The Sovereign-Bank Interaction in the Eurozone Crisis

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Abstract

This paper investigates the relationship between government debt default, the banking sector and the wider economy. It builds a model of the public bond market, the banking sector and the real economy to study the mechanism by which a government default affects the other sectors and shows that this model can explain some "stylized facts" of the Eurozone crisis. The key aspect of the model is a friction in the financial market which forces banks to hold part of their assets in the form of government bonds. In such a model, an exogenous increase in the probability of default can lead to the joint occurrence of a credit crunch (i.e. declining bank lending and rising spreads between loan interest rates and deposit rates) and a decline in output. The paper also shows that an adverse technology shock (an exogenous decline in total factor productivity) cannot fully explain these phenomena.

Keywords: Government default, financial frictions, business cycle model.

JEL Codes E37, E44, H63.

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1 Introduction

The recent economic crisis in the Eurozone had three elements.[1] One the one hand, there was a financial crises in which asset prices plunged, credit spreads widened, financial firms went bankrupt, and the inter-bank market froze up. Second, there was a government debt crisis in some Eurozone countries where interest spreads spiked suddenly and unexpectedly and forced governments to seek fiscal support from other countries and international organizations. Third, there was a deep and long downturn in the real economy producing record highs of unemployment in some member countries.

Although a large literature has developed attempting to explain each element of the crisis in isolation, few attempts have been made to provide a coherent framework for analyzing all three elements of the crisis and their interaction. The goal of this paper is to provide such a framework. First, it develops a fully-specified general equilibrium model which can explain the joint occurrence of a credit crunch, a government debt crisis and a decline in economic activity. Second, it shows that a calibrated version of the model can replicate some basic quantitative features of the Eurozone crisis.

In building this model, we draw on two strands of literature. We borrow from the work of Gertler and Kiyotaki (2010) among others who integrate financial intermediaries (i.e. a banking sector) into a business cycle model. This literature posits that financial intermediation is subject to problems of asymmetric information and limited commitment which give rise to an endogenous constraint on the lending activity of banks. This financial friction acts to amplify the impact of exogenous disturbances ("shocks") to the real economy. Moreover, the financial sector can also be a source of shocks in these models.

The second input for the model comes from the literature on quantitative models of sovereign debt crises. In recent work by Gennaioli et al. (2014) and Sosa-Padilla (2012), banks hold government debt securities on their balance sheets which exposes them to the risk of government default. If the government defaults, financial sector incurs losses on their government bonds which reduces their ability to lend to firms and thus affects and the real economy.

Previous studies have proposed a number of causal mechanisms for the government-bank interaction observed in the Eurozone crisis. Acharya et al. (2014) build a model in which bank bailouts by governments produce a negative effect on public finances which, in turn, reduce the value of the implicit government guarantees for bank debt. They provide empirical evidence for their model by showing that the announcement of bank bailouts was associated

[1] Shambaugh (2012) provides a concise overview of the Eurozone crisis.

with increasing sovereign CDS spreads and that sovereign and bank CDS tended to move in sync after the bailouts. Bolton and Jeanne (2011) provide a detailed theoretical analysis of contagious sovereign defaults in which bank balance sheets play a crucial role. They focus on the incentives of banks to diversify their bond portfolio as well as on the government's incentives to default under different scenarios of financial and fiscal integration.

In a similar vein, Mody and Sandri (2012) show that, beginning with the nationalization of Anglo Irish bank in 2009, sovereign interest spreads were positively correlated with a financial stress indicator. They interpret this finding in the context of a stylized model in which the government faces a trade-off between bailing out weak banks to support the economy and keeping public debt at a sustainable level. Depending on initial levels of debt (among other things), the model predicts bank bailouts can lead to the vicious circle described above. De Bruyckere et al. (2013) investigate risk spillovers between bank and sovereign bonds in the Eurozone using data from the EBA "stress test" and find that banks with a weak capital buffer, a weak funding structure and less traditional banking activities are particularly vulnerable to risk spillovers. Their results further support the view that sovereign default risk and fragile financial institutions mutually reinforce each other.

Recent theoretical work by Gennaioli et al. (2014) shows that government defaults are costly because they destroy the balance sheets of domestic banks, leading to declines in private credit and output losses. Using data from Italy, Bocola (2016) estimates a similar model concludes that an increase in default risk has contributed significantly to the observed decline in output. Sosa-Padilla (2012) employs a similar framework to build a quantitative model of sovereign debt default. He found that his model can replicate some of the key features of the recent Argentine debt crisis.

This paper's main contribution is to incorporate the government-bank interaction into an otherwise standard business cycle model to study the effect of an exogenous increase in the probability of a government default, which we call a "government default shock". It argues that such a shock can explain some stylized facts about the Eurozone crisis. It also shows that a conventional (adverse) technology shock, i.e. an exogenous decrease in total factor productivity, can explain some but not all of these stylized facts. This suggests that attempts to explain the Eurozone crisis as the consequence of ”real shocks” faces serious empirical challenges. However, focusing only on the government interest spread and quantity variables such as output, investment, consumption, bank lending and employment, one cannot differentiate empirically between technology and government default shocks. For that, it is crucial to look at the behavior of bank interest rates. The model thus provides a basis for further empirical research on the sources of the Eurozone crisis.

The remainder of the paper is organized as follows. The next section establishes some "stylized facts" about the Eurozone crisis. Section 3 presents and discusses the model. Section 4 deals with the calibration of the model. Section 5 shows the results of simulations of the calibrated model. Finally, section 6 sums up the main results of the analysis and discusses avenues for future research.

2 The Eurozone crisis: some stylized facts

In this section, we document a set of stylized facts that our model aims to explain. Our focus is on five member states of the Eurozone that have experienced a government debt crisis: Greece, Italy, Ireland, Spain, and Portugal. The data for this section come from the European Central Bank's database unless otherwise indicated. The four stylized facts we wish to establish are:

1. Government interest spreads increased after 2010, then declined beginning in 2012;

2. Banks' lending margins (lending-borrowing spreads) increased after 2010, but with a time lag compared to government spreads;

3. Banks' lending volumes (loans to non-financial businesses) decreased after 2010;

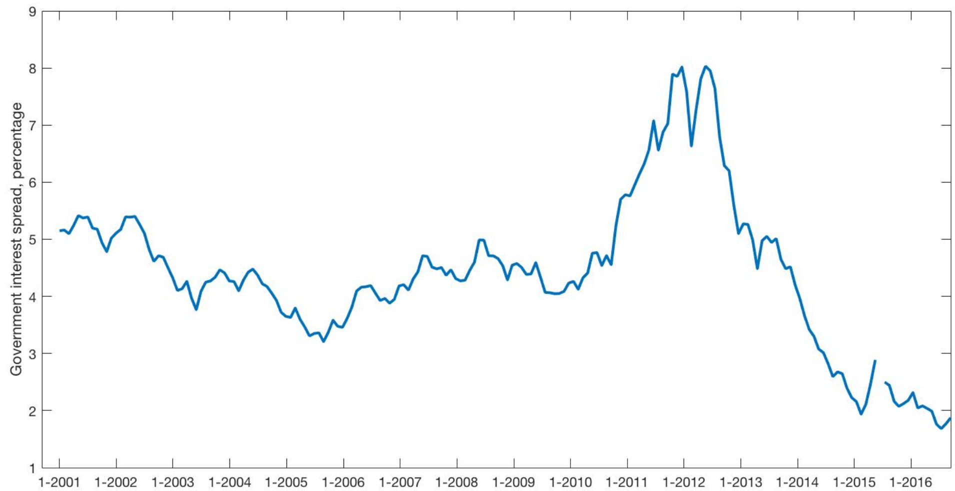
4. Economic activity decreased after 2010 and only slowly recovered beginning in 2013.

2.1 Government interest spreads

The government interest spread is defined as the difference between the yield on (long-term) government bonds and the risk-free interest rate. It can be interpreted as a market-based measure of the perceived risk of a government default insofar as a the larger spread implies a higher (perceived) default probability.[2] In most of the existing literature, the risk-free rate is taken to be the yield of German government bonds, presumably because the German government is supposed to be the "safest" within the eurozone. We depart from this practice and use the interest rate on bank deposits as a proxy of the risk-free rate, since bank deposits in the Eurozone are protected by deposit insurance and can thus be expected to carry no risk premium. This choice is not critical however, because the German interest rate moves in

[2]Risk is not the only reason for interest spreads. Differences in liquidity as well as collateral value also play role.

Figure 1: Government interest spreads



Notes: Difference between the yield on 10-year government bonds and the interest rate on deposits of households in monetary financial institutions (MFIs), GDP-weighted average. Spreads in percentage points. Countries included: Greece, Ireland, Italy, Portugal, Spain. Data from European Central Bank.

