A Structural Investigation of Monetary Policy Shifts

Yoosoon Chang

Indiana University Bloomington

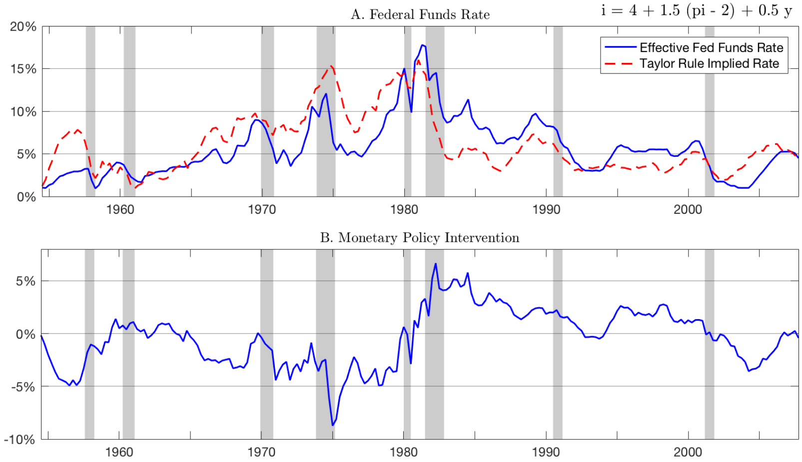
Joint with Fei Tan, Xin Wei

Workshop on Nonlinear Models in Macroeconomics and Finance for an Unstable World

Norges Bank, Oslo, Norway

January 26-27, 2018

Fed Funds Rate and Taylor Rule: 1954-Present



Clear time variation (regime changes) in monetary policy intervention. What are the drivers?

What Is the Paper About?

This work introduces threshold-type switching with endogenous feedback into DSGE models

- how agents form expectations on future regime change
- shed empirical light on how & why policy regime shifts

Substantive finding

- post-WWII U.S. monetary policy shifts have been largely driven by non-policy shocks

Methodological contribution

- derive analytical solution for endogenous switching Fisherian model
- develop an endogenous switching Kalman filter

Main Results

Endogenous switching in Fisherian model

- structural shocks drive regime change through endogenous feedback mechanism
- endogenous feedback induces expectational effect, which helps stabilize price level

Endogenous switching in a New Keynesian model

- we show empirically that U.S. monetary policy shifts are mainly driven by non-policy shocks
- in particular, the markup shocks associated with oil crises were the main driver of monetary policy in 70's, and preference shocks indicating the strong economic recovery in early 80's drove monetary policy regime back to active.

Endogenous Switching in Fisherian Model

Model

Fisher equation:

$$i_t = \mathbb{E}_t \pi_{t+1} + \mathbb{E}_t r_{t+1}$$

Real rate process:

$$r_t = \rho_r r_{t-1} + \sigma_r \epsilon_t^r$$

Monetary policy with endogenous feedback:

$$i_t = \alpha(s_t)\pi_t + \sigma_e \epsilon_t^e$$

$$s_t = 1\{w_t \geq \tau\}$$

$$w_{t+1} = \phi w_t + v_{t+1},$$

$$\begin{pmatrix} \epsilon_t^e \\ v_{t+1} \end{pmatrix} =_d iid\ \mathbb{N}\left(0, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right)$$

as considered in Chang, Choi and Park (2017).

Information Structure

- Agents don't observe the level of latent regime factor w_t, but observe whether or not it crosses the threshold, as reflected in $s_t = 1\{w_t \geq \tau\}$.
- Agents form expectations on future inflation as

$$\mathbb{E}_t \pi_{t+1} = \mathbb{E}(\pi_{t+1} | \mathcal{F}_t), \quad \mathcal{F}_t = \{i_u, \pi_u, r_u, \epsilon_u^r, \epsilon_u^e, s_u\}_{u=0}^t$$

- Monetary authority observes all information in $\mathcal{F}_t$ and also the history of policy regime factor (w_t).

Endogenous Feedback Mechanism

To see the endogenous feedback mechanism, rewrite

$$w_{t+1} = \phi w_t + \underbrace{\rho \epsilon_t^e + \sqrt{1-\rho^2}\eta_{t+1}}_{v_{t+1}}, \quad \eta_{t+1} \sim i.i.d.\mathbb{N}(0,1)$$

From variance decomposition, we see that ρ^2 is the contribution of past intervention to regime change

- $\rho = 0$: fully driven by exogenous non-structural shock

$$w_{t+1} = \phi w_t + \eta_{t+1}$$

- $|\rho| = 1$: fully driven by past monetary policy shock

$$w_{t+1} = \phi w_t + \epsilon_t^e$$

Time-Varying Transition Probabilities

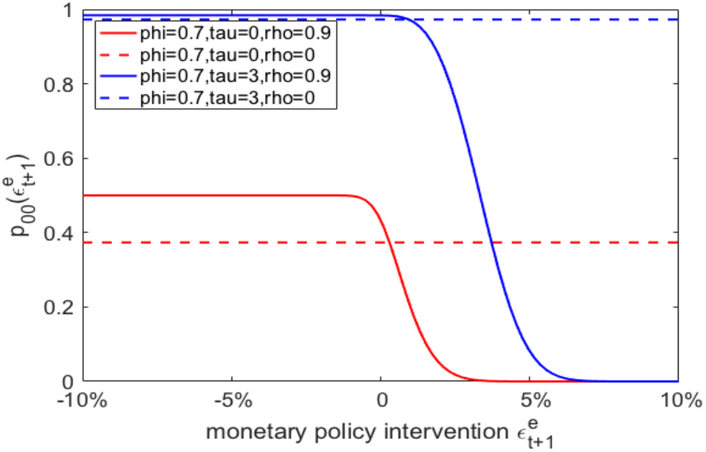
Agents infer TVTP by integrating out the latent factor w_t using its invariant distribution, $\mathbb{N}(0, 1/(1-\phi^2))$, and obtain

