Endogenous Regime Switching Near the Zero Lower Bound[1]

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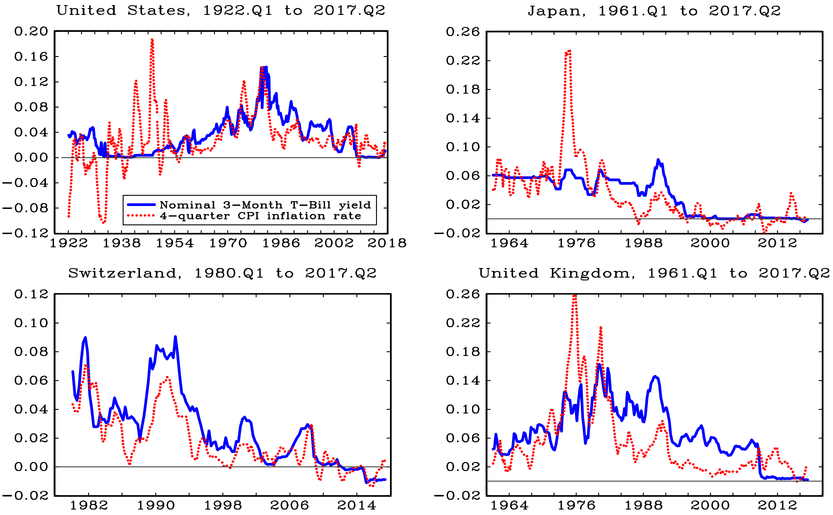
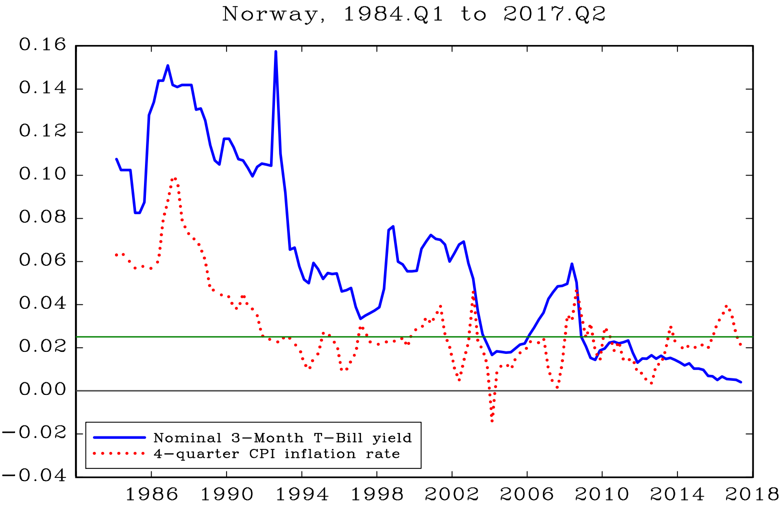
[1] Any opinions expressed here do not necessarily reflect the views of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.

Numerous ZLB (or ELB) episodes in global data



ZLB not (yet) binding in Norway

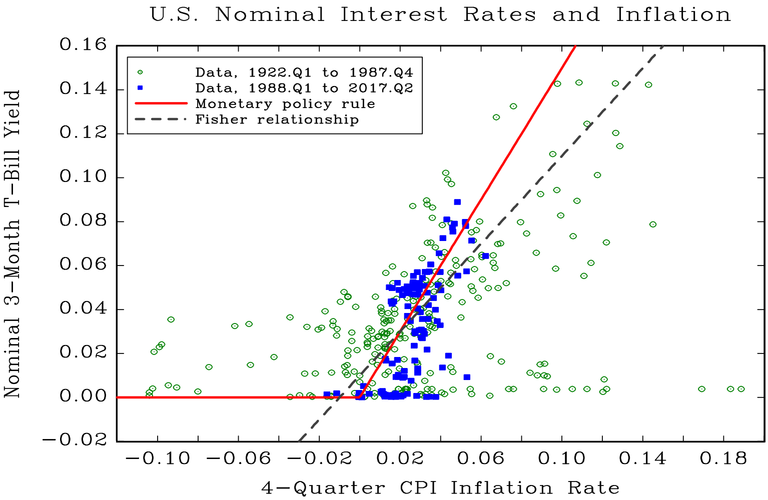


Standard NK model has multiple RE equilibria

- Taylor rule + Fisher Eqn. + ZLB $\Rightarrow$ Two steady states. (Benhabib, Schmitt-Grohé & Uribe *AER*, *JET* 2001a,b).
  - (1) Targeted: $i = r^* + \pi^* > 0$.
  - (2) Deflation: $i = 0$ and $\pi = -r^*$.
- r^* = "natural rate of interest." Evidence: r^* shifts over time and can drop below zero (Laubach & Williams 2016, Eggertsson, Mehrotra & Robbins 2017).

U.S. data: ZLB binding 2008.Q4 to 2015.Q4

Bullard 2010: "Promising to remain at zero for a long time is a double-edged sword."



