# Estimating Macroeconomic Models of Financial Crises: An Endogenous Regime-Switching Approach

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# Motivation

- Global Financial Crisis Proved Costly to Resolve
- Long History of Painful Financial Crises in Emerging Markets
- Large Theoretical Literature in Response
  - Models of Collateral Constraints for Amplification of Shocks
  - Normative Analyses of Inefficiencies from Collateral Constraints
  - *Ex-ante* versus *ex-post* Policies
  - Which Instruments Most Effective
- Still Lack a Concrete Explanation of Why Countries Fall into Crisis
  - Which Shocks (Interest Rate, Technology, Collateral) Trigger Crises?
  - This is an Empirical Issue
  - Can then Return to Policy Questions
- Issue: Models with Occasionally Binding Constraints Hard to Solve
  - Usually Requires Slow Global Solution Methods
  - Makes Likelihood-Based Estimation Infeasible

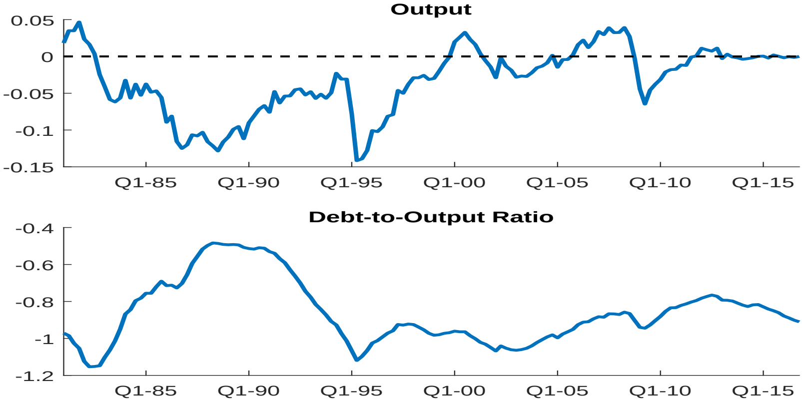
# The Objective of this Paper

- Formulate a Model with Occasionally Binding Constraint
- Quantitative Analysis of Financial Crises in Mexico
- Address Several Questions
  - Which Shocks Drive Crises? The Same Ones that Drive Normal Cycle?
  - Is there Time Variation in the Importance of those Shocks?
  - How do the Dynamic Responses to Shocks Change between Crises and Normal Times?
- Enables Future Steps: Return to the Theoretical Questions
  - Which Instruments Best Address which Shocks?
  - Counterfactuals: Given Shocks that Drove Crisis in Past, would Policy have Helped?

# This Paper

- New Approach to Specifying, Solving, Estimating Models of Crises
  - Financial Crises Rare but Large Events, so Model Must be Non-Linear
  - Provide a Tractable Formulation of Collateral Constraint
  - Develop Methods to Solve and Estimate such a Model
- Collateral Constraint Similar to Kiyotaki and Moore (1997)
  - Limit Total Debt to a Fraction of the Market Value of Physical Capital
  - Unconstrained to Constrained a Stochastic Function of the LTV Ratio
  - Write as Endogenous Regime-Switching Process
    - Two Regimes: Constraint Binds (Crisis) and Doesn't Bind (Normal)
    - Probability of Binding Rises with Leverage (More Debt or Less Collateral)
    - Agents in Model have Rational Expectations
- Estimate via Full-Information Bayesian Methods
  - Estimated Binding Regime Corresponds to Sudden Stop Narrative Dates
  - Fluctuations in Normal Regime Driven by Real Shocks
  - Leverage Shocks most Important in Crisis Regime

# Output and Debt in Mexico



# Model Overview

- Based Largely on Mendoza (2010)
- Small Open Economy that Borrows from Abroad
- Imported Goods used in Production
- Working Capital Constraint for Labor and Import Payments
- Value of Capital Serves as Collateral
- Pecuniary Externality and Overborrowing
- Regime-Specific Borrowing Constraints
- Endogenously Switch Between Regimes
- Four Types of Shocks: 3 Real, 1 Financial