$$p_{00}(\epsilon_t^e) = \frac{\int_{-\infty}^{\tau\sqrt{1-\phi^2}} \Phi_\rho\left(\tau - \frac{\phi x}{\sqrt{1-\phi^2}} - \rho\epsilon_t^e\right)\varphi(x)dx}{\Phi(\tau\sqrt{1-\phi^2})}$$

$$p_{10}(\epsilon_t^e) = \frac{\int_{\tau\sqrt{1-\phi^2}}^{\infty} \Phi_\rho\left(\tau - \frac{\phi x}{\sqrt{1-\phi^2}} - \rho\epsilon_t^e\right)\varphi(x)dx}{1-\Phi(\tau\sqrt{1-\phi^2})}$$

where $\Phi_\rho(x) = \Phi(x/\sqrt{1-\rho^2})$.

Time-Varying Transition Probabilities



- If $\rho = 0$, reduce to exogenous switching model
- ρ governs the fluctuation of transition probabilities

Analytical Solution

We solve the system of expectational nonlinear difference equations using the guess and verify method.

Davig and Leeper (2006) show that the analytical solution for the model with fixed regime monetary policy process is

$$\pi_{t+1} = a_1 r_{t+1} + a_2 \epsilon^e_{t+1}$$

with some constants a_1 and a_2.

Motivated by this, we start with the following guess

$$\pi_{t+1} = a_1(s_{t+1}, p_{s_{t+1},0}(\epsilon^e_{t+1}))r_{t+1} + a_2(s_{t+1})\epsilon^e_{t+1}$$

Analytical Solution

- Solution derivation

$$\pi_{t+1} = \underbrace{\frac{\rho_r}{\alpha(s_{t+1})} \frac{(\alpha_1-\alpha_0)p_{s_{t+1},0}(\epsilon^e_{t+1}) + \alpha_1\left(\frac{\alpha_0}{\rho_r} - \mathbb{E}p_{00}(\epsilon^e_{t+1})\right) + \alpha_0\mathbb{E}p_{10}(\epsilon^e_{t+1})}{(\alpha_1-\rho_r)\left(\frac{\alpha_0}{\rho_r} - \mathbb{E}p_{00}(\epsilon^e_{t+1})\right) + (\alpha_0-\rho_r)\mathbb{E}p_{10}(\epsilon^e_{t+1})}}_{a_1(s_{t+1},p_{s_{t+1},0}(\epsilon^e_{t+1}))} r_{t+1}$$

$$\underbrace{-\frac{\sigma_e}{\alpha(s_{t+1})}}_{a_2(s_{t+1})}\epsilon^e_{t+1}$$

- Limiting case 1: exogenous switching solution ($\rho = 0$)

$$\pi_{t+1} = \underbrace{\frac{\rho_r}{\alpha(s_{t+1})} \frac{(\alpha_1-\alpha_0)\bar{p}_{s_{t+1},0} + \alpha_1\left(\frac{\alpha_0}{\rho_r} - \bar{p}_{00}\right) + \alpha_0\bar{p}_{10}}{(\alpha_1-\rho_r)\left(\frac{\alpha_0}{\rho_r} - \bar{p}_{00}\right) + (\alpha_0-\rho_r)\bar{p}_{10}}}_{a_1(s_{t+1})} r_{t+1} \underbrace{-\frac{\sigma_e}{\alpha(s_{t+1})}}_{a_2(s_{t+1})}\epsilon^e_{t+1}$$

Analytical Solution

- Solution derivation

$$\pi_{t+1} = \underbrace{\frac{\rho_r}{\alpha(s_{t+1})} \frac{(\alpha_1 - \alpha_0) p_{s_{t+1},0}(\epsilon^e_{t+1}) + \alpha_1 \left(\frac{\alpha_0}{\rho_r} - \mathbb{E} p_{00}(\epsilon^e_{t+1}) \right) + \alpha_0 \mathbb{E} p_{10}(\epsilon^e_{t+1})}{(\alpha_1 - \rho_r) \left(\frac{\alpha_0}{\rho_r} - \mathbb{E} p_{00}(\epsilon^e_{t+1}) \right) + (\alpha_0 - \rho_r) \mathbb{E} p_{10}(\epsilon^e_{t+1})}}_{a_1(s_{t+1}, p_{s_{t+1},0}(\epsilon^e_{t+1}))} r_{t+1}$$

$$\underbrace{- \frac{\sigma_e}{\alpha(s_{t+1})}}_{a_2(s_{t+1})} \epsilon^e_{t+1}$$

- Limiting case 2: fixed-regime solution ($\alpha_1 = \alpha_0$)

$$\pi_{t+1} = \underbrace{\frac{\rho_r}{\alpha - \rho_r}}_{a_1} r_{t+1} \underbrace{- \frac{\sigma_e}{\alpha}}_{a_2} \epsilon^e_{t+1}$$

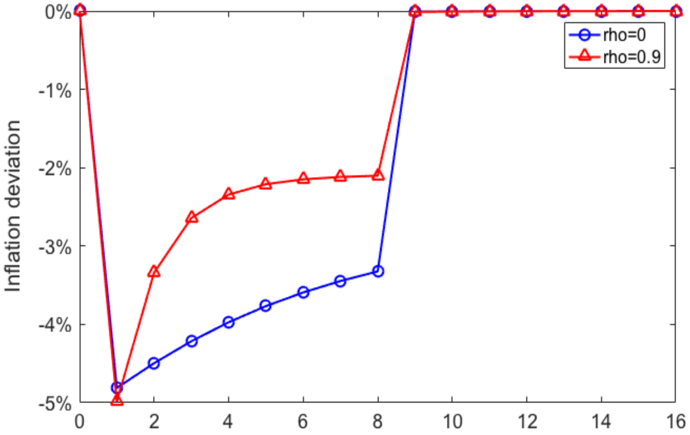
Macro Effects of Policy Intervention

Monetary authority sets future policy intervention $I_t = \{\tilde{\epsilon}^e_{t+1}, \tilde{\epsilon}^e_{t+2}, \ldots, \tilde{\epsilon}^e_{t+K}\}$ and evaluates its effect on future inflation. To illustrate, consider a contractionary intervention as in Leeper and Zha (2003):

$$I_T = \{\underbrace{4\%, \ldots, 4\%}_{8 \text{ periods}}, \underbrace{0, \ldots, 0}_{8 \text{ periods}}\} \text{ with } K = 16, \quad s_T = 0$$