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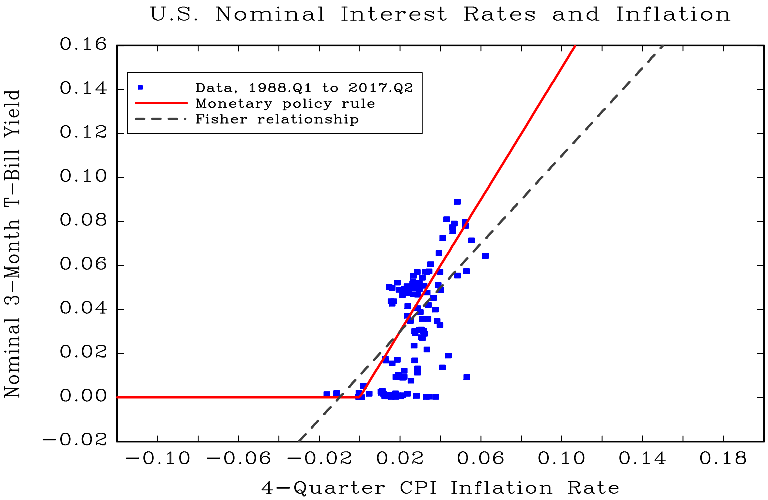

Summary of paper

- NK model with two local equilibria. Agent employs weighted-average of the two sets of local linear forecast rules. Weight optimized to minimize *RMSFE* over past T_w quarters.

- Unlike Arouba et al. (2018), regime switching here is endogenous.

- Results: Adverse shock $\Rightarrow$ more weight on deflation forecast rules $\Rightarrow$ deflation can become self-fulfilling. Episode accompanied by severe recession (highly negative output gap) with nominal rate at ZLB. Similar to 2007-09 Great Recession.

- But even in normal times, agent may place nontrivial weight on deflation forecast rules, causing central bank to consistently undershoot π^* (like now: $\pi_t^{\text{U.S.}} < 0.02$ since mid-2012).

Related literature (partial list)

- Transition between regimes driven by exogenous sunpots
  Aruoba, Cuba-Borda, & Schorfheide (2018, *REStud forthcoming*)
  Aruoba & Schorfheide (2016, *FRBKC Jackson Hole Symposium*
- Infrequent but long-lived ZLB episodes in global data
  Dordal-i-Carreras, Coibion, Gorodnichenko & Wieland (2016))
- Adaptive learning to select among multiple equilibria
  Evans & Honkapohja (2005, *RED*),
  Eusepi (2007, *JME*)
  Benhabib, Evans & Honkapohja (2014, *JEDC)*
  Arifovic, Schmitt-Grohé & Uribe (2018, *JEDC*)
- Optimal monetary policy with shifting natural rate
  Eggertsson and Woodford (2003, *BPEA*)
  Evans, Fisher, Gourio & Krane (2015, *BPEA*)
  Hamilton, Harris, Hatzius, & West (2016. *IMF Econ. Rev.*)
  Gust, Johannsen, López-Salido (2017, *IMF Econ. Rev.*)
  Basu & Bundick (2015, NBER WP 21838)

New Keynesian model with zero lower bound (ZLB)

$$
\begin{aligned}
y_t &= E_t\, y_{t+1} - \alpha \overbrace{\left[i_t - E_t\, \pi_{t+1} - r_t\right]}^{\text{Fisher relationship}} + v_t, \qquad v_t = \rho_v v_{t-1} + \epsilon_{v,t} \\
\pi_t &= \beta E_t\, \pi_{t+1} + \kappa y_t + u_t, \qquad u_t = \rho_u u_{t-1} + \epsilon_{u,t} \\
i_t^* &= \rho i_{t-1}^* + (1-\rho)\left[E_t r_t^* + \pi^* + g_\pi \left(\overline{\pi}_t - \pi^*\right) + g_y \left(y_t - y^*\right)\right] \\
\overline{\pi}_t &= \omega\, \pi_t + (1-\omega)\, \overline{\pi}_{t-1}, \qquad \overline{\pi}_t \simeq \pi_{4,\,t} \equiv \text{ 4-qtr. inflation rate.} \\
i_t &= \max\left\{0,\; i_t^*\right\}.
\end{aligned}
$$

$$
\begin{aligned}
r_t &\equiv -\log \underbrace{\left[\beta \exp\left(\zeta_t\right)\right]}_{\text{Discount factor}} + (1/\alpha) \underbrace{E_t \Delta \bar{y}_{t+1}}_{\text{Expected potential output growth}} \\
r_t &= \rho_r\, r_{t-1} + \left(1-\rho_r\right) r_t^* + \varepsilon_t, \qquad \varepsilon_t \sim N\left(0,\, \sigma_\varepsilon^2\right) \\
r_t^* &= r_{t-1}^* + \eta_t, \qquad \eta_t \sim N\left(0,\, \sigma_\eta^2\right) \\
r_t^* &\equiv \text{Natural rate of interest (long-run endpoint of } r_t\text{)}
\end{aligned}
$$

Two long-run endpoints (steady states)

Targeted Endpoint

$$\pi_t = \pi^*$$
$$y_t = y^* \equiv \pi^* (1 - \beta) / \kappa$$
$$i_t^* = r_t^* + \pi^*$$
$$i_t = i_t^*$$

Deflation Endpoint

$$\pi_t = -r_t^*$$
$$y_t = -r_t^* (1 - \beta) / \kappa$$
$$i_t^* = (r_t^* + \pi^*) \left[1 - g_\pi - \frac{g_y (1-\beta)}{\kappa}\right]$$
$$i_t = 0$$

Two long-run endpoints (steady states)

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$$i_t = 0$$

Shifting Endpoint Time Series Model (Kozicki-Tinsley, JMCB 2012)

$$E_t r_t^* = \lambda \left[\frac{r_t - \rho_r r_{t-1}}{1-\rho_r}\right] + (1-\lambda) E_{t-1} r_{t-1}^*$$

