Bank Runs, Prudential Tools and Social Welfare in a Global Game General Equilibrium Model*

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Abstract

I develop a general equilibrium model of bank runs in a global game framework. The model features banking crises triggered by endogenous system-wide bank runs. The bank run probability – systemic risk – is increasing in bank leverage and decreasing in banks' liquid assets. A market structure in which only interest rates work as a market signal and pecuniary externalities lead to excessive leverage and insufficient liquidity, elevating systemic risk, in a competitive equilibrium. Addressing the inefficiencies requires the implementation of prudential tools on both capital and liquidity. I extend the model to study sectoral capital requirements, risk weights, deposit insurance, shadow banking, risk migration and risk-taking. The model provides a unified framework for analysing banking crises, banks' behaviour and prudential policy.

Keywords: Bank runs; global game; capital requirements; liquidity requirements; sectoral requirements; shadow banking; risk-taking.

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1 Introduction

Historically bank runs were at the centre of financial crises. Financial crises were always bank runs prior to the existence of central banks and most of them involved bank runs in the period since 1970 (Gorton 2012). The global financial crisis that began in the United States in the summer of 2007 was no exception. It was set off by runs to money-like bank debt such as repo and asset-backed commercial papers. In effect, as put by Reinhart and Rogoff (2009), 'for the advanced economies during 1800–2008, the picture that emerges is one of serial banking crises.'

Since the global financial crisis, practice has evolved much faster than theory as policy makers have strived to promote a more resilient financial system. One manifestation of this effort is the establishment of the global regulatory framework for banks and banking systems, so called Basel III (Basel Committee on Banking Supervision 2011, 2013). The framework incorporates prudential instruments on capital and liquidity with their goal of promoting a more resilient banking system to systemic risks.

To catch up with these policy developments, what is needed for theory is a model of prudential instruments that helps us identify externalities and examine the coordination of instruments, especially on capital and liquidity. In light of the enhancement of the gloabl regulatory framework and its objective, three ingredients are essential for developing such a model. The first ingredient is a systemic risk event that triggers a banking crisis. The second is banking system resilience to such an event. The third is externalities that warrant the implementation of prudential instruments.

This paper aims to fill the gap between practice and theory by developing a model of prudential instruments that features the three essential ingredients. Specifically, it embeds a bank run global game model studied by Rochet and Vives (2004) into a two-period general equilibrium model in the spirit of Christiano and Ikeda (2013, 2016). In view of bank runs as an essential aspect of banking crises, the model features bank runs as a systemic event. The probability of bank runs – systemic risk – is endogenously determined as a function of bank leverage and liquidity. Thus, the model has a link between systemic risk and fundamentals of the banking system, the latter of which forms banking system resilience. Using the model, the paper highlights the source of externalities that are unique to the global game.

To identify externalities and conduct a welfare analysis, this paper first presents a benchmark model in which banks choose leverage only. The model consists of three types of agents: households, banks and fund managers. Households and banks receive endowment in the beginning of the first period, which corresponds to household income and bank capital, respectively. Households allocate the income into current consumption and bank deposit. Banks offer a deposit contract such that banks pay a pre-determined interest rate as long as they do not default and that the funds can be withdrawn early in the beginning of the second period at the same interest rate. Banks combine deposits and bank capital to invest in a project that is subject to aggregate risk. If the project return is low enough, the banks, unable to pay the interest rate, default and depositors receive a remained portion of the liquidated value of the banks. To avoid such a loss, households delegate the decision and implementation of early withdrawal to fund managers who have private information about the bank asset return. But early withdrawal is costly for banks because early liquidated assets generate a lower return than assets that are held until the maturity becomes due. This costly liquidation gives rise to the risk of banks default if a large number of fund managers withdraw funds early. This structure leads to a global game in which a bank run is determined uniquely: it occurs if the bank asset return is lower than a certain threshold. Both households and banks take into account the bank run probability in choosing how much to lend and borrow, respectively. In the second period, banks distribute the profits to households, who consume everything in hand.

A unique feature of this model is that bank leverage is pinned down without any binding borrowing constraints. But for bank runs, banks would increase leverage as long as the expected bank asset return is greater than the interest rate, as in the various financial friction models studied by Christiano and Ikeda (2013). But, with bank runs, a higher leverage increases bank-run-led default probability and thereby decreases bank profits. Taking a balance between the two, bank leverage has an interior solution.

The paper analytically shows that bank leverage is excessive in a competitive equilibrium, for two reasons: risk-insensitivity and pecuniary externalities. First, it is assumed that households can observe the riskness of the banking industry as a whole but cannot observe the riskiness of individual banks. This risk-insensitivity leads to a market structure in which only deposit interest rates work as a market signal. Given deposit interest rates, which do not necessarily reflect individual banks' riskiness, banks maximise profits. In doing so, banks ignore the potential effects of leverage on interest rates, which results in excessive leverage. Second, the model has a pecuniary externality that works through the interest rate. The cost of bank runs depends on the interest rate, which, in turn, is affected by the leverage. Banks ignore this externality because they take prices as given in a competitive equilibrium. The source of these two types of inefficiencies is the combination of banks' limited liability, deposit contracts, bank run risk and costly early liquidation.

Excessive leverage warrants prudential instruments that restrict leverage to lower systemic risk. Doing so, however, involves a trade-off by restricting financial intermediation, which has a negative effect on households' consumption smoothing. Prudential policy has to take a right balance between stabilizing the financial system and promoting the real economy.

Next, the paper extends the benchmark model to incorporate a bank liquidity choice. In this model, banks choose how much liquidity to hold in addition to the amount of loans to a risky project, taking into account that such choices will affect their bank run probability. The analytical results reveal that in a competitive equilibrium liquidity is insufficient given leverage; leverage is excessive given liquidity. This warrants the implementation of both prudential instruments on capital/leverage and liquidity.

The two policy tools – a leverage restriction and a liquidity requirement – are strategic substitutes in the following sense. To achieve a certain level of social welfare, a tightened leverage restriction is associated with a loosened liquidity requirement. However, the liquidity requirement is still binding, because a tightened leverage restriction would lead to less liquidity holdings without restrictions. This implies that one instrument only – either leverage or liquidity – will lead to risk migration from one to another, attenuating the intended effects of the instrument. Indeed, a numerical example shows that tightening leverage only can increase systemic risk as banks respond by holding less liquidity. Therefore, although the two instruments are substitutes, both are essential to address system-wide bank run risk.

The benchmark model has rich applications for banks' behaviour and other prudential instruments. This paper further modifies the benchmark model to incorporate heterogeneous sectors/banks to study sectoral capital requirements and risk weights. The modified model is also useful to study shadow banking and risk migration from one type of banks to another. A slightly different version of the modified model allows us to study banks' loan portfolio choice and bank risk-taking. This model shows that banks take more risk than a socially desirable level by not diversifying loan portfolio perfectly. The fact that banks ignore the marginal effect of their choices on bank runs in bank default states induces banks to take more risk to gain from upper-tail returns.

For a further application the paper considers a role of deposit insurance. Deposit insurance has been regarded as an institutional milestone for addressing bank runs by retail depositors. In the model, however, bank runs persist as long as deposit insurance is imperfect, which is the case for large depositors and non-banks in practice. Worse, the paper shows that imperfect deposit insurance exacerbates excessive leverage and elevates systemic risk. This is because households, failing to evaluate the risk associated with bank deposits due to deposit insurance, lend to banks more than otherwise would be the case, leading to a further rise in bank leverage.

The paper is related to recent developments in macroeconomic models of bank runs, which include Ennis and Keister (2003), Martin, et al. (2014), Gertler and Kiyotaki (2015) and Kashyap et al. (2017), all of which build on the idea of Diamond and Dybvig (1983) bank run model, and Angeloni and Faia (2013), which extends Diamond and Rajan (2000, 2001) bank run models. This paper differs by developing a general equilibrium model of banks in a global game framework, building on Rochet and Vives (2004).[1] In addition, it analytically clarifies the source of inefficiencies and studies the role of prudential policy.

The rest of the paper is organized as follows. Section 2 presents the benchmark model in which banks choose leverage only. Section 3 conducts welfare analysis on the model. Section 4 extends the model to incorporate bank liquidity and studies roles and interactions of leverage and liquidity requirements. Section 5 presents further extensions of the benchmark model to study sectoral capital/leverage requirements, risk weights, deposit insurance, shadow banking, risk migration and risk-taking. Section 6 concludes by laying out a plan for future research.

2 Model

2.1 Environment

The model has two periods, $t = 1, 2$. There is a good, which can be used for consumption and investment. The economy is inhabited by three types of agents: households, fund managers and banks. Each type consists of a continuum of agents with measure unity. Banks are owned by households. In period $t = 1$, households and banks receive endowment y and n of the good, respectively. Households consume, and save in banks for next period consumption. Banks invest the sum of bank capital n and deposits in a risky project. Fund managers, as delegates of households, manage funds by deciding whether to withdraw funds

[1]The global game of Rochet and Vives (2004) is incorporated into the Bank of Canada's stress-test model for the banking sector (Fique 2017).

earlier. In period $t = 2$, banks pay interest and transfer their profits to households, who consume all available resources.

2.2 Households

For each household preferences are characterized by quasi-linear utility,

$$u(c_1) + \mathbb{E}(c_2),$$

where c_t is consumption in period t, $\mathbb{E}(\cdot)$ is an expectation operator, and $u(\cdot)$ is a strictly increasing, strictly concave and twice differentiable function and satisfies $\lim_{c_1 \to 0} u'(c_1) = \infty$. In period $t = 1$ households consume c_1 and make a bank deposit of d, subject to the flow budget constraint, $c_1 + d \leq y$. A contract between households and banks is a deposit contract. Specifically, banks pay an interest rate of vR, where R is a promised fixed rate of interest and v is a discount rate which takes 1 if banks do not default and $v < 1$ if they default. Although households can diversify deposits over a continuum of banks, such diversification does not affect default probability, because all banks default, if any, at the same time, as will be shown later. Households delegate the management of deposits to fund managers because fund managers have an information advantage and thereby they can withdraw funds early at a right timing, as will be elaborated in Section 2.3. Households are assumed to diversify the management of their funds in banks over a continuum of fund managers, so that the realization of v is the same for all households, which allows the model to keep the representative agent framework. In period $t = 2$, households consume c_2, subject to $c_2 \leq vRd + \pi$, where π is bank profits. Both R and v are endogenously determined.

Let P denote the probability of bank default, rationally expected by households in period $t = 1$. Then, solving the household problem yields the upward-sloping supply curve of deposits:

$$R = \frac{u'(y - d)}{1 - P + \mathbb{E}(v|\text{default})P}, \tag{1}$$

where $\mathbb{E}(\cdot|\text{default})$ is an expectation operator conditional on bank default.

2.3 Fund managers

Fund managers are risk-neutral. They have information advantage over households about a stochastic bank return on a risky project R^k. In the beginning of period $t = 2$, just after R^k is realized, but before it is known by households, fund manager $i \in (0, 1)$ receives a private noisy signal s_i about R^k, which follows a normal distribution:

$$s_i = R^k + \epsilon_i, \quad \text{with} \quad \epsilon_i \sim N\left(0, \sigma_\epsilon^2\right).$$

As $\sigma_\epsilon \to 0$ fund managers' private information becomes more accurate, being closer to the true value R^k. Although s_i itself is private information, the distribution is public information.

A role of fund managers is to decide whether to withdraw funds early from a bank and to execute a withdrawal, if any, by taking advantage of their private information. If a fund manager, as a delegate of a household, withdraws early and the bank is solvent at this stage, the fund manager secures R per unit of funds and the household receives R per unit of deposit. But if a fund manager does not withdraw and the bank defaults later, the household receives an interest rate strictly less than R. Only fund managers can provide this professional service of early withdrawal with a right timing.

For simplification a contract between households and fund managers is exogenously given as in Rochet and Vives (2004). In particular, a net benefit of withdrawal over non-withdrawal for fund managers is given by $\Gamma_0 > 0$ if the bank defaults and $-\Gamma_1 < 0$ if the bank survives. This benefit structure implies that fund managers are rewarded if they make a right decision, i.e., withdrawing if the bank defaults or not withdrawing if the bank does not default. Also, it implies that executing a withdrawal may be costly because, for example, it requires skills and efforts to do so.

Because of this benefit structure fund managers' decision of withdrawal is made based on their private information. Let P_i denote the probability of bank default, rationally expected by fund manager i by private information s_i. Then, the fund manager withdraws if and only if $P_i\Gamma_0 + (1 - P_i)(-\Gamma) > 0$, that is

$$P_i > \frac{\Gamma_1}{\Gamma_0 + \Gamma_1} \equiv \gamma. \tag{2}$$

This condition implies that fund managers' withdrawal decision depends on a ratio γ, but not the absolute values of Γ_0 and Γ_1. A key assumption is that the values of Γ_0 and

Γ_1 are infinitesimally small, so that these values are ignored in the general equilibrium consideration. One justification could be that perfect competition among fund managers drive down the net benefits of Γ_0 and Γ_1.

As shown by Rochet and Vives (2004) in this environment fund managers employ a threshold strategy such that they withdraw if and only if $s_i < \bar{s}$. The threshold $\bar{s}$ is determined jointly with banks' problem described below.