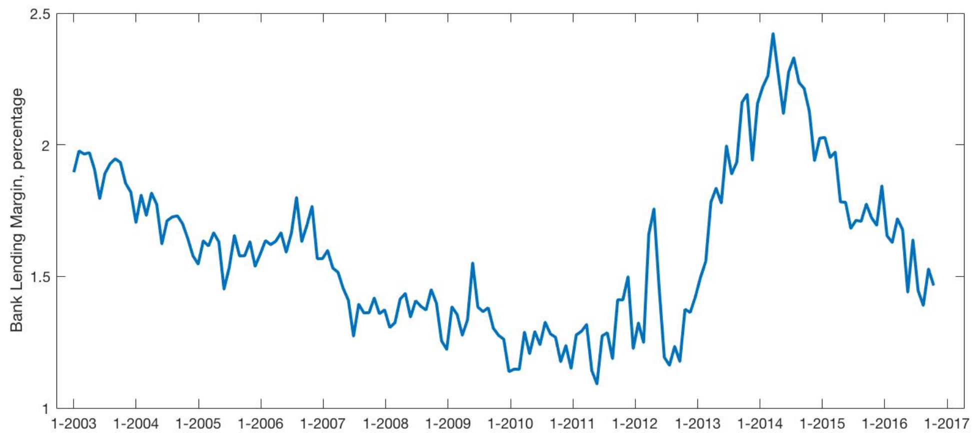
sync with bank deposit rates for most of the period covered here. The main advantage of our definition is that it has a close counterpart in the model we develop in the next section.

Figure (1) shows the GDP-weighted average of the government interest spreads of the five countries in our sample, which can be interpreted as the government spread of the "average" eurozone crisis country. The spread is given in percentage points. Before 2011, the spread fluctuated between 4 and 5 percentage points, and started to rise quickly reaching a peak around 8 percentage points in 2012 and declined thereafter. This behavior of the average spread masks considerable variation between the spreads of specific countries. However, a similar pattern of quickly rising and then falling spreads can be observed in all eurozone crisis countries, although with differences in timing and amplitude. The case of Greece, where the spread reached a maximum of 30 percentage points, certainly stands out in this regard.

2.2 Bank lending margins

The bank lending margin is defined as the difference between the interest rate on bank loans and bank deposits. It's the main source of revenue for commercial banks and measures the economic cost, or "friction", of financial intermediation since, in a perfectly frictionless

Figure 2: Bank lending margins



Notes: Difference (in basis points) between the average interest rate on new MFI loans to non-financial business and the average interest rate on new deposits of households in MFIs. GDP-weighted average. Data from European Central Bank.

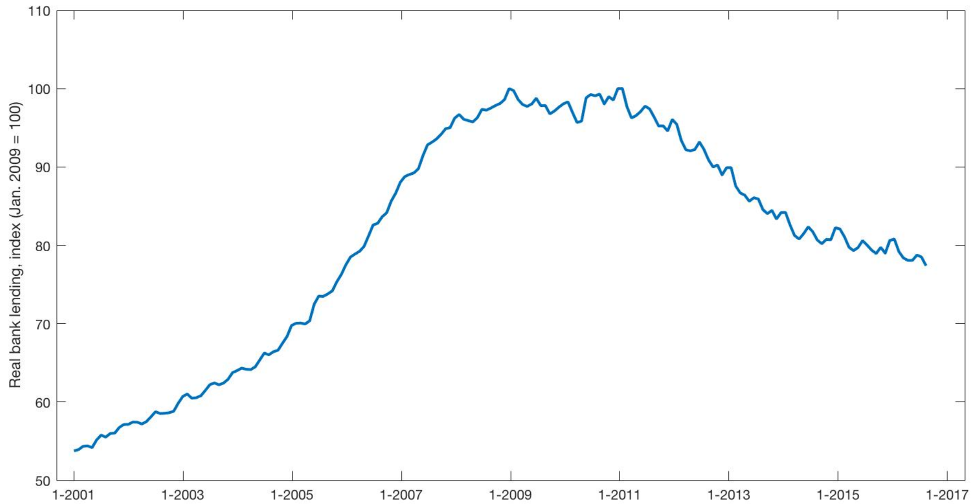
financial system, there would be no difference between lending or borrowing rates. Hence an increase in the lending margin indicates an increase in “financial frictions”. We take the average interest rate on new bank loans to non-financial businesses as our lending rate and the average interest rate on new bank deposits from households as the deposit rate. Again, we weight the country-specific lending margins the GDPs of the respective countries to build an average lending margin.

As one can see in figure (2), the average bank lending margin exhibit a slight downward trend prior to 2011 and a sudden and marked increase thereafter. The average lending margin reaches a peak in early 2014 and then starts to decline. As before, there are important cross-country differences in the behavior of the lending margin. The rise in the lending margin is most pronounced, as well as most persistent, for Greece and Spain.

2.3 Bank lending

Bank loans play a crucial role as a source of financing for firms in the eurozone. Loans to non-financial corporations are also an important asset on banks' balance sheets, accounting for between a third and half of total bank assets in the eurozone. Figure (3) shows the development of the total amount of outstanding loans of monetary financial institutions to non-financial businesses deflated by the harmonized consumer price index. We normalize the amount of loans outstanding at the end of January 2009 to 100 and apply the same GDP-

Figure 3: Volume of bank loans outstanding



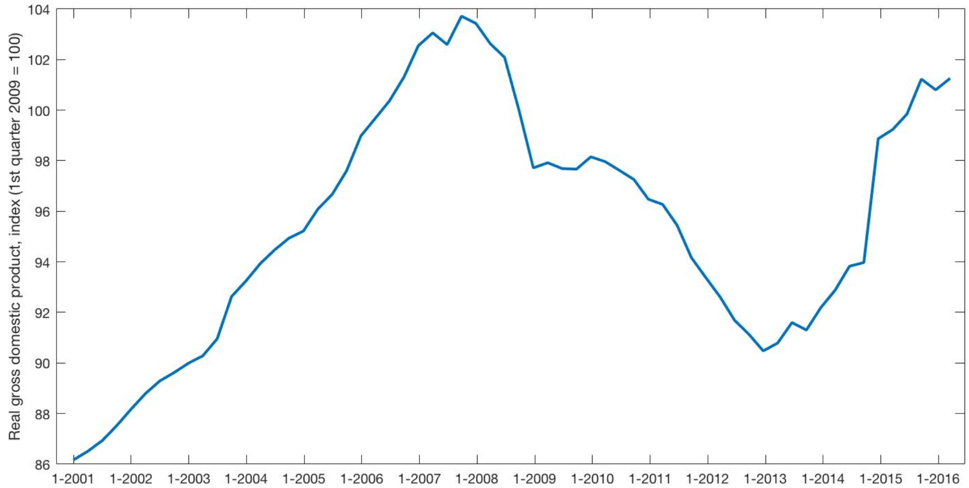
Notes: Outstanding loans of MFIs to non-financial businesses, deflated by the HICP. GDP-weighted average. Normalized to 100 in January 2009. Countries included: Greece, Ireland, Italy, Portugal, Spain. Data from European Central Bank.

weighted averaging procedure across countries which yields an index of real bank lending for the "average" eurozone crisis country. As we can see, real bank lending shows a continuously increasing tendency until 2009, levels off and then starts to decline dramatically after 2011. At the end of the sample period, in September 2016, the real volume of bank loans was 20 percent lower than its peak value. In Greece, Ireland and Spain, the decline in bank lending has been most noticeable, less so in Italy, where bank lending as decreased only slightly between 2009 and now.

2.4 Economic activity

Finally, figure (4) shows the combined real gross domestic product of our five eurozone countries. As can clearly be seen, these countries experienced a sharp decline in real economic activity beginning in 2008 which, after a brief period of stabilization in 2009/10, intensified in 2011. Although a slow recovery is currently underway, real GDP has remained below the pre-crisis level. At the lowest point, real GDP was more than 12 percent below the pre-crisis peak. Again, the decline in real GDP is most pronounced for Greece whose economy never recovered after 2008. Of the countries covered here, only Ireland has seen a substantial recovery in economic activity.

Figure 4: Real gross domestic product



Notes: Combined gross domestic product at constant prices of Greece, Ireland, Italy, Portugal, and Spain. Normalized to 100 in first quarter of 2009. Data from European Central Bank.

3 The model

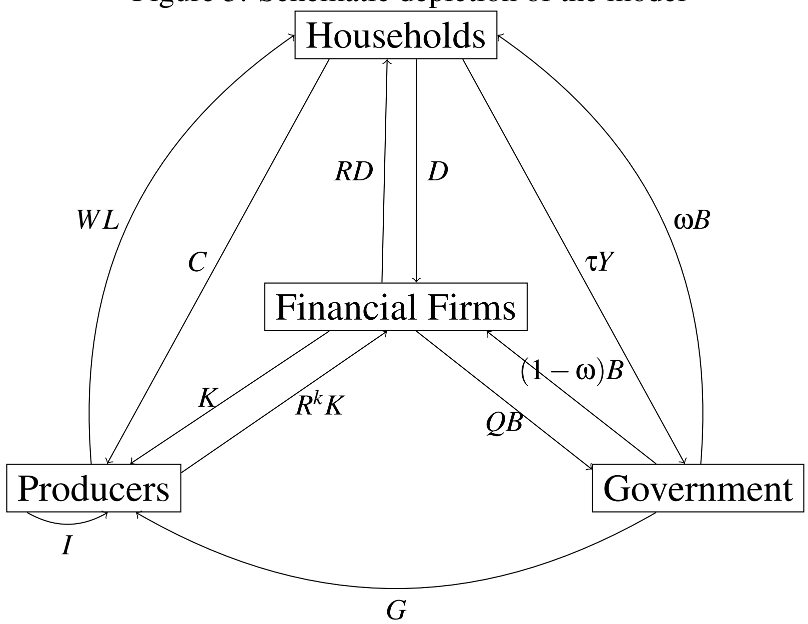
In this section, we build a business cycle model that is able to explain the stylized facts established in the last section.

