Monetary policy and long-term interest rates*

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Abstract

A few recent articles have argued that calibrated DSGE models, provided they are solved nonlinearly, can match well key empirical features of long-term interest rates. This paper studies the implications of these findings for the transmission of monetary policy, based on a model estimated on US macro and yields data over the 1966-2008 period. Regime shifts in the conditional variance of productivity shocks are an important model ingredient. Switches between "normal" and "high" levels of volatility are found to be countercyclical and to play an important role in driving cyclical fluctuations. At the onset of recessions, volatility tends to increase to high levels: this "uncertainty shock" leads both to a persistent increase in precautionary saving, which drives down consumption, inflation and thus current and expected interest rates, and to an increase in risk premia. During the recovery, these dynamics are reversed: volatility returns to normal, low levels, consumption and inflation increase, interest rates are expected to go up persistently, while risk premia become lower. Model-implied 10-year inflation expectations are broadly in line with those based on survey data over the 1980s and 1990s, but less firmly anchored in the 2000s.

JEL classification:

Keywords: monetary policy rules, uncertainty shocks, term structure of interest rates, regime switches, Bayesian estimation.

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1 Introduction

This paper shows that the estimated, nonlinear version of a simple new-Keynesian model provides an internally consistent account of the evolution of macroeconomic and yields data in the United States. Two model ingredients are necessary over and above those used in standard macro applications: non-expected utility preferences and stochastic volatility (in the form of regime switching). Non-expected utility allows us to increase risk-aversion independently of the elasticity of intertemporal substitution. Unexpected switches in volatility produce at the same time variations in bond risk premia and changes in households' precautionary saving over time. Our results suggest that, in contrast to the linearized version of standard macro-models, changes in precautionary saving are a typical feature of U.S. recessions.

While the assumption of non-expected utility preferences is now standard in calibrated analyses of asset pricing in production economies, uncertainty shocks are the distinguishing feature of the model we employ. On the one hand, increases in uncertainty boost households' demand for precautionary saving, which tends to depress consumption and exert downward pressure on real rates. Uncertainty shocks thus contribute to cause economic recession. On the one hand, increases in uncertainty boost bond risk premia. Uncertainty shocks can therefore produce countercyclical risk premia–a stylized fact according to the finance literature (see e.g. Fama and French, 1989, or Cochrane and Piazzesi, 2005).

From an empirical perspective, our simple model specification goes a long way in fitting U.S. data on aggregate consumption, inflation, short and long-term interest rates. The model also fits well dimensions of the data which were not directly used in estimation, such as forward rates at various horizons.

The good model fit relies on a richer monetary policy transmission mechanism than in linearized models. More specifically, once purged of risk-premia the dynamics of long-term rates are not roughly constant, as implied by linearized models. By contrast, risk-adjusted yields are even more volatile than observed yields. This is a direct implication of the countercylicality of risk premia. During recessions, when risk premia increase, risk-adjusted yields must fall more than observed yields. During expansions, risk premia fall and risk-adjusted yields must increase more than actual yields.

This observation begs additional questions. If risk-adjusted long-term rates are highly volatile, what drives their dynamics? What do their variations imply for the expected future path of monetary policy rates over the business cycle? Are long-term inflation expectations implicit in bond yields also highly variable, i.e. not "well-anchored"? Through the lens of our model, we can provide the following answers.

First, risk-adjusted long term rates are importantly driven by fluctuations in uncertainty over future realizations of technology. Periods of high uncertainty boost households' demand for precautionary saving. Since the period of high-variance is estimated to be persistent, current and expected future *real* rates tend to fall to clear the savings market. For roughly constant, long-term inflation expectations, this mechanism also leads to a fall in expected future *nominal* interest rates. Once "confidence" returns and uncertainty over future realizations of technology switches back to normal, lower levels, the demand for precautionary saving falls again. Risk-adjusted nominal (and real) yields return to normal, higher levels.

Changes in the variance of other shocks, including monetary policy shocks, have negligible effects on risk-adjusted yields. However, monetary policy shapes the dynamics of risk-adjusted yields through its systematic reaction to fluctuations in technological uncertainty. This is not the result of an exotic policy rule. A standard Taylor rule also implicitly reacts to uncertainty shocks, because, as described above, these shocks produce fluctuations in the demand for precautionary saving, that are reflected in opposite fluctuations in the demand for consumption goods and, in turn, in inflation. Nevertheless, the standard Taylor rule does not internalize the persistent changes in equilibrium real interest rates after uncertainty shocks. For example, after an increase in uncertainty with the ensuing fall in consumption and inflation, policy rates fall, but they do not fall enough to discourage the increase in precautionary saving. Real rates remain at elevated levels and consumption (and output) remain too low for a prolonged period of time.

While uncertainty shocks affect risk-adjusted long-term rates, their impact on observed rates is masked by contemporaneous variations in risk premia. For example, when a fall in uncertainty is a key driver of the economic recovery, as was the case in 2004 according to our estimates, future policy rates are expected to increase to meet the rising inflationary pressure (which is in turn the result of the fall in precautionary saving and increase in consumption demand). At the same time, however, risk premia fall due to the less uncertain outlook. As a result, observed long-term yields can remain roughly unchanged producing an apparent "conundrum".[1] If, in contrast, other shocks play a dominant role in the recovery and uncertainty remains unchanged when the monetary policy tightening phase begins, as was the case in 1994 according to our model, the response of long-term rates conforms to that of risk-adjusted rates. Yet, real rates remain relatively low, because of the extant high demand for precautionary saving. The increase in nominal long-term rates is associated with an increase in long-term inflation expectations, i.e. an "inflation scare".[2]

How realistic are the fluctuations in long-term inflation expectations implied by our estimates? We can compare them to those available from the Federal Reserve Bank of Philadelphia's quarterly Survey of Professional Forecasters. Over the 1980s, the two measures are quite similar, showing a progressive fall in inflation expectations from the 1980 peaks. Over the 2000s, however, our model suggests a much less tight anchoring of inflation expectations compared to surveys. The latter fall steadily towards 2.5 percent over the 1990s and remain constant at that level thereafter. In contrast, model-implied measures fall faster than surveys during the policy tightening phase which started in spring 1988, then increase sharply during the "inflation scare" of 1993. They hover closely around 2.5 percent at the turn of the millennium, but fall sharply to levels close to 1 percent during the recession of the early 2000s and even below 1 percent ahead of the Great recession. In sum, bond prices suggest that 10-year inflation expectations over the 2000s are less firmly anchored than one would conclude, based on survey data. This conclusion may be affected by the assumption, common to other empirical studies such as Smets and Wouters (2007), of absence of structural canges in the conduct of U.S. monetary policy over the 1966-2008 period. It is however noticeable that inflation developments after the Great recession turned out to be more in line with the expectations implied by our model than with survey expectations.

Our paper is related to a recent literature exploring the term structure implications of macro-models. Many of these papers are theoretical and look at the asset pricing implications of macro models–see e.g. Piazzesi and Schneider (2006), Rudebusch and Swanson (2012), Swanson (2014). Amongst the empirical papers, De Graeve, Emiris and Wouters (2007) estimate a standard DSGE model using both macroeconomic and term

[1] See Greenspan (2005).

[2] Goodfriend (1993) defines an inflation scare as a significant increase in long term nominal interest rates in the absence of an increase in policy rates.

structure data, but rely on the loglinearized version of that model and must therefore introduce additional parameters to allow for constant risk-premia. Christoffel, Jaccard and Kilponen (2011) also estimate the linearized version of a new Keynesian model, and then draw bond pricing implications using a higher order approximation. Bekaert, Cho and Moreno (2010) and Campbell, Pflueger and Viceira (2013) follow an intermediate route and study asset prices in a linearized New Keynesian model assuming a stochastic discount factor that is related to the new Keynesian model's equations in a reduced-form manner. The papers most similar to ours are Doh (2011, 2012), van Binsbergen *et al.* (2012) and Andreasen (2012), which estimate nonlinear models with macroeconomic and term structure data. In contrast to all these papers, we allow for regime switches in the variance of shocks and argue that this is an essential model feature to fit bonds and macro data. Moreover, the focus of all these papers is on the model's ability to fit yields, while we highlight the model's implications for the transmission of monetary policy. From this perspective, we are closer to Cochrane (2008, 2017) and Atkeson and Kehoe (2008).

Our paper is also related to the literature documenting time variation in macroeconomic volatility in a reduced form setting, including e.g. McDonnell and Perez-Quiros (2000), Sims and Zha (2006), Primiceri (2005). Justiniano and Primiceri (2008) allow for shifts in the volatility of structural shocks in a linearized, medium-scale DSGE model applied to the U.S. economy. In contrast, we rely on a smaller, but non-linear model, which allows us to explore the effects of changes in volatility on households' demand for precautionary saving. Conditional on our model, including bond price data in the estimation set also provides us with additional information to sharpen the inference on regime change, since changes in regime have implications on the level of yields.

Finally, our paper is related to the literature on uncertainty shocks spawned from Bloom (2009). In Bloom (2009), an increase in uncertainty induces firms to temporarily reduce investment and hiring. In our model, higher uncertainty over future technology shocks induces households to increase their precautionary saving. Consumption demand will tend to fall. Due to monopolistic competition and sticky prices, this will bring down output and inflation. Uncertainty shocks therefore act like demand shocks. This is consistent with the results in Basu and Bundick (2012), which relies on a more comprehensive, calibrated model of the U.S. economy and analyses uncertainty shocks in both technology and preferences. Bianchi, Ilut and Schneider (2014) put forward a model with ambiguity averse investors, where regime shifts generate large low frequency movements in asset prices.

The rest of the paper is organized as follows. Section 2 describes the model, focusing on its distinguishing features: the distribution of the shocks and the utility function, which is of the class proposed by Epstein and Zin (1989) and Weil (1990), but extended to allow for habit persistence in consumption. The methods that we adopt to solve and estimate the model are described next, in section 3. Such methods are non-standard, because we need to solve the model to a second order approximation in order to capture precautionary savings effects. We demonstrate that the reduced form of the model is quadratic in the state variables with continuous support and includes regime-switching intercepts, as well as variances. We then estimate the non-linear reduced form using Bayesian methods. Section 4 described the estimation results and presents a few goodness-of-fit measures. The implications of our estimates for the relationship between monetary policy and risk premia and for the transmission of monetary policy to long-term rates are discussed in Section 5. Section 6 concludes.

2 The model

We start from a simple version of the new-Keynesian model that has been shown to account relatively well for the dynamics of key nominal and real macroeconomic variables–see e.g. Smets and Wouters (2007). We thus assume nominal price rigidities, external habit persistence, inflation indexation, and a monetary policy rule with partial adjustment–or "interest rate smoothing". Since our interest is on the model's implications for long-term interest rates, we simplify it by abstracting from capital accumulation and real wage rigidities. Our results suggest that even our simple model can go a long way in explaining the data of interest to us.

Compared to the new Keynesian benchmark, we introduce two key modifications.