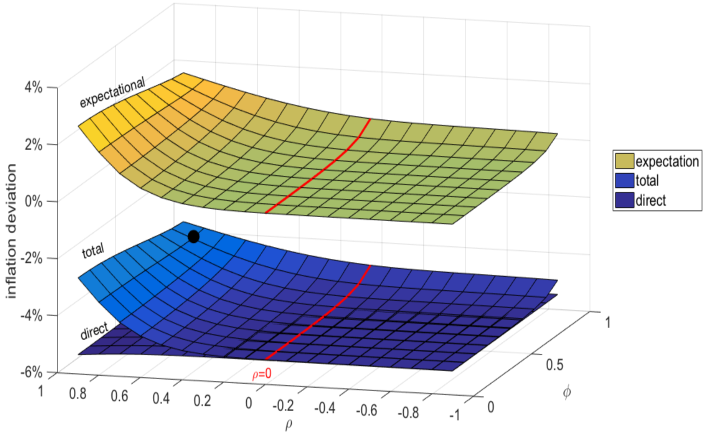
- Baseline = $\mathbb{E}(\pi_{T+K} | \mathcal{F}_T, s_t = s_T, t = T+1, \ldots, T+K)$
- Direct Effects = $\mathbb{E}(\pi_{T+K} | I_T, \mathcal{F}_T, s_t = s_T, t = T+1, \ldots, T+K)$ - Baseline
- Total Effects = $\mathbb{E}(\pi_{T+K} | I_T, \mathcal{F}_T)$ - Baseline
- Expectations Formation Effects = Total Effects - Direct Effects

Impulse Response Function



- $\epsilon_{T+1} > 0 \xrightarrow{\rho>0} w_{T+2} \uparrow, s_{T+2} \nearrow 1 \rightarrow$ more aggressive
- endogenous mechanism helps explain price stabilization

Expectations Formation Effect



- $\epsilon_{T+1} > 0 \xrightarrow{\rho>0} w_{T+2} \uparrow, s_{T+2} \nearrow 1 \rightarrow$ more likely to switch
- price stabilized b/c agents adjust their beliefs on future regimes
- black dot signifies period $T+2$ total effect;

Endogenous Switching in New Keynesian Model

Households and Firms

Households:

$$\max_{\{C_{t+s},N_{t+s},B_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \xi_{t+s} \left(\frac{(C_{t+s}/A_{t+s})^{1-\epsilon}}{1-\epsilon} - N_{t+s} \right)$$

$$\text{s.t. } P_t C_t + B_t + T_t = R_{t-1} B_{t-1} + P_t W_t N_t + P_t D_t$$

Firms:

$$\max_{\{N_{jt+s},P_{jt+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \xi_{t+s} Q_{t+s|t} D_{jt+s}$$

$$\text{s.t. } D_{jt} = \frac{P_{jt} Y_{jt}}{P_t} - W_t N_{jt} - \underbrace{\frac{\phi}{2} \left(\frac{P_{jt}}{\Pi^*_{s_t} P_{jt-1}} - 1 \right)^2 Y_{jt}}_{\text{real price-adjustment cost}}$$

$$Y_{jt} = A_t N_{jt} \quad \text{(Production)}$$

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\theta_t} Y_t \quad \text{(Dixit-Stiglitz aggregation)}$$

Policy and Shocks

Monetary and Fiscal Policy:

$$\frac{R_t}{R^*_{s_t}} = \left(\frac{R_{t-1}}{R^*_{s_{t-1}}}\right)^{\rho_R(s_t)} \left[\left(\frac{\Pi_t}{\Pi^*_{s_t}}\right)^{\psi_\pi(s_t)} \left(\frac{Y_t}{Y^*_t}\right)^{\psi_y(s_t)}\right]^{1-\rho_R(s_t)} e_t$$

$$s_t = \mathbf{1}\{w_t \geq \tau\}$$

$$w_t = \alpha w_{t-1} + v_t$$

$$P_t G_t + R_{t-1} B_{t-1} = T_t + B_t$$

Shocks:

technology: $\ln A_t = \ln \gamma + \ln A_{t-1} + \ln a_t$

$\ln a_t = \rho_a \ln a_{t-1} + \sigma_a \varepsilon^a_t$

preference: $\ln \xi_t = \rho_\xi \ln \xi_{t-1} + \sigma_\xi \varepsilon^\xi_t$

markup: $\ln u_t = (1 - \rho_u) \ln u + \rho_u \ln u_{t-1} + \sigma_u \varepsilon^u_t$

MP: $\ln e_t = \sigma_e \varepsilon^e_t$

FP: $\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \sigma_g \varepsilon^g_t$

Endogenous Feedback Mechanism

$$\begin{pmatrix} \varepsilon_t^a \\ \varepsilon_t^\xi \\ \varepsilon_t^u \\ \varepsilon_t^e \\ \varepsilon_t^g \\ v_{t+1} \end{pmatrix} \sim \mathbb{N}\left(0, \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & \rho_{av} \\ 0 & 1 & 0 & 0 & 0 & \rho_{\xi v} \\ 0 & 0 & 1 & 0 & 0 & \rho_{uv} \\ 0 & 0 & 0 & 1 & 0 & \rho_{ev} \\ 0 & 0 & 0 & 0 & 1 & \rho_{gv} \\ \rho_{av} & \rho_{\xi v} & \rho_{uv} & \rho_{ev} & \rho_{gv} & 1 \end{pmatrix}\right), \quad \rho'\rho < 1$$

i.e. $w_{t+1} = \alpha w_t + \underbrace{\rho'\varepsilon_t + \sqrt{1-\rho'\rho}\,\eta_{t+1}}_{v_{t+1}}$.