2.4 Banks

In period $t = 1$, banks offer a deposit contract to households and take in a deposit of d. Banks combine their net worth n and the deposit d and invest in a risky project with a stochastic return R^k, which follows a normal distribution:

$$R^k \sim N\left(\bar{R}^k, \sigma^2_{R^k}\right),$$

This process is public information. In the beginning of period $t = 2$, R^k is realized. But, before the return $R^k(n + d)$ is finalized, some fund managers may withdraw their funds from banks. This early liquidation is costly: early liquidation of one unit of bank asset generates only a fraction $1/(1 + \lambda)$ of R^k, where $\lambda > 0$. Let x denote the number of fund managers who withdraw funds. Then, to cover the early withdrawal of xRd, banks have to liquidate $(1+\lambda)xRd/R^k$ units of bank assets. After liquidating some assets, banks have $R^k(n + d) - (1 + \lambda)xRd$ in hand. If this amount is less than the promised payment under the deposit contract, $(1 - x)Rd$, banks go bankrupt. That is, banks default if and only if

$$R^k < R\left(1 - \frac{1}{L}\right)(1 + \lambda x), \tag{3}$$

where $L \equiv (n + d)/n$ is bank leverage.

Under the threshold strategy for a withdrawal, $s_i < \bar{s}$, the number of fund managers who withdraw is given by $x(R^k, \bar{s}) = \Pr(s_i < \bar{s}) = \Pr(\epsilon_i < \bar{s} - R^k) = \Phi((\bar{s} - R^k)/\sigma_\epsilon)$, where $\Phi(\cdot)$ is a standard normal distribution function. Condition (3) implies that the probability of bank default perceived by fund manager i is given by

$$P_i = \Pr\left(R^k < R\left(1 - \frac{1}{L}\right)\left[1 + \lambda x(R^k, \bar{s})\right] |s_i\right). \tag{4}$$

Conditions (2)-(4) imply that the equilibrium threshold $\bar{s}^*$ is a solution to the following set of equations as shown by Rochet and Vives (2004):

$$\Pr\left(R^k < R^{k*} | \bar{s}^*\right) = \gamma, \tag{5}$$

$$R^{k*} = R\left(1 - \frac{1}{L}\right)\left[1 + \lambda x(R^{k*}, \bar{s}^*)\right]. \tag{6}$$

Both $\bar{s}^*$ and R^{k*} depend on the interest rate R and the leverage L. In particular, an increase in the leverage raises $\bar{s}^*$ and R^{k*} so that more fund managers withdraw funds and the probability of bank default increases.

Banks choose leverage L to maximize expected profits $\mathbb{E}(\pi)$. This objective is consistent with households' bank ownership and their quasi-linear utility. In choosing leverage banks take into account that they default if and only if $R^k < R^{k*}$. Banks are protected by limited liability so that profits are zero when they default. Banks are subject to a regulatory restriction such that leverage should not be too high: $L \leq L_{\max}$. It is worth noting that this regulatory restriction differs from a macroprundential instrument introduced later. With a high enough $L_{\max}$, the restriction is not binding in equilibrium, but it plays a role of excluding an uninteresting solution of $L = \infty$ as I will discuss shortly. One interpretation of this restriction is that regulators prohibit banks from becoming too risky by having too high leverage. Such an upper bound could be $L_{\max} = (y-1)/n$ at which households lend all their funds to banks.

The problem of banks is written as

$$\max_{\{L\}} \int_{R^{k*}(L)}^{\infty} \left\{R^k L - R\left[1 + \lambda x\left(R^k, \bar{s}^*(L)\right)\right](L-1)\right\} n dF(R^k),$$

subject to $L \leq L_{\max}$, where $F(\cdot)$ is a normal distribution function with mean $\bar{R}^k$ and standard deviation σ_{R^k}, and $\bar{s}^*(L)$ and $R^{k*}(L)$ are solutions for $\bar{s}^*$ and R^{k*} as a function of L, respectively. Assuming that the regulatory restriction is non-binding, the first-order condition is written as

$$\int_{R^{k*}}^{\infty} R^k dF(R^k) = (1-P)R + R\lambda(L-1)\int_{R^{k*}}^{\infty} \frac{\partial x\left(R^k, \bar{s}^*\right)}{\partial \bar{s}^*}\frac{\partial \bar{s}^*(L)}{\partial L} dF\left(R^k\right) + R\lambda \int_{R^{k*}}^{\infty} x\left(R^k, \bar{s}^*(L)\right) dF\left(R^k\right), \tag{7}$$

where P is the probability of bank default, given by

$$P = \Pr(R^k < R^{k*}) = F\left(R^{k*}\right). \tag{8}$$

The left-hand-side of (7) is the expected marginal return of increasing the leverage and the right-hand-side of (7) is the expected marginal cost, which consists of three terms. The initial term is the expected interest cost, which is discounted by $1-P$ due to banks' limited liability. The second term is the expected marginal liquidation cost. An increase in L raises threshold $\bar{s}^*$ and increases the number of fund managers who withdraw, which results in an increase in the liquidation cost. The third term is the expected liquidation cost. Condition (7) does not involve net worth n. .

The banks' problem implies that all banks choose the same level of leverage and default, if any, at the same time. If banks default, they pay to creditors all what they have. Consequently, v in the supply curve for funds (1) is given by

$$v = \min\left\{1, \frac{R^k}{R}\frac{L}{L-1} - \lambda x(R^k, \bar{s}^*)\right\}. \tag{9}$$

2.5 Equilibrium

Competitive equilibrium. A competitive equilibrium for this economy consists of the interest rate R and the leverage L that satisfy the supply curve for funds (1), the demand curve for funds (7) and the market clearing condition, $d = (L-1)n$, where R^{k*}, P and v in these curves are given by (10),(8) and (9), respectively. With a solution of R and L in hand, household consumption series c_1 and c_2 are obtained from the household budget constraints.

Limit equilibrium. For analytical tractability I focus on a limit equilibrium, which is defined as a competitive equilibrium in which the noisy signal vanishes asymptotically, i.e., $\sigma_\epsilon \to 0$. In the limit equilibrium, solutions for $\bar{s}^*$ and R^{k*} are given by $\bar{s}^* = R^{k*}$ where

$$R^{k*} = R\left(1 - \frac{1}{L}\right)\left[1 + \lambda\left(1-\gamma\right)\right]. \tag{10}$$

The optimality condition of the banks' problem (7) is reduced to

$$\int_{R^{k*}}^{\infty} R^k dF(R^k) = \left[1 - F\left(R^{k*}\right)\right] R + \lambda\left(1-\gamma\right) f\left(R^{k*}\right)\left[1 + \lambda\left(1-\gamma\right)\right] R^2 \frac{L-1}{L^2}, \tag{11}$$

where $f(\cdot)$ is the probability density function of R^k.

Given that the solution to (11) attains a local maximum, does it attain a global maximum as well? This is where the regulatory restriction, $L \leq L_{\max}$, bites. Equation (10) implies that $\lim_{L\to\infty} R^{k*} = R[1 + \lambda(1-\gamma)]$, so that even in the limit of $L \to \infty$, the default probability is strictly less than unity: $\lim_{L\to\infty} F\left(R^{k*}\right) < 1$. This and condition (11) suggests $\partial\mathbb{E}(\pi)/\partial L > 0$ for a large value of L. Were it not for $L \leq L_{\max}$, the solution would be $L = \infty$. This issue has to do with the fact that the domain of the distribution for R^k is unbounded above. Should it exist the upper bound $\bar{R}^k$ such that $\bar{R}^k < R[1 + \lambda(1-\gamma)]$ as in, for example, some uniform distributions, there would be no need for such a regulatory restriction.[2]

A unique feature of this model is that bank leverage L is uniquely determined even though there are no financial frictions that directly restrain the leverage. Such frictions include banks' running away moral hazard (Gertler and Kiyotaki 2015), banks' hidden effort as moral hazard (Christiano and Ikeda 2016), asymmetric information and costly state verification (Bernanke et al. 1999), and limited pledgeability (Kiyotaki and Moore 1997). In this model, however, it is an increase in expected liquidation costs that helps pin down bank leverage, which is captured by the second term of the right-hand-side of equation (11). Too high leverage makes banks' liability vulnerable to bank runs, increases the bank run probability, raises expected liquidity costs and lowers profits. Because of this effect banks refrain from choosing too high leverage and by doing so it chooses a profit-maximizing level of bank run probability, given the interest rate R. This endogenous bank run risk is a unique feature of this model that adopts a global game framework.

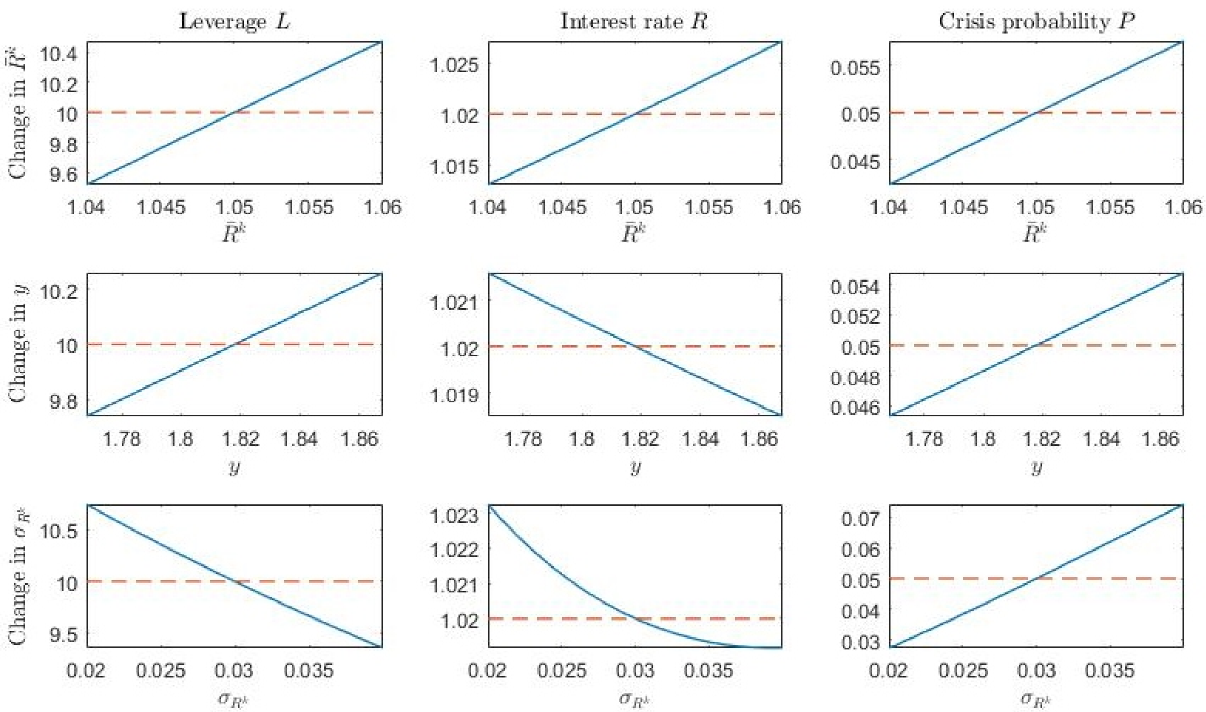
2.6 Comparative Statics

The competitive equilibrium for this economy depends on parameters such as $\bar{R}^k$, γ, λ, y and n. The following proposition summarizes how the demand curve (11) and the supply curve (1) for the funding market are affected with respect to a change in these parameters.

Proposition 1 (Comparative statics). *Consider the funding market described by the demand curve (11) and the supply curve (1), which are plotted in a two-dimension chart where the x-axis is L and the y-axis is R. Assume that bank default probability is not too*

[2] A uniform distribution has also a lower bound, which implies that bank run probability can fall to zero if leverage is sufficiently low. But, with a normal distribution bank run probability is always positive. This is a main reason why this paper consideres a normal distribution. Analytical results on a uniform distribution is available upon request.

Figure 1: Comparative statics for the two-period model



high, $P \leq 0.5$*, and the leverage is not too low,* $L > \left(1 - \frac{0.4}{1+\lambda(1-\gamma)}\right)^{-1}$*. Then, the following results hold.*

(i) An increase in the mean bank asset return $\bar{R}^k$ *shifts the demand curve outward.*

(ii) An increase in the liquidation cost λ *(or a decrease in the threshold probability* γ*) shifts the demand curve inward.*

(iii) An increase in the household endowment y *shifts the supply curve outward.*

(iv) An increase in the bank capital n *shifts the supply curve inward.*

Figure 1 shows changes in the leverage L, the interest rate R and the bank default (crisis) probability P with respect to a change in R^k, y and σ_{R^k} for the calibrated two-period model.[3] Looking at the first row of Figure 1, an increase in the mean bank asset return $\bar{R}^k$ raises the leverage, the interest rate and the crisis probability by shifting the demand curve outward. Turning to the second row of Figure 1, an increase in the household endowment y, by shifting the supply curve outward, raises the leverage, while it reduces the

[3] See the appendix for the calibration.

interest rate. The crisis probability increases as the effect of the leverage on the threshold R^{k*} outweighs that of the interest rate. Finally, in the third row of Figure 1, an increase in the bank asset return volatility σ_{R^k} lowers the leverage and the interest rate, but raises the crisis probability.

