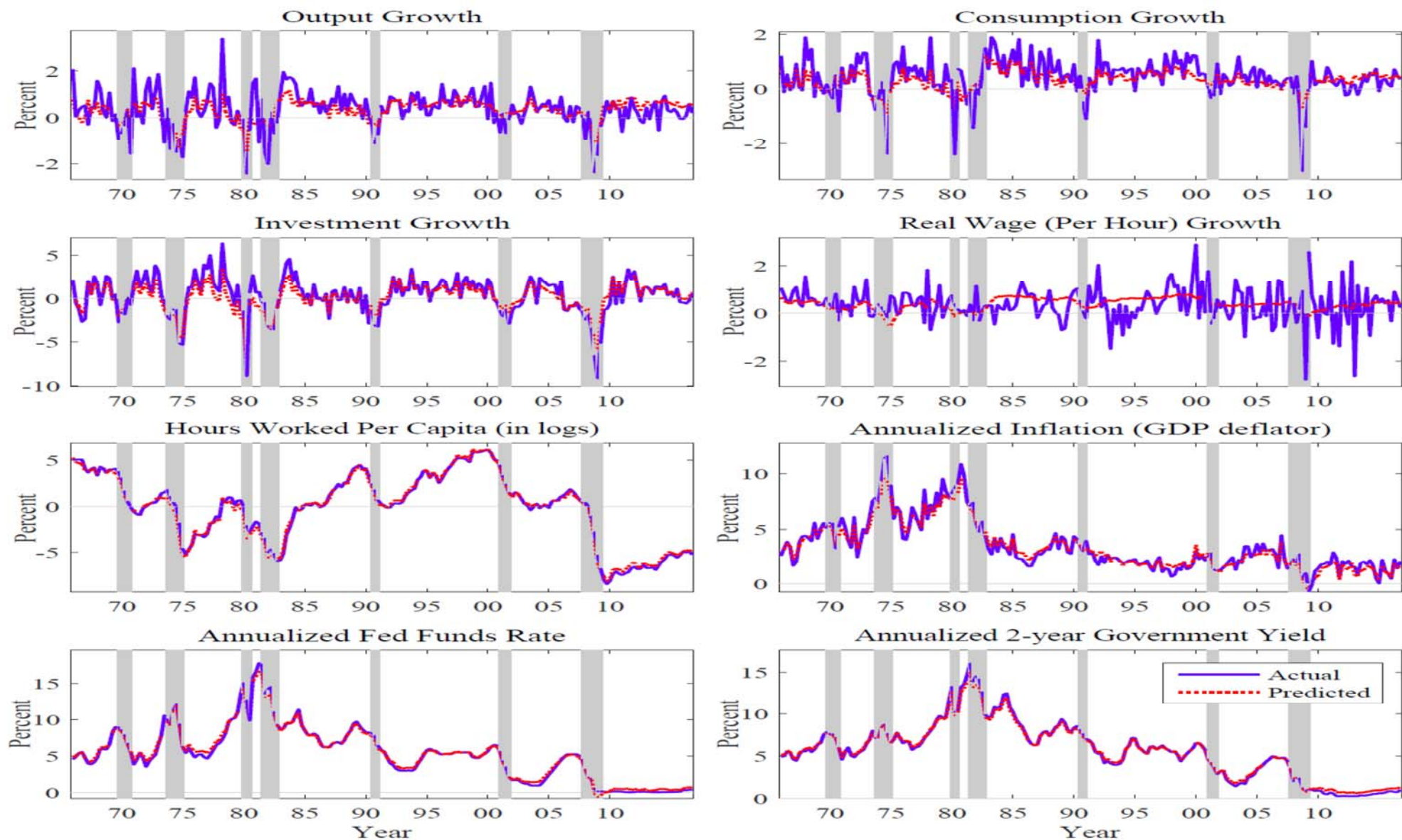
$$\widehat{R}_t^{2Y} = \sum_{h=0,7} \widehat{R}_{t+h}^G \quad \text{with} \quad \widehat{R}_t^G = \widehat{R}_t \cdot \widehat{\varepsilon}_t^{tp}$$

- Allow for a feedback channel of term-premium shocks ( $\varepsilon_t^{tp}$ ) by letting effective interest rate facing households and firms be determined as:

$$\widehat{R}_t^{hh\&f} = \left(\widehat{R}_t\right)^{1-\kappa} \left(\widehat{R}_t^G\right)^{\kappa} \widehat{\varepsilon}_t^{rp}$$

- Distinguish between monetary policy ( $\varepsilon_t^r$ ), term-premium ( $\varepsilon_t^{tp}$ ) and risk premium ( $\varepsilon_t^{rp}$ ) shocks by including both  $R_t$  and  $R_t^G$  as observables (positive **mp** shock increases both  $R_t$  and  $R_t^G$ , positive **tp** shock induces higher wedge  $R_t^G > R_t$ , positive **rp** shock captures residual intert.wedge)

# Data and One-sided Filtered Estimates in **no ZLB model** 66Q1-16Q4



## **Estimation Methodology: endogenous ZLB duration**

- **When estimating the models with anticipated monetary policy shocks to implement the ZLB constraint, we use the Sigma filter to approximate the asymmetry and the time-variation in predictive density (see Binning and Maih 2015).**
  - **The prediction step in the filter is not based on zero future innovations but is averaged over a set of sigma points: we use one period ahead shocks only, with large stdev to assess the impact of the expected ZLB-constraint.**
  - **The mean forecast depends on the asymmetry in the predictive density.**
  - **The covariance matrix for the one-step-ahead prediction errors at the ZLB features an increasing uncertainty for real/nominal variables and decreasing uncertainty around the interest rate as the probability of a ZLB-continuation goes to one.**
  - **The updating step also satisfies the ZLB constraint (~cond. forecasting).**

## **Estimation Methodology: exogenous ZLB duration**

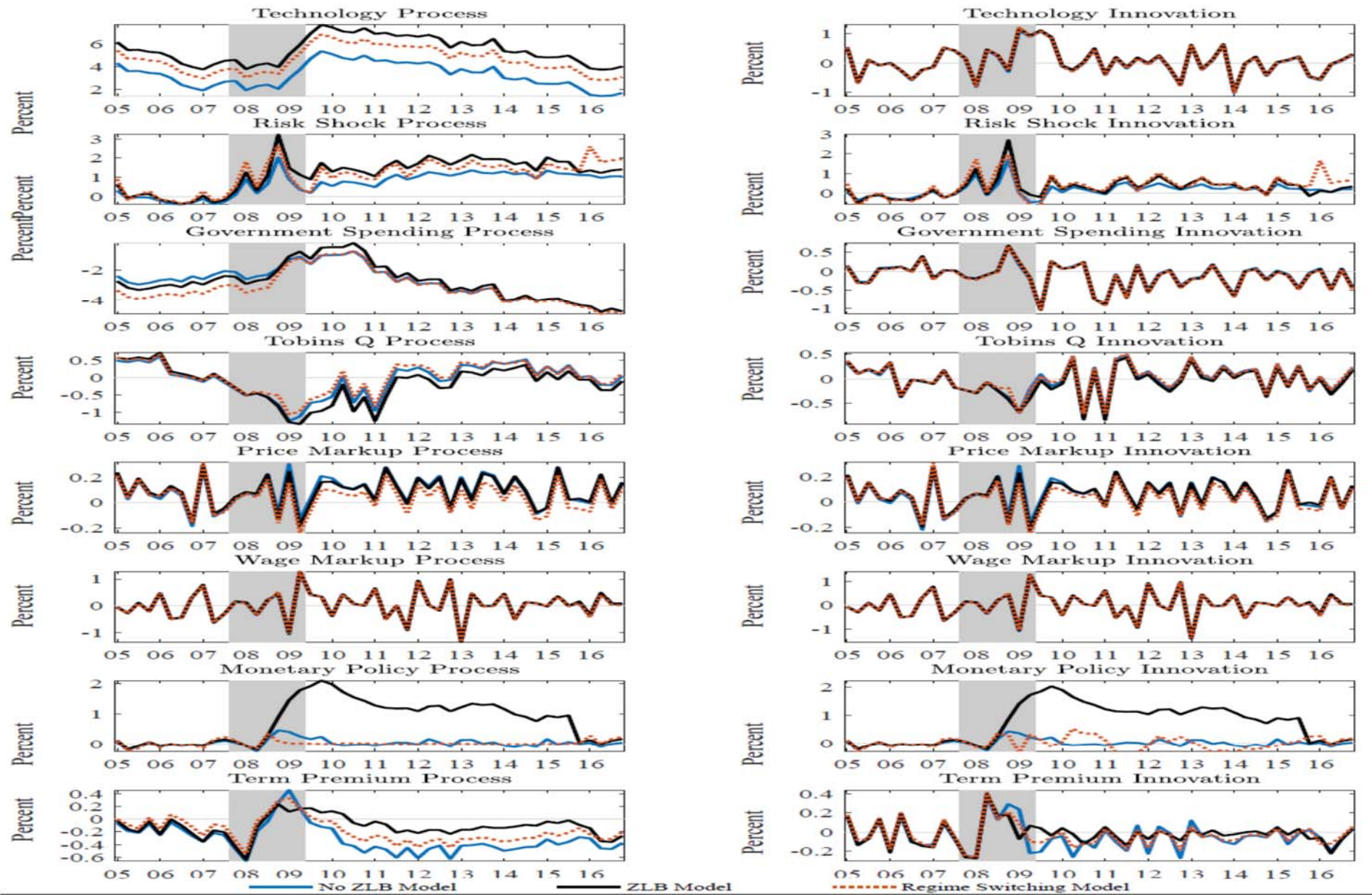
- **When estimating the models with a Regime-Switching approach to implement the ZLB:**
  - ZLB incidence is approximated through RS-methods (see e.g. Farmer, Waggoner and Zha, 2011, and Maih, 2015).
  - ZLB regime is linearized around an imposed steady state (=normal regime steady state)
  - The estimated exogenous probability of switching from “ZLB” to “Normal” regime determines the expected ZLB duration:  $p_{21} = 0.32$  ( $p_{12} = 0.01$ )
- **Extensions:**
  - Endogenous regime switching probability:  $p_{21}, p_{12} = \text{logistic.f}(R^*)$
  - ZLB regime break also implies a break in the risk premium (increased preference for liquidity and safety - see Del Negro et al 2017).