The model consists of four sectors. Figure (5) provides a schematic depiction of the flow of goods and income between the various sectors in the model. The household sector consumes final goods, supplies labor to non-financial firms, pays taxes to the government and lends funds to financial firms (banks). The producer sector consists of non-financial firms who hire labor and borrow from financial firms to produce final goods. The government levies taxes on the household sector and borrows from financial firms. Financial firms intermediate funds between lenders, i.e. households, and borrowers, i.e. private firms and the government.

There are two sources of uncertainty in the model: First, there is random fluctuation in total factor productivity, which we will refer to as "technology shocks". Second, there is uncertainty about the government's willingness to service existing debt which we model as random shocks to the default rate, i.e. the fraction of debt the governments does not repay.

The model is a standard dynamic stochastic general equilibrium model, except for the inclusion of a financial friction and government default. The financial friction consists of an exogenous constraint on the composition of the financial sector's assets, which may arise from asymmetric information in bank lending or legal restrictions enforced by the government. Absent the financial friction, the model would reduce to a standard Real Business Cycle model. There is perfect competition in all markets and all actors have rational expectations.

Figure 5: Schematic depiction of the model



3.1 Households

We consider an economy in discrete time populated by a continuum of identical households each of which lives indefinitely. In each period $t \geq 0$, each household receives labor income $W_t L_t$, where W_t is the wage rate and L_t the household's labor supply, as well as profits from financial and non-financial firms Π_t. Households do not own the economy's capital stock directly, but lend to financial firms in the form of one-period non-contingent securities, which we will call "deposits". The period-t return on securities issued in period $t-1$, i.e. the gross interest rate on deposits, is denoted R_t. Denoting consumption expenditure in period t by C_t, the tax rate on wage income by τ, government transfers by Tr_t and new deposits by D_t, the household's budget constraint is given by

$$C_t + D_t = W_t L_t (1 - \tau) + \Pi_t - Tr_t + R_t D_{t-1}. \tag{1}$$

Note that all price variables are measured in units of the consumption good, which is innocuous since there are no nominal frictions in our model such that purely nominal changes have no effect on real variables.

Households period-t utility $U_t = U(C_t, L_t)$ is increasing in consumption and decreasing in labor and fulfills the usual concavity conditions. Expected lifetime utility is given by

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s U(C_{t+s}, L_{t+s}), \tag{2}$$

where $\beta \in (0,1)$ is the pure time discount factor and $\mathbb{E}_t$ denotes the expected value conditional on information available in period t. The household's optimization problem consists in choosing a sequence $\{(C_{t+s}, L_{t+s}, D_{t+s})\}_{s=0}^{\infty}$ that maximizes lifetime utility (2) subject to a sequence of budget constraints such as (1).[3]

The first-order conditions for this optimization problem are

$$U_C(C_t, L_t) = \lambda_t, \tag{3a}$$

$$-U_L(C_t, L_t) = \lambda_t W_t (1 - \tau), \tag{3b}$$

$$\lambda_t = \mathbb{E}_t \beta \lambda_{t+1} R_{t+1}, \tag{3c}$$

for all $t \geq 0$. Here U_C denotes the marginal utility of consumption, U_L the marginal disutility of labor, and λ_t the marginal utility of real income, i.e. the Lagrange multiplier associated with the period-t budget constraint. In addition, there is a transversality condition

$$\lim_{s \to \infty} \mathbb{E}_t \frac{D_{t+s}}{\Pi_{i=0}^{s} R_{t+i}} = 0, \tag{4}$$

which rules out endlessly growing deposits. For future reference, we define the stochastic discount factor

$$\Lambda_{t,t+s} \equiv \frac{\beta \lambda_{t+s}}{\lambda_t}, \tag{5}$$

[3] We leave out non-negativity conditions on C_t, L_t and D_t, because we will focus on equilibria in which these conditions are always fulfilled with strict inequality.

which represents the period-t value of receiving one unit of consumption in period $t+s$.
For our numerical simulations, we will impose the following functional form on the utility function:

$$U(C_t, L_t) = \frac{1}{1-\sigma}(C_t - \gamma C_{t-1})^{1-\sigma} - \frac{\chi}{1+\psi} L_t^{1+\psi},$$

where $\sigma \in (1, \infty)$ is the elasticity of intertemporal substitution, $\psi \in (0, \infty)$ the inverse Frisch elasticity of labor supply and $\chi \in (0, \infty)$ the utility weight of labor. This utility function exhibits "external habit formation" in consumption, which is included to improve the quantitative performance of the model, and governed by the habit parameter $\gamma \in (0, \infty)$.

3.2 Non-financial firms

The production sector consists of a continuum of perfectly competitive firms of unit mass. Each firm operates the same technology represented by a production function

$$Y_t = A_t F(K_t, L_t), \tag{6}$$

where Y_t is output, A_t is total factor productivity and K_t is the capital stock. F is assumed to obey the standard conditions of constant returns to scale and strict concavity. For the purpose of simulations, we will use the Cobb-Douglas functional form $F(K, L) = K^{\alpha} L^{1-\alpha}$ with $\alpha \in (0, 1)$.
Producers hire labor from households and raise funds from financial firms in competitive markets. We assume that firms raise funds by issuing one-period securities with a gross interest rate R_t^k and use these funds to invest into the capital stock. Denoting investment by I_t, the capital stock evolves according to

$$K_{t+1} = I_t + (1-\delta)K_t, \tag{7}$$

where $\delta \in (0, 1)$ is the physical depreciation rate.
Non-financial firms choose a sequence $\{(K_{t+s}, L_{t+s})\}_{s=0}^{\infty}$ to maximize the present discounted value of future profits using the stochastic discount factor defined in (5):

$$\mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left[A_{t+s} F(K_{t+s}, L_{t+s}) - W_{t+s} L_{t+s} - (R_{t+s}^k - 1 + \delta) K_{t+s} \right]. \tag{8}$$

The corresponding first-order conditions are

$$A_t F_K(K_t, L_t) = R_t^k - 1 + \delta, \tag{9a}$$

$$A_t F_L(K_t, L_t) = W_t, \tag{9b}$$

for all $t \geq 0$, where F_K and F_L denote the marginal products of capital and labor, respectively. Note that firms earn zero pure profits in equilibrium due to the constant returns to scale assumption.

3.3 Financial sector

There is a continuum of unit mass financial firms which raise funds from households and lend them to goods producers and the government. These financial firms may either be interpreted as banks which take deposits and make loans to non-financial firms or as investment firms which sell debt securities to households and buy equity from non-financial firms. On the first interpretation, R_t^k is the gross interest rate on bank loans and $R_t^k - R_t$ is the spread between borrowing and lending rates. In this case, we assume that non-financial firms finance their entire capital stock with bank loans and issue no equity. On the second interpretation R_t^k is the return on equity investments and $R_t^k - R_t$ is the spread between the returns on equity and debt. Accordingly, we must assume that producers issue only equity and no debt securities.

We assume that the financial sector doesn't possess any resources of its own, but merely intermediates funds between households, firms and the public sector. Government debt comes in the form of one-period zero-coupon bonds which are traded for a real price Q_t in period t. Let B_t be the amount of government bonds held by a representative financial firm, then the balance sheet identity of this firm is given by

$$D_t = Q_t B_t + K_{t+1}. \tag{10}$$

In each period t, the financial firm receives payments on its productive assets and its government bonds and pays interest on its liabilities. Each period, a (randomly selected) fraction $(1 - \omega_t)$ of mature government bonds are repaid in full, whereas the rest are not repaid at all. We will refer to ω_t as the default rate.[4] The present value of future profits of the representa-

[4] An alternative interpretation would be that the government imposes a uniform "haircut" ω_t on all outstanding bonds.

tive firm are

$$\mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left[R^k_{t+s} K_{t+s} + (1-\omega_{t+s}) B_{t+s-1} - R_{t+s} D_{t+s-1} \right]. \tag{11}$$

We introduce a simple financial friction into the model which constrains financial firms in their investment decisions. In particular, we assume that financial firms must hold a certain fraction $\phi > 0$ of their assets in the form of government bonds, i.e.

$$Q_t B_t \geq \phi D_t. \tag{12}$$

There are two possible motivations for such a restriction. First, financial firms in developed countries are subject to government regulations which require them to keep a certain fraction of their total assets in liquid securities such as government bonds. For instance, the "Basel III" regulations currently enforced throughout the European Union include a "liquidity coverage ratio" which essentially has that effect. Minimum capital requirements may have similar effects since they treat government bonds preferentially in calculating a bank's "risk weighted assets".

