Discussion Paper

(Un)expected monetary policy shocks and term premia

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Non-technical summary

Research Question

What are the effects of monetary policy on the term structure of interest rates? While a standard question, the empirical literature yet has to reach a definitive conclusion, both quantitatively and qualitatively. We use a quantitative structural model that captures the nonlinearity behind the risk underlying financial variables to answer this question.

Contribution

This paper presents an estimated stochastic dynamic general equilibrium model (DSGE) that replicates key macroeconomic and financial stylized facts during the Great Moderation of 1983-2007. In contrast to standard structural modelling approaches – like the linear New Keynesian models commonly used in policy analysis – our model captures the impact of monetary policy on interest rates beyond the expectation hypothesis.

Results

We find that different monetary policy actions can have substantially different effects on risk premia. First, unexpected transitory changes of the policy rate have limited effects on nominal and real term premia. Second, expected monetary policy shocks, such as unconditional forward guidance, affect households' expectations regarding furture real and nominal variables substantially. This has significant effects on households' precautionary savings motives and, consequentially, on risk premia in the economy. Similarly, shocks to the inflation target have persistent effects on the systematic behavior of monetary policy, generating strong effects in risk premia. By differentiating between these different monetary policy actions, our structural model rationalizes many of the opposing findings on the effects of monetary policy on term premia in the empirical literature

Nichttechnische Zusammenfassung

Fragestellung

Was sind die Auswirkungen von Geldpolitik auf die Zinsstrukturkurve? Obwohl dies eine grundlegende Frage ist, hat die empirische Literatur noch keine abschließende Antwort gefunden. Um diese Frage zu beantworten, nutzen wir in dieser Studie daher ein quantitativ-strukturelles Modell, welches die Risikoprämien - und die damit einhergehenden Nichtlinearitäten - von Finanzvariablen berücksichtigt.

Beitrag

Die Studie präsentiert ein geschätztes dynamisch-stochastisches Gleichgewichtsmodell (DSGE). Dabei repliziert das Model sowohl zentrale makroökonomisches Fakten als auch bekannte Charakteristika von Finanzmärkten während der Great Moderation (1983-2007). Im Gegensatz zu gängigen strukturellen Modellen, wie das lineare neu-keynesianische Modell, welches häufig in der Politikanalyse genutzt wird, berücksichtigt das vorliegende Modell explizit die Effekte von Geldpolitik auf langfristige Zinsen über die Erwartungshypothese hinaus.

Ergebnisse

Wir finden das Geldpolitik substantiell verschiedene Effekte auf Risikoprämien haben kann. Erstens, ein unerwartete vorübergehende Änderung des Leitzinses hat nur begrenzt Einfluss auf die nominalen und realen Laufzeitprämien von Anleihen. Zweitens, erwartete geldpolitische Änderungen (z.B. forward guidance) beeinflussen die zukünftigen Erwartungen der Haushalte und haben daher einen großen Einfluss auf nominale und reale Variablen. Dies hat folglich starke Effekte auf das Vorsorgesparen der Haushalte und somit letztendlich auch auf die Risikoprämien. Ähnlich starke Auswirkungen auf Risikoprämien haben Änderungen des Inflationsziels auf Grund ihres persistenten Effektes auf die systematische Geldpolitik. Durch die Unterscheidung verschiedener geldpolitischer Instrumente ist es möglich viele widersprüchliche Ergebnisse der empirischen Literatur in Einklang zu bringen.

(Un)expected Monetary Policy Shocks and Term Premia[*]

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Abstract

We analyze an estimated stochastic general equilibrium model that replicates key macroeconomic and financial stylized facts during the Great Moderation of 1983-2007. Our model predicts a sizeable and volatile nominal term premium - comparable to recent reduced-form empirical estimates - with real risk two times more important than inflation risk. The model enables us to address salient questions about the effects of monetary policy on the term structure of interest rates. We find that monetary policy can have sizeable and differing effects on nominal and real risk premia, rationalizing many opposing findings in the empirical literature.

Keywords: DSGE model, Bayesian estimation, Term structure, Monetary policy

JEL classification: E13, E31, E43, E44, E52

1 Introduction

What are the effects of monetary policy on the term structure of interest rates? The empirical literature has yet to reach a definitive conclusion on this question, not only quantitatively but also qualitatively. We contribute to this discussion with an estimated stochastic dynamic general equilibrium model (DSGE) which replicates key macroeconomic and financial stylized facts during the Great Moderation of 1983-2007. In contrast to standard structural modelling approaches – like the linear New Keynesian models commonly used in policy analysis – our model captures the impact of monetary policy on interest rates beyond the expectation hypothesis and, therefore, is well positioned to answer our introductory question. We show that different monetary policy actions can have substantially different effects on risk premia. First, unexpected transitory changes of the policy rate have limited effects on nominal and real term premia. Second, expected monetary policy shocks, such as unconditional forward guidance, affect households' future expectation regarding real and nominal variables substantially. This has significant effects on households' precautionary savings motives and, consequentially, on risk premia in the economy. Similarly, shocks to the inflation target have persistent effects on the systematic behavior of monetary policy, generating strong effects in risk premia. By distinguishing between these different monetary policy actions, our structural model rationalizes many of the opposing findings on the effects of monetary policy on term premia in the empirical literature (see, for example, Hanson and Stein, 2015; Nakamura and Steinsson, 2017).

A comprehensive analysis of monetary policy needs a quantitative structural model that captures the nonlinearity behind the risk of underlying financial variables and simultaneously replicates key stylized macroeconomic facts. However, as poignantly phrased by Gürkaynak and Wright (2012, p. 354): "A general problem with a structural model [. . .] is that it is challenging to maintain computational tractability and yet obtain time-variation in term premia." We address this problem and estimate a New Keynesian macro-finance model with U.S. data from 1983:Q1 to 2007:Q4 using a new and computationally efficient procedure that captures both constant and time varying risk premia by maintaining linearity in states and shocks (Meyer-Gohde, 2016). This approach allows us to investigate a structural model in the spirit of Smets and Wouters (2003, 2007) and Christiano, Eichenbaum, and Evans (2005), and is able to provide not only an in-depth analysis of the macroeconomy but also of the term structure of interest rates and their interactions. Figure 1 shows that our structural model predicts a historical 10-year term premium comparable in level, pattern, and volatility with recent reduced-form empirical estimates.[1] Our model predicts both an upward sloping nominal yield curve in line with the data and an upward sloping real yield curve in line with empirical estimates (see, for example, Gürkaynak, Sack, and Wright, 2010; Chernov and Mueller, 2012). Our real yield curve is in contrast to many DSGE models (see, for example, van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez, 2012; Swanson, 2016) that generally attribute a

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[1]The gray area in Figure 1 presents the range (maximum and minimum) of the estimates for the 10-year term premium from models developed by Kim and Wright (2005), Rudebusch and Wu (2008), Bernanke, Reinhart, and Sack (2004), Adrian, Crump, and Moench (2013), and Bauer (2016). The first three measures were calculated by Rudebusch, Sack, and Swanson (2007) and Rudebusch, Swanson, and Wu (2006). A description of the estimates can be found there. We are very thankful to Eric T. Swanson and Michael Bauer for sharing their estimates with us.
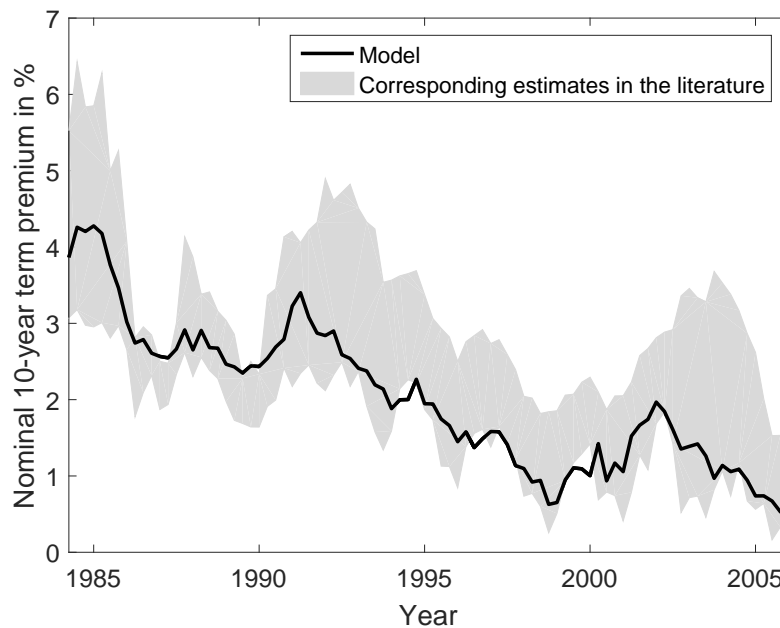
Figure 1: Model implied 10-year nominal term premium (black line) and range of corresponding estimates in the literature (gray area).

stronger insurance-like character to real bonds leading to flat or downward sloping real yield curves. Additionally, our results suggest that 2/3 of the average slope of the nominal term structure is related to real rather than to inflation risk. In this regard, the model implied upward sloping inflation risk premium is consistent with recent estimates in the literature (see, for example, Abrahams, Adrian, Crump, Moench, and Yu, 2016), with our average term structure of inflation risk comfortably between the estimates of Buraschi and Jiltsov (2005) and Chen, Liu, and Cheng (2010). In summary, our model-implied estimates demonstrate a considerable alignment with various empirical estimates in the literature. This alignment is all the more remarkable as these measures, with the exception of nominal yields, were not used in our estimation. This provides us with a high degree of confidence in our model as we proceed to the structural analysis of the effects of monetary policy on the term structure of interest rates and its components.

Our structural analysis contributes to the growing body of empirical investigations into the effects of conventional and, more recently unconventional, monetary policy on the term structure of interest rates.[2] So far the empirical literature disagrees not only on the quantitative effects of monetary policy shocks on term premia (see Hanson and Stein, 2015; Nakamura and Steinsson, 2017), but also on their qualitative effects - i.e., whether interest rates and term premia comove (see Abrahams et al., 2016; Crump et al., 2016). There are many potential reasons for this lack of robustness and an analysis of them is beyond the scope of this paper.[3] Instead, we take our cue from Ramey (2016)

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[2]See for example the pioneering work by Kuttner (2001), Cochrane and Piazzesi (2002), and Gürkaynak, Sack, and Swanson (2005a,b). More recent papers that also place a focus on unconventional monetary policy are, for example, Nakamura and Steinsson (2017), Gertler and Karadi (2015), Gilchrist, López-Salido, and Zakrajšek (2015), Abrahams et al. (2016), and Crump, Eusepi, and Moench (2016).

[3]For example, different underlying samples or identification approaches could be to blame. See, for example, Campbell, Fisher, Justiniano, and Melosi (2016) for a discussion of potential shortcomings in

who notes that the "shocks" identified in the empirical literature are not always the empirical counterparts of shocks from theoretical models. For example, with monetary policy following a Taylor-type rule, we want to disentangle changes in the systematic behavior of monetary policy - due, for example, to changes in the inflation target - from innovations to the Taylor rule and from preannounced monetary actions like forward guidance (see, for example, Woodford, 2012).

We find that an unexpected monetary policy shock via a simple innovation to the Taylor rule affects risk premia at shorter more strongly than longer maturities (see Nakamura and Steinsson, 2017, for a comparable emipirical finding), but overall has limited effects on the term premia at all maturities. This finding is in line with those of other structural models (see, for example, Rudebusch and Swanson, 2012) and confirms some of the empirical findings of Nakamura and Steinsson (2017). Simply put, an uncorrelated innovation to the Taylor rule dies out too quickly to have substantial effects at business cycle frequencies. Therefore, the effects on risk premia, which vary primarily at lower frequencies (see, for example, Piazzesi and Swanson, 2008), are limited. In contrast, a shock to the inflation target has much stronger effects on the term structure of interest rates across all maturities. The reason behind the strong effect on the risk premia, as laid out by Rudebusch and Swanson (2012), is that a change to the inflation target introduces long-run (nominal) risk which is per se longer lasting and so has stronger effects on households' expectation formation, their precautionary savings motives and, thus, on risk premia. The strong quantitative effects of such a monetary action are comparable to the findings of Hanson and Stein (2015). Additionally for longer maturities, the policy rate and risk premia comove on impact (see, for example, Hanson and Stein, 2015; Abrahams et al., 2016) after such a more systematic change of monetary policy. Contrarily, we find for a simple innovation to the Taylor rule that risk premia for long maturities tend to move opposite the policy rate on impact, i.e., a looser monetary policy increases risk premia. In particular in our model, such a looser monetary policy increases the precautionary savings motive of agents as they expect more volatile inflation and output and, therefore, demand higher risk premia. This finding is comparable to the empirical results of Crump et al. (2016) and Nakamura and Steinsson (2017).

Following the approach of Woodford (2012), we analyze the effects of unconditional forward guidance.[4] This is accomplished by adding a sequence of anticipated shocks to the Taylor rule to keep the policy rate upon announcement constant until the announced interest rate change (here a cut) is implemented. We find that this kind of forward guidance affects risk premia substantially, prying bond yields from the expectations hypothesis. In particular, we find that a commitment to a future reduction in the policy rate and constant policy rates until then causes real term premia and inflation risk premia to rise as agents expect more volatile inflation and output in the future. This finding is in line with the empirical finding of Akkaya et al. (2015). Turning to the inflation risk premia, its increase follows what theory would predict: While forward guidance does communicate the expected path of future short rate, it is just as informative about the central bank's

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isolating the effects of monetary policy in the recent literature.

[4]For a discussion of different forms of forward guidance see Campbell, Evans, Fisher, and Justiniano (2012) and Akkaya, Gürkaynak, Kısacıkoğlu, and Wright (2015). Particularly such a distinction is a significant challenge in many empirical approaches (see, for example, the discussion in Nakamura and Steinsson, 2017; Campbell et al., 2016).

commitment to allow higher inflation in the future. This mechanism increases households' precautionary savings motives and their demand for higher inflation risk premia.

The reminder of the paper reads as follows: Section 2 presents the model. Following, section 3 describes the solution method, the data, and the Bayesian estimation approach in greater detail. Section 4 presents the estimation results and discusses the model fit. Section 5 presents the effects of unexpected and expected monetary policy on the term structure. Section 6 concludes the paper.

2 Model

In the following section, we present our dynamic stochastic general equilibrium (DSGE) model. We study a New Keynesian model, in which households have recursive preferences following Epstein and Zin (1989, 1991) and Weil (1989), maximize their utility from consumption relative to a habit and labor, and accumulate capital. The nominal yield curve is derived from the households' stochastic discount factor and no-arbitrage restrictions. Firms are monopolistic competitors selling differentiated products at prices that are allowed to adjust in a stochastic fashion as in Calvo (1983). The central bank follows a Taylor rule which sets the short-term nominal interest rate as a function of the inflation rate and output. The model has a similar structure to Smets and Wouters (2003, 2007) and Christiano et al. (2005) by including nominal and real rigidities which have demonstrated success in replicating stylized facts of the macroeconomy. Additionally, the model incorporates real and nominal long-run risk (Bansal and Yaron, 2004; Gürkaynak et al., 2005b) which, together with recursive preferences, have been highlighted in the literature as important in order to explain many financial moments in consumption-based asset pricing models.

2.1 Firms

A perfect competitive representative firm produces the final good y_t. This final good is an aggregate of a continuum of intermediate goods $y_{j,t}$ and given by the function

$$y_t = \left(\int_0^1 y_{j,t}^{\frac{\theta_p - 1}{\theta_p}} \, dj \right)^{\frac{\theta_p}{1 - \theta_p}}$$

(1)

with $\theta_p > 1$ the intratemporal elasticity of substitution across the intermediate goods. The competitive, representative firm takes the price of output, P_t, and the price of inputs, $P_t(j)$, as given. The resulting demand function for the intermediate good is

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta_p} y_t$$

(2)

and the aggregate price level is defined as

$$P_t = \left(\int_0^1 P_{j,t}^{1 - \theta_p} dj \right)^{\frac{1}{1 - \theta_p}}$$

(3)

and gross inflation is $\pi_t = P_t/P_{t-1}$.

The intermediate good j is produced by a monopolistic competitive firm with the following Cobb-Douglas production function

$$y_{j,t} = \exp\{a_t\}k_{j,t-1}^{\alpha}\left(z_t l_{j,t}\right)^{1-\alpha} - z_t^{+}\Omega_t \tag{4}$$

where $k_{j,t}$ and $l_{j,t}$ denote capital services and the amount of labor used for production by the jth intermediate good producer, respectively. The parameter α denotes the output elasticity with respect to capital and Ω_t the fixed costs of production. The variable $\exp\{a_t\}$ refers to a stationary technology shock, where a_t is described by the following AR(1) process

$$a_t = \rho_a a_{t-1} + \sigma_a \epsilon_{a,t}, \text{ with } \epsilon_{a,t} \overset{iid}{\sim} N\left(0,1\right) \tag{5}$$

The variable z_t depicts a stochastic aggregate productivity trend. We include this non-stationary productivity shock to allow for a source of real long-run risk. As put forward by Bansal and Yaron (2004), the presence of real long-run risk is important in order to explain many financial moments in a consumption-based asset pricing model. We assume that $\exp\{\mu_{z,t}\} = z_t/z_{t-1}$ and let

$$\mu_{z,t} = (1 - \rho_z)\bar{\mu}_z + \rho_z \mu_{z,t-1} + \sigma_z \epsilon_{z,t}, \text{ with } \epsilon_{z,t} \overset{iid}{\sim} N\left(0,1\right). \tag{6}$$

Note that from the perspective of the long-run risk literature, the process a_t reflects short-run and the stochastic trend $\mu_{z,t}$ long-run risk. Indeed, combining the two stochastic processes in production, $\varpi_t \doteq \exp\{a_t\}z_t^{1-\alpha}$, the growth rate $\ln\{\varpi_t/\varpi_{t-1}\}$ recovers Croce's (2014) specification for productivity, which mirrors Bansal and Yaron's (2004) specification for consumption, when $\rho_a = 1$.[5]

The economy has two sources of growth. Alongside the stochastic trend in productivity z_t, the economy also faces a deterministic trend in the relative price of investment Υ_t with $\exp\{\bar{\mu}_{\Upsilon}\} = \Upsilon_t/\Upsilon_{t-1}$. We follow Altig, Christiano, Eichenbaum, and Linde (2011) and define $z_t^{+} = \Upsilon_t^{\frac{\alpha}{1-\alpha}}z_t$, which can be interpreted as an overall measure of technological progress in the economy. The overall trend in the economy is characterized by

$$\mu_{z^{+},t} = \frac{\alpha}{1-\alpha}\bar{\mu}_{\Upsilon} + \mu_{z,t} \tag{7}$$

Finally, we scale Ω_t by z_t^{+} to ensure the existence of a balanced growth path and let production costs be time-varying as proposed by Andreasen (2011). In our model, such variations in firms' fixed production costs represent real supply shocks by assuming that

$$\log\left(\frac{\Omega_t}{\bar{\Omega}}\right) = \rho_{\Omega}\log\left(\frac{\Omega_{t-1}}{\bar{\Omega}}\right) + \sigma_{\Omega}\epsilon_{\Omega,t}, \text{ with } \epsilon_{\Omega,t} \overset{iid}{\sim} N\left(0,1\right) \tag{8}$$

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[5]To see this, note that

$$\begin{aligned} \ln\{\varpi_t/\varpi_{t-1}\} &= (1-\alpha)\ln\{z_t/z_{t-1}\} + a_t - a_{t-1} \\ &= \underbrace{(1-\alpha)\mu_{z,t}}_{LLR} + \underbrace{(\rho_a - 1)a_{t-1} + \sigma_a\epsilon_{a,t}}_{SRR} \end{aligned}$$

where short-run risk (SSR) is white noise as in Croce (2014) and Bansal and Yaron (2004) if $\rho_a = 1$.

