Latent Variables and Real-Time Forecasting in DSGE Models with Occasionally Binding Constraints. Can Non-Linearity Improve Our Understanding of the Great Recession?

Massimo Giovannini and Marco Ratto

JRC, European Commission

Outline

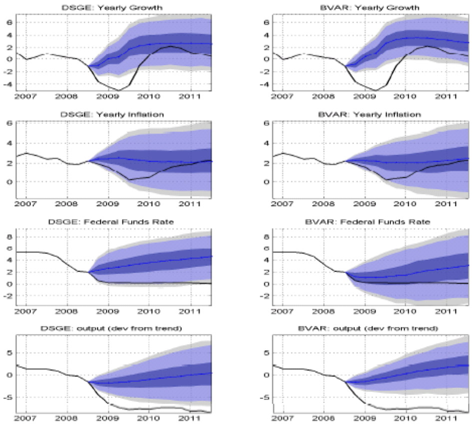
- Motivation
- Contribution
- Methods
- Implementation
- Model set-up
- Conclusions

Motivation

- *Methodological*: recently lots of effort in modeling non-linearities, in particular Occasionally Binding Constraints (OBC) especially for the study of ZLB (Guerrieri and Iacoviello, 2015; Holden)
  - But need to understand shocks contributions in explaining observables in this non-linear context, especially for policy analysis.
  - Additivity of shocks no longer holds in non-linear models.
- *Theoretical*: Linde et al. show the need of non Gaussian models to "predict" the Global Financial Crisis (GFC)

Motivation (cont'd)

Figure 4.1: Forecast 2008Q4-2011Q3 conditional on state in 2008Q3.



European Commission

Contribution

- We propose an algorithm that allows us to compute historical contribution of smoothed shocks onto observables in models with piecewise linear solution.
- We implement the algorithm in a model with financial OBC and ask whether non-linearities (in the financial sector) may allow us to predict extreme events within Gaussian assumptions

European Commission

Methodology- Obtaining estimates of latent variables

1. Guess an initial sequence of regimes for each historical period $R_t^{(0)}$ for $t = 1, .., T$. (similar to Anzoategui et al. 2015)
2. Given the sequence of regimes, compute the sequence of state space matrices $\Upsilon_t^{(0)}$ following the piecewise linear solution method of Guerrieri et al. (2015).
3. For each iteration $j = 1, .., n$:
   - 3.1 feed the state space matrices $\Upsilon_t^{(j-1)}$to a Kalman Filter / Fixed interval smoothing algorithm to determine initial conditions, smoothed variables $y_t^{(j)}$ and shocks $\epsilon_t^{(j)}$. (Kulish 2014)
   - 3.2 given initial conditions and shocks perform Occbin simulations that endogenously determine a new sequence of regimes $R_t^{(j)}$, from which a new sequence of states space matrices is derived $\Upsilon_t^{(j)}$.
4. The algorithm stops when $R_t^{(j)} = R_t^{(j-1)}$ for all $t = 1, .., T$.

Methodology-caveat with piecewise linear solution

In this environment the contribution of individual smoothed shocks, is not the mere additive superposition of each shock propagated by the sequence of state space matrices Υ_t estimated with the smoother.

The occurrence of a specific regime at time t, in fact, is a non-linear function of the states in $t-1$, y_{t-1} and of the whole set of shocks simultaneously affecting the economy, that is $\Upsilon_t = f\left(\epsilon_{1t}, .., \epsilon_{kt}, y_{t-1}\right)$, $t = 1, .., T$. Hence, the sequence of regimes will change when taking subsets of shocks or individual shocks alone.

Methodology-proposal

We propose two definitions, Main and Total effects, that generalize the concept of shock contributions to the non-linear case, and that will be based on simulations conditional to given shock patterns, i.e. performing counterfactuals opportunely choosing combinations of shocks and initial conditions.