The first is to allow for stochastic regime switching in the variance of structural shocks. The evidence of time variation in the variance of macroeconomic shocks is well-established–see e.g. Justiniano and Primiceri (2008), McDonnell and Perez-Quiros (2000), Primiceri (2005) and Sims and Zha (2006). The novelty in our paper is to explore the implications of time varying variances on bond prices.

Our second modification, which is already common in the consumption-based asset pricing literature, is to adopt the non-expected utility specification for preferences proposed by Epstein and Zin (1989) and Weil (1990). Here we extend this specification to a general equilibrium model in which we also allow for habit persistence in consumption and labour-leisure choice.

2.1 Structural shocks

A key distinguishing feature of our model are changes in the demand for precautionary saving induced by variations in the conditional variance of the structural shocks. We therefore start the description of our model from the distribution of structural shocks.

In macroeconomic applications, exogenous shocks are almost always assumed to be (log-)normal, partly because models are typically log-linearized and researchers are mainly interested in characterizing conditional means. However, Hamilton (2008) argues that a correct modelling of conditional variances is always necessary, for example because inference on conditional means can be inappropriately influenced by outliers and high-variance episodes. The need for an appropriate treatment of heteroskedasticity becomes even more compelling when models are solved nonlinearly, because conditional variances have a direct impact on conditional means.

In this paper, we assume that variances are subject to stochastic regime switches. We will allow for shocks to the level and growth rates of technology, to mark-ups, to the monetary policy rule and to a non-interest-rate-sensitive component of output G_t. G_t will enter GDP like government spending, but we do not model it explicitly since our interest is not on fiscal policy. We only use G_t to allow for a demand-type shock and we therefore refer to it generically as "demand shock". The conditional variance of any of these shocks could in principle be subject to regime switching, but in this paper we adopt a parsimonious specification such that only (level) productivity, monetary policy and demand shocks have regime switching variances. These assumptions are loosely inspired by the finding of the literature on the "Great moderation" (see e.g. McDonnell and Perez-Quiros, 2000) that has emphasized the reduction in the volatility of real aggregate variables starting in the second half of the 1980s, and by the large increase in interest rate volatility in the early 1980s, the time of the so-called "monetarist experiment" of the Federal Reserve.

More specifically, we will assume that the technology shock z_t, the monetary policy shocks η_t and the shock G_t have standard deviations that can independently switch between a high and a low regime.[3] Denoting the low variance regime by 1 and the high variance regime by 0, we write

$$
\begin{aligned}
\sigma_{z,s_{z,t}} &= \sigma_{z,0}s_{z,t} + \sigma_{z,1}\left(1 - s_{z,t}\right) \\
\sigma_{G,s_{G,t}} &= \sigma_{G,0}s_{G,t} + \sigma_{G,1}\left(1 - s_{G,t}\right) \\
\sigma_{\eta,s_{\eta,t}} &= \sigma_{\eta,0}s_{\eta,t} + \sigma_{\eta,1}\left(1 - s_{\eta,t}\right)
\end{aligned}
$$

where the variables $s_{z,t}$, $s_{G,t}$ and $s_{\eta,t}$ can assume the discrete values 0 and 1. For each variable $s_{j,t}$ $(j = z, G, \eta)$, the probabilities of remaining in states 0 and 1 are constant and equal to $p_{j,0}$ and $p_{j,1}$, while the probabilities of switching to the other state will be $1 - p_{j,0}$ and $1 - p_{j,1}$, respectively.

2.2 Households

We assume that each household i provides $N(i)$ hours of differentiated labor services to firms in exchange for a labour income $w_t(i)\,N_t(i)$. Each household owns an equal share of all firms j and receives profits $\int_0^1 \Psi_t(j)\mathrm{d}j$. As in Erceg, Henderson and Levin (2000), an employment agency combines households' labor hours in the same proportions as firms would choose. The agency's demand for each household's labour is therefore equal to the sum of firms' demands. The labor index L_t has the Dixit-Stiglitz form $L_t = \left[\int_0^1 N_t(i)^{\frac{\theta_{w,t}-1}{\theta_{w,t}}}\mathrm{d}i\right]^{\frac{\theta_{w,t}}{\theta_{w,t}-1}}$, where $\theta_{w,t} > 1$ is subject to exogenous shocks. At time t, the employment agency minimizes the cost of producing a given amount of the aggregate labor index, taking each household's wage rate $w_t(i)$ as given, and then sells units of the labor index to the production sector at the aggregate wage index $w_t = \left[\int_0^1 w(i)^{1-\theta_{w,t}}\mathrm{d}i\right]^{\frac{1}{1-\theta_{w,t}}}$. The employment agency's demand for the labor hours of household i is given by

$$N_t(i) = L_t\left(\frac{w_t(i)}{w_t}\right)^{-\theta_{w,t}} \tag{1}$$

Each household i maximizes its intertemporal utility with respect to consumption, the wage rate and holdings of contingent claims, subject to the demand for its labour (1) and the budget constraint

$$P_tC_t(i) + \mathrm{E}_tQ_{t,t+1}W_{t+1}(i) \leq W_t(i) + w_t(i)\,N_t(i) + \int_0^1 \Psi_t(j)\,\mathrm{d}j \tag{2}$$

[3] We have also estimated versions of the model allowing for regime-switching in the variance of mark-up and technology growth shocks. These dimensions of regime switching receive little support from the data.

where C_t is a consumption index satisfying

$$C_t = \left(\int_0^1 C_t(z)^{\frac{\theta-1}{\theta}} \,\mathrm{d}z \right)^{\frac{\theta}{\theta-1}} \tag{3}$$

In the budget constraint, W_t denotes the beginning-of-period value of a complete portfolio of state contingent assets, $Q_{t,t+1}$ is their price and $\Psi_t(j)$ are the profits received from investment in firm j. The price level P_t is defined as the minimal cost of buying one unit of C_t, hence equal to

$$P_t = \left(\int_0^1 p(z)^{1-\theta} \,\mathrm{d}z \right)^{\frac{1}{1-\theta}}. \tag{4}$$

Equation (2) states that each household can only consume or hold assets for amounts that must be less than or equal to its salary, the profits received from holding equity in all the existing firms and the revenues from holding a portfolio of state-contingent assets.

Households' preferences are described by the Kreps and Porteus (1978) specification proposed by Epstein and Zin (1989). In that paper, utility is defined recursively through the aggregator U such that

$$U\left[C_t, \left(\mathrm{E}_t V_{t+1}^{1-\gamma}\right)\right] = \left\{(1-\beta) C_t^{1-\psi} + \beta \left(\mathrm{E}_t V_{t+1}^{1-\gamma}\right)^{\frac{1-\psi}{1-\gamma}}\right\}^{\frac{1}{1-\psi}}, \qquad \psi, \gamma \neq 1 \tag{5}$$

where β, ψ and γ are positive constants. Using a specification equivalent to that in equation (5), Weil (1990) shows that β is, under certainty, the subjective discount factor, but time preference is in general endogenous under uncertainty. The parameter γ is the relative risk aversion coefficient for timeless gambles. The parameter $1/\psi$ measures the elasticity of intertemporal substitution for deterministic consumption paths.

The distinguishing feature of the Epstein-Zin-Weil preferences, compared to the standard expected utility specification, is that the coefficient of relative risk aversion can differ from the reciprocal of the intertemporal elasticity of substitution. In addition, Kreps and Porteus (1978) show that, again contrary to the expected utility specification, the timing of uncertainty is relevant in their class of preferences. The specification in equation (5) displays preferences for an early resolution of uncertainty when the aggregator is convex in its second argument, i.e. when $\gamma > \psi$. Any source of risk will be reflected in asset prices not only if it makes consumption more volatile, but also if it affects the temporal distribution of consumption volatility.

We generalize the utility function in equation (5) by allowing for habit formation and a labour-leisure choice, as in standard, general equilibrium macro-models. The generalization to allow for the labour-leisure choice has already been used, for example, in Rudebusch and Swanson (2012). We additionally allow for habit formation because it has been shown to be important to match the dynamic behavior of aggregate consumption–see e.g. Fuhrer (2000).

As a result, time-t utility will not only depend on consumption C_t but it will be a more general function of consumption and leisure

$$U_t(j) = u\left\{C_t(j) - h\Xi_t C_{t-1}, 1 - N_t(j)\right\}$$

where leisure is written as $1 - N_t$ because total hours are normalized to 1, the h parameter represents the force of external habits and Ξ_t is the rate of growth of technology.[4]

With our more general preferences specification, γ is no-longer related one-to-one to risk aversion. Swanson (2012) discusses the appropriate measures of risk aversion in a dynamic setting with consumption and leisure entering the utility function. However, $1/\psi$ continues to measure the long-run elasticity of intertemporal substitution of consumption.

The first order conditions include

$$\frac{u_{N,t}}{u_{c,t}} = \mu_{w,t}\frac{w_t(j)}{P_t}$$

and

$$Q_{t,t+1} = \beta\left[\mathrm{E}_t\left(\frac{J_{t+1}}{J_t}\right)^{1-\gamma}\right]^{\frac{\gamma-\psi}{1-\gamma}}\left(\frac{J_{t+1}}{J_t}\right)^{-(\gamma-\psi)}\left(\frac{u_{t+1}}{u_t}\right)^{-\psi}\frac{u_{c,t+1}}{u_{c,t}}\frac{1}{\Pi_{t+1}} \tag{6}$$

where Π_t is the inflation rate between t and $t-1$, and the mark-up $\mu_{w,t} \equiv (\theta_{w,t} - 1)/\theta_{w,t}$ follows an exogenous autoregressive process

$$\mu_{w,t+1} = \mu_w^{1-\rho_\mu}\left(\mu_{w,t}\right)^{\rho_\mu} e^{\varepsilon^\mu_{t+1}}, \qquad \varepsilon^\mu_{t+1} \approx N\left(0, \sigma_\mu\right)$$

The gross interest rate, I_t, equals the conditional expectation of the stochastic discount factor, i.e.

$$I_t^{-1} = \mathrm{E}_t Q_{t,t+1} \tag{7}$$

Note that we will focus on a symmetric equilibrium in which nominal wage rates are all allowed to change optimally at each point in time, so that individual nominal wages will equal the average w_t.

[4] Guariglia and Rossi (2002) also use expected utility preferences combined with habit formation to study precautionary savings in UK consumption. Koskievic (1999) studies an intertemporal consumption-leisure model with non-expected utility.

Equation (6) highlights how our model nests the standard power utility case, in which $\psi = \gamma$ and the maximum value function J_t disappears from the first order conditions. The same equations also demonstrate that the parameter γ only affects the dynamics of higher order approximations. It is straightforward to see that, to first order, the term $\left[\mathrm{E}_t \left(J_{t+1}/J_t\right)^{1-\gamma}\right]^{(\gamma-\psi)/(1-\gamma)} \left(J_{t+1}/J_t\right)^{-(\gamma-\psi)}$ cancels out in the interest rate equation (7).