# Estimation Results: parameters and shocks

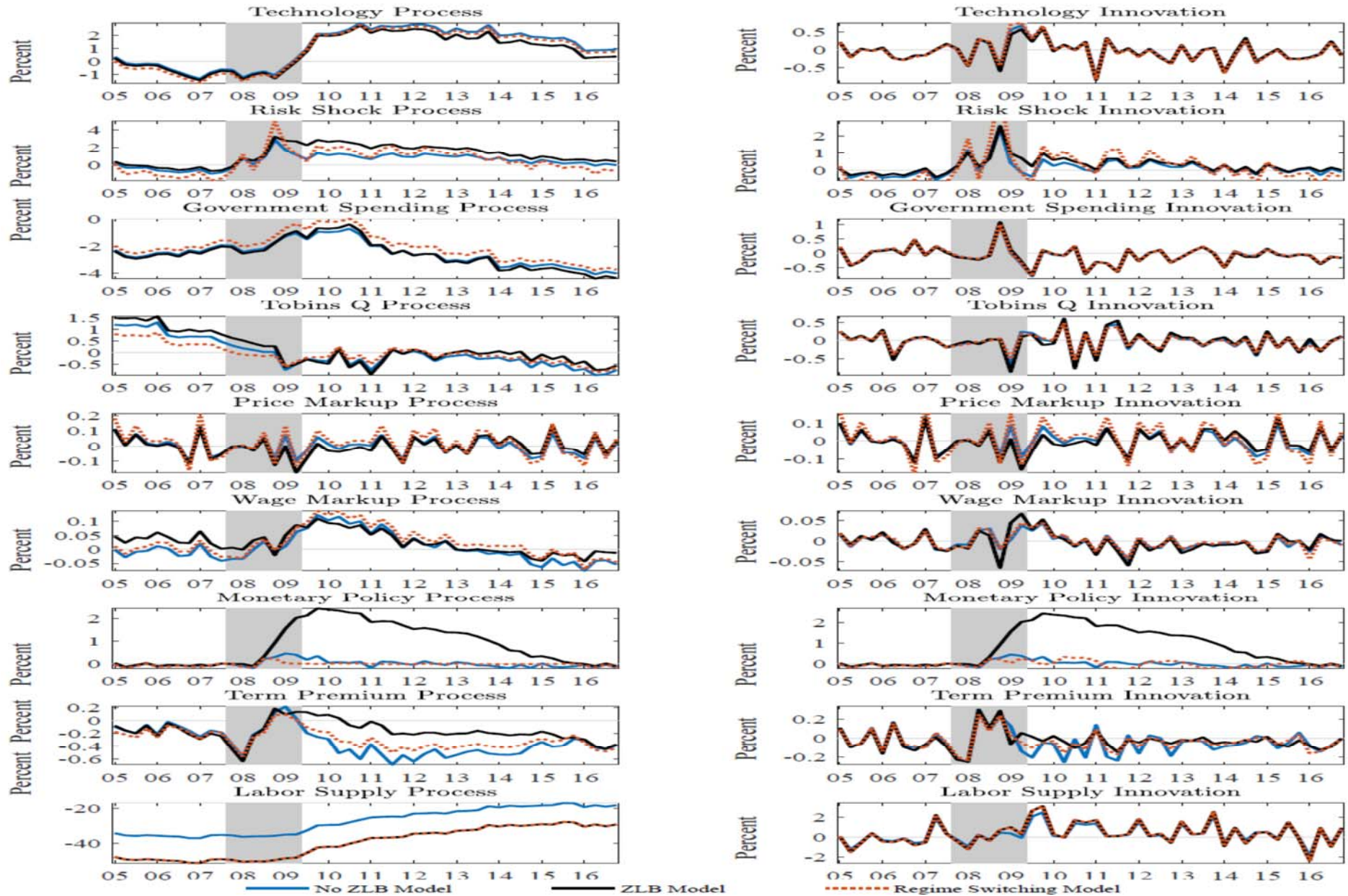
- **By and large, parameters are not much affected by the ZLB.**
  - Higher price and wage stickiness when including the Great Recession in the estimation sample is independent of the ZLB-approach ( $\Leftrightarrow$  LSW 2016).
- **All models identify a similar mix of shocks to account for the Great Recession and the subsequent slow recovery:**
  - An increase in the risk premium
  - Negative shocks to investment-specific technology
  - Positive MP shocks
  - Positive TFP shocks during recession
- **The specific ZLB-treatment determines mainly the relative contribution of RP versus anticipated MP shock: these are close substitutes in SW-context.**



# Filtered Shocks SW



# Filtered Shocks GSW





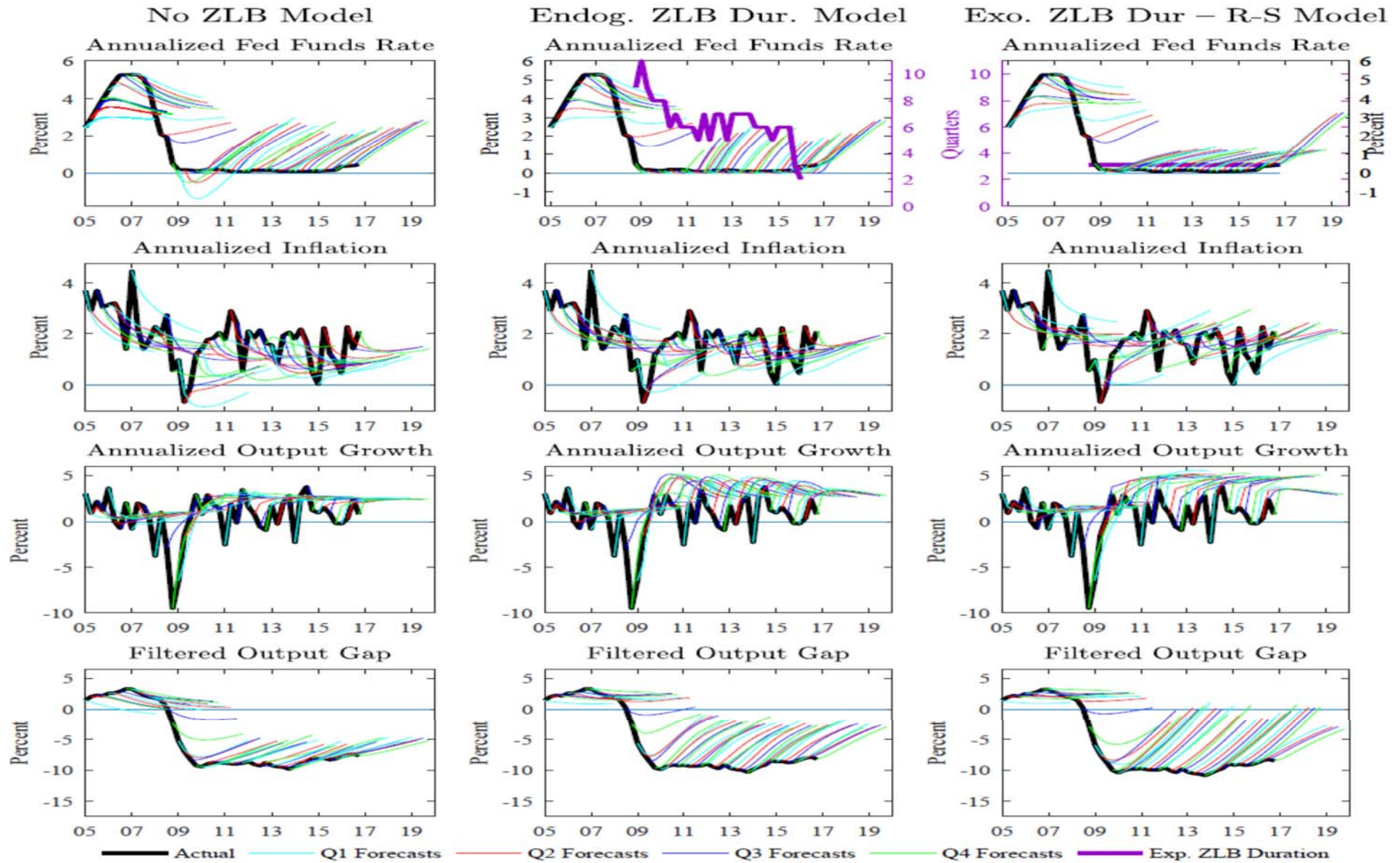
## Estimation Results: marginal likelihood

- ML improves considerably when explicitly accounting for ZLB:

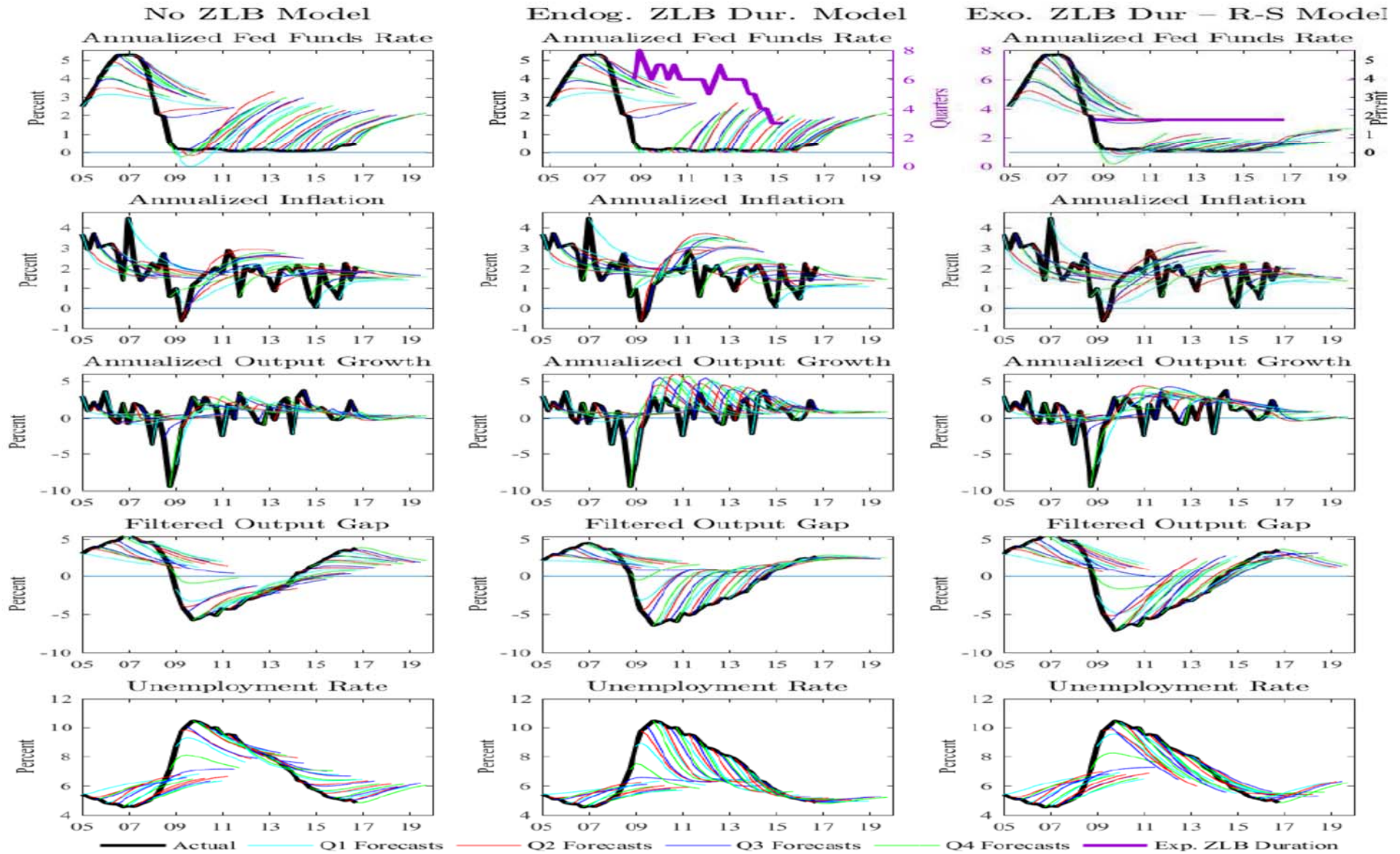
	SW	GSW
No ZLB	-1290.5	-1269.5
Endogenous ZLB duration	-1242.7	-1232.2
Exogenous ZLB duration	-1248.1	-1234.4

- Smoothing over the **shadow rate** is crucial for improved ML in endogenous ZLB model ( $\Leftrightarrow$  LSW 2016).
- Further gains are possible for RS with endogenous switching probabilities & break in risk premium during ZLB period.
- What is the source of this gain? Not parameters, not shocks, but changes in the propagation mechanism during ZLB period!