Following Calvo (1983), intermediate good firms are subject to staggered price setting, i.e., they are allowed to adjust their prices only with probability $(1 - \gamma_p)$ each period. If a firm cannot re-optimize, its price evolves according to the indexation rule: $P_{j,t} = P_{j,t-1}\pi_{t-1}^{\xi_p}$. When the firm is able to optimally adjust its price, the firm sets the price $\tilde{p}_t = P_{j,t}$ to maximize the value of its expected future dividend stream subject to the demand it faces and taking into account the indexation rule and the probability of not being able to readjust. The first order conditions of this maximization problem are

$$\mathcal{K}_t = y_t \tilde{p}_t^{-\theta_p} + \gamma_p E_t \left[M_{t+1} \left(\frac{\pi_t^{\xi_p}}{\pi_{t+1}} \right)^{1-\theta_p} \left(\frac{\tilde{p}_t}{\tilde{p}_{t-1}} \right)^{-\theta_p} \mathcal{K}_{t+1} \right] \tag{9}$$

and

$$\frac{\theta_p - 1}{\theta_p} \mathcal{K}_t = y_t mc_t \tilde{p}_t^{-\theta_p - 1} + \gamma_p E_t \left[M_{t+1} \left(\frac{\pi_t^{\xi_p}}{\pi_{t+1}} \right)^{-\theta_p} \left(\frac{\tilde{p}_t}{\tilde{p}_{t-1}} \right)^{-\theta_p - 1} \frac{\theta_p - 1}{\theta_p} \mathcal{K}_{t+1} \right] \tag{10}$$

which is the same for all firms that can adjust their price in period t. Moreover, the variable M_{t+1} represents the real stochastic discount factor of the representative household from period t to $t+1$ and mc_t the real marginal costs of the intermediate good firm. In sum, the aggregate price index evolves according to

$$1 = \gamma_p \left(\frac{\pi_{t-1}^{\xi_p}}{\pi_t} \right)^{1-\theta_p} + (1 - \gamma_p) \left(\tilde{p}_t \right)^{1-\theta_p} \tag{11}$$

2.2 Households

We assume that the representative household has recursive preferences as postulated by Epstein and Zin (1989, 1991) and Weil (1989). Following Rudebusch and Swanson (2012), the value function of the household can be written as

$$V_t = \begin{cases} u_t + \beta \left(E_t \left[V_{t+1}^{1-\sigma_{EZ}} \right] \right)^{\frac{1}{1-\sigma_{EZ}}} & \text{if } u_t > 0 \text{ for all } t \\ u_t - \beta \left(E_t \left[(-V_{t+1})^{1-\sigma_{EZ}} \right] \right)^{\frac{1}{1-\sigma_{EZ}}} & \text{if } u_t < 0 \text{ for all } t \end{cases} \tag{12}$$

where u_t is the household's period utility kernel and $\beta \in (0,1)$ the subjective discount factor. For $\sigma_{EZ} > 0$, these preferences allow us to disentangle the household's risk aversion from its intertemporal elasticity of the substitution (IES). For $\sigma_{EZ} = 0$, eq. (12) reduces to standard expected utility.

Similarly to Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017), the utility kernel takes the following functional form

$$u_t = \exp\{\varepsilon_{b,t}\} \left[\frac{1}{1-\gamma} \left(\left(\frac{c_t - bh_t}{z_t^+} \right)^{1-\gamma} - 1 \right) + \frac{\psi_L}{1-\chi} (1 - l_t)^{1-\chi} \right] \tag{13}$$

with consumption c_t, the predetermined stock of consumption habits h_t, hours worked l_t, and preference parameters γ, χ, and ψ_L. The habit stock is external to the household,

thus we set $h_t = C_{t-1}$, the level of aggregate consumption in the previous period. The parameter $b \in (0, 1)$ controls the degree of external habit formation. The presence of habit formation enables the model to match macroeconomic as well as asset pricing moments jointly as discussed in the literature (see, for example, Hördahl, Tristani, and Vestin, 2008; van Binsbergen et al., 2012). The variable $\exp\{\varepsilon_{b,t}\}$ represents a preference shock, where $\varepsilon_{b,t}$ evolves according to the process

$$\varepsilon_{b,t} = \rho_b \varepsilon_{b,t-1} + \sigma_b \epsilon_{b,t}, \text{ with } \epsilon_{b,t} \overset{iid}{\sim} N(0,1) \tag{14}$$

As described above, the variable z_t^+ represents the overall level of technology in the economy and, by expressing habit-adjusted consumption relative to this trend, the utility kernel ensures a balanced growth path (see, for example, An and Schorfheide, 2007).

The household's real period-by-period budget constraint reads

$$c_t + \frac{I_t}{\Upsilon_t} + b_t + T_t = w_t l_t + r_t^k k_{t-1} + \frac{b_{t-1} \exp\left\{R_{t-1}^f\right\}}{\pi_t} + \int_0^1 \Pi_t(j)\, dj \tag{15}$$

where the left-hand side represents the household's resources spent on consumption, investment I_t, a lump-sum tax T_t, and a one-period bond b_t that accrues the risk-free nominal interest R_t^f in the following period. The right-hand side of eq. (15) describes the income of the household in period t. It consists of labor income $w_t l_t$ with w_t the real wage, income from capital services sold to firms last period $r_t^k k_{t-1}$, the pay-off from bonds issued one period before b_{t-1}. Finally, the term $\Pi(j)$ represents the income from dividends of monopolistically competitive intermediate firms – indexed by j – owned by households.

The households own the economy-wide physical capital stock, which accumulates according to the following law of motion

$$k_t = (1 - \delta)k_{t-1} + \exp\{\varepsilon_{i,t}\}\left(1 - \frac{\nu}{2}\left(\frac{I_t}{I_{t-1}} - \exp\{\bar{\mu}_{z^+} + \bar{\mu}_\Upsilon\}\right)^2\right) I_t \tag{16}$$

where δ is the depreciation rate and $\nu \geq 0$ introduces investment adjustment costs as in Christiano et al. (2005). The term $\exp\{\bar{\mu}_{z^+} + \bar{\mu}_\Upsilon\}$ ensures that the investment adjustment costs are zero along the balanced growth path. Following Justiniano, Primiceri, and Tambalotti (2010), the variable $\exp\{\varepsilon_{i,t}\}$ represents an investment shock which measures the exogenous variation in the efficiency with which the final good can be transformed into physical capital and thus into tomorrow's capital input, where $\varepsilon_{i,t}$ evolves according to the process:

$$\varepsilon_{i,t} = \rho_i \varepsilon_{i,t-1} + \sigma_i \epsilon_{i,t}, \text{ with } \epsilon_{i,t} \overset{iid}{\sim} N(0,1) \tag{17}$$

2.3   Monetary Policy

We follow Rudebusch and Swanson (2008, 2012) and assume that monetary policy sets the one-period nominal interest rate R_t^f by following a Taylor-type policy rule expressed

annually

$$4R_t^f = 4 \cdot \rho_R R_{t-1}^f + (1 - \rho_R) \left(4\bar{r}^{real} + 4 \log \pi_t + \eta_y \log \left(\frac{y_t}{z_t^+ \bar{y}} \right) + \eta_\pi \log \left(\frac{\pi_t}{\pi_t^*} \right) \right) + \sigma_m \epsilon_{m,t} \tag{18}$$

where $\bar{r}^{real}$ is the real interest rate at the deterministic steady state and ρ_R, η_y, and η_π are policy parameters that characterize the systematic response of the central bank. The term $\epsilon_{m,t}$ represents a shock to the nominal interest rate which is assumed to be iid normally distributed with mean 0 and variance 1. Monetary policy aims to stabilize the inflation gap, $\log (\pi_t/\pi_t^*)$, and the output gap, $\log (y_t/z_t^+ \bar{y})$. The output gap is characterized by the deviation of actual output from its balanced growth path. The inflation gap is characterized by the deviation of inflation from the central bank's inflation target π_t^*. Rudebusch and Swanson (2012) interpret changes in the inflation target as long-run nominal (inflation) risk and show that the existence of such long-run risk is helpful in explaining the historical U.S. term premium. We follow Gürkaynak et al. (2005b) and Rudebusch and Swanson (2012) and assume that the inflation target is time-varying and is described by the following law of motion

$$\log \pi_t^* - 4 \log \bar{\pi} = \rho_\pi \left(\log \pi_{t-1}^* - 4 \log \bar{\pi} \right) + 4\zeta_\pi \left(\log \pi_{t-1} - \log \bar{\pi}, \right) + \sigma_\pi \epsilon_{\pi,t} \tag{19}$$

with $\epsilon_{\pi,t}$ representing a shock to the inflation target, assumed iid normal with mean 0 and variance 1.

2.4 Aggregation and Market Clearing

The aggregate resource constraint in the goods market is given by

$$p_t^+ y_t = \exp\{a_t\} k_{t-1}^\alpha (z_t l_t)^{1-\alpha} - z_t^+ \Omega_t \tag{20}$$

where $l_t = \int_0^1 l(j,t) \, dj$ and $k_t = \int_0^1 k(j,t) \, dj$ are the aggregate labor and capital inputs, respectively. The term $p_t^+ = \int_0^1 \left(\frac{P_{j,t}}{P_t} \right)^{-\theta_p} dj$ measures the price dispersion arising from staggered price setting. Price distortion follows the law of motion

$$p_t^+ = (1 - \gamma_p) (\tilde{p}_t)^{-\theta_p} + \gamma_p \left(\frac{\pi_{t-1}^{\xi_p}}{\pi_t} \right)^{-\theta_p} p_{t-1}^+ \tag{21}$$

Finally, the economy's aggregate resource constraint implies that

$$y_t = c_t + \frac{I_t}{\Upsilon_t} + g_t \tag{22}$$

where $g_t = \bar{g} z_t^+ \exp\{\varepsilon_{g,t}\}$ represents government consumption expenditures, which are growing with the economy and are financed by lump-sum taxes $g_t = T_t$. The variable $\exp\{\varepsilon_{g,t}\}$ represents an exogenous shock to government consumption with $\varepsilon_{g,t}$ evolving according to the following AR(1) process

$$\varepsilon_{g,t} = \rho_g \varepsilon_{g,t-1} + \sigma_g \epsilon_{g,t}, \text{ with } \epsilon_{g,t} \overset{iid}{\sim} N(0,1) \tag{23}$$

2.5 The Nominal and Real Term Structures

The derivation of the nominal and real term structure in our model is identical to the procedure described, for example, by Rudebusch and Swanson (2008, 2012) and Andreasen (2012a). Specifically, the price of any financial asset equals the sum of the stochastically discounted state-contingent payoffs of the asset in period $t + 1$ following standard no-arbitrage arguments. For example, the price of a default free n-period zero-coupon bond that pays one unit of cash at maturity satisfies

$$P_{n,t} = E_t \left[M^\$_{t,t+n} 1 \right] \tag{24}$$
$$= E_t \left[M^\$_{t,t+1} P_{n-1,t+1} \right]$$

where $M^\$_{t,t+1}$ is the household's nominal stochastic discount factor, which has the following functional form

$$M^\$_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}} (V_{t+1})^{-\sigma_{EZ}} E_t \left[V_{t+1}^{\frac{\sigma_{EZ}}{1-\sigma_{EZ}}} \right] \tag{25}$$

with λ_t the marginal utility of consumption. Additionally, the continuously compounded yield to maturity on the n-period zero-coupon nominal bond is defined as

$$\exp \left\{ -n R^\$_{n,t} \right\} = P^\$_{n,t} \tag{26}$$

Following the literature (e.g. Rudebusch and Swanson, 2012), we define the term premium on a long-term bond as the difference between the yield on the bond and the unobserved risk-neutral yield for that same bond. Similarly to eq. (24), this risk-neutral bond price, $\hat{P}_{n,t}$, which pays also one unit of cash at maturity, is defined as

$$\hat{P}_{n,t} = \exp \left\{ -R^f_t \right\} E_t \left[\hat{P}_{n-1,t+1} \right] \tag{27}$$

In contrast to eq. (24), discounting is performed using the risk-free rate (with R^f_t equal to the expression $R_{1,t}$) rather than the stochastic discount factor. Accordingly, the nominal term premium on a bond with maturity n is given by

$$TP^\$_{n,t} = \frac{1}{n} \left(\log \hat{P}_{n,t} - \log P^\$_{n,t} \right) \tag{28}$$

Similarly, we can derive the yield to maturity of a real bond $R_{n,t}$ as well a the price of risk-neutral real bond. Hence, it is straightforward to solve also for the real term premium $TP_{n,t}$ of a bond with maturity n. Finally, we follow the literature and define inflation risk premia $TP^\pi_{n,t}$ in our model as

$$TP^\pi_{n,t} = TP^\$_{n,t} - TP_{n,t} \tag{29}$$

3 Model Solution and Estimation

3.1 Solution Method

We adopt the method of Meyer-Gohde (2016) to solve the model. This approximation adjusts a linear in states approximation for risk and provides derivations for the approximation around the means of the endogenous variables approximated out to a finite moment of the underlying stochastic driving forces.[6] This allows us to use the standard set of macroeconometric tools for estimation and analysis of linear models, without limiting the approximation to the certainty-equivalent approximation around the deterministic steady state. We adjust the points and slopes of the decision rules for risk out to the second moments of the underlying stochastics to capture both constant and time-varying risk premium, as well as the effects of conditional heteroskedasticity (e.g. van Binsbergen et al., 2012). Unlike standard higher order polynomial perturbations[7] or affine approximation methods,[8] this linear in states approximation gives us significant computational advantages for iterative calculations such as the Metropolis-Hastings algorithm we will use to sample from the posterior distribution of the parameters while maintaining the endogenous pricing of risk implied by agents' optimizing behavior. Our resulting linear in states approximation is

$$y_t \simeq \tilde{y}(\sigma) + y_y(\tilde{y}(\sigma), 0, \sigma)\left(y_{t-1} - \tilde{y}(\sigma)\right) + y_\varepsilon(\tilde{y}(\sigma), 0, \sigma)\varepsilon_t \qquad (30)$$

where y_t are endogenous and ε_t exogenous variables, $\tilde{y}(\sigma)$ the means of the endogenous variables, and $y_y(\tilde{y}(\sigma), 0, \sigma)$ and $y_\varepsilon(\tilde{y}(\sigma), 0, \sigma)$ the first derivatives of the policy function evaluated at the means $\tilde{y}(\sigma)$. The hyperparameter σ scales the distribution of the exogenous variables with $\sigma = 1$ returning the stochastic model we analyze and $\sigma = 0$ its deterministic counterpart. We employ perturbation techniques to construct Taylor series in σ (out to second order, which approximates out to the second moments of the distributions of the exogenous variables) of the $\sigma = 1$ means, $\tilde{y}(\sigma)$, and slopes at theses means, $y_y(\tilde{y}(\sigma), 0, \sigma)$ and $y_\varepsilon(\tilde{y}(\sigma), 0, \sigma)$, using the derivative information available at the $\sigma = 0$ mean, i.e., deterministic steady state, from standard higher order polynomial perturbations. Appendix A.2 provides a self-contained overview of the derivations involved in this approximation.

The tension between the nonlinearity need to capture the time varying effects of risk underlying asset prices on the one hand and the difficulties bringing nonlinear estimation routines such as the particle filter to bear on such models on the other is highlighted

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[6]Meyer-Gohde (2016) provides derivations for adjustments around the deterministic and stochastic steady states, along with those around the mean that we derive and apply here, accuracy checks and formal justifications for the method.

[7]Among others, recent third order perturbation approximations for DSGE models of the term structure include Rudebusch and Swanson (2008, 2012), van Binsbergen et al. (2012) Andreasen (2012a), and Andreasen et al. (2017).

[8]These approaches separate the macro and financial variables, generally using a (log) linear approximation of the former and an affine approximation for the yield curve following the empirical finance literature. Bonds are priced in an arbitrage free setup using either the endogenous pricing kernel implied by households' stochastic discount factors, as Dew-Becker (2014), Bekaert, Cho, and Moreno (2010), and Palomino (2012), or an estimated exogenously specified kernel, as Hördahl, Tristani, and Vestin (2006) , Hördahl and Tristani (2012), Ireland (2015), Rudebusch and Wu (2007), Rudebusch and Wu (2008).

by van Binsbergen et al. (2012), who model inflation as exogenous in a New Keynesian model to make their Bayesian likelihood estimation tractable. The advantage of a linear in state approximation for estimation has also been noted by, e.g., Ang and Piazzesi (2003), Hamilton and Wu (2012), Dew-Becker (2014). Our approach compromises between the goals of nonlinearity in risk to capture financial variables and the endogenous stochastic discount factor to price financial variables consistent with the macroeconomy on the one hand, and the need for linearity in states to make the estimation of medium scale policy relevant models feasible on the other. To further reduce the computational burden, we apply the PoP method of Andreasen and Zabczyk (2015) that solves the model in a two-step fashion. First, the policy rules for the macro side, including the pricing kernel and the nominal short rate, are approximated and then the financial variables are solved for using this policy function. It is important to note that this is not a further approximation, but rather the recognition that the equations that price different maturities such as eq. (24) are forward recursions that do not enlarge the state space.

3.2   Data

We estimate the model with quarterly U.S. data from 1983:q1 to 2007:q4. Thus, our sample covers the Great Moderation, stopping right before the onset of the Great Recession. This period is chosen specifically for two reasons. First, it is widely accepted in the literature that the U.S. faced a systematic change in monetary policy after Paul Volcker became chairman of the Federal Reserve (e.g. Clarida, Galí, and Gertler, 2000). Second, the start of the Great Recession, the financial crisis of 2008, along with the zero interest policy rates that prevailed from December 2008 onward marks another structural change in U.S. monetary policy. While the systematic behavior of monetary policy is an important driver of the yield curve, as pointed out, for example, by Rudebusch and Swanson (2012), we chose a time episode which is characterized by a relatively stable monetary policy regime.[9]

Our estimation is based on four macroeconomic time series complemented by six time series from the nominal yield curve and two time series of survey data on interest rate forecasts.[10] The macroeconomic dynamics are characterized by real GDP growth, real private investment growth, real private consumption growth, and annualized GDP deflator inflation rates. While the last is measured in levels, the remaining variables are expressed in per capita log-differences using the civilian noninstitutional population over 16 years (CNP16OV) series from the U.S. Department of Labor, Bureau of Labor Statistics.

The nominal yield curve is measured by the 1-quarter, 1-year, 3-year, 5-year, and 10-year annualized interest rates of U.S. Treasury bonds. With the exception of the 1-quarter interest rate, the data are from Adrian et al. (2013) which are identical to the otherwise often used time series by Gürkaynak, Sack, and Wright (2007). For the 1-quarter maturity, we use the 3-month Treasury Bill rate from the Board of Governors of the Federal Reserve System. To have a consistent description of the yield curve, we use this interest rate as the policy rate ($R_t^f = R_{1,t}^\$$) in our model instead of the effective Fed funds rate.

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[9]See, for example, Bikbov and Chernov (2013) and Bianchi, Kung, and Morales (2016) for an investigation of policy regime changes and the term structure of interest rates.

[10]See Appendix A.3 for details on the source and a description of all data used in this paper.