# Preferences and Production

- Representative Household-Firm with Preferences

$$U \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{1}{1-\rho} \left( C_t - \frac{H_t^{\omega}}{\omega} \right)^{1-\rho} \right\}$$

- Production uses Capital, Labor, and Imported Intermediate Goods

$$Y_t = A_t K_{t-1}^{\eta} H_t^{\alpha} V_t^{1-\alpha-\eta}$$

- Investment with Adjustment Costs

$$I_t = \delta K_{t-1} + (K_t - K_{t-1}) \left( 1 + \frac{\iota}{2} \left( \frac{K_t - K_{t-1}}{K_{t-1}} \right)^2 \right)$$

- Budget Constraint, with $B_t < 0$ as Debt

$$C_t + I_t = Y_t - P_t V_t - \phi r_t \left( W_t H_t + P_t V_t \right) - \frac{1}{(1+r_t)} B_t + B_{t-1}$$

# Collateral Constraint: Motivation

- The Agent Faces a Regime-Specific Collateral Constraint
  - When $s_t = 1$, Borrowing is Constrained (Crisis Regime)
  - When $s_t = 0$, Borrowing is Unconstrained (Normal)
- International Lenders have Stochastic Monitoring
  - In Crisis, Actively Monitor and Enforce Borrowing Constraint
  - In Normal, Don't Actively Monitor and Allow Borrowing
  - Decision to Monitor or Not Depends on Previous Borrowing and Monitoring Shock
  - Key Timing: Monitoring Shock Orthogonal to Structural Shocks

# Collateral Constraint: Crisis Regime

- In Crisis Regime, Total Borrowing is a Fraction of Value of Collateral

$$\frac{1}{(1+r_t)} B_t - \phi (1+r_t) (W_t H_t + P_t V_t) = -\kappa_t q_t K_t$$

  - Debt and Working Capital Restricted
  - Collateral in the Model is Defined over the Value of Capital
  - Pecuniary Externality: Price and Quantity of Collateral are Endogenous
- Multiplier Associated with Constraint is $\lambda_t$

# Collateral Constraint: Normal Regime

- In Normal Regime, Borrowing is Unconstrained
  - Collateral Value is Sufficient for International Lenders to Finance all Desired Borrowing
  - No Explicit Constraint on Borrowing
  - Two Forces Limiting Infinite Borrowing
    - Small Debt Elastic Interest Rate Premium
    - Expectations
- The "Borrowing Cushion" is Debt Less the Collateral Value

$$B_t^* = \frac{1}{(1+r_t)} B_t - \phi (1+r_t) (W_t H_t + P_t V_t) + \kappa_t q_t K_t$$

- Small Borrowing Cushion Implies High Leverage Ratio

# Endogenous Switching

- In Normal Regime, Probability that Constraint Binds or Not Next Period Depends on Borrowing Cushion and Monitoring Shock

$$s_{t+1} = \Pi\left(B_t^*, \epsilon_{t+1}^M | s_t = 0\right)$$

- In Crisis Regime, Probability that Constraint Binds or Not Next Period Depends on Multiplier

$$s_{t+1} = \Pi\left(\lambda_t, \epsilon_{t+1}^M | s_t = 1\right)$$

- Reformulates Kiyotaki-Moore Idea that Increased Leverage Leads to Binding Collateral Constraints as a Probabilistic Statement
- Note the Difference in Timing

# Endogenous Switching

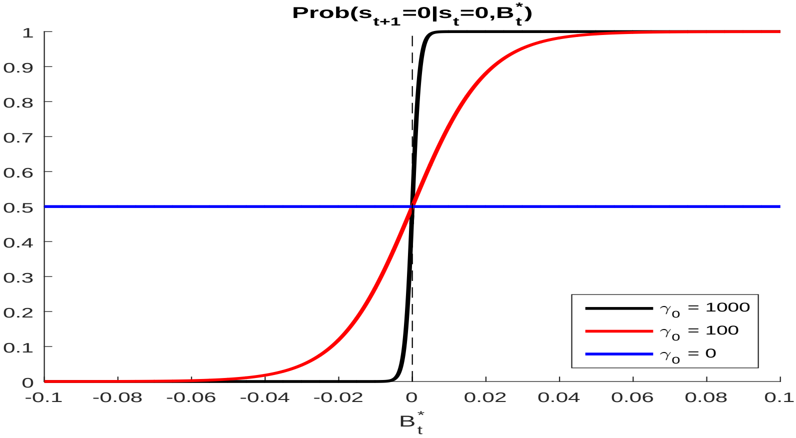
- Assume $\Pi$ and $\epsilon^M_{t+1}$ Generate Logistic Distributions