Kalman gain

$$\lambda = \frac{-(1-\rho_r)^2 \phi + (1-\rho_r)\sqrt{(1-\rho_r)^2\phi^2 + 4\phi}}{2}, \qquad \phi \equiv \frac{\sigma_\eta^2}{\sigma_\varepsilon^2}$$

$$E_t(r_{t+h} - r_{t+h}^*) = (\rho_r)^h (r_t - E_t r_t^*), \qquad \rho_r = 0.86$$

Two local rational expectations equilibria

Targeted (Unique). Forecast rules assume $i_t^* = i_t > 0$ for all t

$$\begin{bmatrix} y_t - \pi^* (1-\beta)/\kappa \\ \pi_t - \pi^* \\ i_t^* - (E_t\, r_t^* + \pi^*) \end{bmatrix} = \mathbf{A} \times \begin{bmatrix} r_t - E_t\, r_t^* \\ \overline{\pi}_{t-1} - \pi^* \\ i_{t-1}^* - (E_t\, r_t^* + \pi^*) \\ v_t \\ u_t \end{bmatrix}$$

Deflation (MSV). Forecast rules assume $i_t^* \leq 0$, $i_t = 0$ for all t

$$\begin{bmatrix} y_t - (-E_t\, r_t^*)(1-\beta)/\kappa \\ \pi_t - (-E_t\, r_t^*) \\ i_t^* - (E_t\, r_t^* + \pi^*)\left[1 - g_\pi - g_y (1-\beta)/\kappa\right] \end{bmatrix} =$$

$$\mathbf{B} \times \begin{bmatrix} r_t - E_t\, r_t^* \\ \overline{\pi}_{t-1} - (-E_t\, r_t^*) \\ i_{t-1}^* - (E_t\, r_t^* + \pi^*)\left[1 - g_\pi - g_y (1-\beta)/\kappa\right] \\ v_t \\ u_t \end{bmatrix}$$

Two local rational expectations equilibria

Targeted (Unique). Forecasts assume $i_t^* = i_t > 0$ for all t

$$\mathbf{A} = \begin{bmatrix} 0.594 & -0.153 & -0.386 & 3.221 & -0.174 \\ 0.069 & -0.017 & -0.033 & 0.275 & 1.396 \\ 0.128 & 0.129 & 0.718 & 0.682 & 0.158 \end{bmatrix}$$

Deflation (MSV). Forecasts assume $i_t^* \le 0$, $i_t = 0$ for all t

$$\mathbf{B} = \begin{bmatrix} 1.247 & 0 & 0 & 5.397 & 0.092 \\ 0.213 & 0 & 0 & 0.661 & 1.429 \\ 0.279 & 0.162 & 0.8 & 1.171 & 0.215 \end{bmatrix}$$

Local solution coefficients for state variable $r_t - E_t r_t^*$:

$$\frac{\mathbf{B}_{11}}{\mathbf{A}_{11}} = 2.1 \qquad \frac{\mathbf{B}_{21}}{\mathbf{A}_{21}} = 3.1 \qquad \frac{\mathbf{B}_{31}}{\mathbf{A}_{31}} = 2.2$$

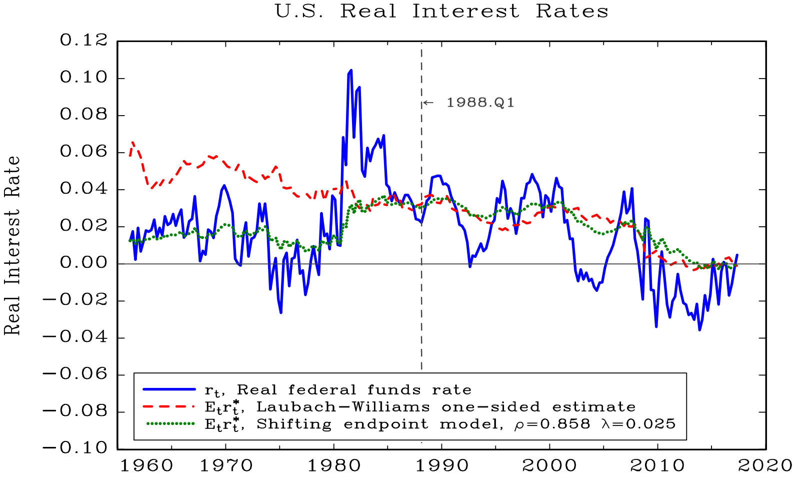
$\Rightarrow$ Deflation equilibrium exhibits much more volatility.

Model parameter values

α	0.15	Interest rate coefficient in Euler equation.
β	0.995	Discount factor in Phillips curve.
κ	0.025	Output gap coefficient in Phillips curve.
σ_v	0.010	Std. dev. of demand shock.
σ_u	0.005	Std. dev. of cost push shock.
ρ_v	0.8	Persistence of demand shock.
ρ_u	0.3	Persistence of cost push shock.
π^*	0.02	Central bank inflation target.
ω	0.459	$\overline{\pi}_t \simeq$ 4-quarter inflation rate.
g_π	1.5	Policy rule response to inflation.
g_y	1.0	Policy rule response to output gap.
ρ	0.80	Interest rate smoothing parameter.
ρ_r	0.858	Persistence parameter for r_t.
σ_ε	0.010	Std. dev. temporary shock to r_t.
σ_η	0.002	Std. dev. permanent shock to r_t.
λ	0.025	Optimal Kalman gain for $E_t r_t^*$.