Survey data on interest rate forecasts have shown to be helpful to improve the identification of term structure models (see, for example, Kim and Orphanides, 2012; Andreasen, 2011). For this reason, we incorporate 1 and 4-quarter ahead expectations of the 3-month Treasury Bill into the estimation. The data are taken from the Survey of Professional Forecasters.

3.3 Bayesian Estimation

In this subsection, we present the prior choices for the estimated parameters as well as the calibration of the parameters we choose not to estimate.

Given the choice of our observable variables and the characteristics of our model, for example, the highly stylized labor market, some of the model parameters can hardly be expected to be identified. These parameters are calibrated either following the literature or related to our observables. In particular, we calibrate the steady state growth rates, $\bar{z}^+$ and $\bar{\Psi}$ to $0.54/100$ and $0.08/100$ which implies growth rates of 0.54 and 0.62 percent for GDP and investment as in our sample. Moreover, we calibrate the capital depreciation rate, δ, to 10% per year and the share of capital, α, in the production function to $1/3$. We also assume that in the deterministic steady state, the labor supply $\bar{l}$ and government consumption to GDP ratio $\bar{g}/\bar{y}$ are $1/3$ and 0.19, respectively. The discount rate β is set equal to 0.99 and the steady state of the elasticity of substitution between the intermediate goods θ_p is equal to 6, implying a markup of 20%. Following Andreasen et al. (2017), we set the price indexation $\xi_p = 0$ and calibrate the Frisch elasticity of labor supply FE to 0.5. Hence, we can solve recursively for $\chi = 1/FE \cdot \left(1/\bar{l} - 1\right)$. Table 1 summarizes the parameter calibration.

Description	Symbol	Value
Technology trend in percent	$\bar{z}^+$	$0.54/100$
Investment trend in percent	$\bar{\Psi}$	$0.08/100$
Capital share	α	$1/3$
Depreciation rate	δ	0.025
Price markup	$\theta_p/(\theta_p - 1)$	1.2
Price indexation	ξ_p	0
Discount factor	β	0.99
Frisch elasticity of labor supply	FE	0.5
Labor supply	$\bar{l}$	$1/3$
Ratio of government consumption to output	$\bar{g}/\bar{y}$	0.19

Table 1: Parameter calibration.

The remaining parameters of the model are estimated. Since the focus of the paper is to jointly explain macroeconomic and asset pricing facts, we pay special attention to selected first and second moments when estimating the DSGE model. As described in Kliem and Uhlig (2016), the practical problem boils down to having just one observation on the means, e.g., of the slope, curvature, and level of the yield curve, while there are many observations to identify parameters crucial for the macroeconomic dynamics of the

model. To mitigate this imbalance, we apply an endogenous prior approach similar to Del Negro and Schorfheide (2008) and Christiano, Trabandt, and Walentin (2011). In particular, we use a set of initial priors, $p(\theta)$, where the priors are independent across parameters. Then, we use two sets of first and second moments from a pre-sample.[11] We treat the first and second moments of interest separably in two blocks to capture potentially different precisions of beliefs regarding first and second moments. Finally, the product of the initial priors, the likelihood of selected first moments, and the likelihood of selected second moments forms the endogenous prior distribution which we use for the estimation of the model. In the subsequent paragraphs, we describe the method of endogenously formed priors regarding first and second moments as well as its practical application in the paper.

Following Del Negro and Schorfheide (2008), we assume $\hat{F}$ to be a vector that collects the first moments of interest from our pre-sample and $F_M(\theta)$ be a vector-valued function which relates model parameters and ergodic means

$$\hat{F} = F_M(\theta) + \eta \tag{31}$$

where η is a vector of measurement errors. In our application, we assume that the error terms η are independently and normally distributed. Hence, we express eq. (31) as a quasi-likelihood function which can be interpreted as the conditional density

$$\mathcal{L}\left(F_M(\theta)\,|\hat{F}, T^*\right) = \exp\left\{-\frac{T^*}{2}\left(\hat{F} - F_M(\theta)\right)'\Sigma_\eta^{-1}\left(\hat{F} - F_M(\theta)\right)\right\} \tag{32}$$

$$= p\left(\hat{F}|F_M(\theta), T^*\right)$$

This quasi-likelihood is small for values of θ that lead the DSGE model to predict first moments that strongly differ from the measures of the pre-sample. The parameter T^* captures, along with the standard deviation of η, the precision of our beliefs about the first moments. In practice we set T^* to the length of the pre-sample.

For the application in this paper, we assume that the vector $\hat{F}$ contains the mean of inflation and the means of proxies for the level, slope, and curvature factors of the yield curve. We include the mean of inflation because the non-linearities in our model impose strong precautionary motives that push the predicted ergodic mean of inflation away from its deterministic steady state, $\bar{\pi}$, as is also discussed by Tallarini (2000) and Andreasen (2011). Regarding $\mathcal{L}\left(F_M(\theta)\,|\hat{F}\right)$, we assume that $E_t[400\pi|\theta]$ is normally distributed with mean 2.5 and variance 0.1.

We follow, e.g., Diebold, Rudebusch, and Aruoba (2006) and specify common proxies for the level, slope, and curvature factors of the yield curve. Specifically, the proxy for the level factor is $\left(R_{1,t}^\$ + R_{8,t}^\$ + R_{40,t}^\$\right)/3$, with all yields expressed in annualized terms and the nominal yield of the 1-quarter Treasury Bond equal to the policy rate in the model. Additionally, the proxies for the slope and curvature factors are defined as $R_{1,t}^\$ - R_{40,t}^\$$ and $2R_{8,t}^\$ - R_{1,t}^\$ - R_{40,t}^\$$, respectively. Regarding $\mathcal{L}\left(F_M(\theta)\,|\hat{F}\right)$, we assume that the

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[11]In practice, we follow Christiano et al. (2011) and use the actual sample as our pre-sample as no other suitable data is available because of the monetary regime changes immediately before and after our sample.

ergodic mean of each factor is normally distributed, with the mean equal to its empirical counterpart of the pre-sample. Moreover, we assume that the means of level, slope, and curvature have a variance of 22, 12, and 9 basis points respectively. Thus, the means and variances can be interpreted as $\hat{F}$ value and the variance of the measurement error η in eq. (31).

Additionally, we use the second moments of macroeconomic variables, about which we have a priori knowledge, to inform our prior distribution and apply the approach of Christiano et al. (2011). This approach uses classical large sample theory to form a large sample approximation to the likelihood of the pre-sample statistics. The approach is conceptually similar to the one proposed by Del Negro and Schorfheide (2008), but differs in some important respects. Specifically, Del Negro and Schorfheide (2008) focus on the model-implied p-th order vector autoregression, which implies that the likelihood of the second moments is known exactly conditional on the DSGE model parameters and requires no large-sample approximation in contrast to the approach by Christiano et al. (2011). Yet, the latter approach is more flexible insofar as the statistics to target are concerned. Accordingly, let S be a column vector containing the second moments of interest, then, as shown by Christiano et al. (2011) under the assumption of large sample, the estimator of S is

$$\hat{S} \sim N \left(S^0, \frac{\hat{\Sigma}_S}{T} \right) \tag{33}$$

with S^0 the true value of S, T the sample length, and $\hat{\Sigma}_S$ the estimate of the zero-frequency spectral density. Now, let $S_M(\theta)$ be a function which maps our DSGE model parameters θ into S. Then, for n targeted second moments and sufficiently large T, the density of $\hat{S}$ is given by

$$p\left(\hat{S}|\theta\right) = \left(\frac{T}{2\pi}\right)^{\frac{n}{2}} \left\|\hat{\Sigma}_S\right\|^{-\frac{1}{2}} \exp\left\{-\frac{T}{2}\left(\hat{S} - S_M(\theta)\right)' \hat{\Sigma}_S^{-1} \left(\hat{S} - S_M(\theta)\right)\right\} \tag{34}$$

In our application, S is a set of variances of macroeconomic variables (GDP growth, consumption growth, investment growth, inflation, and the policy rate). In sum, the overall endogenous prior distribution takes the following form

$$p\left(\theta|\hat{F}, \hat{S}, T^*\right) = C^{-1} p\left(\theta\right) p\left(\hat{F}|F_M(\theta), T^*\right) p\left(\hat{S}|\theta\right) \tag{35}$$

where $p(\theta)$ is the initial prior distribution and C a normalization constant. Two points are noteworthy. First, while the initial priors are independent across parameters, as is typical in Bayesian analysis, the endogenous prior is not independent across parameters. Second, the normalization constant C is necessary for, e.g., posterior odds calculation but not for estimating the model. Accordingly, we do not calculate this constant, which has otherwise to be approximated (see, for example, Del Negro and Schorfheide, 2008; Kliem and Uhlig, 2016). So, the posterior distribution is given by

$$p\left(\theta|X, \hat{F}, \hat{S}, T^*\right) \propto p\left(\theta|\hat{F}, \hat{S}, T^*\right) p\left(X|\theta\right) \tag{36}$$

with $p(X|\theta)$ the likelihood of the data conditional on DSGE model parameters θ.

Name	Symbol	Domain	Density	Para(1)	Para(2)
Relative risk aversion	$RRA/100$	$\mathbb{R}^+$	Uniform	0	20
Calvo parameter	γ_p	$[0,1)$	Beta	0.5	0.1
Investment adjustment	ν	$\mathbb{R}^+$	Gamma	4.0	0.75
Habit formation	b	$[0,1)$	Beta	0.5	0.1
Intertemporal elas. substitution	IES	$[0,1)$	Beta	0.25	0.1
Steady state inflation	$100\left(\bar{\pi}-1\right)$	$\mathbb{R}^+$	Uniform	0	6
Interest rate AR coefficient	ρ_R	$[0,1)$	Beta	0.8	0.1
Interest rate inflation coefficient	η_π	$\mathbb{R}^+$	Gamma	1	0.15
Interest rate output coefficient	η_y	$\mathbb{R}^+$	Gamma	0.5	0.1
Inflation target coefficient	$100\zeta_\pi$	$[0,1)$	Beta	0.3	0.1
AR coefficient technology	ρ_a	$[0,1)$	Beta	0.75	0.1
AR coefficient preference	ρ_b	$[0,1)$	Beta	0.75	0.1
AR coefficient investment	ρ_i	$[0,1)$	Beta	0.75	0.1
AR coefficient gov. spending	ρ_g	$[0,1)$	Beta	0.75	0.1
AR coefficient inflation target	ρ_π	$[0,1)$	Beta	0.95	0.025
AR coefficient long-run growth	ρ_z	$[0,1)$	Beta	0.75	0.1
AR coefficient fixed costs	ρ_Ω	$[0,1)$	Beta	0.75	0.1
S.d. technology	$100\sigma_a$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. preference	$100\sigma_b$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. investment	$100\sigma_i$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. monetary policy shock	$100\sigma_m$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. government spending	$100\sigma_g$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. inflation target	$100\sigma_\pi$	$\mathbb{R}^+$	InvGam	0.06	0.03
S.d. long-run growth	$100\sigma_z$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. fixed costs	$100\sigma_\Omega$	$\mathbb{R}^+$	InvGam	0.5	2
ME 1-year T-Bill	$4R_{4,t}^{\$}$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 2-year T-Bill	$4R_{8,t}^{\$}$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 3-year T-Bill	$4R_{12,t}^{\$}$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 5-year T-Bill	$4R_{20,t}^{\$}$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 10-year T-Bill	$4R_{40,t}^{\$}$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 1Q-expected policy rate	$4E_t\left[R_{t,t+1}^f\right]$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 4Q-expected policy rate	$4E_t\left[R_{t,t+4}^f\right]$	$\mathbb{R}^+$	InvGam	0.005	∞

Table 2: Initial prior distribution. Para(1) and Para(2) correspond to means and standard deviations for the Beta, Gamma, Inverted Gamma, and Normal distributions and to the lower and upper bounds for the Uniform distribution.

Table 2 summarizes the initial prior distributions of the remaining parameters. While the prior distributions for most of the parameters are chosen following the literature, it is noteworthy to highlight some deviations. First, we do not use a prior for the preference parameters, γ and α_{EZ}, directly, but rather impose priors for the intertemporal elasticity of substitution, IES, and the coefficient relative risk aversion, RRA, and solve for the underlying parameters. The intertemporal elasticity of substitution, IES, in our model with external habit formation is

$$IES = \frac{1}{\gamma}\left[1 - \frac{b}{\exp\left(\bar{z}^+\right)}\right] \tag{37}$$

We follow Swanson (2012) by using his closed-form expressions for risk aversion, RRA, which takes into account that households can vary their labor supply. Hence, our model implies

$$RRA = \frac{\gamma}{1 - \frac{b}{\exp(\bar{z}^+)} + \frac{\gamma}{\chi}\left(1 - \bar{l}\right)\frac{\bar{w}}{\bar{c}}} + \alpha_{EZ}\frac{1 - \gamma}{1 - \frac{b}{\exp(\bar{z}^+)} - \left(1 - \frac{b}{\exp(\bar{z}^+)}\right)^{\gamma}\bar{c}^{\gamma-1} + \frac{\bar{w}\left(1-\bar{l}\right)}{\bar{c}}\frac{1-\gamma}{1-\chi}} \tag{38}$$

where $\bar{l}$ is the steady state labor supply, while $\bar{c}$ and $\bar{w}$ are consumption and the real wage in the deterministic steady state, respectively. Given the wide range of different estimates for relative risk aversion in the macro- and finance literatures, we initially assume a uniform prior with support over the interval 0 to 2000; our endogenous prior approach, however, does impose an informative prior. We proceed analogously for the deterministic steady state of inflation and choose an uninformative initial prior distribution. Finally, we add measurement errors to the 1-year, 2-year, 3-year, 5-year, and 10-year Treasury bond yields as well as to the expected policy rate expected 1 and 4-quarters ahead. By adding measurement errors along the yield curve, we are following the empirical term structure literature (see, for example, Diebold et al., 2006) and the measurement errors on the expectations of the short rate align the imperfect fit of the data with the model's rational expectation assumption.

4 Estimation Results

In the following section, we present the estimated parameters and discuss the predicted first and second moments of endogenous variables. Additionally, we compare the historical components of the ten-year yield predicted by our model with estimates from the literature.

4.1 Parameter Estimates

As discussed in section 3.1, unlike standard perturbations (e.g. Andreasen et al., 2017), our solution method maintains linearity in states and shocks which allows us to use standard Bayesian techniques to estimate the model. In particular, we estimate the posterior mode of the distribution and employ a random walk Metropolis-Hasting algorithm to simulate the posterior distribution of the parameters and to quantify the uncertainty of our estimates of the same. In particular, we run two chains, each with 100,000 parameter

vector draws where the fist 50% have been discarded. Table 3 provides posterior statistics of the estimated parameters, e.g., the posterior mode, posterior mean and the 90% posterior credible set.[12] The results indicate that the posterior distributions of all structural parameters are well approximated and differ from the initial prior distribution. In the following, we discuss some key parameters in greater detail.

Name	Symbol	Mode	Mean	5%	95%
Relative risk aversion	RRA	89.860	91.427	75.581	108.489
Calvo parameter	γ_p	0.853	0.855	0.843	0.866
Investment adjustment	ν	1.417	1.440	1.204	1.667
Habit formation	b	0.685	0.679	0.614	0.741
Intertemporal elas. substitution	IES	0.089	0.089	0.077	0.101
Steady state inflation	$100\,(\bar{\pi}-1)$	1.038	1.034	0.981	1.091
Interest rate AR coefficient	ρ_R	0.754	0.752	0.718	0.786
Interest rate inflation coefficient	η_π	3.124	3.164	2.839	3.491
Interest rate output coefficient	η_y	0.156	0.159	0.114	0.204
Inflation target coefficient	$100\zeta_\pi$	0.210	0.242	0.109	0.366
AR coefficient technology	ρ_a	0.366	0.356	0.304	0.412
AR coefficient preference	ρ_b	0.820	0.817	0.793	0.843
AR coefficient investment	ρ_i	0.956	0.955	0.949	0.961
AR coefficient gov. spending	ρ_g	0.910	0.909	0.880	0.937
AR coefficient inflation target	ρ_π	0.934	0.925	0.901	0.950
AR coefficient long-run growth	ρ_z	0.630	0.611	0.500	0.729
AR coefficient fixed cost	ρ_Ω	0.928	0.928	0.922	0.933
S.d. technology	$100\sigma_a$	2.333	2.460	1.929	2.985
S.d. preference	$100\sigma_b$	4.878	4.880	4.180	5.570
S.d. investment	$100\sigma_i$	2.516	2.523	2.337	2.689
S.d. monetary policy shock	$100\sigma_m$	0.561	0.572	0.494	0.653
S.d. government spending	$100\sigma_g$	2.010	2.018	1.825	2.220
S.d. inflation target	$100\sigma_\pi$	0.167	0.180	0.130	0.226
S.d. long-run growth	$100\sigma_z$	0.345	0.353	0.253	0.446
S.d. fixed cost	$100\sigma_\Omega$	9.766	9.705	9.022	10.372
ME 1-year T-Bill	$400R_{4,t}^{\$}$	0.185	0.188	0.161	0.214
ME 2-year T-Bill	$400R_{8,t}^{\$}$	0.084	0.085	0.071	0.100
ME 3-year T-Bill	$400R_{12,t}^{\$}$	0.078	0.081	0.067	0.095
ME 5-year T-Bill	$400R_{20,t}^{\$}$	0.152	0.156	0.130	0.181
ME 10-year T-Bill	$400R_{40,t}^{\$}$	0.287	0.297	0.251	0.346
ME 1Q-expected policy rate	$400E_t\left[R_{t,t+1}^{f}\right]$	0.456	0.464	0.408	0.522
ME 4Q-expected policy rate	$400E_t\left[R_{t,t+4}^{f}\right]$	0.738	0.750	0.660	0.842

Table 3: Posterior statistics. Posterior means and parameter distributions are based on a standard MCMC algorithm with two chains of 100,000 parameter vector draws each, 50% of the draws used for burn-in, and a draw acceptance rates about 1/3.