In view of the model, a typical credit boom features increases in the bank mean return $\bar{R}^k$, the household endowment y and the bank capital n. On the demand side, a perception of low liquidation costs may add a further outward shift in the demand curve. On the supply side, if the effect of y dominates the effect of n, the supply curve shift outward, which, combined with an increase in the demand, leads to a rise in the leverage and the crisis probability, as implied by the first and second rows of Figure 1. Hence, a credit boom builds up financial system vulnerability that causes system-wide bank defaults – a financial crisis.

3 Welfare Analysis

This section conducts a welfare analysis on the benchmark model. To this end, I first define a social planner's problem and characterise the solution. Then, I analytically show that leverage is excessive in a competitive equilibrium and pin down the source of inefficiencies.

3.1 Social Planner Problem

Is the bank leverage excessive from social welfare perspective? To address this question I formulate a constrained social planner problem in which the planner chooses leverage L to maximize social welfare subject to bank runs and the supply curve for funds (1). In other words, in place of banks the planner chooses L, but unlike banks the planner maximizes the social welfare and takes into account the general equilibrium effect of the choice of L on R. The social welfare, SW, is given by the expected households' utility, $SW = u(c_1) + \mathbb{E}(c_2)$, because banks are owned by the households.

The social planner's problem is $\max_{\{L\}} SW$, which is explicitly written as

$$\max_{\{L\}} u\left(y - (L-1)\, n\right) + \left\{\mathbb{E}(R^k)L - \lambda \mathbb{E}\left[x(R^k, \bar{s}^*(L))\right] R(L)(L-1)\right\} n,$$

subject to $L \leq L_{\max}$, where $R(L)$ is given by the supply curve (1). In the limit equilibrium, $\mathbb{E}(x) \rightarrow P$. Hence, the social planner balances the expected benefit of financial intermediation, $u(c_1) + \mathbb{E}(R^k)Ln$, and the expected cost of a banking crisis, which is given by the crisis probability times the associated cost, $P \times \lambda R(L-1)n$.

The first-order condition in the limit equilibrium is given by

$$\mathbb{E}(R^k) = R\left[1 - P + \mathbb{E}(v|\text{default})P\right] + \lambda PR + \lambda f\left(R^{k*}\right)\left[1 + \lambda\left(1-\gamma\right)\right] R^2 \frac{L-1}{L^2}, \\ + \lambda P(L-1)\frac{dR(L)}{dL}. \quad (12)$$

There are a few notable differences between the banks' optimality condition (11) and the social planner's optimality condition (12). One is that the social planner takes into account all possible states including bank run states, but the banks focus only on non-default states because of their limited liability. Another is that the social planner considers the general equilibrium effect of L on R, which is captured by the last term in the right-hand-side of (12) that involves $dR(L)/dL$, while the banks do not as they take R as given.

3.2 Role of leverage restrictions

Now we are in a position to study whether the competitive equilibrium feature excessive leverage or not. If the slope of the social welfare (12) evaluated at the competitive equilibrium is negative, the leverage is excessive: restricting leverage improves welfare. Let $\partial SW/\partial L|_{\text{CE}}$ denote the planner's optimality condition evaluated at the competitive equilibrium. Because the competitive equilibrium solves the banks' optimal condition, it has to be $\partial \mathbb{E}(\pi)/\partial L|_{\text{CE}} = 0$. Then, $\partial SW/\partial L|_{\text{CE}}$ is written and expanded as

$$\left.\frac{\partial SW}{\partial L}\right|_{\text{CE}} = \left.\frac{\partial SW}{\partial L}\right|_{\text{CE}} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial L}\right|_{\text{CE}} \\ \propto -\frac{\int_{-\infty}^{R^{k*}} R^k dF(R^k)}{L-1} - \lambda\gamma f\left(R^{k*}\right)\left[1 + \lambda\left(1-\gamma\right)\right] R^2 \frac{L-1}{L^2} - \lambda P(L-1)\frac{dR(L)}{dL}. \quad (13)$$

The term $\int_{-\infty}^{R^{k*}} R^k dF(R^k)$ in the first term in the right-hand-side of (13) is likely to be positive. It can be negative in theory, but it should be infinitestimally small because the probability of the gross return R^k falling below 0 under standard parameter values of σ_{R^k} is essentially zero, so that this term can be ignored. Then, if the supply curve (1) is upward-sloping, which is true if $P < (1+\lambda)^{-1}$ as shown in the appendix, equation (13) implies $\partial SW/\partial L|_{\text{CE}} < 0$, implying that bank leverage in a competitive equilibrium is excessive and restraining the leverage can improve welfare. I summarize this result in the following proposition.

Proposition 2 (Excessive leverage)**.** *Consider the model in which the supply curve (1) is upward sloping. Then, in a competitive equilibrium, bank leverage is excessive. Lowering leverage can improve social welfare.*

Equation (13) shows that the bank leverage in a competitive equilibrium is too high for two reasons. First, because of the market structure in which only interest rates work as a market signal, banks compete for attracting deposits by using interest rates only. Even if a bank attempts to become safe by lowering leverage, the bank cannot lower the interest rate because it would lose customers. Hence, such an attempt cannot be a profitable deviation from the equilibrium. Instead, if households can observe individual banks' riskiness, the market works through bank riskiness as well as interest rates. In this case, banks maximise profits subject to the constraint that the expected return offered by banks is equal or greater than a certain level. Banks now take into account of the effect of leverage on the interest rate. Indeed, in this case, the first two terms in equation (13) will vanish.

Second, the third term in the right-hand-side of (13) captures a pecuniary externality that arises from the interest rate R. An increase in bank leverage raises the interest rate and increases the liquidation cost of λRx per unit of funds. This effect is ignored by bankers who take R as given in a competitive equilibrium.

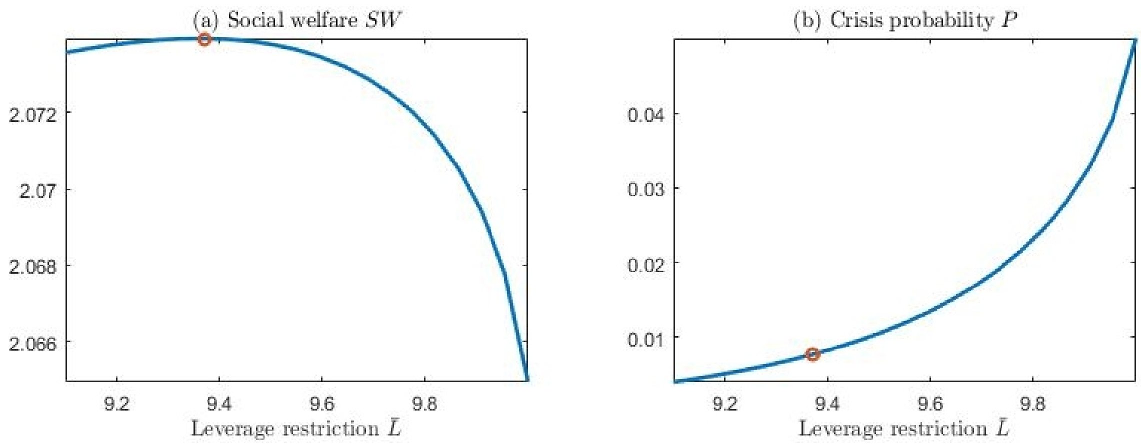
A corollary of proposition 2 is that the probability of bank runs and default is too high in the competitive equilibrium. High leverage implies a high threshold R^{k*} given by (10), which, in turn, leads to a high bank default probability $P = F(R^{k*})$.

Excessive leverage and a resulting high crisis probability in a competitive equilibrium provides a rational for policy makers to introduce prudential policy to improve welfare. The second best allocation, which solves the constrained social planner's problem, can be achieved, for example, by imposing a leverage restriction on banks, $L \leq \bar{L} = L^*$, where $\bar{L}$ is an upper bound of leverage under the restriction and L^* is a solution to equation (12). Similarly, it is achieved by restricting a capital ratio, $n/(n + d)$, such that it is no less than $1/L^*$.

Figure 2 illustrates the effects of leverage restrictions on the social welfare SW and the crisis probability P in the calibrated two-period model.[4] In the competitive equilibrium without the leverage restrictions, the leverage is 10 and the crisis probability is 5 percent. As the restrictions are tightened, the social welfare increases and the crisis probability

[4]See appendix for the calibration.

Figure 2: Effects of leverage restrictions



decreases. Around the leverage of 9.4, the social welfare achieves the maximum. Further tightening of the restrictions is counterproductive as it reduces the social welfare, although it lowers the crisis probability further. This is because the leverage restrictions involve a trade-off between the crisis probability and financial intermediation. Too restrictive leverage reduces the amount of financial intermediation and thus hampers households' consumption smoothing between periods $t = 1$ and $t = 2$.

4 Liquidity and Leverage

I extend the benchmark model presented in Section 2 to incorporate liquidity in the asset side of a bank balance sheet. I first present the extended model. Next, I study roles and interactions of liquidity and leverage requirements regarding social welfare and systemic risk.

4.1 Model with Liquidity and Leverage

In this model, a bank balance sheet consists of liquidity as well as lending, while it consists of lending only in the benchmark model. For simplicity, I assume that banks have a liquidity technology such that liquidity is drawn at any time without any costs but liquidity yields no interest rate so that the return on holding liquidity is unity.

In period $t = 1$, banks allocate the sum of their net worth n and the deposit d to lending and liquidity M. In response to fund managers' early withdrawal claim of xRd, banks use liquidity first, because it is not costly, and liquidate their assets if the amount of liquidity is not enough to cover the amount of the claim: $xRd > M$. In this case, the banks have to liquidate $(1+\lambda)(xRd - M)/R^k$ units of bank assets. If the banks revenue, $R^k(n+d-M) - (1+\lambda)(xRd - M)$, cannot cover the promised payment to the depositors who have not withdraw early, $(1-x)Rd$, they go bankrupt. Hence, banks default if and only if

$$R^k < \frac{R-m}{\frac{L}{L-1} - m}\left(1 + \lambda \frac{xR - m}{R - m}\right), \tag{14}$$

where $m \equiv M/D$ is a liquidity-deposit ratio and $L \equiv (n+d)/n$ is leverage. This condition is reduced to condition (3) if $m = 0$. Condition (14) implies that thresholds $\bar{s}^*$ and R^{k*} are determined by equation (5) and

$$R^{k*} = \frac{R-m}{\frac{L}{L-1} - m}\left[1 + \lambda \frac{x(R^{k*}, \bar{s}^*)R - m}{R - m}\right], \tag{15}$$

where $x(R^{k*}, \bar{s}^*) = \Phi((\bar{s}^* - R^{k*})/\sigma_\epsilon$. Equation (15) is the extension of equation (6) to include liquidity m.

The problem of banks is to maximize the expected profits $\mathbb{E}(\pi)$ by choosing leverage and liquidity,

$$\max_{\{L,m\}} \int_{R^{k*}(L,m)}^{\infty} \left\{R^k L - (R^k - 1)(L-1)m - R\left[1 + \lambda x\left(R^k, \bar{s}^*(L,m)\right)\right](L-1)\right\} ndF(R^k),$$

subject to $L \leq L_{\max}$ and $0 \leq m \leq L/(L-1)$, where the thresholds $\bar{s}^*(L,m)$ and $R^{k*}(L,m)$ are a solution to equations (5) and (15), expressed as a function of L and m.

I focus on a limit equilibrium where $\sigma_\epsilon \to 0$ as in the benchmark model presented in Section 2. In the limit equilibrium, equations (5) and (15) imply that the thresholds are given by $\bar{s}^* = R^{k*}$, where

$$R^{k*} = \frac{R-m}{\frac{L}{L-1} - m}\left[1 + \lambda \frac{(1-\gamma)R - m}{R - m}\right]. \tag{16}$$

The threshold R^{k*} is decreasing in m if the interest rate is not high enough to satisfy

$$R < 1 + \frac{\lambda\gamma}{1 + \lambda(1-\gamma)}. \tag{17}$$

An increase in liquidity m reduces the threshold and lowers the bank run probability $F(R^{k*})$ and thereby increases the resiliency of the financial system when the interest rate satisfies condition (17). Instead, if condition (17) is violated, the interest cost on the bank liability is so high that a decrease in the expected revenue due to an increase in liquidity holding causes the banks more vulnerable to bank runs, raising the threshold R^{k*} and the bank run probability $F(R^{k*})$.