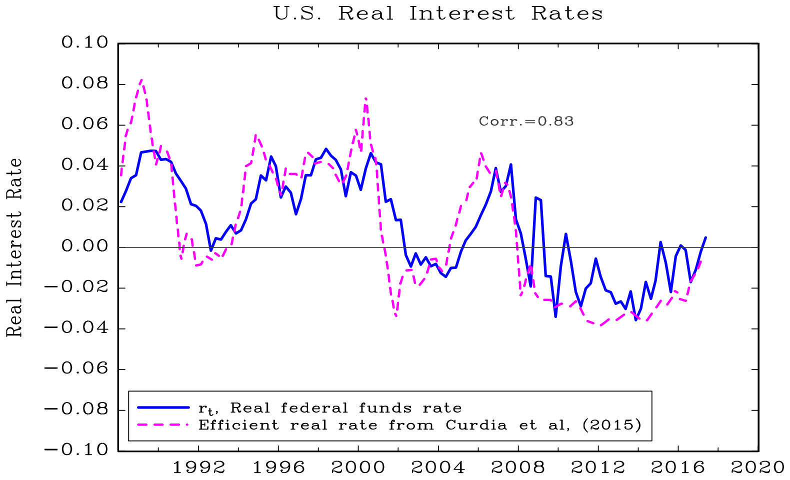
Natural rate process approximates Laubach-Williams r-star

Bounds for simulations: $-0.004 \leq r_t^* \leq 0.037$



Real federal funds rate versus efficient real rate

<u>Efficient real rate</u>: Cúrdia, Ferrero, Ng & Tambalotti (*JME*, 2015)



Endogenous forecast rule switching based on past RMSFE

Variables that the agent must forecast: y_{t+1} and π_{t+1}

$$\widehat{E}_t\, y_{t+1} = \mu_t E_t^{\text{targ}}\, y_{t+1} + (1-\mu_t)\, E_t^{\text{defl}}\, y_{t+1}$$
$$\widehat{E}_t\, \pi_{t+1} = \mu_t E_t^{\text{targ}}\, \pi_{t+1} + (1-\mu_t)\, E_t^{\text{defl}}\, \pi_{t+1}$$

Choose μ_t to minimize $RMSFE_{t-1}$ for moving window of recent data

$$\min_{\mu_t} \frac{1}{T_w} \sum_{j=1}^{T_w} \left\{ \left[y_{t-j} - \mu_t E_{t-j-1}^{\text{targ}}\, y_{t-j} - (1-\mu_t)\, E_{t-j-1}^{\text{defl}}\, y_{t-j} \right]^2 + \left[\pi_{t-j} - \mu_t E_{t-j-1}^{\text{targ}}\, \pi_{t-j} - (1-\mu_t)\, E_{t-j-1}^{\text{defl}}\, \pi_{t-j} \right]^2 \right\}^{0.5}$$

For simulations, impose $0 \leq \mu_t \leq 1$, with $T_w = 8$ qtrs.

Endogenous forecast rule switching based on past RMSFE

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$$\min_{\mu_t} \frac{1}{T_w} \sum_{j=1}^{T_w} \left\{ \left[y_{t-j} - \mu_t E_{t-j-1}^{\text{targ}}\, y_{t-j} - (1-\mu_t)\, E_{t-j-1}^{\text{defl}}\, y_{t-j} \right]^2 + \left[\pi_{t-j} - \mu_t E_{t-j-1}^{\text{targ}}\, \pi_{t-j} - (1-\mu_t)\, E_{t-j-1}^{\text{defl}}\, \pi_{t-j} \right]^2 \right\}^{0.5}$$

For simulations, impose $0 \leq \mu_t \leq 1$, with $T_w = 8$ qtrs.

Alternative (Binning and Maih 2017):

$$\mu_t = \exp\left(\psi i_{t-1}^*\right) / \left[1 + \exp\left(\psi i_{t-1}^*\right)\right], \qquad \psi = 2000.$$

Given current forecasts, solve for equilibrium variables

$$y_t = \widehat{E}_t\, y_{t+1} - \alpha \left[i_t - \widehat{E}_t\, \pi_{t+1} - r_t \right] + v_t$$

$$\pi_t = \beta\, \widehat{E}_t\, \pi_{t+1} + \kappa y_t + u_t$$

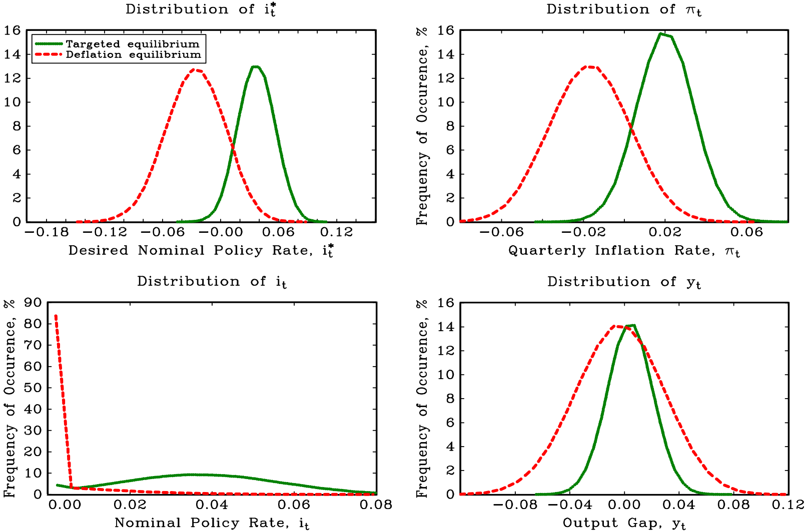
$$i_t^* = \rho i_{t-1}^* + (1-\rho)\left[E_t r_t^* + \pi^* + g_\pi \left(\overline{\pi}_t - \pi^* \right) + g_y \left(y_t - y^* \right) \right]$$