We find a low intertemporal elasticity of substitution ($IES = 0.089$) and a high relative risk aversion ($RRA \approx 90$). Our estimate of the intertemporal elasticity of substitution is in line with the finding of e.g., Hall (1988). However, it differs, for example, from estimates

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[12]Figures 10 and 11 in the appendix illustrate the posterior distribution of each parameter in comparison to its initial prior distribution.

in the long-run and valuation risk literature (e.g. Albuquerque, Eichenbaum, Luo, and Rebelo, 2016), both of which argue for an IES above one. With this in mind, it is not surprising that the model needs a high relative risk aversion to fit the data. Nevertheless, our estimate is still in line with much of the existing macro-finance literature (see, for example, van Binsbergen et al., 2012; Rudebusch and Swanson, 2012). However, it is difficult to compare these numbers. First, all these studies use different samples for the estimation, whereas our study covers just the Great Moderation. Second, the models differ regarding the underlying structural shocks of the economy. As pointed out by van Binsbergen et al. (2012), models that feature a higher volatility of shocks (higher risk) that increase the volatility of the stochastic discount factor need a smaller amount of, e.g., relative risk aversion to match average bond yields. Nevertheless, our estimates are high in comparison with risk aversion used in endowment economies or in comparison with micro-studies (Barsky, Juster, Kimball, and Shapiro, 1997). However, Malloy, Moskowitz, and Vissing-Jørgensen (2009) show that risk aversion estimated for stockholders in the U.S. is substantially lower than a representative agent using aggregate consumption. The authors find that the estimated relative risk aversion increases to 81 when using aggregate consumption. Alternatively, Barillas, Hansen, and Sargent (2009) argue that a small amount of model uncertainty can substitute for the large degree relative risk aversion often found in the literature.
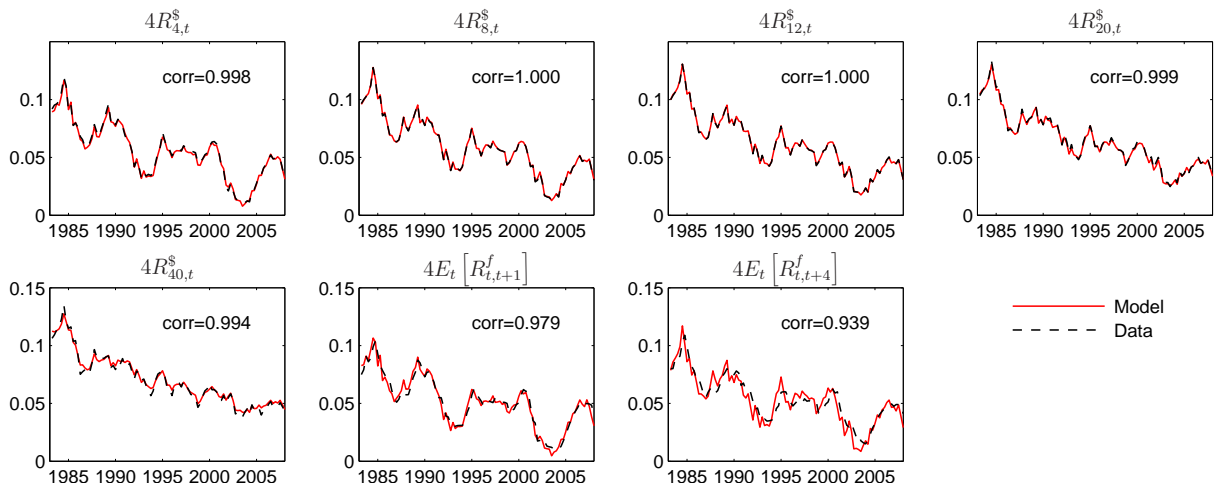

Figure 2: Observed and model implied nominal returns of treasury bills and returns of expected short rates.

We estimate a quarterly deterministic steady state inflation of around 1.04% which is substantially higher than the average observed inflation rate (0.64%). As mentioned before, due to the non-linearities in our model, the difference is related to the household's precautionary motive in our model as also discussed by Tallarini (2000). However, we show in the subsequent subsection, that the approximated ergodic mean of inflation is similar to the average U.S. inflation over our sample.

For the inflation target, we estimate $\rho_\pi = 0.93$ and $\zeta_\pi = 0.002$. The latter coefficient is similar to Rudebusch and Swanson (2012), while the former coefficient is slightly smaller, implying a less persistent effect of nominal risk in our model. Moreover, we estimate a moderate size of investment adjustment costs ($\nu = 1.4$) and comparable estimates to the

literature for price stickiness ($\gamma_p = 0.85$) and external habit formation ($b = 0.67$). Finally, we find that monetary policy puts more weight on stabilizing the inflation gap ($\eta_\pi = 3.13$) than on the output gap ($\eta_y = 0.16$) and smoothes changes in the policy rate ($\rho_R = 0.75$).

Figure 2 shows the historical time series (dash-dotted line) and the model implied smoothed time series (solid line) for the seven variables estimated with measurement error. Note that we estimate small measurement errors along the yield curve. In particular, the measurement errors range between 7 and 29 basis points, implying a correlation between the smoothed model implied yields and the data of 0.99 or higher. The measurement errors for the 1-quarter ahead and 1-year ahead expectations of the 3-month T-Bill are 45 and 74 basis points, respectively, delivering high correlations (0.94 and 0.98) of our model-based expectations with the data from the Survey of Professional Forecasters.

4.2 Predicted Moments

In the following subsection, we begin our posterior analysis with respect to the predicted first and second moments. Figure 3 shows the predicted ergodic means of the nominal yields in relation to the means of the corresponding data. The figure illustrates the success of our estimation approach, with the a priori information about the level, slope, and curvature, based on only 3-month, 2-year, and 10-year nominal yields, sufficient to estimate first moments for all maturities.
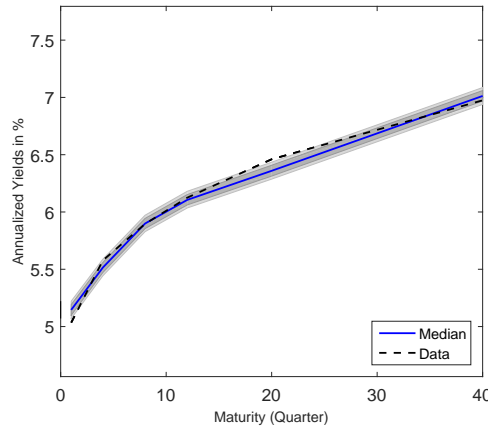

Figure 3: Nominal yield curve

Backus, Gregory, and Zin (1989) and den Haan (1995) formalized the bond-pricing puzzle with the question of why the yield curve is upward sloping. This question refers to the idea that long-term bond should carry an insurance-like negative risk premium, and therefore the yield curve should be downward sloping. However, the data for nominal yields as well as estimates for the nominal term premium suggest the opposite as does our model (see Figure 4(b)). The mechanism behind this has already been described by, e.g., Rudebusch and Swanson (2012): supply shocks move consumption and inflation in opposite directions, imposing a negative correlation between the two. Thus, inflation reduces the real value of nominal bonds precisely in states of low consumption when agents would particularly value higher payouts, thereby generating a positive term premium. To this end, Piazzesi and Schneider (2007) show that consumption and inflation were negatively correlated in the period 1952-2004 for the U.S., which suggests that supply

(a) Real yield curve

(b) Nominal term premium

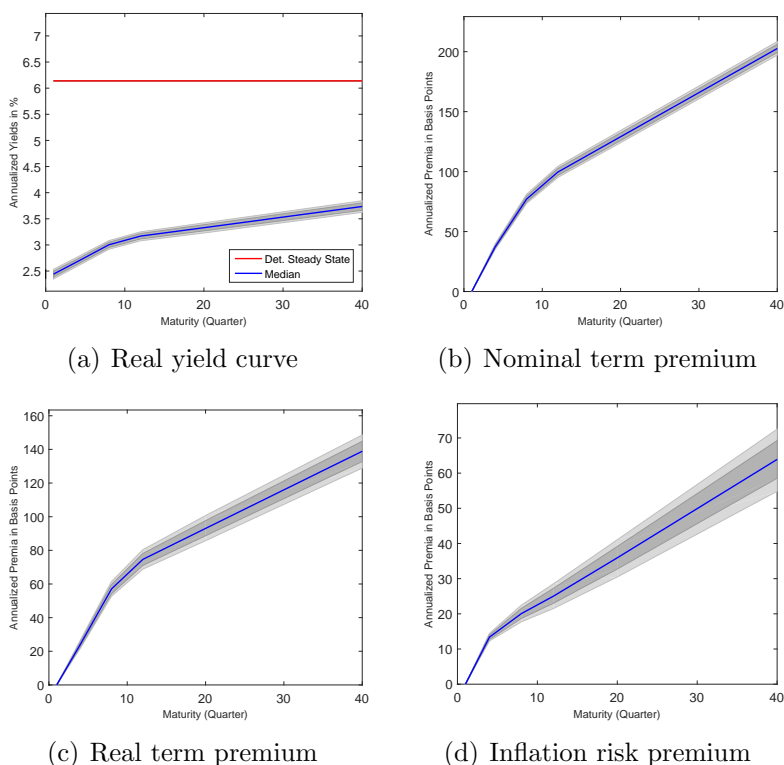(c) Real term premium

(d) Inflation risk premium

Figure 4: Term structure of interest rates

shocks play a relatively important role in generating the upward sloping nominal term structure in the data and in our model.

The negative correlation between consumption growth and inflation can explain the positive slope in the nominal term structure by appealing to inflation risk, but absent another mechanism cannot account for the real term structure. If it is solely inflation risk driving the upward slope of the nominal term structure, then the real term structure should be downward sloping as spells of low consumption growth will be associated with low real rates (and hence high prices for real bonds). This gives agents a higher payout precisely when they would value it highly and implying that real bonds should carry negative, insurance-like risk premia. Nevertheless, as illustrated by Figures 4(a) and 4(c), our model also predicts an upward-sloping real term structure which is in line with the literature (see, for example, Gürkaynak et al., 2010; Chernov and Mueller, 2012). The mechanism in our model follows that described in Wachter (2006) and Hördahl et al. (2008), as our households' habit formation introduces a hump-shaped response of consumption. This makes consumption growth positively autocorrelated while reducing agents' precautionary saving motive for longer maturities: households will seek to maintain their habit in the face of a slowdown in consumption, drawing down their precautionary savings and driving down real bond prices, implying that payouts on real bonds are negatively correlated with marginal utility and that real bonds demand a positive risk premium. The precautionary motive is illustrated in Figure 4(a), where the red line shows the real yield curve in absence of risk, i.e., at the deterministic steady state. When confronted with risk, agents accumulate additional capital, driving down its return. This reduction, however, is decreasing in the maturity due to the positive real risk premium, resulting in our

20

estimated upward sloping real term structure.

Figure 4(d) shows that our model predicts an upward sloping inflation risk premium consistent with recent estimates in the literature (see, for example, Abrahams et al., 2016), with our ergodic mean term structure of inflation risk comfortably between the estimates of Buraschi and Jiltsov (2005) and Chen et al. (2010). The ergodic mean of inflation risk is approximately half the size of the real term premia for all maturities, consistent with Kim and Wright's (2005) estimates for the ten year inflation and real risk premia. Consequentially, our results suggest that most of the average slope of the nominal term structure is related to real rather than to inflation risk. Again, this finding is consistent with recent estimates for the U.S. (see, for example, Kim and Wright, 2005) and is also qualitatively comparable to the results by Hördahl and Tristani (2012) for the Euro area. So far most of the DSGE models (see, for example, van Binsbergen et al., 2012; Swanson, 2016) generally attribute a stronger insurance-like character to real bonds, that lead to flat or downward sloping real yield curves.

Name	Data/BVAR		Model	
	Mean	S.d.	Mean	S.d.
GDP growth	0.540	0.593	**0.540**	0.803*
		[0.515, 0.764]		[0.761, 0.838]
Consumption growth	0.610	0.435	**0.540**	0.559*
		[0.383, 0.515]		[0.528, 0.587]
Investment growth	0.620	2.096	**0.620**	2.292*
		[1.796, 2.744]		[2.120, 2.438]
Annualized inflation	2.496	1.022	2.469*	1.198*
		[0.840, 1.493]	[2.418, 2.515]	[1.136, 1.254]
Annualized policy rate	5.034	2.069	5.144*	2.861*
		[1.521, 3.927]	[5.070, 5.222]	[2.733, 3.026]
1-year T-Bill	5.577	2.334	5.515	2.574
		[1.724, 4.417]	[5.443, 5.588]	[2.453, 2.733]
2-year T-Bill	5.896	2.373	5.900*	2.257
		[1.699, 4.435]	[5.828, 5.972]	[2.144, 2.389]
3-year T-Bill	6.124	2.384	6.106	2.019
		[1.699, 4.580]	[6.035, 6.181]	[1.914, 2.137]
5-year T-Bill	6.460	2.311	6.359	1.662
		[1.611, 4.643]	[6.287, 6.435]	[1.582, 1.760]
10-year T-Bill	6.974	2.101	7.013*	1.150
		[1.480, 4.634]	[6.939, 7.086]	[1.120, 1.253]

Table 4: Predicted first and second moments of selected macro and financial variables. Bold moments are calibrated and moments appended with * were used directly or indirectly to form the endogenous prior.

Table 4 presents the first and second moments of the observables predicted by the model. As the predicted moments from the model are population moments, we compare them with first moments of the data and second moments calculated from a Bayesian vector autoregression model (BVAR) with two lags.[13] The results illustrate that our es-

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[13]We fit a BVAR(2) to the observables by assuming a weak Normal-Wishart prior for the coefficients

timation approach delivers an ergodic mean of inflation comparable to the mean of the data and, as a result, captures households' precautionary savings motives. Moreover, the predicted second moments regarding the macroeconomic variables are in line with those of the BVAR, highlighting the ability of our New Keynesian DSGE model to match financial and macroeconomic moments jointly (see also Andreasen et al., 2017).[14] Regarding treasury bonds, our model misses the high volatility for longer maturities, but matches the monotonic decrease in volatility with the maturity. This result in general equilibrium models has been described in den Haan (1995) and is related to some missing source of persistence in the model (see Hördahl et al., 2008). We do not see this, however, as a fatal shortcoming of our analysis. Firstly, the uncertainty related to these population moments is quite high and, secondly, it rather illustrates the tension in the competing goals the model faces: matching highly volatile nominal treasury bonds while predicting a very smooth inflation rate.

4.3   Model Implied Historical Fit

In the following subsection, we discuss our model implied historical time series for the nominal term premium, break-even inflation rate, real rate, and inflation risk premium. It is important to stress that these measures did not enter into our estimation and, instead, are produced as estimated latent variables in our analysis. To judge the quality of our estimated model, we contrast our estimates with various estimates from the literature. Following the majority of the empirical literature, we limit our discussion to 10-year maturities.
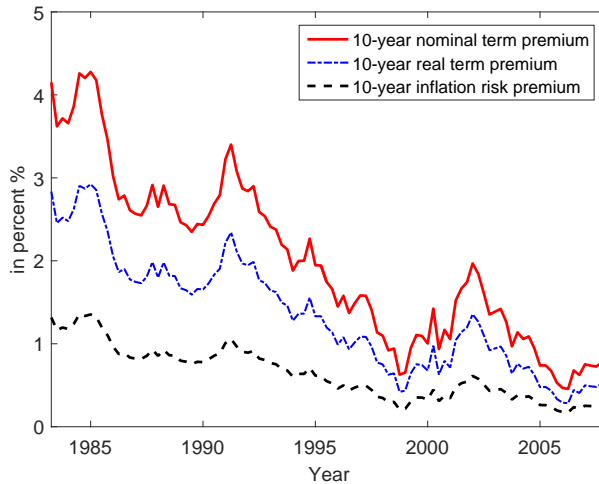

Figure 5: Decomposition of model implied 10-year nominal term premium over the full sample.

Figure 5 shows the smoothed 10-year nominal term premium predicted by our model (see also Figure 1) and its decomposition into real term premium and inflation risk pre-

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and covariance of the BVAR. For the comparison, we draw 1200 parameter vector draws from the posterior of the BVAR as well as 1200 parameter vector draws from posterior distribution of the DSGE model. Appendix A.4 presents further statistics for the DSGE model.

[14]Additionally, figure 12 in appendix A.4 presents the autocorrelation of hp-filtered macroeconomic variables for up to four lags which also illustrates the good fit relative to a corresponding BVAR.

mium. All risk premia show the same steadily declining pattern. Over the sample, real term premia contributed between 62% and 68% to the nominal term premium. Moreover, the inflation risk premium declines until 1998, consistent with steadily declining inflation expectations over this period.

In Figure 1, we compare our 10-year nominal term premium with several different prominent estimates from the literature. As Rudebusch et al. (2007) show, all of the estimated term premia, which they investigate, follow a similar pattern and are highly correlated. This is also true for our extented sample which includes two more recent estimates by Adrian et al. (2013) and Bauer (2016).[15] Table 5 presents the correlations between these five measures of the term premium and the estimate of our model. Our estimate shows also a remarkably high correlation with all measures, but especially with those of Kim and Wright (2005) and Bauer (2016) (0.94 and 0.93, respectively). Given that our model is arguably closest in structure to the model used by Rudebusch and Wu (2008), we would have expected our model to display a much higher correlation with their measure than it actually does. Also, while the model by Rudebusch and Wu (2008) predicts a smooth term premium, all other models including the model presented in this paper predict a much more volatile measure.

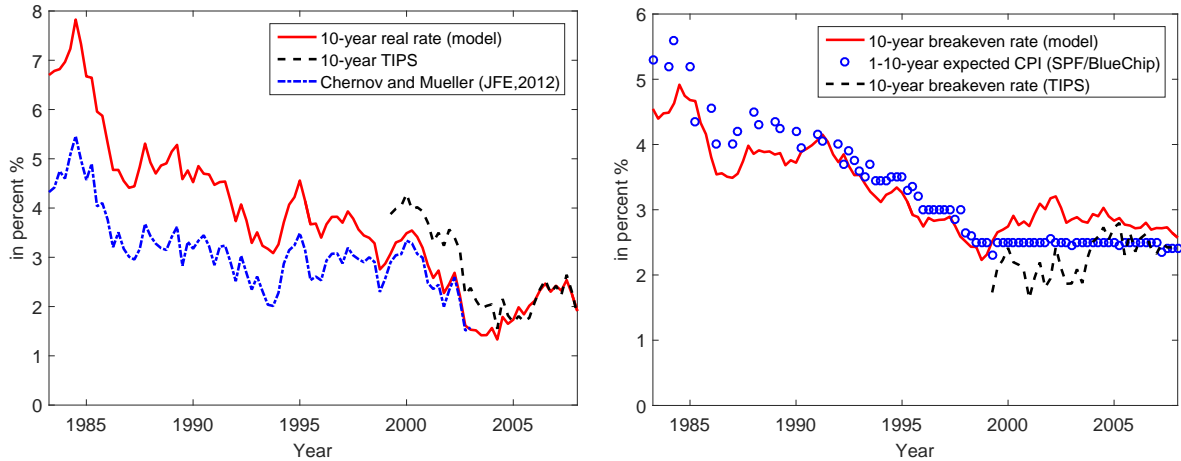|	Bernanke et al.	Rudebusch and Wu	Kim and Wright	Adrian et al.	Bauer	S.d.
Bernanke et al. (2004)	1.000					1.294
Rudebusch and Wu (2008)	0.763	1.000				0.336
Kim and Wright (2005)	0.976	0.811	1.000			0.981
Adrian et al. (2013)	0.817	0.941	0.891	1.000		1.033
Bauer (2016)	0.853	0.734	0.936	0.885	1.000	1.182
Model	0.904	0.800	0.940	0.868	0.932	0.943

Table 5: Correlations among six measures of the 10-year term premium from 1984:q1-2005:q4. The last column presents the standard deviation over the sample. Statistics related to the estimates by Bauer (2016) are based on a shorter sample starting 1990.

The reason that our model produces a large and volatile term premium is similar to explanations postulated in the recent literature (see, for example, Andreasen et al., 2017). Beside the role of supply shocks in our model that generate a sizable term premium, the presence of long-run nominal risk is important in generating a volatile term premium (see Rudebusch and Swanson, 2012). Additionally, our model captures a channel recently postulated by Andreasen et al. (2017), namely the role of steady-state inflation for the mean and volatility of risk premia. In particular, steady-state inflation generates more heteroscedasticity in the stochastic discount factor which eventually produces more volatile

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[15]The estimates by Bauer (2016) start in 1990, so all calculations using this estimate are restricted to a shorter sample.

risk premia. This channel is present despite the fact that the shocks in our model are all homoscedastic. More specifically, the endogenously generated heteroscedasticity in the pricing kernel is a byproduct of the heteroscedasticity in price dispersion due to positive steady-state inflation.