The first-order conditions of the banks' problem in the limit equilibrium characterize an interior solution for leverage L and liquidity m as

$$\begin{aligned} 0 =& \int_{R^{k*}}^{\infty} [R^k - (R^k - 1)m] dF(R^k) - \left[1 - F\left(R^{k*}\right)\right] R \\ & -\lambda (1-\gamma) f\left(R^{k*}\right) \left[1 + \lambda \frac{R(1-\gamma) - m}{R - m}\right] \frac{R(R-m)}{\left(\frac{L}{L-1} - m\right)^2 (L-1)}, \end{aligned} \tag{18}$$

$$\begin{aligned} 0 =& -\int_{R^{k*}}^{\infty} (R^k - 1) dF(R^k) \\ & +\lambda(1-\gamma) f(R^{k*}) \left[\frac{R\left(\frac{L}{L-1} - R\right)\left[1 + \lambda \frac{R(1-\gamma)-m}{R-m}\right]}{\left(\frac{L}{L-1} - m\right)^2} + \frac{\lambda \gamma R^2}{\left(\frac{L}{L-1} - m\right)(R-m)} \right]. \end{aligned} \tag{19}$$

Equation (18) corresponds to $0 = \partial \mathbb{E}(\pi)/\partial L$, which is the extension of equation (11) to include m. Equation (19) corresponds to $0 = \partial \mathbb{E}(\pi)/\partial m$. The first term in the right-hand-side of equation (19) is the opportunity cost associated with holing liquidity, i.e. the net expected return on the risky project which the banks would have earned if they had not held liquidity but invested in the project. The second term in the right-hand-side of equation (19) is the marginal benefit of holding liquidity by lowering the threshold, $\bar{s}^*(L,m)$, and decreasing the number of fund managers who withdraw early, $x(R^k, \bar{s}^*(L,m))$.

A necessary condition for an unique solution for (19) that is optimal is that the right-hand-side of equation (19) is positive when $m = 0$. This condition, combined with equation (18), is written as

$$-(R-1)[1 - F(R^{k*})] + \lambda(1-\gamma) f(R^{k*}) R \frac{L-1}{L} \left[1 + \lambda - R(1 + \lambda(1-\gamma))\right] > 0. \tag{20}$$

This condition holds only if the interest rate is not high enough to satisfy condition (17). The interest rate serves as a measure of the opportunity cost of holding liquidity. If the opportunity cost is not too high, the banks have an incentive to hold liquidity in equilibrium.

Another observation about liquidity is that condition (20) is less likely to hold as leverage L is reduced in the region where the bank run probability is not so large, $P = F(R^{k*}) < 1/2$. This observation implies that banks may want to reduce liquidity holding if their leverage is restrained by regulations. I will numerically explore this possibility in Section 4.3.

The supply side of funds – the household problem – is the same as in the benchmark model except for the recovery rate v. A fraction, x, of fund managers who withdraw early receive R per unit of deposit. When banks default, a remaining fraction, $1-x$, of fund managers divide banks' return $[R^k(n+d-M)-\lambda(xRd-M)]$ equally and receive $[R^k(n+d-M)-\lambda(xRd-M)]/[(1-x)d]$ per unit of deposit. Because households diversify over fund managers, households receive a weighted sum of the returns when banks default. Consequently, the recovery rate is given by

$$v = \min\left\{1, \frac{R^k}{R}\frac{L}{L-1} - \lambda x + \frac{(1+\lambda-R^k)m}{R}\right\}.$$

The recovery rate is increasing in liquidity m as long as $R^k < 1+\lambda$.

4.2 Roles of Liquidity and Leverage Requirements

Is liquidity in a competitive equilibrium lower than the socially optimal level? Does leverage continue to be excessive in the model with liquidity? To address these questions, as in Section 3, I set up a social planner problem in which a benevolent planner chooses leverage L and liquidity m to maximize social welfare

$$\begin{aligned}\max_{\{L,m\}} \quad & u(y-(L-1)n) \\ & + \left\{\mathbb{E}(R^k)L - \left[\mathbb{E}(R^k)-1\right](L-1)m - \lambda\mathbb{E}\left[x(R^k, \bar{s}^*(L,m))\right]R(L)(L-1)\right\}n,\end{aligned}$$

subject to $L \leq L_{\max}$, where $R(L)$ is given by the supply curve (1) and $\bar{s}^*(L,m)$ is given by a solution to equations (5) and (15).

The social planner takes into account bank-run states of $R^k < R^{k*}$ in addition to no-bank-run states of $R^k \geq R^{k*}$, as opposed to the bankers who consider no-bank-run states only. This observation leads to the following proposition that shows insufficient liquidity in the competitive equilibrium. As in Section 3, I continue to focus on a limit equilibrium in which $\sigma_\epsilon \to 0$.

Proposition 3 (Insufficient liquidity)**.** *Consider the model with liquidity in which condition (19) holds with positive liquidity holding. Assume that the threshold R^{k*} is low enough to satisfy $\int_{-\infty}^{R^{k*}}(1-R^k)dF(R^k) > 0$. Then, for given leverage, banks choose insufficient liquidity. Increasing liquidity can improve social welfare.*

Proposition 3 does not require that L is the competitive equilibrium level of leverage. Indeed, Proposition 3 holds for an arbitrary value of L. Then, the corollary of Proposition 3 is that bank liquidity is insufficient not only in a competitive equilibrium but also in an equilibrium with $m > 0$ in which leverage is restrained by the corresponding prudential policy. This result suggests that a prudential liquidity tool is essential by improving welfare even if a prudential capital/leverage tool is already in place.

Next I turn to welfare implications of bank capital/leverage in this extended model with liquidity. As in the benchmark model, bank leverage is excessive as formally stated in the following proposition.

Proposition 4 (Excessive leverage in the model with liquidity)**.** *Consider the model with liquidity in which the supply curve (1) is upward sloping. Then, for given liquidity, banks choose excessive leverage. Lowering leverage can improve social welfare.*

Propositions 3 and 4 imply that the competitive equilibrium for the model with liquidity features both excessive leverage and insufficient liquidity. This warrants prudential policy on capital and liquidity, which I will consider below.

4.3 Policy Coordination: Leverage and Liquidity

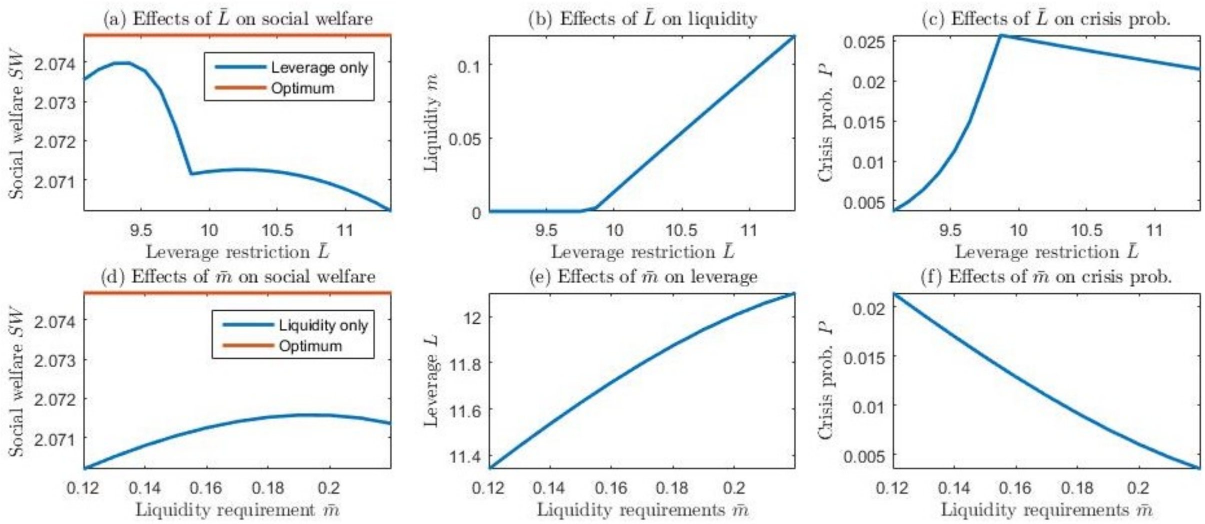
This section numerically explores policy coordination between capital and liquidity requirements. The model employs the same parameter values as in the benchmark model presented in Section 2. In the benchmark model, leverage was $L = 10$ and the crisis probability was $P = 0.05$ in the competitive equilibrium. In the model with liquidity, leverage rises to $L = 11.34$, liquidity is $m = 0.12$ and the crisis probability drops to $P = 0.0214$ in the competitive equilibrium. On the one hand, liquidity holding makes banks more resilient to a bank run risk, as is clear from the lower probability of bank runs. On the other hand, it allows the banks to take risk by increasing leverage.

To understand the joint impact of capital and liquidity requirements, I first consider the case of capital requirements only and next the case of liquidity requirements only. Then, I proceed to analyse their joint effect.

4.3.1 Leverage restrictions only

The upper panels of Figure 3 show the impacts of capital requirements in the form of the leverage restriction, $L \leq \bar{L}$, on social welfare, liquidity and the crisis probability. The

Figure 3: Effects of leverage (upper panels) and liquidity (lower panels) requirements



Note: 'Optimum' in panels (a) and (d) represents the highest level of social welfare when both leverage and liquidity requirements are active.

effect of $\bar{L}$ on social welfare is not monotonic (Figure 3(a)). As the leverage restriction is tightened from the competitive equilibrium level of $L = 11.34$, social welfare increases, but starts decreasing slightly at around $\bar{L} = 10.2$. Then, social welfare resumes increasing sharply when $\bar{L}$ is tightened to just below 10. And for the lower values of $\bar{L}$ social welfare shows an inverse U-shape.

What is behind this non-monotonic effect of the leverage restriction? It is 'risk migration' from leverage to liquidity. Although the leverage is restricted, the banks still have a free variable that affects their profits, namely, liquidity holding. As the leverage restriction is tightened, the banks choose to hold less liquidity (Figure 3(b)) Surprisingly the crisis probability increases as the leverage restriction is tightened from the competitive equilibrium level (Figure 3(c)). This adverse effect of risk migration vanishes when liquidity holding drops to zero, hitting the lower bound. With no further risk migration, the leverage restriction becomes more effective as it lowers the crisis probability and increases social welfare until the adverse effect of tight leverage restriction – a decrease in financial intermediation – starts dominating.

4.3.2 Liquidity requirements only

Now I introduce a liquidity policy tool, $m \geq \bar{m}$, such that banks are required to hold liquidity at least a fraction $\bar{m}$ of deposits. The lower panels of Figure 3 show the impacts of the liquidity tool on social welfare, leverage and the crisis probability. As the liquidity requirement is tightened, social welfare increases (Figure 3(d)). But the degree of welfare improvement is attenuated, as banks take more risk by increasing leverage (Figure 3(e)). Still, the effect of the liquidity requirement dominates the risk migration effect, decreasing the crisis probability (Figure 3(f)). Social welfare attains the maximum at $\bar{m} = 0.18$, but the level is below the maximum when only the leverage restriction is put in place.

Why is the impact of the liquidity requirement on social welfare weaker than that of the leverage restriction? The answer has to do with risk migration. In the case of the liquidity requirement, risk continues to migrate through an increase in leverage. With this risk migration a higher level of liquidity is needed to achieve a low crisis probability than what would be a case without the risk migration. Such a high liquidity holding lowers the bank asset return, attenuating the impact on social welfare.

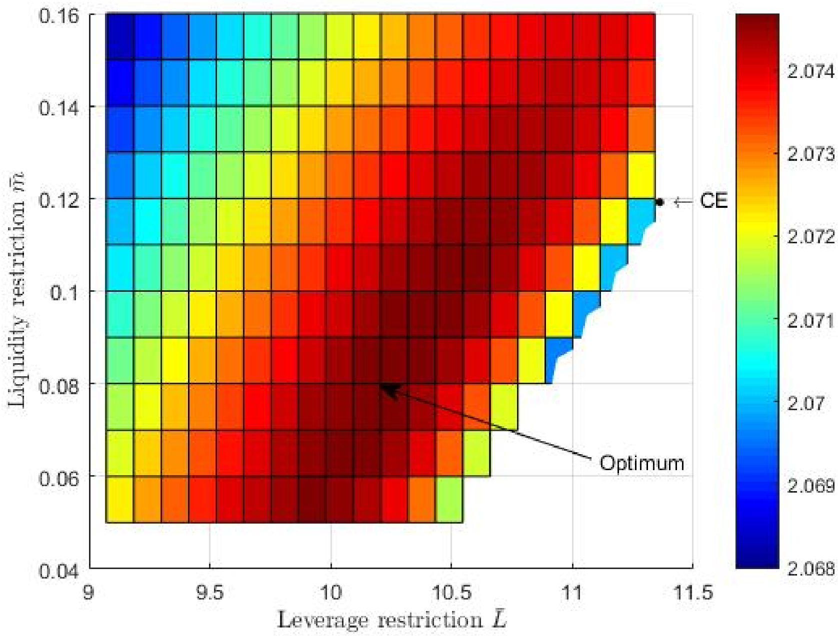
4.3.3 Risk migration

A key takeaway of this analysis is that one policy instrument is not enough when risk migrates through other areas. Even worse, a crisis probability may rise as a leverage restriction is tightened as banks lower a liquidity holding and risk is migrated to liquidity (Figure 3(c)). This situation calls for policy coordination between capital and liquidity requirements.