Methodology-Main and Total effects

Suppose we have a non-linear model $y = f(x_1, .., x_n)$

From ANOVA theory, we can decompose y into main effects and interactions:

$$y = f_0 + \sum_{i=1}^{n} f_i(x_i) + \sum_{i=1}^{n}\sum_{j>i} f_{ij}(x_i x_j) + .. + f_{1,..,n}(x_1, .., x_n)$$

If we are interested into the main effect of $x_i$that would simply be:

$$E(y \mid x_i) = f_0 + f_i(x_i)$$

If we are interested into the joint main effect of x_i, x_j that would be:

$$E(y \mid x_i, x_j) = f_0 + f_i(x_i) + f_j(x_j) + f_{ij}(x_i x_j)$$

European Commission

Methodology-Main and Total effects (cont'd)

So, again given our decomposition

$$y = f_0 + \sum_{i=1}^{n} f_i(x_i) + \sum_{i=1}^{n}\sum_{j>i} f_{ij}(x_i x_j) + .. + f_{1,..,n}(x_1, .., x_n)$$

we can define the Total effect of x_i as

$$y^{tot}(x_i) = f_i(x_i) + \sum_{j\neq i} f_{ij}(x_i x_j) + .. + f_{1,..,n}(x_1, .., x_n)$$

that is the complement of all other x's main effects:

$$y^{tot}(x_i) = y - E(y \mid x_{j\neq i})$$

Methodology-Main effect contribution

- Denote with ϵ_{lt} the shock or group of shocks of interest, while $\tilde{\epsilon}_{lt}$ indicates the complementary set of shocks in the model.
- We define the *Main effect contribution*, the effect computed via Monte Carlo counterfactuals drawing respectively $\tilde{\epsilon}_{lt}$and the initial conditions $\mathbf{y}_0$from their normal distributions, or $E\left(\mathbf{y}_t \mid \epsilon_{lt}\right)$which can be simplified as $\mathbf{y}_t\left(\epsilon_{lt}, \tilde{\epsilon}_{lt} = 0, \mathbf{y}_0 = 0\right)$.

European Commission

Methodology-Total Effect Contribution

- Denote with ϵ_{It} the shock or group of shocks of interest, while $\tilde{\epsilon}_{It}$ indicates the complementary set of shocks in the model.
- We define the *Total Effect contribution,* the effect computed as the difference of the states variables $\mathbf{y}_t$ and the contributions of $\tilde{\epsilon}_{It}$and of $\mathbf{y}_0$ obtained by integrating out ϵ_{It} via Monte Carlo counterfactuals drawing $\epsilon_{It}$from its normal distribution, or $\mathbf{y}_t - E\left(\mathbf{y}_t \mid \tilde{\epsilon}_{It}, \mathbf{y}_0\right)$which can be simplified as $\mathbf{y_t} - \mathbf{y}_t\left(\tilde{\epsilon}_{It}, \mathbf{y}_0, \epsilon_{It} = 0\right)$.

The model

- We apply the above methods to an estimated closed economy version of Kollmann et al. (EER, 2016) for the Euro Area
- The model is a standard NK model, with public sector, and with non Ricardian households
- The "twist" is represented by financial frictions which translate into lending (borrowing) constraints, and by OBC for the ZLB.
- One type of constraint is a constraint on total risky private assets held by the households: always binding
- A second constraint limits the amount of loans between households and firms: occasionally binding (2 model settings: 1) constraint internalized by lenders (Justiniano et al. 2015) 2) internalized by borrowers)

The model - Financial constraints

- As in Jermann et al. (2012), firms may raise funds either by issuing equity or through a debt contract with limited enforceability
- We assume an always binding constraints on total (nominal) risky private assets, equity shares $P_t^s S_t$ plus loans to the firms L_t, held by Ricardian households. In particular we assume an upper bound proportional to the beginning of period firms' capital value:

$$L_t + P_t^S S_t = m^{tot} z_t^F \left(P_t^I K_{t-1} \right)$$

The model -Financial constraints (cont'd)

- We also assume the presence of an OBC tying the amount of loans to the stock of capital, which in period of financial distress, reduces the possibility to substitute between risky assets:

$$L_t \leq m^I z_t^F \left(P_t^I K_{t-1} \right)$$

- z_t^F is an AR(1) process describing the financial conditions of the economy
- Under one exercise this constraint will be part of the HHs' problem (lending constraint), in a second exercise it will be part of the firms' problem (borrowing constraint).