# Estimation Results: recursive forecasts SW

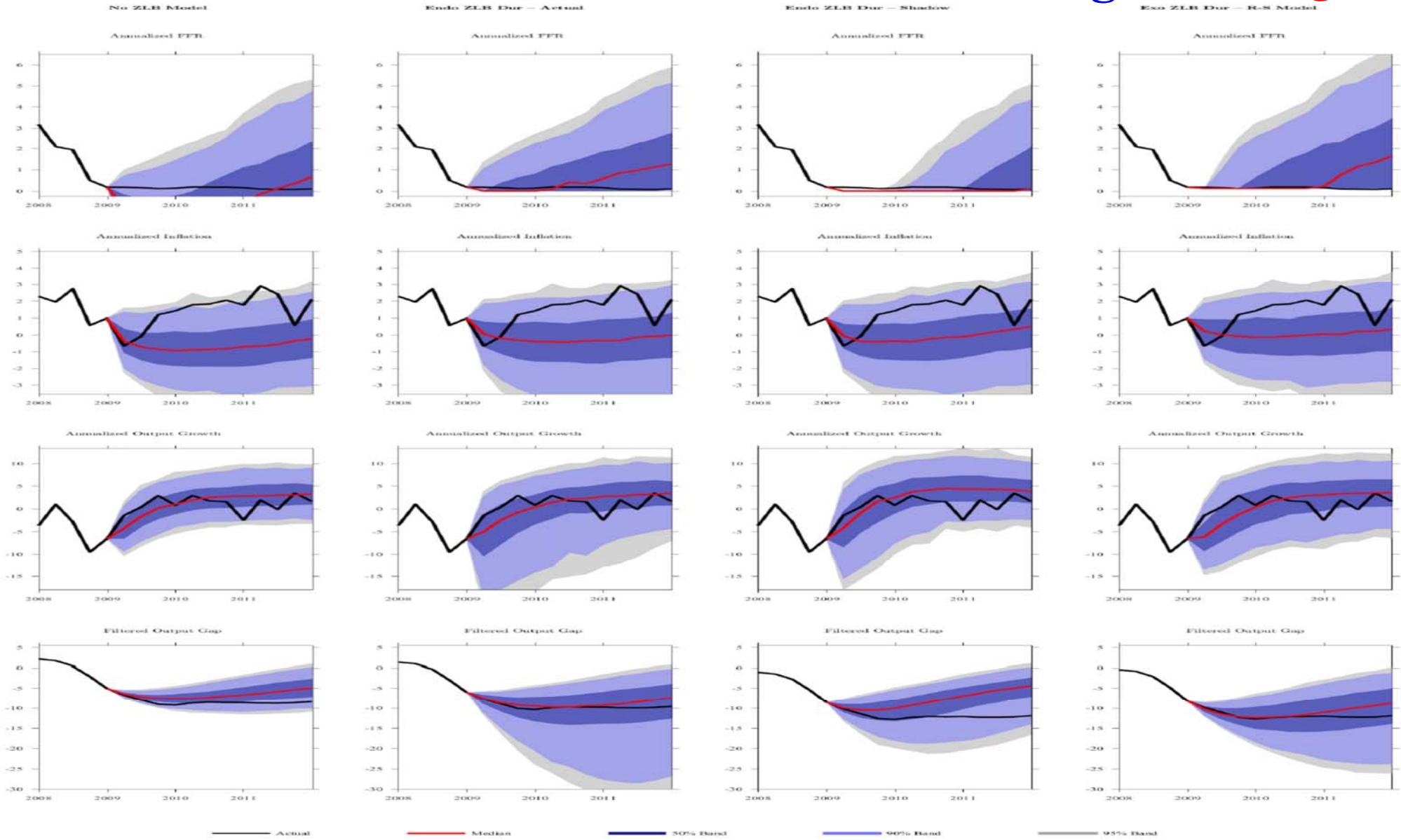


# Estimation Results: recursive forecasts GSW

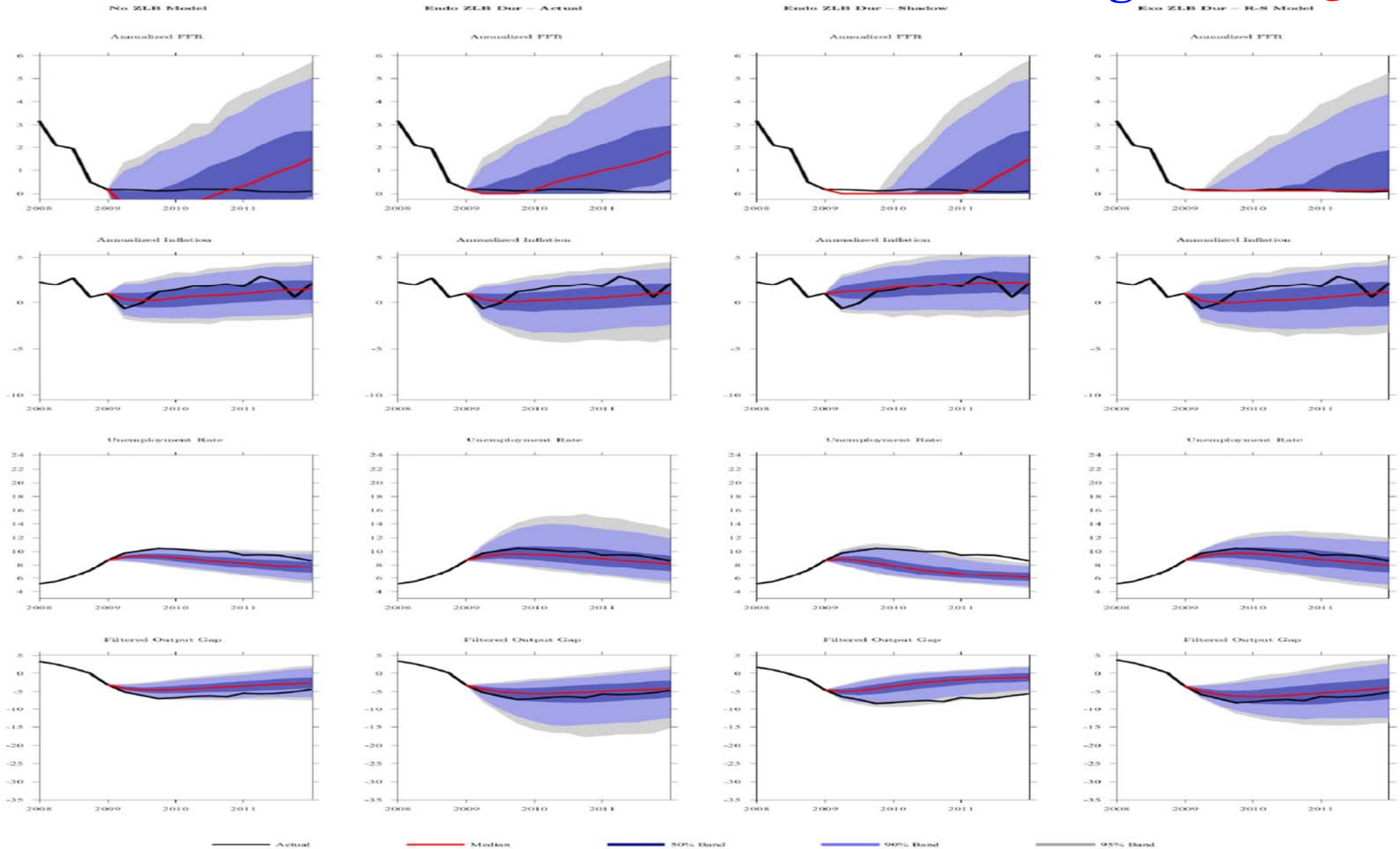




# Estimation Results: SW Post. Pred. Dens. given 09Q1



# Estimation Results: GSW Post. Pred. Dens. given 09Q1

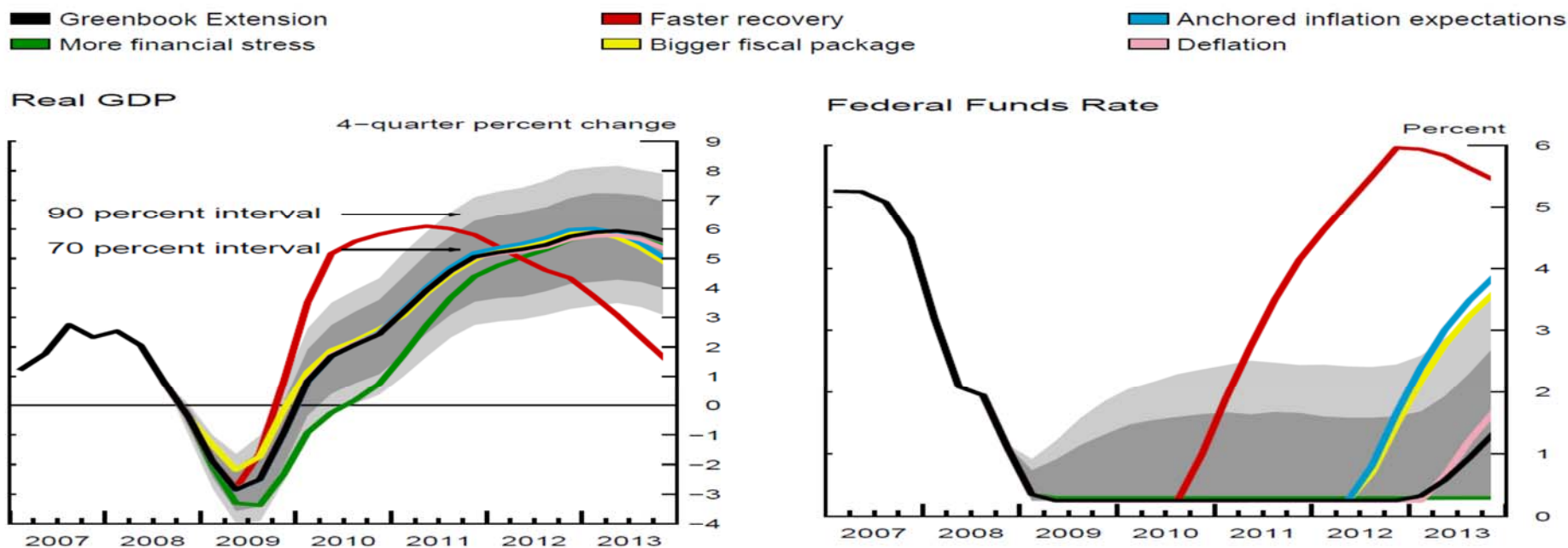


# Estimation Results: Comparison with the Fed

- The endogenous duration ZLB model predicts long ZLB durations in 2009-2010H1, and quicker lift-off afterwards. OIS rates imply exactly the opposite with long zlb-durations first in 2011H2-2012. Our model results are in between OIS (Shadow Rate) and Fed Greenbook forecasts in December 2008.