$$\Pr\left(s_{t+1}=1|s_t=0\right)=\frac{\exp\left(-\gamma_0 B^*_t\right)}{1+\exp\left(-\gamma_0 B^*_t\right)}$$

$$\Pr\left(s_{t+1}=0|s_t=1\right)=\frac{\exp\left(-\gamma_1 \lambda_t\right)}{1+\exp\left(-\gamma_1 \lambda_t\right)}$$

- Similar to Davig, et al (2010), Bi and Traum (2014), and Kumhof et al (2015)
- Evidence for $\gamma_0$ and $\gamma_1$ Key in Estimation
- Slackness Condition is $B^*_t \lambda_t = 0$, will Return to This Later

# Form of the Logistic Function



# Interest Rates and Exogenous Processes

- Interest Rate Process

$$r_t = r^* + \psi_r \left( e^{\overline{B} - B_t} - 1 \right) + \sigma_w (s_t) \varepsilon_{w,t}$$

- Productivity

$$\log A_t = (1 - \rho_A (s_t)) a (s_t) + \rho_A (s_t) \log A_{t-1} + \sigma_A (s_t) \varepsilon_{A,t}$$

- Terms of Trade

$$\log P_t = (1 - \rho_P (s_t)) p (s_t) + \rho_P (s_t) \log P_{t-1} + \sigma_P (s_t) \varepsilon_{P,t}$$

- Leverage

$$\kappa_t = (1 - \rho_\kappa (s_t)) \kappa (s_t) + \rho_\kappa (s_t) \kappa_{t-1} + \sigma_\kappa (s_t) \varepsilon_{\kappa,t}$$

# Solution

- Full Set of Structural Equations: 16 Equilibrium Conditions
    - First-Order Conditions
    - Constraints
    - Exogenous Processes
- Nonlinear Model that Can in Principle be Solved with Global Methods
- This Paper: Compute an Approximate Solution via Perturbation
    - Very Fast Solution that Allows for Likelihood-Based Estimation
    - Show How Rewrite Slackness Condition as Regime-Switching
    - Endogenously Determined Approximation Point between Normal and Crisis Regimes

# Regime Switching Slackness Condition

- Recall the Slackness Condition $B_t^* \lambda_t = 0$
- This Condition is Hard to Implement via Local Approximations
- Introduce Indicator Variables $\varphi(s_t) = \nu(s_t) = s_t$
- Slackness Constraint Becomes

$$\varphi(s_t) B_{ss}^* + \nu(s_t)(B_t^* - B_{ss}^*) = (1 - \varphi(s_t)) \lambda_{ss} + (1 - \nu(s_t))(\lambda_t - \lambda_{ss})$$

- Modified Slackness Condition
  - In Normal Regime, $\varphi(0) = \nu(0) = 0$, so $\lambda_t = 0$
  - In Crisis Regime, $\varphi(1) = \nu(1) = 1$, so $B_t^* = 0$

# Properties of the Solution

- Extend Perturbation Method of Foerster, et. al. (2016)
- Other Approaches: Lind (2014), Maih (2015), Barthelemy and Marx (2017)
- Approximation Point Ergodic Mean of Regimes

$$P_{ss} = \begin{bmatrix} 1 - \frac{\exp(-\gamma_0 B^*_{ss})}{1+\exp(-\gamma_0 B^*_{ss})} & \frac{\exp(-\gamma_0 B^*_{ss})}{1+\exp(-\gamma_0 B^*_{ss})} \\ \frac{\exp(-\gamma_1 \lambda_{ss})}{1+\exp(-\gamma_1 \lambda_{ss})} & 1 - \frac{\exp(-\gamma_1 \lambda_{ss})}{1+\exp(-\gamma_1 \lambda_{ss})} \end{bmatrix}$$