4.3.4 Coordination of leverage and liquidity tools

What is an optimal policy coordination between leverage and liquidity tools? Is the two requirements a substitute, meaning that a tightening in one policy tool is associated with a loosening in other tool? Figure 4 answers these questions by showing the joint effect of the two requirements on social welfare. The welfare is maximized around $\bar{L} = 1.2 < L_{CE}$ and $\bar{m} = 0.08 < m_{CE}$, where subscript CE denotes a competitive equilibrium. At the optimum, the leverage is restrained but the liquidity holing is lower than the competitive equilibrium level. Yet, the liquidity holding is constrained too, because it would be lower around $m = 0.02$ without the liquidity requirement, as can be seen from Figure 3(b). In Figure 4 the optimum level of social welfare is coloured by dark red and this colour spreads out diagonally. This is also true for other levels (colours) of social welfare in Figure 4. This pattern implies that capital and liquidity requirements are a substitute to some degree: a tightening in the capital/leverage requirement is associated with a loosening in the liquidity

Figure 4: Joint effects of capital and liquidity requirements on social welfare



requirement to achieve a certain level of social welfare.

5 Extensions

The benchmark model presented in Section 2 has various extensions, serving as a unified framework for analysing banking crises, banks' behaviour and prudential instruments. In this section, on the policy front, I analyse sectoral capital requirements, risk weights and deposit insurance. On banks' behaviour, I touch on shadow banking and banks' risk-taking.

5.1 Sectoral Capital Requirements and Risk Weights

5.1.1 Model with Two Sectors

I extend the benchmark model presented in Section 2 to incorporate two sectors and two types of banks. Bank $j \in \{1, 2\}$ specializes in lending to sector j and cannot lend to other sector. Lending to sector j yields return R_j^k, which follows $N(\bar{R}_j^k, \sigma^2_{R_j^k})$. Fund manager i, who specializes in monitoring bank j and sector j, receives noisy signal s_{ij}, which is given by $s_{ij} = R_j^k + \epsilon_{ij}$ with $\epsilon_{ij} \sim N(0, \sigma^2_{\epsilon_j})$. For simplicity, probability threshold γ for fund managers' withdrawal decision, given by (2), and bank net worth n are assumed to be identical between the two types of banks, but liquidation cost parameter λ_j is assumed to differ. The remaining part of the model is essentially the same as in the benchmark model.

The limit equilibrium for this economy is characterized by the following four equations with four unknowns $\{R_j, L_j\}_{j=1}^2$: for $j = 1, 2$

$$R_j = \frac{u'(y - (L_1 - 1)n - (L_2 - 1)n)}{1 - P_j + \mathbb{E}(v_j|\text{default})P_j}, \tag{21}$$

$$\int_{R_j^{k*}}^{\infty} R^k dF_j(R^k) = (1 - P_j)R_j + \lambda_j (1 - \gamma) f_j \left(R_j^{k*}\right) [1 + \lambda_j (1 - \gamma)] R_j^2 \frac{L_j - 1}{L_j^2}, \tag{22}$$

where $P_j = F_j(R_j^{k*})$ is a default probability for bank j, $F_j(\cdot)$ is the normal distribution function with mean $\bar{R}_j^k$ and standard deviation $\sigma_{R_j^k}$ and $f_j(\cdot)$ is its probability density function. The threshold R_j^{k*} and the recovery rate v_j are given by (10) and (9), respectively, with a modification to add sector specific subscript j.

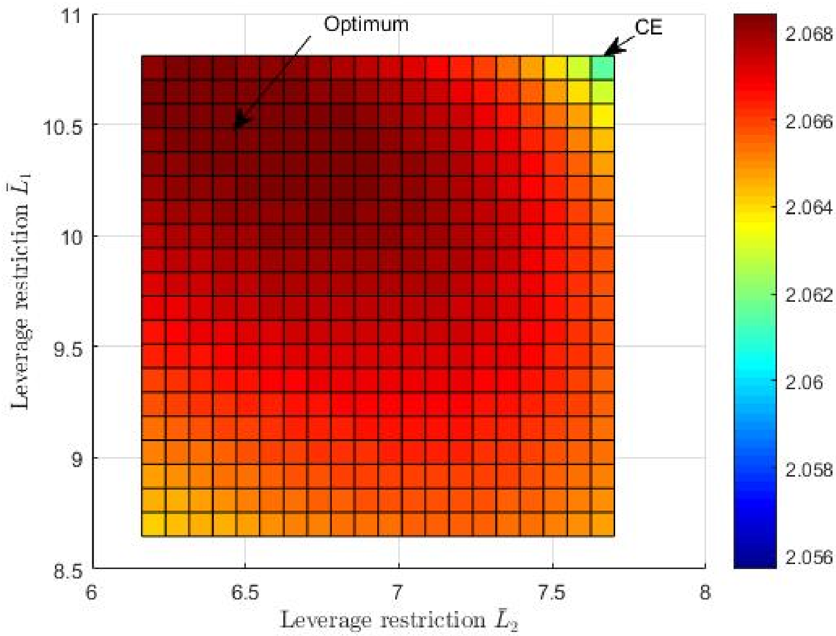
For a numerical illustration of two sectors in which one is risky and the other is less risky, I assume that the two sectors and the two types of banks are identical and the same parameter values used in Section 2 are assigned except that (i) the standard deviation of the sector-2 return is twice as big as that of the sector-1 return and (ii) the bank endowment is a half of the value used in Section 2 for each type of banks.

By assumption sector 2 is riskier than sector 1 and so are type-2 banks than type-1 banks. In a competitive equilibrium leverage and a bank run probability are $L_1 = 10.8$ and $P_1 = 0.065$ for type-1 banks, and $L_2 = 7.7$ and $P_2 = 0.096$ for type-2 banks. If $\sigma_{R_2^k}$ were the same as $\sigma_{R_1^k}$, the model would be essentially reduced to the benchmark model presented in Section 2, so that leverage and a probability of bank runs would be $L_1 = L_2 = 10$ and $P_1 = P_2 = 0.05$. However, reflecting higher riskiness on their loans to sector 2, type-2 banks have lower leverage but a higher probability of bank runs relative to type-1 banks. Compensating the type-2 banks' low capacity in intermediation, type-1 banks have higher leverage and a higher probability of bank runs than in the benchmark model.

5.1.2 Role of Sectoral Capital Requirements and Risk Weights

A heterogeneity in sectoral riskiness gives rise to a need for sectoral capital requirements. Figure 5 shows the joint effects of sectoral capital (leverage) requirements on social welfare. The optimum is attained around $L_1 = L_1^* \equiv 10.5$ and $L_2 = L_2^* \equiv 6.5$. Relative to the competitive equilibrium – the upper right corner in Figure 5 – the type-2 bank leverage is more restrained than the type-1 bank leverage. Both types of banks have the same source

Figure 5: Joint effects of sectoral capital requirements on social welfare



of inefficiencies: they ignore asset liquidation cost in the bank runs that result in bank default. However, Figure 5 suggests that the degree of inefficiencies is severer for type-2 banks than type-1 banks. Type-2 banks, exposed to a higher loan risk, ignore a tail risk more than type-1 banks. Given the same expected returns for the two sectors, it intuitively makes sense to restrain lending to the riskier sector and promote lending to the less-safe sector. At the optimum, the probability of bank runs drop to around 1 percent for both types of banks.

The above analysis assumes that risk weights are 100 percent for both sectoral loans. If risk weights are appropriately set, a risk-weighted-based capital requirement can address the problem caused by a heterogeneity in sectoral riskiness. Suppose that a risk weight is 100 percent for sector-1 loans and 100ω percent for sector-2 loans. Suppose further that a risk-weighted-based capital requirement is $1/L_1^*$. To achieve $L_2 = L_2^*$, the risk weight has to be such that it is binding, $n/(n + \omega d_2) = 1/L_1^*$, and the non-risk weighted capital ratio is $1/L_2^*$, $n/(n + d_2) = 1/L_2^*$. Solving the equations for ω yields

$$\omega = \omega^* \equiv \frac{L_1^* - 1}{L_2^* - 1} > 1.$$

Thus, the optimal risk weight for sector-2 loans is more than 100 percent, reflecting their high riskiness. With $\omega = \omega^*$, the risk-weight-based capital requirement achieves the same outcome as the sectoral capital requirements.

It is worth mentioning that in practice the riskiness of loans can vary across sectors over time. Hence, to keep up with this change, either sectoral capital requirements or risk weights need to be adjusted to achieve the optimal level of welfare. It would depend on timely implementability which policy tool should be employed to address a change in sectoral riskiness.

5.1.3 Shadow banking

Shadow banks, by definition, lie outside the reach of regulations and prudential policy on the banking system. Type-2 banks, which specialize in lending to a riskier sector, can be interpreted as shadow banks if regulations and prudential policy cannot be directly implemented on them. In this case, risk migration from type-1 banks (commercial banks) to type-2 banks (shadow banks) can occur as capital requirements are imposed only on type-1 banks.

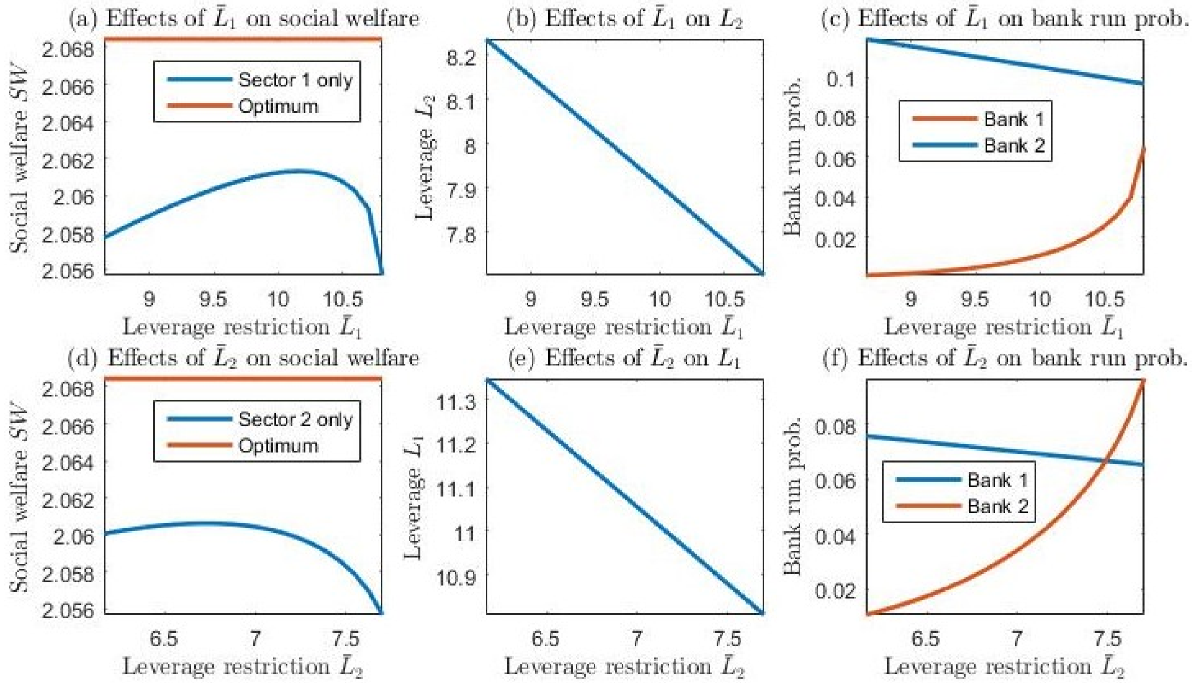
The upper panels of Figure 6 plot the impacts of capital/leverage requirements for type-1 banks only on social welfare, type-2 bank leverage and the probabilities of bank runs. As expected, as the leverage restriction on type-1 banks is tightened, the leverage of type-2 banks increases and so does the probability of bank runs for these banks. With only one policy instrument being active, social welfare is improved for somewhat, but its achievable level is far below the optimum attained when both tools are active. Implications are similar for the opposite case when capital requirements are imposed only on type-2 banks (lower panels of Figure 6).

5.2 Risk Taking

5.2.1 Model with Portfolio Selection

Banks may take more risk in making loans than what would be desirable from a social welfare view point. To explore this risk-taking behaviour, I modify the model presented in Section 5.1 to allow for banks to choose portfolio of loans. Specifically, there is one type of banks which make loans to two sectors, indexed by $j \in \{1, 2\}$. The returns of the two sectors follow a joint normal distribution, $\boldsymbol{R}^k \sim N(\bar{\boldsymbol{R}}^k, \Sigma_{R^k})$, where $\boldsymbol{R}^k \equiv [R_1^k, R_2^k]'$ is a vector of returns of the two sectors. The variance-covariance matrix Σ_{R^k} implies that the two returns can be correlated.

Figure 6: Effects of sector-1 (upper panels) and sector-2 (lower panels) capital requirements



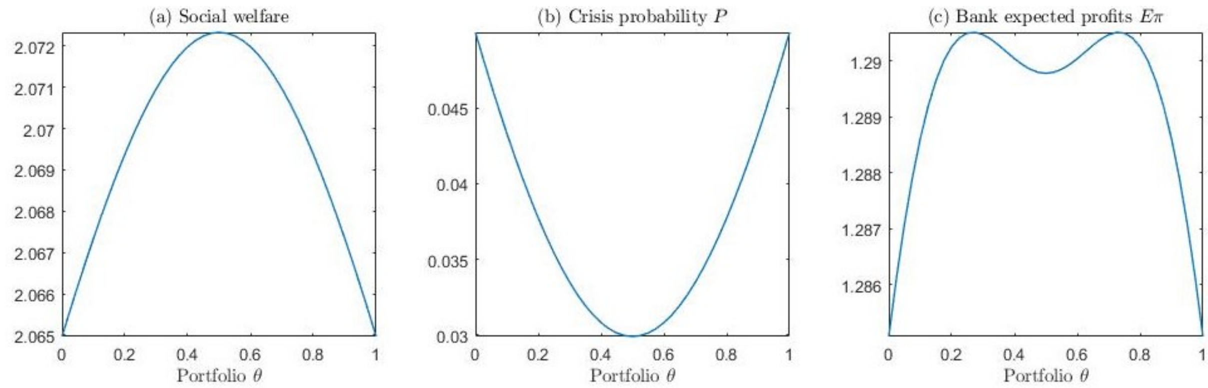
Note: 'Optimum' in panels (a) and (d) represents the highest level of social welfare when both capital requirements are active.