2.3 Firms

We assume a continuum of monopolistically competitive firms (indexed on the unit interval by j), each of which produces a differentiated good. Demand arises from households' consumption and from the exogenous component G_t, which is an aggregate of differentiated goods of the same form as households' consumption. It follows that total demand for the output of firm i takes the form $Y_t^D(j) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t^D$. Y_t^D is an index of aggregate demand which satisfies $Y_t^D = C_t + G_t$.

Firms have the production function

$$Y_t(j) = A_t L_t^{\alpha}(j)$$

where L_t is the labour index L_t defined above and A_t is a mixture of two shocks $A_t = Z_t B_t$ such that, in logs,

$$\begin{aligned} b_t &= b_{t-1} + \xi + \varepsilon_t^{\xi}, \qquad \varepsilon_{t+1}^{z} \approx N(0, \sigma_{\xi}) \\ z_t &= \rho_z z_{t-1} + \varepsilon_t^{z}, \qquad \varepsilon_{t+1}^{z} \approx N\left(0, \sigma_{z, s_{z,t}}\right) \end{aligned}$$

where ξ is the long run productivity growth rate. This specification allows for both a standard, stationary technology shock and for a stochastic trend, represented by B_t. For the solution and estimation of the model, we will work with de-trended variables.

As in Rotemberg (1982), we assume the firms face quadratic costs in adjusting their prices. This assumption is also adopted, for example, by Schmitt-Grohé and Uribe (2004b) and it is known to yield first-order inflation dynamics around a zero inflation steady state equivalent to those arising from the assumption of Calvo pricing.[5] From our viewpoint, it has the advantage of greater computational simplicity, as it allows us to avoid having

[5] The equivalence does not hold exactly around a positive inflation steady state – see Ascari and Rossi (2010). Moreover two pricing models have in general different welfare implications – see Lombardo and Vestin (2008).

to include an additional state variable in the model, i.e. the cross-sectional dispersion of prices across firms.

The specific assumption we adopt is that firm j faces a quadratic cost when changing its prices in period t, compared to period $t-1$. Consistently with what is typically done in the Calvo literature, we modify the original Rotemberg (1982) formulation for partial indexation of prices to lagged inflation. More specifically, we assume that

$$\frac{\zeta}{2}\left(\frac{P_t^j}{P_{t-1}^j} - (\Pi^*)^{1-\iota}\,\Pi_{t-1}^{\iota}\right)^2 Y_t$$

where Π^* is the central bank's inflation target. In a symmetric equilibrium, firms' profits maximization problem leads to

$$(\theta-1)\,Y_t + \zeta\left(\Pi_t - (\Pi^*)^{1-\iota}\,\Pi_{t-1}^{\iota}\right)Y_t\Pi_t = \frac{\theta}{\alpha}\frac{w_t}{P_t}\left(\frac{Y_t}{A_t}\right)^{\frac{1}{\alpha}} + \mathrm{E}_t Q_{t,t+1}\zeta\left(\Pi_{t+1} - (\Pi^*)^{1-\iota}\,\Pi_t^{\iota}\right)Y_{t+1}\Pi_{t+1}$$

2.4 Monetary policy and market clearing

We close the model with the simple Taylor-type policy rule

$$I_t = \left(\frac{\Pi^*}{\beta}\right)^{1-\rho_I}\left(\frac{\Pi_t}{\Pi^*}\right)^{\psi_\Pi}\left(\frac{\widetilde{Y}_t}{\widetilde{Y}}\right)^{\psi_Y} I_{t-1}^{\rho_I} e^{\eta_{t+1}} \tag{8}$$

where $\widetilde{Y}_t \equiv Y_t/B_t$ is detrended aggregate output, $\widetilde{Y}$ its steady state level, Π^* is the constant inflation target and η_{t+1} is a policy shock such that

$$\eta_{t+1} = e^{\varepsilon_{t+1}^{\eta}}, \qquad \varepsilon_{t+1}^{\eta} \approx N\left(0, \sigma_{\eta,s_{\eta,t}}\right).$$

Market clearing in the goods market requires

$$Y_t = C_t + G_t + \frac{\zeta}{2}\left(\Pi_t - (\Pi^*)^{1-\iota}\,\Pi_{t-1}^{\iota}\right)^2 Y_t$$

where G_t is an exogenous stochastic process which captures additional non-interest-rate-sensitive components of output and which we specify in deviation from the stochastic growth trend B_t, so that

$$\frac{G_t}{B_t} = \left(\frac{gY}{B}\right)^{1-\rho_g}\left(\frac{G_{t-1}}{B_{t-1}}\right)^{\rho_g} e^{\varepsilon_t^g} \qquad \varepsilon_{t+1}^{G} \approx N\left(0, \sigma_{G,s_{G,t}}\right)$$

where the long run level g is specified in percent of output, so that $g \equiv G/Y$.

In the labour market, labour demand will have to equal labour supply. In addition, the total demand for hours worked in the economy must equal the sum of the hours worked by all individuals. Taking into account that at any point in time the nominal wage rate is identical across all labor markets because all wages are allowed to change optimally, individual wages will equal the average w_t. As a result, all households will chose to supply the same amount of labour and labour market equilibrium will require that

$$L_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{\alpha}}$$

3 Solution and estimation methods

3.1 Solution

To solve the model, we first approximate the system around a deterministic steady state in which all real variables are detrended by the technological level B_t. For example, detrended output is $\widetilde{Y}_t \equiv Y_t/B_t$. In the solution, we expand variables around their natural logarithms, which are denoted by lower-case letters.

We collect all detrended, predetermined variables (including both lagged endogenous predetermined variables and exogenous states with continuous support) in a vector $\mathbf{x}_t$ and all the non-predetermined variables in a vector $\mathbf{y}_t$ (note that $\mathbf{y}_t$ is different from output y_t).

The macroeconomic system can thus be written in compact form as

$$\mathbf{y}_t = g\left(\mathbf{x}_t, \widetilde{\sigma}, \mathbf{s}_t\right) \quad (9)$$

$$\mathbf{x}_{t+1} = h\left(\mathbf{x}_t, \widetilde{\sigma}, \mathbf{s}_t\right) + \widetilde{\sigma}\Sigma\left(\mathbf{s}_t\right)\mathbf{u}_{t+1} \quad (10)$$

for matrix functions $g(\cdot)$, $h(\cdot)$, and $\Sigma(\cdot)$ and a vector of i.i.d. innovations $\mathbf{u}_t$. The vector $\mathbf{s}_t$ includes the state variables that index the discrete regimes and $\widetilde{\sigma}$ is a perturbation parameter.

Following Hamilton (1994), we can write the law of motion of the discrete processes $\mathbf{s}_t$ as

$$\mathbf{s}_{t+1} = \boldsymbol{\kappa}_0 + \boldsymbol{\kappa}_1\mathbf{s}_t + \boldsymbol{\nu}_{t+1} \quad (11)$$

for a vector $\boldsymbol{\kappa}_0$ and a matrix $\boldsymbol{\kappa}_1$. The law of motion of state $s_{z,t}$, for example, is written as $s_{z,t+1} = (1 - p_{z,0}) + (-1 + p_{z,1} + p_{z,0})\, s_{z,t} + \nu_{z,t+1}$, where $\nu_{z,t+1}$ is an innovation with mean zero and heteroskedastic variance.

For the solution, we follow the approach described in Amisano and Tristani (2011), which exploits the model property that regime switches only affect the shock variances.

We can therefore apply standard perturbation methods (as in, for example, Schmitt-Grohé and Uribe, 2004a, or Gomme and Klein, 2011) and approximate the solution as a function of the state vector $\mathbf{x}_t$ and perturbation parameter $\widetilde{\sigma}$, but keep it fully nonlinear as a function of the vector $\mathbf{s}_t$. More specifically, we seek a second-order approximation to the functions $g(\mathbf{x}_t, \widetilde{\sigma}, \mathbf{s}_t)$ and $h(\mathbf{x}_t, \widetilde{\sigma}, \mathbf{s}_t)$ around the non-stochastic steady state, namely the point where $\mathbf{x}_t = \overline{\mathbf{x}}$ and $\widetilde{\sigma} = 0$.

Due to the presence of the discrete regimes in the system, both the steady state and the coefficients of the second order approximation could potentially depend on $\mathbf{s}_t$ in a nonlinear fashion. Since the discrete states only affect the variance of the shocks, however, they disappear when $\widetilde{\sigma} = 0$ so that the non-stochastic steady state is not regime-dependent. Amisano and Tristani (2011) demonstrate that the second order approximation can be written as

$$g(\mathbf{x}_t, \widetilde{\sigma}, \mathbf{s}_t) = F\widehat{\mathbf{x}}_t + \frac{1}{2}\left(I_{n_y} \otimes \widehat{\mathbf{x}}_t'\right) E\widehat{\mathbf{x}}_t + k_{y,s_t}\widetilde{\sigma}^2 \tag{Sol1}$$

and

$$h(\mathbf{x}_t, \widetilde{\sigma}, \mathbf{s}_t) = P\widehat{\mathbf{x}}_t + \frac{1}{2}\left(I_{n_x} \otimes \widehat{\mathbf{x}}_t'\right) G\widehat{\mathbf{x}}_t + k_{x,s_t}\widetilde{\sigma}^2 \tag{Sol2}$$

where F, E, P and G are constant vectors and matrices and only the vectors k_{y,s_t} and k_{x,s_t} are regime dependent.

Note that regime-switching plays no role to a first order approximation. The quadratic terms in the vector of predetermined variables with continuous support are also regime invariant. Changes in volatility only have an impact on the quadratic terms in the perturbation parameter $\widetilde{\sigma}$. Such terms would be constant in a model with homoskedastic shocks.

3.2 Estimation

Exploiting this feature of the solution, the reduced form system of equations (9) and (10) can be re-written as

$$\mathbf{y}_{t+1}^{o} = k_{y,j} + F\hat{\mathbf{x}}_{t+1} + \frac{1}{2}\left(I_{n_y} \otimes \hat{\mathbf{x}}_{t+1}^{\prime}\right) E\hat{\mathbf{x}}_{t+1} + D\mathbf{v}_{t+1} \tag{12}$$

$$\mathbf{x}_{t+1} = k_{x,i} + P\hat{\mathbf{x}}_t + \frac{1}{2}\left(I_{n_x} \otimes \hat{\mathbf{x}}_t^{\prime}\right) G\hat{\mathbf{x}}_t + \widetilde{\sigma}\Sigma_i\mathbf{w}_{t+1} \tag{13}$$

$$\mathbf{s}_t \backsim \text{Markov switching} \tag{14}$$

where

$$\begin{aligned} k_{y,j} &= k_{y,s_{t+1}=j} \\ k_{x,i} &= k_{x,s_t=i} \\ \Sigma_i &= \Sigma(s_t = i). \end{aligned}$$

The vector $\mathbf{y}_t^o$ includes all observable variables, and $\mathbf{v}_{t+1}$ and $\mathbf{w}_{t+1}$ are measurement and structural shocks, respectively. In this representation, the regime switching variables affect the system by changing the intercepts $k_{y,j}$, $k_{x,i}$ and the loadings of the structural innovations Σ_i (we indicate here with i the value of the discrete state variables at t and with j the value of the discrete state variables at $t+1$).

