Latent Variables and Real-Time Forecasting in DSGE Models with Occasionally Binding Constraints. Can Non-Linearity Improve Our Understanding of the Great Recession?*

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Abstract

We introduce an algorithm that measures the contribution of smoothed estimates of historical shocks onto observed variables in models with occasionally binding constraints solved in a piecewise fashion with the Occbin toolkit developed by Guerrieri and Iacoviello (2015). We implement the algorithm on a estimated DSGE model for the Euro Area in which both financial constraints and the Zero Lower Bound are occasionally binding. In a real-time forecast exercise, we show that the introduction of financial constraints alone may be responsible for a degree of non-linearity sufficient to encompass in the predictive density of the model extreme events such as the Great Recession. We find that households' "lending" constraints imply larger departures from the linear solution than firms' borrowing constraints.

1 Introduction

In recent years, and as a consequence of the outburst of the Global Financial Crisis (GFC), a great deal of effort has been put into place both methodologically and theoretically to enrich the existing (and criticized) workhorse DSGE model in a way that could provide us with a deeper understanding of the macroeconomic developments in the aftermath of the crisis itself, namely the Great Recession (GR), and possibly help us prevent or at least anticipate similar events in the future.

From a methodological standpoint, the bulk of the new advances have either pursued departures from the linear framework, or moved beyond the Gaussian assumption. On the theoretical side, instead, and given the nature of GFC, new extensions of the financial sector have been mostly considered,

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involving firms' investment decisions, housing investment and the role of bank capital in the financial intermediation sector.[1].

In this paper, we intend to contribute along both the dimensions above: methodologically, we introduce an algorithm that allows us to estimate shocks contributions in the context of the piecewise linear solution of Guerrieri and Iacoviello (2015)in presence of occasionally binding constraints, whereas theoretically, we show that the introduction of an occasionally binding lending constraint in the Ricardian households' optimization problem, as opposed to a borrowing constraint in the firms' problem, may imply sufficient non-linearities that can encompass extreme events such as the GFC while maintaining the Gaussian assumption.

The non-linearities stemming from the piecewise linear solution of a model with occasionally binding constraints violate the additive property of shocks contributions of linearized models. We propose a simulation method which can measure the total effect contribution of a given group of shocks of interest by computing the residual of the contribution of the complement set of shocks and the initial condition.

We implement this algorithm on a closed economy estimated DSGE model of the Euro Area in which we allow for occasionally binding constraints on the monetary policy rule, the Zero Lower Bound (ZLB), and on the debt contracts between Ricardian households and monopolistically competitive intermediate goods producing firms in the spirit of Jermann and Quadrini (2012) . Regarding the specification of the financial friction, we analyze the implications of two alternative approaches: in the first case, we assume that the limited enforceability of debt contracts it is internalized by the lenders in the form of a lending constraint into the households optimization problem, in the second, and following the standard practice in the literature, we assume that the default option on debt obligations translates into an enforcement constraint of the borrower profit maximization problem.

SUMMARY OF INTUITION

The remainder of the paper is structured as follows.

2 Shocks contributions with occasionally binding constraints

In order to address the presence of occasionally binding constraints, we adopt the Occbin solution method developed by Guerrieri and Iacoviello (2015) which provides a piecewise linear solution. The first necessary step towards the estimation of shocks contributions onto observables in this context, is the computation of estimated latent variables and of historical regimes (i.e. binding vs not binding regimes). To this end we employ an algorithm similar to Anzoategui et al. (2016)as follows.

1. Guess an initial sequence of regimes for each historical period $R_t^{(0)}$ for $t = 1, .., T$

2. Given the sequence of regimes, compute the sequence of state space matrices $\boldsymbol{\Upsilon}_t^{(0)}$ following the piecewise linear solution method of Guerrieri and Iacoviello (2015).

3. For each iteration $j = 1, .., n$:

[1] A complete review is clearly beyond the scope of the paper, we point the interested reader to Wieland et al. (2016)and Lindé et al. (2016), and the references therein.

(a) feed the state space matrices $\mathbf{\Upsilon}_t^{(j-1)}$to a Kalman Filter[2] / Fixed interval smoothing algorithm to determine initial conditions, smoothed variables $\mathbf{y}_t^{(j)}$ and shocks $\epsilon_t^{(j)}$.

(b) given initial conditions and shocks perform Occbin simulations that endogenously determine a new sequence of regimes $R_t^{(j)}$, from which a new sequence of states space matrices is derived $\mathbf{\Upsilon}_t^{(j)}$.

4. The algorithm stops when $R_t^{(j)} = R_t^{(j-1)}$ for all $t = 1, .., T$.

Hence, this algorithm provides initial conditions, smoothed variables and shocks, consistent with the occasionally binding constraint, i.e. it also estimates a sequence of regimes along the historical periods. One caveat of this environment is that the contribution of individual smoothed shocks, is not the mere additive superposition of each shock propagated by the sequence of state space matrices $\mathbf{\Upsilon}_t$ estimated with the smoother. The occurrence of a specific regime at time t, in fact, is a non-linear function of the states in $t-1$, $\mathbf{y}_{t-1}$ and of the whole set of shocks simultaneously affecting the economy, that is $\mathbf{\Upsilon}_t = f(\epsilon_{1t}, .., \epsilon_{kt}, \mathbf{y}_{t-1})$, $t = 1, .., T$.

This circumstance calls for the extension of the standard linear historical shock decompositions to the case of occasionally binding regimes consistent with its piecewise linear solution.

It is straightforward to show that the sequence of occasionally binding regimes will change when taking subsets of shocks or individual shocks alone. One way of measuring the effect of shocks in this non-linear context is to consider simulations conditional to given shock patterns, i.e. performing counterfactuals opportunely choosing combinations of shocks and initial conditions. In particular, we can consider two definitions that generalize the concept of shock contributions to the non-linear case, which degenerate to the standard shock decompositions for the linear case.

2.1 Main and total effects in shocks decompositions

Denote with ϵ_{lt} the shock or group of shocks of interest, while $\tilde{\epsilon}_{lt}$ indicates the complementary set of shocks in the model[3]. We define the *Main effect contribution,* the effect computed via Monte Carlo counterfactuals drawing respectively $\tilde{\epsilon}_{lt}$and the initial conditions $\mathbf{y}_0$from their normal distributions, or $E(\mathbf{y}_t \mid \epsilon_{lt})$which can be simplified as $\mathbf{y}_t(\epsilon_{lt}, \tilde{\epsilon}_{lt} = 0, \mathbf{y}_0 = 0)$.

We define the *Total Effect contribution,* the effect computed as the difference of the states variables $\mathbf{y}_t$ and the contributions of $\tilde{\epsilon}_{lt}$and of $\mathbf{y}_0$ obtained by integrating out ϵ_{lt} via Monte Carlo counterfactuals drawing $\epsilon_{lt}$from its normal distribution, or $\mathbf{y}_t - E(\mathbf{y}_t \mid \tilde{\epsilon}_{lt}, \mathbf{y}_0)$which can be simplified as $\mathbf{y_t} - \mathbf{y}_t(\tilde{\epsilon}_{lt}, \mathbf{y}_0, \epsilon_{lt} = 0)$.

It is important to stress that each of these simulations provides a different sequence of regimes, which in general will be different from the historical one. The *Total Effect contribution*, triggers key non-linear features associated to the interaction between shock realization and the occasionally binding constraints. In what follows, we use the *Total Effect contribution* to measure the contribution of shocks to observed variables under occasionally binding constraints.

[2]Kulish et al. (2014)also apply the piecewise linear solution in the Kalman filter to estimate DSGE models with forward guidance.

[3]To gain intuition, it may be useful to think of a set of shocks of interest of relatively small size relatively to the total number of shocks in the model.

3 The model

We apply the above methodology to a closed economy version of Kollmann et al. (2016)in which we introduce financial frictions. The model features, a private sector composed by two types of households, Ricardians and hand to mouth, monopolistically competitive intermediate goods producing firms, and perfectly competitive final good producers. Moreover, it allows for nominal price and wage rigidities as well as real wage rigidities. Furthermore there is a public sector which finances public consumption, public investment and transfers through distortionary taxes on firms profits, labor and consumption and a residual component of lump sum taxes.Finally there is a monetary authority who follows a Taylor rule and is subject to an occasionally binding ZLB.

The financial frictions involve the relationship between the intermediate good firms and the Ricardian households[4]. We assume, as in Jermann and Quadrini (2012), that firms may raise funds either by issuing equity or through a debt contract with limited enforceability. Moreover we assume that savers will face an always binding constraint on total risky assets: the sum of loans and equity is in every period a constant fraction (plus a financial shock) of the value of firms' capital which serves as collateral. Finally, we assume that there is an occasionally binding constraint tying the amount of loans to the stock of capital, which in period of financial distress, reduces the possibility to substitute between risky assets. Regarding the latter constraint, we consider two alternative formulations. In the first case we will assume the constraint being internalized by the savers, in the second case by the firms.

3.1 The households' problem

The population is constituted by a continuum of households indexed by $j \in [0,1]$and they may belong to two types, a share ω^s of Ricardians, indexed with superscript s, and a share $1-\omega^s$ of hand-to-mouth households which we denote with superscript c. We formulate the two optimization problems and the consequent optimality conditions in the two subsections below.

3.1.1 Ricardian households

Ricardian households have full access to financial markets, subject to the constraints described in Section 3. Their preferences are defined over consumption (with external habits) X^s_{jt}, hours worked

[4]We will use the terms Ricardian households, savers, lenders interchangeably throughout the paper.

(with external habits) H^s_{jt} and assets holding A_{jt-1} as follows:

$$
\begin{aligned}
U^s\left(X^s_{jt}, H^s_{jt}, A_{jt-1}\right) = & \frac{1}{1-\theta}\left(C^s_{jt} - h^C C^s_{t-1}\right)^{1-\theta} - z^N_t C^{1-\theta}_t \frac{s^N}{1-\theta^N}\left(N^s_{jt} - h^N N^s_{t-1}\right)^{1-\theta^N} \\
& - \left(C^s_t - h^C C^s_{t-1}\right)^{-\theta}\left[\frac{s^{B^g}\left(\left(\alpha^{B^g}_0 + z^{B^g}_{t-1}\right) B^g_{jt-1} + \frac{1}{2}\alpha^{B^g}_1 \frac{\left(B^g_{jt-1}\right)^2}{P_{t-1}Y_{t-1}}\right)}{\left(1+\tau^C\right)P_t}\right. \\
& + \frac{s^S\left(\left(\alpha^S_0 + z^S_{t-1}\right) P^S_{t-1} S_{jt-1} + \frac{1}{2}\alpha^S_1 \frac{\left(P^S_{t-1}S_{jt-1}\right)^2}{P_{t-1}Y_{t-1}}\right)}{\left(1+\tau^C\right)P_t} \\
& \left. + \frac{s^L\left(\left(\alpha^L_0 + z^L_{t-1}\right) L_{jt-1} + \frac{1}{2}\alpha^L_1\left(P^I_{t-1}K_{t-1}\right)\left(\frac{L_{jt-1}}{P^I_{t-1}K_{t-1}} - \frac{\overline{L}}{\overline{P^I K}}\right)^2\right)}{\left(1+\tau^C\right)P_t}\right]
\end{aligned} \tag{1}
$$

where $C^s_{jt}$and N^s_{jt} indicate respectively consumption and hours worked, while $B^g_{jt-1}, S_{jt-1}, L_{jt-1}$are holdings of Government bonds paying a nominal interest i^g_t, equity shares and firms' loans with nominal interest i^l_t. Moreover P_t is the GDP deflator, $\tau^C$is a consumption tax (VAT),Y_t is GDP, $P^S_t$is price of equity shares S_{jt-1}, L_{jt} are the loans in nominal terms, $P^I_t$is the nominal price of physical capital K_t. The parameters $\alpha^{\cdot}_0$, $\alpha^{\cdot}_1$ measure the intercept and the slope of the risk premia associated to the different assets, whereas the shocks $z^{\cdot}_t$in the square bracket of (1) represent asset specific risk premium shocks.

Their problem is to:

$$
\max_{\left\{C^s_{jt}, B_{jt}, B^g_{jt}, L_{jt}, S_{jt}\right\}} E_0 \sum_{t=0}^{\infty}\left(\beta z^C_{t-1}\right)^t U^s\left(X^s_{jt}, H^s_{jt}, A_{jt-1}\right)
$$

subject to the budget constraint:

$$
\begin{aligned}
\left(1+\tau^C\right)P_t C^s_{jt} + B_{jt} + B^g_{jt} + L_{jt} + P^S_t S_{jt} = & \left(1-\tau^N\right)W_t N^s_{jt} + \left(1+i_{t-1}\right)B_{jt-1} + \left(1+i^g_{t-1}\right)B^g_{jt-1} \\
& + \left(1+i^l_{t-1}\right)L_{jt-1} + \left(P^S_t + P_t d_t\right)S_{jt-1} + T^s_{jt} - tax^s_{jt}
\end{aligned}
$$

and the lending constraints:

$$
L_{jt} + P^S_t S_{jt} = m^{tot} z^F_t\left(P^I_t K_{t-1}\right) \tag{2}
$$

$$
L_{jt} \leq m^l z^F_t\left(P^I_t K_{t-1}\right). \tag{3}
$$

where $B_{jt}$is a risk free bond which pays a nominal interest i_t, $W_t$is the nominal wage, T^s_{jt} are public transfers and tax^s_{jt} lump sum taxes. The term z^C_t refers to a saving shock that makes the discount factor time-varying whereas $z^F_t$is a financial shock hitting the lending constraints.