The key equations

Under the lending constraints the equations affected by the OBC are the Euler equations for loans and equity shares :

$$1 + \mu_t^{s,tot} + \mu_t^{s,l} = \beta E_t \left[\tilde{z}_t^C \frac{\lambda_{t+1}^s}{\lambda_t^s} \frac{P_t}{P_{t+1}} \left(1 + i_t^l \right.\right.$$

$$\left.\left. - s^L \left(\alpha_0^L + z_t^L + \alpha_1^L \left(\frac{L_t}{P_t^l K_t} - \frac{\overline{L}}{\overline{P^l}\,\overline{K}} \right) \right) \right) \right]$$

$$1 + \mu_t^{s,tot} = \beta E_t \left[\tilde{z}_t^C \frac{\lambda_{t+1}^s}{\lambda_t^s} \frac{P_t}{P_{t+1}} \left(1 + i_{t+1}^s - s^S \left(\alpha_0^S + z_t^S + \alpha_1^S \frac{P_t^S S_t}{P_t Y_t} \right) \right) \right]$$

The key equations (cont'd)

Under the borrowing constraint specification, the multiplier on the OBC will disappear from the HH FOC for loans, and show up in the Firms' FOC for Capital and Loans:

$$Q_t = E_t \left[\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t} \frac{P^I_{t+1}}{P_{t+1}} \frac{P_t}{P^I_t} \left(\tau^K \delta - \gamma_0^u \left(CU_{t+1} - 1 \right) - \frac{\gamma_1^u}{2} \left(CU_{t+1} - 1 \right)^2 \right. \right.$$

$$\left. \left. + \left(1 - \delta \right) Q_{t+1} + \left(1 - \alpha \right) \mu^Y_{t+1} \frac{P_{t+1}}{P^I_{t+1}} \frac{Y_{t+1}}{K^{tot}_t} + \mu^I_{t+1} P_{t+1} m^I z^F_{t+1} \right) \right]$$

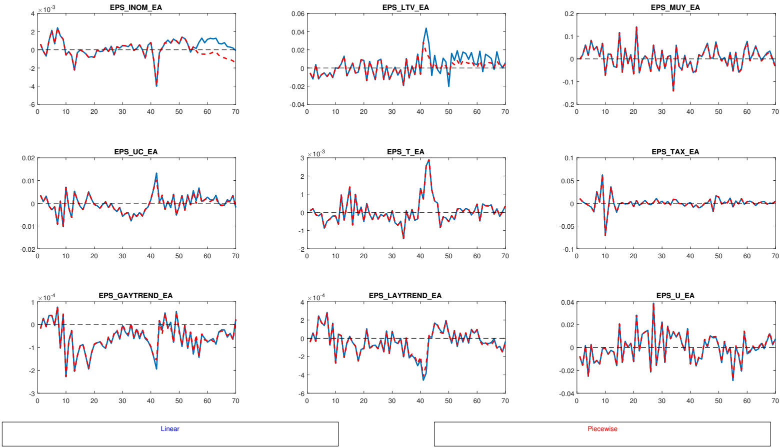
$$E_t \left[\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t} \frac{P_t}{P_{t+1}} \frac{1}{1 - \mu^I_t P_t} \left(1 + i^I_t \right) \right] = 1$$