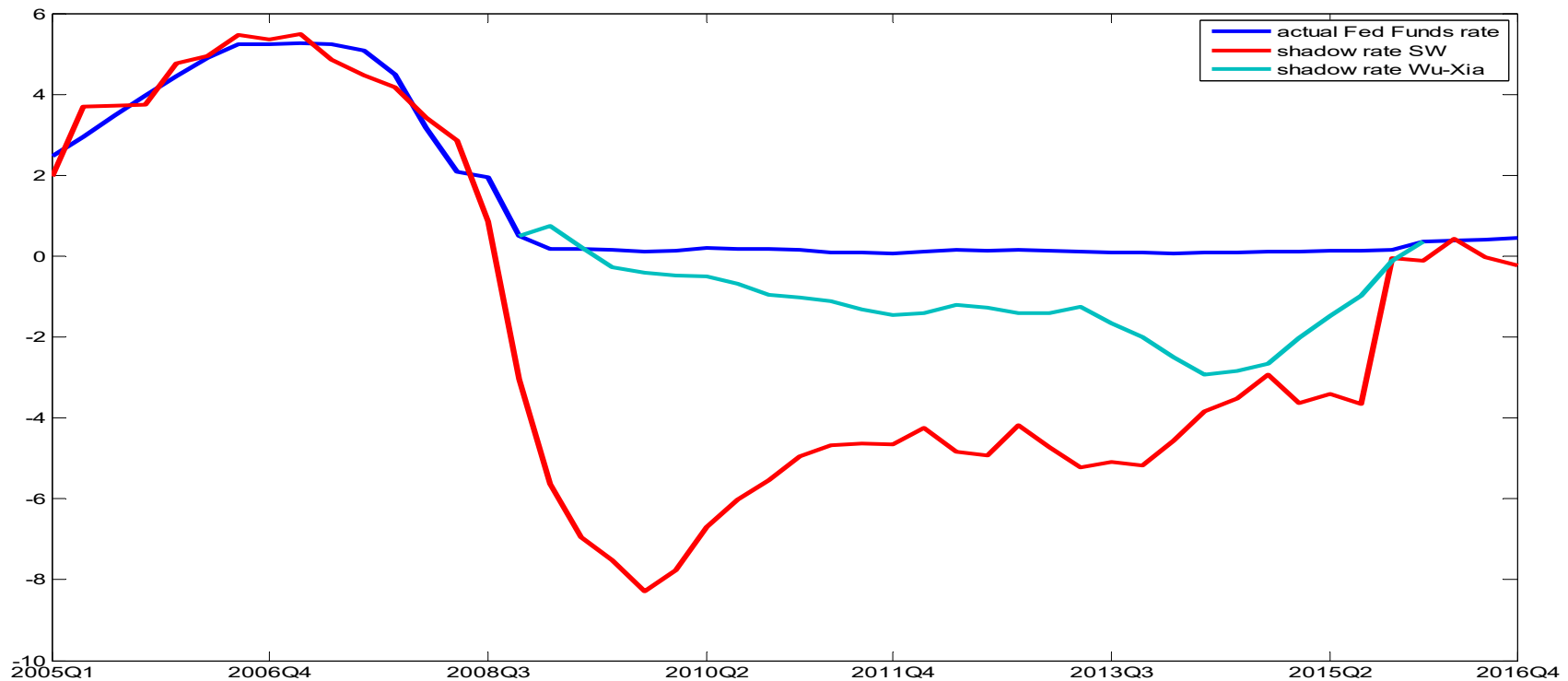
## Forecast Confidence Intervals and Alternative Scenarios

Confidence Intervals Based on FRB/US Stochastic Simulations



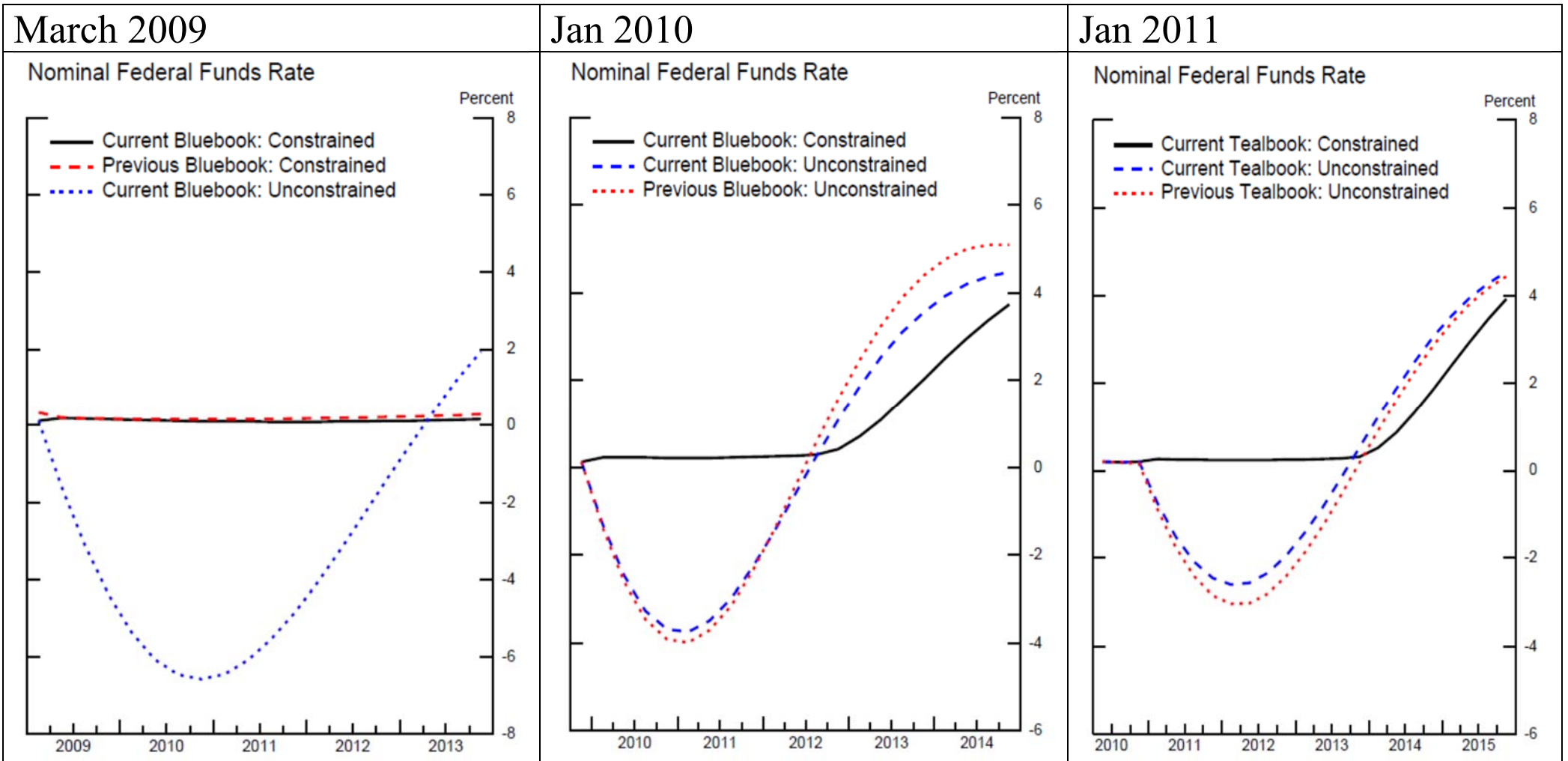
# Estimation Results: Shadow rate series

- Shadow rate implied by Endogenous ZLB duration models and the yield curve based models (Wu&Xia 2015) behave very differently:



# Estimation Results: Shadow rate series

Constrained vs. Unconstrained Monetary Policy  
(2 Percent Inflation Goal)



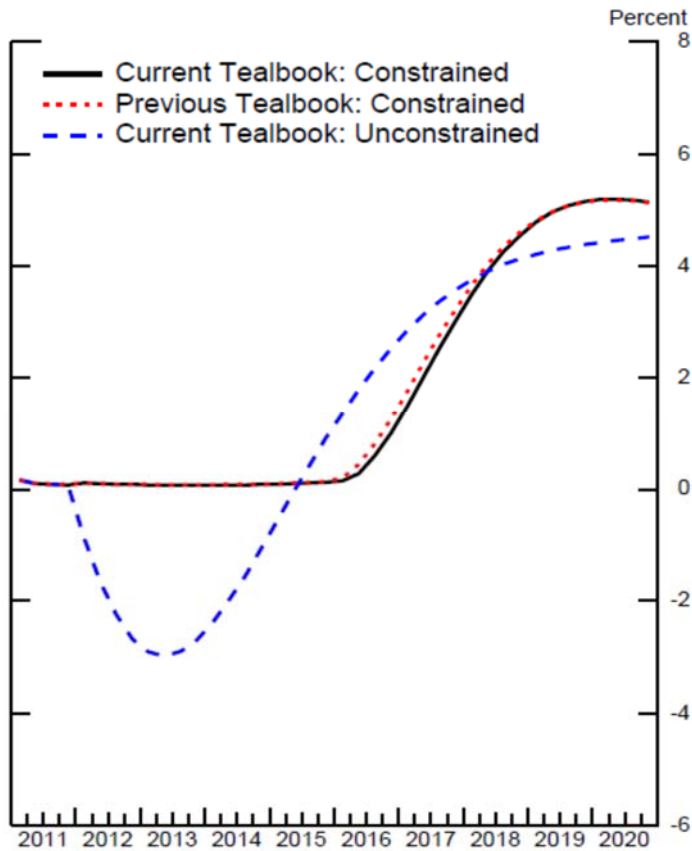


# Estimation Results: Shadow rate series

Constrained vs. Unconstrained Monetary Policy  
(2 Percent Inflation Goal)