The first order conditions with respect to C^s_{jt},B_{jt},B^g_{jt},L_{jt},S_{jt}, read respectively:

$$\left(C^s_{jt} - h^C C^s_{t-1}\right)^{-\theta} = \lambda^s_t \tag{4}$$

$$\beta E_t \left[\widetilde{z}^C_t \frac{\lambda^s_{t+1}}{\lambda^s_t} \frac{P_t}{P_{t+1}} (1+i_t)\right] = 1 \tag{5}$$

$$\beta E_t \left[\widetilde{z}^C_t \frac{\lambda^s_{t+1}}{\lambda^s_t} \frac{P_t}{P_{t+1}} \left(1 + i^g_t - s^{B^g}\left(\alpha^{B^g}_0 + z^{B^g}_{t+1} + \alpha^{B^g}_1 \frac{B^g_t}{P_t Y_t}\right)\right)\right] = 1 \tag{6}$$

$$1 + \mu^{s,tot}_t + \mu^{s,l}_t = \beta E_t \left[\widetilde{z}^C_t \frac{\lambda^s_{t+1}}{\lambda^s_t} \frac{P_t}{P_{t+1}} \left(1 + i^l_t - s^L\left(\alpha^L_0 + z^L_t + \alpha^L_1 \left(\frac{L_t}{P^I_t K_t} - \frac{\overline{L}}{\overline{P^I K}}\right)\right)\right)\right] \tag{7}$$

$$1 + \mu^{s,tot}_t = \beta E_t \left[\widetilde{z}^C_t \frac{\lambda^s_{t+1}}{\lambda^s_t} \frac{P_t}{P_{t+1}} \left(1 + i^s_{t+1} - s^S\left(\alpha^S_0 + z^S_t + \alpha^S_1 \frac{P^S_t S_t}{P_t Y_t}\right)\right)\right] \tag{8}$$

where $\mu^{s,tot}_t$ and $\mu^{s,l}_t$ are the Lagrange multipliers associated to (2) and (3), respectively. Clearly from the Kuhn Tucker theorem, $\mu^{s,l}_t$ is going to be positive only when (3) binds.

3.1.2 Hand-to-mouth Households

The second type of households cannot be engaged in financial activities, therefore they consume all their disposable income in each period. Their preferences are described by:

$$U^c\left(X^c_{jt}, H^c_{jt}\right) = \frac{1}{1-\theta}\left(C^c_{jt} - h^C C^c_{t-1}\right)^{1-\theta} - z^N_t C^{1-\theta}_t \frac{s^N}{1-\theta^N}\left(N^c_{jt} - h^N N^c_{t-1}\right)^{1-\theta^N}$$

and their consumption path follows their budget constraint:

$$\left(1+\tau^C\right) P_t C^c_{jt} = \left(1-\tau^N\right) W_t N^c_{jt} + +T^c_{jt} - tax^c_{jt}$$

3.2 The labor supply

We assume the presence of a trade union which sets the wage taking into account the preferences of both types of households and their budget constraints. More formally the trade union solves:

$$\max E_0 \sum_{t=0}^{\infty} \left(\beta z^C_{t-1}\right)^t \left[U\left(X_{jt}, H_{jt}, A_{jt-1}\right)\right]$$

with $X_{jt} \equiv \omega^s X^s_{jt} + (1-\omega^s) X^c_{jt}$, and $H_{jt} \equiv \omega^s H^s_{jt} + (1-\omega^s) H^c_{jt} = \omega^s \left(N^s_{jt} - h^N N^s_{t-1}\right) + (1-\omega^s)\left(N^c_{jt} - h^N N^c_{t-1}\right) = \left(N_{jt} - h^N N_{t-1}\right)$ where we imposed : $N^s_{jt} = N^c_{jt} = N_{jt}$, subject to the weighted sum of the budget constraints, inclusive of an adjustment cost term involving an inflation indexation parameter sf^w:

$$\begin{aligned}
&\left(1+\tau^{C}\right) P_t \left(\omega C^s_{jt} + \left(1-\omega^s\right) C^c_{jt}\right) \\
&\quad +\omega^s \left(B_{jt} + B^g_{jt} + L_{jt} + P^S_t S_{jt}\right) = \left(1-\tau^N\right) W_{jt} N_{jt} \\
&\qquad + \omega^s \left[\left(1+i_{t-1}\right) B_{jt-1} + \left(1+i^g_{t-1}\right) B^g_{jt-1}\right. \\
&\qquad \left. + \left(1+i^l_{t-1}\right) L_{jt-1} + \left(P^S_t + P_t d_t\right) S_{jt-1}\right] \\
&\qquad + \omega^s \left(T^s_{jt} - tax^s_{jt}\right) + \left(1-\omega^s\right)\left(T^c_{jt} - tax^c_{jt}\right) \\
&\qquad - \frac{\gamma^w}{2} W_t N_t \left(\frac{W_{jt}}{W_{jt-1}} - 1 - \left(1 - sf^w\right)\left(\pi_{t-1}\text{-}\pi\right) - \pi^w\right)^2
\end{aligned}$$

and the the intermediate good producing firms' demand of differentiated labor:

$$N_{jt} = \left(\frac{W_{jt}}{W_t}\right)^{-\sigma^n} N_t.$$

After having allowed for real wage rigidity as in Blanchard and Galí (2007) through the inclusion of the parameter γ^{w_r} , the wage setting equation describing the labor supply becomes:

$$\begin{aligned}
&\left[-\frac{U_H}{\lambda_t}\frac{\sigma^n}{\sigma^n-1}\left(1+\tau^C\right)\right]^{1-\gamma^{w_r}} \\
&\quad \left[\left(1-\tau^N\right)\frac{W_{t-1}}{P_{t-1}}\right]^{\gamma^{w_r}} = \left(1-\tau^N\right)\frac{W_t}{P_t} \\
&\qquad + \frac{\gamma^w}{\sigma^n-1}\left(\frac{W_t}{W_{t-1}} - 1 - \left(1-sf^w\right)\left(\pi_{t-1}\text{-}\pi\right) - \pi^w\right)\frac{W_t}{W_{t-1}}\frac{W_t}{P_t} \\
&\qquad - \frac{\gamma^w}{\sigma^n-1} E_t\left[\beta z^C_t \frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}\frac{N_{t+1}}{N_t}\right. \\
&\qquad \left.\left(\frac{W_{t+1}}{W_t} - 1 - \left(1-sf^w\right)\left(\pi_t\text{-}\pi\right) - \pi^w\right)\frac{W_{t+1}}{W_t}\frac{W_{t+1}}{P_t}\right]
\end{aligned}$$

where we defined $\lambda_t = \omega^s \left(C^s_{jt} - h^C C^s_{t-1}\right)^{-\theta} + \left(1-\omega^s\right)\left(C^c_{jt} - h^C C^c_{t-1}\right)^{-\theta}$ and $U_H = -z^N_t C^{1-\theta}_t s^N \left(N_{jt} - h^N N_{t-1}\right)^{-\theta^N}$.

3.3 The firms' problem

The production sector is fairly standard: there is a continuum of identical perfectly competitive firms who assemble the final good using differentiated intermediate goods produced by a continuum of $i \in [0,1]$ monopolistically competitive firms as inputs. The intermediate goods producing firms use labor and capital (private and public) inputs and are subject to adjustment costs affecting labor and investment demand, pricing decision, the utilization of capital and dividends payout. Moreover, under the alternative, although more standard specification of financial frictions, they will face an occasionally binding constraint on debt issuance.

3.3.1 Final good firms

Perfectly competitive firms endowed with Dixit-Stiglitz production technology maximize their profits choosing the intermediate inputs Y_{it} taking their prices P_{it} as given. As a result of their optimization problem we obtain the demand of intermediate goods and the definition of final good price index:

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\sigma^Y} Y_t$$
$$P_t = \left(\int_0^1 P_{it}^{1-\sigma^Y} di\right)^{\frac{1}{1-\sigma^Y}}.$$

3.3.2 Intermediate good firms

Monopolistically competitive firms maximize the stream of expected future profits (dividends D_{it}) evaluated through the stochastic discount factor of the savers, subject to the intermediate good demand, the law of motion of capital, the Cobb-Douglas production function and a sequence of adjustment costs. Formally given the stochastic discount factor, defined as$\mathcal{M}_{is} \equiv \left(\frac{1+r_t^s}{\prod_{r=t}^s(1+r_r^s)}\right) = \left(\frac{\frac{1+i_t^s}{1+\pi_t}}{\prod_{r=t}^s\left(\frac{1+i_r^s}{1+\pi_r}\right)}\right)$ their problem is:

$$\max_{\{D_{it},P_{it},N_{it},I_{it},K_{it},CU_{it},L_{it}\}} E_0 \sum_{t=0}^{\infty} \mathcal{M}_{it} D_{it}$$

subject to:

$$D_{it} = \left(1-\tau^K\right)\left(\frac{P_{it}}{P_t}Y_{it} - \frac{W_t}{P_t}N_{it}\right) + \tau^K\delta\frac{P_t^I}{P_t}K_{it-1} - \frac{P_t^I}{P_t}I_{it} + \frac{L_{it}}{P_t} - \frac{L_{it-1}}{P_t}\left(1+i_{t-1}^l\right) - adj_{it}$$
$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\sigma^Y} Y_t$$
$$K_{it} = I_{it} + (1-\delta)K_{it-1}$$
$$Y_{it} = \left(A_t^Y N_{it}\right)^{\alpha}\left(CU_{it}K_{it-1}^{tot}\right)^{1-\alpha}$$

where $adj_{it} = adj_{it}^P + adj_{it}^N + adj_{it}^{CU} + adj_{it}^I + adj_{it}^D$and[5],

$$adj_{it}^P = \frac{\sigma^Y\gamma^p}{2}Y_t\left(\frac{P_{it}}{P_{it-1}} - 1 - \pi\right)^2$$
$$adj_{it}^N = \frac{\gamma^n}{2}Y_t\left(\frac{N_{it}}{N_{it-1}} - \exp\left(g^{pop}\right)\right)^2$$
$$adj_{it}^{CU} = \frac{P_t^I}{P_t}K_{it-1}^{tot}\left(\gamma_0^u\left(CU_{it}-1\right) + \frac{\gamma_1^u}{2}\left(CU_{it}-1\right)^2\right)$$
$$adj_{it}^I = \frac{\gamma_0^i}{2}\frac{P_t^I}{P_t}K_{t-1}\left(\frac{I_{it}}{K_{t-1}} - \delta_t\right)^2 + \frac{\gamma_1^i}{2}\frac{P_t^I}{P_t}\frac{\left(I_{it} - I_{it-1}\exp\left(g^Y + g^{P^I}\right)\right)^2}{K_{t-1}}$$
$$adj_{it}^D = \frac{\gamma^d}{2}\left(D_{it} - D\right)^2.$$

[5]The pricing adjustment cost is multiplied by σ^Yin order to improve the identification of γ^p.

The optimality equilibrium conditions resulting from the problem are:

$$\lambda_t^f = \frac{1}{\left(1+\gamma^d \left(D_t - D\right)\right)} \tag{9}$$

$$\begin{aligned} \mu_t^Y \sigma^Y = {} & \lambda_{it}^f \left(\left(1-\tau^K\right)\left(\sigma^Y - 1\right) + \sigma^Y \gamma^p \frac{P_t}{P_{t-1}} \left(\frac{P_t}{P_{t-1}} - 1 - \pi \right) \right) \\ & - \sigma^Y \gamma^p E_t \left[\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t} \lambda_{t+1}^f \frac{Y_{t+1}}{Y_t} \left(\frac{P_{t+1}}{P_t} \right) \left(\frac{P_{t+1}}{P_t} - 1 - \pi \right) \right] \end{aligned} \tag{10}$$

$$\alpha \frac{\mu_t^Y}{\lambda_{it}^f} \frac{Y_t}{N_t} - \gamma^n \frac{Y_t}{N_{t-1}} \left(\frac{N_t}{N_{t-1}} - \exp\left(g^{pop}\right) \right) + \gamma^n E_t \left[\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t} \frac{\lambda_{t+1}^f}{\lambda_t^f} \frac{Y_{t+1}}{N_t} \frac{N_{t+1}}{N_t} \left(\frac{N_{t+1}}{N_t} - \exp\left(g^{pop}\right) \right) \right] = \left(1-\tau^K\right) \frac{W_t}{P_t} \tag{11}$$

$$\begin{aligned} Q_t = {} & \lambda_{it}^f \left(1 + \gamma_0^i \left(\frac{I_t}{K_{t-1}} - \delta_t \right) + \gamma_1^i \frac{\left(I_t - I_{t-1} \exp\left(g^Y + g^{P^I} \right) \right)}{K_{t-1}} \right) \\ & - E_t \left[\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t} \lambda_{t+1}^f \frac{P_{t+1}^I}{P_t^I} \frac{P_t}{P_{t+1}} \gamma_1^i \frac{\left(I_{t+1} - I_t \exp\left(g^Y + g^{P^I} \right) \right)}{K_t} \exp\left(g^Y + g^{P^I} \right) \right] \end{aligned} \tag{12}$$

$$\begin{aligned} Q_t = {} & E_t \left[\frac{\mathcal{M}_{it+1}}{\mathcal{M}_{it}} \frac{P_{t+1}^I}{P_{t+1}} \frac{P_t}{P_t^I} \left(\lambda_{it+1}^f \left(\tau^K \delta - \gamma_0^u \left(CU_{it+1} - 1 \right) - \frac{\gamma_1^u}{2} \left(CU_{it+1} - 1 \right)^2 \right) \right. \right. \\ & \left. \left. + \left(1-\delta\right) Q_{t+1} + \left(1-\alpha\right) \mu_{t+1}^Y \frac{P_{t+1}}{P_{t+1}^I} \frac{Y_{it+1}}{K_{it}^{tot}} \right) \right] \end{aligned} \tag{13}$$

$$\mu_t^Y \left(1-\alpha\right) \frac{Y_t}{CU_t} = \lambda_{it}^f \frac{P_t^I}{P_t} K_{t-1}^{tot} \left(\gamma_0^u + \gamma_1^u \left(CU_t - 1 \right) \right) \tag{14}$$

$$E_t \left[\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t} \frac{\lambda_{t+1}^f}{\lambda_t^f} \frac{P_t}{P_{t+1}} \left(1 + i_t^l \right) \right] = 1 \tag{15}$$

where (9) reflects the distortion introduced by the adjustment cost in the financial structure of the firm, (10) is the standard pricing equation, (11) is the labor demand, (12) is the investment demand, (13) is the Tobin's Q equation, (14) is the first order condition with respect to capacity utilization, and (15) is the demand for loans.