Results - Regimes sequence

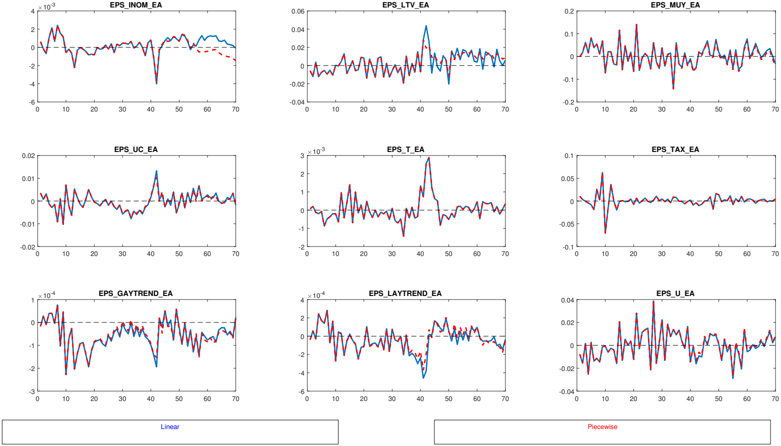
time	regime sequence 1	starting period of regime 1	regime sequence 2	starting period of regime 2
2008	0	1	0	1
2008.25	0	1	0	1
2008.5	0	1	0	1
2008.75	0	1	0	1
2009	0 1 0	1 4 7	1 0	1 6
2009.25	0 1 0	1 3 6	1 0	1 6
2009.5	0 1 0	1 2 5	1 0	1 5
2009.75	1 0	1 4	1 0	1 6
2010	1 0	1 5	1 0	1 5
2010.25	1 0	1 2	1 0	1 4
2010.5	1 0	1 2	1 0	1 5
2010.75	0	1	1 0	1 5
2011	0	1	1 0	1 4
2011.25	0	1	1 0	1 5
2011.5	0	1	1 0	1 5
2011.75	0	1	1 0	1 6
2012	0	1	1 0	1 5
2012.25	0 1 0	1 2 4	1 0	1 6
2012.5	1 0	1 4	1 0	1 6
2012.75	1 0	1 4	1 0	1 6
2013	1 0	1 4	1 0	1 6
2013.25	1 0	1 4	1 0	1 6
2013.5	1 0	1 5	1 0	1 6
2013.75	1 0	1 4	1 0	1 6
2014	1 0	1 4	1 0	1 5
2014.25	1 0	1 5	1 0	1 6
2014.5	1 0	1 5	1 0	1 6
2014.75	1 0	1 4	1 0	1 6
2015	1 0	1 4	1 0	1 5
2015.25	1 0	1 4	1 0	1 6
2015.5	1 0	1 4	1 0	1 5
2015.75	1 0	1 4	1 0	1 5
2016	1 0	1 3	1 0	1 5

time	regime sequence 1	starting period of regime 1	regime sequence 2	starting period of regime 2
2008	0	1	0	1
2008.25	0	1	0	1
2008.5	0	1	0	1
2008.75	0	1	0	1
2009	0 1 0	1 5 7	1 0	1 6
2009.25	0 1 0	1 4 5	1 0	1 5
2009.5	0 1 0	1 3 4	1 0	1 4
2009.75	1 0 1 0	1 2 3 4	1 0	1 3
2010	1 0	1 5	1 0	1 2
2010.25	1 0	1 2	0	1
2010.5	1 0	1 2	1 0	1 3
2010.75	0	1	1 0	1 2
2011	0	1	0	1
2011.25	0	1	1 0	1 2
2011.5	0	1	1 0	1 3
2011.75	0	1	1 0	1 2
2012	0	1	1 0	1 4
2012.25	0 1 0	1 3 4	1 0	1 3
2012.5	1 0	1 4	1 0	1 2
2012.75	1 0	1 4	1 0	1 4
2013	1 0	1 4	1 0	1 3
2013.25	1 0	1 4	1 0	1 2
2013.5	1 0	1 5	1 0	1 3
2013.75	1 0	1 4	1 0	1 2
2014	1 0	1 4	1 0	1 3
2014.25	1 0	1 5	1 0	1 2
2014.5	1 0	1 4	1 0	1 4
2014.75	1 0	1 4	1 0	1 3
2015	1 0	1 4	1 0	1 2
2015.25	1 0	1 3	1 0	1 4
2015.5	1 0	1 4	1 0	1 3
2[illegible]	1 0	1 4	1 0	1 2
[illegible]	1 0	1 3	1 0	1 3