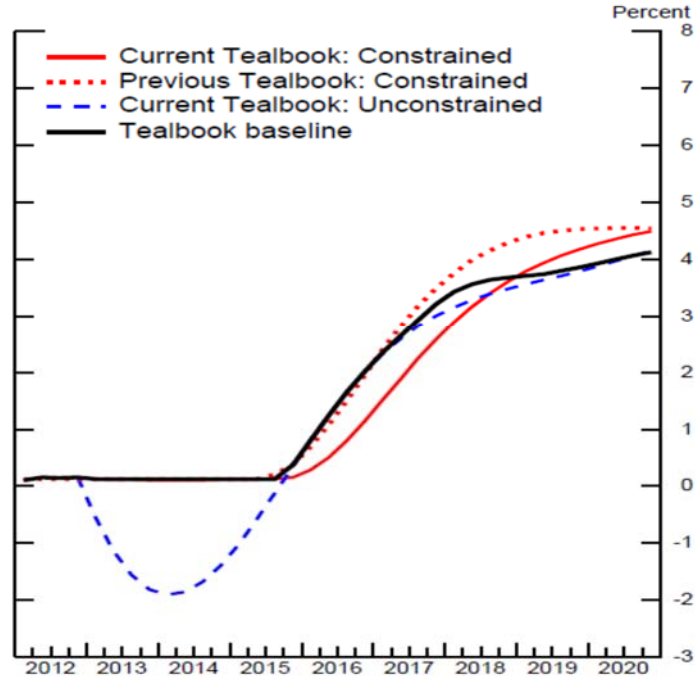
March 2012

Nominal Federal Funds Rate



Dec 2012

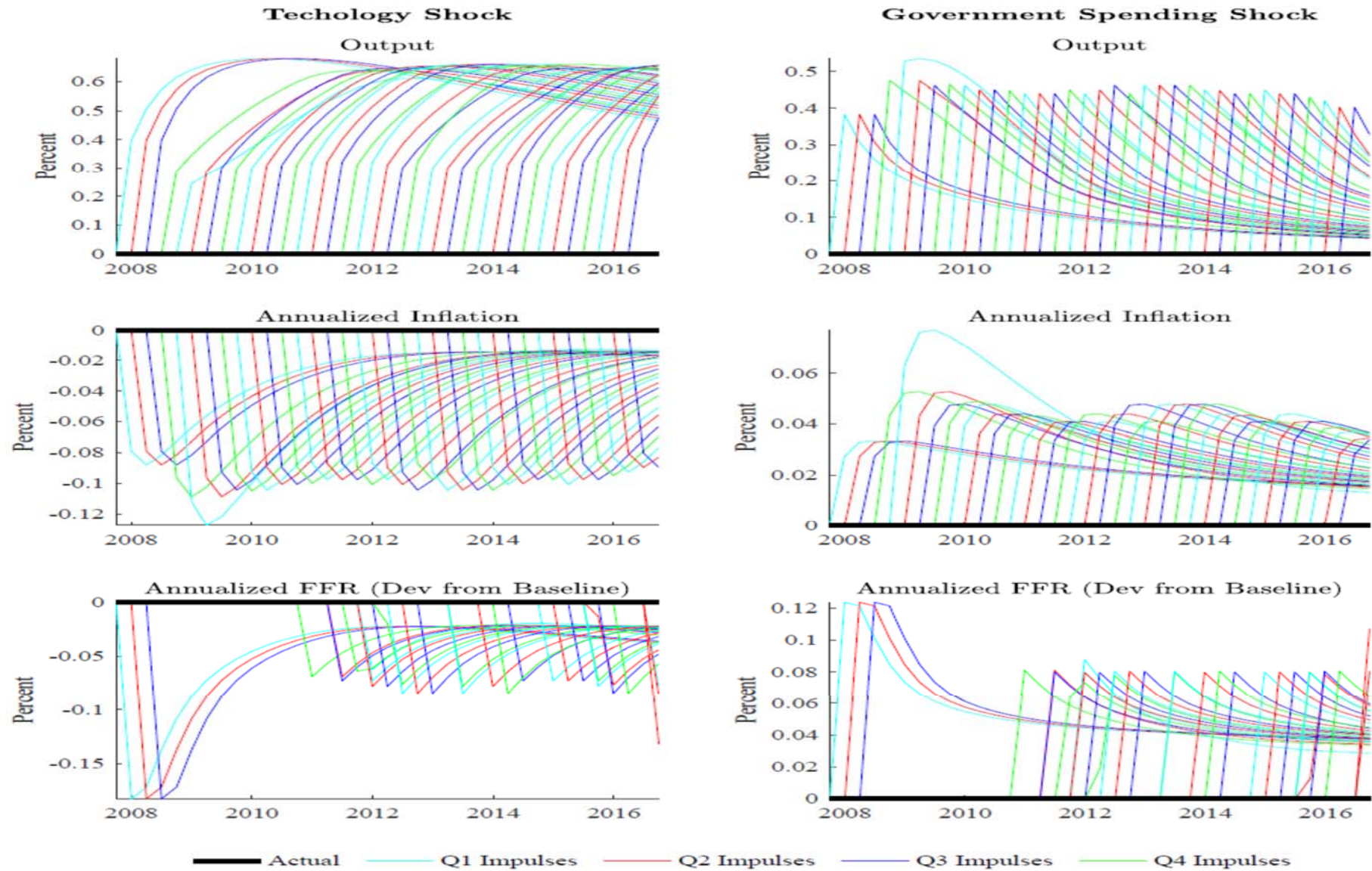
Nominal Federal Funds Rate



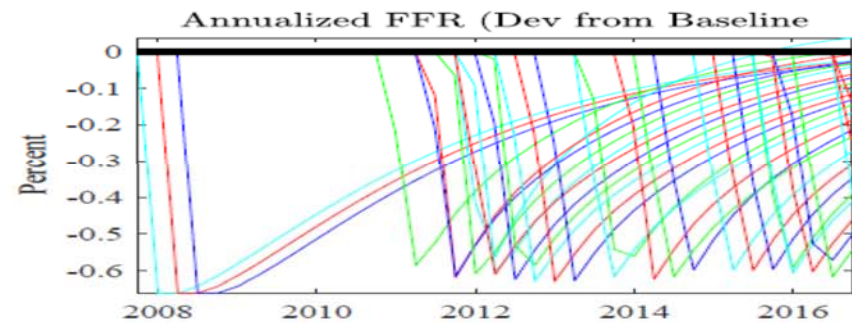
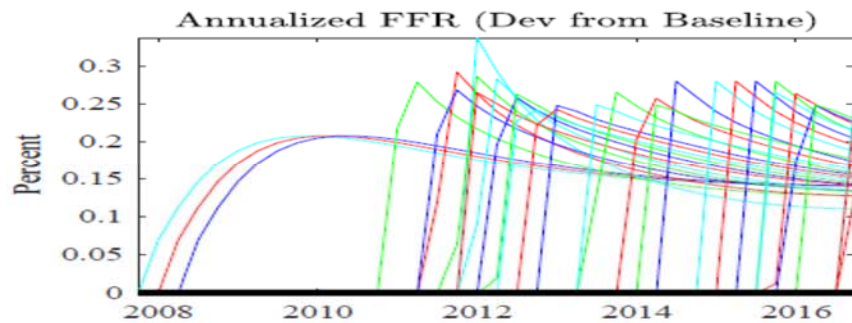
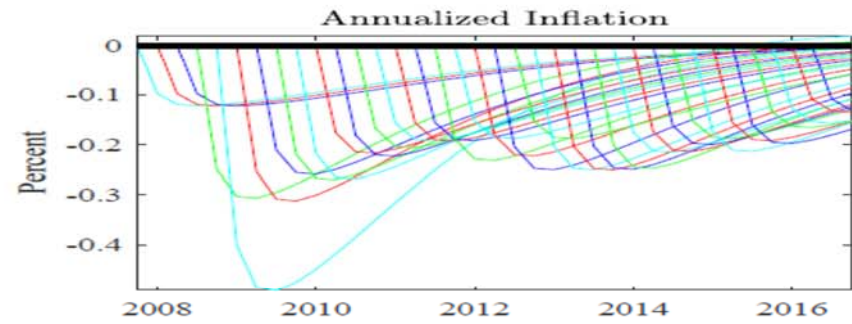
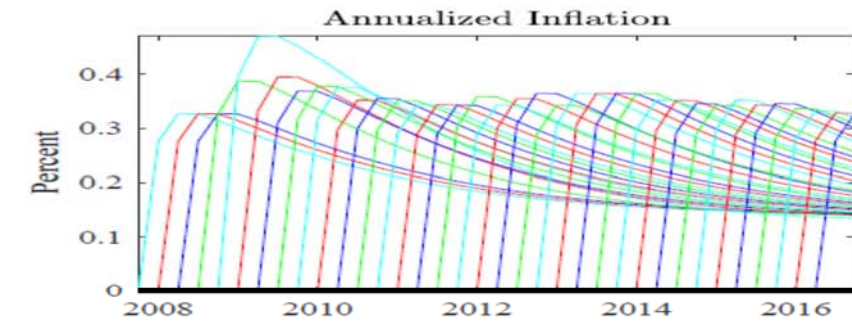
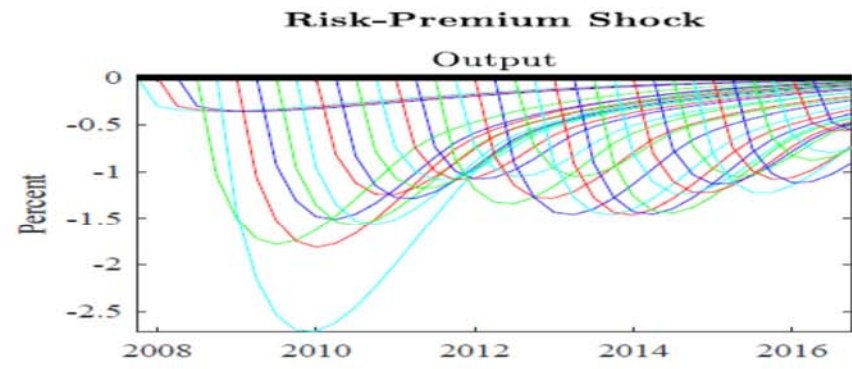
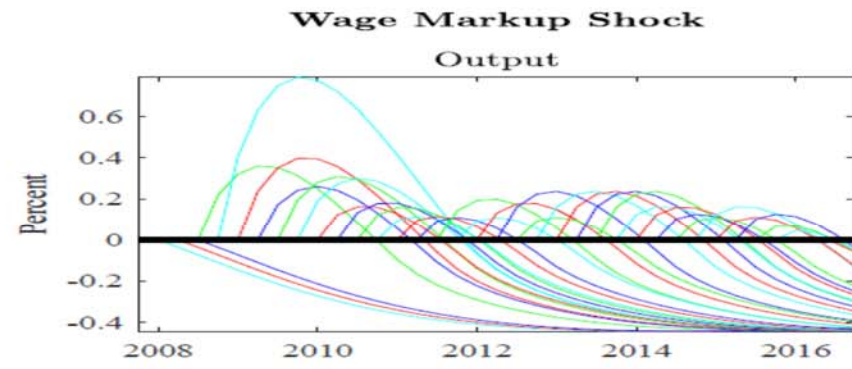
## **Estimation Results: SW endogenous ZLB duration - irfs**

- **To document the time variation of the propagation mechanism induced by the ZLB, we simulate the impulse response functions of the various shocks period by period:**

# Estimation Results: SW endogenous ZLB duration - irfs



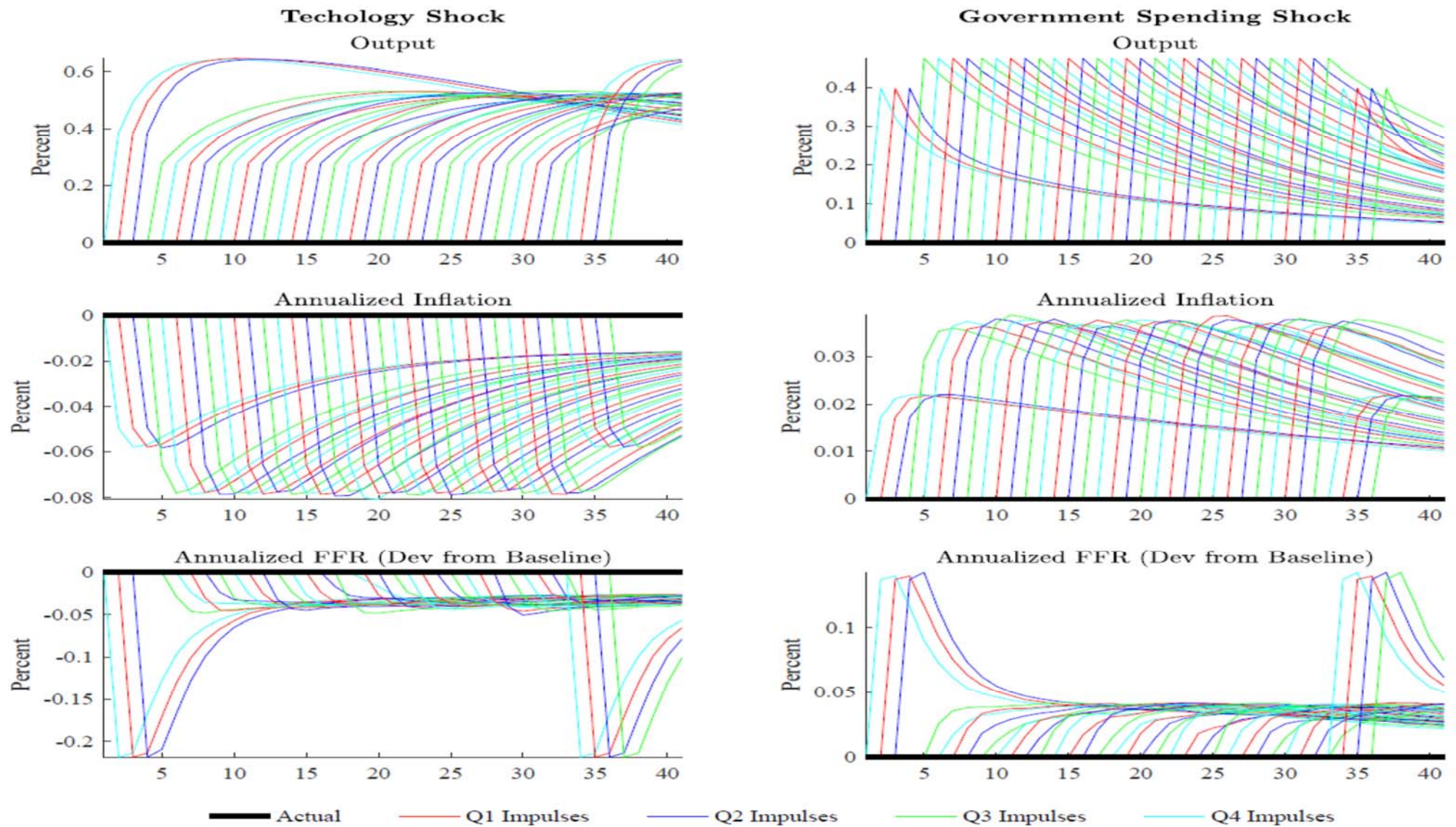
# Estimation Results: SW endogenous ZLB duration - irfs