Variance decomposition:

$$\begin{aligned} FEV(w_{t,h}) &= \sum_{j=1}^{h} \alpha^{2(h-j)} \\ &= \sum_{k=1}^{5} \underbrace{\sum_{j=1}^{h} \rho_k^2 \alpha^{2(h-j)}}_{k\text{-th structural}} + \underbrace{\sum_{j=1}^{h} \left(1 - \sum_{k=1}^{5} \rho_k^2\right) \alpha^{2(h-j)}}_{\text{non-structural}} \end{aligned}$$

Equilibrium Conditions

Euler: $$\mathbb{E}_t\left[\frac{\beta R_t}{\Pi_{t+1}}\left(\frac{C_t/A_t}{C_{t+1}/A_{t+1}}\right)^{\epsilon}\left(\frac{A_t}{A_{t+1}}\right)\left(\frac{\xi_{t+1}}{\xi_t}\right)\right]=1$$

NKPC: $$u_t\left(\frac{C_t}{A_t}\right)^{\epsilon}-\phi\left(\frac{\Pi_t}{\Pi^*_{s_t}}-1\right)\left[\left(\frac{u_t}{2}-1\right)\frac{\Pi_t}{\Pi^*_{s_t}}+\frac{u_t}{2}\right]$$

$$+\beta\phi(u_t-1)\mathbb{E}_t\left[\frac{\xi_{t+1}}{\xi_t}\frac{Y_{t+1}/A_{t+1}}{Y_t/A_t}\left(\frac{C_{t+1}/A_{t+1}}{C_t/A_t}\right)^{-\epsilon}\frac{\Pi_{t+1}}{\Pi^*_{s_{t+1}}}\left(\frac{\Pi_{t+1}}{\Pi^*_{s_{t+1}}}-1\right)\right]=1$$

Mkt Clear: $$Y_t=C_t+G_t+\frac{\phi}{2}\left[\frac{\Pi_t}{\Pi^*_{s_t}}-1\right]^2 Y_t$$

MP: $$\frac{R_t}{R^*_{s_t}}=\left(\frac{R_{t-1}}{R^*_{s_{t-1}}}\right)^{\rho_R(s_t)}\left[\left(\frac{\Pi_t}{\Pi^*_{s_t}}\right)^{\psi_\pi(s_t)}\left(\frac{Y_t}{Y^*_t}\right)^{\psi_y(s_t)}\right]^{1-\rho_R(s_t)} e_t$$

TVTP1: $$p_{00}(\varepsilon_t)=\frac{\int_{-\infty}^{\tau\sqrt{1-\alpha^2}}\Phi_\rho\left(\tau-\frac{\alpha x}{\sqrt{1-\alpha^2}}-\rho'\varepsilon_t\right)\varphi(x)dx}{\Phi(\tau\sqrt{1-\alpha^2})}$$

TVTP2: $$p_{10}(\varepsilon_t)=\frac{\int_{\tau\sqrt{1-\alpha^2}}^{\infty}\Phi_\rho\left(\tau-\frac{\alpha x}{\sqrt{1-\alpha^2}}-\rho'\varepsilon_t\right)\varphi(x)dx}{1-\Phi(\tau\sqrt{1-\alpha^2})}$$

Steady States

- Detrending: $c_t = C_t/A_t$, $y_t = Y_t/A_t$
- Define steady states as an equilibrium where shocks are turned off and inflation is at its target rate.
- Eliminate c_t by the market clearing condition, and obtain steady states as

$$
\begin{aligned}
&\left(\bar{y}, \overline{\Pi}_{s_t}, \overline{R}_{s_t}, \bar{a}, \bar{\xi}, \bar{u}, \bar{e}, \bar{g}\right) \\
&= \left(g\left(\frac{\theta-1}{\theta}\right)^{1/\epsilon}, \Pi^*_{s_t}, \frac{\gamma}{\beta}\left(\frac{\bar{p}_{s_t,0}}{\Pi^*_0} + \frac{\bar{p}_{s_t,1}}{\Pi^*_1}\right)^{-1}, 1, 1, u, 1, g \right)
\end{aligned}
$$

where $\Pi^*_{s_t}$ is regime-dependent inflation targets.

- Write all variables in log-deviations: $\hat{x} = \log\left(\frac{x}{\bar{x}}\right)$

First-Order Perturbation Solution

Model variables: $Z_t = (\hat{y}_t, \hat{\Pi}_t, \hat{R}_t, \hat{a}_t, \hat{\xi}, \hat{u}_t, \hat{e}_t, \hat{g}_t)'$

Shocks: $\varepsilon_t = (\varepsilon_t^a, \varepsilon_t^\xi, \varepsilon_t^u, \varepsilon_t^e, \varepsilon_t^g)'$

Parameters Θ assumed to be known

Obtain the solution using the first-order perturbation method by Barthelemy and Marx (2017):

$$Z_t = \underbrace{A_1(s_t, \Theta)}_{8\times 8} Z_{t-1} + \underbrace{A_2(s_t, \Theta)}_{8\times 5} \varepsilon_t$$

where $A_2(s_t, \Theta)\varepsilon_t$ combines the direct effect and the linear approximation of the nonlinear effect of endogenous feedback mechanism from the structural shocks to the regime change.