Second, in addition to regulatory reasons, banks hold government bonds as collateral for short-term refinancing operations in the inter-bank market and with central banks. The need for collateral in turn may arise from imperfect commitment or a moral hazard friction in the inter-bank market. Bolton and Jeanne (2011) develop a model in which banks cannot commit to repay inter-bank loans in excess of the value of collateral, which consists of government bonds. Their model implies a collateral restriction equivalent to (12) on the fraction of banks which are (net) borrowers in the inter-bank market. A similar constraint also arises in Gertler and Kiyotaki (2010). In their model, a moral hazard problem in financial intermediation implies an endogenous constraint on bank lending which depends on the value of the financial firm's net worth. Our formulation may be interpreted as a reduced-form version of these models, which is why we will refer to (12) as the collateral constraint.

The representative financial intermediary maximizes (11) subject to a sequence of constraints (10) and (12). The first-order conditions are as follows:

$$\mu_t = \mathbb{E}_t \Lambda_{t,t+1} (R^k_{t+1} - R_{t+1}), \tag{13a}$$

$$Q_t = \frac{\phi}{\phi - \mu_t (1-\phi)} \mathbb{E}_t \Lambda_{t,t+1} (1-\omega_{t+1}), \tag{13b}$$

$$\mu_t (Q_t B_t - \phi D_t) = 0, \tag{13c}$$

where $\mu_t \geq 0$ is the Lagrange multiplier on the period-t collateral constraint (12). It may be interpreted as the shadow price of collateral, i.e. the value to the financial firm of having one more consumption unit worth of collateral.

A couple of comments about these first order conditions are warranted. First, the complementary slackness condition (13c) implies that either the shadow price of collateral is zero or the collateral constraint is binding. Equation (13a) states that the expected discounted excess return on productive assets over the deposit rate must be equal to the shadow price of collateral. If the latter is positive, i.e. the collateral constraint is binding, the spread between the bank's lending rate and borrowing rate must also be positive. Only when the collateral constraint is slack, the return on productive assets will be equal to the deposit rate. Finally, equation (13b) is an asset pricing equation for government bonds. Note that the price of government bonds decreases with the default rate and increases in the shadow price of collateral. If the latter drops to zero, the bond price just equals the expected discounted repayment rate. It is also worth pointing out that, whether the collateral constraint is binding or not, financial firms earn zero economic profits in equilibrium. This can easily be verified by inserting (13a) and (13b) into (11).

3.4 Government

The government receives revenue from income taxes and issues new bonds to finance current government spending G_t and the repayment of outstanding bonds $(1-\omega_t)B_{t-1}$. Hence the period-t government budget constraint can be written

$$Q_t B_t = (1-\omega_t)B_{t-1} + G_t - T_t. \tag{14}$$

Taxes are proportional to wage income, i.e. $\tau W_t L_t$ with $\tau > 0$. In the case of a default, the government transfers the resources saved through the default directly to households in lump-sum fashion: $Tr_t = \omega_t B_{t-1}$. Hence total net tax revenue is given by

$$T_t = \tau W_t L_t - \omega_t B_{t-1}. \tag{15}$$

The default-related transfer payment neutralizes the effect of a default on the debt level ex post. This does not mean, of course, that a default has no impact on the government's finances, since the default will affect the price of newly issues bonds. One may think of the transfer as a temporary tax rebate that occurs whenever the government decides to default.

We assume that the government keeps a constant stock of bonds $\bar{B}$ in existence and simply rolls over all its debt from period to period. At the beginning of each period, the government announces its repayment decision, i.e. the level of ω_t, and auctions off new bonds. Then the government adjusts spending G_t so as to satisfy its budget constraint (14).

Here, the repayment/default decision of the government is not modeled as the result of an optimization problem as much of the literature does. Instead, we treat default as a random event resulting from discretionary government action which cannot be (perfectly) anticipated by private actors. In particular, we assume that the default rate ω_t evolves according to the law of motion

$$\omega_t = \rho_\omega \omega_{t-1} + f\left(\frac{B_{t-1}}{Y_t} - b^*\right) + \varepsilon_t^\omega, \tag{16}$$

where ε_t^ω is a random "default shock" and ρ_ω is a parameter measuring the persistence of the default shock. The function $f()$ captures the idea that default becomes more likely once the debt-to-output ratio exceeds a certain critical value b^*. This assumption is motivated by a broad class models of sovereign debt defaults originating with Eaton and Gersovitz (1981). This function is assumed to have the following form:

$$f\left(\frac{B_{t-1}}{Y_t} - b^*\right) = \max\left\{\left(\frac{B_{t-1}}{Y_t} - b^*\right)^2, 0\right\}.$$

The purpose of modeling a government default in this way is to put a sharp focus on the effect of government default on the rest of the economy, which is the main interest of the paper, while remaining agnostic about the reasons why defaults occur. In a more complete model, we would have to take a stance on the objective function of the government when deciding whether to default or not. But such an analysis is beyond the scope of the present paper. It has been carried out in the context of a similar model by Sosa-Padilla (2012).

3.5 Equilibrium

The model is closed by a market clearing condition for the goods market,

$$Y_t = C_t + I_t + G_t. \tag{17}$$

Other market clearing conditions for the labor, capital and bond markets are implicit in the model.[5]

In addition to the uncertainty about the repayment of government debt, we introduce a second exogenous source on uncertainty, namely technology shocks. In line with traditional business cycle literature, we assume that total factor productivity evolves according to

$$\ln A_t = (1-\rho_A)\ln \bar{A} + \rho_A \ln A_{t-1} + \varepsilon_t^A, \tag{18}$$

where ε_t^A is a random variable with zero mean, $\bar{A} > 0$ is the long-run value of total factor productivity and $\rho_A \in (0,1)$ measures the persistence of the technology shock.

An equilibrium in our model is defined as a sequence of quantities $\{(Y_{t+s}, C_{t+s}, L_{t+s}, D_{t+s}, I_{t+s}, K_{t+s}, G_{t+s}, T_{t+s})\}_{s=0}^{\infty}$ and prices $\{W_{t+s}, R_{t+s}, R_{t+s}^k, Q_{t+s}, \mu_{t+s}\}_{s=0}^{\infty}$ such that, given a sequence of exogenous variables $\{(A_{t+s}, \omega_{t+s})\}_{s=0}^{\infty}$,

1. the household's first-order conditions (3a-3c),
2. the producer's first-order conditions (9a-9b),
3. the financial sector's first order conditions (13a-13c),
4. the government's budget constraint (14) and tax rule (15) as well as
5. the market clearing conditions (7), (10) and (17)

are all fulfilled in every period $s \geq 0$.

4 Numerical Solution

We begin to analyze the model by looking at a steady state equilibrium, in which all exogenous variables are constant, i.e. $A_{t+s} = \bar{A}$ and $\omega_{t+s} = \bar{\omega}$ for all s. We set the critical debt-output ratio b^* to the stead-state level, so that the default rate $\bar{\omega}$ is zero at the steady state. Hence we refer to it as a "no-default" steady state.

In such a steady state, all quantity and price variables are constant. As a result, the deposit interest rate is given via (3c) by

$$R = 1/\beta$$

[5] Indeed, the capital accumulation equation in conjunction with the financial firm's balance sheet identity could also be read as market clearing conditions for financial assets.

and the return on productive assets (the lending rate) is given via (13a) by

$$R^k = (1+\mu)/\beta.$$

The government bond price is pinned down by (13b):

$$Q = \frac{\beta\phi}{\phi - \mu(1-\phi)}.$$

Given the bond price, the collateral constraint (12) determines the amount of deposits and the capital stock:

$$D = Q\bar{B}/\phi,$$

$$K = (1-\phi)Q\bar{B}/\phi.$$

Steady state investment is then given by

$$I = \delta(1-\phi)Q\bar{B}/\phi.$$

Government spending is found via (14) to be

$$G = (Q-1)\bar{B} + \tau\bar{A}F_L(K,L)L$$

Using these results and taking into account (3b), (9b) and (17) one can find steady-state consumption and labor by solving the non-linear system:

$$C = \bar{A}F(K,L) - I - G.$$

$$-U_L(C,L) = U_C(C,L)\bar{A}F_L(K,L)(1-\tau).$$

Finally, the solution must also satisfy (9a), i.e.

$$1+\mu = \beta(\bar{A}F_L(K,L) + 1 - \delta).$$

For our numerical solution, we use an iterative procedure to find the steady state. Guessing initial values for μ and b^*, we solve for all the endogenous variables and use the last equation to update μ at every step until convergence. We also update b^* at every iteration. It must be noted, that the collateral constraint need not be binding in the steady state. In that case, the model reduces essentially to a standard Real Business Cycle model with a government

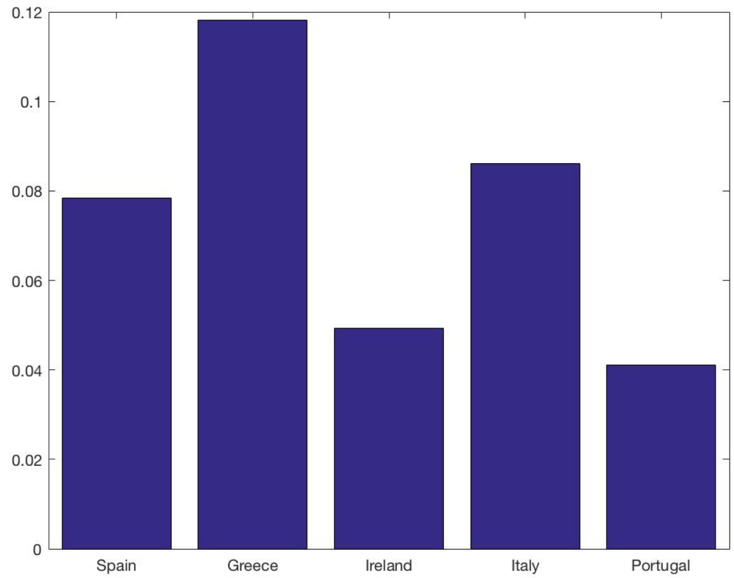
sector. In our calibration, however, we can exclude this possibility.
Because our model is highly nonlinear, an exact analysis of its dynamic behavior is not feasible. Therefore, we use a second-order approximation around the steady state just described to investigate the dynamic effects of exogenous shocks – in particular, a shock to the default rate and to the growth rate of total factor productivity.