$$i_t = 0.5\, i_t^* + 0.5 \sqrt{\left(i_t^* \right)^2}$$

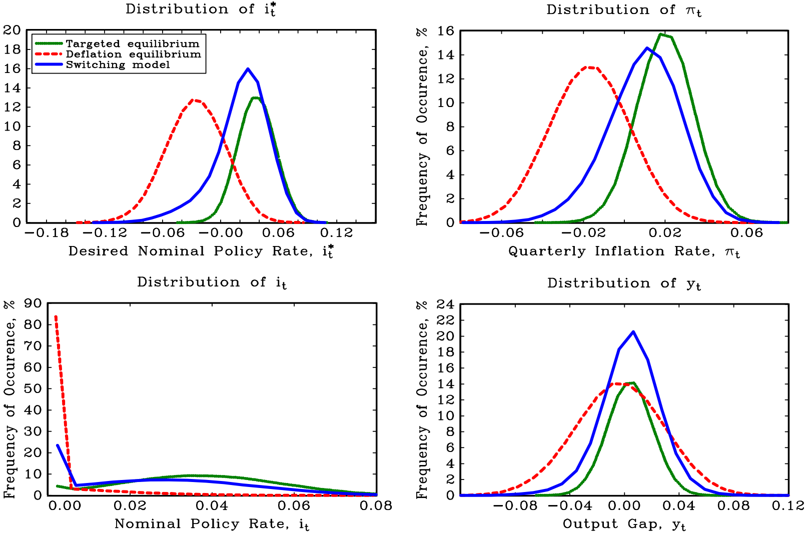
$$\overline{\pi}_t = \omega\, \pi_t + (1-\omega)\, \overline{\pi}_{t-1}$$

Given forecasts $\widehat{E}_t\, y_{t+1}$, $\widehat{E}_t\, \pi_{t+1}$, and $E_t r_t^*$, solve nonlinear system each period for y_t, π_t, and i_t^*.

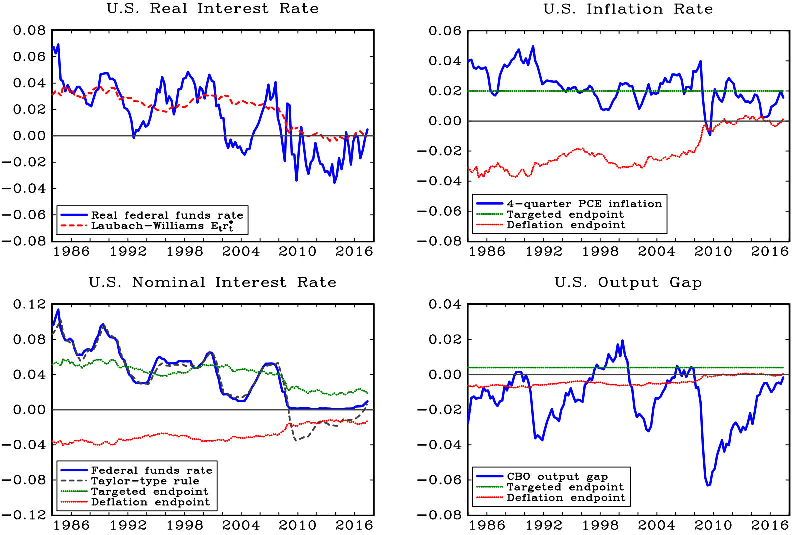
Overlapping distributions induce endogenous regime shifts



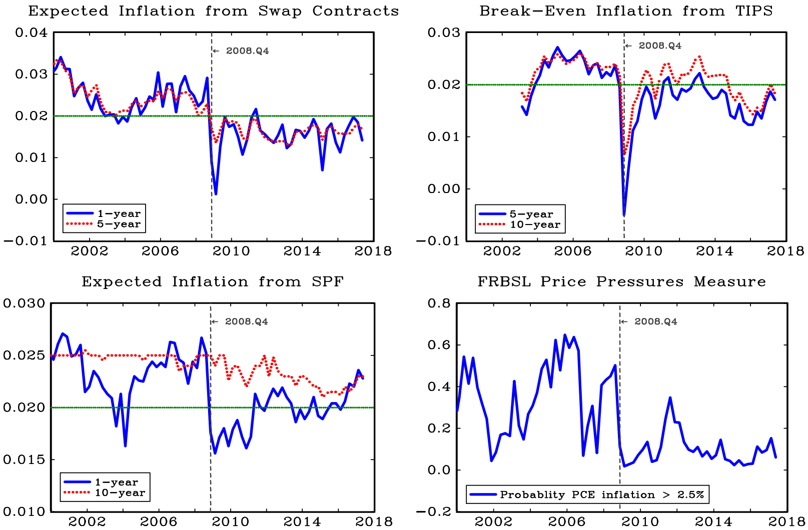
Switching model: Inflation distribution shifts left