Under the alternative specification of the model, in which the occasionally binding constraint enters the intermediate firms' problem equations (13) and (15) become:

$$Q_t = E_t \left[\frac{\mathcal{M}_{it+1}}{\mathcal{M}_{it}} \frac{\lambda^f_{it+1}}{\lambda^f_{it}} \frac{P^I_{t+1}}{P_{t+1}} \frac{P_t}{P^I_t} \left(\tau^K \delta - \gamma^u_0 \left(CU_{it+1} - 1\right) - \frac{\gamma^u_1}{2} \left(CU_{it+1} - 1\right)^2 \right.\right.$$
$$\left.\left. + \left(1-\delta\right) Q_{t+1} + \left(1-\alpha\right) \mu^Y_{t+1} \frac{P_{t+1}}{P^I_{t+1}} \frac{Y_{it+1}}{K^{tot}_{it}} + \mu^l_{it+1} P_{t+1} m^l z^F_{t+1} \right) \right] \tag{16}$$

$$E_t \left[\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t} \frac{\lambda^f_{t+1}}{\lambda^f_t} \frac{P_t}{P_{t+1}} \frac{1}{1-\mu^l_{it} P_t} \left(1 + i^l_t\right) \right] = 1 \tag{17}$$

4 Results

In this section we implement the algorithms presented in Section 2to the model of Section 3under the two alternative specifications, in which the occasionally binding constraint on loans enters either in the households' problem or in the firms' one.

The analysis is carried out starting from parameter estimates, obtained with Bayesian methods, under the assumption that the lending constraint is not binding. The observable variables are: population, employment, GDP deflator, real GDP, hours worked, participation rate, nominal policy rate, consumption, investment, public investment, transfers, government debt, interest payments on public debt, wages, physical capital. The sample goes from 1991Q1 to 2016Q1.

In Tables 1and 2we show the regimes sequence for the ZLB constraint and for the financial constraint under the two alternative models. The key to read the table is the following: in the" regime sequence" columns 0 means non binding, 1 binding. So a sequence 1 0 means that the constraint is binding at time t and is expected to be non binding in the future. A sequence 0 1 0 means that the constraint is not binding at time t but is expected to be binding in the future and then be not binding again later on. The columns "starting period of regime" indicate the actual periods in which regimes switch. For instance a sequence 1 3, means that the constraint is binding at time t and that will be binding for 3 periods. A sequence 1 4 6 means that the constraint is not going to be binding until 4 periods from now and that it is expected to be binding for 2 periods.

Comparing the two Tables we can see that the regimes sequences are very similar, but on average the financial constraint seems to be expected to bind for longer periods under the lending constraint assumption.

In Figures 1 and 2 we plot the smoothed series for the the loan to value, $\frac{L_t}{P^I_t K_t} \frac{1}{z^F_t}$, for the latent AR(1) process describing the financial conditions, z^F_t, for the Lagrangian multiplier on the always binding constraint on total risky assets $\mu^{s,tot}_t$, and for the multipliers on the occasionally binding constraint on loans, $\mu^{s,l}_t$and μ^l_t depending on the model version analyzed. Again we notice that both models show similar departures from the linear solution (in blue). The most significant difference between the two models involves the financial variable z^F_t which during the GR and under the model with lending constraints it's closer to its mean, implying that the non-linearity of the model goes a longer way in describing the behavior of the observables during the financial crisis.

In Figures 3 and 4 for the model with lending constraints and in Figures 5 and 6 for the alternative specification, we plot the smoothed shocks of the model. Again, by looking at the smoothed series of the financial shocks ε_t^F we reach the same conclusion in terms of the underlying non-linearities under the two settings. The model with lending constraints, by implying a longer expected duration of binding constraints, displays a higher degree of non-linearity which translates in innovations closer to the Gaussian assumption. It is also worth noticing, for both models, the relevance of the ZLB in the last periods of the sample.

In Figures 7 and 8 we show the shock decompositions of the growth rate of GDP under the linear model (dark blue), and under the piecewise linear solution using the "Main effect" (light blue) and the "Total effect" (red). In both model versions, and under the "Total effect" decomposition we notice an amplification of the role played by fiscal shocks following the GFC. Moreover, price mark-up shocks play a bigger role than in the linear model, but less so in the model with borrowing constraints. This may be explained by the fact that the Tobin's Q equation inherits the multiplier of the borrowing constraint since a "purchase" of one unit of capital today makes coeteris paribus the borrowing limit looser tomorrow. Again by looking at the role played by financial innovations one can notice that their impact on GDP growth is somewhat smaller under the model with lending constraints.

Finally, similar to Lindé et al. (2016) in Figures and we plot the real time forecast densities performed in the last quarter of 2008, and computed with Quasi-Montecarlo methods. The dashed black line is the data, the blue line is the mean forecast under linearity with gray area indicating 95% forecast interval, the red line is the mean forecast under occasionally binding constraints with pink area indicating also 95% forecast intervals. Coherent with the previous results we notice that the extreme drop in GDP growth occurred in the first quarter of 2009 is, contrary to the linear model, and under both specifications of the piecewise linear model, within the 95% forecast interval. Moreover, the same holds true for investment, clearly indicating that financial frictions with occasionally binding constraints, may be a promising way to account for extreme events maintaining the Gaussian assumption.

5 Conclusions

We have introduced an algorithm which allows to measure the contributions of smoothed shocks onto observable variables in the context of models with occasionally binding constraints solved with piecewise solution methods. In this regard we introduced two ways to account for non-linearities, namely the "Main effect contribution" and the "Total effect contribution". We implemented this algorithm on a closed economy model estimated for the Euro area, which allows for two sources of non-linearity. First we allow for the ZLB, second we introduce a constraint on loan contracts between consumption smoothing households and firms. We proposed two alternative formulations, allowing for the enforceability constraint to be either internalized by the lender or by the borrower. We show that both versions, contrary to the linear model, allow for predictive densities which include the huge drop of GDP occurred in the first quarter of 2009, indicating that it may not be necessary to depart from Gaussian assumption to be able to predict extreme events. We also show that the model with lending constraint may display a higher degree of non-linearity, suggesting that further research in the designing of financial contracts in macroeconomic models may be pursued.

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6 Tables

time	regime sequence 1	starting period of regime 1	regime sequence 2	starting period of regime 2
2008	0	1	0	1
2008.25	0	1	0	1
2008.5	0	1	0	1
2008.75	0	1	0	1
2009	0 1 0	1 4 7	1 0	1 6
2009.25	0 1 0	1 3 6	1 0	1 6
2009.5	0 1 0	1 2 5	1 0	1 5
2009.75	1 0	1 4	1 0	1 6
2010	1 0	1 5	1 0	1 5
2010.25	1 0	1 2	1 0	1 4
2010.5	1 0	1 2	1 0	1 5
2010.75	0	1	1 0	1 5
2011	0	1	1 0	1 4
2011.25	0	1	1 0	1 5
2011.5	0	1	1 0	1 5
2011.75	0	1	1 0	1 6
2012	0	1	1 0	1 5
2012.25	0 1 0	1 2 4	1 0	1 6
2012.5	1 0	1 4	1 0	1 6
2012.75	1 0	1 4	1 0	1 6
2013	1 0	1 4	1 0	1 6
2013.25	1 0	1 4	1 0	1 6
2013.5	1 0	1 5	1 0	1 6
2013.75	1 0	1 4	1 0	1 6
2014	1 0	1 4	1 0	1 5
2014.25	1 0	1 5	1 0	1 6
2014.5	1 0	1 5	1 0	1 6
2014.75	1 0	1 4	1 0	1 6
2015	1 0	1 4	1 0	1 5
2015.25	1 0	1 4	1 0	1 6
2015.5	1 0	1 4	1 0	1 5
2015.75	1 0	1 4	1 0	1 5
2016	1 0	1 3	1 0	1 5

Table 1: Regimes sequence under lending occasionally binding constraints

time	regime sequence 1	starting period of regime 1	regime sequence 2	starting period of regime 2
2008	0	1	0	1
2008.25	0	1	0	1
2008.5	0	1	0	1
2008.75	0	1	0	1
2009	0 1 0	1 5 7	1 0	1 6
2009.25	0 1 0	1 4 5	1 0	1 5
2009.5	0 1 0	1 3 4	1 0	1 4
2009.75	1 0 1 0	1 2 3 4	1 0	1 3
2010	1 0	1 5	1 0	1 2
2010.25	1 0	1 2	0	1
2010.5	1 0	1 2	1 0	1 3
2010.75	0	1	1 0	1 2
2011	0	1	0	1
2011.25	0	1	1 0	1 2
2011.5	0	1	1 0	1 3
2011.75	0	1	1 0	1 2
2012	0	1	1 0	1 4
2012.25	0 1 0	1 3 4	1 0	1 3
2012.5	1 0	1 4	1 0	1 2
2012.75	1 0	1 4	1 0	1 4
2013	1 0	1 4	1 0	1 3
2013.25	1 0	1 4	1 0	1 2
2013.5	1 0	1 5	1 0	1 3
2013.75	1 0	1 4	1 0	1 2
2014	1 0	1 4	1 0	1 3
2014.25	1 0	1 5	1 0	1 2
2014.5	1 0	1 4	1 0	1 4
2014.75	1 0	1 4	1 0	1 3
2015	1 0	1 4	1 0	1 2
2015.25	1 0	1 3	1 0	1 4
2015.5	1 0	1 4	1 0	1 3
2015.75	1 0	1 4	1 0	1 2
2016	1 0	1 3	1 0	1 3