Figure 6(a) compares our 10-year real rate with the estimates provided by Gürkaynak et al. (2010) using TIPS data and those of Chernov and Mueller (2012) using survey-based forecasting data. Both measures are not fully identical with the real rate measured by our model, for example, while our real rates are based on GDP inflation the aforementioned measures are based on CPI data. Also, our model has no role for a liquidity premium component that is arguably a non-negligible component of TIPS (see, for example, Abrahams et al., 2016). Nevertheless, our estimate captures the downward trend since the 1980s found likewise in Chernov and Mueller (2012). Additionally, our estimate demonstrates a high correlation with both (0.9 with Gürkaynak et al. (2010) and 0.94 with Chernov and Mueller (2012)) of these alternative measures, derived from empirical reduced-form models.

(a) Model implied 10-year real rates (red solid), 10-year TIPS of Gürkaynak et al. (2010) (black dashed), and 10-year real rate of Chernov and Mueller (2012)(blue dash-dotted).

(b) Model implied 10-year break-even inflation rate (red solid), 10-year break-even inflation rate of Gürkaynak et al. (2010) (black dashed), and 1 to 10-year average expected CPI inflation from SPF and BlueChip (blue circle).

Figure 6: 10-year real interest rate and 10-year break-even rate.

Figure 6(b) shows the model implied 10-year break-even inflation rate. At the beginning of the sample, the breakeven inflation rate declines continuously until 1998. From 1999 onward we find a stable breakeven rate fluctuating around 3 percent. Over this period, our estimate is comparable in levels and pattern with those by Gürkaynak et al. (2010). Moreover, the continuous decline in the model's breakeven rate until 1998 is accompanied by a decreasing inflation risk premium (see figure 5). This pattern is commensurate with declining inflation expectations in this period.

In summary, our model-implied estimates of the components of 10-year bond yields demonstrate a considerable alignment with various empirical estimates in the literature. This alignment is all the more remarkable as these components of the yields were not used in our estimation procedure. This provides us with a high degree of confidence in our

model's ability to replicate stylized term structure facts as we now turn to the structural analysis of the effects of monetary policy on the term structure of interest rates and its components.

5 Monetary Policy Through the Lens of Our Model

In this section, we analyze the effects of monetary policy on term premia by distinguishing between three different policy actions. First, a surprise shock to the policy rate via the residual of the Taylor-rule. Second, a shock to the inflation target that might be interpreted as a change in the systematic component of monetary policy as it affects agents' perception of inflation in the long run. Third, we investigate the effects of a commitment by the monetary authority to a path for future short rates; i.e., forward guidance by means of a credible announcement to change the policy rate in the future while holding it constant until then. While this may seem a narrow aspect of recent experience with unconventional monetary policy, Woodford (2012), for example, argues that even quantitative easing itself can at least partially be interpreted as forward guidance through the signalling channel, building on results by Krishnamurthy and Vissing-Jorgensen (2011) and Bauer and Rudebusch (2014). Furthermore, forward guidance has been a component of standard monetary policy at major central banks even before its explicit implementation since the financial crisis (see Gürkaynak et al., 2005a). Technically, we implement this forward guidance scenario by altering the Taylor rule in eq. (18) following Laséen and Svensson (2011), Del Negro, Giannoni, and Patterson (2015), and others by adding a sequence of anticipated shocks to the Taylor rule that allow the monetary authority to keep the policy rate upon announcement constant until the announced interest rate change (here a cut) is implemented as follows

$$ r_t^f = R\left(r_{t-1}^f, \pi_t, y_t \right) + \sigma_m \left(\epsilon_{m,t} + \sum_{k=1}^{K} \epsilon_{m,t+k} \right), \;\; \epsilon_{m,t+k} \overset{iid}{\sim} N(0,1) \qquad (39) $$

where $R\left(\cdot\right)$ characterizes the systematic response of monetary policy, $\epsilon_{m,t}$ is the usual contemporaneous policy shock, and $\sum_{k=1}^{K} \epsilon_{m,t+k}$ a sequence of policy shocks known to agents at time t but that affect the policy rule k periods later, i.e., at time $t+k$.

The three columns in Figure 7 contain the IRFs of macroeconomic variables to a surprise shock to the policy rate (left column), to a surprise inflation target shock (middle column), and to a four-quarter ahead forward guidance shock (right column). All shocks are normalized to yield a median lowering of the policy rate by 50 basis points on impact (or in four quarters for the forward guidance shock).

As is standard in the literature, the expansionary policy due to surprise policy rate cut (left column of Figure 7) leads to an increase in aggregate demand and its components as well as inflation. As the policy rate begins to return to its mean level with inflation still elevated, the resulting increase in expected real rates reverses the expansion, depressing aggregate demand and its components, before the macroeconomy then settles back to its mean position after around 10 quarters.

The middle column of Figure 7 shows the impulse responses to a surprise inflation target shock. The reduction in the inflation target is accompanied with a nearly two
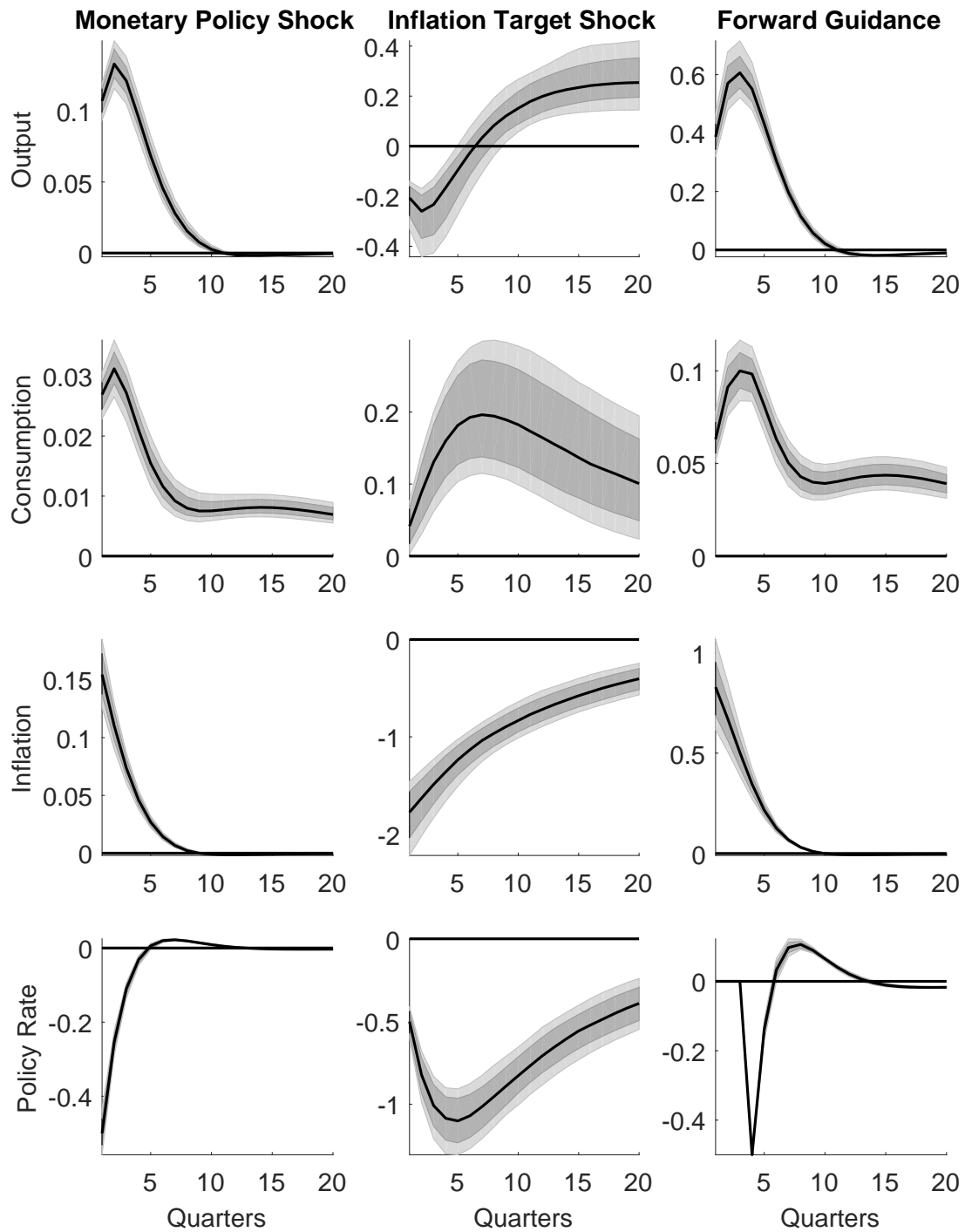
Figure 7: Posterior impulse responses of macro variables to a surprise 50 basis point policy rate cut, a surprise cut in the inflation target leading to a 50 basis point policy rate cut, and forward guidance of a 50 basis point policy rate cut in 4 quarters. Shaded areas represent the 90% and 68% posterior credible sets.

annualized percentage point reduction in inflation, roughly the same magnitude as the reduction of the target, which corresponds to a substantial change in the systematic behavior of monetary policy. The lowering of the policy rate is hump shaped with the maximal decrease of about 110 annualized basis points occurring about a year after the lowering of the inflation target. This lowering of the policy rate is not sufficient to overcome the initial contractionary effects of the lowered inflation target and associated disinflation as can be seen by the negative responses on aggregate demand. Moreover, our results illustrate that a shock to the inflation target is much more long lasting and therefore has stronger effects on business cycle and lower frequencies, in contrast to a simple innovation to the Taylor-rule which quickly dissipates. This confirms the interpretation of Rudebusch and Swanson (2012) that a change in the inflation target, or more generally a change in the systematic behavior of monetary policy, introduces long-run nominal risk into the economy.

The right column in Figure 7 shows the evolution of macroeconomic variables following the forward guidance experiment. Similarly to most studies, we find that forward guidance increases macroeconomic activity and substantially increases inflation. Output and inflation both increase on impact with output reaching its peak after 3 quarters and falling slightly below its mean value after 12 quarters. The response to the announcement is driven by expectations of lower nominal short term interest rates and of future inflation. Expected higher inflation leads to a rise in current inflation through forward looking price setting, with a consequential fall in current and expected real interest rates and associated increase in economic activity on impact. Therefore, comparable to a change in the inflation target, forward guidance communicates the central bank's commitment to allow higher inflation in the future, which has more stronger and more long lasting effects on households' expectation and so on their precautionary savings motives.

Figure 8 shows the impact responses of the nominal and real term structures while Figure 9 presents the dynamic responses of 1-year and 10-year maturities. The unexpected monetary policy shock (the left column of both figures) shows that the response on impact of the term structure becomes more muted with the maturity, as would be expected in accordance with the expectations hypothesis and the path of the policy rate (assumed identical to the short rate). Similarly, the response on impact of the real yield curve, see the second row of Figure 8, is driven primarily by the expectations hypothesis and the Fisher equation with the response likewise becoming more muted with the maturity. This too is reflected in the impulse responses over time for short and long maturities contained in the second row of Figure 9. With the expectations hypothesis being the predominate driver of the impact on real and nominal rates, an unexpected monetary policy shock – a simple innovation to the Taylor rule – has limited, though nonzero, effects on the risk premia along all maturities. This finding is in line with those of other structural models (see, for example, Rudebusch and Swanson, 2012). On impact, see the third row of Figure 8, bond holders demand higher total premia for holding nominal bonds for longer maturities and lower total premia for shorter maturities which is qualitatively in line with the findings of Nakamura and Steinsson (2017). The stimulative effects in the short run generate increased confidence in the absence of downside risks to the economy, reflecting the fall in the short run premia. The delayed contractionary effects of the loosening of monetary policy are reflected in the higher medium to long run premia demanded on impact. When the contractionary effects are realized two quarters after the shock, the economy is
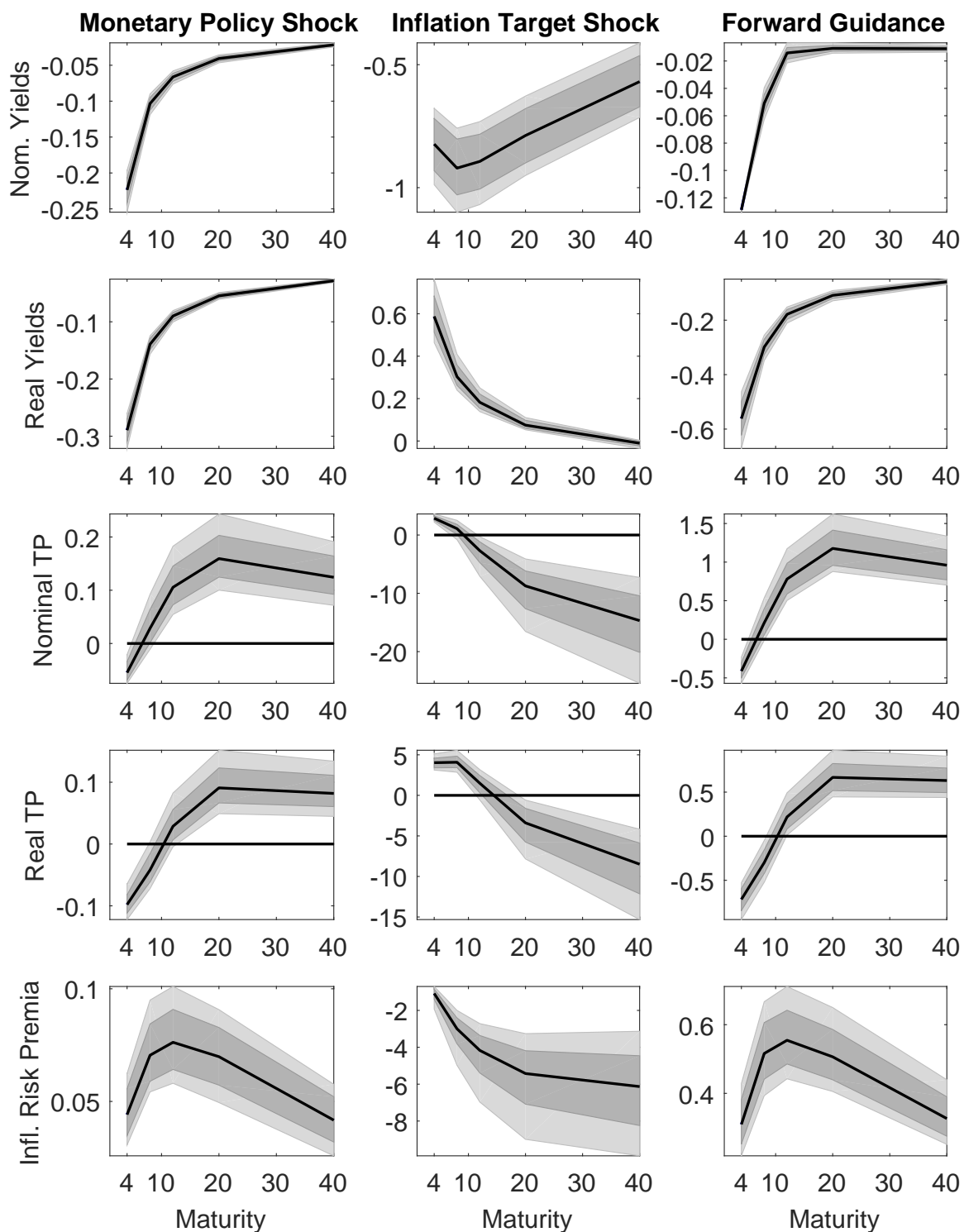
Figure 8: Impact responses of nominal and real term structures across all maturities to a surprise 50 basis point policy rate cut, a surprise cut in the inflation target leading to a 50 basis point policy rate cut, and forward guidance of a 50 basis point policy rate cut in 4 quarters. The deviations of yields are in percentage points while the deviations of risk premia are presented in basis points. Shaded areas represent the 90% and 68% posterior credible sets.

Figure 9: Posterior impulse responses of nominal and real term structure at the short and long ends to a surprise 50 basis point policy rate cut, a surprise cut in the inflation target leading to a 50 basis point policy rate cut, and forward guidance of a 50 basis point policy rate cut in 4 quarters. The deviations of yields are in percentage points while the deviations of risk premia are presented in basis points. Shaded areas represent the 90% and 68% posterior credible sets.
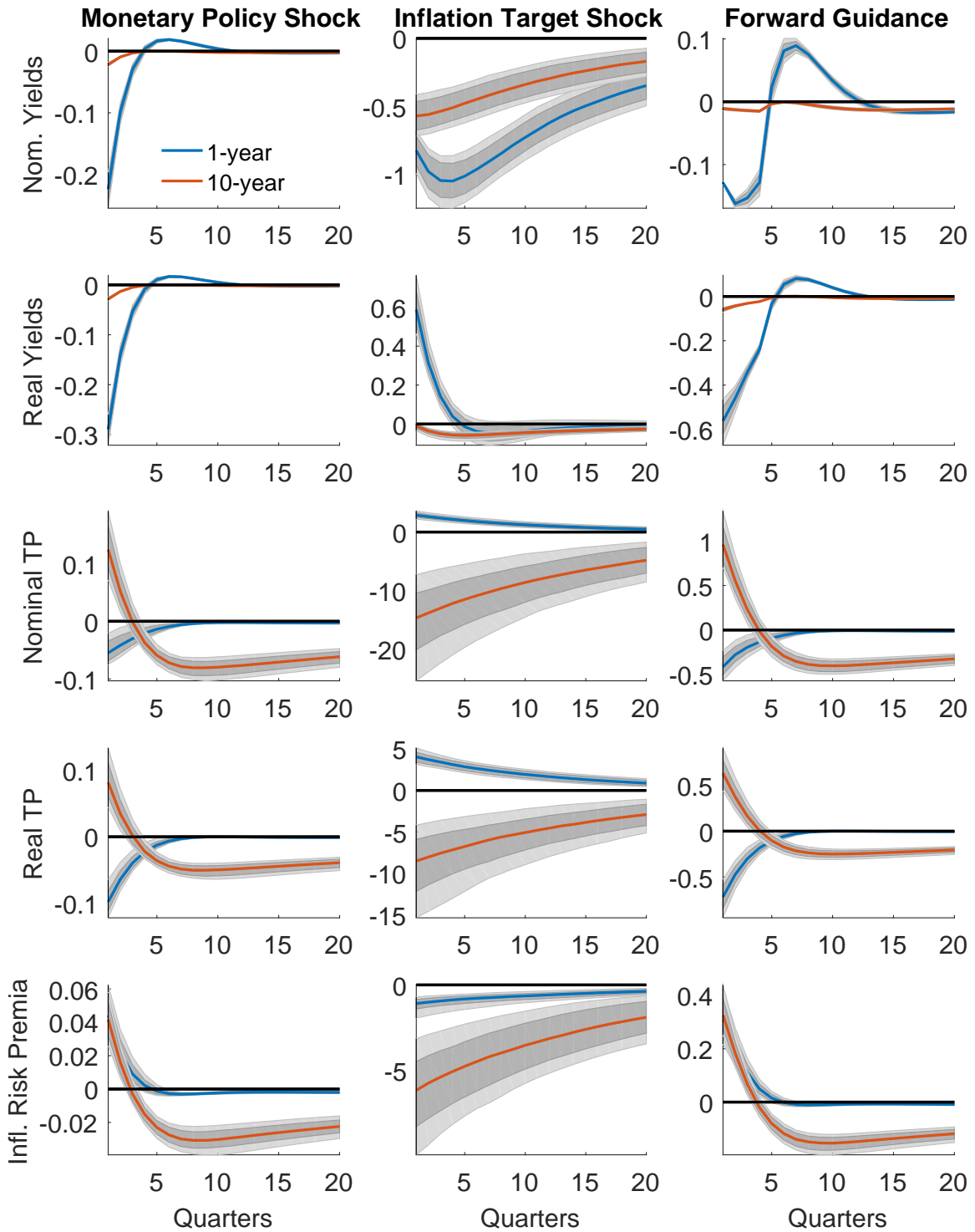
looking towards a recovery and the downside risks are decreased at all horizons, which is reflected in a reduction in the premia demanded at both the short and long ends of the term structure of nominal term premia (the third row of Figure 9). In this regard, our findings are qualitatively similar to those by Crump et al. (2016). The effects on impact and the dynamic responses of the short and long ends for the real term premia qualitatively mirror those of the nominal term premia, confirming that the primary driver of the nominal term premia is indeed the real economy and associated risks. On impact, the real term premia, see the fourth row of Figure 9, are shifted downward across all maturities relative to the impact response of the nominal term premia, reflecting the elevation in the inflation risk premia, see the bottom row of Figure 9, demanded by investors in response to the inflationary effects of the expansionary monetary policy. The negative initial response of real term premia associated with shorter maturities and positive response of those associated with longer maturities can be understood roughly from the comovement of the real yields and the consumption relative to its habit in the pricing kernel. Yields on real bonds at all maturities drop on impact whereas consumption relative to its habit initially rises but then falls. This generates a positive comovement between the kernel and yields on shorter maturities that thus contain a negative, insurance-like premium. At longer maturities, this comovement becomes negative as consumption drops relative to its habit and thus real bonds of longer maturities bear a positive risk premium to induce households to hold these bonds that pay less when payoffs are more highly valued. The timing of when the ten year real term premium turns negative coincides with the onset of the contraction in the real economy. On impact, investors demand a higher premium across all maturities to compensate them for the upside risks in inflation associated with the surprise change in monetary policy. This upside risk is quickly reversed as the delayed contractionary effects of the monetary policy shock are realized and the inflation premia demanded at both the short and long ends of the term structure are reduced.