If a linear approximation were used, we would have a linear state space model with Markov switching (see Kim, 1994, Kim and Nelson, 1999, and Schorfheide, 2005). In the quadratic case, however, the likelihood cannot be obtained in closed form. One possible approach to compute the likelihood is to rely on Sequential Monte Carlo techniques. The convergence of these methods, however, can be very slow in a case, such as the one of our model, in which both nonlinearities and non-Gaussianity of the shocks characterise the economy. Based on the observation that quadratic terms $1/2\left(I_{n_y} \otimes \hat{\mathbf{x}}_{t+1}'\right) E\hat{\mathbf{x}}_{t+1}$ and $1/2\left(I_{n_x} \otimes \hat{\mathbf{x}}_t'\right) G\hat{\mathbf{x}}_t$ in equations (12) and (13) tend to be small, we therefore proceed as follows.

At any point in time, we first linearise the two quadratic terms around the conditional mean of the continuous state variables. In a homoskedastic setting, this would correspond to applying the extended Kalman filter. In our model with regime switching, the linearisation must be conditional on the prevailing regime. As a result, at any point in time we can rewrite equations (12) and (13) as

$$\begin{aligned} \mathbf{y}_{t+1}^o &= \widetilde{k}_{y,t+1}^{(i,j)} + \widetilde{F}_{t+1}^{(i,j)} \hat{\mathbf{x}}_{t+1} + Dv_{t+1} \\ \widehat{\mathbf{x}}_{t+1} &= \widetilde{k}_{x,t}^{(i)} + \widetilde{P}_t^{(i)} \widehat{\mathbf{x}}_t + \Sigma_i \mathbf{w}_{t+1} \end{aligned} \tag{15}$$

for suitably defined coefficients $\widetilde{k}_{y,t+1}^{(i,j)}$, $\widetilde{F}_{t+1}^{(i,j)}$, $\widetilde{k}_{x,t}^{(i)}$ and $\widetilde{P}_t^{(i)}$. Note that, in contrast to the original system (9)-(10), in the above equations both the intercepts $\widetilde{k}_{y,t+1}^{(i,j)}$, $\widetilde{k}_{x,t}^{(i)}$ and the slope coefficients $\widetilde{F}_{t+1}^{(i,j)}$, $\widetilde{P}_t^{(i)}$ become regime-dependent. Nevertheless, we are still in the world of linear state space models with Markov switching. To compute the likelihood, we can therefore apply Kim's (1994) approximate filter. We then combine the likelihood with a prior and sample from the posterior using a tuned Metropolis-Hastings algorithm. This approach based on the extended Kalman Filter linearisation is computationally much faster than using sequential Monte Carlo methods.

4 Empirical results

4.1 Functional forms

In our empirical analysis we need to choose a functional form for the utility aggregator $u\{C_t(j) - h\Xi_t C_{t-1}, 1 - N_t(j)\}$. As shown by King, Plosser and Rebelo (1988), consistency with long run growth requires a functional form of the following type

$$u = (C_t - h\Xi_t C_{t-1})\, v(N_t)$$

where $v(N_t)$ is a decreasing function. Various options are available for $v(N_t)$. We rely on the particular specification proposed by Trabandt and Uhlig (2011), which implies a constant Frisch elasticity of labour supply in the absence of habits and with standard, expected-utility preferences. The utility aggregator that we use is therefore

$$u = (C_t - h\Xi_t C_{t-1}) \left(1 - \eta(1-\psi) N_t^{1+\frac{1}{\phi}}\right)^{\frac{\psi}{1-\psi}}$$

4.2 Data and prior distributions

We estimate the model on quarterly US data over the sample period from 1966Q1 to 2009Q1. Our estimation sample starts in 1966, because this is often argued to be the date when a Taylor rule begins providing a reasonable characterization of Federal Reserve policy.[6] We end in 2009Q1 when the zero bound constraint, which we do not explicitly include in our model, is likely to have become binding.

Concerning the macro data, we use per capita consumption, per capita GDP and inflation. We use both GDP and consumption to impose some discipline on our estimates of the demand shock G_t. Given that we abstract from investment, consumption in our

[6] According to Fuhrer (1996), "since 1966, understanding the behaviour of the short rate has been equivalent to understanding the behaviour of the Fed, which has since that time essentially set the federal Funds rate at a target level, in response to movements in inflation and real activity". Goodfriend (1991) argues that even under the period of official reserves targeting, the Federal Reserve had in mind an implicit target for the Funds rate.

model captures all interest-sensitive components of private expenditure. As argued by Giannoni and Woodford (2005), assuming habit persistence for the whole level of private expenditure is a reasonable assumption, given that models with capital typically need adjustment costs that imply inertia in the rate of investment spending. We therefore use total real personal consumption per-capita in the information set. Inflation is measured as the logarithmic first-difference in the consumption deflator (all macro variables are from the FRED database of the St. Louis Fed).

We use continuously compounded yields on 3-month, 3-year and 10-year zero-coupon bonds (from the Federal Reserve Board). Prior to the analysis, we take logarithmic first differences for consumption and GDP, which are assumed to follow a stochastic trend. No other data transformations are applied. All variables are expressed in decimal terms per quarter, so that 0.0025 represents an annualized interest rate, inflation rate, or growth rate equal to 1 percent.

Prior and posterior distributions for our model are presented in Table 1.

Concerning regime switching processes, we assume beta priors for transition probabilities. We expect the states to be relatively persistent, so we centre all distributions around a value of 0.9, which implies a persistence of 2.5 years for each state.

We use inverse gamma priors for the standard deviations of the shocks. With the exception of the technology growth shock, which has a tighter prior centred around a small value because the process is a random walk, we keep the prior distribution relatively dispersed around a mean value around 0.003. The regime-switching standard deviations also have the same prior distribution in the high and low regimes. To ensure identification, however, all draws from the prior are first ordered and then assigned to the high or low state. Table 1 reports the resulting empirical distribution for the prior of regime-switching standard deviations. Concerning the persistence of the shocks, we use beta priors centred around the value of 0.85.

For the policy rule, we use relatively loose priors centred around parameter values estimated from quarterly data over a pre-sample period running from 1953 to 1965, namely $\rho_I = 0.85$, $\psi_{\Pi} = 0.2$ and $\psi_Y = 0.02$.

The priors for all utility parameters are specified broadly in line with the rest of the literature. For the ϕ parameter we rely on a normal prior centred around 1.0, a value in between macro estimates and micro estimates of the Frisch elasticity of labour supply (see e.g. the evidence reviewed in Chetty *et al.*, 2011). We use a translated Gamma distribution for ψ and γ, to ensure that $\psi, \gamma > 1$. We centre the distribution of ψ around a value above but close to 1. For the γ parameter, which contributes to shape risk aversion, we use a very large standard deviation whose 95 percent confidence set goes from 2 to 30. The habit parameter has a beta prior centred around 0.5. Finally, for β we use a relatively tight prior with a mean of 0.9985. This is consistent with assumptions made in models with growth–see e.g. Christiano, Motto and Rostagno (2014).

For the long run parameters Ξ and Π^* we rely on more dogmatic priors. For Ξ, which determines the growth rate of the economy in the non-stochastic steady state, we use a tight prior centred around 0.005. This implies an annualized growth rate of 2 percent, which is consistent with the average per-capita U.S. GDP/GNP growth from the 1870s to the 1950s–see Maddison (2013). For the inflation target, we choose a prior centred around 1.0063 that gives most mass to annualized values between 2 and 3 percent.

The price adjustment cost ζ is typically calibrated based on the implied frequency of adjustment of prices in linearized models. In our model, however, the relationship is more complex due to both the nonlinearity of the model and the presence of steady state inflation. We therefore centre the prior around 15, which is roughly consistent, for example, with the value used in Schmitt-Grohé and Uribe (2004b), but allow for a relatively large standard deviation. For inflation indexation, we rely on a beta prior centred around 0.5.

The elasticity of intratemporal substitution θ, which is weakly identified, is set dogmatically at 6. Similarly, we set $\mu_w = 1.2$.

4.3 Posterior distributions and goodness of fit

The posterior distributions of structural parameters in Table 1 suggest that the data are informative about the estimation of most parameters, as witnessed by the typically smaller standard deviation of the posterior distribution compared to the prior distribution.

More specifically, the different regimes in the volatilities of monetary policy, technology and government spending shocks are clearly identified. For monetary policy, the standard deviations in the low and high regimes are equal to 0.13 percent and 0.39 percent respectively. These values straddle the constant standard deviation of 0.24 percent estimated in Smets and Wouters (2007). The standard deviation of technology shocks change between 1.1 percent in the low volatility regime and 2.7 percent in the high volatility regime. The difference between the two volatility regimes is largest for demand shocks: their standard deviation shifts between 0.33 and 3.2 percent.

The posterior mode of the transition probabilities suggests that the low-volatility states are more persistent for monetary policy and technology shocks. For policy shocks, the ergodic probability of being in the low-volatility state is approximately 0.69, which is consistent with the idea that policy shocks were small over most of the sample, except for the Volcker disinflation period. Both the low and the high volatility states are more persistent for technology shocks. These states are countercyclical, being persistently high during recessions and low over expansions. The ergodic probability of the low volatility state for technology shocks is 0.71. By contrast, the volatility of demand shocks is more persistent in the high state, whose ergodic probability is 0.68. Based on these results, we refer to low-volatility regimes for policy and technology shocks as "normal regimes".

As in estimates solely based on macro data, shock processes tend be highly serially correlated. At 0.99, the correlation of the level technology shock process is especially high. Together with the features of the monetary policy rule, this implies that technology shock have very persistent effects.

The estimated parameter values of the coefficients of the monetary policy rule are of particular interest. *Ceteris paribus* different parameters of the policy rule will be associated with different expectations of the future path of short-term interest rates, thus different configurations of the yield curve. Since we explicitly use yields data when estimating the model, our estimates of the policy rule parameters should be more informative than those obtained without including yields in the econometrician's information set. Given the well-known problems of general equilibrium models to match the unconditional volatility of long-term yields (see e.g. Den Haan, 1995), one would expect the degree of interest rate smoothing to be higher than in estimates ignoring yields data. A higher smoothing coefficient would impart persistence to any movements in the short-term rate. Its variability would thus be transmitted to longer rates (for a discussion of this point, see Hördahl, Tristani and Vestin, 2008).