U.S. data: Severe recession, deflation, ZLB binding

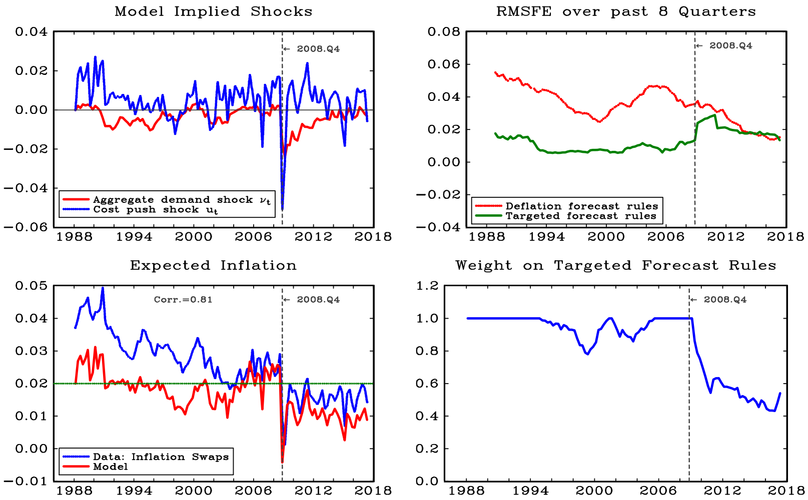


Measures of expected inflation declined after 2008.Q4



Replicating U.S. data with the switching model

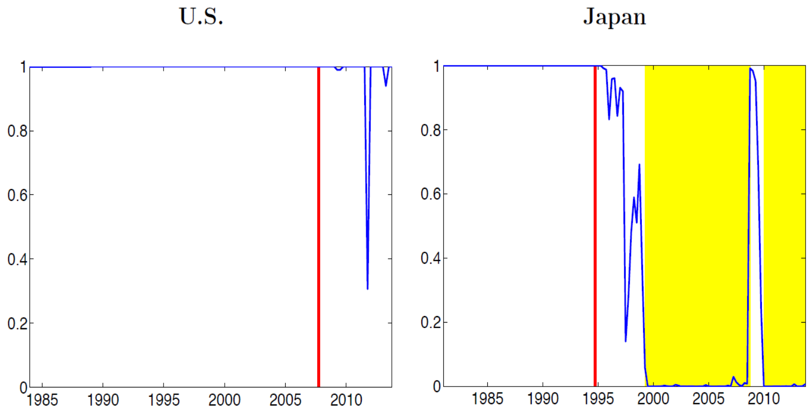
Given r_t, $E_t\, r_t^*$, i_t, i_t^*, y_t, π_t in U.S. data, solve for v_t, u_t, and μ_t.



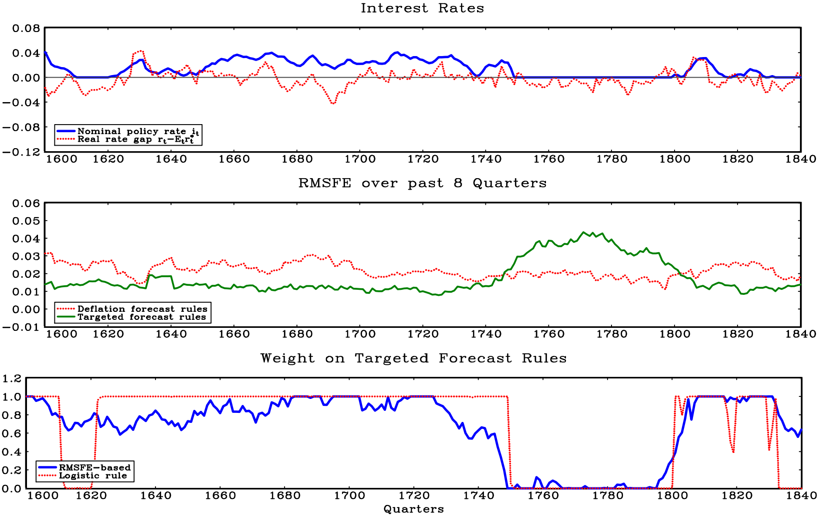
Aruoba, Cuba-Borda, & Schorfheide (2018, forthcoming)

"With the exception of 2011:Q4, when the probability of the deflation regime increased to about 70%, the U.S. has been in the targeted inflation regime."

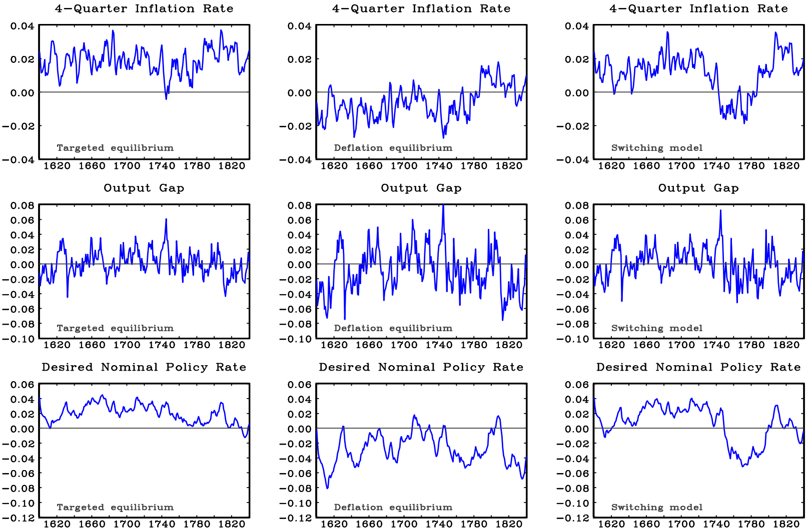
Figure 5: Filtered Probability of Targeted-Inflation Regime