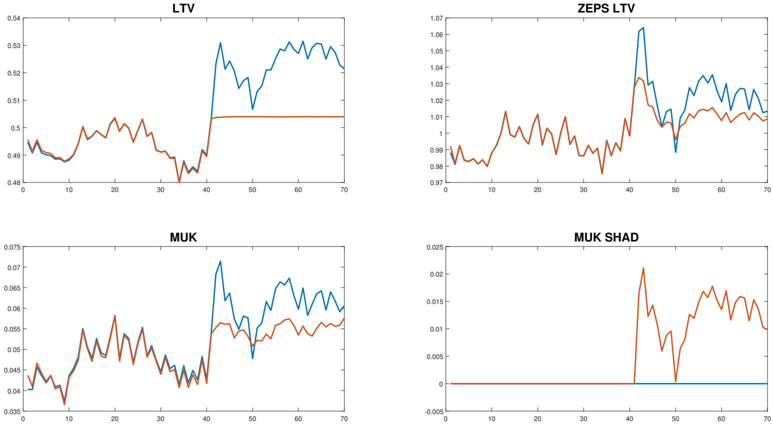
Smoothed Shocks - lending



Smoothed Shocks - borrowing

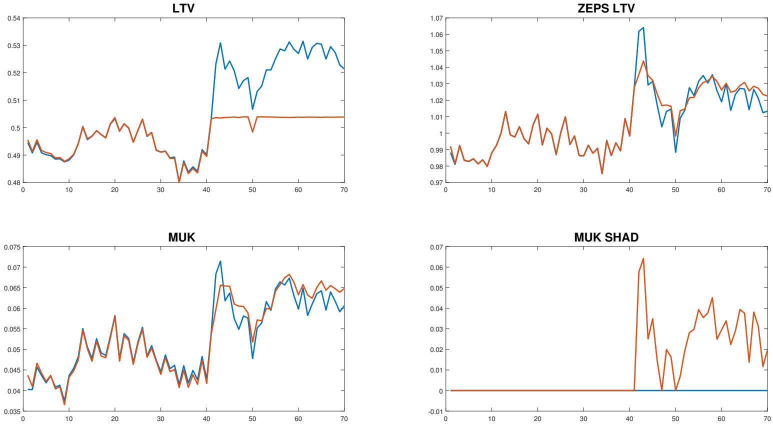


Smoothed latent - lending

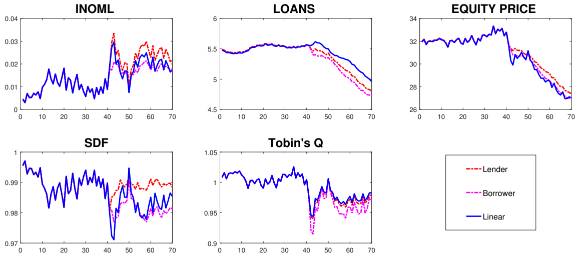


European Commission

Smoothed latent - borrowing



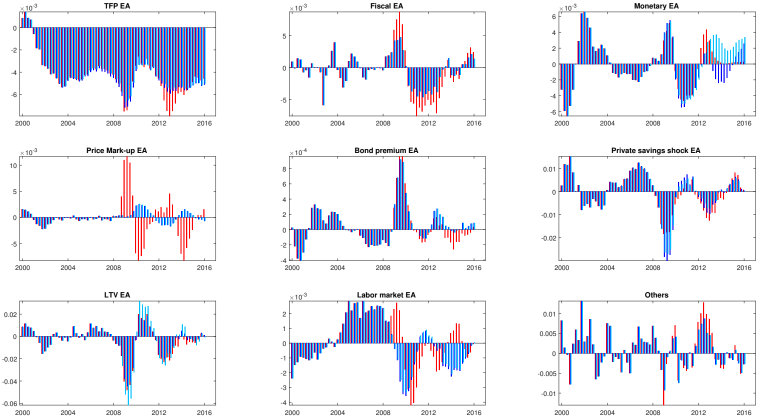
Lending vs Borrowing OBC



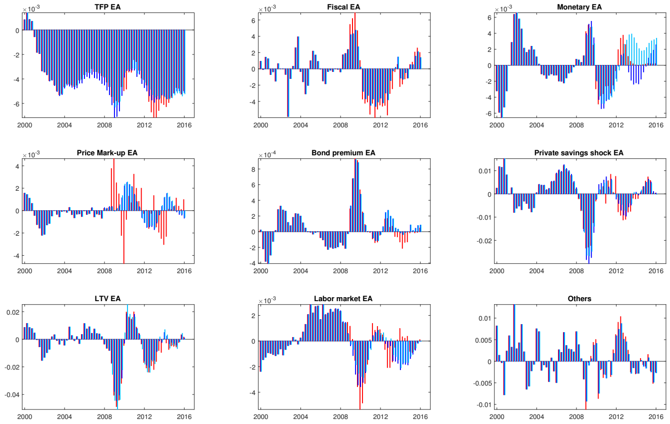
Lending vs Borrowing OBC (cont'd)