- General Result: Endogenous Switching Doesn't Appear in First Order
  - First-Order Dynamics Same with Endogenous and Exogenous Probabilities of $P_{ss}$
  - Precautionary Behavior in the Second Order Solution is Critical
- Expectational Effects Matter for Response to Shocks in Normal Regime
  - Sensitivity of Crises to Debt Cushion
  - Magnitude of Crises
  - Note that this Makes Policy Implications Interesting/Relevant

# Estimating the Nonlinear Model

- Second-Order plus Endogenous Probabilities Complicates Estimation
- Rational Expectations
  - Links Parameters Across Regimes and Economic Behavior
  - Two-Step Procedures Inappropriate
  - Agents in the Model Fully Understand Crises Occur and Adjust Behavior
  - Estimated Model Useful for Normative Analysis Precisely because of this Feature of the Model Solution/Estimation
- Identification of Parameters Helped by Rational Expectations
- Procedure for Simultaneous Estimation of Regimes and Parameters
  - Metropolis-Hastings Algorithm
  - Binning and Maih (2015): Unscented Kalman Filter with Sigma Points
- Bayesian Estimation with Diffuse Priors

# Data for Estimation

- Data for Mexico from 1981Q1 to 2016Q1
  - Includes Financial Crises of 1982, 1994, 2007
  - Also Periods of Expansion and Recession
- Observables
  - Real GDP Growth
  - Investment Growth
  - Consumption Growth
  - Import Price Growth
  - Interest Rate: EMBI Global + World Interest Rate
- Measurement Errors for all Observables

# Quick Recap

- Set up a Small Open Economy Model
  - Hit with 4 Types of Shocks
  - Borrow to Smooth Consumption, Pay for Inputs
  - As Debt Increases Relative to Capital, Probability of a Crisis Increases
  - Crisis Constrains Borrowing
- Developed Solution and Estimation Procedures
  - Endogenous Regime Switching
  - Second Order Solution and Estimation
- Objectives for Estimation
  - Estimate Key Structural Parameters
  - Characterize When in Crisis Regime, Which Shocks Drive Crises
  - Determine which Shocks Drive Standard Fluctuations

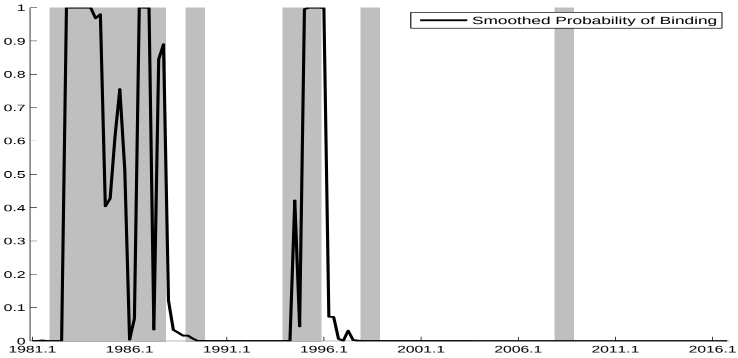
# Calibrated Parameters

| Parameter | Value |
|---|---|
| Discount Factor | $\beta = 0.97959$ |
| Risk Aversion | $\rho = 2$ |
| Labor Share | $\alpha = 0.592$ |
| Capital Share | $\eta = 0.306$ |
| Wage Elasticity of Labor Supply | $\omega = 1.846$ |
| Capital Depreciation (8.8% Annually) | $\delta = 0.022766$ |
| Interest Rate Intercept | $r^* = 0.0208352$ |
| Interest Rate Elasticity | $\psi_r = 0.05$ |
| Neutral Debt Level | $\bar{B} = -1.7517$ |
| Mean of TFP Process, Normal Regime | $a(0) = 0$ |
| Mean of Import Price Process, Normal Regime | $p(0) = 0$ |
| Mean of Leverage Process, Normal Regime | $\kappa(0) = 0.15$ |