In addition to leverage banks choose a portfolio of loans, $\boldsymbol{\theta} \equiv [\theta, 1-\theta]'$, where $\theta \in [0,1]$ is a fraction of total loans invested in sector $j = 1$. Then, the return of the portfolio is given by $R^k(\theta) \equiv \boldsymbol{\theta}'\boldsymbol{R}^k$, which follows $N(\bar{R}^k(\theta), \sigma_{R^k}(\theta)^2)$, where $\bar{R}^k(\theta) \equiv \boldsymbol{\theta}'\bar{\boldsymbol{R}}^k$ is the mean return and $\sigma_{R^k}(\theta) \equiv (\boldsymbol{\theta}'\Sigma\boldsymbol{\theta})^{\frac{1}{2}}$ is the standard deviation of the portfolio. Each fund manager observes bank portfolio $\boldsymbol{\theta} \equiv [\theta, 1-\theta]'$ as well as leverage L and receives independent signals $s_{ij} = R_j^k + \epsilon_{ij}$ with $\epsilon_{ij} \sim N(0, \sigma_{\epsilon_j}^2)$ for $j = 1, 2$. Given θ, this model works essentially the same way as in the benchmark model presented in Section 2. Fund manager i withdraws deposits early if and only if $\boldsymbol{\theta}'\boldsymbol{s}_i$ is less than the threshold $\bar{s}^*(L,\theta)$, where $\boldsymbol{s}_i \equiv [s_{i1}, s_{i2}]'$ is a vector of noisy signals. A difference is that now the threshold depends bank asset portfolio θ as well as leverage L.

The problem of banks is to choose leverage L and asset portfolio θ to maximize the expected profits, taking into account that the choices of L and θ affect a bank run probability facing the banks:

$$\max_{\{L,\theta\}} \int_{R^{k*}(L,\theta)}^{\infty} \left\{R^k(\theta)L - R\left[1 + \lambda x\left(R^k(\theta), \bar{s}^*(L,\theta)\right)\right](L-1)\right\} n dF(R^k;\theta),$$

Figure 7: The effects of risk taking



subject to $L \leq L_{\max}$, where $F(R^k;\theta)$ is the normal cumulative distribution function with mean $\bar{R}^k(\theta)$ and standard deviation $\sigma_{R^k}(\theta)$.

5.2.2 An Example of Risk Taking

As a simple example of risk-taking, I consider the model in which the two sectors are identical. The only difference from the benchmark model presented in Section 2 is that banks can reduce their loan risk by diversifying over loans to the two sectors. Specifically, banks are able to minimize the risk of their loan portfolio by setting $\theta = 1/2$. Social welfare achieves a maximum at $\theta = 1/2$, where the crisis probability P is also minimized (Figure 7(a) and (b)).

However, banks do not choose the portfolio that minimizes their loan portfolio risk. Figure 7(c) shows that the banks lend to one sector more than the other by choosing θ around 0.3 or 0.7 to maximize the profits. By doing so, the banks take more loan portfolio risk than the minimized level that would be attained under $\theta = 1/2$. The banks engage in this risk-taking because, protected by limited liability, they focus only on non-bank run states. Some volatility in the loan portfolio return is beneficial for banks as they can capture the upper outcome of their loan portfolio. But this risk-taking is harmful for the economy as a whole as it increases the crisis probability and reduces social welfare.

5.3 Deposit Insurance

Perfect deposit insurance, which ensures $v = 1$ for all states, will eliminate bank runs in theory, but such an insurance is hardly institutionalized in practice. Specifically, wholesale deposits are only partially insured at most.

In the benchmark model presented in Section 2, bank runs and a resulting bank default can persist even if deposit insurance is put in place, as long as such an insurance is imperfect. Imperfect deposit insurance implies that households who withdraw early benefit from doing so when banks are in trouble. Households continue delegating their deposit management to fund managers and as a result bank runs persist. Worse, imperfect deposit insurance would exacerbate excessive leverage and a too-high systemic risk.

Suppose that a government introduces a deposit insurance such that in the case of bank default households receive $100\bar{v}$ percent of the interest rate R, where the covered rate of $\bar{v}$ is assumed to be greater than the actual recovery rate of v. The government finances $(\bar{v} - v)R$ per unit of funds by imposing lump-sum taxes on households period $t = 2$. Then, the supply curve of funds (1) is changed to

$$R = \frac{u'(y - (L - 1)n)}{1 - (1 - \bar{v})P}.$$

An increase in the covered rate of $\bar{v}$ shifts the supply curve outward and increases the leverage and thereby the systemic risk.

6 Conclusion

This paper has developed a bank run model in a global game general equilibrium framework. The model highlights banks' limited liability and their ignorance of liquidation costs resulting from bank default as a source of inefficiencies that leads to excessive bank leverage and insufficient bank liquidity holdings. This warrants prudential policy on capital and liquidity. The model has rich applications including sectoral capital requirements, deposit insurance, risk migration, shadow banks and risk-taking. The model provides a unified framework for studying banking crises, banks' behaviour and prudential policy tools.

I conclude the paper by laying out a plan for future research. There are mainly three directions. First, by using the two-period model framework, I plan to study the role of unconventional government policy – policy ex-post a banking crisis – as in Christiano and Ikeda (2011) and its possible interaction and coordination with ex-ante prudential policy studied in this paper. Second, I plan to extend the two-period model to a dynamic model in an infinite horizon economy. In the dynamic model, the bank asset return, bank capital and household income, which were taken as given in this paper, will be determined within in a model endogenously. The dynamic model will shed light on the dynamic effects of banking crises on the real economy and a role of a countercyclical capital buffer and a liquidity requirement. Third, after studying the dynamic model, I plan to extend the model to incorporate nominal rigidities to study monetary policy and its possible interaction with macroprudential policy tools. Some analyses in these directions are already under way.

Appendix

Derivation of equation (10). As shown in Section 2 the threshold R^{k*} is a solution to equations (5) and (6). These equations are written explicitly as:

$$\Phi\left(\sqrt{\frac{1}{\sigma_{R^k}^2}+\frac{1}{\sigma_\epsilon^2}}R^{k*}-\frac{\frac{1}{\sigma_{R^k}^2}\bar{R}^k+\frac{1}{\sigma_\epsilon^2}\bar{s}^*}{\sqrt{\frac{1}{\sigma_{R^k}^2}+\frac{1}{\sigma_\epsilon^2}}}\right)=\gamma, \tag{23}$$

$$R^{k*}=R\left(1-\frac{1}{L}\right)\left[1+\lambda\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\right], \tag{24}$$

where $\Phi(\cdot)$ is the standard normal distribution function. Equation (23) implies that $\lim_{\sigma_\epsilon\to 0}\Phi((R^{k*}-\bar{s}^*)/\sigma_\epsilon)=\gamma$. Therefore, $\lim_{\sigma_\epsilon\to 0}\Phi((\bar{s}^*-R^{k*})/\sigma_\epsilon)=1-\gamma$. Substituting this result into equation (24) leads to equation (10).

Derivation of equation (11). Equation (11) is the limiting case of equation (7) where $\sigma_\epsilon\to 0$. First, we derive an expression for $\partial\bar{s}^*(L)/\partial L$ in equation (7). Totally differentiating equations (23) and (24) yields

$$dR^{k*}=\frac{1}{\frac{\sigma_\epsilon^2}{\sigma_{R^k}^2}+1}d\bar{s}^*,$$

$$dR^{k*}=\frac{R}{L^2}\left[1+\lambda\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\right]dL+R\left(1-\frac{1}{L}\right)\lambda\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}(d\bar{s}^*-dR^{k*})$$

Combining these equations yields

$$\frac{d\bar{s}^*}{dL}=\frac{\left(\sigma_{R^k}^2+\sigma_\epsilon^2\right)\frac{R}{L^2}\left[1+\lambda\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\right]}{\sigma_{R^k}^2-\left(1-\frac{1}{L}\right)\lambda\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\sigma_\epsilon},$$

where $\phi(\cdot)$ is the standard normal pdf. Note that $\lim_{\sigma_\epsilon\to 0}\phi((\bar{s}^*-R^{k*})/\sigma_\epsilon)=\phi(\lim_{\sigma_\epsilon\to 0}(\bar{s}^*-R^{k*})/\sigma_\epsilon)=\phi(\Phi^{-1}(1-\gamma))$. Then, in the limit, $d\bar{s}^*/dL$ is given by

$$\lim_{\sigma_\epsilon\to 0}\frac{d\bar{s}^*}{dL}=\frac{R}{L^2}\left[1+\lambda(1-\gamma)\right].$$

Next, consider $\int_{R^{k*}}^{\infty}[\partial x(R^k,\bar{s}^*)/\partial\bar{s}^*]dF(R^k)$ in equation (7), where $F(\cdot)$ is the normal distribution function with mean $\bar{R}^k$ and variance $\sigma^2_{R^k}$. It is explicitly written as

$$\begin{aligned}\int_{R^{k*}}^{\infty} \frac{\partial x(R^k, \bar{s}^*)}{\partial \bar{s}^*} dF(R^k) &= \int_{R^{k*}}^{\infty} \phi\left(\frac{\bar{s}^* - R^k}{\sigma_\epsilon}\right) \frac{1}{\sigma_\epsilon} dF(R^k) \\ &= \int_{R^{k*}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\bar{s}^* - R^k}{\sigma_\epsilon}\right)^2} \frac{1}{\sigma_\epsilon} \frac{1}{\sqrt{2\pi}\sigma_{R^k}} e^{-\frac{1}{2}\left(\frac{R^k - \bar{R}^k}{\sigma_{R^k}}\right)^2} dR^k.\end{aligned}$$

The terms in the power of e are arranged as

$$\begin{aligned}& -\frac{1}{2}\left(\frac{\bar{s}^* - R^k}{\sigma_\epsilon}\right)^2 - \frac{1}{2}\left(\frac{R^k - \bar{R}^k}{\sigma_{R^k}}\right)^2 \\ =& -\frac{1}{2}\left[\frac{\bar{s}^{*2} - 2\bar{s}^* R^k + R^{k2}}{\sigma^2_\epsilon} + \frac{R^{k2} - 2R^k\bar{R}^k + \bar{R}^{k2}}{\sigma^2_{R^k}}\right] \\ =& -\frac{1}{2}\left[\left(\frac{1}{\sigma^2_\epsilon} + \frac{1}{\sigma^2_{R^k}}\right) R^{k2} - 2\left(\frac{\bar{s}^*}{\sigma^2_\epsilon} + \frac{\bar{R}^k}{\sigma^2_{R^k}}\right) R^k + \frac{\bar{s}^*}{\sigma^2_\epsilon} + \frac{\bar{R}^{k2}}{\sigma^2_{R^k}}\right] \\ =& -\frac{1}{2}\left(\frac{1}{\sigma^2_\epsilon} + \frac{1}{\sigma^2_{R^k}}\right)\left[R^{k2} - 2\frac{\frac{\bar{s}^*}{\sigma^2_\epsilon} + \frac{\bar{R}^k}{\sigma^2_{R^k}}}{\frac{1}{\sigma^2_\epsilon} + \frac{1}{\sigma^2_{R^k}}} R^k + \frac{\frac{\bar{s}^*}{\sigma^2_\epsilon} + \frac{\bar{R}^{k2}}{\sigma^2_{R^k}}}{\frac{1}{\sigma^2_\epsilon} + \frac{1}{\sigma^2_{R^k}}}\right] \\ =& -\frac{1}{2}\left(\frac{1}{\sigma^2_\epsilon} + \frac{1}{\sigma^2_{R^k}}\right)\left[\left(R^k - \frac{\frac{\bar{s}^*}{\sigma^2_\epsilon} + \frac{\bar{R}^k}{\sigma^2_{R^k}}}{\frac{1}{\sigma^2_\epsilon} + \frac{1}{\sigma^2_{R^k}}}\right)^2 - \left(\frac{\frac{\bar{s}^*}{\sigma^2_\epsilon} + \frac{\bar{R}^k}{\sigma^2_{R^k}}}{\frac{1}{\sigma^2_\epsilon} + \frac{1}{\sigma^2_{R^k}}}\right)^2 + \frac{\frac{\bar{s}^*}{\sigma^2_\epsilon} + \frac{\bar{R}^{k2}}{\sigma^2_{R^k}}}{\frac{1}{\sigma^2_\epsilon} + \frac{1}{\sigma^2_{R^k}}}\right] \\ =& -\frac{1}{2}\left(\frac{R^k - \frac{\frac{\bar{s}^*}{\sigma^2_\epsilon} + \frac{\bar{R}^k}{\sigma^2_{R^k}}}{\frac{1}{\sigma^2_\epsilon} + \frac{1}{\sigma^2_{R^k}}}}{\sqrt{\frac{\sigma^2_\epsilon \sigma^2_{R^k}}{\sigma^2_\epsilon + \sigma^2_{R^k}}}}\right)^2 + \frac{1}{2}\left[\frac{\left(\frac{\bar{s}^*}{\sigma^2_\epsilon} + \frac{\bar{R}^k}{\sigma^2_{R^k}}\right)^2}{\frac{1}{\sigma^2_\epsilon} + \frac{1}{\sigma^2_{R^k}}} - \frac{\bar{s}^*}{\sigma^2_\epsilon} - \frac{\bar{R}^{k2}}{\sigma^2_{R^k}}\right].\end{aligned}$$