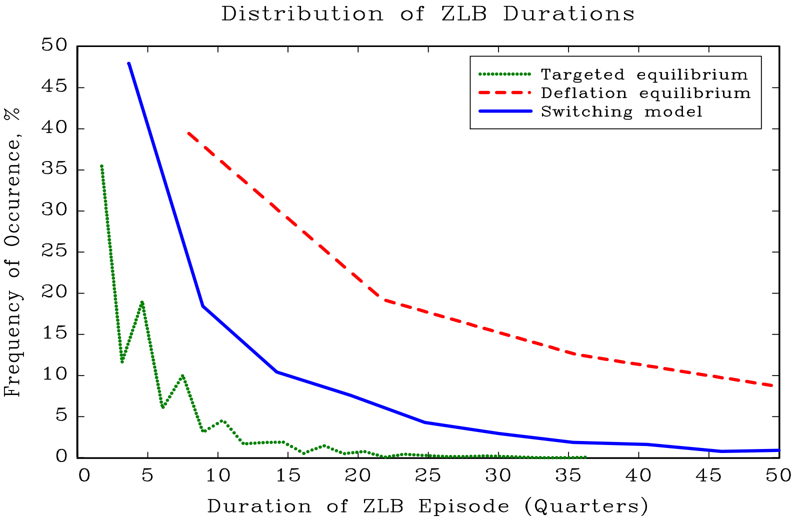
Weight on targeted forecast rules can decline rapidly



Comparing simulations: Targeted, Deflation, Switching



Switching model: Infrequent but long-lived ZLB episodes



Quantitative comparison: Data versus models

	U.S. Data	Model Simulations		
Statistic	1988.Q1-2017.Q2	Targeted	Deflation	Switching
% periods $i_t = 0$	24.6%	2.52%	80.2%	19.6%
Mean ZLB duration	29 qtrs.	5.3 qtrs.	34.7 qtrs.	12.5 qtrs.
Max. ZLB duration	29 qtrs.	37 qtrs.	346 qtrs.	133 qtrs.
Mean y_t	−1.44%	0.40%	−0.38%	0.42%
Std. Dev.	1.75%	1.65%	3.21%	2.19%
Mean $\pi_{4,t}$	2.16%	1.99%	−1.70%	0.93%
Std. Dev.	1.09%	0.85%	1.58%	1.46%

Model results computed from 300,000 period simulation.

Properties of representative agent's forecast errors

Statistic	Targeted	Deflation	Switching
$Corr\left(err^{y}_{t+1},\ err^{y}_{t}\right)$	0.002	−0.007	0.019
$Corr\left(err^{\pi}_{t+1},\ err^{\pi}_{t}\right)$	0.003	0.002	0.074
$E\left(err^{y}_{t+1}\right)$	−0.001%	−0.045%	0.008%
$E\left(err^{\pi}_{t+1}\right)$	−0.003%	−0.004%	0.003%
$\sqrt{E\left[\left(err^{y}_{t+1}\right)^2\right]}$	1.11%	1.87%	1.35%
$\sqrt{E\left[\left(err^{\pi}_{t+1}\right)^2\right]}$	1.31%	1.35%	1.34%

Model results computed from 300,000 period simulation.

- $err^{x}_{t+1} = x_{t+1} - F_t\, x_{t+1}$ for $x_{t+1} \in \left\{y_{t+1},\ \pi_{t+1}\right\}$.
- Agent employs linear forecast rules in a nonlinear environment with an occasionally binding ZLB.
- Nevertheless, agent's forecast errors in all three model versions are close to white noise.

Effect of natural rate range in switching model

Statistic	$-0.004 \leq r_t^* \leq 0.037$	$-0.015 \leq r_t^* \leq 0.037$
% periods $i_t = 0$	19.6%	23.2%
Mean ZLB duration	12.5 qtrs.	12.4 qtrs.
Mean y_t	0.42%	0.38%
Std. Dev.	2.19%	2.23%
Mean $\pi_{4,t}$	0.93%	1.08%
Std. Dev.	1.46%	1.40%

Model results computed from 300,000 period simulation.

- Wide uncertainty bands around empirical estimates of r_t^*
- Eggertsson, Mehrotra, & Robbins (2017): Steady state r^* in a life cycle model calibrated to U.S. data in 2015 is -1.5%.
- Endpoint of π_t in deflation equilibrium is $-r_t^*$. So negative r_t^* $\Rightarrow$ positive inflation in the "deflation" equilibrium.

Effect of higher inflation target in switching model

Yellen, 6-14-2017: "This is one of the most important questions facing monetary policy."

Statistic	$\pi^* = 0.02$	$\pi^* = 0.03$	$\pi^* = 0.04$
% periods $i_t = 0$	19.6%	14.2%	9.5%
Mean ZLB duration	12.5 qtrs.	12.4 qtrs.	11.7 qtrs.
Std. Dev. y_t	2.19%	2.12%	2.04%
Std. Dev. $\pi_{4,t}$	1.46%	1.56%	1.61%
Loss value, $\theta = 1$	2.84%	**2.66%**	2.75%
Loss value, $\theta = 0.25$	2.12%	**1.91%**	2.04%

Model results computed from 300,000 period simulation.

- Higher π^* can reduce switching to volatile deflation equilibrium where recessions are more severe.
- Similar to Kiley and Roberts (BPEA, 2017):

$$Loss = E\left\{\left[\pi_{4,\,t} - 0.02\right]^2 \;+\; \theta\left[y_t - 0.02\left(1-\beta\right)/\kappa\right]^2\right\}.$$

Effect of interest rate smoothing in switching model

Statistic	$\pi^* = 0.02$	$\pi^* = 0.03$	$\pi^* = 0.04$
$\rho = 0.8$			
% periods $i_t = 0$	19.6%	14.2%	9.5%
Mean ZLB duration	12.5 qtrs.	12.4 qtrs.	11.7 qtrs.
$\rho = 0$			
% periods $i_t = 0$	30.0%	24.4%	19.6%
Mean ZLB duration	4.9 qtrs.	5.0 qtrs.	5.0 qtrs.