- The Lagrange multiplier on the lending OBC induces an increase in the loan rate whereas the opposite occurs from the firms loans demand
- This means that, in equilibrium, loans under the lending OBC will be higher (but clearly lower than in the linear case)
- Combining the two financial constraints when both are binding one obtains a constant share $\frac{L_t}{L_t+P_t^S S_t} = \frac{m^l}{m^{tot}}$ hence under lending OBC also P_t^swill be higher
- In turn this will reduce the expectedd real return from equity and increase the SDF under lending OBC
- On the contrary, the lower SDF under borrowing OBC is responsible of the reduction in Tobin's Q.

Shocks contributions - lending



Shocks contribution - borrowing



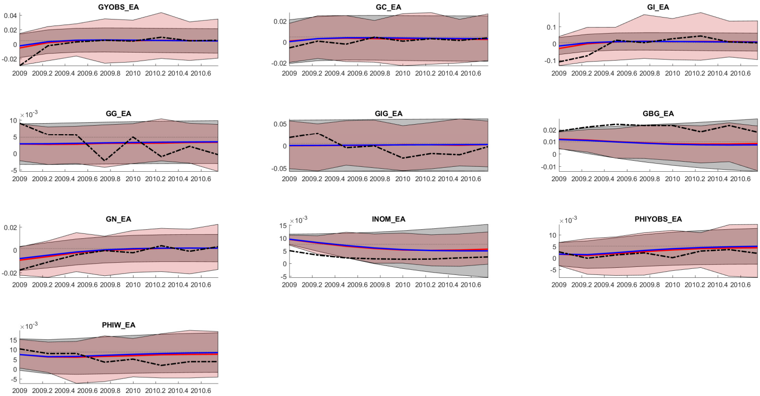
Real Time forecasting

What we are interested in is $E\left(y_{t+k} \mid y_{t-1}\right)$integrating over all possible ϵ_{t+k}, $k = 0, .., T$ with (Quasi) Monte Carlo simulations. So given an initial condition $y_{t-1|t-1}$:

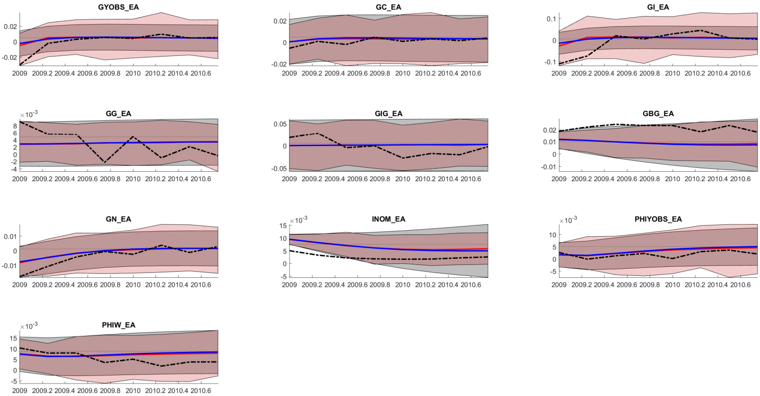
1. For each j Monte Carlo simulation, $j = 1, .., N$
2. For $k = 0, .., T$
3. Draw one realization of shocks ϵ^j_{t+k}, and run Occbin simulations to get $y^j_{t+k} = g\left(y^j_{t+k-1}, \epsilon^j_{t+k}\right)$

The Monte Carlo sample y^j_{t+k} with $j = 1, .., N$ provides us an estimate of the predictive density of the piecewise linear model.

Real Time forecast 2009Q1- lending



Real Time forecast 2009Q1- borrowing



Conclusions

- We proposed an algorithm which allows for the measuring of historical shock decomposition of observables in models with OBC solved with piecewise linear solution.
- We applied the algorithm to a closed economy model with OBC in the financial relationship between households and firms
- We showed that the degree of non-linearity caused by OBC may allow us to include extreme events such as the GFC into the model's predictive density, without invoking non-Gaussian exogenous processes
- Thanks!