Table 2: Regimes sequence under borrowing occasionally binding constraints

7 Figures

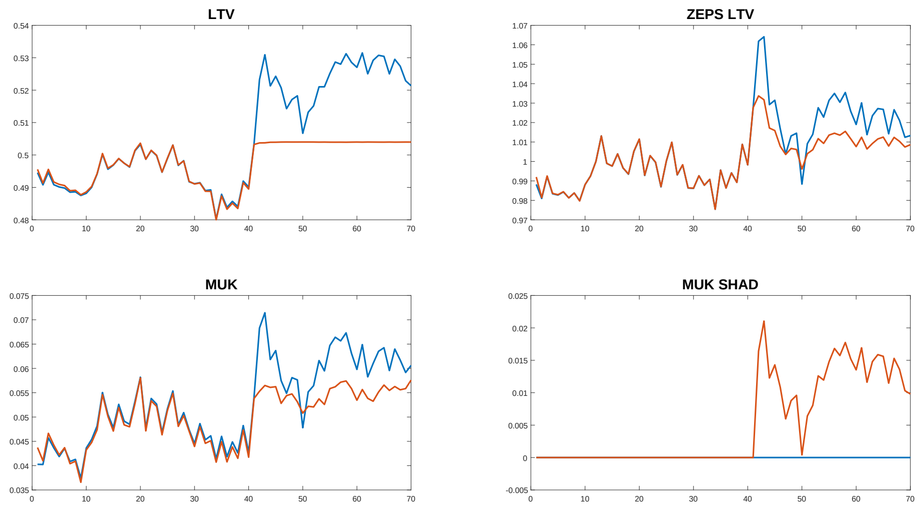

Figure 1: Smoothed unobserved under lending occasionally binding constraints

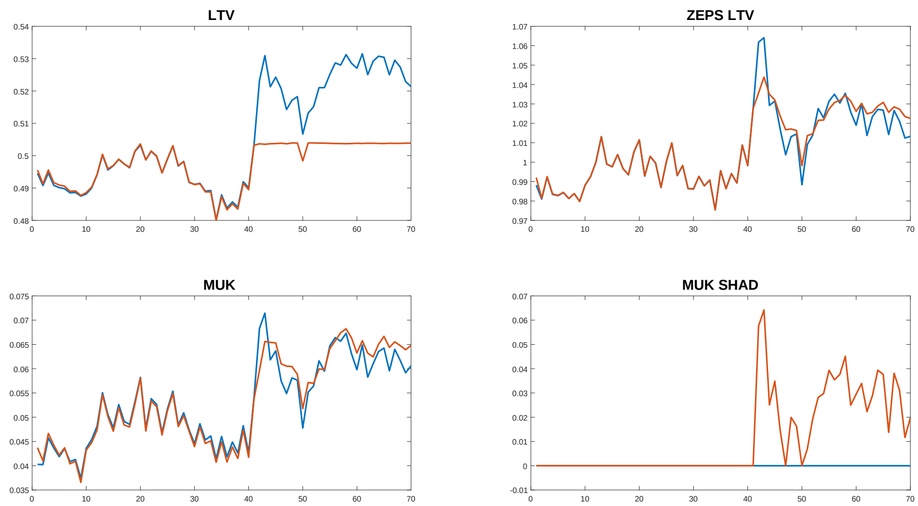

Figure 2: Smoothed unobserved under borrowing occasionally binding constraints

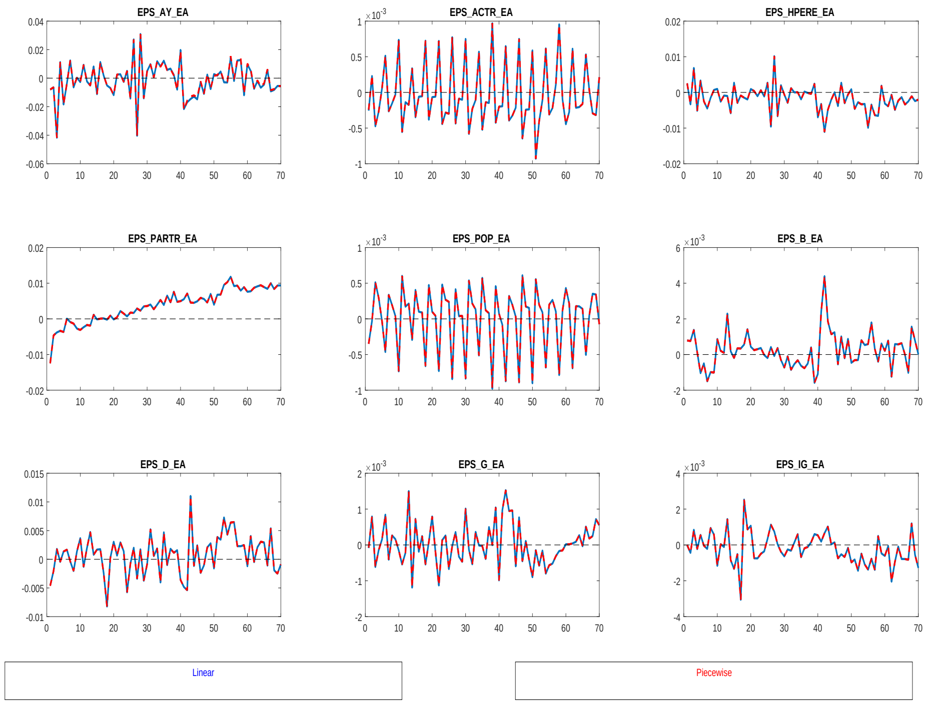

Figure 3: Smoothed shocks under lending occasionally binding constraints

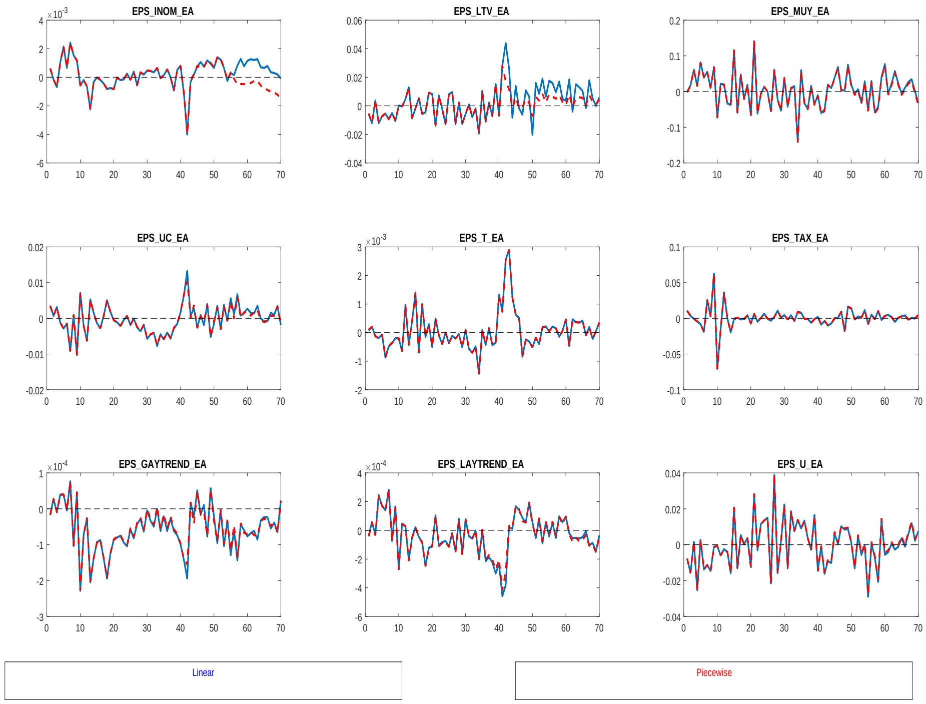

Figure 4: Smoothed shocks under lending occasionally binding constraints

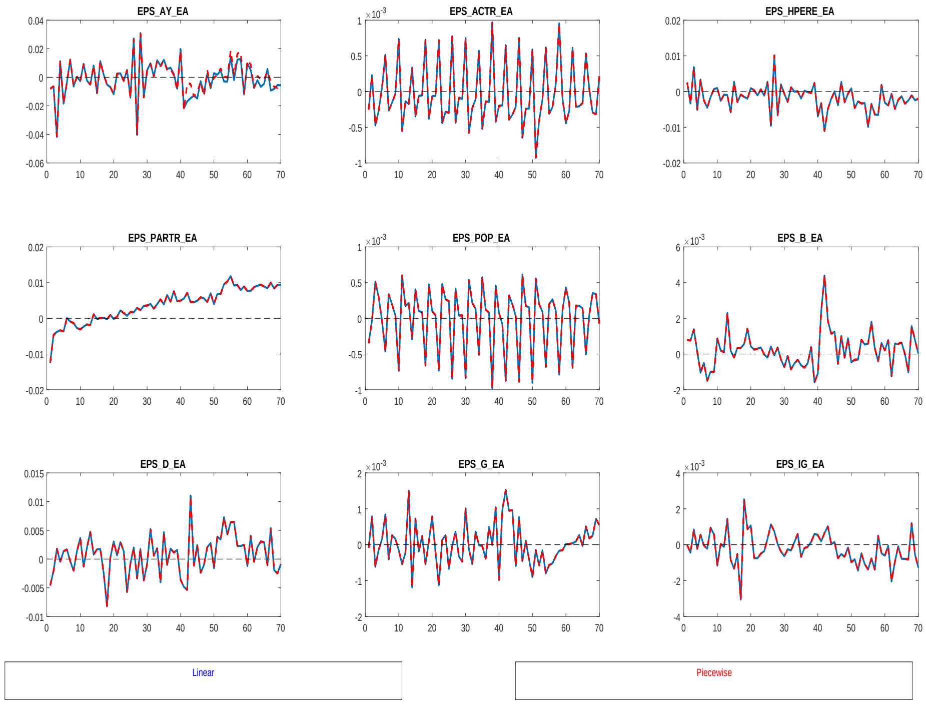

Figure 5: Smoothed shocks under borrowing occasionally binding constraints

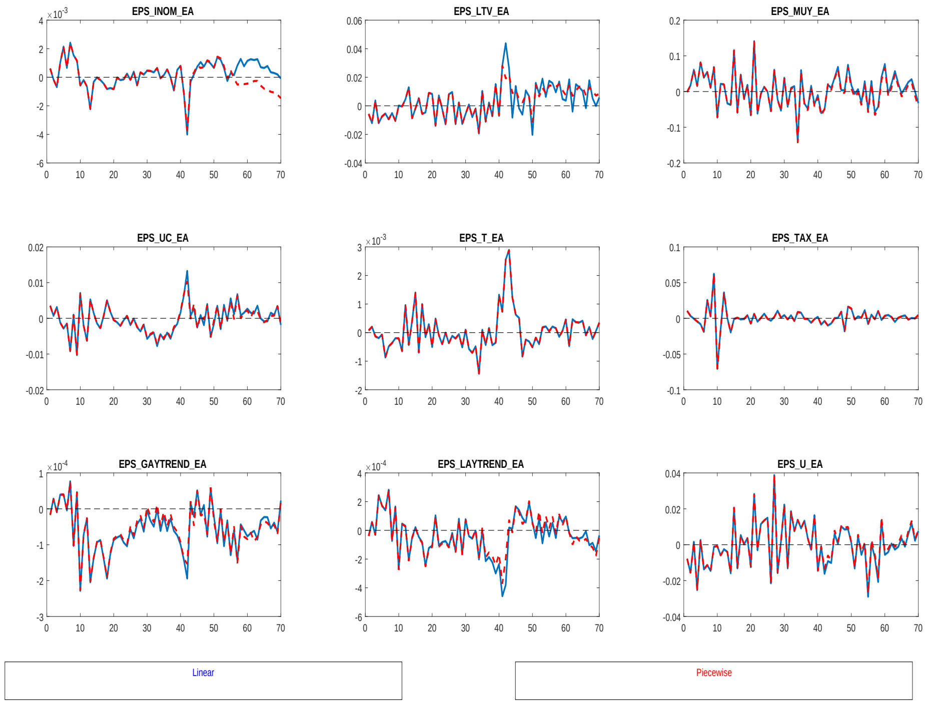

Figure 6: Smoothed shocks under borrowing occasionally binding constraints

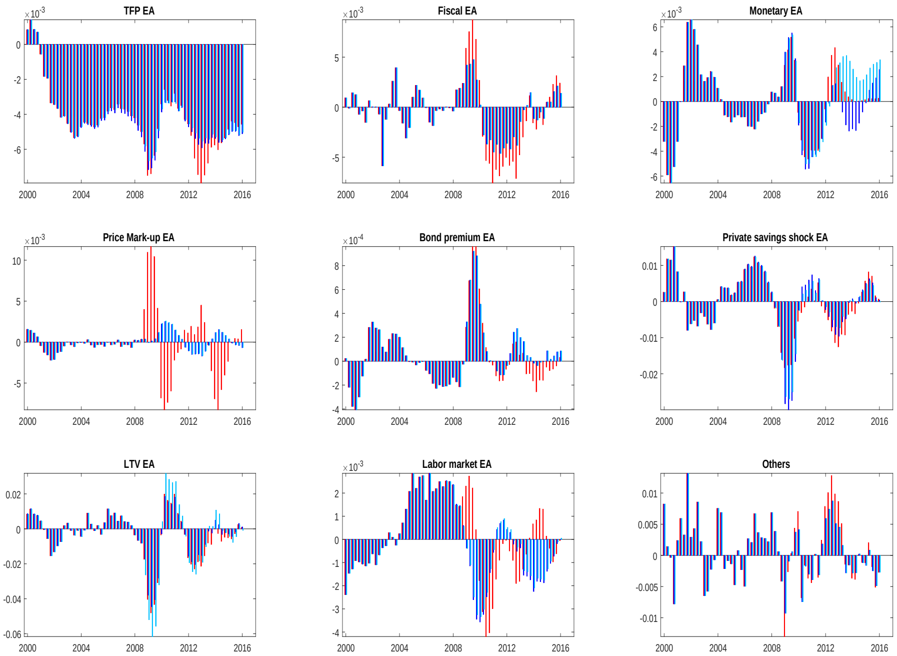

Figure 7: Shock decomposition of the growth rate of GDP under lending occasionally binding constraint

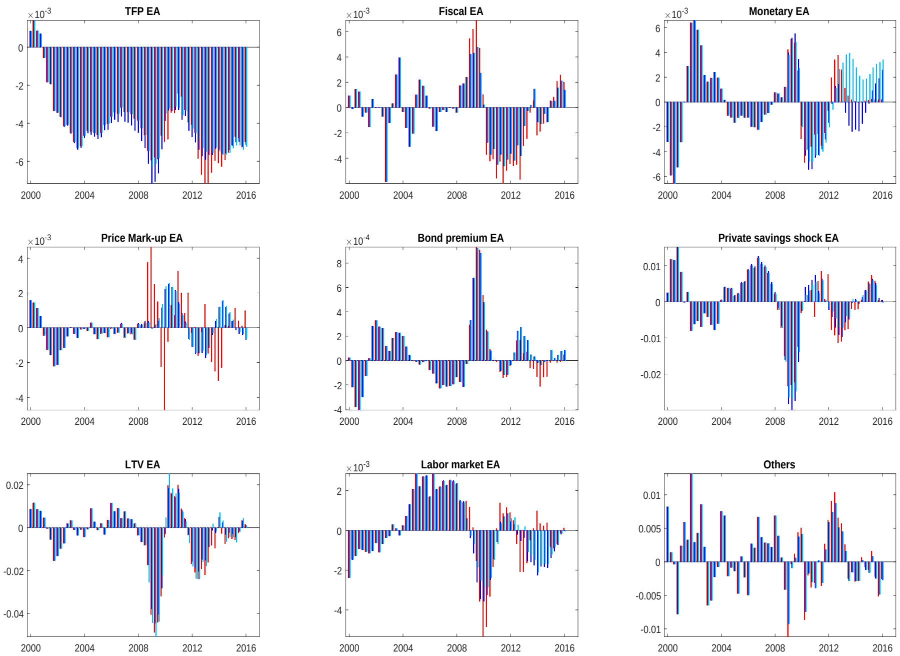

Figure 8: Shock decomposition of the growth rate of GDP under borrowing occasionally binding constraint

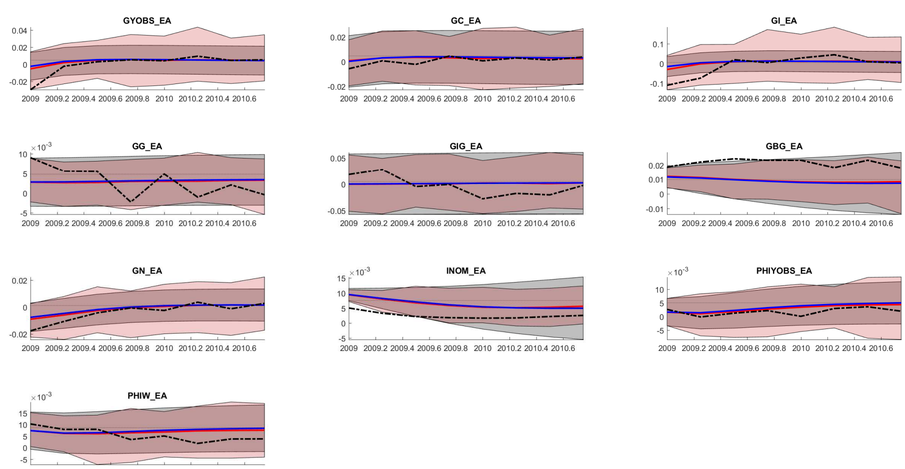

Figure 9: Real time forecast at 2018Q4 with 95% predictive density interval under lending occasionally binding constraints

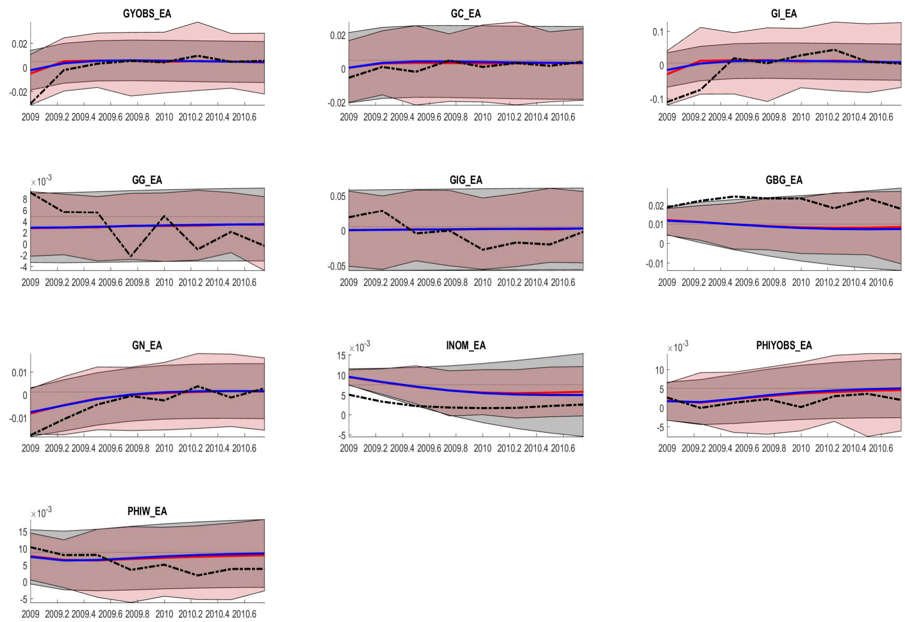

Figure 10: Real time forecast at 2018Q4 with 95% predictive density interval under borrowing occasionally binding constraints