Then, $\int_{R^{k*}}^{\infty}[\partial x(R^k, \bar{s}^*)/\partial \bar{s}^*] dF(R^k)$ is written as

$$\int_{R^{k*}}^{\infty} \frac{\partial x(R^k, \bar{s}^*)}{\partial \bar{s}^*} dF(R^k) = \left(\int_{z^*}^{\infty} \phi(z) dz\right) \frac{1}{\sqrt{2\pi}} \sqrt{\frac{1}{\sigma^2_\epsilon + \sigma^2_{R^k}}} \exp\left\{\frac{1}{2}\left[\frac{\left(\frac{\bar{s}^*}{\sigma^2_\epsilon} + \frac{\bar{R}^k}{\sigma^2_{R^k}}\right)^2}{\frac{1}{\sigma^2_\epsilon} + \frac{1}{\sigma^2_{R^k}}} - \frac{\bar{s}^*}{\sigma^2_\epsilon} - \frac{\bar{R}^{k2}}{\sigma^2_{R^k}}\right]\right\},$$

where

$$z^* = \frac{R^{k*} - \frac{\frac{\bar{s}^*}{\sigma^2_\epsilon} + \frac{\bar{R}^k}{\sigma^2_{R^k}}}{\frac{1}{\sigma^2_\epsilon} + \frac{1}{\sigma^2_{R^k}}}}{\sqrt{\frac{\sigma^2_\epsilon \sigma^2_{R^k}}{\sigma^2_\epsilon + \sigma^2_{R^k}}}}$$

Note that $\lim_{\sigma_\epsilon \to 0} = \Phi^{-1}(\gamma)$ and

$$\lim_{\sigma_\epsilon \to 0} \frac{1}{2} \left[\frac{\left(\frac{\bar{s}^*}{\sigma_\epsilon^2} + \frac{\bar{R}^k}{\sigma_{R^k}^2} \right)^2}{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_{R^k}^2}} - \frac{\bar{s}^*}{\sigma_\epsilon^2} - \frac{\bar{R}^{k2}}{\sigma_{R^k}^2} \right] = -\frac{1}{2} \left(\frac{\bar{s}^* - \bar{R}^k}{\sigma_{R^k}} \right)^2 .$$

Therefore, the limit of $\int_{R^{k*}}^{\infty} [\partial x(R^k, \bar{s}^*)/\partial \bar{s}^*] dF(R^k)$ is given by

$$\lim_{\sigma_\epsilon \to 0} \int_{R^{k*}}^{\infty} \frac{\partial x(R^k, \bar{s}^*)}{\partial \bar{s}^*} dF(R^k) = (1-\gamma) f(\bar{s}^*),$$

where $f(\cdot)$ is the pdf of the normal distribution with mean $\bar{R}^k$ and variance $\sigma_{R^k}^2$.

Finally, the term, $\int_{R^{k*}}^{\infty} x(R^k, \bar{s}^*(L)) dF(R^k)$, in equation (7) goes to zero as $\sigma_\epsilon \to 0$. Therefore, in the limit of $\sigma_\epsilon \to 0$, equation (7) is reduced to equation (11).

Proof of Proposition 1.

(i) The first-order condition of the banks' problem in the limit equilibrium (11) is written as $0 = \partial \mathbb{E}(\pi) / \partial L$, where

$$\begin{aligned} \frac{\partial \mathbb{E}(\pi)}{\partial L} &= \int_{\frac{R^{k*} - \bar{R}^k}{\sigma_{R^k}}}^{\infty} \left(\bar{R}^k + \sigma_{R^k} z \right) d\Phi(z) \\ &- \left\{ \left[1 - \Phi\left(\frac{R^{k*} - \bar{R}^k}{\sigma_{R^k}} \right) \right] R + \lambda (1-\gamma) [1 + \lambda (1-\gamma)] \phi \left(\frac{R^{k*} - \bar{R}^k}{\sigma_{R^k}} \right) R^2 \frac{L-1}{L^2} \right\}. \end{aligned}$$

A marginal change in this derivative with respect to a marginal increase in $\bar{R}^k$ is given by

$$\begin{aligned} \frac{\partial^2 \mathbb{E}(\pi)}{\partial L \partial \bar{R}^k} &= 1 - \Phi\left(\frac{R^{k*} - \bar{R}^k}{\sigma_{R^k}} \right) + \left[R^{k*} - R\phi\left(\frac{R^{k*} - \bar{R}^k}{\sigma_{R^k}} \right) \right] \frac{1}{\sigma_{R^k}} \\ &+ \frac{\lambda (1-\gamma) [1 + \lambda (1-\gamma)]}{\sigma_{R^k}} \phi' \left(\frac{R^{k*} - \bar{R}^k}{\sigma_{R^k}} \right) R^2 \frac{L-1}{L^2}. \end{aligned}$$

Because $\max_z \phi(z) < 0.4$, the assumptions of this proposition imply $R^{k*} > R\phi\left(\frac{R^{k*} - \bar{R}^k}{\sigma_{R^k}} \right)$, and thereby the sign of the above derivative is positive: $\partial^2 \mathbb{E}(\pi) / \left(\partial L \partial \bar{R}^k \right) > 0$. Given that the solution L is an optimal solution, the $\partial \mathbb{E}(\pi) / \partial L$ curve is downward sloping. Then, $\partial^2 \mathbb{E}(\pi) / \left(\partial L \partial \bar{R}^k \right) > 0$ implies that the $\partial \mathbb{E}(\pi) / \partial L$ curve shifts upward, implying that the optimal L increases. Hence, the demand curve shifts outward.

(ii) Similarly, a marginal change of $\partial \mathbb{E}(\pi) / \partial L$ with respect to a marginal increase in λ is given by

$$\frac{\partial^2 \mathbb{E}(\pi)}{\partial L \partial \lambda} = -\left[R^{k*} - R\phi\left(\frac{R^{k*}-\bar{R}^k}{\sigma_{R^k}}\right)\right]\frac{1}{\sigma_{R^k}}\frac{\partial R^{k*}}{\partial \lambda}$$
$$-\frac{\lambda(1-\gamma)[1+\lambda(1-\gamma)]}{\sigma_{R^k}}\phi'\left(\frac{R^{k*}-\bar{R}^k}{\sigma_{R^k}}\right)R^2\frac{L-1}{L^2}\frac{\partial R^{k*}}{\partial \lambda},$$

where $\partial R^{k*}/\partial\lambda = R(1-1/L)(1-\gamma) > 0$. Hence, $\partial^2\mathbb{E}(\pi)/\partial L\partial\lambda < 0$, which implies that an increase in λ shifts the demand curve inward.

(iii) The supply curve (1) is written as

$$R = \frac{u'(y-(L-1)n)}{1-P+\mathbb{E}(v|\text{default})P}.$$

From this it is clear that an increase in y shifts the supply curve outward.

(iv) Similarly, the supply curve implies that an increase n shits the curve inward.

Calibration: the two-period model. The period of time is annual. The calibration strategy is to set target values for L, R and P and pin down parameter values for γ, y and σ_{R^k} jointly. The target values are the leverage of $L = 15$, the interest rate of $R = 1.01$ and the default probability of $P = 0.03$. For other parameters, the liquidation cost is set as $\lambda = 0.3$, the mean return on bank asset is set as $\bar{R}^k = 1.05$, implying a four percent interest rate spread, and the banks' endowment n is set to 0.1. Finally, the utility function in period 1 is assumed to be $u(c_1) = \frac{c_1^{1-\sigma}}{1-\sigma}$ with $\sigma = 0.1$.

The three parameters, γ, y and σ_{R^k}, are set as follows. Fix γ. Calculate R^{k*} from equation (10) as $R^{k*} = R(1-1/L)[1+\lambda(1-\gamma)]$. From equation (8) calculate σ_{R^k} as

$$\sigma_{R^k} = \frac{R^{k*}-\bar{R}^k}{\Phi^{-1}(P)}.$$

Because σ_{R^k} has to be strictly positive, the initial guess for γ has to be such that $R^{k*} - \bar{R}^k > 0$, i.e.,

$$\gamma < \bar{\gamma} \equiv 1 - \frac{1}{\lambda}\left(\frac{\bar{R}^k}{R}\frac{L}{L-1} - 1\right).$$

Under the parameter values set above, the upper bound is $\bar{\gamma} = 0.6205$. Then, given $\gamma < \bar{\gamma}$, adjust γ so that it satisfies condition (11):

$$\int_{R^{k*}}^{\infty} R^k dF(R^k) = \left[1 - F\left(R^{k*}\right)\right]R + \lambda(1-\gamma)[1+\lambda(1-\gamma)]f\left(R^{k*}\right)R^2\frac{L-1}{L^2}.$$

This process pins down values for γ and σ_{R^k}.

Next, y is set to satisfy equation (1), i.e.,

$$y = (L-1)\,n + \frac{1}{R\,(1 - P + \mathbb{E}(v|\text{default})P)},$$

where $\mathbb{E}(v|\text{default})P$ is given by

$$\mathbb{E}(v|\text{default})P = \int_{-\infty}^{R^{k*}} \left(\frac{R^k}{R} \frac{L}{L-1} - \lambda \right) dF\left(R^k\right).$$

Derivation of equation (13). The first-order condition of the constrained social planner problem is $\partial \text{SW}/\partial L = 0$, where

$$\begin{aligned}\frac{\partial \text{SW}}{\partial L} =& \Bigg\{ \mathbb{E}(R^k) - R\,[1 - P + \mathbb{E}(v|\text{default})P] - \lambda \mathbb{E}(x) R - \lambda\,[1 + \lambda(1-\gamma)]\, f(R^{k*}) R^2 \frac{L-1}{L^2} \\ & - \lambda \mathbb{E}(x)(L-1)\frac{dR(L)}{dL} \Bigg\} n.\end{aligned}$$

The first-order condition of the bank's problem in the competitive equilibrium is $\partial \mathbb{E}(\pi)/\partial L = 0$, where

$$\frac{\partial \mathbb{E}(\pi)}{\partial L} = \left[\int_{R^{k*}}^{\infty} R^k dF(R^k) - (1-P)R - \lambda(1-\gamma)\,[1 + \lambda(1-\gamma)]\, f(R^{k*}) R^2 \frac{L-1}{L^2} \right] n.$$

Then, $\partial \text{SW}/\partial L$ evaluated at the competitive equilibrium is given by

$$\begin{aligned}\left.\frac{\partial \text{SW}}{\partial L}\right|_{\text{CE}} =& \left.\frac{\partial \text{SW}}{\partial L}\right|_{\text{CE}} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial L}\right|_{\text{CE}} \\ \propto & \int_{-\infty}^{R^{k*}} R^k dF(R^k) - R\mathbb{E}(v|\text{default})P - R\lambda \mathbb{E}(x) \\ & - \lambda\gamma\,[1 + \lambda(1-\gamma)]\, f(R^{k*}) R^2 \frac{L-1}{L} - \lambda \mathbb{E}(x)(L-1)\frac{dR(L)}{dL}.\end{aligned}$$

Note that $R\mathbb{E}(v|\text{default})P$ is given by

$$\begin{aligned}R\mathbb{E}(v|\text{default})P =& R \int_{-\infty}^{R^{k*}} \left[\frac{R^k}{R} \frac{L}{L-1} - \lambda x(R^k, \bar{s}^*) \right] dF(R^k) \\ =& \frac{L}{L-1} \int_{-\infty}^{R^{k*}} R^k dF(R^k) - R\lambda \int_{-\infty}^{R^{k*}} x(R^k, \bar{s}^*) dF(R^k).\end{aligned}$$

Then, the first-order condition of the constrained social planner problem, evaluated at the competitive equilibrium, is written as

$$\left.\frac{\partial \mathrm{SW}}{\partial L}\right|_{\mathrm{CE}} \propto -\frac{1}{L-1}\int_{-\infty}^{R^{k*}} R^k dF(R^k) - R\lambda \int_{R^{k*}}^{\infty} x(R^k, \bar{s}^*) dF(R^k)$$
$$-\lambda\gamma\left[1+\lambda(1-\gamma)\right] f(R^{k*}) R^2 \frac{L-1}{L} - \lambda \mathbb{E}(x)(L-1)\frac{dR(L)}{dL}.$$

In the limit equilibrium $\int_{R^{k*}}^{\infty} x(R^k, \bar{s}^*) dF(R^k) = 0$. This completes the derivation of (13).