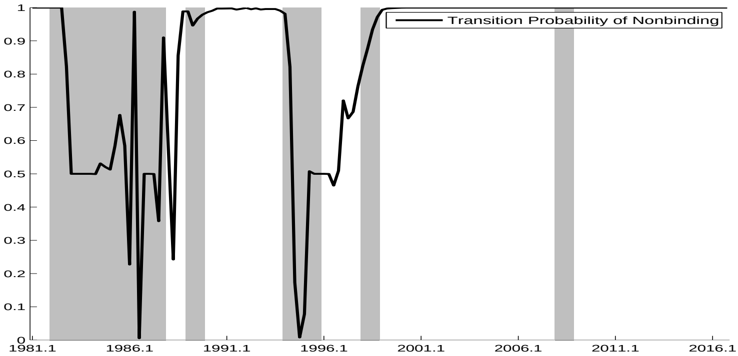
# Estimation Results: Key Structural Parameters

| Parameter | | Posterior Mean | q5 | q95 |
|---|---|---|---|---|
| TFP Persistence | $\rho_a(0)$ | 0.8134 | 0.7208 | 0.8843 |
| | $\rho_a(1)$ | 0.7746 | 0.5543 | 0.8968 |
| TOT Persistence | $\rho_p(0)$ | 0.9637 | 0.9340 | 0.9876 |
| | $\rho_p(1)$ | 0.9260 | 0.8258 | 0.9941 |
| Lev Persistence | $\rho_\kappa(0)$ | 0.6656 | 0.4152 | 0.8946 |
| | $\rho_\kappa(1)$ | 0.7804 | 0.6728 | 0.8872 |
| TFP Mean, Crisis | $a(1)$ | -0.0059 | -0.0072 | -0.0047 |
| TOT Mean, Crisis | $p(1)$ | 0.0005 | 0.0000 | 0.0013 |
| Lev Mean, Crisis | $\kappa(1)$ | 0.2305 | 0.2203 | 0.2440 |
| Capital Adjust Cost | $\iota$ | 2.8233 | 2.8144 | 2.8360 |
| Working Capital | $\phi$ | 0.3036 | 0.2697 | 0.3217 |
| Normal to Crisis Prob | $\gamma_0$ | 89.0076 | 73.2143 | 108.1845 |
| Crisis to Normal Prob | $\gamma_1$ | 1.9676 | 0.0892 | 5.8921 |

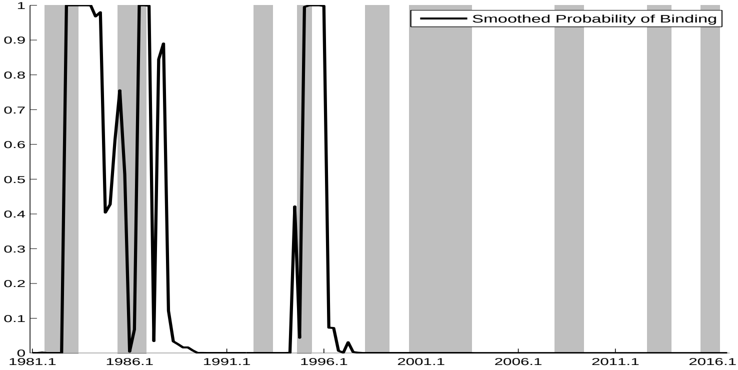
# Crises Estimates vs. Reinhart-Rogoff Currency Crisis Dates