A Model derivations

A.1 Ricardian Households

Households who have access to financial markets solve the following infinite horizon utility maximization problem:

$$\max_{\left\{C^s_{jt},B_{jt},B^g_{jt},L_{jt},S_{jt}\right\}} E_0 \sum_{t=0}^{\infty} \left(\beta z^C_{t-1}\right)^t U^s\left(X^s_{jt}, H^s_{jt}, A_{jt-1}\right)$$

with $X^s_{jt} \equiv C^s_{jt} - h^C C^s_{t-1}$, $H^s_{jt} \equiv N^s_{jt} - h^N N^s_{t-1}$, where $A_{jt-1} \equiv \left[B^g_{jt-1}, S_{jt-1}, L_{jt-1}\right]$ is a vector of all assets held entering period t, and $\tilde{z}^C_{t-1} \equiv \exp\left(z^C_{t-1}\right)$, subject to the budget constraint,

$$\begin{aligned}\left(1+\tau^C\right) P_t C^s_{jt} + B_{jt} + B^g_{jt} + L_{jt} + P^S_t S_{jt} = \left(1-\tau^N\right) W_t N^s_{jt} + \left(1+i_{t-1}\right) B_{jt-1} + \left(1+i^g_{t-1}\right) B^g_{jt-1} \\ + \left(1+i^l_{t-1}\right) L_{jt-1} + \left(P^S_t + P_t d_t\right) S_{jt-1} + T^s_{jt} - tax^s_{jt}\end{aligned}$$

to the constraint on total firms lending,

$$L_{jt} + P^S_t S_{jt} = m^{tot}_t \left(P^I_t K_{t-1}\right)$$

and to the lending constraint,

$$L_{jt} \leq m^l_t \left(P^I_t K_{t-1}\right).$$

The functional form adopted for preferences is:

$$\begin{aligned}U^s\left(X^s_{jt}, H^s_{jt}, A_{jt-1}\right) = &\frac{1}{1-\theta}\left(C^s_{jt} - h^C C^s_{t-1}\right)^{1-\theta} - z^N_t C^{1-\theta}_t \frac{s^N}{1-\theta^N}\left(N^s_{jt} - h^N N^s_{t-1}\right)^{1-\theta^N} \\ &- \left(C^s_t - h^C C^s_{t-1}\right)^{-\theta}\left[\frac{s^{B^g}\left(\left(\alpha^{B^g}_0 + z^{B^g}_{t-1}\right) B^g_{jt-1} + \frac{1}{2}\alpha^{B^g}_1 \frac{\left(B^g_{jt-1}\right)^2}{P_{t-1}Y_{t-1}}\right)}{\left(1+\tau^C\right) P_t}\right. \\ &+ \frac{s^S\left(\left(\alpha^S_0 + z^S_{t-1}\right) P^S_{t-1} S_{jt-1} + \frac{1}{2}\alpha^S_1 \frac{\left(P^S_{t-1}S_{jt-1}\right)^2}{P_{t-1}Y_{t-1}}\right)}{\left(1+\tau^C\right) P_t} \\ &\left. + \frac{s^L\left(\left(\alpha^L_0 + z^L_{t-1}\right) L_{jt-1} + \frac{1}{2}\alpha^L_1\left(P^I_{t-1}K_{t-1}\right)\left(\frac{L_{jt-1}}{P^I_{t-1}K_{t-1}} - \frac{\overline{L}}{\overline{P^I K}}\right)^2\right)}{\left(1+\tau^C\right) P_t}\right]\end{aligned}$$

The first order conditions with respect to C^s_{jt},B_{jt},B^g_{jt},L_{jt},S_{jt}, read respectively:

$$\left(C^s_{jt} - h^C C^s_{t-1}\right)^{-\theta} = \tilde{\lambda}^s_t \left(1+\tau^C\right) P_t \equiv \lambda^s_t \tag{18}$$

$$\tilde{\lambda}^s_t = \beta E_t \left(\tilde{z}^C_t \tilde{\lambda}^s_{t+1}\left(1+i_t\right)\right)$$

$$\beta E_t\left[\widetilde{z}_t^C\frac{\lambda_{t+1}^s}{\lambda_t^s}\frac{P_t}{P_{t+1}}\left(1+i_t\right)\right]=1 \tag{19}$$

$$-\widetilde{\lambda}_t^s+\beta E_t\left[-\frac{\widetilde{z}_t^C\left(C_{t+1}^s-h^C C_t^s\right)^{-\theta}}{\left(1+\tau^C\right)P_{t+1}}s^{B^g}\left(\alpha_0^{B^g}+z_t^{B^g}+\alpha_1^{B^g}\frac{B_{jt}^g}{P_tY_t}\right)+\widetilde{z}_t^C\widetilde{\lambda}_{t+1}^s\left(1+i_t^g\right)\right]=0$$

$$-\widetilde{\lambda}_t^s+\beta E_t\left[-\widetilde{z}_t^C\tilde{\lambda}_{t+1}^s s^{B^g}\left(\alpha_0^{B^g}+z_t^{B^g}+\alpha_1^{B^g}\frac{B_{jt}^g}{P_tY_t}\right)+\widetilde{z}_t^C\widetilde{\lambda}_{t+1}^s\left(1+i_t^g\right)\right]=0$$

$$\frac{\lambda_t^s}{\left(1+\tau^C\right)P_t}=\beta E_t\left[-\frac{\lambda_{t+1}^s\widetilde{z}_t^C}{\left(1+\tau^C\right)P_{t+1}}s^{B^g}\left(\alpha_0^{B^g}+z_{t+1}^{B^g}+\alpha_1^{B^g}\frac{B_{jt}^g}{P_tY_t}\right)+\frac{\lambda_{t+1}^s\widetilde{z}_t^C}{\left(1+\tau^C\right)P_{t+1}}\left(1+i_t^g\right)\right]$$

$$\beta E_t\left[\widetilde{z}_t^C\frac{\lambda_{t+1}^s}{\lambda_t^s}\frac{P_t}{P_{t+1}}\left(1+i_t^g-s^{B^g}\left(\alpha_0^{B^g}+z_{t+1}^{B^g}+\alpha_1^{B^g}\frac{B_{jt}^g}{P_tY_t}\right)\right)\right]=1 \tag{20}$$

$$-\widetilde{\lambda}_t^s-\widetilde{\mu}_t^{s,tot}-\widetilde{\mu}_t^{s,l}+\beta E_t\left[-\frac{\widetilde{z}_t^C\left(C_{t+1}^s-h^C C_t^s\right)^{-\theta}}{\left(1+\tau^C\right)P_{t+1}}s^L\left(\alpha_0^L+z_t^L+\alpha_1^L\left(\frac{L_{jt}}{P_t^IK_t}-\frac{\overline{L}}{\overline{P^I}\overline{K}}\right)\right)+\widetilde{z}_t^C\widetilde{\lambda}_{t+1}^s\left(1+i_t^l\right)\right]=0$$

$$-\widetilde{\lambda}_t^s-\widetilde{\mu}_t^{s,tot}-\widetilde{\mu}_t^{s,l}+\beta E_t\left[-\widetilde{z}_t^C\tilde{\lambda}_{t+1}^s s^L\left(\alpha_0^L+z_t^L+\alpha_1^L\left(\frac{L_{jt}}{P_t^IK_t}-\frac{\overline{L}}{\overline{P^I}\overline{K}}\right)\right)+\widetilde{z}_t^C\widetilde{\lambda}_{t+1}^s\left(1+i_t^l\right)\right]=0$$

$$-\widetilde{\lambda}_t^s-\widetilde{\mu}_t^{s,tot}-\widetilde{\mu}_t^{s,l}+\beta E_t\left[\widetilde{z}_t^C\widetilde{\lambda}_{t+1}^s\left(1+i_t^l-s^L\left(\alpha_0^L+z_t^L+\alpha_1^L\left(\frac{L_{jt}}{P_t^IK_t}-\frac{\overline{L}}{\overline{P^I}\overline{K}}\right)\right)\right)\right]=0$$

$$1+\frac{\widetilde{\mu}_t^{s,tot}+\widetilde{\mu}_t^{s,l}}{\widetilde{\lambda}_t^s}=\beta E_t\left[\widetilde{z}_t^C\frac{\widetilde{\lambda}_{t+1}^s}{\widetilde{\lambda}_t^s}\left(1+i_t^l-s^L\left(\alpha_0^L+z_t^L+\alpha_1^L\left(\frac{L_{jt}}{P_t^IK_t}-\frac{\overline{L}}{\overline{P^I}\overline{K}}\right)\right)\right)\right]$$

$$1+\frac{\widetilde{\mu}_t^{s,tot}+\widetilde{\mu}_t^{s,l}}{\lambda_t^s}\left(1+\tau^C\right)P_t=\beta E_t\left[\widetilde{z}_t^C\frac{\lambda_{t+1}^s}{\lambda_t^s}\frac{P_t}{P_{t+1}}\left(1+i_t^l-s^L\left(\alpha_0^L+z_t^L+\alpha_1^L\left(\frac{L_{jt}}{P_t^IK_t}-\frac{\overline{L}}{\overline{P^I}\overline{K}}\right)\right)\right)\right]$$

$$1+\mu_t^{s,tot}+\mu_t^{s,l}=\beta E_t\left[\widetilde{z}_t^C\frac{\lambda_{t+1}^s}{\lambda_t^s}\frac{P_t}{P_{t+1}}\left(1+i_t^l-s^L\left(\alpha_0^L+z_t^L+\alpha_1^L\left(\frac{L_{jt}}{P_t^IK_t}-\frac{\overline{L}}{\overline{P^I}\overline{K}}\right)\right)\right)\right] \tag{21}$$

where $\mu_t^{s,tot}\equiv\widetilde{\mu}_t^{s,tot}\frac{(1+\tau^C)P_t}{\lambda_t^s}$ and $\mu_t^{s,l}\equiv\widetilde{\mu}_t^{s,l}\frac{(1+\tau^C)P_t}{\lambda_t^s}$

$$-\widetilde{\lambda}_t^sP_t^S-\widetilde{\mu}_t^{s,tot}P_t^S+\beta E_t\left[-\frac{\widetilde{z}_t^C\left(C_{t+1}^s-h^C C_t^s\right)^{-\theta}}{\left(1+\tau^C\right)P_{t+1}}s^S\left(\left(\alpha_0^S+z_t^S\right)P_t^S+\alpha_1^S\frac{\left(P_t^S\right)^2S_{jt}}{P_tY_t}\right)+\widetilde{z}_t^C\widetilde{\lambda}_{t+1}^s\left(P_{t+1}^S+P_{t+1}d_{t+1}\right)\right]=0$$

$$-\widetilde{\lambda}_t^s P_t^S - \widetilde{\mu}_t^{s,tot} P_t^S + \beta E_t \left[-\widetilde{z}_t^C \widetilde{\lambda}_{t+1}^s s^S \left(\left(\alpha_0^S + z_t^S\right) P_t^S + \alpha_1^S \frac{\left(P_t^S\right)^2 S_{jt}}{P_t Y_t} \right) + \widetilde{z}_t^C \widetilde{\lambda}_{t+1}^s \left(P_{t+1}^S + P_{t+1} d_{t+1} \right) \right] = 0$$

$$1 + \frac{\widetilde{\mu}_t^{s,tot} P_t^S}{\widetilde{\lambda}_t^s P_t^S} = \beta E_t \left[\widetilde{z}_t^C \frac{\widetilde{\lambda}_{t+1}^s}{\widetilde{\lambda}_t^s P_t^S} \left(P_{t+1}^S + P_{t+1} d_{t+1} - s^S \left(\left(\alpha_0^S + z_t^S\right) P_t^S + \alpha_1^S \frac{\left(P_t^S\right)^2 S_{jt}}{P_t Y_t} \right) \right) \right]$$

$$1 + \frac{\widetilde{\mu}_t^{s,tot}}{\widetilde{\lambda}_t^s} = \beta E_t \left[\widetilde{z}_t^C \frac{\widetilde{\lambda}_{t+1}^s}{\widetilde{\lambda}_t^s} \left(\frac{P_{t+1}^S + P_{t+1} d_{t+1}}{P_t^S} - s^S \left(\alpha_0^S + z_t^S + \alpha_1^S \frac{P_t^S S_{jt}}{P_t Y_t} \right) \right) \right]$$

$$1 + \frac{\widetilde{\mu}_t^{s,tot}}{\lambda_t^s} \left(1 + \tau^C\right) P_t = \beta E_t \left[\widetilde{z}_t^C \frac{\lambda_{t+1}^s}{\lambda_t^s} \frac{P_t}{P_{t+1}} \left(\frac{P_{t+1}^S + P_{t+1} d_{t+1}}{P_t^S} - s^S \left(\alpha_0^S + z_t^S + \alpha_1^S \frac{P_t^S S_{jt}}{P_t Y_t} \right) \right) \right]$$

$$1 + \mu_t^{s,tot} = \beta E_t \left[\widetilde{z}_t^C \frac{\lambda_{t+1}^s}{\lambda_t^s} \frac{P_t}{P_{t+1}} \left(1 + i_{t+1}^s - s^S \left(\alpha_0^S + z_t^S + \alpha_1^S \frac{P_t^S S_t}{P_t Y_t} \right) \right) \right] \tag{22}$$

where $1 + i_{t+1}^s \equiv \frac{P_{t+1}^S + P_{t+1} d_{t+1}}{P_t^S}$ and $\mu_t^{s,tot} \equiv \widetilde{\mu}_t^{s,tot} \frac{\left(1+\tau^C\right) P_t}{\lambda_t^s}$.
Using the definition above we can define the real return from stocks as:

$$1 + r_{t+1}^s \equiv \frac{1 + i_{t+1}^s}{1 + \pi_{t+1}} = \frac{P_t}{P_{t+1}} \frac{P_{t+1}^S + P_{t+1} d_{t+1}}{P_t^S}$$

or, rearranging:

$$P_t^S = \frac{P_t}{P_{t+1}} \frac{P_{t+1}^S + P_{t+1} d_{t+1}}{1 + r_{t+1}^s}$$

iterating forward for two periods we obtain:

$$P_{t+1}^S = \frac{P_{t+1}}{P_{t+2}} \frac{P_{t+2}^S + P_{t+2} d_{t+2}}{1 + r_{t+2}^s}$$

$$P_{t+2}^S = \frac{P_{t+2}}{P_{t+3}} \frac{P_{t+3}^S + P_{t+3} d_{t+3}}{1 + r_{t+3}^s}$$

substituting back:

$$P_t^S = \frac{P_t}{P_{t+1}} \frac{\frac{P_{t+1}}{P_{t+2}} \frac{\frac{P_{t+2}}{P_{t+3}} \frac{P_{t+3}^S + P_{t+3} d_{t+3}}{1 + r_{t+3}^s} + P_{t+2} d_{t+2}}{1 + r_{t+2}^s} + P_{t+1} d_{t+1}}{1 + r_{t+1}^s}$$

rearranging:

$$P_t^S = \frac{P_t}{P_{t+1}} \left[\frac{P_{t+1}}{P_{t+2}} \left(\frac{P_{t+2}}{P_{t+3}} \frac{P_{t+3}^S + P_{t+3} d_{t+3}}{\left(1 + r_{t+2}^s\right)\left(1 + r_{t+3}^s\right)\left(1 + r_{t+1}^s\right)} + \frac{P_{t+2} d_{t+2}}{\left(1 + r_{t+1}^s\right)\left(1 + r_{t+2}^s\right)} \right) + \frac{P_{t+1} d_{t+1}}{\left(1 + r_{t+1}^s\right)} \right]$$

$$
\begin{aligned}
P_t^S &= \frac{P_t}{P_{t+1}} \frac{P_{t+1} d_{t+1}}{\left(1+r_{t+1}^s\right)} + \frac{P_t}{P_{t+1}} \frac{P_{t+1}}{P_{t+2}} \frac{P_{t+2} d_{t+2}}{\left(1+r_{t+1}^s\right)\left(1+r_{t+2}^s\right)} \\
&+ \frac{P_t}{P_{t+1}} \frac{P_{t+1}}{P_{t+2}} \frac{P_{t+2}}{P_{t+3}} \frac{P_{t+3} d_{t+3}}{\left(1+r_{t+2}^s\right)\left(1+r_{t+3}^s\right)\left(1+r_{t+1}^s\right)} \\
&+ \frac{P_t}{P_{t+1}} \frac{P_{t+1}}{P_{t+2}} \frac{P_{t+2}}{P_{t+3}} \frac{P_{t+3}^S}{\left(1+r_{t+2}^s\right)\left(1+r_{t+3}^s\right)\left(1+r_{t+1}^s\right)}
\end{aligned}
$$

or more generally:

$$
\frac{P_t^S}{P_t} = \sum_{s=t+1}^{\infty} \left(\frac{d_s}{\prod_{r=t+1}^{s} \left(1+r_r^s\right)} \right) + \lim_{n\to\infty} \frac{P_{t+n}^S}{P_{t+n}} \frac{1}{\prod_{r=t+1}^{n} \left(1+r_r^s\right)}
$$

where the limit term tends to zero. Hence we can define the stochastic discount factor as :

$$
\mathcal{M}_{is} \equiv \left(\frac{1+r_t^s}{\prod_{r=t}^{s} \left(1+r_r^s\right)} \right) = \left(\frac{\frac{1+i_t^s}{1+\pi_t}}{\prod_{r=t}^{s} \left(\frac{1+i_r^s}{1+\pi_r}\right)} \right). \tag{23}
$$

A.2 Non Ricardian Households

Households who cannot transfer resources intertemporally have the following preferences:

$$
U^c\left(X_{jt}^c, H_{jt}^c\right) = \frac{1}{1-\theta}\left(C_{jt}^c - h^C C_{t-1}^c\right)^{1-\theta} - z_t^N C_t^{1-\theta} \frac{s^N}{1-\theta^N}\left(N_{jt}^c - h^N N_{t-1}^c\right)^{1-\theta^N}
$$

with $X_{jt}^c \equiv C_{jt}^c - h^C C_{t-1}^c$, $H_{jt}^c \equiv N_{jt}^c - h^N N_{t-1}^c$,and consume their disposable income in each period:

$$
\left(1+\tau^C\right) P_t C_{jt}^c = \left(1-\tau^N\right) W_t N_{jt}^c + +T_{jt}^c - tax_{jt}^c
$$

Still it is possible to define the shadow price of an extra unit of income as:

$$
\left(C_{jt}^c - h^C C_{t-1}^c\right)^{-\theta} = \tilde{\lambda}_t^c \left(1+\tau^C\right) P_t \equiv \lambda_t^c
$$

A.3 Labor Supply

We assume the presence of a labor union which sets the wage rate by maximizing:

$$
\max E_0 \sum_{t=0}^{\infty} \left(\beta z_{t-1}^C\right)^t \left[U\left(X_{jt}, H_{jt}, A_{jt-1}\right)\right]
$$

with $X_{jt} \equiv \omega^s X_{jt}^s + (1-\omega^s) X_{jt}^c$, and $H_{jt} \equiv \omega^s H_{jt}^s + (1-\omega^s) H_{jt}^c = \omega^s \left(N_{jt}^s - h^N N_{t-1}^s\right) + (1-\omega^s)\left(N_{jt}^c - h^N N_{t-1}^c\right) = \left(N_{jt} - h^N N_{t-1}\right)$ where we imposed : $N_{jt}^s = N_{jt}^c = N_{jt}$, subject to the weighted sum of the budget constraints

$$\left(1+\tau^{C}\right) P_t \left(\omega C_{jt}^{s} + \left(1-\omega^{s}\right) C_{jt}^{c}\right) + \omega^{s} \left(B_{jt} + B_{jt}^{g} + L_{jt} + P_t^{S} S_{jt}\right) = \left(1-\tau^{N}\right) W_{jt} N_{jt}$$
$$+ \omega^{s} \left[\left(1+i_{t-1}\right) B_{jt-1} + \left(1+i_{t-1}^{g}\right) B_{jt-1}^{g}\right.$$
$$\left. + \left(1+i_{t-1}^{l}\right) L_{jt-1} + \left(P_t^{S} + P_t d_t\right) S_{jt-1}\right]$$
$$+ \omega^{s} \left(T_{jt}^{s} - tax_{jt}^{s}\right) + \left(1-\omega^{s}\right) \left(T_{jt}^{c} - tax_{jt}^{c}\right)$$
$$- \frac{\gamma^{w}}{2} W_t N_t \left(\frac{W_{jt}}{W_{jt-1}} - 1 - \left(1 - sf^{w}\right)\left(\pi_{t-1}\text{-}\pi\right) - \pi^{w}\right)^{2}$$

and the the demand of differentiated labor:

$$N_{jt} = \left(\frac{W_{jt}}{W_t}\right)^{-\sigma^{n}} N_t$$

The first order condition with respect to W_{jt} reads:

$$U_H \frac{\partial H_{jt}}{\partial N_{jt}} \frac{\partial N_{jt}}{\partial W_{jt}} + \widetilde{\lambda}_t \left(1-\tau^{N}\right) \left(N_{jt} + \frac{\partial N_{jt}}{\partial W_{jt}} W_{jt}\right) - \widetilde{\lambda}_t \gamma^{w} W_t N_t \left(\frac{W_{jt}}{W_{jt-1}} - 1 - \left(1-sf^{w}\right)\left(\pi_{t-1}\text{-}\pi\right) - \pi^{w}\right) \frac{1}{W_{jt-1}}$$
$$+ E_t \left[\widetilde{\lambda}_{t+1} \left(\beta z_t^{C}\right) \gamma^{w} W_{t+1} N_{t+1} \left(\frac{W_{jt+1}}{W_{jt}} - 1 - \left(1-sf^{w}\right)\left(\pi_t\text{-}\pi\right) - \pi^{w}\right) \frac{W_{jt+1}}{W_{jt}^{2}}\right] = 0$$

$$U_H \left(-\sigma^{n} \left(\frac{W_{jt}}{W_t}\right)^{-\sigma^{n}-1} \frac{N_t}{W_t}\right) + \widetilde{\lambda}_t \left(1-\tau^{N}\right) \left(\left(\frac{W_{jt}}{W_t}\right)^{-\sigma^{n}} N_t + \left(-\sigma^{n} \left(\frac{W_{jt}}{W_t}\right)^{-\sigma^{n}-1} \frac{N_t}{W_t} W_{jt}\right)\right)$$
$$- \widetilde{\lambda}_t \gamma^{w} W_t N_t \left(\frac{W_{jt}}{W_{jt-1}} - 1 - \left(1-sf^{w}\right)\left(\pi_{t-1}\text{-}\pi\right) - \pi^{w}\right) \frac{1}{W_{jt-1}}$$
$$+ E_t \left[\widetilde{\lambda}_{t+1} \left(\beta z_t^{C}\right) \gamma^{w} W_{t+1} N_{t+1} \left(\frac{W_{jt+1}}{W_{jt}} - 1 - \left(1-sf^{w}\right)\left(\pi_t\text{-}\pi\right) - \pi^{w}\right) \frac{W_{jt+1}}{W_{jt}^{2}}\right] = 0$$

$$-U_H \left(\sigma^{n} \left(\frac{W_{jt}}{W_t}\right)^{-\sigma^{n}-1} \frac{W_{jt} N_t}{W_t}\right) + \widetilde{\lambda}_t \left(1-\tau^{N}\right) \left(1-\sigma^{n}\right) \left(\frac{W_{jt}}{W_t}\right)^{-\sigma^{n}} W_{jt} N_t$$
$$- \widetilde{\lambda}_t \gamma^{w} W_t N_t \left(\frac{W_{jt}}{W_{jt-1}} - 1 - \left(1-sf^{w}\right)\left(\pi_{t-1}\text{-}\pi\right) - \pi^{w}\right) \frac{W_{jt}}{W_{jt-1}}$$
$$+ E_t \left[\widetilde{\lambda}_{t+1} \left(\beta z_t^{C}\right) \gamma^{w} W_{t+1} N_{t+1} \left(\frac{W_{jt+1}}{W_{jt}} - 1 - \left(1-sf^{w}\right)\left(\pi_t\text{-}\pi\right) - \pi^{w}\right) \frac{W_{jt+1}}{W_{jt}}\right] = 0$$

$$-U_H\left(\sigma^n\left(\frac{W_{jt}}{W_t}\right)^{-\sigma^n}N_t\right)+\widetilde{\lambda}_t\left(1-\tau^N\right)\left(1-\sigma^n\right)\left(\frac{W_{jt}}{W_t}\right)^{-\sigma^n}W_{jt}N_t$$
$$-\widetilde{\lambda}_t\gamma^w W_t N_t\left(\frac{W_{jt}}{W_{jt-1}}-1-\left(1-sf^w\right)\left(\pi_{t-1}\text{-}\pi\right)-\pi^w\right)\frac{W_{jt}}{W_{jt-1}}$$
$$+E_t\left[\widetilde{\lambda}_{t+1}\left(\beta z_t^C\right)\gamma^w W_{t+1}N_{t+1}\left(\frac{W_{jt+1}}{W_{jt}}-1-\left(1-sf^w\right)\left(\pi_t\text{-}\pi\right)-\pi^w\right)\frac{W_{jt+1}}{W_{jt}}\right]=0$$

imposing ex post symmetry, and dividing by $\widetilde{\lambda}$and by N_t :

$$-\frac{U_H}{\widetilde{\lambda}_t}\left(\sigma^n\right)+\left(1-\tau^N\right)\left(1-\sigma^n\right)W_t-\gamma^w\left(\frac{W_t}{W_{t-1}}-1-\left(1-sf^w\right)\left(\pi_{t-1}\text{-}\pi\right)-\pi^w\right)\frac{W_t^2}{W_{t-1}}$$
$$+E_t\left[\frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t}\left(\beta z_t^C\right)\gamma^w\frac{N_{t+1}}{N_t}\left(\frac{W_{t+1}}{W_t}-1-\left(1-sf^w\right)\left(\pi_t\text{-}\pi\right)-\pi^w\right)\frac{W_{t+1}^2}{W_t}\right]=0$$

The intuition is that the nominal wage minus the costs of changing the wage today plus the expected benefit that a change today has on the (lower) costs to be faced tomorrow is equal to a markup on the marginal rate of substitution between consumption and leisure. Note that in the absence of nominal rigidities (i.e. $\gamma^w=0$) we would have:

$$\frac{U_H}{\widetilde{\lambda}_t}\frac{\sigma^n}{1-\sigma^n}=\left(1-\tau^N\right)W_t$$

Also note that we assume:

$$U_X=\left[\omega^s\left(C_{jt}^s-h^C C_{t-1}^s\right)^{-\theta}+\left(1-\omega^s\right)\left(C_{jt}^c-h^C C_{t-1}^c\right)^{-\theta}\right]=\widetilde{\lambda}_t\left(1+\tau^C\right)P_t=\lambda_t$$

so that we can rewrite:

$$-\frac{U_H}{\lambda_t}\left(\sigma^n\right)\left(1+\tau^C\right)+\left(1-\tau^N\right)\left(1-\sigma^n\right)\frac{W_t}{P_t}-\gamma^w\left(\frac{W_t}{W_{t-1}}-1-\left(1-sf^w\right)\left(\pi_{t-1}\text{-}\pi\right)-\pi^w\right)\frac{W_t}{W_{t-1}}\frac{W_t}{P_t}$$
$$+E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}\left(\beta z_t^C\right)\gamma^w\frac{N_{t+1}}{N_t}\left(\frac{W_{t+1}}{W_t}-1-\left(1-sf^w\right)\left(\pi_t\text{-}\pi\right)-\pi^w\right)\frac{W_{t+1}}{W_t}\frac{W_{t+1}}{P_t}\right]=0$$

$$-\frac{U_H}{\lambda_t}\frac{\sigma^n}{\sigma^n-1}\left(1+\tau^C\right)=\left(1-\tau^N\right)\frac{W_t}{P_t}+\frac{\gamma^w}{\sigma^n-1}\left(\frac{W_t}{W_{t-1}}-1-\left(1-sf^w\right)\left(\pi_{t-1}\text{-}\pi\right)-\pi^w\right)\frac{W_t}{W_{t-1}}\frac{W_t}{P_t}$$
$$-\frac{\gamma^w}{\sigma^n-1}E_t\left[\beta z_t^C\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}\frac{N_{t+1}}{N_t}\left(\frac{W_{t+1}}{W_t}-1-\left(1-sf^w\right)\left(\pi_t\text{-}\pi\right)-\pi^w\right)\frac{W_{t+1}}{W_t}\frac{W_{t+1}}{P_t}\right]$$

Moreover we assume

$$U_H=-z_t^N\left[\omega^s C_t^{1-\theta}s^N\left(N_{jt}^s-h^N N_{t-1}^s\right)^{-\theta^N}+\left(1-\omega^s\right)C_t^{1-\theta}s^N\left(N_{jt}^c-h^N N_{t-1}^c\right)^{-\theta^N}\right]$$

which given the assumption above becomes:

$$U_H = -z_t^N C_t^{1-\theta} s^N \left(N_{jt} - h^N N_{t-1}\right)^{-\theta^N}$$