State Space Representation

Augment the state vector Z_t with $\hat{y}_{t-1}$, shocks ε_t, η_t and regime factor w_t given by $w_t = \alpha w_{t-1} + \rho' \varepsilon_{t-1} + \sqrt{1 - \rho'\rho}\, \eta_t$ as

$$\varsigma_t = (\hat{y}_t, \hat{\Pi}_t, \hat{R}_t, \hat{a}_t, \hat{\xi}, \hat{u}_t, \hat{e}_t, \hat{g}_t, \hat{y}_{t-1}, \varepsilon_t', \eta_t, w_t)'$$

Accordingly, also augment ε_t with η_t as

$$\xi_t = (\varepsilon_t', \eta_t)'$$

Then, our nonlinear state space model is written with

- Transition Equations: $\varsigma_t = \widetilde{G}(s_t, \Theta)\varsigma_{t-1} + \widetilde{M}(s_t, \Theta)\xi_t$
- Measurement Equations: $y_t = D(s_t, \Theta) + \widetilde{Z}(s_t, \Theta)\varsigma_t + F\eta_t$

where $\widetilde{Z}(s_t) = [Z(s_t), 0_{l\times n}, F, 0_{l\times 1}]$, and the observable y_t includes per capita real output growth rate, net inflation rate, and net nominal interest rate in percentage.

Endogenous-Switching Kalman Filter

Initialization: Initialize $(\varsigma^j_{0|0}, P^j_{0|0})$ and $p^j_{0|0}$ from invariant dist'n.

Forecasting: Apply Kalman filter forecasting step to obtain

$$\varsigma^{(i,j)}_{t|t-1} = \widetilde{G}(s_t = j)\varsigma^i_{t-1|t-1}$$

$$P^{(i,j)}_{t|t-1} = \widetilde{G}(s_t = j)P^i_{t-1|t-1}\widetilde{G}(s_t = j)' + \widetilde{M}(s_t = j)\widetilde{M}(s_t = j)'$$

Approximate $w_t|s_{t-1} = i, Y_{1:t-1}$ by normal dist'n

$$p(w_t|s_{t-1} = i, Y_{1:t-1}) = \mathbb{N}(\varsigma^{(i,j)}_{w,t|t-1}, P^{(i,j)}_{w,t|t-1})$$

for any j. Thus,

$$p^{(i,0)}_{t|t-1} = \Phi\left((\tau - \varsigma^{(i,0)}_{w,t|t-1})/\sqrt{P^{(i,0)}_{w,t|t-1}}\right)p^i_{t-1|t-1}$$

$$p^{(i,1)}_{t|t-1} = p^i_{t-1|t-1} - p^{(i,0)}_{t|t-1}$$

Endogenous-Switching Kalman Filter(cont'd)

Likelihood: Apply Kalman filter forecasting step to obtain

$$y_{t|t-1}^{(i,j)} = D(s_t = j) + \widetilde{Z}(s_t = j)\varsigma_{t|t-1}^{(i,j)}$$
$$F_{t|t-1}^{(i,j)} = \widetilde{Z}(s_t = j)P_{t|t-1}^{(i,j)}\widetilde{Z}(s_t = j)' + \Sigma_u$$

Then the period-t likelihood contribution can be computed as

$$p(y_t|Y_{1:t-1}) = \sum_{j=0}^{1}\sum_{i=0}^{1} p_{\mathbb{N}}(y_t|y_{t|t-1}^{(i,j)}, F_{t|t-1}^{(i,j)})p_{t|t-1}^{(i,j)}$$

Updating: First, apply the Bayes formula to update

$$p_{t|t}^{(i,j)} = \frac{p_{\mathbb{N}}(y_t|y_{t|t-1}^{(i,j)}, F_{t|t-1}^{(i,j)})p_{t|t-1}^{(i,j)}}{p(y_t|Y_{1:t-1})}$$

and compute $p_{t|t}^{j} = \sum_{i=0}^{1} p_{t|t}^{(i,j)}$. Next, use Kalman filter to obtain

$$\varsigma_{t|t}^{(i,j)} = \varsigma_{t|t-1}^{(i,j)} + P_{t|t-1}^{(i,j)}\widetilde{Z}(s_t = j)'(F_{t|t-1}^{(i,j)})^{-1}(y_t - y_{t|t-1}^{(i,j)})$$
$$P_{t|t}^{(i,j)} = P_{t|t-1}^{(i,j)} - P_{t|t-1}^{(i,j)}\widetilde{Z}(s_t = j)'(F_{t|t-1}^{(i,j)})^{-1}\widetilde{Z}(s_t = j)P_{t|t-1}^{(i,j)}$$

Endogenous-Switching Kalman Filter(cont'd)

Collapse: Collapse $(\varsigma_{t|t}^{(i,j)}, P_{t|t}^{(i,j)})$ into

$$\varsigma_{t|t}^{j} = \sum_{i=0}^{1} \frac{p_{t|t}^{(i,j)}}{p_{t|t}^{j}} \varsigma_{t|t}^{(i,j)}, \quad P_{t|t}^{j} = \sum_{i=0}^{1} \frac{p_{t|t}^{(i,j)}}{p_{t|t}^{j}} \left[P_{t|t}^{(i,j)} + (\varsigma_{t|t}^{j} - \varsigma_{t|t}^{(i,j)})(\varsigma_{t|t}^{j} - \varsigma_{t|t}^{(i,j)})' \right]$$

Further collapse $(\varsigma_{t|t}^{j}, P_{t|t}^{j})$ into

$$\varsigma_{t|t} = \sum_{j=0}^{1} p_{t|t}^{j} \varsigma_{t|t}^{j}, \quad P_{t|t} = \sum_{j=0}^{1} p_{t|t}^{j} \left[P_{t|t}^{j} + (\varsigma_{t|t} - \varsigma_{t|t}^{j})(\varsigma_{t|t} - \varsigma_{t|t}^{j})' \right]$$

which gives the extracted filtered states.