4.1 Calibration

To perform the numerical simulations, we calibrate the model parameters to match data from the Eurozone. In particular, we want the steady state of the model to mimic the pre-crisis period between 2000 and 2009. A period in the model is equivalent to one quarter.
Household parameters are chosen in line with Kollmann et al. (2016) who estimate a DSGE model with a household sector similar to ours using euro area data from 1999 to 2014. The intertemporal elasticity of substitution σ is 1.4 and the habit persistence in consumption γ is 0.8. The inverse Frisch elasticity of labor supply is set to 0.45. Furthermore, we set the pure time discount factor β to 0.99 which implies a long-run deposit rate of about 1 percent (about 4 percent on a per-annum basis).
For technology parameters, we rely on standard values from the quantitative business cycle literature. Thus, the capital income share α is set to 0.33, the physical depreciation rate δ is 0.025 such that the capital stock lasts for ten years (40 model periods).
Turning to the financial sector, the key parameter is the ratio of collateral to total assets ϕ. As shown in figure (6), the average share of government bonds in total bank assets for is in the range between 4 and 12 percent. Taking a GDP-weighted of this share yields a value of about 9 percent. Hence, in our baseline calibration, we set to $\phi = 1/11$. Since this parameter plays a crucial role in our model, we will also show simulations based on different values of ϕ as a robustness check.
The stock of government debt $\bar{B}$ which is rolled over each period is set to 1. The critical debt-output ratio b^* is set to 0.79 and is equal to the steady-state value of $\bar{B}/Y$. We set the tax rate τ to 0.25 which broadly mimics the Eurozone average income tax share. Finally, we choose both ρ_ω and ρ_A to be 0.75 implying a moderate persistence in the shock processes.

Table (1) shows the steady state values of our calibrated model. All quantity variables in this table are divided by the steady-state output so that they can be interpreted as GDP shares (the labor supply can be interpreted as the inverse of labor productivity). Different values of ϕ imply different steady states. Generally, the lower ϕ, the higher the capital-

Figure 6: Share of government bonds in total bank assets



Notes: Share of government bonds in total assets of MFIs in selected Eurozone countries, average value of observations between 1997 and 2015. Data from the European Central Bank.

output ratio, the lower the government debt-output ratio, the lower the bank lending spread and the higher the government interest spread are in the steady state. The inverse steady-state relationship between the debt-output ratio and the government interest spread is explained by the behavior of the collateral premium: the larger ϕ, the higher the collateral premium pushing up the price of government bonds. For $\phi = 1/13$, the collateral constraint becomes very close to being non-binding in the steady state, as can be seen by a very low value of μ.

The steady-state values reported in table (1) are broadly in line with their empirical counterparts in the Eurozone. Government debt is in the range of 73 to 81 percent of national income, close to the GDP-weighted average value of 75 percent in the Eurozone crisis countries in 2008. Consumption, investment and government spending shares are also reasonably close to the data and are relatively stable across different choices of ϕ. The capital-output ratio is about 8 and the deposit-output ratio, i.e. the ratio of households' financial wealth to total income is about 9 in the baseline calibration.

One issue with our calibration is that it implies a counterfactually low steady-state value of the government interest spread. Indeed, $1/Q - R$, i.e. the spread between the interest rate on government bonds and the deposit rate is slightly negative in the steady state for

Table 1: Steady State Values, Calibrated Model

Variable	Description	Steady state values		
		$\phi = 1/8$	$\phi = 1/11$	$\phi = 1/13$
Y	Output	1	1	1
C	Private consumption	0.58	0.59	0.60
L	Labor supply	0.40	0.36	0.34
D	Deposits	7.26	8.78	9.66
K	Capital stock	6.35	7.98	8.91
I	Investment	0.16	0.20	0.22
B	Government debt	0.81	0.76	0.73
G	Government spending	0.26	0.21	0.18
T	Net taxes	0.16	0.17	0.17
W	Wage rate	1.67	1.86	1.97
R	Gross deposit rate	1.010	1.010	1.010
R^k	Gross return on capital	1.027	1.016	1.012
Q	Price of gov. bonds	1.114	1.055	1.013
μ	Shadow price of collateral	0.016	0.006	0.002
ω	Default rate	0	0	0

Notes: all quantity variables ($Y, C, L, D, K, I, B, G, T$) are scaled by output.

all calibrations. This is due to the fact that government debt pays a "collateral premium": banks are willing to hold government bonds despite their low return, because they can use the bonds to enlarge their stock of loans which, due to the fact that the bank lending margin is positive, increases their profits. One explanation for this discrepancy is that the maturity of government debt and bank deposits is the same in our model (one period), but different in reality. Since debt of longer maturity pays a term premium, the government interest spread reported in figure (1) is not a perfect counterpart to the government interest spread in the model.

5 Effects of a government default shock

We are now ready to study the effect of an exogenous shock to the default rate. Our focus is on the dynamic response of the government interest spread, the bank lending margin, the volume of bank lending, and output.

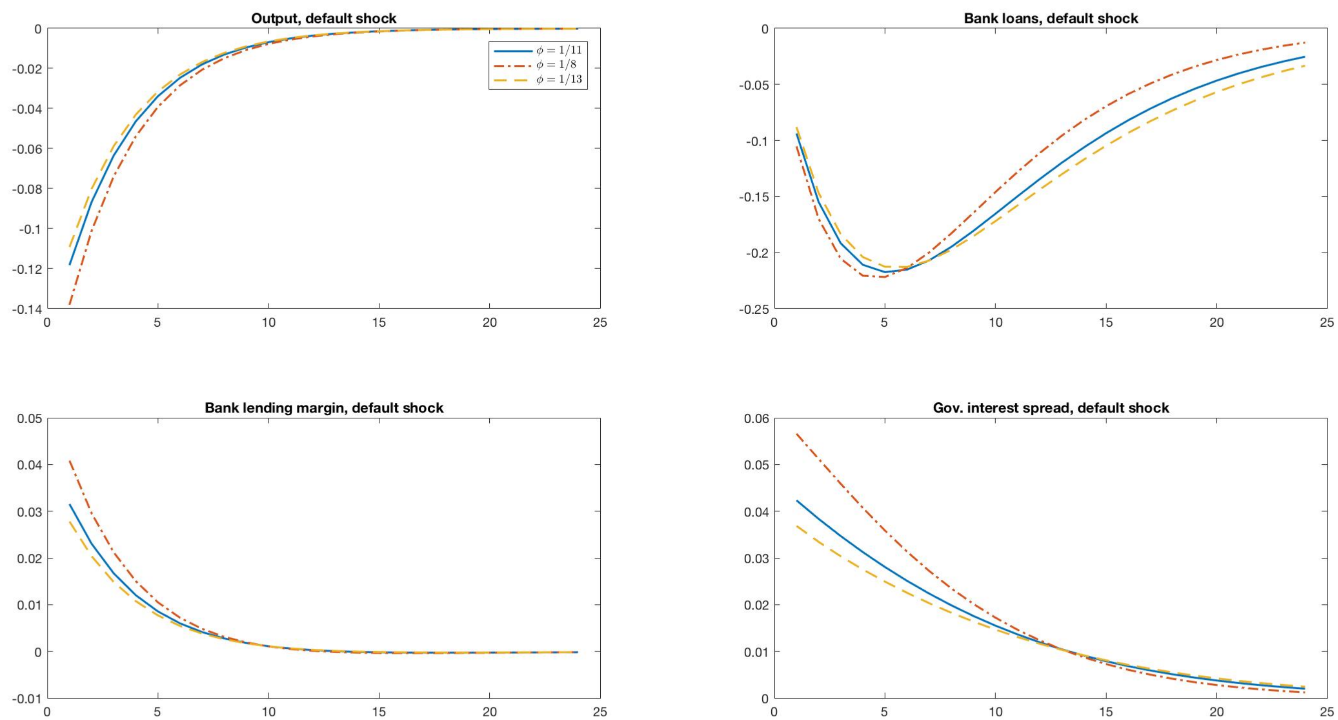
In order to study the dynamic behavior of the model, we take a first-order approximation

around the steady state equilibrium described in the previous section.[6] In what follows, we study the impulse-response functions of the model's endogenous variables with respect to a shock to ε_t^{ω}. The impulse-response functions are calculated under the assumption that the collateral constraint remains binding at all times. We can verify this assumption by checking that the shadow price of collateral is positive in all periods of the simulation.
The size of the default shock is chosen in such a way that the implied government interest spread $1/Q_t - R_t$ increases to 400 basis points (4 percentage points) on impact under the baseline calibration. This matches approximately the average increase in government interest spreads observed in the Eurozone in 2011 as seen in figure (1).