Condition for the upper-sloping supply curve. Consider the limit equilibrium. The slope of the supply curve (1) is given by

$$\frac{dR(L)}{dL} = \frac{-u'' n}{1 - P + \mathbb{E}(v|\text{default})P} - \frac{u'\left[-\frac{dP(L)}{dL} + \frac{d\mathbb{E}(v|\text{default})P}{dL}\right]}{\left[1 - P + \mathbb{E}(v|\text{default})P\right]^2},$$

where

$$\frac{dP(L)}{dL} = f(R^{k*})\frac{dR^{k*}(L)}{dL},$$
$$\frac{d\mathbb{E}(v|\text{default})P}{dL} = \left[\frac{R^{k*}}{R}\frac{L}{L-1} f(R^{k*}) - \lambda(1-\gamma)\right]\frac{dR^{k*}(L)}{dL} - \frac{1}{R}\frac{1}{(L-1)^2}\int_{-\infty}^{R^{k*}} R^k dF(R^k)$$
$$-\frac{1}{R^2}\frac{L}{L-1}\int_{-\infty}^{R^{k*}} R^k dF(R^k)\frac{dR(L)}{dL}.$$

Rearranging for $dR(L)/dL$ yields

$$\begin{aligned}\frac{dR(L)}{dL} &= \frac{-u'' n + \frac{1}{R}\frac{1}{(L-1)^2}\int_{-\infty}^{R^{k*}} R^k dF(R^k)}{1 - P + \mathbb{E}(v|\text{default})P - \int_{-\infty}^{R^{k*}} \frac{R^k}{R}\frac{L}{L-1} dF(R^k)} \\ &= \frac{-u'' n + \frac{1}{R}\frac{1}{(L-1)^2}\int_{-\infty}^{R^{k*}} R^k dF(R^k)}{1 - P - \lambda\int_{-\infty}^{R^{k*}} x(R^k, \bar{s}^*) dF(R^k)} \\ &= \frac{-u'' n + \frac{1}{R}\frac{1}{(L-1)^2}\int_{-\infty}^{R^{k*}} R^k dF(R^k)}{1 - (1+\lambda)P}\end{aligned}$$

Therefore, the supply curve is upward-sloping if and only if $P < (1+\lambda)^{-1}$.

Derivation of condition (17). From equation (16) the partial derivative of R^{k*} with respect to m is given by

$$\frac{\partial R^{k*}}{\partial m} = \frac{1}{\left(\frac{L}{L-1} - m\right)^2}\left[-\frac{L}{L-1}(1+\lambda) + R(1+\lambda(1-\gamma))\right].$$

Hence, $\partial R^{k*}/\partial m < 0$ if and only if $R < (L/(L-1))[1+\lambda\gamma/(1+\lambda(1-\gamma))]$. This condition holds for any $L > 1$ if $R < 1+\lambda\gamma/(1+\lambda(1-\gamma))$, which is condition (17).

Derivation of equation (18). Equation (18) is derived similarly to deriving equation (11). A main difference lies in the calculation of $\partial\bar{s}^*/\partial L$. Calculating $\partial\bar{s}^*/\partial L$ requires a solution for $\bar{s}^*$, which is characterized by equations (5) and (15). These equations are explicitly written as (23) and

$$R^{k*} = \frac{R-m}{\frac{L}{L-1}-m}\left[1+\lambda\frac{R\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)-m}{R-m}\right]. \tag{25}$$

Totally differentiating equations (23) and (25) with respect to $\bar{s}^*$, R^{k*} and L yields

$$\begin{aligned} dR^{k*} =& \frac{1}{\frac{\sigma_\epsilon^2}{\sigma_{R^k}^2}+1}d\bar{s}^*,\\ dR^{k*} =& \frac{1}{(L-1)^2}\frac{R-m}{\left(\frac{L}{L-1}-m\right)^2}\left[1+\lambda\frac{R\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)-m}{R-m}\right]dL\\ &+\lambda\frac{R-m}{\frac{L}{L-1}-m}\frac{R\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)-m}{R-m}\frac{1}{\sigma_\epsilon}\left(d\bar{s}^*-dR^{k*}\right). \end{aligned}$$

Combining these two equations yields

$$\frac{d\bar{s}^*}{dL} = \frac{\frac{\sigma_\epsilon^2+\sigma_{R^k}^2}{(L-1)^2}\frac{R-m}{\left(\frac{L}{L-1}-m\right)^2}\left[1+\lambda\frac{R\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)-m}{R-m}\right]}{\sigma_{R^k}^2-\sigma_\epsilon\lambda\frac{R-m}{\frac{L}{L-1}-m}\frac{R\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)-m}{R-m}}.$$

This implies that in the limit equilibrium,

$$\lim_{\sigma_\epsilon\to 0}\frac{d\bar{s}^*}{dL} = \frac{1}{(L-1)^2}\frac{R-m}{\left(\frac{L}{L-1}-m\right)^2}\left[1+\lambda\frac{R(1-\gamma)-m}{R-m}\right].$$

The rest of the derivation of equation (18) is the same as the derivation of equation (11).

Derivation of equation (19). The first-order condition of the banks' problem with respect to liquidity m is

$$0 = -\int_{R^{k*}}^{\infty}(R^k-1)dF(R^k) - R\lambda\int_{R^{k*}}^{\infty}\frac{\partial x(R^k,\bar{s}^*)}{\partial\bar{s}^*}\frac{\partial\bar{s}^*(L,m)}{\partial m}dF(R^k). \tag{26}$$

To calculate $\partial\bar{s}^*(L,m)/\partial m$, totally differentiating equations (23) and (25) with respect to $\bar{s}^*$, R^{k*}

and m yields

$$dR^{k*} = \frac{1}{\frac{\sigma_\epsilon^2}{\sigma_{R^k}^2}+1} d\bar{s}^*,$$

$$dR^{k*} = -\frac{\frac{L}{L-1}-R}{\left(\frac{L}{L-1}-m\right)^2}\left[1+\lambda\frac{R\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)-m}{R-m}\right]dm - \frac{\lambda R\left[1-\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\right]}{\left(\frac{L}{L-1}-m\right)(R-m)}dm$$
$$+\frac{\lambda R\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)}{\frac{L}{L-1}-m}\frac{1}{\sigma_\epsilon}\left(d\bar{s}^*-dR^{k*}\right).$$

Combining these two equations yields

$$\frac{d\bar{s}^*}{dm} = -\frac{\frac{\frac{L}{L-1}-R}{\left(\frac{L}{L-1}-m\right)^2}\left[1+\lambda\frac{R\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)-m}{R-m}\right]+\frac{\lambda R\left[1-\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\right]}{\left(\frac{L}{L-1}-m\right)(R-m)}}{\frac{\sigma_{R^k}^2}{\sigma_\epsilon^2+\sigma_{R^k}^2}-\frac{\sigma_\epsilon}{\sigma_\epsilon^2+\sigma_{R^k}^2}\frac{\lambda R\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)}{\frac{L}{L-1}-m}}$$

Therefore,

$$\lim_{\sigma_\epsilon\to 0}\frac{d\bar{s}^*}{dm} = -\left\{\frac{\frac{L}{L-1}-R}{\left(\frac{L}{L-1}-m\right)^2}\left[1+\lambda\frac{R(1-\gamma)-m}{R-m}\right]+\frac{\lambda R\gamma}{\left(\frac{L}{L-1}-m\right)(R-m)}\right\}. \tag{27}$$

In the limit equilibrium, the term, $\int_{R^{k*}}^{\infty}\partial x(R^k,\bar{s}^*)/\partial\bar{s}^* dF(R^k)$, in (26) is given by $(1-\gamma)f(R^{k*})$ as shown in deriving equation (11). Substituting this and (27) into the first-order condition (26) leads to equation (19).

Proof of Proposition 3. The assumption of $m > 0$ in the limit equilibrium implies that the first-order condition (19) holds:

$$0 = \frac{\partial\mathbb{E}(\pi)}{\partial m} = \left[-\int_{R^{k*}}^{\infty}(R^k-1)dF(R^k) - \lambda(1-\gamma)f(R^{k*})R\frac{\partial\bar{s}^*}{\partial m}\right](L-1)n,$$

where

$$\frac{\partial\bar{s}^*}{\partial m} = -\left[\frac{\left(\frac{L}{L-1}-R\right)\left[1+\lambda\frac{R(1-\gamma)-m}{R-m}\right]}{\left(\frac{L}{L-1}-m\right)^2}+\frac{\lambda\gamma R}{\left(\frac{L}{L-1}-m\right)(R-m)}\right].$$

For this condition to hold, it must be $\partial\bar{s}^*/\partial m < 0$. The first-order condition of the social planner problem is written as

$$0 = \frac{\partial\text{SW}}{\partial m} = \left[-(\mathbb{E}(R^k)-1) - \lambda f(R^{k*})R\frac{\partial\bar{s}^*}{\partial m}\right](L-1)n.$$

Evaluating $\partial \text{SW}/\partial m$ at the competitive equilibrium, I obtain:

$$\begin{aligned}\left.\frac{\partial \text{SW}}{\partial m}\right|_{\text{CE}} =& \left.\frac{\partial \text{SW}}{\partial m}\right|_{\text{CE}} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial m}\right|_{\text{CE}} \\ &\propto \int_{-\infty}^{R^{k*}} (1-R^k)dF(R^k) - \lambda\gamma f(R^{k*})R\frac{\partial \bar{s}^*}{\partial m} > 0.\end{aligned}$$

The inequality holds because $\partial \bar{s}^*/\partial m < 0$ and because $\int_{-\infty}^{R^{k*}}(1-R^k)dF(R^k) > 0$ by assumption. The fact that $\partial \text{SW}/\partial m > 0$ implies that bank liquidity in a limit equilibrium is insufficient. This completes the proof of Proposition 3.

Proof of Proposition 4. The first-order condition, with respect to bank leverage, of the social planner problem in the model with liquidity is given by

$$\begin{aligned}0 = \frac{\partial \text{SW}}{\partial L} =& \Big[-u'(y-(L-1)n) + \mathbb{E}(R^k) - (\mathbb{E}(R^k)-1)m \\ &-\lambda\mathbb{E}\left(\frac{\partial x}{\partial \bar{s}^*}\right)\frac{\partial \bar{s}^*}{\partial L}R(L-1) - \lambda\mathbb{E}(x)R - \lambda\mathbb{E}(x)\frac{\partial R}{\partial L}(L-1)\Big]n,\end{aligned}$$

where $u'(y-(L-1)n) = R(1-P+\mathbb{E}(v|\text{default})P)$. In the limit equilibrium, $\lim_{\sigma_\epsilon \to 0}\mathbb{E}(\partial x/\partial \bar{s}^*) = f(R^{k*})$. The first-order condition of the banks problem is given by equation (18), which is equal to $0 = (\partial \mathbb{E}(\pi)/\partial L)(1/n)$. Evaluating $\partial \text{SW}/\partial L$ at the allocation implied by condition (18), I obtain

$$\begin{aligned}\left.\frac{\partial \text{SW}}{\partial L}\right|_{\text{CE}} =& \left.\frac{\partial \text{SW}}{\partial L}\right|_{\text{CE}} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial L}\right|_{\text{CE}} \\ &\propto - R\mathbb{E}(v|\text{default})P + \int_{-\infty}^{R^{k*}} R^k dF(R^k) - \int_{-\infty}^{R^{k*}} (R^k-1)dF(R^k) \\ &- \lambda\gamma f(R^{k*})\frac{\partial \bar{s}^*}{\partial L}R(L-1) - \lambda R\mathbb{E}(x) - \lambda\mathbb{E}(x)\frac{\partial R}{\partial L}(L-1)\end{aligned}$$

Note that $R\mathbb{E}(v|\text{default})P$ is given by

$$\begin{aligned}R\mathbb{E}(v|\text{default})P =& R\int_{-\infty}^{R^{k*}} \left[\frac{R^k}{R}\frac{L}{L-1} - \lambda x + \frac{(1+\lambda-R^k)m}{R}\right] dF(R^k) \\ =& \frac{L}{L-1}\int_{-\infty}^{R^{k*}} R^k dF(R^k) - \lambda R\int_{-\infty}^{R^{k*}} x dF(R^k) + \int_{-\infty}^{R^{k*}} (1+\lambda-R^k)m dF(R^k).\end{aligned}$$

Therefore,

$$\left.\frac{\partial \text{SW}}{\partial L}\right|_{\text{CE}} = -\frac{1}{L-1}\int_{-\infty}^{R^{k*}} R^k dF(R^k) - \lambda mP - \lambda\gamma f(R^{k*})\frac{\partial \bar{s}^*}{\partial L}R(L-1) - \lambda\mathbb{E}(x)\frac{\partial R}{\partial L}(L-1),$$

where $\int_{R^{k*}}^{\infty} x dF(R^k) = 0$ was imposed. Thus, the under the assumption of the upward-sloping supply curve, i.e., $\partial R/\partial L > 0$, the sign of $\partial \mathrm{SW}/\partial L|_{\mathrm{CE}}$ is negative, implying that the leverage chosen by banks is excessive from the social view point. Lowering leverage can increase social welfare. This completes the proof of Proposition 4.

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