Aggregation: The likelihood function is given by

$$p(Y_{1:T}) = \prod_{t=1}^{T} p(y_t | Y_{1:t-1})$$

Quasi-Bayesian MLE

- Widely used to induce desired curvature in likelihood surface.
- For a given log-likelihood function

$$\log L(Y_{1:T}|\Theta) = \sum_{t=1}^{T} \log p(y_t|Y_{1:t-1})$$

  where $Y_{1:T}$ denotes data, Θ parameters, the quasi-Bayesian MLE is defined as

$$\hat{\Theta} = \underset{\Theta \in R(\Theta)}{\arg\max} \log L(Y_{1:T}|\Theta) + \log p(\Theta)$$

- Used as the initial guess in our MCMC procudure with standard random walk Metropolis-Hastings.

MCMC

Step 1. Initialize the Markov chain with the quasi-Bayesian ML estimates $x^{(0)} = \hat{\Theta}$. Also, obtain the inverse of negative Hessian Σ from the quasi-Bayesian MLE

Step 2. Repeat Steps 2.1-2.3 for $j = 1, 2, \ldots, N$.

Step 2.1. Generate y from $q(x^{(j-1)}, \cdot) =_d \mathbb{N}(x^{(j-1)}, c\Sigma)$ and u from $\mathcal{U}(0, 1)$.

Step 2.2. Compute the probability of move

$$\alpha(x^{(j-1)}, y) = \min\left[\frac{p(y|Y_{1:T})q(y, x^{(j)})}{p(x^{(j)}|Y_{1:T})q(x^{(j)}, y)}, 1\right]$$

Step 2.3. If $u \le \alpha(x^{(j-1)}, y)$

– Set $x^{(j)} = y$.

Else

– Set $x^{(j)} = x^{(j-1)}$.

Step 3. Return the values $\{x^{(1)}, x^{(2)}, \ldots, x^{(N)}\}$.

Prior and Posterior Estimates

Parameter	Prior			Posterior		
	Density	π_1	π_2	Mean	5%	95%
ϵ	G	2.00	0.50	3.01	2.22	3.90
κ	G	0.50	0.20	0.05	0.03	0.08
ψ_π^0	G	1.00	0.20	0.92	0.65	1.22
$\psi_\pi^1 - \psi_\pi^0$	G	1.00	0.30	1.07	0.66	1.57
ψ_y^0	G	0.25	0.15	0.96	0.64	1.34
ψ_y^1	G	0.25	0.15	0.72	0.48	1.02
ρ_R^0	B	0.60	0.20	0.14	0.05	0.24
ρ_R^1	B	0.60	0.20	0.83	0.78	0.86
$\pi_0^{(A)}$	G	4.00	1.00	4.31	3.55	5.07
$\pi_1^{(A)}$	G	4.00	1.00	4.55	3.82	5.29
$r^{(A)}$	G	0.50	0.20	0.53	0.25	0.88
$\gamma^{(Q)}$	N	0.40	0.20	0.41	0.31	0.51
ρ_a	B	0.50	0.20	0.46	0.22	0.71
ρ_ξ	B	0.50	0.20	0.86	0.80	0.90
ρ_u	B	0.50	0.20	0.96	0.93	0.98
ρ_g	B	0.50	0.20	0.54	0.17	0.90

Prior and Posterior Estimates(cont'd)

Parameter	Prior			Posterior		
	Density	π_1	π_2	Mean	5%	95%
$100\sigma_a$	I	2.00	0.10	0.57	0.31	0.75
$100\sigma_\xi$	I	2.00	0.10	2.78	2.23	3.44
$100\sigma_u$	I	2.00	0.10	0.99	0.51	1.63
$100\sigma_e$	I	2.00	0.10	0.04	0.03	0.06
$100\sigma_g$	I	2.00	0.10	0.27	0.07	0.57
$1/g$	B	0.85	0.10	0.86	0.66	0.98
α	B	0.90	0.05	0.93	0.89	0.96
τ	N	0.00	0.50	-0.75	-1.33	-0.10
ρ_{av}	U	-1.00	1.00	0.38	0.11	0.63
$\rho_{\xi v}$	U	-1.00	1.00	-0.24	-0.43	-0.04
ρ_{uv}	U	-1.00	1.00	-0.80	-0.93	-0.62
ρ_{ev}	U	-1.00	1.00	0.03	0.01	0.05
ρ_{gv}	U	-1.00	1.00	0.04	-0.38	0.45
f_y	N	0.00	0.18	-0.03	-0.16	0.08
f_Π	N	0.00	0.59	1.17	1.04	1.31
f_R	N	0.00	0.66	-0.03	-0.07	0.01

Model Fit

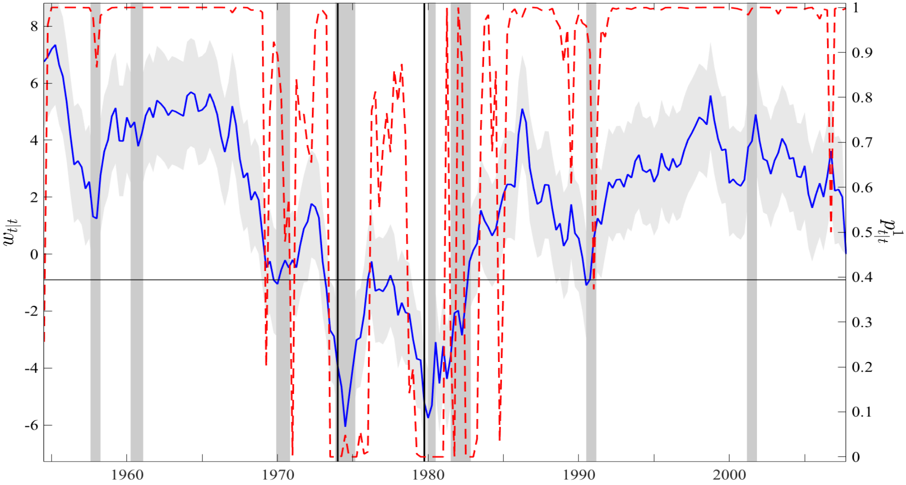
Use Geweke(1999)'s harmonic mean estimator to compute marginal data density:

	exogenous	endogenous
$\ln \hat{p}(Y)$	-1051.29	-1034.51
	(0.02)	(0.07)

The log-likelihood difference is roughly 17, larger than 4.6. By Jeffrey(1998) criterion, endogenous model is decisively preferred.