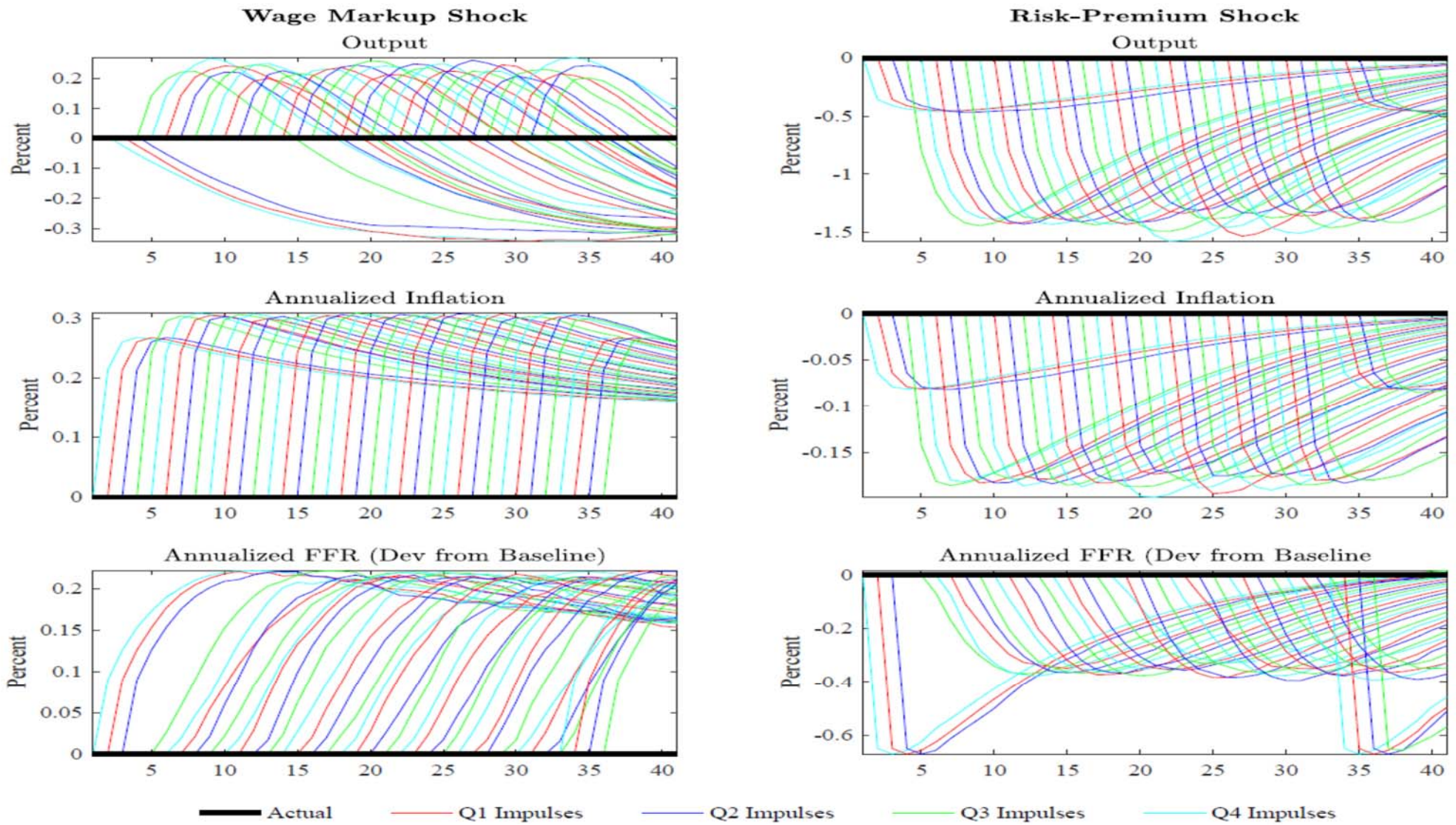
— Actual    — Q1 Impulses    — Q2 Impulses    — Q3 Impulses    — Q4 Impulses



# Estimation Results: SW exogenous ZLB duration - irfs



# Estimation Results: SW exogenous ZLB duration - irfs

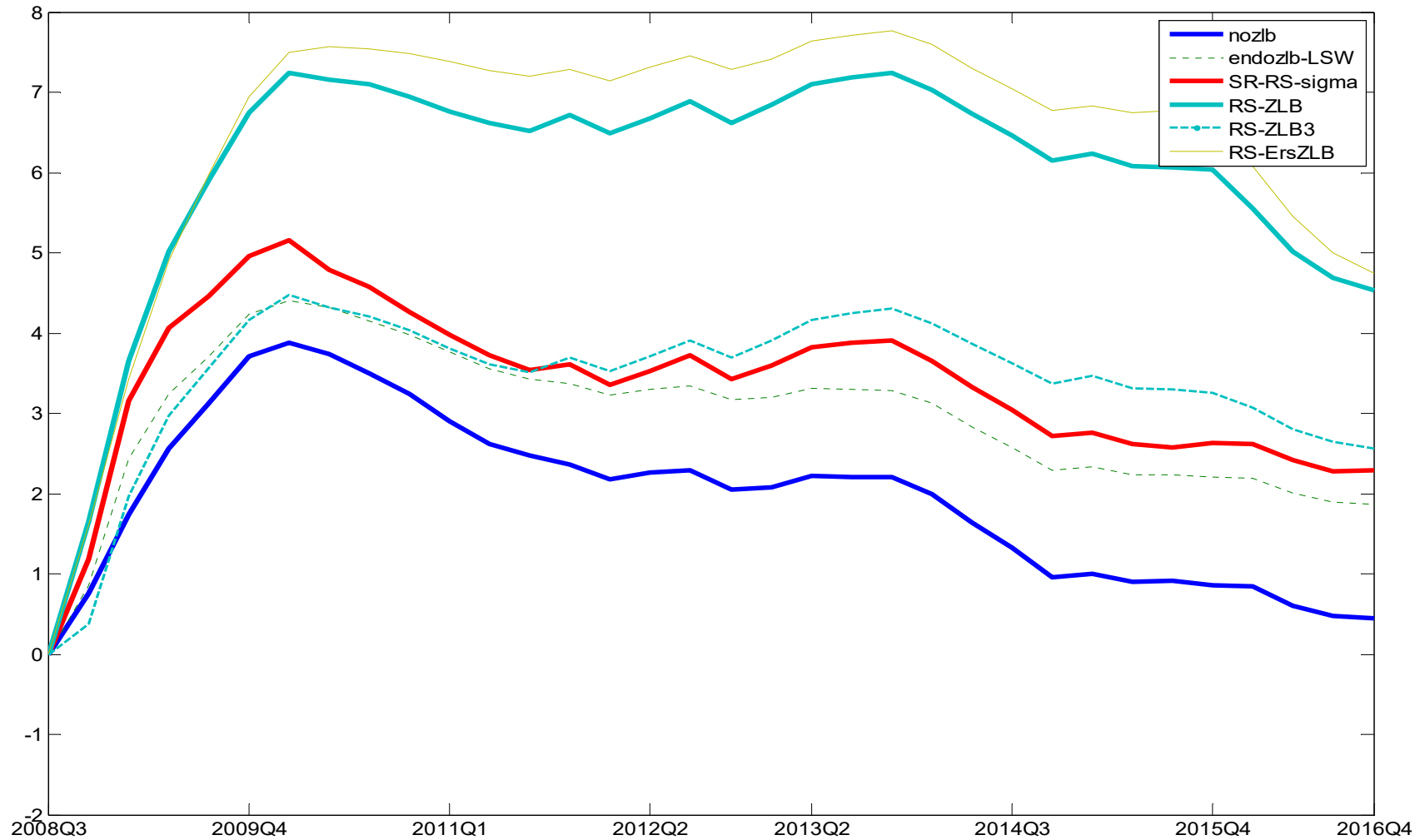




# Evaluate costs of the ZLB

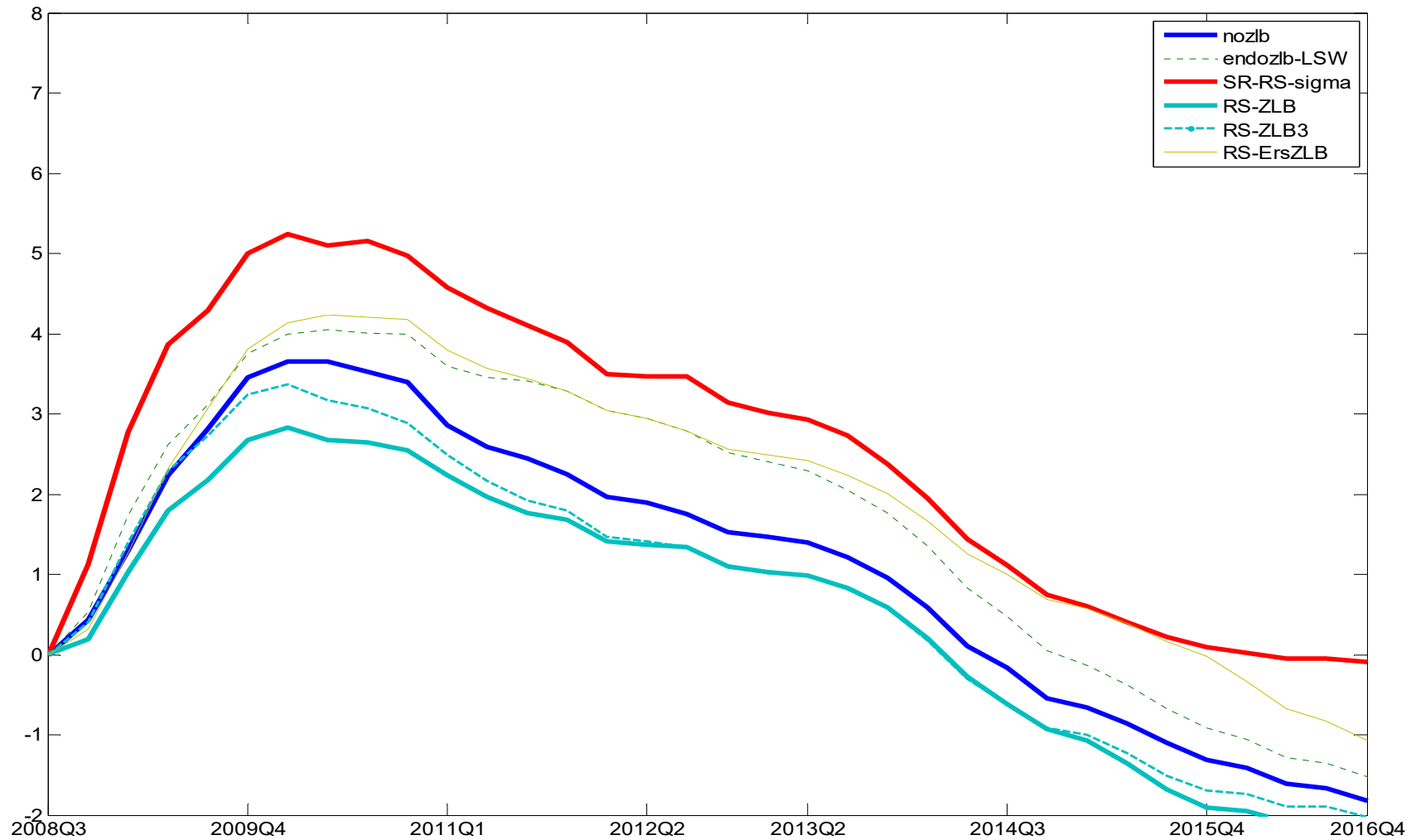
- **Compute cost of ZLB by making a counterfactual dynamic simulation of how much higher output would have been without constraints on monetary policy from 2008Q4 and onwards.**
  - **No ZLB model**, no mp shocks from this quarter.
  - In the **endogenous ZLB model**, we turn off the max operator (i.e. all current and anticipated mp shocks) and simulate the impact of all other shocks as if policy was unconstrained.
  - In **R-S model**, we assume “Normal” regime prevails and turn off policy shocks but use all other shocks in counterfactual simulation.
- **Cost = Counterfactual with No ZLB / Unconstrained Path**
  - **Simulated path with constraints and all shocks (~history).**