In the code we have $U_H = z_t^N \left[\omega^s C_t^{1-\theta} s^N \left(N_{jt}^s - h^N N_{t-1}^s\right)^{-\theta^N} + (1-\omega^s) C_t^{1-\theta} s^N \left(N_{jt}^c - h^N N_{t-1}^c\right)^{-\theta^N}\right]$

Finally we assume real wage rigidity, whose extent is measured by γ^{w_r} so that:

$$\begin{aligned}\left[-\frac{U_H}{\lambda_t}\frac{\sigma^n}{\sigma^n-1}\left(1+\tau^C\right)\right]^{1-\gamma^{w_r}} \left[\left(1-\tau^N\right)\frac{W_{t-1}}{P_{t-1}}\right]^{\gamma^{w_r}} &= \left(1-\tau^N\right)\frac{W_t}{P_t} \\ &+ \frac{\gamma^w}{\sigma^n-1}\left(\frac{W_t}{W_{t-1}} - 1 - (1-sf^w)\left(\pi_{t-1}\text{-}\pi\right) - \pi^w\right)\frac{W_t}{W_{t-1}}\frac{W_t}{P_t} \\ &- \frac{\gamma^w}{\sigma^n-1}E_t\left[\beta z_t^C \frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}\frac{N_{t+1}}{N_t}\right. \\ &\left.\left(\frac{W_{t+1}}{W_t} - 1 - (1-sf^w)\left(\pi_t\text{-}\pi\right) - \pi^w\right)\frac{W_{t+1}}{W_t}\frac{W_{t+1}}{P_t}\right]\end{aligned}$$

A.4 Intermediate goods producing firms' problem

Firms maximize the stream of expected future profits subject to the intermediate good demand, the law of motion of capital the production function and adjustment costs. Formally given the stochastic discount factor, $\mathcal{M}_{it}$

$$\max_{\{P_{it},N_{it},I_{it},K_{it},CU_{it},L_{it}\}} E_0 \sum_{t=0}^{\infty} \mathcal{M}_{it} D_{it}$$

subject to:

$$\begin{aligned}D_{it} &= \left(1-\tau^K\right)\left(\frac{P_{it}}{P_t}Y_{it} - \frac{W_t}{P_t}N_{it}\right) + \tau^K \delta \frac{P_t^I}{P_t}K_{it-1} - \frac{P_t^I}{P_t}I_{it} + \frac{L_{it}}{P_t} - \frac{L_{it-1}}{P_t}\left(1+i_{t-1}^l\right) - adj_{it} \\ Y_{it} &= \left(\frac{P_{it}}{P_t}\right)^{-\sigma^Y} Y_t \\ K_{it} &= I_{it} + (1-\delta) K_{it-1} \\ Y_{it} &= \left(A_t^Y N_{it}\right)^{\alpha}\left(CU_{it} K_{it-1}^{tot}\right)^{1-\alpha}\end{aligned}$$

where $adj_{it} = adj_{it}^P + adj_{it}^N + adj_{it}^{CU} + adj_{it}^I + adj_{it}^D$ and[6],

[6] The pricing adjustment cost is multiplied by σ^Y in order to improve the identification of γ^p.

$$
\begin{aligned}
adj_{it}^{P} &= \frac{\sigma^Y \gamma^p}{2} Y_t \left(\frac{P_{it}}{P_{it-1}} - 1 - \pi \right)^2 \\
adj_{it}^{N} &= \frac{\gamma^n}{2} Y_t \left(\frac{N_{it}}{N_{it-1}} - \exp\left(g^{pop}\right) \right)^2 \\
adj_{it}^{CU} &= \frac{P_t^I}{P_t} K_{it-1}^{tot} \left(\gamma_0^u \left(CU_{it} - 1\right) + \frac{\gamma_1^u}{2} \left(CU_{it} - 1\right)^2 \right) \\
adj_{it}^{I} &= \frac{\gamma_0^i}{2} \frac{P_t^I}{P_t} K_{t-1} \left(\frac{I_{it}}{K_{t-1}} - \delta_t \right)^2 + \frac{\gamma_1^i}{2} \frac{P_t^I}{P_t} \frac{\left(I_{it} - I_{it-1} \exp\left(g^Y + g^{P^I} \right) \right)^2}{K_{t-1}} \\
adj_{it}^{D} &= \frac{\gamma^d}{2} \left(D_{it} - D \right)^2
\end{aligned}
$$

The first order conditions with respect to to P_{it}yields:

$$
\begin{aligned}
\mathcal{M}_{it} \Bigg[\lambda_{it}^f \Bigg(\left(1 - \tau^K\right) \left(1 - \sigma^Y\right) \left(\frac{P_{it}}{P_t} \right)^{-\sigma^Y} \frac{Y_t}{P_t} - \sigma^Y \gamma^p \frac{Y_t}{P_{it-1}} \left(\frac{P_{it}}{P_{it-1}} - 1 - \pi \right) \Bigg) - \mu_t^Y \left(-\sigma^Y \left(\frac{P_{it}}{P_t} \right)^{-\sigma^Y - 1} \frac{Y_t}{P_t} \right) \Bigg] \\
- E_t \left[\mathcal{M}_{it+1} \lambda_{it+1}^f \sigma^Y \gamma^p \left(- \frac{P_{it+1} Y_{t+1}}{P_{it}^2} \right) \left(\frac{P_{it+1}}{P_{it}} - 1 - \pi \right) \right] = 0
\end{aligned}
$$

$$
\begin{aligned}
\lambda_{it}^f \Bigg(\left(1 - \tau^K\right) \left(1 - \sigma^Y\right) \left(\frac{P_{it}}{P_t} \right)^{-\sigma^Y} - \sigma^Y \gamma^p \frac{Y_t}{P_{it-1}} \frac{P_t}{Y_t} \left(\frac{P_{it}}{P_{it-1}} - 1 - \pi \right) \Bigg) - \mu_t^Y \left(-\sigma^Y \left(\frac{P_{it}}{P_t} \right)^{-\sigma^Y - 1} \right) \\
- E_t \left[\frac{\mathcal{M}_{it+1}}{\mathcal{M}_{it}} \frac{P_t}{Y_t} \lambda_{it+1}^f \sigma^Y \gamma^p \left(- \frac{P_{it+1} Y_{t+1}}{P_{it}^2} \right) \left(\frac{P_{it+1}}{P_{it}} - 1 - \pi \right) \right] = 0
\end{aligned}
$$

$$
\begin{aligned}
\left[-\mu_t^Y \left(-\sigma^Y \left(\frac{P_{it}}{P_t} \right)^{-\sigma^Y - 1} \right) \right] = \lambda_{it}^f \Bigg(\left(1 - \tau^K\right) \left(\sigma^Y - 1\right) \left(\frac{P_{it}}{P_t} \right)^{-\sigma^Y} + \sigma^Y \gamma^p \frac{Y_t}{P_{it-1}} \frac{P_t}{Y_t} \left(\frac{P_{it}}{P_{it-1}} - 1 - \pi \right) \Bigg) \\
+ E_t \left[\frac{\mathcal{M}_{it+1}}{\mathcal{M}_{it}} \lambda_{it+1}^f \sigma^Y \gamma^p \frac{P_t}{Y_t} \left(- \frac{P_{it+1} Y_{t+1}}{P_{it}^2} \right) \left(\frac{P_{it+1}}{P_{it}} - 1 - \pi \right) \right]
\end{aligned}
$$

$$
\begin{aligned}
\mu_t^Y \sigma^Y = \lambda_{it}^f \left(\left(1 - \tau^K\right) \left(\sigma^Y - 1\right) + \sigma^Y \gamma^p \frac{P_t}{P_{t-1}} \left(\frac{P_t}{P_{t-1}} - 1 - \pi \right) \right) \\
- \sigma^Y \gamma^p E_t \left[\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t} \lambda_{t+1}^f \frac{Y_{t+1}}{Y_t} \left(\frac{P_{t+1}}{P_t} \right) \left(\frac{P_{t+1}}{P_t} - 1 - \pi \right) \right] \qquad (24)
\end{aligned}
$$

The first order conditions with respect to to N_{it}yields:

$$
\begin{aligned}
\mathcal{M}_{it} \left[\lambda_{it}^f \left(- \left(1 - \tau^K\right) \frac{W_t}{P_t} - \gamma^n \frac{Y_t}{N_{it-1}} \left(\frac{N_{it}}{N_{it-1}} - \exp\left(g^{pop}\right) \right) \right) + \alpha \mu_t^Y \frac{Y_{it}}{N_{it}} \right] \\
- E_t \left[\mathcal{M}_{it+1} \lambda_{it+1}^f \gamma^n \left(- \frac{N_{it+1} Y_{t+1}}{N_{it}^2} \right) \left(\frac{N_{it+1}}{N_{it}} - \exp\left(g^{pop}\right) \right) \right] = 0
\end{aligned}
$$

$$\alpha\frac{\mu_t^Y}{\lambda_{it}^f}\frac{Y_t}{N_t}-\gamma^n\frac{Y_t}{N_{t-1}}\left(\frac{N_t}{N_{t-1}}-\exp\left(g^{pop}\right)\right)+\gamma^n E_t\left[\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t}\frac{\lambda_{t+1}^f}{\lambda_t^f}\frac{Y_{t+1}}{N_t}\frac{N_{t+1}}{N_t}\left(\frac{N_{t+1}}{N_t}-\exp\left(g^{pop}\right)\right)\right]=\left(1-\tau^K\right)\frac{W_t}{P_t} \tag{25}$$

The first order condition with respect to I_{it} reads:

$$\mathcal{M}_{it}\left[\lambda_{it}^f\left(-\frac{P_t^I}{P_t}-\gamma_0^i\frac{P_t^I}{P_t}\frac{K_{t-1}}{K_{t-1}}\left(\frac{I_{it}}{K_{t-1}}-\delta_t\right)-\gamma_1^i\frac{P_t^I}{P_t}\frac{\left(I_{it}-I_{it-1}\exp\left(g^Y+g^{P^I}\right)\right)}{K_{t-1}}\right)+\mu_{it}^k\right]$$
$$-E_t\left[-\mathcal{M}_{it+1}\lambda_{it+1}^f\frac{P_{t+1}^I}{P_{t+1}}\gamma_1^i\frac{\left(I_{it+1}-I_{it}\exp\left(g^Y+g^{P^I}\right)\right)}{K_t}\exp\left(g^Y+g^{P^I}\right)\right]=0$$

$$\mu_{it}^k=\lambda_{it}^f\left(\frac{P_t^I}{P_t}+\gamma_0^i\frac{P_t^I}{P_t}\frac{K_{t-1}}{K_{t-1}}\left(\frac{I_{it}}{K_{t-1}}-\delta_t\right)+\gamma_1^i\frac{P_t^I}{P_t}\frac{\left(I_{it}-I_{it-1}\exp\left(g^Y+g^{P^I}\right)\right)}{K_{t-1}}\right)$$
$$-E_t\left[\frac{\mathcal{M}_{it+1}}{\mathcal{M}_{it}}\lambda_{it+1}^f\frac{P_{t+1}^I}{P_{t+1}}\gamma_1^i\frac{\left(I_{it+1}-I_{it}\exp\left(g^Y+g^{P^I}\right)\right)}{K_t}\exp\left(g^Y+g^{P^I}\right)\right]$$

$$Q_t=\lambda_{it}^f\left(1+\gamma_0^i\left(\frac{I_t}{K_{t-1}}-\delta_t\right)+\gamma_1^i\frac{\left(I_t-I_{t-1}\exp\left(g^Y+g^{P^I}\right)\right)}{K_{t-1}}\right)$$
$$-E_t\left[\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t}\lambda_{t+1}^f\frac{P_{t+1}^I}{P_t^I}\frac{P_t}{P_{t+1}}\gamma_1^i\frac{\left(I_{t+1}-I_t\exp\left(g^Y+g^{P^I}\right)\right)}{K_t}\exp\left(g^Y+g^{P^I}\right)\right] \tag{26}$$

where $Q_t\equiv\frac{\mu_{it}^k}{P_t^I/P_t}$is Tobin's marginal Q.
The first order condition with respect to K_{it}solves:

$$\mathcal{M}_{it}\left[-\mu_{it}^k\right]+E_t\left[\mathcal{M}_{it+1}\lambda_{it+1}^f\frac{P_{t+1}^I}{P_{t+1}}\left(\tau^K\delta\right.\right.$$
$$\left.\left.-\frac{\partial K_{it}^{tot}}{\partial K_{it}}\left(\gamma_0^u\left(CU_{it+1}-1\right)+\frac{\gamma_1^u}{2}\left(CU_{it+1}-1\right)^2\right)\right)+\mathcal{M}_{it+1}\left(\left(1-\delta\right)\mu_{it+1}^k+\left(1-\alpha\right)\mu_{t+1}^Y\frac{Y_{it+1}}{K_{it}^{tot}}\right)\right]=0$$

$$\mu_{it}^k=E_t\left[\frac{\mathcal{M}_{it+1}}{\mathcal{M}_{it}}\left(\lambda_{it+1}^f\frac{P_{t+1}^I}{P_{t+1}}\left(\tau^K\delta-\gamma_0^u\left(CU_{it+1}-1\right)-\frac{\gamma_1^u}{2}\left(CU_{it+1}-1\right)^2\right)\right.\right.$$
$$\left.\left.+\left(1-\delta\right)\mu_{it+1}^k+\left(1-\alpha\right)\mu_{t+1}^Y\frac{Y_{it+1}}{K_{it}^{tot}}\right)\right]$$

$$\mu_{it}^{k} = E_t\left[\frac{\mathcal{M}_{it+1}}{\mathcal{M}_{it}}\frac{P_{t+1}^{I}}{P_{t+1}}\left(\lambda_{it+1}^{f}\left(\tau^{K}\delta-\gamma_0^{u}\left(CU_{it+1}-1\right)-\frac{\gamma_1^{u}}{2}\left(CU_{it+1}-1\right)^2\right)\right.\right.$$
$$\left.\left.+\left(1-\delta\right)\frac{\mu_{it+1}^{k}}{\frac{P_{t+1}^{I}}{P_{t+1}}}+\left(1-\alpha\right)\mu_{t+1}^{Y}\frac{P_{t+1}}{P_{t+1}^{I}}\frac{Y_{it+1}}{K_{it}^{tot}}\right)\right]$$

$$\frac{\mu_{it}^{k}}{\frac{P_t^{I}}{P_t}} = E_t\left[\frac{\mathcal{M}_{it+1}}{\mathcal{M}_{it}}\frac{P_{t+1}^{I}}{P_{t+1}}\frac{P_t}{P_t^{I}}\left(\lambda_{it+1}^{f}\left(\tau^{K}\delta-\gamma_0^{u}\left(CU_{it+1}-1\right)-\frac{\gamma_1^{u}}{2}\left(CU_{it+1}-1\right)^2\right)\right.\right.$$
$$\left.\left.+\left(1-\delta\right)\frac{\mu_{it+1}^{k}}{\frac{P_{t+1}^{I}}{P_{t+1}}}+\left(1-\alpha\right)\mu_{t+1}^{Y}\frac{P_{t+1}}{P_{t+1}^{I}}\frac{Y_{it+1}}{K_{it}^{tot}}\right)\right]$$

$$Q_t = E_t\left[\frac{\mathcal{M}_{it+1}}{\mathcal{M}_{it}}\frac{P_{t+1}^{I}}{P_{t+1}}\frac{P_t}{P_t^{I}}\left(\lambda_{it+1}^{f}\left(\tau^{K}\delta-\gamma_0^{u}\left(CU_{it+1}-1\right)-\frac{\gamma_1^{u}}{2}\left(CU_{it+1}-1\right)^2\right)\right.\right.$$
$$\left.\left.+\left(1-\delta\right)Q_{t+1}+\left(1-\alpha\right)\mu_{t+1}^{Y}\frac{P_{t+1}}{P_{t+1}^{I}}\frac{Y_{it+1}}{K_{it}^{tot}}\right)\right] \tag{27}$$