Note: The estimates are based on essentially the same model, but without markup and preference shocks, and with only monetary policy shock driving the regime change.

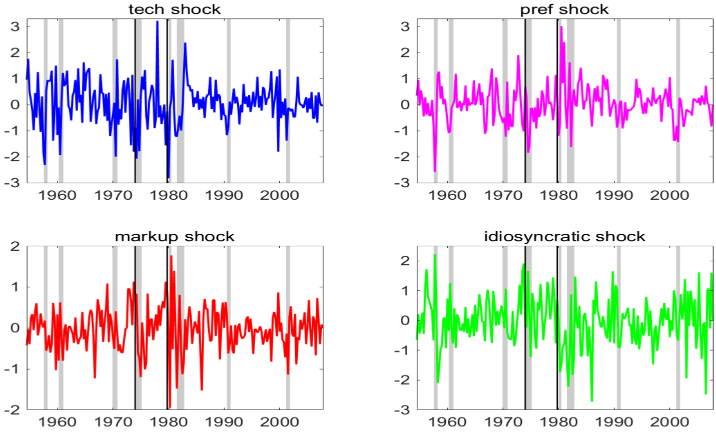
Extracted Regime Factor and Regime-1 Probability



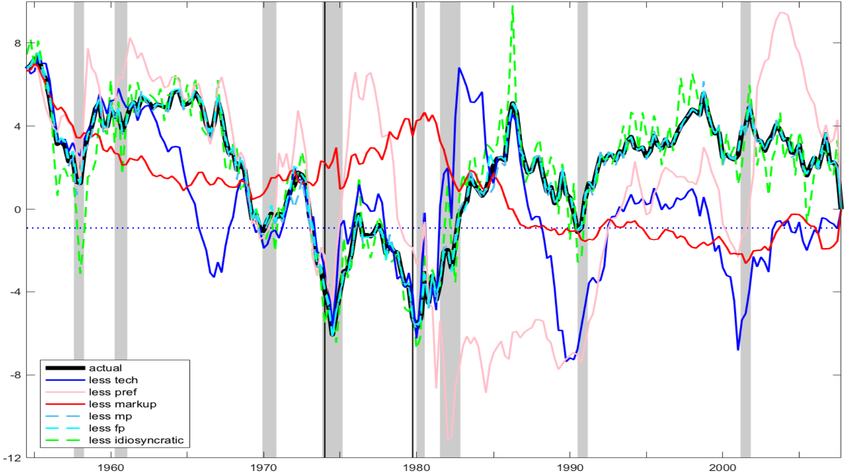
Shaded areas: NBER recessions

Two vertical lines: oil shocks in 1974.Q1 and 1979.Q3

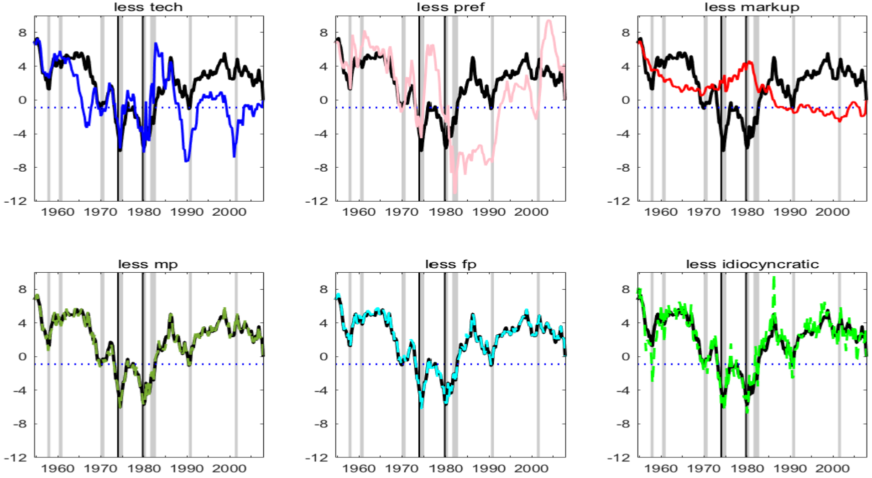
Filtered Shocks



Counterfactual Analysis



Counterfactual Analysis (cont'd)



Findings

- Regime factor was larger without the markup shock in the 70's, which implies that without markup shock, monetary policy would be tighter. This maybe relates to oil shock in the 70's which pushed up inflation and pushed down output. Fed reacted to this stagflation by becoming less aggressive.
- Without the preference shock, monetary policy would be significantly passive during early 80' and 90'. This may result in a prolonged period of the Great Inflation and the Great Moderation might have happened much later.
- Monetary and fiscal policy shocks contribute insignificantly to regime change compared to other non-policy shocks.