Model results computed from 300,000 period simulation..

- No smoothing ($\rho = 0$) implies higher frequency of hitting ZLB, but episodes are shorter on average.
- From ZLB perspective, no clear advantage from reducing the degree of interest rate smoothing in the policy rule.

Adaptive learning in a simplified model

- Impose $\rho = \rho_v = \rho_u = 0$ (no persistence), $\omega = 1$ (policy targets quarterly inflation), and $\sigma_\eta = 0$ (r^* is constant).
- Version 1: Agent estimates correctly-specified decision rules:

$$
\begin{aligned}
y_t &= c_{0,t} + c_{1,t}\left(r_t - r^*\right) + c_{2,t}\, v_t + c_{3,t}\, u_t \\
\pi_t &= d_{0,t} + d_{1,t}\left(r_t - r^*\right) + d_{2,t}\, v_t + d_{3,t}\, u_t
\end{aligned}
$$

- Version 2: Agent estimates misspecified decision rules:

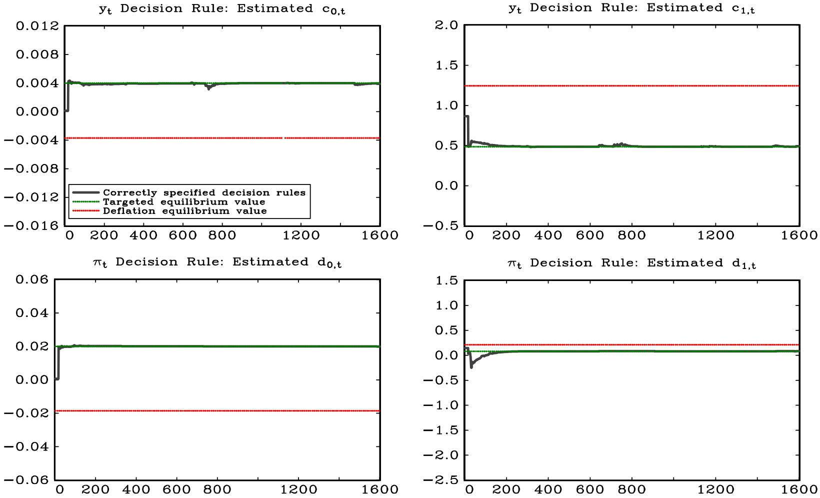
$$
\begin{aligned}
y_t &= c_{0,t} + c_{1,t}\left(r_t - r^*\right) \\
\pi_t &= d_{0,t} + d_{1,t}\left(r_t - r^*\right)
\end{aligned}
$$

- Subjective forecasts:

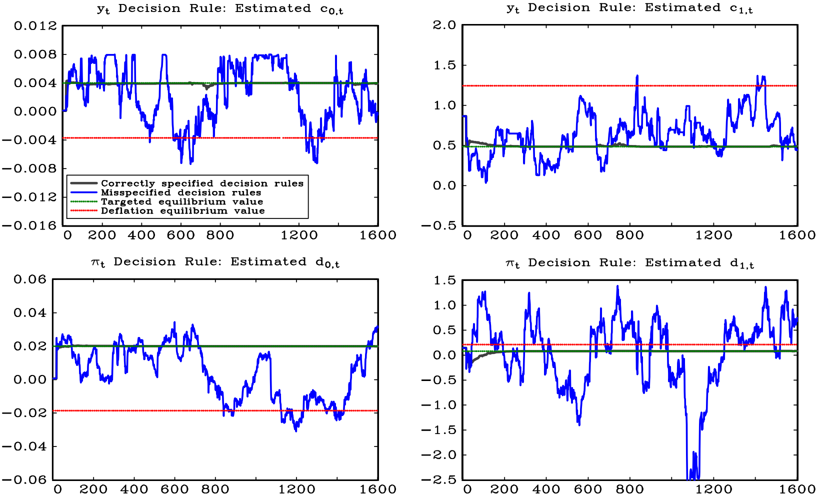
$$
\begin{aligned}
\widehat{E}_t\, y_{t+1} &= c_{0,t-1} + c_{1,t-1}\rho_r\left(r_t - r^*\right) \\
\widehat{E}_t\, \pi_{t+1} &= d_{0,t-1} + d_{1,t-1}\rho_r\left(r_t - r^*\right)
\end{aligned}
$$

- $c_{i,t}$ and $d_{i,t}$ estimated each period using OLS for a rolling window of 16 quarters (4 years) of model-generated data.

Learning with correctly specified decision rules



Learning with misspecified decision rules



Conclusion

- Most NK studies ignore the deflation equilibrium. But no clear reason why this equilibrium should be ruled out.
- Switching model can produce Great Recessions when $r_t - E_t r_t^*$ is persistently negative, causing agent to place large weight on deflation forecast rules. Escape from ZLB occurs endogenously when $r_t - E_t r_t^*$ eventually starts rising.
- In normal times, non-trivial weight on deflation forecast rules may cause central bank to undershoot π^* (like today?).
- Model (with shocks) can replicate U.S. data since 1988.
- A simple loss function approach favors a modest increase in π^* to around 3%. But even with $\pi^* = 4\%$, the ZLB binding frequency is 9.5% and the average duration of a ZLB episode is 11.7 quarters.