To compare our estimates to those in the literature, it is useful to rewrite the rule in partial adjustment form. Our parameter estimates then imply:

$$\widehat{i}_t = 0.09 \ \left[3.09 \ (\pi_t - \pi^*) + 0.57 \ (\widetilde{y}_t - \widetilde{y})\right] + 0.91 \ \widehat{i}_{t-1} + \eta_{t+1}. \tag{16}$$

Equation (16) confirms the above intuition. Compared to the estimates in Smets and

Wouters (2007), our parameters imply a somewhat higher, but not exceedingly high, inflation response coefficient.[7] The more striking feature of our estimates, however, is the increase in the degree of interest rate smoothing (0.91 vs. 0.81 in Smets and Wouters). More inertial movements in short-term rates imply that longer-term yields can be systematically affected by monetary policy. This feature is important for the model to be able to generate variation at longer maturities in the term-structure of interest rates.[8]

The estimates of the other structural parameters are roughly consistent with the existing literature.

Concerning long-run means, the mode of the quarterly trend growth rate of technology is 0.45 and the quarterly inflation target is 0.61, both within the posterior distribution of estimates obtained in Smets and Wouters (2007).

Amongst preference parameters, the posterior mean of ϕ is 0.6. Our estimate of ψ implies a long-run elasticity of intertemporal substitution of consumption of 0.76, which is in line with other available estimates (see e.g. Basu and Kimball, 2002). The γ parameter is equal to 11.5 and the habit parameter $h = 0.86$. Together, these two parameters are suggestive of a high level of risk aversion, which is in line with the results in Piazzesi and Schneider (2006), or in Rudebusch and Swanson (2012).

The model fit is good for both macroeconomic and yields data. This claim is supported by three pieces of evidence.

First, measurement errors on all variables are small. This is perhaps not surprising for macro variables and for the short-term interest rate, given the results in Smets and Wouters (2007). For longer-term yields, however, one could expect a worse performance. Nevertheless, both 3-year and 12-year rates are fit rather well. The measurement errors on these two variables are equal to 29 and 18 basis points, respectively. This is a comparable fit to the results in more empirically flexible models such as Ang and Piazzesi (2003).[9]

Second, we check the implications of our model in terms of the dynamic correlations it implies between observable variables–see panels (a) and (b) in figure 1. Model-implied

[7] The parameter estimates are not fully comparable, because the policy rule used in Smets and Wouters (2007) includes additional arguments.

[8] De Graeve, Emiris and Wouters (2009) also uses yields data in estimation, but obtains interest rate smoothing estimates similar to Smets and Wouters (2007). In De Graeve, Emiris and Wouters (2009), however, persistent movements in policy interest rates are driven by changes in a stochastic inflation target, which is almost a random walk.

[9] Ang and Piazzesi (2003) is however estimated on more volatile, monthly data.

dynamic correlations at lags and leads up to 20 quarters are compared to sample correlations. Model-implied correlations are computed for all posterior draws and error bands corresponding to a 95 percent confidence set are also displayed in figure 1.

By and large, the figure indicates that our model captures reasonably well the dynamic cross-correlations between all variables. The distribution of model-implied dynamic correlations tends to always include its empirical counterpart. This is specifically the case for autocorrelations, that start from an appropriately high value and tend to decay in line with the empirical measures.

Third, we test the implications of our model for dimensions of the data which were not directly used in estimation, notably for forward rates at various horizons. Model-implied and actual 3-month forward rates in 1, 3 and 10 years are reported in figure 2. Note that the 1-year rate was not used in estimation. Nevertheless, the model tracks well the evolution of all forward rates. More specifically, the model can track well the variations in the long-term forward rate that are a puzzle for linearized models.

4.4 Volatility regimes and uncertainty shocks

Figure 3 displays filtered and smoothed estimates of the probability of being in a low-variance regime for the three heteroskedastic shocks.

The demand shock has high variance in the first part of the sample and lower variance during the Great moderation period.

Concerning the monetary policy shock, our results are consistent with those in Justiniano and Primiceri (2008), where heteroskedasticity takes the form of stochastic volatility, rather than regime switching. The policy shock has a high variance during the mid-1970s and again during the so-called "Volcker disinflation" period in 1979-83. One marginally different feature of our results, is that the increase in volatility in 1979 is estimated to be very rapid in real time. This is arguably consistent with the spikes which can be observed in the short term interest rate over this period. Such sudden increases in volatility can more easily be captured by a regime-switching model than by a stochastic volatility model.

The most striking feature of the regimes for the variance of technology shocks in Figure 3 is that they are strongly cyclical. Starting in 1980, the standard deviation of these shocks tends to increase at the beginning of each recessions and to fall again after a few quarters. This pattern is quite systematic, especially over the 1990s and the 2000s. The period of the Volcker disinflation is therefore unique in being characterized by high variance of government spending, policy and technology shocks.

We next focus on the impulse responses to uncertainty shocks. To help understand the impact of these shocks, consider the second-order approximation to the Euler equation

$$\begin{aligned}
\widehat{\widehat{c}}_t &= \frac{1}{1+h}\mathrm{E}_t\widehat{\widehat{c}}_{t+1} + \frac{h}{1+h}\widehat{\widehat{c}}_{t-1} - \frac{1}{\overline{\psi}}\left(\widehat{i}_t - \mathrm{E}_t\left[\widehat{\pi}_{t+1}\right]\right) + \frac{1-h}{1+h}\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}\mathrm{E}_t\Delta\widehat{l}_{t+1} \\
&\quad -(\gamma-\psi)\frac{1-h}{1+h}\mathrm{Cov}_t\left[\frac{1}{1-h}\left(\Delta\widehat{\widehat{c}}_{t+1} - h\Delta\widehat{\widehat{c}}_t\right) + \left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}\Delta\widehat{l}_{t+1}, \widehat{\xi}_{t+1} + \widehat{\widehat{j}}_{t+1}\right] \qquad (17) \\
&\quad -\frac{\gamma-\psi}{\overline{\psi}}\mathrm{Cov}_t\left[\psi\widehat{\xi}_{t+1} + \widehat{\pi}_{t+1}, \widehat{\xi}_{t+1} + \widehat{\widehat{j}}_{t+1}\right] + \frac{1}{2}(\gamma-\psi)\frac{\psi-1}{\overline{\psi}}\mathrm{Var}_t\left[\widehat{\xi}_{t+1} + \widehat{\widehat{j}}_{t+1}\right] - \frac{1}{2}\mathrm{Var}_t\Omega_{t+1}
\end{aligned}$$

where $\overline{\psi} \equiv \psi\frac{1+h}{1-h}$ is the inverse of the short-run elasticity of intertemporal substitution, $\overline{n} \equiv \eta(1-\psi)N^{1+1/\phi}$ and where $\widehat{\widehat{j}}_{t+1} + \widehat{\xi}_{t+1}$ are expectations about the present discounted stream of future utility

$$\widehat{\widehat{j}}_{t+1} + \widehat{\xi}_{t+1} = \sum_{i=0}^{\infty}\left(\beta\Xi^{1-\psi}\right)^i \mathrm{E}_{t+1}\left[\widehat{\xi}_{t+1+i} + \left(1-\beta\Xi^{1-\psi}\right)\widehat{\widehat{u}}_{t+1+i}\right]$$

The first row on the right-hand-side of equation (17) includes first-order terms. As in the standard new-Keynesian model with habits (see e.g. Woodford, 2003), consumption is partly backward-looking, partly forward looking, and it is negatively related to the real interest rate. The following terms in the equation arise under Epstein-Zin-Weil utility and involve conditional variance and covariances of expected future utility news, $\widehat{\widehat{j}}_{t+1} + \widehat{\xi}_{t+1}$. The covariance term in the second row is positive when news about expected future utility growth tend to be associated with high expected growth in consumption and hours worked at $t+1$. When this covariance increases, i.e. expected future utility growth becomes riskier, households increase their precautionary saving and reduce their consumption. This trasmission channel is stronger, the more agents are unwilling to adjust their level of utility across states (i.e. the higher γ). Note, however, that this channel is dampenend by the adjustment in the real interest rate, which falls, thus stimulating the demand for consumption goods, to restore the equilibium in the savings market. A similar effect is produced by increases in the first covariance on the last row of the equation, which is positive when news about expected future utility growth are associated with high inflation and/or high productivity growth at $t+1$. When $\psi \neq 1$ increases in the variance of revisions of expected future utility also play a role and tend to boost consumption, according to the following term in the last row of the equation. This effect is, however, small for our estimated parameter values. Finally, the $\text{Var}_t\Omega_{t+1}$ term denotes other second order terms such as $\text{Var}_t\widehat{\widetilde{c}}_{t+1}$, or $\text{Var}_t\widehat{\xi}_{t+1}$ whose increase tends to reduce consumption. Also these terms tend to be quantitatively small.

To summarize, increases in the volatility of structural shocks which bring about an increase in the conditional covariance terms in equation (17) tend to produce an increase in precautionary saving and a fall in consumption. For our estimated parameter values, these effects are small in reaction to changes in the conditional variance of demand and policy shocks, but non-negligible after switches in the standard deviation of (level) technology shocks.

The impulse responses to an increase in the variance of technology from the low to the high regime is displayed in Figure 4. For illustrative purposes, in this figures we assume that no further changes in the variance regime occur after the shock. The increase in the variance of technology shocks generates an increase in the demand for precautionary saving. As a result, the demand for consumption goods falls. Given that prices are sticky and output is demand determined, lower demand for consumption goods generates a fall in output and inflation. The policy rate also falls according to the Taylor rule. To clear the savings market, however, real rates must fall at all future horizons, because the uncertainty shock is expected to be persistent. The fall is marked at short horizons, more muted at longer horizons, when the conditional variance of technology shocks is expected to decline again, due to the probability of it switching back to the low-variance regime. As a result, the expected policy rate remains persistently low as long as the detrended level of output remains below its steady state, which is long after the negative inflationary shock has been reabsorbed.

All in all, and consistently with the results in Basu and Bundick (2012), an uncertainty shock in technology looks like a demand shock, in the sense of being associated with a fall in output, consumption and prices at the same time. Our results also corroborates, in the context of an estimated model, Basu and Bundick's finding that a persistent fall in nominal interest rates is an important part of the macroeconomic adjustment mechanism, following an uncertainty shock. If the fall in the nominal interest rate were prevented by the zero lower bound, the macro-economic effects of the shock would be even larger.

5 Monetary policy and long term rates

We have shown that uncertainty shocks have macroeconomic effects. We now investigate their effects on bond prices.

5.1 Monetary policy and risk premia

Nominal bonds reflect risk premia associated with both consumption risk and with inflation risk. Hördahl, Tristani and Vestin (2008) demonstrate that models with homoskedastic shocks solved to a second order approximation can only generate constant risk premia. Consistently with this result, our model can produce changes in risk premia only when there is a change in the standard deviation of the structural shocks. In other words, time variation in risk premia is associated with switches in the variance regimes.

A typically used measure of risk premia which is independent of expected changes in the future path of short term interest rate is the expected excess holding period return on a bond of maturity n. This corresponds to the expected return that can be earned by holding an n-maturity bond for one quarter in excess of the one quarter interest rate.