The first order condition with respect to CU_{it} yields:

$$\mathcal{M}_{it}\left[-\lambda_{it}^{f}\frac{P_t^{I}}{P_t}K_{it-1}^{tot}\left(\gamma_0^{u}+\gamma_1^{u}\left(CU_{it}-1\right)\right)+\mu_t^{Y}\left(1-\alpha\right)\frac{Y_{it}}{CU_{it}}\right]=0$$

$$\mu_t^{Y}\left(1-\alpha\right)\frac{Y_t}{CU_t} = \lambda_{it}^{f}\frac{P_t^{I}}{P_t}K_{t-1}^{tot}\left(\gamma_0^{u}+\gamma_1^{u}\left(CU_t-1\right)\right) \tag{28}$$

The first order condition with respect toL_{it} reads:

$$\mathcal{M}_{it}\lambda_{it}^{f}\frac{1}{P_t}-E_t\left[\mathcal{M}_{it+1}\lambda_{it+1}^{f}\frac{1}{P_{t+1}}\left(1+i_t^{l}\right)\right]=0$$

$$E_t\left[\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t}\frac{\lambda_{t+1}^{f}}{\lambda_t^{f}}\frac{P_t}{P_{t+1}}\left(1+i_t^{l}\right)\right]=1 \tag{29}$$

The first order condition with respect to D_{it}solves:

$$\mathcal{M}_{it}+\mathcal{M}_{it}\lambda_{it}^{f}\left(-1-\gamma^{d}\left(D_{it}-D\right)\right)=0$$

$$\lambda_t^{f} = \frac{1}{\left(1+\gamma^{d}\left(D_t-D\right)\right)} \tag{30}$$

A.5 Intermediate goods producing firms' problem (with borrowing constraints)

Firms maximize the stream of expected future profits subject to the intermediate good demand, the law of motion of capital the production function and adjustment costs. Formally given the stochastic discount factor, $\mathcal{M}_{it}$

$$\max_{\{P_{it},N_{it},I_{it},K_{it},CU_{it},L_{it}\}} E_0 \sum_{t=0}^{\infty} \mathcal{M}_{it} D_{it}$$

subject to:

$$\begin{aligned}
D_{it} &= \left(1-\tau^K\right)\left(\frac{P_{it}}{P_t}Y_{it} - \frac{W_t}{P_t}N_{it}\right) + \tau^K \delta \frac{P_t^I}{P_t} K_{it-1} - \frac{P_t^I}{P_t} I_{it} + \frac{L_{it}}{P_t} - \frac{L_{it-1}}{P_t}\left(1+i_{t-1}^l\right) - adj_{it} \\
Y_{it} &= \left(\frac{P_{it}}{P_t}\right)^{-\sigma^Y} Y_t \\
K_{it} &= I_{it} + (1-\delta) K_{it-1} \\
Y_{it} &= \left(A_t^Y N_{it}\right)^{\alpha}\left(CU_{it} K_{it-1}^{tot}\right)^{1-\alpha} \\
L_{jt}. &\leq m_t^l \left(P_t^I K_{it-1}\right)
\end{aligned}$$

where $adj_{it} = adj_{it}^P + adj_{it}^N + adj_{it}^{CU} + adj_{it}^I + adj_{it}^D$ and[7],

$$\begin{aligned}
adj_{it}^P &= \frac{\sigma^Y \gamma^p}{2} Y_t \left(\frac{P_{it}}{P_{it-1}} - 1 - \pi\right)^2 \\
adj_{it}^N &= \frac{\gamma^n}{2} Y_t \left(\frac{N_{it}}{N_{it-1}} - \exp\left(g^{pop}\right)\right)^2 \\
adj_{it}^{CU} &= \frac{P_t^I}{P_t} K_{it-1}^{tot}\left(\gamma_0^u \left(CU_{it}-1\right) + \frac{\gamma_1^u}{2}\left(CU_{it}-1\right)^2\right) \\
adj_{it}^I &= \frac{\gamma_0^i}{2}\frac{P_t^I}{P_t} K_{t-1}\left(\frac{I_{it}}{K_{t-1}} - \delta_t\right)^2 + \frac{\gamma_1^i}{2}\frac{P_t^I}{P_t}\frac{\left(I_{it} - I_{it-1}\exp\left(g^Y + g^{P^I}\right)\right)^2}{K_{t-1}} \\
adj_{it}^D &= \frac{\gamma^d}{2}\left(D_{it} - D\right)^2
\end{aligned}$$

The first order conditions which change with respect to the benchmark setting are: the first order condition with respect to K_{it}:

$$\begin{aligned}
&\mathcal{M}_{it}\left[-\lambda_{it}^f \mu_{it}^k\right] + E_t\Bigg[\mathcal{M}_{it+1}\lambda_{it+1}^f \frac{P_{t+1}^I}{P_{t+1}}\Bigg(\tau^K \delta \\
&-\frac{\partial K_{it}^{tot}}{\partial K_{it}}\left(\gamma_0^u\left(CU_{it+1}-1\right) + \frac{\gamma_1^u}{2}\left(CU_{it+1}-1\right)^2\right)\Bigg) + \mathcal{M}_{it+1}\left((1-\delta)\lambda_{it+1}^f \mu_{it+1}^k + (1-\alpha)\lambda_{it+1}^f \mu_{t+1}^Y \frac{Y_{it+1}}{K_{it}^{tot}}\right) \\
&+\mathcal{M}_{it+1}\left(\lambda_{it+1}^f \mu_{it+1}^l P_{t+1}^I m_{t+1}^l\right)\Bigg] = 0
\end{aligned}$$

[7]The pricing adjustment cost is multiplied by σ^Y in order to improve the identification of γ^p.

$$\mu_{it}^{k} = E_t\left[\frac{\mathcal{M}_{it+1}}{\mathcal{M}_{it}}\frac{\lambda_{it+1}^{f}}{\lambda_{it}^{f}}\left(\frac{P_{t+1}^{I}}{P_{t+1}}\left(\tau^{K}\delta-\gamma_0^{u}\left(CU_{it+1}-1\right)-\frac{\gamma_1^{u}}{2}\left(CU_{it+1}-1\right)^2\right)\right.\right.$$
$$\left.\left.+\left(1-\delta\right)\mu_{it+1}^{k}+\left(1-\alpha\right)\mu_{t+1}^{Y}\frac{Y_{it+1}}{K_{it}^{tot}}+\mu_{it+1}^{l}P_{t+1}^{I}m_{t+1}^{l}\right)\right]$$

$$\mu_{it}^{k} = E_t\left[\frac{\mathcal{M}_{it+1}}{\mathcal{M}_{it}}\frac{\lambda_{it+1}^{f}}{\lambda_{it}^{f}}\frac{P_{t+1}^{I}}{P_{t+1}}\left(\tau^{K}\delta-\gamma_0^{u}\left(CU_{it+1}-1\right)-\frac{\gamma_1^{u}}{2}\left(CU_{it+1}-1\right)^2\right.\right.$$
$$\left.\left.+\left(1-\delta\right)\frac{\mu_{it+1}^{k}}{\frac{P_{t+1}^{I}}{P_{t+1}}}+\left(1-\alpha\right)\mu_{t+1}^{Y}\frac{P_{t+1}}{P_{t+1}^{I}}\frac{Y_{it+1}}{K_{it}^{tot}}+\mu_{it+1}^{l}P_{t+1}^{I}\frac{P_{t+1}}{P_{t+1}^{I}}m_{t+1}^{l}\right)\right]$$

$$\frac{\mu_{it}^{k}}{\frac{P_t^{I}}{P_t}} = E_t\left[\frac{\mathcal{M}_{it+1}}{\mathcal{M}_{it}}\frac{\lambda_{it+1}^{f}}{\lambda_{it}^{f}}\frac{P_{t+1}^{I}}{P_{t+1}}\frac{P_t}{P_t^{I}}\left(\tau^{K}\delta-\gamma_0^{u}\left(CU_{it+1}-1\right)-\frac{\gamma_1^{u}}{2}\left(CU_{it+1}-1\right)^2\right.\right.$$
$$\left.\left.+\left(1-\delta\right)\frac{\mu_{it+1}^{k}}{\frac{P_{t+1}^{I}}{P_{t+1}}}+\left(1-\alpha\right)\mu_{t+1}^{Y}\frac{P_{t+1}}{P_{t+1}^{I}}\frac{Y_{it+1}}{K_{it}^{tot}}+\mu_{it+1}^{l}P_{t+1}m_{t+1}^{l}\right)\right]$$

$$Q_t = E_t\left[\frac{\mathcal{M}_{it+1}}{\mathcal{M}_{it}}\frac{\lambda_{it+1}^{f}}{\lambda_{it}^{f}}\frac{P_{t+1}^{I}}{P_{t+1}}\frac{P_t}{P_t^{I}}\left(\tau^{K}\delta-\gamma_0^{u}\left(CU_{it+1}-1\right)-\frac{\gamma_1^{u}}{2}\left(CU_{it+1}-1\right)^2\right.\right.$$
$$\left.\left.+\left(1-\delta\right)Q_{t+1}+\left(1-\alpha\right)\mu_{t+1}^{Y}\frac{P_{t+1}}{P_{t+1}^{I}}\frac{Y_{it+1}}{K_{it}^{tot}}+\mu_{it+1}^{l}P_{t+1}m_{t+1}^{l}\right)\right] \quad (31)$$

And the first order condition with respect toL_{it} which becomes:

$$\mathcal{M}_{it}\lambda_{it}^{f}\frac{1}{P_t}-\mathcal{M}_{it}\lambda_{it}^{f}\mu_{it}^{l}-E_t\left[\mathcal{M}_{it+1}\lambda_{it+1}^{f}\frac{1}{P_{t+1}}\left(1+i_t^{l}\right)\right]=0$$

$$E_t\left[\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t}\frac{\lambda_{t+1}^{f}}{\lambda_t^{f}}\frac{P_t}{P_{t+1}}\frac{1}{1-\mu_{it}^{l}P_t}\left(1+i_t^{l}\right)\right]=1 \quad (32)$$

In case we write the borrowing constraints as:

$$L_{jt}+P_t^{S}S_{jt}=m_t^{tot}\left(P_{t+1}^{I}K_{it}\right)$$
$$L_{jt}.\leq m_t^{l}\left(P_{t+1}^{I}K_{it}\right)$$

the first order condition with respect to K_{it} would read:

$$\mathcal{M}_{it}\lambda_{it}^{f}\left[-\mu_{it}^{k}+\mu_{it}^{tot}m_{t}^{tot}P_{t+1}^{I}+\mu_{it}^{l}m_{t}^{l}P_{t+1}^{I}\right]+E_{t}\Bigg[\mathcal{M}_{it+1}\lambda_{it+1}^{f}\frac{P_{t+1}^{I}}{P_{t+1}}\Big(\tau^{K}\delta$$

$$-\frac{\partial K_{it}^{tot}}{\partial K_{it}}\left(\gamma_{0}^{u}\left(CU_{it+1}-1\right)+\frac{\gamma_{1}^{u}}{2}\left(CU_{it+1}-1\right)^{2}\right)\Big)+\mathcal{M}_{it+1}\left(\left(1-\delta\right)\lambda_{it+1}^{f}\mu_{it+1}^{k}+\left(1-\alpha\right)\lambda_{it+1}^{f}\mu_{t+1}^{Y}\frac{Y_{it+1}}{K_{it}^{tot}}\right)\Bigg]=0$$

$$\left[\mu_{it}^{k}\frac{P_{t}}{P_{t}^{I}}-\mu_{it}^{tot}m_{t}^{tot}P_{t+1}^{I}\frac{P_{t}}{P_{t}^{I}}-\mu_{it}^{l}m_{t}^{l}P_{t+1}^{I}\frac{P_{t}}{P_{t}^{I}}\right]=E_{t}\Bigg[\frac{\mathcal{M}_{it+1}}{\mathcal{M}_{it}}\frac{\lambda_{it+1}^{f}}{\lambda_{it}^{f}}\frac{P_{t+1}^{I}}{P_{t+1}}\Big(\tau^{K}\delta-\gamma_{0}^{u}\left(CU_{it+1}-1\right)-\frac{\gamma_{1}^{u}}{2}\left(CU_{it+1}-1\right)^{2}$$

$$+\left(1-\delta\right)\frac{\mu_{it+1}^{k}}{\frac{P_{t+1}^{I}}{P_{t+1}}}+\left(1-\alpha\right)\mu_{t+1}^{Y}\frac{P_{t+1}}{P_{t+1}^{I}}\frac{Y_{it+1}}{K_{it}^{tot}}\Big)\Bigg]$$