# Evaluate cost of ZLB: SW with alternative methods

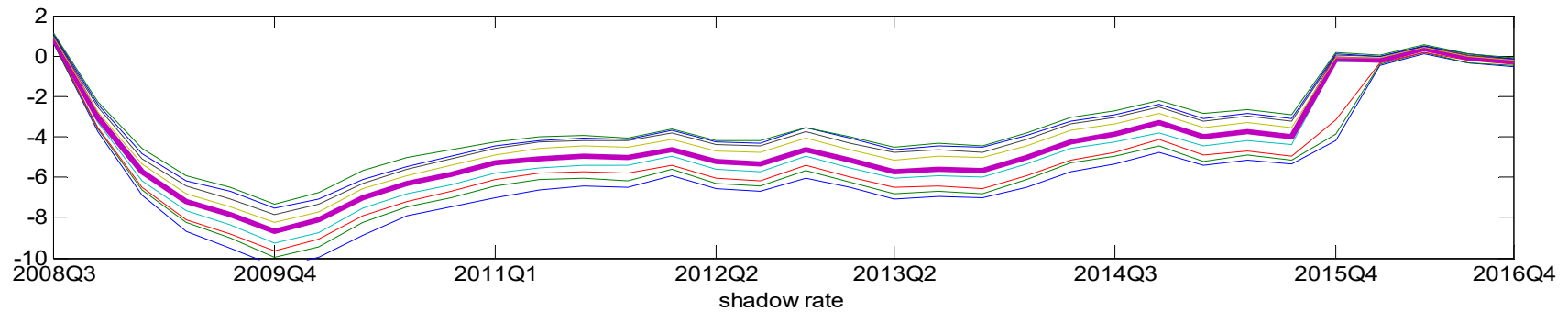
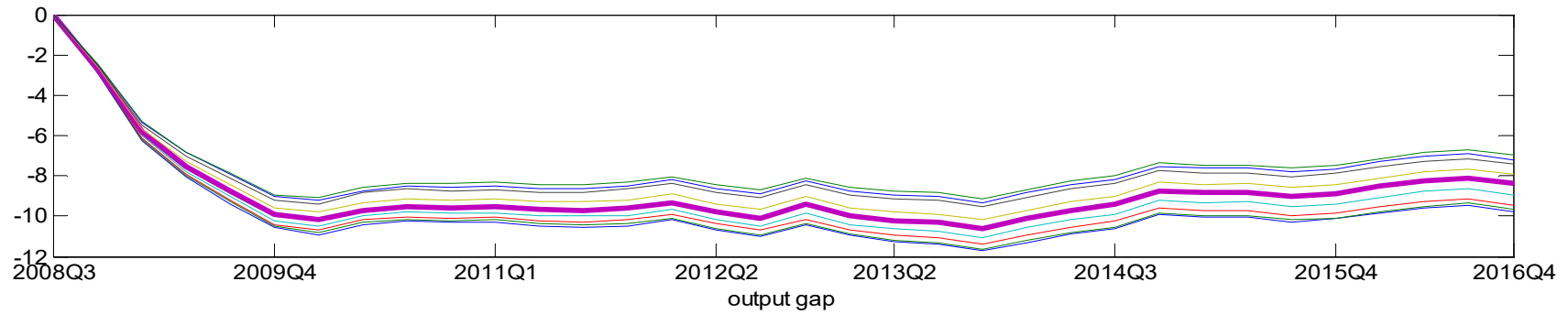
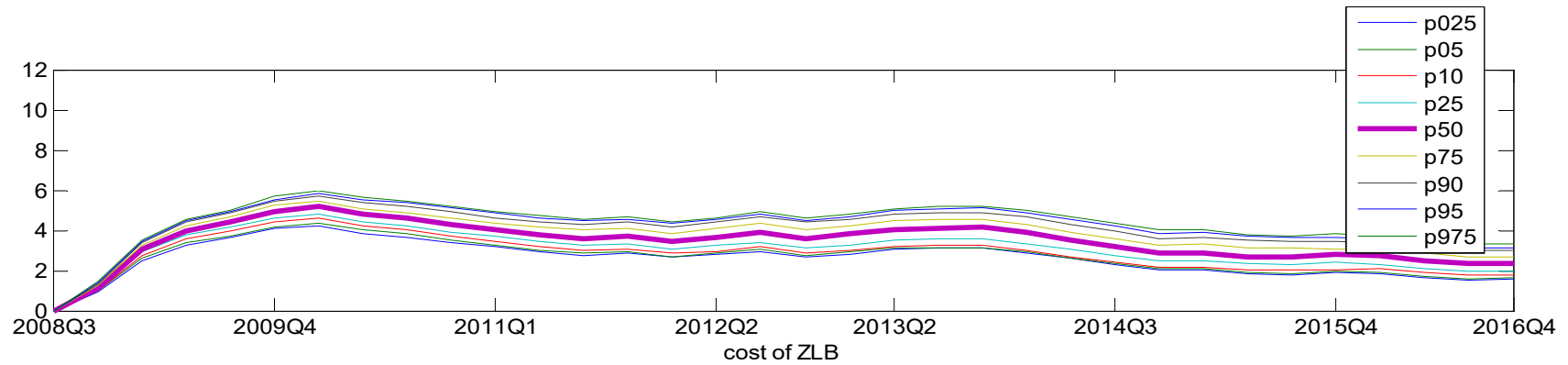




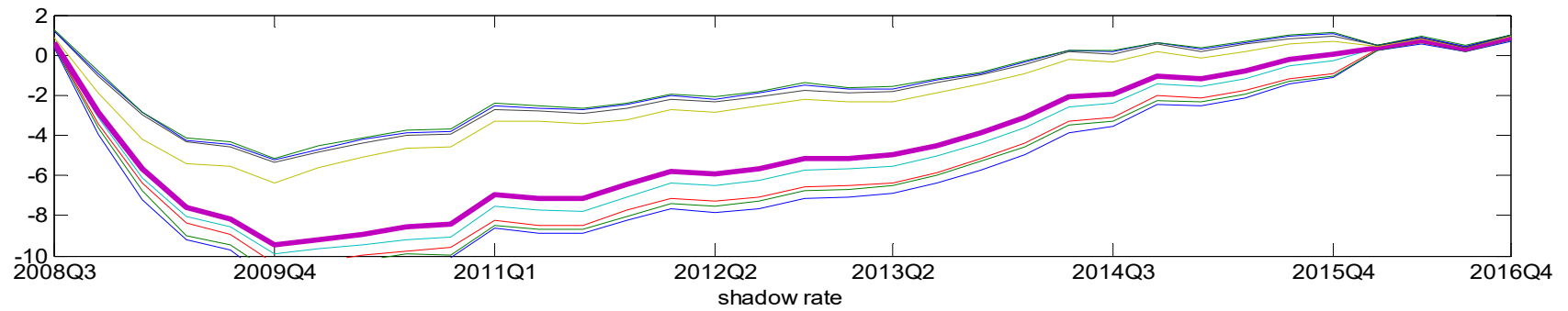
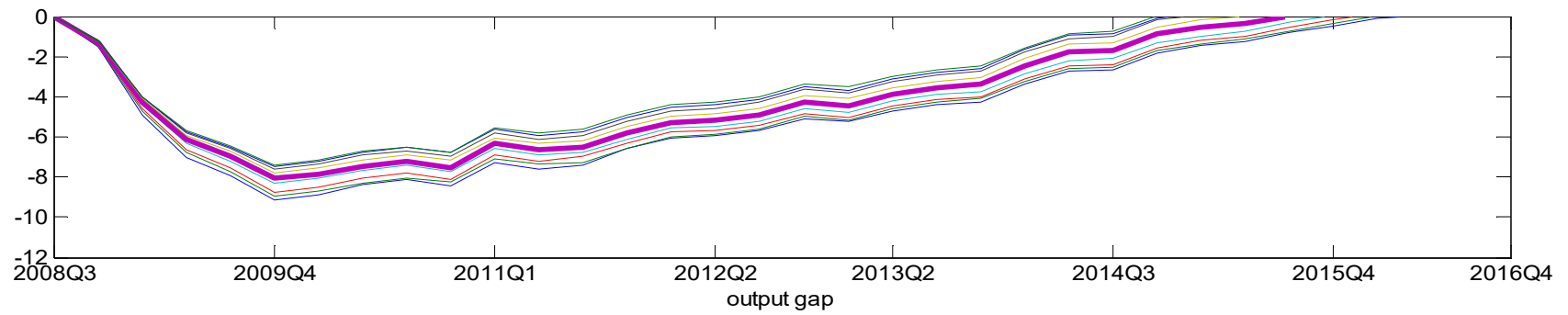
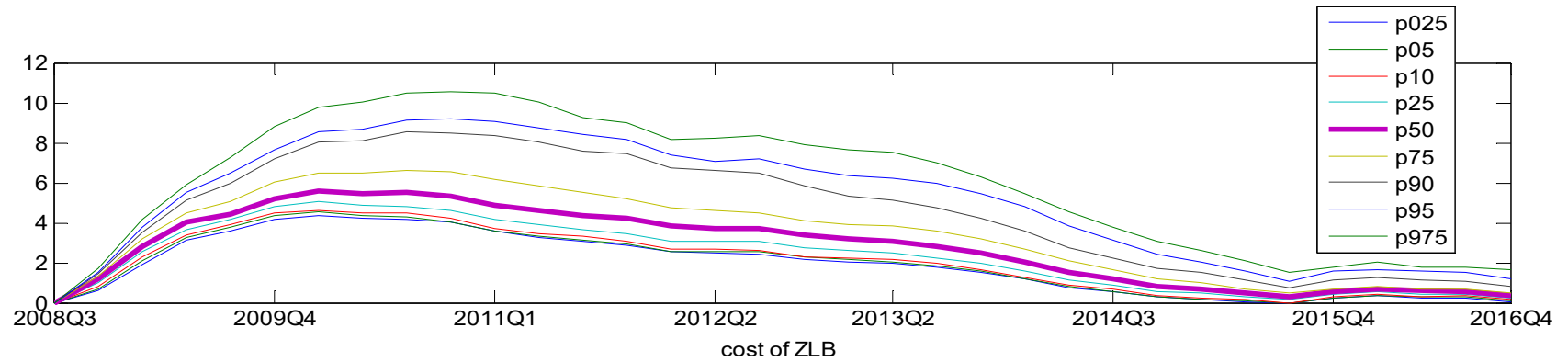
# Evaluate cost of ZLB: GSW with alternative methods