Figure (7) shows the impulse-response functions of the four main variables of interest to the default shock. The full set of impulse-responses can be found in the appendix (see figure (9)). The impulse, or "shock", is a one-off increase of 0.5 in ε_t^{ω}, which can be thought of as an increase in the default probability from zero to 50 percent. As shown in the lower right panel, the response of the government interest spread on impact is in the range of 360 to 570 basis points, depending on the collateral parameter ϕ. The effect slowly dies out and reverts to the steady-state value after 24 quarters. The protracted increase in the government interest spread is partly due to the feedback effect of lower output, which raises the debt-output ratio and hence endogenously increases the default probability.
The volume of bank loans, which equals the capital stock in the model, follows a U-shaped dynamic response. The initial drop in bank lending is reinforced in the subsequent periods until a slow recovery sets in after about 5 periods. In order to explain this behavior, recall that bank lending is determined by the value of the banks' collateral, i.e. the price of government bonds Q_t, whose dynamic response to the default shock is the result of two counteracting forces. The direct effect of the increase in the default probability is to *lower* Q_t. However, as explained above, the shadow price of collateral increases on impact which acts to *raise* Q_t. Since the latter effect dies our more rapidly than the direct effect, Q_t shows a U-shaped response, which is inherited by the volume of bank loans.
The bank lending margin, $R_t^k - R_t$, increases on impact by about 300 basis points and then reverts back to the steady state value relatively quickly. This reflects the dynamic behavior of the shadow price of collateral μ_t, which increases due to the fact that a depreciated value of government bonds in effect makes collateral "scarcer", tightens the collateral constraint of the financial sector and thus raises its price. The banking sector passes on this price

[6] Second-order approximation has been tested as well. The results were virtually identical.

Figure 7: Impulse-responses to a default shock



Notes: default shock is a 0.5 increase in ε_t^{ω}, impulse-response functions calculated from first-order approximation around no-default steady state.

increase by raising the spread between the loan interest rate and the deposit rate. On closer examination, the increase in the bank lending margin is brought about almost entirely by a drop in the deposit rate R_t while the loan rate R_t^k remains relatively flat. The reason for this drop is that banks, confronted with a fall in the value of collateral, reduce their borrowing from households so the demand for households' loanable funds decreases which lowers the deposit interest rate.

Output falls in response to the default shock by about 10 percent relative to its steady-state value to which it reverts within the subsequent 16 periods (4 years). This compares well with the observed decline in GDP of about 12 percent in the Eurozone crisis countries as seen in figure (4). The decline in output results mainly from the decrease in bank lending (capital stock) and is reinforced by the falling labor supply. Both business investment and government spending falls, while private consumption increases slightly on impact. The initial rise in consumption is due to the transfer to households which boosts after-tax income and an intertemporal substitution effect resulting from an initial drop in the deposit interest rate. It should be noted, however, that the size of the movement in consumption is less than 1 percent of steady-state consumption.

In summary, the default shock is able to reproduce the key stylized facts of the Eurozone crisis, leading to an increase in the government interest spread, a decrease in bank credit, an increase in the loan-deposit interest differential and a decrease in output and employment. The magnitude of the impulse response functions seem to be broadly similar to the magnitude of the observed movements in the data. It bears pointing out that the contractionary effect of the default shock has nothing to do with an increase in "uncertainty" or the fear of future tax increases. It comes entirely and directly from the deterioration of the balance sheet of financial intermediaries and its subsequent effect on bank credit, capital accumulation and labor supply.

Overall, the results do not appear to be sensitive to the calibrated value of the collateral-to-asset ratio ϕ. As one might expect, the effects of a default shock are generally stronger (weaker) for higher (lower) values of ϕ. However, one should bear in mind that decreasing ϕ below a certain value renders the collateral constraint non-binding in the steady state, in which case the dynamic behavior of the model would change substantially. Indeed, if the collateral constraint is sufficiently lax so that it never binds in steady state as well as for small deviations from the steady state, a government default shock is neutral to output, bank credit and the bank lending margin. Hence, the presence of the financial friction is crucial for our results.

5.1 Effects of an adverse technology shock

One might ask, however, whether the model allows for a different explanation of the same stylized facts along more "conventional" lines. In particular, the business cycle literature has found a significant role for "technology shocks", i.e. exogenous shifts in the (growth rate of) total factor productivity, in explaining economic fluctuations. Can a technology shock explain the stylized facts of the Eurozone crisis too?

To address this question we simulate the effect of an adverse technology shock. Stock and Watson (2012) have found that a real shock (in particular a shock to the oil price) contributed to the Great Recession in the United States and it is arguable that the Eurozone could have been hit in a similar fashion. We calibrate the size of the shock so as to give the same response to output as the default shock. This turns out to be a shock of -0.06 to ε_t^A, i.e. a 6 percent decrease in total factor productivity.

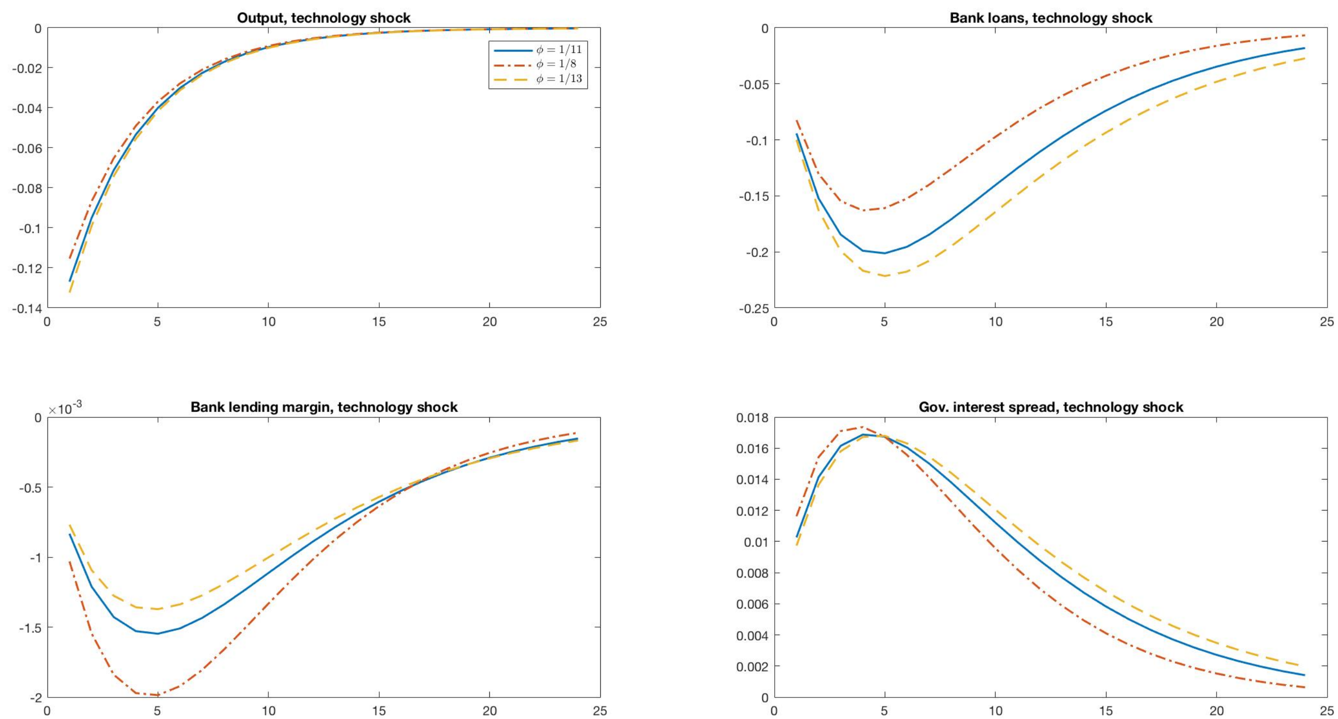
As shown in figure (8), the technology shock depresses output, by design, by about 12 percent relative to steady state on impact. Output remains depressed for about 16 periods, just as in the case of the default shock. As a result, the government debt-to-output ratio increases which raises the default rate and thus the government interest spread. But that is not all. The decline in total factor productivity also reduces the demand for capital by goods producers and hence the demand for bank loans. This in turn reduces the shadow price of collateral for financial intermediaries, which reinforces the negative effect on the price of government bonds, thus increasing the government interest spread further. Notice that the response to the technology shock follows the same logic as the response to the default shock, only in reverse. In both cases, the shadow price of collateral plays the decisive role in the transmission mechanism.