To second order, the expected excess holding period return can be written as

$$\widehat{h}_{n,t}-\widehat{i}_t = F_{B_{n-1}}\mathrm{E}_t\left[\mathbf{u}_{t+1}\mathbf{u}_{t+1}'\right]\left(\psi\frac{1}{1-h}F_c'+\psi\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}F_l'+\psi F_\xi'+F_\pi'+\left(\gamma-\psi\right)F_j'\right)$$

where F_z denotes the vector of parameters of the first order approximation to the law of motion of any variable z. This equation highlights that all terms in the excess holding period return are constant, except for the variace-covariance matrix of the structural shocks $\mathrm{E}_t\left[\mathbf{u}_{t+1}\mathbf{u}_{t+1}'\right]$. As a result, changes in expected excess holding period returns occur as a result of regime switches in conditional variances. In terms of the finance literature, our model approximation generates differences in the market prices of risk across variance regimes, but regime-switching risk is not priced. The prices of risk are increasing in the sensitivity to changes in the state vector of different variables, including consumption, labour supply, inflation and the present discounted value of future utility.

The expected excess holding period return generated by our model for the 3-year and 10-year bonds is displayed in Figure 5. A notable feature of Figure 5 is that excess holding period returns can be large. At the 10-year maturity, they oscillate between 2 and 13 percentage points per year. This is in the same order of magnitude as some estimates from the finance literature–see e.g. Figure 1 in Duffee (2002). Also consistently with the finding in that literature (see e.g. Fama and French, 1989), risk premia are countercyclical.

In contrast to the finance literature, however, we estimate variations in risk premia to be much more infrequent. Our results suggest that they were constant up until the end of the 1970s. Thereafter they were characterized by prolonged periods of either high or low level, but not by high-frequency fluctuations.

One would expect that regime switches in the volatility of all shocks lead to variation in risk premia. In the model, however, variations in risk premia must be associated with uncertainty about revisions in the rate of growth of future utility and with their correlations with inflation and with the marginal utility of consumption–see also Restoy and Weil (2011) and Piazzesi and Schneider (2006). From a quantitative perspective, monetary policy and demand shocks have a small impact on the rate of growth of utility over long future horizons. Changes in their variance have therefore a small impact on the size of risk premia. This can be observed through a comparison of Figures 5 and 7.

The key source of quantitatively sizable time-variation in risk premia are switches in the variance of technology shocks. Since these variance regimes are estimated very precisely, also in real time, risk premia oscillate mostly between a high and a low value. Consistently with the cyclicality of technological uncertainty shocks, risk premia increase during every NBER-dated recession, then fall again after a few years.

It goes without saying that considerable uncertainty characterizes any estimates of risk premia, because of estimation and model uncertainty. Figure 5 shows that filtering uncertainty is around 5 percentage points at the 10-year maturity. In a classical econometric setting, the small sample bias in maximum likelihood estimates also plays a role–see e.g. Bauer, Rudebusch and Wu (2014), and Wright (2014).

5.2 Yields and the monetary policy transmission mechanism

We have shown that changes in the conditional variances of technology shocks play an important cyclical role.

At the beginning of recessions, the increase in volatility is tantamount to a persistent fall in confidence. Risk premia become larger, precautionary saving increases, consumption, output and inflation fall. Both current and expected future nominal interest rates also fall, but actual long term yields can increase, due to an increase in risk premia at longer horizons. After the recovery sets in, however, the conditional variance of technology switches back to lower levels and confidence returns. The demand for precautionary saving falls back to normal levels, expected future policy interest rates increase, but long term yields are also affected by the marked reduction in risk premia. During cyclical turning points, a model with constant premia does not provide a good description of long-term yields. Movements in long-term yields are primarily the result of changes in *real* yields and risk premia.

More specifically, changes in long-term yields need not be related to expected future monetary policy moves. This occurred famously in 2004, when long-term yields did not increase in the face of the increase in expected future policy rates. Such behavior of long term yields would be entirely standard in a linearized version of the new Keynesian model, but it represents an anomaly compared to typical cyclical developments in bond yields. In his semiannual Monetary Policy Report to the Congress, Chairman Greenspan stated that "the broadly unanticipated behavior of world bond markets remains a conundrum"–see Greenspan (2005). From the perspective of our model, the only unanticipated features of bond developments over the conundrum period is the timing of the downward volatility shock–see Figure 6. Otherwise, a reduction in volatility, and thus a reduction in risk premia, during a cyclical upswing is entirely to be expected.

It is important to note that the cyclical fluctuations of real yields and of the term spread are not only a feature of the past. Figure 3 shows that non-negligible changes in 3-month forward rates 10-year ahead are also visible over the 1990s and the 2000s, i.e. periods of low inflation.

Over the prolonged periods in which volatility stays constant, however, long-term rates react to changes in monetary policy rates according to the expectations hypothesis. Changes in long term rates reflect variations in *long-term inflation expectations*.

This result sheds light on the ability of the linearized, new Keynesian model to account for the monetary policy transmissions mechanism. Over the years between occasional changes in volatility, that model works well: up to a constant risk premium, long-term yields correspond to average expected future short term rates. The connection between long term yields and monetary policy can then be well understood via a linearized model.

This logic may explain the acceptable forecasting performance of linearized models over specific periods of time. For example, De Graeve *et al.* (2009) finds that a linearized model is competitive with the random walk in forecasting 1-year yields up to 3-year ahead over the 1990:Q1-2007Q1 period, but less successful in forecasting longer maturity yields. This is not so surprising given that, according to our estimates, risk premia tend to be smaller at short horizons and they only increased and fell four times over the 1990:Q1-2007Q1 period.

Over periods of constant risk premia, variations in long-term interest rates and long-term inflation expectations must be accounted for by standard shocks. More specifically, our model relies also on standard Gaussian shocks to account for the secular changes in long-term interest rates and long-term forward rates documented in Figure 3. To produce changes in long-term rates, such shocks must be extremely persistent and they need to be coupled with a high degree of inertia in the monetary policy rule. A single shock plays this role in our model: the level technology shock z_t.

Figure 7 shows an impulse response to the technology shock. The shock has the usual opposite effects on output and inflation: real variables increase, while inflation falls. Contrary to typical results, however, the shock generates extremely persistent responses of macroeconomic variables. This is due, first, to the extremely high persistence of the autoregressive process for z_t, whose half-life is of about 15 years.[10] Second, it is due to the high interest rate smoothing coefficient of the Taylor rule, which keeps the short-term real interest rate positive over many quarters after the shock. The increase in the real interest rates reinforces the initial fall in inflation and requires an increasingly loose monetary policy stance over the first year after the shock. It is only after two years that all variables slowly start returning towards their long-run value in a monotonic fashion. In the absence of regime switches, the expectations hypothesis holds and long-term rates fall alongside the policy interest rate.

Both uncertainty shocks and level technology shocks affect inflation over a prolonged period. It is therefore instructive to analyze the overall implications of our estimates for long-term inflation expectations–i.e. expected inflation over the next 10-year. These expectations are important as their stability, or "anchoring", is often interpreted as a measure of central banks' anti-inflationary credibility. As a benchmark for comparison, we use expectations by the Federal Reserve Bank of Philadelphia's quarterly Survey of Profes-

[10] The half-life is defined as the number of periods over which the effect of a unit shock remains above 0.5. For an autoregressive process with serial correlation coefficient ρ, the half-life is $hl = \ln(0.5) / \ln(\rho)$.

sional Forecasters combined with the Blue Chip Economic Indicators, which is available since 1979:Q4.[11]

From a secular perspective, a downward trend can clearly be identified in long-term inflation expectations over the 1980s. Over this period, model-implied 10-year inflation expectations are roughly consistent with the survey data–see Figure 8. The high volatility of the early 1980s kept risk premia and yields high, even as expected future inflation and expected future policy rates were coming down. From this long-term perspective, the improved anchoring of inflation expectations in the U.S. is undoubtable.

From a more cyclical perspective, however, survey and model-implied results differ. Survey expectations fall steadily towards 2.5 percent over the 1990s and then remain constant at that level through the 2000s. In contrast, yields dynamics interpreted through the lens of our model suggest a much less tight anchoring of inflation expectations.

Model-implied measures fall faster than surveys during the policy tightening phase which started in spring 1988 and was followed by the 1990 Gulf War and the ensuing recession. The fall in long-term inflation expectations is smaller than the fall in 10-year yields, because it is accompanied by a surge in volatility and a fall in real rates.

Model-implied inflation expectations increase again sharply in 1993. An increase in long-term inflation expectations is in line with the idea of an "inflation scare", which was put forward by some commentators in this period. For example, Goodfriend (2002) states: "Starting from a level of 5.9 percent [in October 1993], the 30-year bond rate rose through 1994 to peak at 8.2 percent just before election day in November. The nearly 2 1/2 percentage point increase in the bond rate indicated that the Fed's credibility for low inflation was far from secure in 1994."

Following this period, model-implied inflation expectations remain roughly close to the survey measures. However, model-implied expectations diverge again during the recession of the early 2000s, when they fall sharply to levels around 1 percent. These dynamics are arguably consistent with the views of Federal Reserve officials, who expressed concerns about the, albeit remote, possibility of deflation from late 2002 through 2003. In November 2002, the then Governor Bernanke (2002) judged that "the chance of significant deflation

[11] Both surveys report forecasts for the average rate of CPI inflation over the next 10 years. The Blue Chip survey reports long-term inflation forecasts taken twice a year (March and October). Prior to 1983, and in 1983:4, the variable was the GNP deflator rather than the CPI. As of 1991:Q4, we rely on the Philadelphia Survey of Professional Forecasters.

in the United States in the foreseeable future is extremely small", but added that "having said that deflation in the United States is highly unlikely, I would be imprudent to rule out the possibility altogether."

After a return towards 2.5 percent, model-implied long-term inflation expectations fall again ahead of the Great recession, i.e. a period when the possibility of a protracted, low-inflation period was difficult to rule out.

To summarise, our model-implied estimate of long-term inflation expectations implicit in bond prices complements comparable information available from survey data. It suggests that long-term inflation expectations are less firmly anchored than one would conclude, based on survey data.

Since our estimates are model-based, they may of course be affected by model misspecification. One particular source of misspecification may be due to our assumption, common to other empirical studies such as Smets and Wouters (2007), of absence of structural canges in the conduct of U.S. monetary policy over the 1966-2008 period. However the literature has highlighted the possibility of a break in the U.S. monetary policy rule in the late-1970s or early-1980s (e.g. Clarida, Galí and Gertler, 2000, and Lubik and Schorfeide, 2004), while our estimates show a large discrepancy with survey data starting only in the mid-2000. It is therefore not clear that allowing for a break in the policy rule during the Volker tenure would affect our model-based inflation expectations in the late-2000. Another possibility is that the transition probability, or the exact values of the standard deviation of technology shocks changed in the mid-2000, so that, for example, the normal volatility shock has become and absorbing state. If so, this evidence should reveal itself over time, as more and more data are accumulated. In the meantime, it is noticeable that inflation developments after the Great recession turned out to be more in line with the expectations implied by our model than with survey expectations.