Analytical Solution

- Conditional expectation

$$
\begin{aligned}
\mathbb{E}_t\pi_{t+1} =& [\mathbb{E}(a_1(s_{t+1}=0, p_{00}(\epsilon^e_{t+1}), p_{01}(\epsilon^e_{t+1}))) \cdot p_{s_t,0}(\epsilon^e_t) \\
&+ \mathbb{E}(a_1(s_{t+1}=1, p_{10}(\epsilon^e_{t+1}), p_{11}(\epsilon^e_{t+1}))) \cdot p_{s_t,1}(\epsilon^e_t)] \cdot \rho_r r_t
\end{aligned}
$$

- Combining Fisher equation

$$
\begin{aligned}
i_t =& [\mathbb{E}(a_1(s_{t+1}=0, p_{00}(\epsilon^e_{t+1}), p_{01}(\epsilon^e_{t+1}))) \cdot p_{s_t,0}(\epsilon^e_t) \\
&+ \mathbb{E}(a_1(s_{t+1}=1, p_{10}(\epsilon^e_{t+1}), p_{11}(\epsilon^e_{t+1}))) \cdot p_{s_t,1}(\epsilon^e_t) + 1] \cdot \rho_r r_t \\
=& \alpha(s_t)\pi_t + \sigma_e \epsilon^e_t
\end{aligned}
$$

Analytical Solution

- Solving π_{t+1}

$$\begin{aligned}\pi_{t+1} =& \frac{\rho_r}{\alpha(s_{t+1})}[\mathbb{E}(a_1(s_{t+2}=0, p_{00}(\epsilon^e_{t+2}), p_{01}(\epsilon^e_{t+2}))) \cdot p_{s_{t+1},0}(\epsilon^e_{t+1}) \\ &+ \mathbb{E}(a_1(s_{t+2}=1, p_{10}(\epsilon^e_{t+2}), p_{11}(\epsilon^e_{t+2}))) \cdot p_{s_{t+1},1}(\epsilon^e_{t+1}) + 1]r_{t+1} \\ &- \frac{\sigma_e}{\alpha(s_{t+1})}\epsilon^e_{t+1}\end{aligned}$$

- Comparing with initial guess to match unknown coefficients

$$\begin{aligned}&a_1(s_{t+1}, p_{s_{t+1},0}(\epsilon^e_{t+1}), p_{s_{t+1},1}(\epsilon^e_{t+1})) \\ &= \frac{\rho_r}{\alpha(s_{t+1})}[\mathbb{E}(a_1(s_{t+2}=0, p_{00}(\epsilon^e_{t+2}), p_{01}(\epsilon^e_{t+2}))) \cdot p_{s_{t+1},0}(\epsilon^e_{t+1}) \\ &+ \mathbb{E}(a_1(s_{t+2}=1, p_{10}(\epsilon^e_{t+2}), p_{11}(\epsilon^e_{t+2}))) \cdot p_{s_{t+1},1}(\epsilon^e_{t+1}) + 1]\end{aligned} \quad (1)$$

$$a_2(s_{t+1}) = -\frac{\sigma_e}{\alpha(s_{t+1})}$$

Analytical Solution

- To determine a_1, we define

$$C_0 = \mathbb{E}(a_1(s_{t+2} = 0, p_{00}(\epsilon^e_{t+2}), p_{01}(\epsilon^e_{t+2}))) \tag{2}$$

$$C_1 = \mathbb{E}(a_1(s_{t+2} = 1, p_{10}(\epsilon^e_{t+2}), p_{11}(\epsilon^e_{t+2}))) \tag{3}$$

- Considering $s_{t+1} = 0, 1$ for LHS of (**??**), taking expectation with respect to ϵ^e_{t+1}, then combining (**??**) and (**??**), we obtain

$$C_0 = \frac{\alpha_1 + \rho_r(\mathbb{E}p_{10}(\epsilon^e_{t+1}) - \mathbb{E}p_{00}(\epsilon^e_{t+1}))}{(\alpha_1 - \rho_r)\left(\frac{\alpha_0}{\rho_r} - \mathbb{E}p_{00}(\epsilon^e_{t+1})\right) + (\alpha_0 - \rho_r)\mathbb{E}p_{10}(\epsilon^e_{t+1})} > 0$$

$$C_1 = \frac{\rho_r \mathbb{E}p_{10}(\epsilon^e_{t+1})}{\alpha_1 - \rho_r + \rho_r \mathbb{E}p_{10}(\epsilon^e_{t+1})} C_0 + \frac{\rho_r}{\alpha_1 - \rho_r + \rho_r \mathbb{E}p_{10}(\epsilon^e_{t+1})}$$

Analytical Solution

- Numerical evaluation of $\mathbb{E}p_{00}(\epsilon^e_{t+1})$ and $\mathbb{E}p_{10}(\epsilon^e_{t+1})$

$$\mathbb{E}p_{00}(\epsilon^e_{t+1}) = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\tau\sqrt{1-\phi^2}} \Phi_\rho\left(\tau - \frac{\phi x}{\sqrt{1-\phi^2}} - \rho\epsilon^e_{t+1}\right)\varphi(x)\varphi(\epsilon^e_{t+1})dx d\epsilon^e_{t+1}}{\Phi(\tau\sqrt{1-\phi^2})}$$

$$= \frac{\int_{-\infty}^{\tau\sqrt{1-\phi^2}}\int_{-\infty}^{\tau/\sqrt{1-\rho^2}}\int_{-\infty}^{\infty} f_3(x,y,\epsilon)d\epsilon dy dx}{\Phi(\tau\sqrt{1-\phi^2})}$$

with

$$f_3(x,y,\epsilon) = \mathbb{N}\left(0, \begin{pmatrix} 1 & \frac{\phi}{\sqrt{1-\rho^2}\sqrt{1-\phi^2}} & 0 \\ \frac{\phi}{\sqrt{1-\rho^2}\sqrt{1-\phi^2}} & \frac{1}{(1-\rho^2)(1-\phi^2)} & \frac{\rho}{\sqrt{1-\rho^2}} \\ 0 & \frac{\rho}{\sqrt{1-\rho^2}} & 1 \end{pmatrix}\right)$$

Solution