In contrast, a surprise shock to the inflation target has a much stronger effect on the risk premia of interest rates across all maturities, see the second columns of Figures 8 and 9, with the effects roughly two orders of magnitude larger. While this stronger effect on the nominal term premia can also be found in Rudebusch and Swanson (2012), our findings show that monetary policy substantially affects real term premia as postulated by Hanson and Stein (2015). In contrast to the nominal yield curve, the response of the real yields is decreasing in the maturity. This is consistent with a Fisher equation perspective on the real rates, noting the delayed reduction in the nominal short rate in response to the decrease in inflation. The drop in yields is driven by households drawing down their precaution stock of capital, thereby driving up real yields, to smooth consumption in the face of the initial contraction in output. The short end of the real yield curve falls below zero when the nominal short rate recovers from its trough one year after the impact of the shock and remains there as the policy rate converges more quickly to its mean value than inflation. On impact and through time, the effect on real rates of longer maturities is limited and almost entirely driven by term premia. On impact, shorter maturities are associated with increased nominal and real term premia and longer maturities with decreased nominal and real term premia, see the third and fourth rows of Figure 8. This coincides by and large with the initial expansion and delayed contraction in the aggregate real economy and is consistent with the comovement of the pricing kernel, driven partially by the initial rise and decrease later in consumption relative to its habit, and the initial

increase in yields at all maturities. This again confirms that the primary driver of the nominal term premia is indeed the real economy. The downside risks to the real economy are perceived on impact to be longer lived than the nominal risks, which can be seen in the larger positive impact effect of the reduction in the inflation target on the premia for longer maturities demanded by investors. Rows three and four of Figure 9 show that the premia at the short and long ends of the term structure remain diverged in their entire dynamic responses. This is consistent with the interpretation of the shock to the inflation target as being a shift in the systematic monetary policy: long run downside risks to the economy are reduced by the more aggressive response of monetary policy at the cost of heightened short run risks. With both the inflation target and realized inflation reduced by the more aggressive posture of monetary policy towards inflation, investors' perception of upside risks to inflation are ameliorated, leading to a reduction in the inflation risk premia that they demand at all horizons on impact, as well as dynamically at the short and long ends of the associated term structure, see the bottom middle panels of Figures 8 and 9. While the nominal term premia are still primarily driven by risks associated with the real economy in response to the inflation target shock, the effects of inflation risk premia are disproportionately increased in magnitude, consistent with the interpretation of this experiment being not only a change in the systematic response of monetary policy, but more specifically a more aggressive posture towards inflation.

Alongside the expectation channel from above, forward guidance propagates through an additional channel, the movements in the nominal long rates, which the recent literature has argued plays a nontrivial role (e.g., Woodford, 2012; Del Negro et al., 2015). From both a theoretical and empirical perspective, it is not obvious a priori which maturities in the nominal term structure should fall in our forward guidance experiment. From the perspective of our model, the dynamic responses of interest rates are driven by the countervailing effects of the expectations hypotheses and risk premia. As in standard models under the expectations hypothesis, the dynamics of interest rates with longer maturities reflect the dynamic adjustment of the risk free short rate, determined by the monetary authority's Taylor rule. The large effects on inflation and output imply that the policy rate rises quickly above its ergodic mean only few quarters after its announced fall. This explains, at least in part, why we observe only a mild drop on impact in nominal bonds with a maturity longer than 2 years (see the upper right panel of Figure 8). While the yield of a 1-quarter real bond falls by around 30 basis points on impact, the yield of a 10-year real bond falls by around 3 basis points (see the second row of the right column in Figure 8). The right columns in Figures 8 and 9 show the impact responses and impulse responses over time, respectively, for the nominal and real term premia as well as the inflation risk premia. They illustrate that bondholders demand higher nominal premia on impact for all maturities from 2 years onward to compensate them for the downside risks they perceive in the nominal economy. This is in line with the empirical findings of Akkaya et al. (2015). While there is some increased short to medium term confidence in the real economy, as can be seen by the fall in the real premium demanded for two year real bonds on impact, this is outweighed by the larger increase in inflation risk perceived by the bondholders, see the bottom two rows of the right column in Figure 8. This overall increase in nominal premia prevents nominal rates from falling as strongly as the expectations hypothesis would predict and therefore dampens the expansionary effects of the announced cut in the policy rate. Finally, the increase in inflation risk premia fol-

lows what theory would predict. While forward guidance does communicate the expected path of future short rate, it is just as informative about the central bank's commitment to allow higher inflation in the future. This commitment drives households' demand for higher inflation risk premia.

In sum, our findings show that different monetary policy actions affect the term structure of interest rates differently. In particular, changes to the inflation target (or, more generally, changes to the systematic response of monetary policy, as is also a component of forward guidance) have stronger and more long lasting effects on households' precautionary savings motives and, therefore, on risk premia. In contrast, unexpected monetary policy shocks die out quite quickly, limiting their effects on business cycle frequencies and, consequentially, on risk premia. While some of our qualitative results have already been discussed in the literature (see, for example, Rudebusch and Swanson, 2012), we are able to quantify these effects and the uncertainty surrounding them with our structural macro-finance model. Furthermore, our estimated model does provide new qualitative as well as quantitative insights regarding the effects on the term structure on interest rates of monetary policy announcements. In this light, our model can rationalize the seemingly contradictory findings in the empirical literature (see, for example Hanson and Stein, 2015; Nakamura and Steinsson, 2017).

6 Conclusion

The role of monetary policy in shaping the term structure has been gaining increased prominence. Yet the empirical literature has yet to reach a definitive conclusion on either the qualitative or quantitative effects of monetary policy on the term structure and the standard structural alternative – the linear New Keynesian model – has been criticized for lacking effects on interest rates beyond the expectations hypothesis (Hanson and Stein, 2015). Newer structural modelling approaches that go beyond the expectations hypothesis face significant computational challenges (van Binsbergen et al., 2012). We ameliorate these challenges by using the risk adjusted approximation of Meyer-Gohde (2016), allowing our model to capture the salient features of risk while remaining linear in states such that Bayesian estimation and posterior analysis using standard macroeconometric techniques is tractable. Our estimated structural framework is consistent with a wide variety of asset pricing and macroeconomic facts, making it well suited to investigate the impact of monetary policy on term structure of interest rates. Specifically, our medium scale New Keynesian macro-finance model produces sizable and time varying risk premia comparable to historical estimates from affine term structure models (e.g. Kim and Wright, 2005; Adrian et al., 2013) without sacrificing the fit of macroeconomic or other financial variables.

We show that distinguishing between different monetary policy actions rationalizes many of the seemingly contradictory findings on the effects of monetary policy on term premia in the empirical literature (see, for example, Hanson and Stein, 2015; Nakamura and Steinsson, 2017). In particular, we find that a shock to the inflation target has strong effects on risk premia and that these premia are the primary drivers of real interest rates in the long run. In contrast, the effect of an unexpected monetary policy shock via a simple innovation to the Taylor rule has limited effects on the term premia at all maturities as it dissipates too quickly to have meaningful effects at business cycle

frequencies. Consequentially, the effects on risk premia, which vary primarily at lower frequencies (see, for example, Piazzesi and Swanson, 2008), are limited. This is in stark contrast to shocks that affect monetary policy much more systematically, such as a shock to the inflation target. They affect households' precautionary savings motives much more strongly and so have much stronger effects on the term structure of interest rates across all maturities. Similarly, we find that unconditional forward guidance affects risk premia substantially in a sizable separation from the expectations hypothesis. Specifically, we find that a commitment to a future reduction in the policy rate and constant policy rates until then causes real term premia and inflation risk premia to rise. This follows as agents expect more volatile inflation and output in the future and is in line with the empirical findings of Akkaya et al. (2015).

The present paper offers a first step toward understanding the transmission of monetary policy on the term structure of interest rates from a structural Bayesian perspective, but many salient questions need further investigation. For example, while our model features a frictionless asset trade, a model featuring market segmentation could affect the policy conclusions of our paper (see, for example, Fuerst, 2015). Additionally, a further extension would be the incorporation of the zero lower bound for interest rates, which remains a not fully resolved methodological challenge for nonlinear DSGE models as well affine term structure models. Moreover, investigating the impact of unconventional monetary policy on risk premia or the impact of monetary policy on asset valuation more generally are natural questions of currently high interest. We acknowledge but leave these extensions for future work, providing an estimated macro-finance model in this paper able to provide a structural analysis of the impact of monetary policy on the term structure of interest rates.

A   Appendix

A.1   Model Solution

A.1.1   Stationarized Model

Household:

$$V_t = \left[\frac{e^{\varepsilon_{b,t}}}{1-\gamma} \left(\left(c_t - \frac{bc_{t-1}}{e^{z_t^+}} \right)^{1-\gamma} - 1 \right) + \frac{e^{\varepsilon_{b,t}} \psi_L \left(1 - L_t \right)^{1-\chi}}{1-\chi} \right]$$
$$+ \beta \left(E_t \left[V_{t+1}^{1-\sigma_{EZ}} \right] \right)^{\frac{1}{1-\sigma_{EZ}}} \tag{A-1}$$

$$\lambda_t = e^{\varepsilon_{b,t}} \left(c_t - \frac{bc_{t-1}}{e^{z_t^+}} \right)^{-\gamma} \tag{A-2}$$

$$q_t = \frac{1 - E_t \left[M_{t+1} q_{t+1} \nu \left(\frac{I_{t+1} e^{z_{t+1}^+ + \Psi_{t+1}}}{I_t} - e^{\bar{z}^+ + \bar{\Psi}} \right) e^{\varepsilon_{I,t+1}} \left(\frac{I_{t+1} e^{z_{t+1}^+ + \Psi_{t+1}}}{I_t} \right)^2 \right]}{\left(1 - \frac{\nu}{2} \left(\frac{I_t e^{z_t^+ + \Psi_t}}{I_{t-1}} - e^{\bar{z}^+ + \bar{\Psi}} \right)^2 - \nu \left(\frac{I_t e^{z_t^+ + \Psi_t}}{I_{t-1}} - e^{\bar{z}^+ + \bar{\Psi}} \right) \frac{I_t e^{z_t^+ + \Psi_t}}{I_{t-1}} \right) e^{\varepsilon_{I,t}}} \tag{A-3}$$

$$q_t = E_t \left[\frac{M_{t+1}}{e^{\Psi_{t+1}}} \left(r_{t+1}^k + q_{t+1} \left(1 - \delta \right) \right) \right]$$

$$w_t \lambda_t = e^{\varepsilon_{b,t}} \psi_L \left(1 - L_t \right)^{-\chi} \tag{A-4}$$

$$M_t = \beta e^{-z_{t+1}^+} \frac{\lambda_t}{\lambda_{t-1}} \left(V_t \right)^{-\sigma_{EZ}} E_{t-1} \left[V_t^{\frac{\sigma_{EZ}}{1-\sigma_{EZ}}} \right] \tag{A-5}$$

$$1 = M_{t+1} \frac{\exp \left(R_t^f \right)}{\pi_{t+1}} \tag{A-6}$$

Price setting:

$$K_t^p = e^{\varepsilon_{p,t}} y_t \tilde{p}_t^{-\theta_p} + \gamma_p E_t \left[M_{t+1} \left(\frac{\pi_t^{\xi_p}}{\pi_{t+1}} \right)^{1-\theta_p} \left(\frac{\tilde{p}_t}{\tilde{p}_{t-1}} \right)^{-\theta_p} e^{z_t^+ + 1} K_{t+1}^p \right] \tag{A-7}$$

$$\frac{\theta_p - 1}{\theta_p} K_t^p = y_t mc_t \tilde{p}_t^{-\theta_p - 1} + \gamma_p E_t \left[M_{t+1} \left(\frac{\pi_t^{\xi_p}}{\pi_{t+1}} \right)^{-\theta_p} \left(\frac{\tilde{p}_t}{\tilde{p}_{t-1}} \right)^{-\theta_p - 1} \frac{\theta_p - 1}{\theta_p} e^{z_t^+ + 1} K_{t+1}^p \right] \tag{A-8}$$

$$1 = \gamma_p \left(\frac{\pi_{t-1}^{\xi_p}}{\pi_t} \right)^{1-\theta_p} + \left(1 - \gamma_p \right) \left(\tilde{p}_t \right)^{1-\theta_p} \tag{A-9}$$

INTERMEDIATE GOODS PRODUCER:

$$p_t^+ y_t = e^{a_t} \left(\frac{k_{t-1}}{e^{z_t^+ + \Psi_t}} \right)^\alpha (L_t)^{1-\alpha} - \Phi_t \tag{A-10}$$

$$w_t = mc_t e^{a_t} (1-\alpha) \left(\frac{k_{t-1}}{e^{z_t^+ + \Psi_t}} \right)^\alpha L_t^{-\alpha} \tag{A-11}$$

$$r_t^k = mc_t e^{a_t} \alpha \left(\frac{k_{t-1}}{e^{z_t^+ + \Psi_t}} \right)^{\alpha-1} L_t^{1-\alpha} \tag{A-12}$$

AGGREGATION:

$$k_t = (1-\delta) \frac{k_{t-1}}{e^{z_t^+ + \Psi_t}} + e^{\varepsilon_{I,t}} \left(1 - \frac{\nu}{2} \left(\frac{I_t e^{z_t^+ + \Psi_t}}{I_{t-1}} - e^{\bar{z}^+ + \bar{\Psi}} \right)^2 \right) I_t \tag{A-13}$$

$$p_t^+ = (1-\gamma_p) (\tilde{p}_t)^{-\theta_p} + \gamma_p \left(\frac{\pi_{t-1}^{\xi_p}}{\pi_t} \right)^{-\theta_p} p_{t-1}^+ \tag{A-14}$$

$$y_t = c_t + I_t + \bar{g} e^{g_t} \tag{A-15}$$

$$z_t^+ = \frac{\alpha}{1-\alpha} \bar{\Psi} + z_t \tag{A-16}$$

MONETARY POLICY:

$$R_t^f = \rho_R R_{t-1}^f + (1-\rho_R) \left(\bar{r}^{real} + \frac{\eta_y}{4} \log \left(\frac{y_t}{\bar{y}} \right) + \eta_\pi \log \left(\frac{\tilde{\pi}_t}{\pi_t^*} \right) \right) + \frac{\sigma_m \epsilon_{m,t}}{4} \tag{A-17}$$

$$\log (\pi_t^*) = (1-\rho_\pi) \bar{\pi} + \rho_\pi \log (\pi_{t-1}^*) + \zeta_\pi \log \left(\frac{\pi_{t-1}}{\bar{\pi}} \right) + \sigma_\pi \epsilon_{\pi,t} \tag{A-18}$$

SHOCK PROCESSES:

$$g_t = \rho_g g_{t-1} + \sigma_g \epsilon_{g,t} \tag{A-19}$$

$$a_t = \rho_a a_{t-1} + \sigma_a \epsilon_{a,t} \tag{A-20}$$

$$\varepsilon_{I,t} = \rho_I \varepsilon_{I,t-1} + \sigma_I \epsilon_{I,t} \tag{A-21}$$

$$\varepsilon_{b,t} = \rho_b \varepsilon_{b,t-1} + \sigma_b \epsilon_{b,t} \tag{A-22}$$

$$z_t - \bar{z} = \rho_z (z_{t-1} - \bar{z}) + \sigma_z \epsilon_{z,t} \tag{A-23}$$

$$\log (\Omega_t / \bar{\Omega}) = \rho_\Omega \log (\Omega_{t-1} / \bar{\Omega}) + \sigma_\Omega \epsilon_{\Omega,t} \tag{A-24}$$

A.1.2 Deterministic Steady State

Given our parameterizations for $\frac{\bar{g}}{\bar{y}}$, $\bar{\pi}$, and $\bar{L}$, we can solve for the deterministic steady state as follows:

$$\bar{\tilde{p}} = \left[\frac{1 - \gamma_p \bar{\pi}^{(\xi_p - 1)(1 - \theta_p)}}{1 - \gamma_p} \right]^{\frac{1}{1 - \theta_p}} \tag{A-25}$$

$$\bar{p}^+ = \frac{(1 - \gamma_p)\, \bar{\tilde{p}}^{-\theta_p}}{1 - \gamma_p \bar{\pi}^{(1 - \xi_p)\theta_p}} \tag{A-26}$$

$$\bar{R}^f = \log\left(\frac{\bar{\pi}}{\beta}\right) - (-\gamma(1 - \phi) - \phi)\, \bar{z}^+ \tag{A-27}$$

$$\bar{M} = \beta \exp\left(-\bar{z}^+\right) \tag{A-28}$$

$$\bar{r}^k = \frac{\exp\left(\bar{z}^+ + \bar{\Psi}\right)}{\beta} - (1 - \delta) \tag{A-29}$$

$$\overline{mc} = \bar{\tilde{p}} \frac{\theta_p - 1}{\theta_p} \frac{\left(1 - \gamma_p \beta \bar{\pi}^{(1 - \xi_p)\theta_p}\right)}{\left(1 - \gamma_p \beta \bar{\pi}^{(\xi_p - 1)\theta_p}\right)} \tag{A-30}$$

$$\bar{k} = \bar{L}\left(\frac{\bar{r}^k}{\overline{mc}\,\alpha} \frac{1}{\exp\left(\bar{z}^+ + \bar{\Psi}\right)}\right)^{-\frac{1}{1 - \alpha}} \tag{A-31}$$

$$\bar{w} = \overline{mc}\,(1 - \alpha)\left(\exp\left(\bar{z}^+ + \bar{\Psi}\right)\right)^{-\alpha}\left(\frac{\bar{r}^k}{\overline{mc}\,\alpha} \frac{1}{\exp\left(\bar{z}^+ + \bar{\Psi}\right)}\right)^{\frac{\alpha}{1 - \alpha}} \tag{A-32}$$

$$\bar{y} = \bar{r}^k\left(\frac{\bar{k}}{\exp\left(\bar{z}^+ + \bar{\Psi}\right)}\right) + \bar{w}\bar{L} \tag{A-33}$$

$$\Phi = \left(\frac{\bar{k}}{\exp\left(\bar{z}^+ + \bar{\Psi}\right)}\right)^{\alpha} \bar{L}^{1 - \alpha} - \bar{y}\bar{p}^+ \tag{A-34}$$

$$\bar{I} = \left(1 - \frac{1 - \delta}{\exp\left(\bar{z}^+ + \bar{\Psi}\right)}\right)\bar{k} \tag{A-35}$$

$$\bar{g} = \left(\frac{\bar{g}}{\bar{y}}\right)\bar{y} \tag{A-36}$$

$$\bar{c} = \bar{y} - \bar{g} - \bar{I} \tag{A-37}$$

$$\bar{\lambda} = \left(\bar{c} - \frac{b\bar{c}}{\exp\left(\bar{z}^+\right)}\right)^{-\gamma} \tag{A-38}$$

$$\psi_L = \bar{w}\bar{\lambda}\left(1 - \bar{L}\right)^{\chi} \tag{A-39}$$

$$\bar{K}^p = \frac{\bar{y}\bar{\tilde{p}}^{-\theta_p}}{1 - \gamma_p \beta \bar{\pi}^{(\xi_p - 1)(1 - \theta_p)}} \tag{A-40}$$

$$\bar{V} = \frac{1}{1 - \beta}\left(\frac{\left(\bar{c} - \frac{b\bar{c}}{\exp(\bar{z}^+)}\right)^{1 - \gamma} - 1}{1 - \gamma} - \frac{\psi_L\left(1 - \bar{L}\right)^{1 - \chi}}{1 - \chi}\right) \tag{A-41}$$

A.2 Approximation

A.2.1 Risk-Adjusted Linear Approximation

The method of Meyer-Gohde (2016) differs from others in constructing an approximation centered around a risk-adjusted critical point, such as Juillard (2010), Kliem and Uhlig (2016), and Coeurdacier, Rey, and Winant (2011). First, it is direct and noniterative relying entirely on perturbation methods to construct the approximation. Second, it enables us to construct the approximation around (an approximation of) the ergodic mean of the true policy function instead of its stochastic or "'risky" steady state, placing the locality of our approximation in a region with a likely high (model-based) data density. The closest methods in the macro-finance term structure literature are Dew-Becker (2014) and Lopez, Lopez-Salido, and Vazquez-Grande (2015), who both approximate the nonlinear macro side of the model to obtain a linear in states approximation with adjustments for risk and then derive affine approximation of the yield curve taking this macro approximation as given. The exact meaning of these risk adjustments remains unclear, however, whereas Meyer-Gohde's (2016) method adjusts the coefficients out to the second moments in shocks around the mean of the endogenous variables, itself approximated out to the second moments in shocks.