6 Conclusions

We have presented the results of the estimation of a nonlinear macro-yield curve model with Epstein-Zin-Weil preferences, in which the variance of structural shocks is subject to changes of regime. We have argued that the model fits the data well: measurement errors are small; the dynamic cross-correlation matrix of the data is closely replicated; long-term forward rates are matched.

An important role to account for this performance is played by changes in variance regimes, or uncertainty shocks, which tends to occur during recessions. On the one hand, uncertainty shocks induce changes in the demand for precautionary savings. Expected real and nominal yields also fall, which is consistent with the empirical evidence. On the other hand, the increase in volatility during recessions also boosts uncertainty over future consumption growth. Risk premia increase in a countercyclical fashion, which is consistent with results from the finance literature.

Our results suggest that movements in long-term yields can reflect both variations in long-term inflation expectations, and changes in *real* yields induced by uncertainty shocks. Compared to survey evidence, our measures of long-term inflation expectations are more variable over the economic cycle. They fall to low levels over the 1980s, but are subject to cyclical "scares"–either upwards or downwards. They suggest that the Federal Reserve's credibility for low inflation is less firmly established than one would conclude, based on survey data.

References

[1] Amisano, G. and O. Tristani (2011), "Exact likelihood computation for nonlinear DSGE models with heteroskedastic innovations," Journal of Economic Dynamics and Control 35, 2167-2185.

[2] Andreasen, Martin M. (2012), "An Estimated DSGE Model: Explaining Variation in Nominal Term Premia, Real Term Premia, and Inflation Risk Premia", European Economic Review 56, 1656–1674.

[3] Ang, Andrew and Monika Piazzesi (2003), "A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables", Journal of Monetary Economics 50, 745–787.

[4] Atkeson, A. and P. Kehoe (2008), "On the need for a new approach to analyzing monetary policy", in D. Acemoglu, K. Rogoff and M. Woodford, eds., NBER Macroeconomics Annual 23, 389-425.

[5] Basu, Susanto and Brent Bundick (2012), "Uncertainty Shocks in a Model of Effective Demand", Boston College Working Paper 774.

[6] Basu, Susanto and Miles S. Kimball (2002), "Long-run labour supply and the elasticity of intertemporal substitution for consumption", mimeo, University of Michigan.

[7] Bauer, Michael D., Glenn D. Rudebusch and Jing Cynthia Wu (2014), "Term Premia and Inflation Uncertainty: Empirical Evidence from an International Panel Dataset: Comment," American Economic Review 104, 323–337.

[8] Bekaert, G., Seonghoon Cho and Antonio Moreno (2010). "New Keynesian Macroeconomics and the Term Structure," Journal of Money, Credit and Banking 42, 33–62.

[9] Bernanke, Ben S. (2002), "Deflation: Making sure "it" doesn't happen here", Remarks before the National Economist Club, Washington, 21 November.

[10] Bianchi, Francesco, Cosmin Ilut and Martin Schneider (2014), Uncertainty shocks, asset supply and pricing over the business cycle, NBER Working Paper No. 20081.

[11] Bloom, Nicholas (2009), "The impact of uncertainty shocks", Econometrica 77, 623–685

[12] Campbell, John Y., Carolin Pflueger and Luis M. Viceira (2013), "Monetary Policy Drivers of Bond and Equity Risks ", mimeo.

[13] Chetty, Raj, Adam Guren, Day Manoli and Andrea Weber (2011), "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins", American Economic Review Papers and Proceedings 101, 471-75.

[14] Christiano, Lawrence J., Roberto Motto and Massimo Rostagno (2014), "Risk shocks", American Economic Review 104, 27–65.

[15] Christoffel, Kai, Ivan Jaccard and Juha Kilponen (2011), "Government bond risk premia and the cyclicality of fiscal policy", ECB Working Paper No. 1411.

[16] Clarida, R., J. Galí and M. Gertler (2000), "Monetary policy rules and macroeconomic stability: evidence and some theory", Quarterly Journal of Economics, 147-180.

[17] Cochrane, John H. (2008), "Comment on "On the need for a new approach to analyzing monetary policy"", in D. Acemoglu, K. Rogoff and M. Woodford, eds., NBER Macroeconomics Annual 23, 427-448.

[18] Cochrane, John H. (2017), "Macro-Finance", Review of Finance, forthcoming.

[19] Cochrane, John H. and Monika Piazzesi (2005), "Bond risk premia", American Economic Review 94, 138-160.

[20] Dai, G. and K. Singleton (2002), "Expectations puzzles, time-varying risk premia, and affine models of the term structure," *Journal of Financial Economics* 63, 415-441.

[21] De Graeve, Ferre, Marina Emiris and Raf Wouters (2009), "A Structural Decomposition of the US Yield Curve", Journal of Monetary Economics 56, pp. 545-559.

[22] Den Haan, W.J. (1995), "The term structure of interest rates in real and monetary economies", Journal of Economic Dynamics and Control 19, pp. 909-40.

[23] Doh, Taeyoung (2011), "Yield Curve in an Estimated Nonlinear Macro Model", Journal of Economic Dynamics and Control 35, 1229-1244.

[24] Doh, Taeyoung (2012), "What Does the Yield Curve Tell Us About the Federal Reserve's Implicit Inflation Target?", Journal of Money, Credit, and Banking 44, 469-486.

[25] Duffee, Gregory R. (2002), "Term premia and interest rate forecasts in affine models", Journal of Finance 57, 405-443.

[26] Fama, Eugene F. and Kenneth R. French (1989), "Business conditions and expected returns on stocks and bonds", Journal of Financial Economics 25, 23-49.

[27] Fuhrer, Jeffrey C. (1996), "Monetary policy shifts and long-term interest rates," *Quarterly Journal of Economics* 111, pp. 1183-1209.

[28] Fuhrer, Jeffrey C. (2000), "Habit Formation in Consumption and Its Implications for Monetary-Policy Models," American Economic Review 90, 367-390.

[29] Giannoni, Marc P. and Michael Woodford (2005), "Optimal Inflation Targeting Rules," in: B.S. Bernanke and M. Woodford, eds., Inflation Targeting, University of Chicago Press.

[30] Gomme, Paul and Paul Klein (2011), "Second-order approximation of dynamic models without the use of tensors", Journal of Economic Dynamics and Control 35, 604-615.

[31] Goodfriend, M. (1991), "Interest Rates and the Conduct of Monetary Policy," *Carnegie-Rochester Conference Series on Public Policy* 34, 7–30.

[32] Goodfriend, Marvin (1993), "Interest Rate Policy and the Inflation Scare Problem: 1979-1992", Federal Reserve Bank of Richmond Economic Quarterly 79(1), 1–24.

[33] Goodfriend, Marvin (2002), “The phases of U.S. monetary policy: 1987 to 2001”, Federal Reserve Bank of Richmond Economic Quarterly 88(4), 1–17.

[34] Greenspan, Alan (2005), Federal Reserve Board semiannual Monetary Policy Report to the Congress Before the Committee on Banking, Housing, and Urban Affairs, U.S. Senate, February 16.

[35] Gürkaynak, Refet S., Brian Sack and Eric Swanson (2005), "The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models," American Economic Review 95, 425-436.

[36] Hördahl, P., O. Tristani and D. Vestin (2008), "The yield curve and macroeconomic dynamics", Economic Journal 118, 1937-70.

[37] Justiniano, Alejandro and Giorgio Primiceri (2008), "The Time Varying Volatility of Macroeconomic Fluctuations", American Economic Review 98, 604-641.

[38] Kim, Chang-Jin (1994), "Dynamic linear models with Markov-switching", Journal of Econometrics 60, 1-22.

[39] Kim, Chang-Jin and Charles R. Nelson (1999), *State-Space Models with Regime-Switching: Classical and Gibbs-Sampling Approaches with Applications*, MIT Press.

[40] King, Robert G., Charles I. Plosser and Sergio T. Rebelo (1988), "Production, Growth and Business Cycles I. The Basic Neoclassical Model", Journal of Monetary Economics 21, 195–232

[41] Lubik, Thomas A. and Frank Schorfheide (2004), "Testing for Indeterminacy: An Application to U.S. Monetary Policy", American Economic Review 94, pp. 190-217.

[42] Maddison, Angus (2003), *The World Economy: Historical Statistics*, Paris: OECD Publishing.

[43] McDonnell, M. M. and Perez-Quiros, G. (2000) "Output Fluctuations in the United States: What has Changed Since the Early 1980s?", American Economic Review 90, 1464-1467

[44] Piazzesi, M. and M. Schneider (2006), "Equilibrium yield curves", in D. Acemoglu, K. Rogoff and M. Woodford, eds., NBER Macroeconomics Annual 21, 389 - 472.

[45] Primiceri, Giorgio (2005), "Time Varying Structural Vector Autoregressions and Monetary Policy", Review of Economic Studies 72, 821-852.

[46] Restoy, Fernando and Philippe Weil (2011), “Approximate Equilibrium Asset Prices”, Review of Finance 15, 1-28.

[47] Rudebusch, Glenn D. and Eric Swanson (2012), “The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks”, American Economic Journal: Macroeconomics 4, 105-43.

[48] Schmitt-Grohé, Stephanie and Martín Uribe (2004a), "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function", Journal of Economic Dynamics and Control 28, 755-775.

[49] Schmitt-Grohé, Stephanie and Martín Uribe (2004b), "Optimal fiscal and monetary policy under sticky prices", Journal of Economic Theory 114, 198–230.

[50] Schorfeide, F. (2005), "Learning and Monetary Policy Shifts,"Review of Economic Dynamics 8, 392-419.

[51] Sims, Christopher A. and Tao Zha (2006), "Were There Regime Switches in U.S. Monetary Policy?", American Economic Review 96, 54 - 81.

[52] Swanson, Eric T. (2012), "Risk Aversion and the Labor Margin in Dynamic Equilibium Models," American Economic Review 102, 1663-1691.

[53] Swanson, Eric T. (2014), "A macroeconomic model of equities and real, nominal, and defaultable debt", mimeo.

[54] Trabandt, M. and H. Uhlig (2011), "The Laffer Curve Revisited", Journal of Monetary Economics 58, 305-327.

[55] van Binsbergen, Jules H., Jesús Fernández-Villaverde, Ralph S.J. Koijen and Juan Rubio-Ramírez (2012), "The term structure of interest rates in a DSGE model with recursive preferences", Journal of Monetary Economics 59, 634–648.

[56] Woodford, Michael (2003), Interest and Prices: Foundations of a Theory of Monetary Policy, Princeton University Press.

[57] Wright, Jonathan H. (2014), "Term Premia and Inflation Uncertainty: Empirical Evidence from an International Panel Dataset: Reply", American Economic Review 104, 338–341.