The dynamic behavior of other quantity variables such as the labor supply, investment, government spending and net taxes are very similar to the response to the default shock: a decrease on impact followed by a relatively fast return to the steady-state level.[7] An exception is the behavior of consumption, which falls on impact due to the negative wealth effect implied by the technology shock. One can conclude that a technology shock is not distinguishable from a default shock only by observing their effects on quantities and the government spread.

The shocks can be distinguished, however, by looking at the response of the bank lending margin. Whereas, as we have seen above, the default shock increases the bank lending

[7] See figures (11) and (12) in the appendix.

Figure 8: Impulse-responses to a technology shock



Notes: technology shock is a 0.06 decrease in ε_t^A, impulse-response functions calculated from first-order approximation around no-default steady state.

margin, the technology shock has the opposite effect. The reason is the differential impact these shocks have on the shadow price of collateral. As the technology shock lowers demand for bank loans, the loan interest rate drops. The interest rate on deposit falls as well but by a smaller amount than the loan rate, owing to the fact that collateral has become "less scarce" relative to total bank assets, which is reflected in a smaller shadow price of collateral. This results indicate that interest rate data, and data on financial assets more broadly, play a crucial role in identifying the source of economic fluctuations. In particular, the technology shock cannot explain why bank lending margins have increased during the crisis.

6 Discussion and Conclusion

This paper has introduced a financial friction in an otherwise standard business cycle model by assuming that banks have to hold government bonds as collateral on their balance sheets. We used this model to study the effect of a "government default shock" - an exogenous increase in the probability of debt default - and concluded that such a shock produces movements in the government interest spread, bank credit, bank lending margin and output that roughly match the observed behavior of these variables in the Eurozone crisis. We have also shown that a negative technology shock - an exogenous decrease in total factor productivity - can explain some but not all of these observations.

The shadow price of collateral - the marginal value to financial firms of having an additional unit of collateral - plays a key role in the transmission of shocks. The negative consequences of a government default are driven only by the effect on banks' balance sheets and the associated increase in the effective scarcity of the collateral assets which is reflected in its shadow price, not by any increase in economic "uncertainty" or the fear of future tax increases. While the shadow price of collateral cannot be directly observed, the papers shows that it is reflected in the spread between the interest rates on loans and deposit, the bank lending margin. This underscores the importance of financial price variables in identifying the sources of structural economic shocks.

We have kept the model in this paper deliberately simple in that we left out many of the frictions that have been found important in explaining business cycles. For instance, we have abstracted entirely from nominal rigidities and foreign trade. Incorporating these features in the model would no doubt improve its usefulness for empirical work as well as provide additional insights in the transmission of government default to other macroeconomic variables. One of the open questions in that respect is to what extend monetary policy can help to shield the economy from the detrimental effects of government default in the presence of financial frictions. Another is how government default can spill over into foreign countries. These could be worthwhile endeavors for future research.

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7 Appendix

The full non-linear model is described by equations (3a-3c), (7), (9a-9b), (10), (13a-13c), (14), (15), (17) as well as (16) and (18). It can be written in the form

$$\mathbb{E}_t F(x_{t+1}, x_t, x_{t-1}, u_t) = 0,$$

where x_t is the vector of endogenous variables, and u_t is the vector of shocks ε_t^{ω} and ε_t^{A}. The function $F()$ is implicitly defined by the model equations. Solving the model involves finding a policy function $g()$ such that

$$x_t = g(x_{t-1}, u_t),$$

relating current endogenous variables to past realizations and current shocks. Plugging in the policy functions repeatedly in the above equation yields

$$\mathbb{E}_t F(g(g(x_{t-1}, u_t), u_{t+1}), g(x_{t-1}, u_t), x_{t-1}, u_t) = 0,$$

We take a first-order Taylor series expansion around the steady-state point $F(x,x,x,0)$:

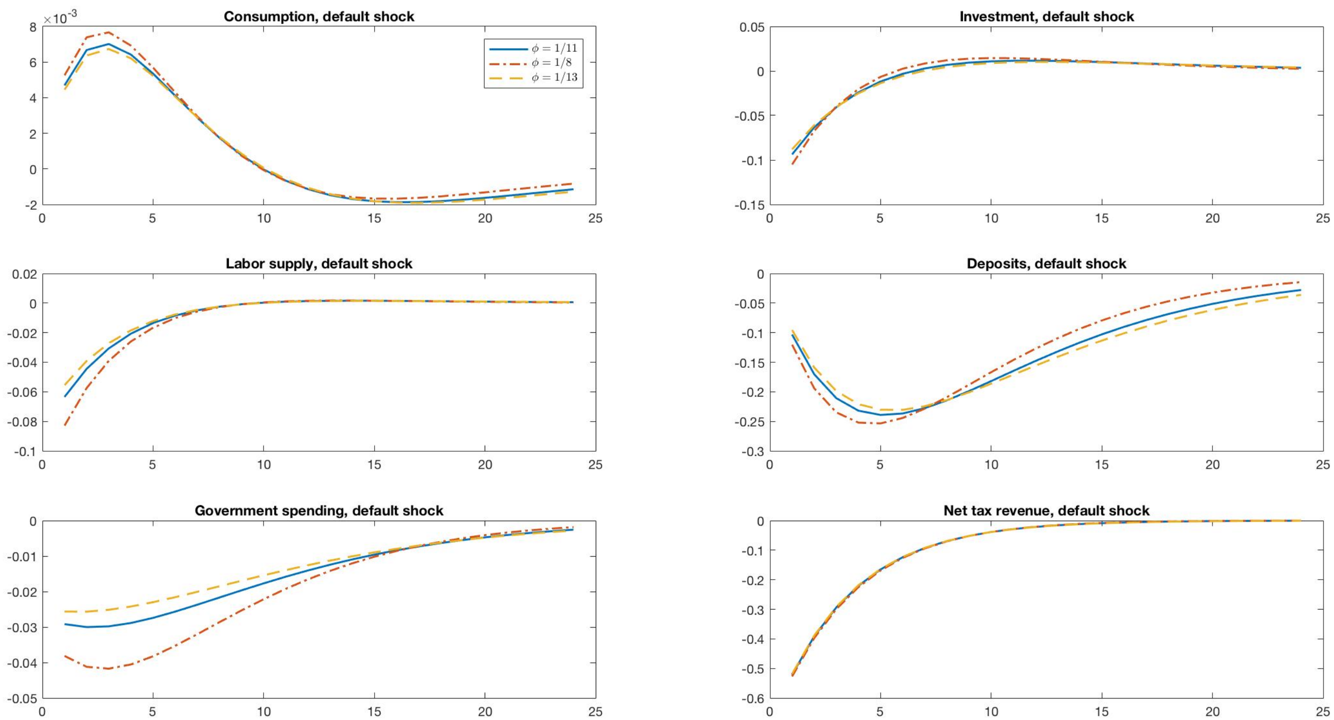
$$\mathbb{E}_t[F_{x,+}(g_x g_x \hat{x}_{t-1} + g_u u_t + g_u u_{t+1}) + F_{x,0}(g_x \hat{x} + g_u u_t) + F_{x,-}\hat{x}_{t-1} + F_u u_t] = 0,$$

where $\hat{x}_{t-1} = x_{t-1} - x$, $F_{x,+} = \partial F / \partial x_{t+1}$, $F_{x,0} = \partial F / \partial x_t$, $F_{x,-} = \partial F / \partial x_{t-1}$, $g_x = \partial F / \partial x_{t-1}$ and $g_u = \partial g / \partial u_t$. The derivatives g_x and g_u can be recovered using the techniques described in Fernandez-Villaverde et al. (2016) yielding a linear approximation of the form