Thus instead of either a linear certainty-equivalent or nonlinear non-certainty-equivalent approximation, the method constructs a linear non-certainty-equivalent approximation. By using higher order derivatives of the policy function at the deterministic steady state, it approximates the ergodic mean of endogenous variables and the first derivatives of the policy function around this ergodic mean.

Stacking our n_y endogenous variables into the vector y_t and our n_ε normally distributed exogenous shocks into the vector ε_t, we collect our equations into the following vector of nonlinear rational expectations difference equations

$$0 = E_t[f(y_{t+1}, y_t, y_{t-1}, \varepsilon_t)] = \hat{F}(y_{t-1}, \varepsilon_t) \tag{B-1}$$

where f is an $(n_{eq} \times 1)$ vector valued function, continuously M-times differentiable in all its arguments and with as many equations as endogenous variables ($n_{eq} = n_y$).

The solution to the functional problem in (B-1) is the policy function

$$y_t = g^0(y_{t-1}, \varepsilon_t) \tag{B-2}$$

Generally, a closed form for (B-2) is not available, so recourse to numerical approximations is necessary.

We assume that the related deterministic model

$$0 = f(y_{t+1}, y_t, y_{t-1}, 0) = \overline{F}(y_{t-1}, 0) \tag{B-3}$$

admits the calculation of a fix point, the deterministic steady state, defined as $\overline{y} \in \mathbb{R}^{n_y}$ such that $0 = \overline{F}(\overline{y}, 0)$. We are, however, interested in the stochastic version of the model and will now proceed to nest the deterministic model, for which we can recover a fix point, and the stochastic model, for which we cannot, within a larger continuum of models, following standard practice in the perturbation DSGE literature.

We introduce an auxiliary variable $\sigma \in [0, 1]$ to scale the stochastic elements in

the model. The value $\sigma = 1$ corresponds to the "true" stochastic model and $\sigma = 0$ returns the deterministic model in (B-3). Accordingly, the stochastic model, (B-1), and the deterministic model, (B-3), can be nested inside the following continuum of models

$$0 = E_t[f(y_{t+1}, y_t, y_{t-1}, \tilde{\varepsilon}_t)] = F(\sigma, y_{t-1}, \tilde{\varepsilon}_t), \; \tilde{\varepsilon}_t \equiv \sigma \varepsilon_t \qquad \text{(B-4)}$$

with the associated policy function

$$y_t = g(y_{t-1}, \tilde{\varepsilon}_t, \sigma) \qquad \text{(B-5)}$$

Notice that this reformulation allows us to express the deterministic steady state as the fix point of (B-4) for $\sigma = 0$, i.e., $\overline{y} \in \mathbb{R}^{n_y}$ such that $0 = F(0, \overline{y}, 0) = \overline{F}(\overline{y}, 0)$ and, as a consequence $\overline{y} = g(\overline{y}, 0, 0)$. We use this deterministic steady state and derivatives of the policy function in (B-5), recovered by the implicit function theorem,[16] evaluated at at $\overline{y}$ (both in the deterministic model, (B-3), and towards our stochastic model, (B-1), to construct our approximation of and around the ergodic mean.

Since y in the policy function (B-5) is a vector valued function, its derivatives form a hypercube.[17] Adopting an abbreviated notation, we write $g_{z^j \sigma^i} \in \mathbb{R}^{n_y \times n_z^j}$ as the partial derivative of the vector function g with respect to the state vector z_t j times and the perturbation parameter σ i times evaluated at the deterministic steady state.

Instead of using the partial derivatives to construct a Taylor series as is the standard procedure,[18] we would like to construct a more accurate linear approximation of the true policy function (B-2), centered at the mean of y_t. Accordingly, we will construct a linear approximation of (B-2) around the ergodic mean, which we formalize in the following.

Proposition 1. *Linear Approximation around the Ergodic Mean*
Nest the means of the stochastic model ($\sigma = 1$) and of the deterministic model ($\sigma = 0$) through

$$\tilde{y}(\sigma) \equiv E\left[g(y_{t-1}, \sigma \varepsilon_t, \sigma)\right] = E\left[y_t\right] \qquad \text{(B-8)}$$

Then for any $\sigma \in [0, 1]$, the linear approximation of the policy function, (B-2), around

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[16] See Jin and Judd (2002).

[17] We use the method of Lan and Meyer-Gohde (2014) that differentiates conformably with the Kronecker product, allowing us to maintain standard linear algebraic structures to derive our results as follows: Let $A(B) : \mathbb{R}^{s \times 1} \to \mathbb{R}^{p \times q}$ be a matrix-valued function that maps an $s \times 1$ vector B into a $p \times q$ matrix $A(B)$, the derivative structure of $A(B)$ with respect to B is defined as

$$A_B \equiv \mathscr{D}_{B^T}\{A\} \equiv \begin{bmatrix} \frac{\partial}{\partial b_1} & \cdots & \frac{\partial}{\partial b_s} \end{bmatrix} \otimes A \qquad \text{(B-6)}$$

where b_i denotes i'th row of vector B, ^{T} indicates transposition; n'th derivatives are

$$A_{B^n} \equiv \mathscr{D}_{(B^T)^n}\{A\} \equiv \left(\begin{bmatrix} \frac{\partial}{\partial b_1} & \cdots & \frac{\partial}{\partial b_s} \end{bmatrix}^{\otimes[n]} \right) \otimes A \qquad \text{(B-7)}$$

[18] The Taylor series approximation at a deterministic steady state, assuming (B-5) is C^M with respect to all its arguments, can be written as $y_t = \sum_{j=0}^{M} \frac{1}{j!} \left[\sum_{i=0}^{M-j} \frac{1}{i!} g_{z^j \sigma^i} \sigma^i \right] (z_t - \overline{z})^{\otimes[j]}$

the mean of y_t defined in (B-8) and that of ε_t is

$$y_t \simeq \tilde{y}(\sigma) + y_y(\tilde{y}(\sigma), 0, \sigma)(y_{t-1} - \tilde{y}(\sigma)) + y_\varepsilon(\tilde{y}(\sigma), 0, \sigma)\varepsilon_t \tag{B-9}$$

Furthermore, the mean of y_t defined in (B-8) and the two additional unknown functions in this linear approximation

$$\tilde{y}_y(\sigma) \equiv g_y(\tilde{y}(\sigma), 0, \sigma) \tag{B-10}$$
$$\tilde{y}_\varepsilon(\sigma) \equiv g_\varepsilon(\tilde{y}(\sigma), 0, \sigma) \tag{B-11}$$

can be approximated, assuming that they are all analytic in a neighborhood around $\sigma = 0$ with a radius of at least one,[19] using the partial derivatives of (B-5) from the standard nonlinear perturbation around the deterministic steady state.

Proof. See the next subsection. □

A.2.2   Proof of Proposition 1

We will recover the first order partial derivatives by applying the implicit function theorem on (B-4) and higher order partials through successive differentiation.[20]

Beginning with the unknown point of approximation, the ergodic mean, construct a Taylor series around the deterministic steady state

$$\tilde{y}(\sigma) = \tilde{y}(0) + \tilde{y}'(0)\sigma + \frac{1}{2}\tilde{y}''(0)\sigma^2 \dots \tag{B-12}$$

under the assumption of analyticity, the ergodic mean $\tilde{y}(1)$ can be approximated by

$$\tilde{y}(1) \approx \tilde{y}(0) + \tilde{y}'(0) + \frac{1}{2}\tilde{y}''(0) + \dots + \frac{1}{n!}\tilde{y}^{(n)}(0) \tag{B-13}$$

Analogously for the two first derivatives of the policy function (B-2)

$$\tilde{y}_y(1) \approx \tilde{y}_y(0) + \tilde{y}_y{}'(0) + \frac{1}{2}\tilde{y}_y{}''(0) + \dots + \frac{1}{(n-1)!}\tilde{y}_y{}^{(n-1)}(0) \tag{B-14}$$

$$\tilde{y}_\varepsilon(1) \approx \tilde{y}_\varepsilon(0) + \tilde{y}_\varepsilon{}'(0) + \frac{1}{2}\tilde{y}_\varepsilon{}''(0) + \dots + \frac{1}{(n-1)!}\tilde{y}_\varepsilon{}^{(n-1)}(0) \tag{B-15}$$

Note that the approximations of $\tilde{y}_\varepsilon(1)$ and $\tilde{y}_y(1)$ are expressed up to order $n-1$, whereas the approximation of $\tilde{y}(1)$ is expressed up to order n. As the first two are derivatives of the third, terms of the order of $n-1$ in these two are actually of the order n with respect to derivatives of the underlying policy function (B-5), from which we will construct the approximations. Additionally, the assumption of analyticity, here in a domain encompassing both the deterministic steady state and ergodic mean of (B-5), while hardly innocuous,

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[19]This ensures that the Taylor series in these functions converge to the true functions for values of σ including the value of one that transitions to the true stochastic problem.

[20]See Jin and Judd (2002) for a local existence theorem as well as Juillard and Kamenik (2004) for derivations with successive differentiation and Lan and Meyer-Gohde (2014) for solvability conditions for perturbations of arbitrary order.

underlies standard perturbations methods that approximate the stochastic model using derivatives of the meta policy function (B-5) evaluated at the deterministic steady state.

Now we will show that the Taylor series representations of (B-8), (B-10), and (B-11) can be recovered from the derivatives of the policy function (B-5) evaluated at the deterministic steady state used in standard perturbations. We will derive the expressions out to $n = 3$ order, consistent with the goals laid out in the main text.

We will start with (B-8), the point of approximation,

$$\tilde{y}(1) \approx \tilde{y}(0) + \tilde{y}'(0) + \frac{1}{2}\tilde{y}''(0) + \frac{1}{6}\tilde{y}^{(3)}(0) \tag{B-16}$$

we need the four terms on the right hand side—$\tilde{y}(0)$, $\tilde{y}'(0)$, $\tilde{y}''(0)$, and $\tilde{y}^{(3)}(0)$—to construct this approximation. Proceeding in increasing order of differentiation, we begin with $\tilde{y}(0)$. From (B-8),

$$\tilde{y}(0) = E\left[g(y_{t-1}, 0, 0)\right] = g(\overline{y}, 0, 0) = \overline{y} \tag{B-17}$$

the first derivative, $\tilde{y}'(\sigma)$, is

$$\tilde{y}'(0) = \mathscr{D}_\sigma\{E\left[y_t\right]\}\Big|_{\sigma=0} = \mathscr{D}_\sigma\{E\left[g(y_{t-1}, \sigma\varepsilon_t, \sigma)\right]\}\Big|_{\sigma=0} = E\left[\mathscr{D}_\sigma\{g(y_{t-1}, \sigma\varepsilon_t, \sigma)\}\right]\Big|_{\sigma=0} \tag{B-18}$$

where the expectation is with respect to the infinite sequence of $\{\varepsilon_{t-j}\}_{j=0}^\infty$ with invariant i.i.d. distributions, thus and assuming stability of y_t, gives the final equality. Taking derivatives and expectations and evaluating at the deterministic steady state

$$\mathscr{D}_\sigma\{E\left[y_t\right]\}\Big|_{\sigma=0} = g_y\mathscr{D}_\sigma\{E\left[y_{t-1}\right]\}\Big|_{\sigma=0} + g_\varepsilon E\left[\varepsilon_t\right] + g_\sigma \tag{B-19}$$

$$= g_y\mathscr{D}_\sigma\{E\left[y_{t-1}\right]\} \tag{B-20}$$

where the second line follows from the assumption of ε_t being mean zero.[21] Thus,

$$\tilde{y}'(0) = 0 \tag{B-21}$$

as g_y has all its eigenvalues inside the unit circle. The second derivative, $\tilde{y}''(\sigma)$, is

$$\tilde{y}''(0) = \mathscr{D}_{\sigma^2}\{E\left[y_t\right]\}\Big|_{\sigma=0} = E\left[\mathscr{D}_{\sigma^2}\{g(y_{t-1}, \sigma\varepsilon_t, \sigma)\}\right]\Big|_{\sigma=0} \tag{B-22}$$

Taking derivatives and expectations, evaluating at the deterministic steady state, and

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[21]Thus, $E\left[\varepsilon_t\right] = 0$ follows directly and g_σ consequentially, see Schmitt-Grohe and Uribe (2004), Jin and Judd (2002), or Lan and Meyer-Gohde (2014).

recalling results from the first derivative above[22]

$$\mathscr{D}_{\sigma^2}\{E[y_t]\}\Big|_{\sigma=0} = E\left[g_y\mathscr{D}_{\sigma^2}\{y_{t-1}\} + g_{y^2}\mathscr{D}_\sigma\{y_{t-1}\}^{\otimes[2]} + 2g_{y\varepsilon}\varepsilon_t \otimes \mathscr{D}_\sigma\{y_{t-1}\}\right.$$
(B-23)

$$\left.+ 2g_{y\sigma}\mathscr{D}_\sigma\{y_{t-1}\} + 2g_{\varepsilon\sigma}\varepsilon_t + g_{\varepsilon^2}\varepsilon_t^{\otimes[2]} + g_{\sigma^2}\right]\Big|_{\sigma=0}$$

$$= g_y\mathscr{D}_{\sigma^2}\{E[y_{t-1}]\}\Big|_{\sigma=0} + g_{y^2}E\left[\mathscr{D}_\sigma\{y_{t-1}\}^{\otimes[2]}\right]\Big|_{\sigma=0}$$

$$+ g_{\varepsilon^2}E\left[\varepsilon_t^{\otimes[2]}\right] + g_{\sigma^2}$$

$$= g_y\mathscr{D}_{\sigma^2}\{E[y_{t-1}]\}\Big|_{\sigma=0} + g_{y^2}\left(I_{n_y} - g_y^{\otimes[2]}\right)^{-1}g_\varepsilon^{\otimes[2]}E\left[\varepsilon_t^{\otimes[2]}\right]$$

$$+ g_{\varepsilon^2}E\left[\varepsilon_t^{\otimes[2]}\right] + g_{\sigma^2}$$

$$\tilde{y}''(0) = \mathscr{D}_{\sigma^2}\{E[y_t]\}\Big|_{\sigma=0} = \left(I_{n_y} - g_y\right)^{-1}\left(\left(g_{\varepsilon^2} + \left(I_{n_y} - g_y^{\otimes[2]}\right)^{-1}g_\varepsilon^{\otimes[2]}\right)E\left[\varepsilon_t^{\otimes[2]}\right] + g_{\sigma^2}\right)$$

where the second to last equality follows[23]—taking expectations, evaluating at the deterministic steady state, and recalling results from the first derivative above—as

$$E\left[\mathscr{D}_\sigma\{y_t\}^{\otimes[2]}\right]\Big|_{\sigma=0} = E\left[\left(g_y\mathscr{D}_\sigma\{y_{t-1}\} + g_\varepsilon\varepsilon_t + g_\sigma\right)^{\otimes[2]}\right]\Big|_{\sigma=0}$$

$$= g_y^{\otimes[2]}E\left[\mathscr{D}_\sigma\{y_{t-1}\}^{\otimes[2]}\right]\Big|_{\sigma=0} + g_\varepsilon^{\otimes[2]}E\left[\varepsilon_t^{\otimes[2]}\right]$$
(B-24)

Thus, $\tilde{y}''(0)$ adjusts the zeroth order mean $\tilde{y}(0)$ or deterministic steady state for the cumulative—$\left(I_{n_y} - g_y\right)^{-1}$—influence of the variance of shocks, directly through $E\left[\varepsilon_t^{\otimes[2]}\right]$ and indirectly through the influence of risk on the policy function captured by g_{σ^2}. The third derivative, $\tilde{y}^{(3)}(0)$, is

$$\tilde{y}(3)(0) = \mathscr{D}_{\sigma^3}\{E[y_t]\}\Big|_{\sigma=0} = E\left[\mathscr{D}_{\sigma^3}\{g(y_{t-1}, \sigma\varepsilon_t, \sigma)\}\right]\Big|_{\sigma=0}$$
(B-25)

Taking derivatives and expectations, evaluating at the deterministic steady state, and recalling results from the first two derivatives above

$$\mathscr{D}_{\sigma^3}\{E[y_t]\}\Big|_{\sigma=0} = E\left[g_{y^3}\mathscr{D}_\sigma\{y_{t-1}\}^{\otimes[3]} + 3g_{y^2\varepsilon}\mathscr{D}_\sigma\{y_{t-1}\}^{\otimes[2]} \otimes \varepsilon_t + 3g_{y^2\sigma}\mathscr{D}_\sigma\{y_{t-1}\}^{\otimes[2]}\right.$$

$$+ 3g_{y\varepsilon^2}\mathscr{D}_\sigma\{y_{t-1}\} \otimes \varepsilon_t^{\otimes[2]} + 6y_{y\varepsilon\sigma}\mathscr{D}_\sigma\{y_{t-1}\} \otimes \varepsilon_t + 3g_{y\sigma^2}\mathscr{D}_\sigma\{y_{t-1}\}$$

$$+ y_{\varepsilon^3}\varepsilon_t^{\otimes[3]} + 3g_{\varepsilon^2\sigma}\varepsilon_t^{\otimes[2]} + 3g_{\varepsilon\sigma^2}\varepsilon_t + g_{\sigma^3} + 3g_{y^2}\mathscr{D}_{\sigma^2}\{E[y_t]\} \otimes \mathscr{D}_\sigma\{E[y_t]\}$$

$$\left.+ 3g_{y\varepsilon}\mathscr{D}_{\sigma^2}\{E[y_t]\}\varepsilon_t + 3y_{y\sigma}\mathscr{D}_{\sigma^2}\{E[y_t]\} + g_y\mathscr{D}_{\sigma^3}\{y_{t-1}\}\right]\Big|_{\sigma=0}$$
(B-26)

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[22]The notation $x^{\otimes[n]}$ represents Kronecker powers, $x^{\otimes[n]}$ is the n'th fold Kronecker product of x with itself: $x \otimes x \cdots \otimes x$.