# Transition Prob. vs. Reinhart-Rogoff Currency Crisis Dates



# Crises Estimates vs. OECD Recession Dates



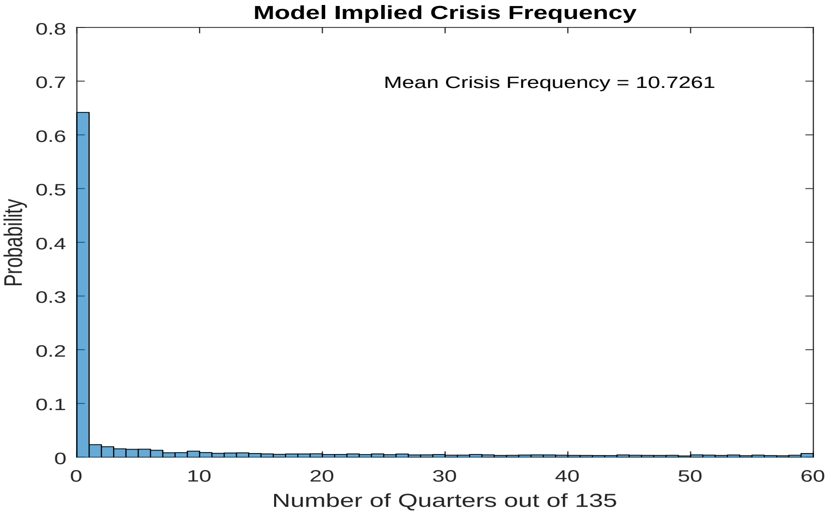
# Estimation Results: Shock Standard Deviations

| Parameter | | Posterior Mean | q5 | q95 |
|---|---|---|---|---|
| World Interest Rate | $\sigma_w(0)$ | 0.0007 | 0.0001 | 0.0015 |
| | $\sigma_w(1)$ | 0.0438 | 0.0332 | 0.0496 |
| TFP | $\sigma_a(0)$ | 0.0056 | 0.0043 | 0.0068 |
| | $\sigma_a(1)$ | 0.0091 | 0.0062 | 0.0123 |
| TOT | $\sigma_p(0)$ | 0.0401 | 0.0338 | 0.0478 |
| | $\sigma_p(1)$ | 0.0487 | 0.0218 | 0.0766 |
| Leverage | $\sigma_\kappa(0)$ | 0.0012 | 0.0001 | 0.0030 |
| | $\sigma_\kappa(1)$ | 0.0248 | 0.0072 | 0.0419 |

# Importance of Shocks

| Shock | | Regime | C | I | r | Y |
|---|---|---|---|---|---|---|
| World Interest Rate | $\varepsilon_{w,t}$ | Normal | 0.0001 | 0.0128 | 0.0066 | 0.0000 |
| Technology | $\varepsilon_{a,t}$ | Normal | 0.3087 | 0.2670 | 0.6390 | 0.3158 |
| Import Price | $\varepsilon_{p,t}$ | Normal | 0.6817 | 0.3777 | 0.1971 | 0.6814 |
| Leverage | $\varepsilon_{\kappa,t}$ | Normal | 0.0095 | 0.3424 | 0.1572 | 0.0027 |
| World Interest Rate | $\varepsilon_{w,t}$ | Crisis | 0.0074 | 0.0044 | 0.3701 | 0.0145 |
| Technology | $\varepsilon_{a,t}$ | Crisis | 0.0106 | 0.0003 | 0.0004 | 0.0705 |
| Import Price | $\varepsilon_{p,t}$ | Crisis | 0.0124 | 0.0002 | 0.0003 | 0.0630 |
| Leverage | $\varepsilon_{\kappa,t}$ | Crisis | 0.9696 | 0.9951 | 0.6291 | 0.8520 |

# Crisis Frequency



# What Drives the Crisis Frequency

| Shock | | Frequency |
|---|---|---|
| All Shocks | | 10.7261 |
| Individual | | |
| World Interest Rate Only | $\varepsilon_{w,t}$ | 0.0095 |
| Technology Only | $\varepsilon_{a,t}$ | 1.8908 |
| Import Price Only | $\varepsilon_{p,t}$ | 4.5550 |
| Leverage Only | $\varepsilon_{\kappa,t}$ | 3.0736 |
| Sum | | 9.5289 |

# Conclusion

- New Approach to Specifying, Solving, Estimating Models of Financial Crises
- Probability Regime Switch Depends on State of Economy
- Endogenous Switching Impacts the Economic Behavior in Qualitatively and Quantitatively Important Ways
- Crisis Regime Corresponds to Narrative Dates
- Leverage Shocks Drive Fluctuations during Financial Crises
- Real Shocks Drive Fluctuations in Normal Regime
- Future Work: Conditional Policy Counterfactuals