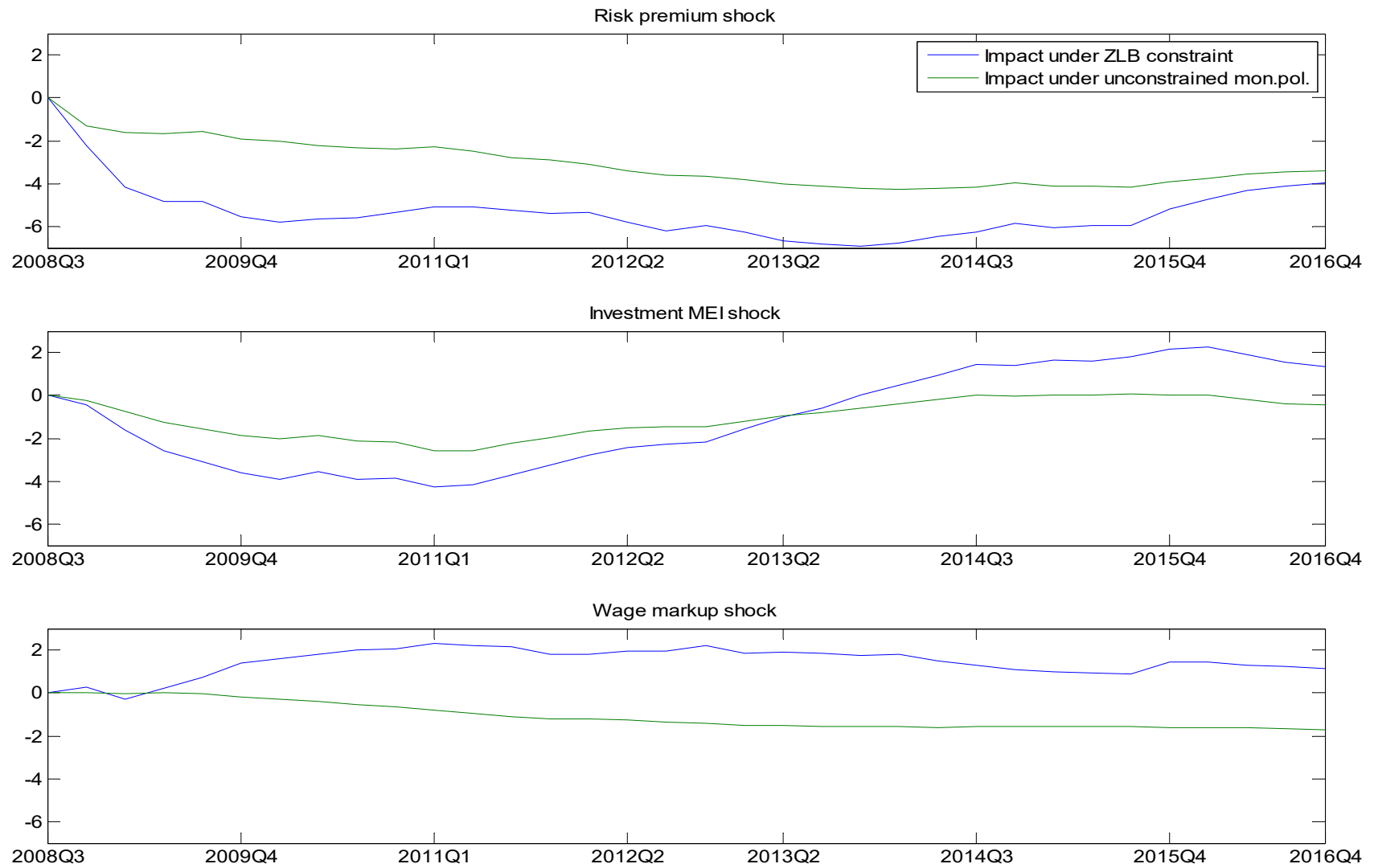
# Cost of ZLB in SW: posterior distr. for Endo. ZLB



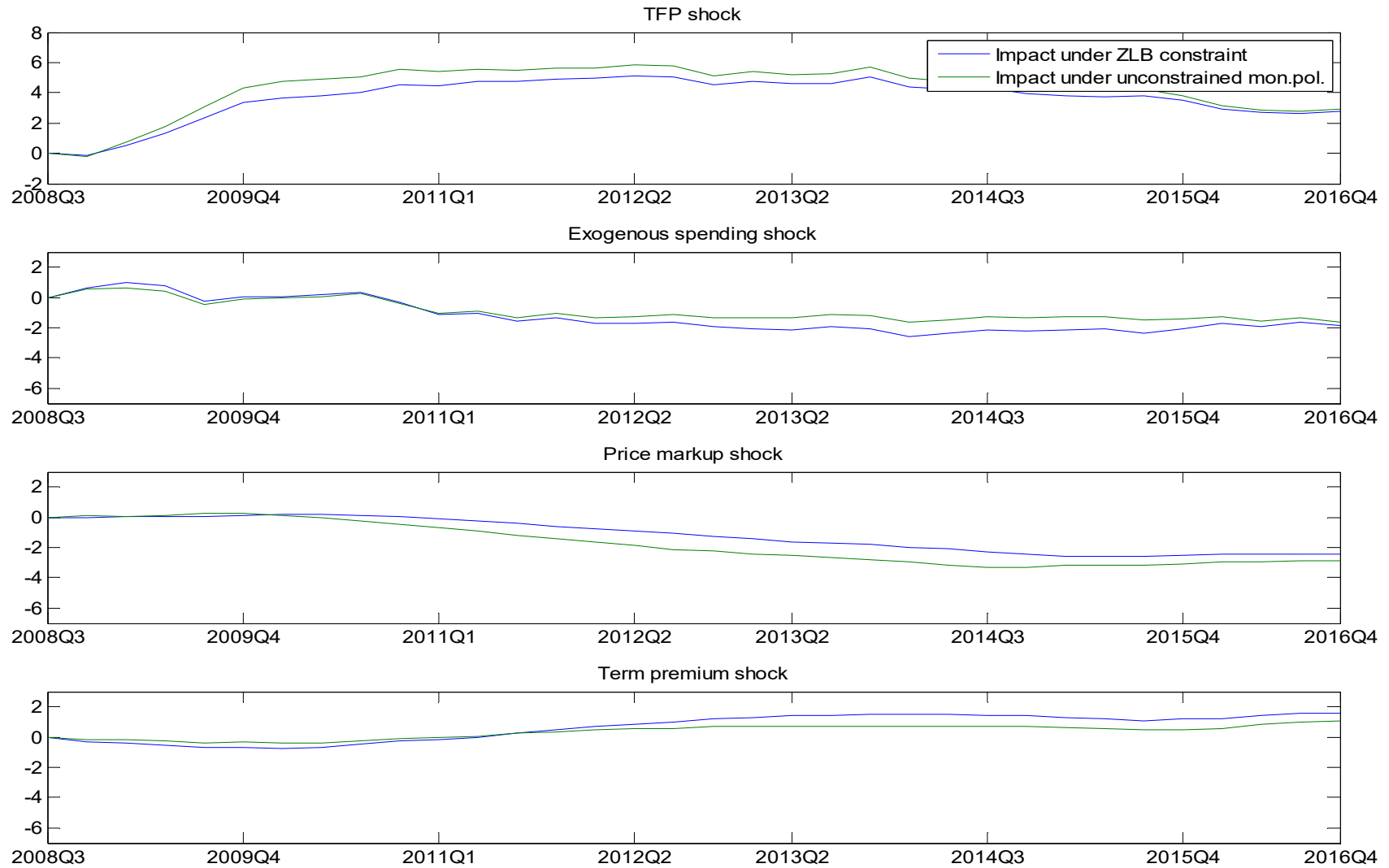
# Cost of ZLB in GSW: uncertainty under endo. ZLB



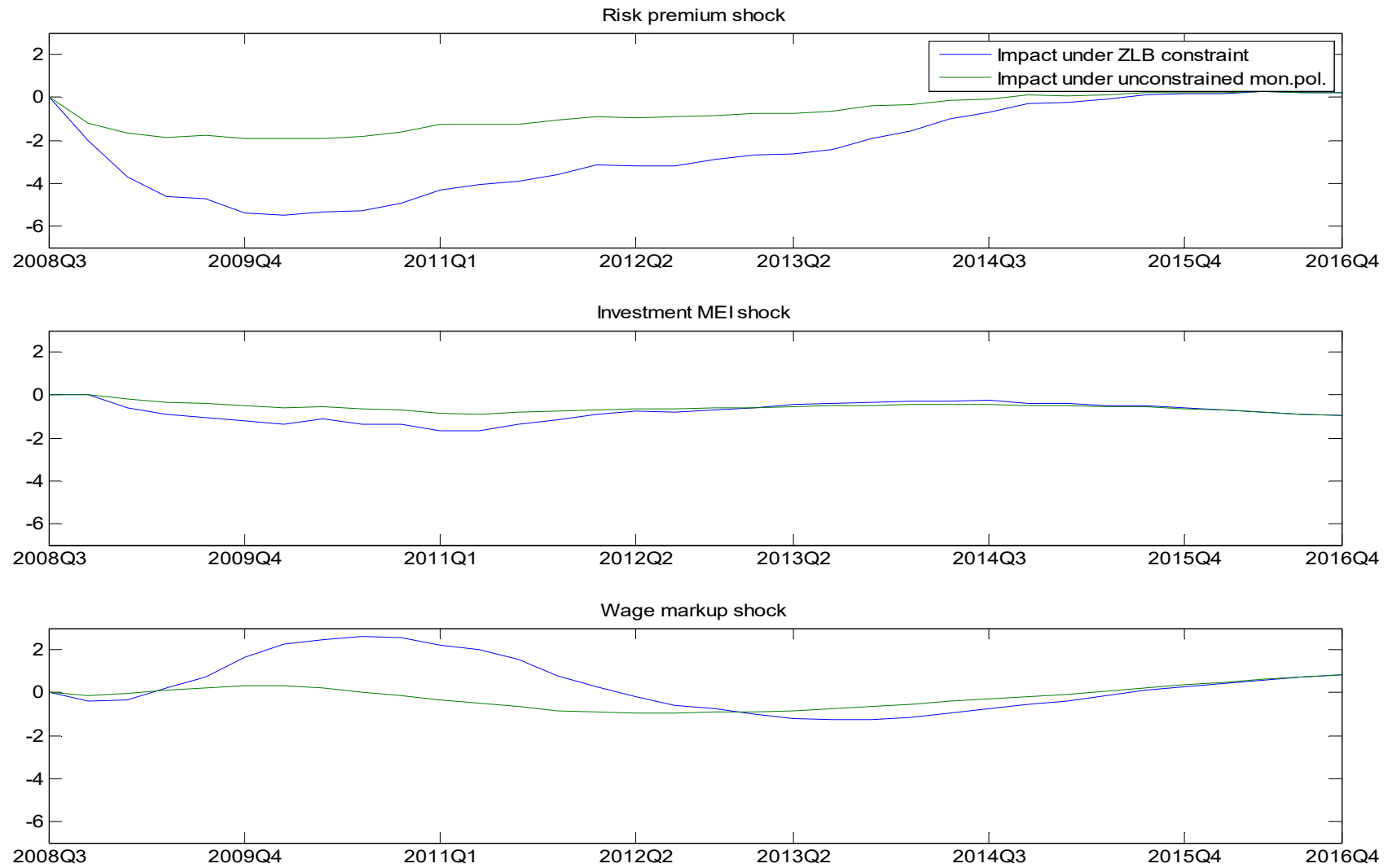
# Cost of ZLB in SW: decomposition to shocks



# Cost of ZLB in SW: decomposition to shocks (cont.)



# Cost of ZLB in GSW: decomposition to shocks





## Tentative Conclusions

- **We have presented and applied techniques that can be used to take ZLB incidents into account in operational large-scale macro models.**
- **Our results suggest that explicit treatment of the ZLB is important, but its exact influence depends on the experiments and in particular on how the CB behaves during ZLB incidents.**
- **A robust finding, across different estimation methods and across different models, is the substantial change in the propagation of shocks: risk premium and wage markup shocks are most affected.**
- **Also, our results indicate that the macro-economic cost of the interest rate lower bound is substantial and explains a major share of the negative output gap since the beginning of the GR.**



# Tentative Conclusions

- **In our model specification, the potential of UMP to offset these costs is weak.**
- **Given the importance of the non-linear dynamics, it is questionable whether we can abstract from other non-linear adjustment dynamics elsewhere in the model. Both our approaches have the flexibility to incorporate non-linear constraints in other blocks as well. But RS is computationally more efficient.**