[23]The second line follows as $g_{y\sigma}$ and $g_{\varepsilon\sigma}$ are zero, see Schmitt-Grohe and Uribe (2004), Jin and Judd (2002), or Lan and Meyer-Gohde (2014).

From our assumption of mean-zero, normally distributed shocks, it follows that

$$\tilde{y}(3)(0) = \mathscr{D}_{\sigma^3}\{E\left[y_t\right]\}\Big|_{\sigma=0} = 0 \tag{B-27}$$

as third derivatives of g involving derivatives with respect of σ only once are zero,[24] terms cubic in ε_t (either directly or through products involving $\mathscr{D}_{\sigma}\{y_{t-1}\}$, which is linear in ε_t, or $\mathscr{D}_{\sigma^2}\{y_{t-1}\}$, which is quadratic in ε_t), and g_{σ^3} are all zero in accordance with the symmetry of the normal distribution.[25]

Moving on to the derivative of the policy function with respect to y_{t-1}, (B-10), for small deviations of y_{t-1} and ε_t from their respective means

$$\tilde{y}_y(1) \approx \tilde{y}_y(0) + \tilde{y}_y{}'(0) + \frac{1}{2}\tilde{y}_y{}''(0) \tag{B-28}$$

we need the three terms on the right hand side—$\tilde{y}_y(0)$, $\tilde{y}_y{}'(0)$, and $\tilde{y}_y{}''(0)$. Starting with $\tilde{y}_y(0)$,

$$\tilde{y}_y(0) = \mathscr{D}_{y_{t-1}}\{y_t\}\Big|_{\sigma,\varepsilon_t=0} = \mathscr{D}_{y_{t-1}}\{g(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma,\varepsilon_t=0} = g_y \tag{B-29}$$

Turning to $\tilde{y}_y{}'(0)$

$$\begin{aligned}
\tilde{y}_y{}'(0) = \mathscr{D}_{\sigma y_{t-1}}\{y_t\}\Big|_{\sigma,\varepsilon_t=0} &= \mathscr{D}_{\sigma y_{t-1}}\{g(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma,\varepsilon_t=0} \\
&= \mathscr{D}_{\sigma}\{g_y(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma,\varepsilon_t=0} \\
&= g_{y^2}\mathscr{D}_{\sigma}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0} \otimes I_{n_y} + g_{\sigma y} \\
&= 0 \tag{B-30}
\end{aligned}$$

The first term is zero as $\mathscr{D}_{\sigma}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0}$ was shown to be zero above and the second is equal to zero following standard results in the perturbation literature as discussed above. Finally, $\tilde{y}_y{}''(0)$

$$\begin{aligned}
\tilde{y}_y{}''(0) = \mathscr{D}_{\sigma^2 y_{t-1}}\{y_t\}\Big|_{\sigma,\varepsilon_t=0} &= \mathscr{D}_{\sigma^2 y_{t-1}}\{g(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma,\varepsilon_t=0} \\
&= \mathscr{D}_{\sigma^2}\{g_y(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma=0} \\
&= g_{y^3}\mathscr{D}_{\sigma}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0}^{\otimes[2]} \otimes I_{n_y} + 2g_{\sigma y^2}\mathscr{D}_{\sigma}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0} \otimes I_{ny} \\
&\quad + g_{y^2}\mathscr{D}_{\sigma^2}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0} \otimes I_{n_y} + g_{\sigma^2 y} \\
&= g_{y^2}\mathscr{D}_{\sigma^2}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0} \otimes I_{n_y} + g_{\sigma^2 y} \tag{B-31}
\end{aligned}$$

The final equality follows as $\mathscr{D}_{\sigma}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0}$ and $g_{\sigma y^2}$ are both zero following the results

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[24]See Andreasen (2012b), Jin and Judd (2002), or Lan and Meyer-Gohde (2014).

[25]See Andreasen (2012b) for perturbations with skewed distributions.

and discussions above.

Finally, the derivative of the policy with respect to ε_t, (B-11), follows analogously to the derivative with respect to y_{t-1},

$$\tilde{y}_\varepsilon(1) \approx \tilde{y}_\varepsilon(0) + \tilde{y}_\varepsilon{}'(0) + \frac{1}{2}\tilde{y}_\varepsilon{}''(0) \tag{B-32}$$

Again, we need the three terms on the right hand side—$\tilde{y}_\varepsilon(0)$, $\tilde{y}_\varepsilon{}'(0)$, and $\tilde{y}_\varepsilon{}''(0)$. Starting with $\tilde{y}_\varepsilon(0)$,

$$\tilde{y}_\varepsilon(0) = \mathscr{D}_{\varepsilon_t}\{y_t\}\Big|_{\sigma,\varepsilon_t=0} = \mathscr{D}_{\varepsilon_t}\{g(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma,\varepsilon_t=0} = g_\varepsilon \tag{B-33}$$

then $\tilde{y}_\varepsilon{}'(0)$

$$\begin{aligned}
\tilde{y}_\varepsilon{}'(0) = \mathscr{D}_{\sigma\varepsilon_t}\{y_t\}\Big|_{\sigma,\varepsilon_t=0} &= \mathscr{D}_{\sigma\varepsilon_t}\{g(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma,\varepsilon_t=0} \\
&= \mathscr{D}_\sigma\{g_\varepsilon(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma,\varepsilon_t=0} \\
&= g_{y\varepsilon}\mathscr{D}_\sigma\{\tilde{y}(\sigma)\}\Big|_{\sigma=0} \otimes I_{n_\varepsilon} + g_{\sigma\varepsilon} \\
&= 0 \tag{B-34}
\end{aligned}$$

The first term is zero as $\mathscr{D}_\sigma\{\tilde{y}(\sigma)\}\Big|_{\sigma=0}$ was shown to be zero above and the second is equal to zero following standard results in the perturbation literature as discussed above. Finally, $\tilde{y}_y{}''(0)$

$$\begin{aligned}
\tilde{y}_\varepsilon{}''(0) = \mathscr{D}_{\sigma^2\varepsilon_t}\{y_t\}\Big|_{\sigma,\varepsilon_t=0} &= \mathscr{D}_{\sigma^2\varepsilon_t}\{g(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma,\varepsilon_t=0} \\
&= \mathscr{D}_{\sigma^2}\{g_\varepsilon(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma=0} \\
&= g_{y^2\varepsilon}\mathscr{D}_\sigma\{\tilde{y}(\sigma)\}\Big|_{\sigma=0}^{\otimes[2]} \otimes I_{n_\varepsilon} + 2g_{\sigma y\varepsilon}\mathscr{D}_\sigma\{\tilde{y}(\sigma)\}\Big|_{\sigma=0} \otimes I_{n_\varepsilon} \\
&\quad + g_{y\varepsilon}\mathscr{D}_{\sigma^2}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0} \otimes I_{n_\varepsilon} + g_{\sigma^2\varepsilon} \\
&= g_{y\varepsilon}\mathscr{D}_{\sigma^2}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0} \otimes I_{n_\varepsilon} + g_{\sigma^2\varepsilon} \tag{B-35}
\end{aligned}$$

The final equality follows as $\mathscr{D}_\sigma\{\tilde{y}(\sigma)\}\Big|_{\sigma=0}$ and $g_{\sigma y\varepsilon}$ are both zero following the results and discussions above.

43

A.3 Data

In this paper we use several macro and financial time series. This appendix describes some modifications and especially the source of the raw data.

Real GDP: This series is *BEA NIPA table 1.1.6 line 1 (A191RX1)*.

Nominal GDP: This series is *BEA NIPA table 1.1.5 line 1 (A191RC1)*.

Implicit GDP Deflator: The implicit GDP deflator is calculated as the ratio of **Nominal GDP** to **Real GDP**.

Private Consumption: Real consumption expenditures for non-durables and services is the sum of the respective nominal values of the *BEA NIPA table 1.1.5 line 5 (DNDGRC1)* and *BEA NIPA table 1.1.5 line 6 (DNDGRC1)* and finally deflated by the deflator mentioned above.

Private Investment: Total real private investment is the sum of the respective nominal values of the series Gross Private Investment *BEA NIPA table 1.1.5 line 7 (A006RC1)* and Personal Consumption Expenditures: Durable Goods *BEA NIPA table 1.1.5 line 4 (DDURRC1)* and finally deflated by the deflator mentioned above.

Civilian Population: This series is calculated from monthly data of *civilian noninstitutional population over 16 years (CNP16OV)* from the U.S. Department of Labor: Bureau of Labor Statistics.

Policy Rate: The quarterly policy rates is the 3-Month Treasury Bill: Secondary Market Rate *TB3MS* provided by Board of Governors of the Federal Reserve System. The quarterly aggregation is end of period.

Treasury Bond Yields: The quarterly series for 1-year, 2-year, 3-year, 5-year, and 10-year zero-coupon bond yields re measured end of quarter. The original series are daily figures based on the updated series by Adrian et al. (2013).
Source: https://www.newyorkfed.org/research/data_indicators/term_premia.html

Nominal Interest Rate Forecasts: The quarterly series for 1-quarter (TBILL3) and 4-quarter (TBILL6) ahead forecasts of the nominal 3month Treasury Bill. The time series are the median responses by the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia.
Source: https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/data-files

A.4 Supplementary Results
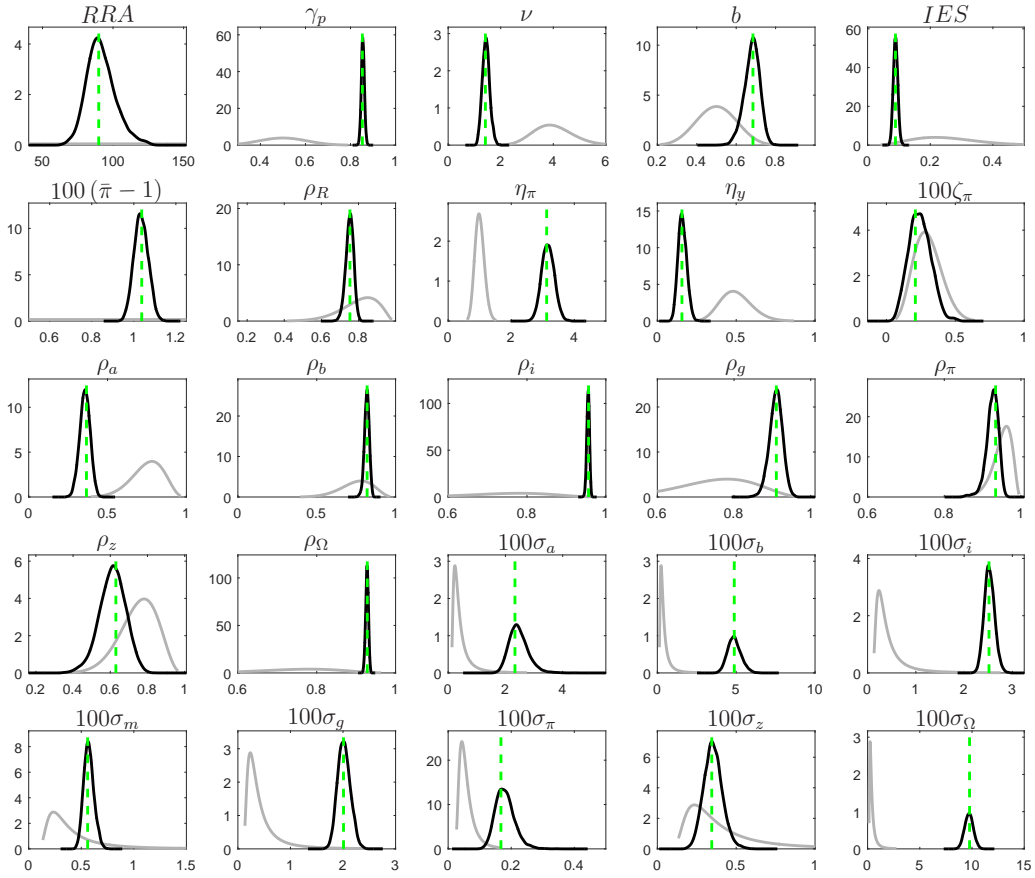
A.4.1 Initial Prior vs Posterior Plots

Figure 10: Prior (gray) and posterior (black) distribution of the model parameters, the green dashed line indicates the posterior mode.
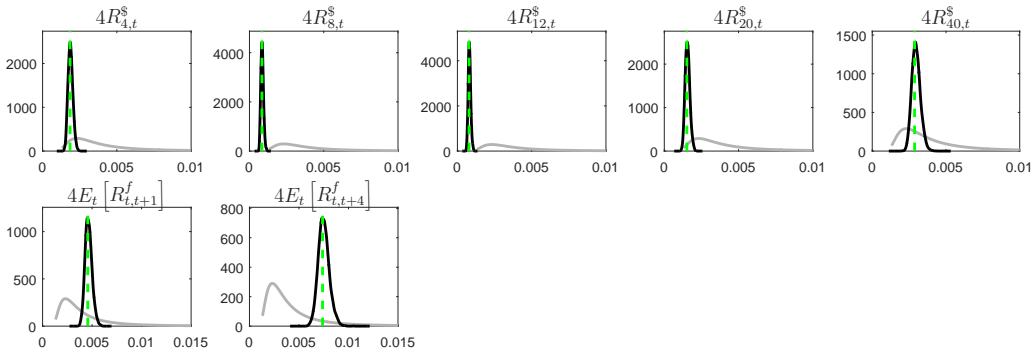

Figure 11: Prior (gray) and posterior (black) distribution of measurement errors, the green dashed line indicates the posterior mode.
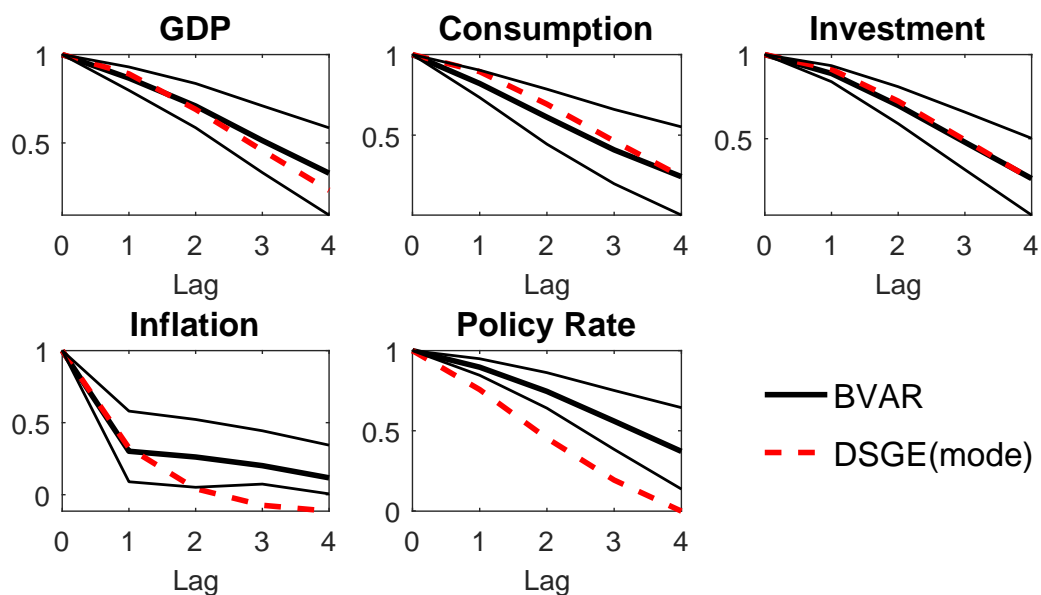
A.4.2 Predicted Moments

Figure 12: Predicted autocorrelation of selected HP-filtered macro variables at the posterior mode and the corresponding population moments of the data calculated by using a Bayesian vector autoregression model with two lags. The thin black lines represent the 90% probability bands.

Name	Symbol	Mean			S.d.		
		50%	5%	95%	50%	5%	95%
1-year real T-Bill	$R_{4,t}$	2.68	2.59	2.77	2.13	2.02	2.25
2-year real T-Bill	$R_{8,t}$	3.00	2.92	3.08	1.83	1.73	1.94
3-year real T-Bill	$R_{12,t}$	3.17	3.08	3.25	1.62	1.53	1.72
5-year real T-Bill	$R_{20,t}$	3.33	3.23	3.42	1.32	1.25	1.39
10-year real T-Bill	$R_{40,t}$	3.74	3.62	3.85	0.86	0.82	0.91
1-year nominal term premium	$TP^{\$}_{4,t}$	37.35	35.20	39.81	10.86	9.57	12.35
2-year nominal term premium	$TP^{\$}_{8,t}$	77.08	73.46	81.15	24.18	21.78	27.06
3-year nominal term premium	$TP^{\$}_{12,t}$	99.63	95.13	104.39	32.04	28.95	35.55
5-year nominal term premium	$TP^{\$}_{20,t}$	129.06	124.09	134.27	40.38	36.57	44.62
10-year nominal term premium	$TP^{\$}_{40,t}$	202.62	197.14	208.41	52.22	46.83	58.84
1-year real term premium	$TP_{4,t}$	24.01	21.68	26.49	5.85	4.95	6.97
2-year real term premium	$TP_{8,t}$	57.10	52.77	61.54	16.22	14.21	18.58
3-year real term premium	$TP_{12,t}$	74.64	68.83	80.41	22.36	20.04	25.17
5-year real term premium	$TP_{20,t}$	93.03	85.72	100.71	27.91	25.44	30.97
10-year real term premium	$TP_{40,t}$	138.88	128.89	148.57	35.77	32.74	39.28
1-year inflation risk premium	$TP^{\pi}_{4,t}$	13.37	12.28	14.43	5.17	4.61	5.79
2-year inflation risk premium	$TP^{\pi}_{8,t}$	20.07	17.74	22.47	8.18	7.15	9.33
3-year inflation risk premium	$TP^{\pi}_{12,t}$	25.01	21.50	28.53	9.84	8.39	11.55
5-year inflation risk premium	$TP^{\pi}_{20,t}$	35.93	30.47	41.07	12.48	10.34	14.94
10-year inflation risk premium	$TP^{\pi}_{40,t}$	63.90	54.83	72.49	16.52	13.49	20.22

Table 6: Predicted first and second moments of further financial variables. All returns are measured in annualized percentage points and all risk premia are measured in annualized basis points.

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