$$\hat{x}_t = g_x \hat{x}_{t-1} + g_u u_t,$$

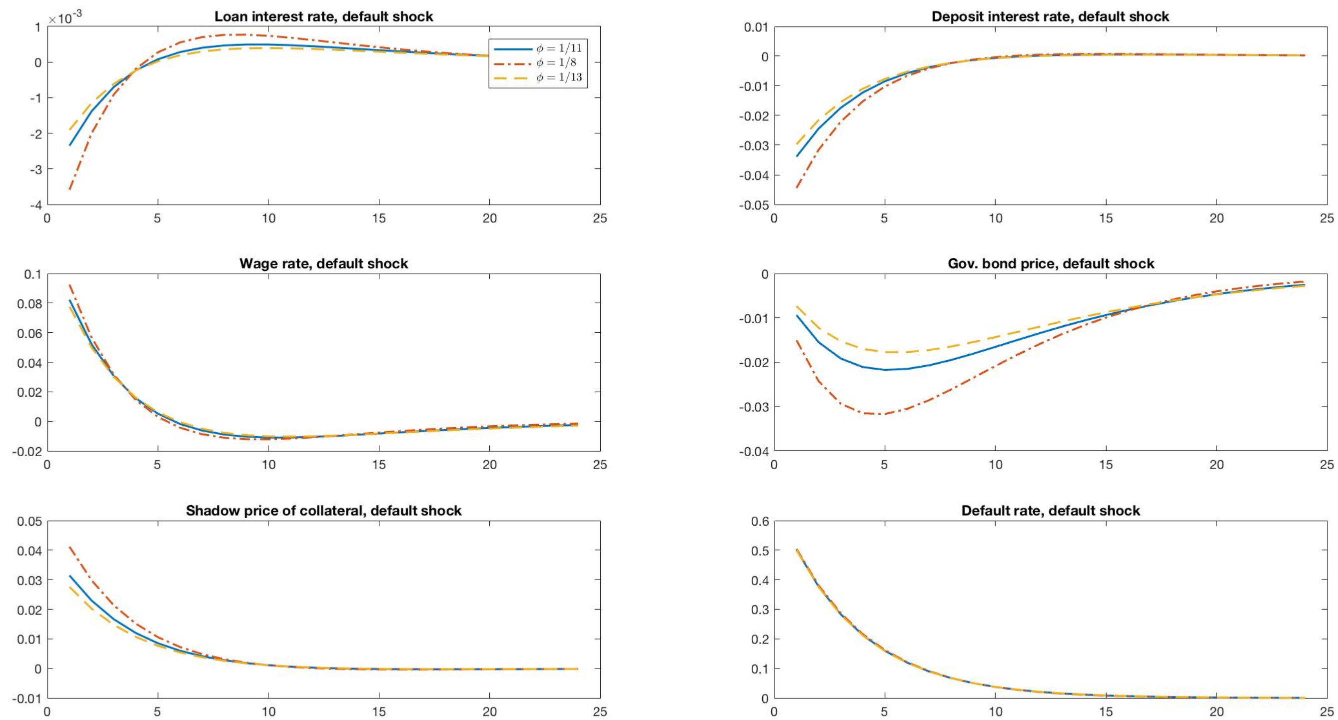
from which it is straightforward to calculate impulse-response functions. It should be noted that the linear approximation is only good for small perturbations from the initial point. All calculations in these paper are carried out using the Dynare tool on MATLAB. The code is available from the author upon request.

Figure 9: Impulse-responses to a technology shock



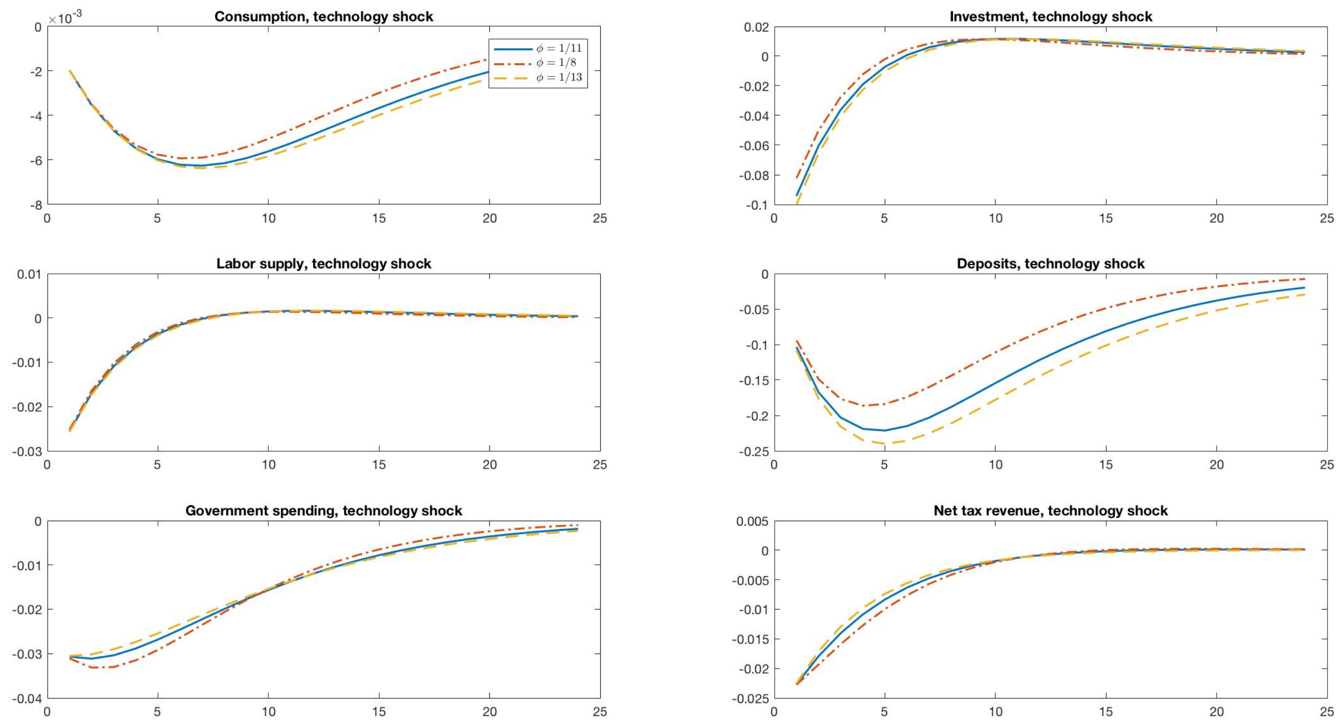
Notes: default shock is a 0.5 increase in ε_t^A, impulse-response functions calculated from first-order approximation around no-default steady state.

Figure 10: Impulse-responses to a technology shock



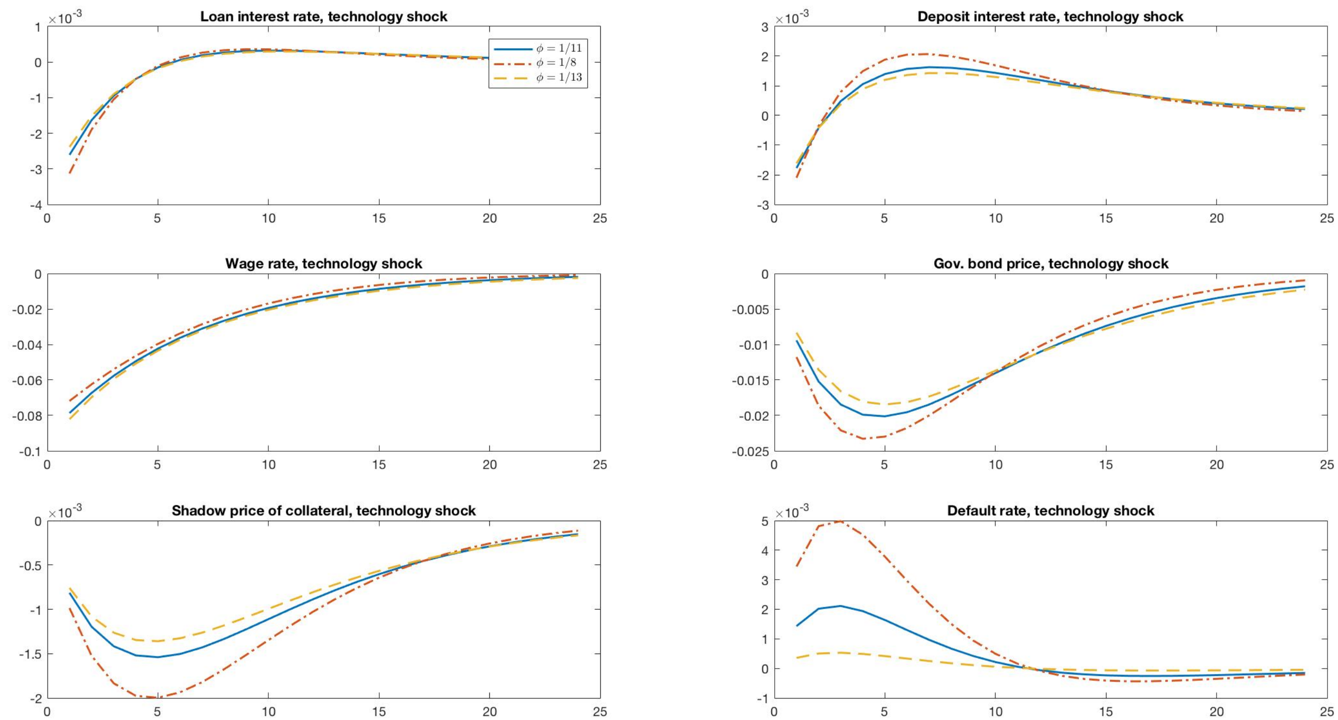
Notes: default shock is a 0.5 increase in ε_t^A, impulse-response functions calculated from first-order approximation around no-default steady state.

Figure 11: Impulse-responses to a technology shock



Notes: technology shock is a 0.06 decrease in ε_t^A, impulse-response functions calculated from first-order approximation around no-default steady state.

Figure 12: Impulse-responses to a technology shock



Notes: technology shock is a 0.06 decrease in ε_t^A, impulse-response functions calculated from first-order approximation around no-default steady state.