Table 1(a): Structural parameter estimates

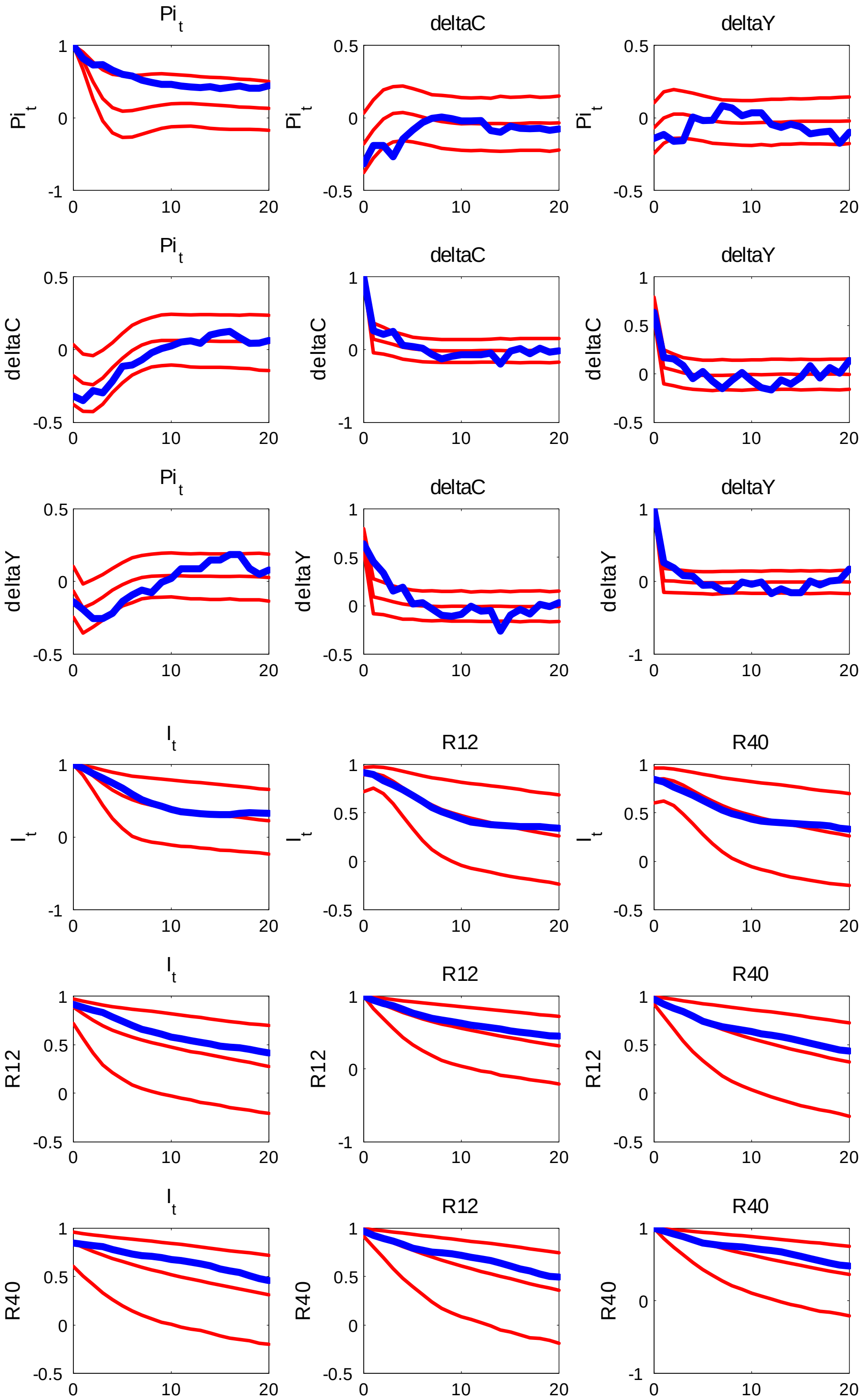
	post mean	**post sd**	**prior mean**	**prior sd**
$p_{G,11}$	0.875997	0.055635	0.899727	0.065687
$p_{G,00}$	0.941294	0.035121	0.899437	0.066248
$p_{\eta,11}$	0.959538	0.019619	0.899591	0.065683
$p_{\eta,00}$	0.907894	0.044664	0.899823	0.065774
$p_{z,11}$	0.972819	0.009089	0.901282	0.065122
$p_{z,00}$	0.931666	0.019045	0.899314	0.066243
$\sigma_{G,1}$	0.003269	0.002076	0.002144	0.0008
$\sigma_{G,0}$	0.031624	0.003721	0.003929	0.002788
$\sigma_{\eta,1}$	0.001279	0.000131	0.002135	0.000788
$\sigma_{\eta,0}$	0.003882	0.000538	0.00391	0.002723
$\sigma_{z,1}$	0.010891	0.002059	0.002152	0.000801
$\sigma_{z,0}$	0.02705	0.004998	0.003945	0.002854
σ_{μ}	0.17681	0.021744	0.003056	0.002584
σ_{ξ}	0.006191	0.000326	0.00119	0.0005
ρ_{μ}	0.548747	0.058116	0.855175	0.091594
ρ_z	0.988924	0.001815	0.858245	0.089933
ρ_G	0.909108	0.029819	0.855938	0.090583
Π	1.006143	0.00069	1.006255	0.00072
ψ_{π}	0.267607	0.024073	0.199031	0.10011
ψ_y	0.049662	0.007535	0.02004	0.009968
ρ_i	0.913538	0.016879	0.849432	0.10022
Ξ	1.004527	0.000413	1.005008	0.001003
ι	0.733358	0.111614	0.500288	0.189923
ϕ	0.615584	0.084646	1.002252	0.504916
γ	11.51852	3.674717	10.95369	6.972984
ψ	1.307529	0.086758	1.203535	0.28997
ζ	33.80709	3.134418	14.97439	6.981933
h	0.861861	0.026101	0.499611	0.188565
β	0.998395	0.000567	0.998567	0.001429

Table 1(b): Measurement errors

	post mean	**post sd**	**prior mean**	**prior sd**
$\sigma_{me,\pi}$	1.4E-06	1.6E-06	1.4E-06	1.3E-06
$\sigma_{me,\Delta c}$	1.3E-06	6.8E-07	1.4E-06	1.1E-06
$\sigma_{me,\Delta y}$	0.003607	0.000617	0.000505	0.00027
$\sigma_{me,i}$	1.3E-06	7.5E-07	1.4E-06	1.0E-06
$\sigma_{me,i_{12}}$	0.00072	7.6E-05	0.001378	0.001037
$\sigma_{me,i40}$	0.000437	5.0E-05	0.001381	0.000999

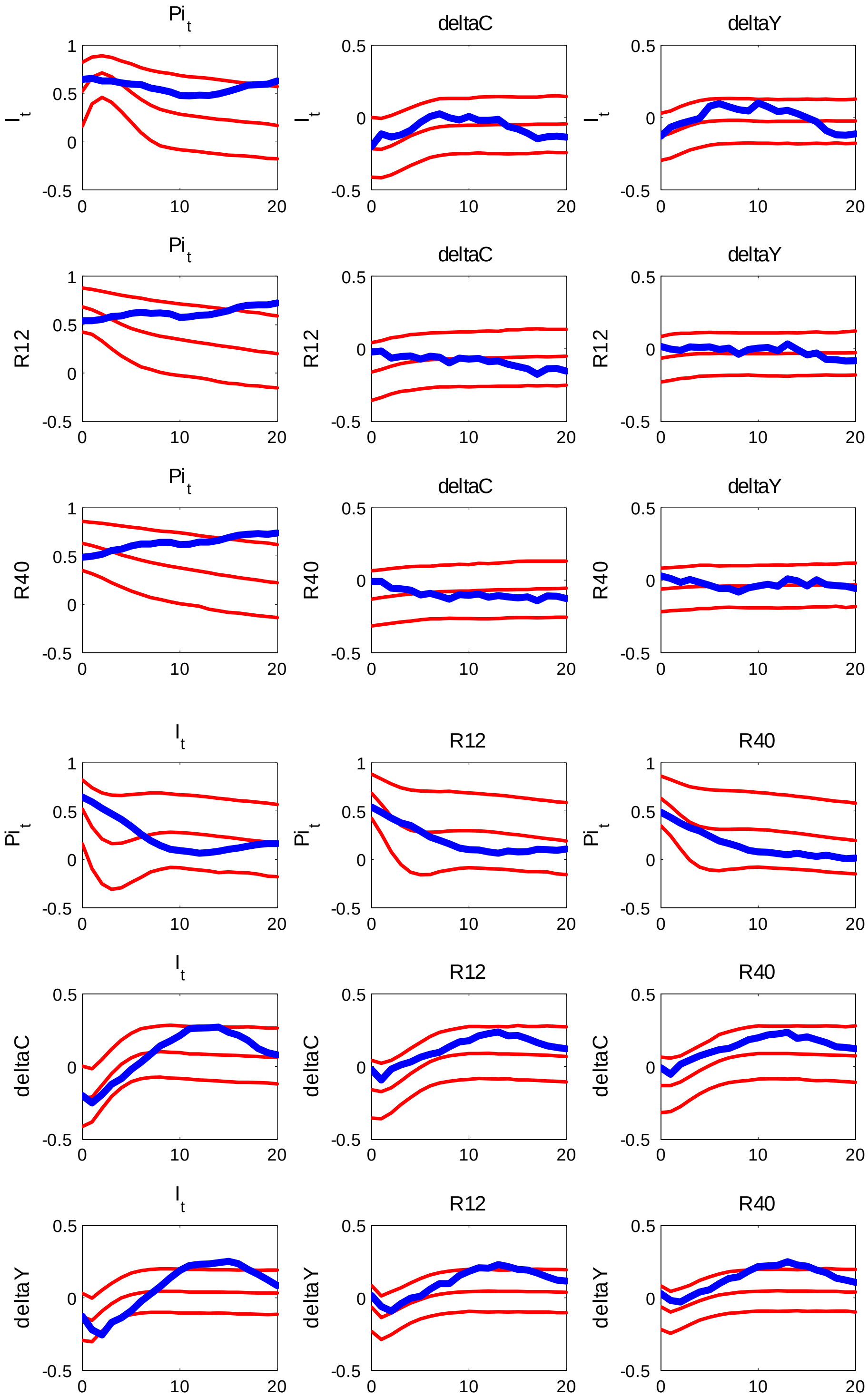
Legend: "sd" denotes the standard deviation; "low q" and "up q" denote the 5th and 95th percentiles of the distribution. Priors: beta distribution for β, h, ι, ζ, ρ_{ξ}, ρ_g, ρ_{π}; gamma distribution for ψ_{π}, ψ_y and all standard deviations; shifted gamma distribution (domain from 1 to ∞) for γ, ϕ, Ξ, Π^*; normal distribution for ρ_i. Posterior distributions are based on 50,000 draws.

Figure 1(a): Dynamic correlations



Note: the green lines display correlation coefficients from the data; the red lines report the mean and the 5th and 95th percentiles of the distribution across parameter draws of the theoretical correlation coefficients implied by the model.

Figure 1(b): Dynamic correlations



Note: the green lines display correlation coefficients from the data; the red lines report the mean and the 5th and 95th percentiles of the distribution across parameter draws of the theoretical correlation coefficients implied by the model.

Figure 2: Actual and model-implied 3-month forward rates (annualised percentage points)

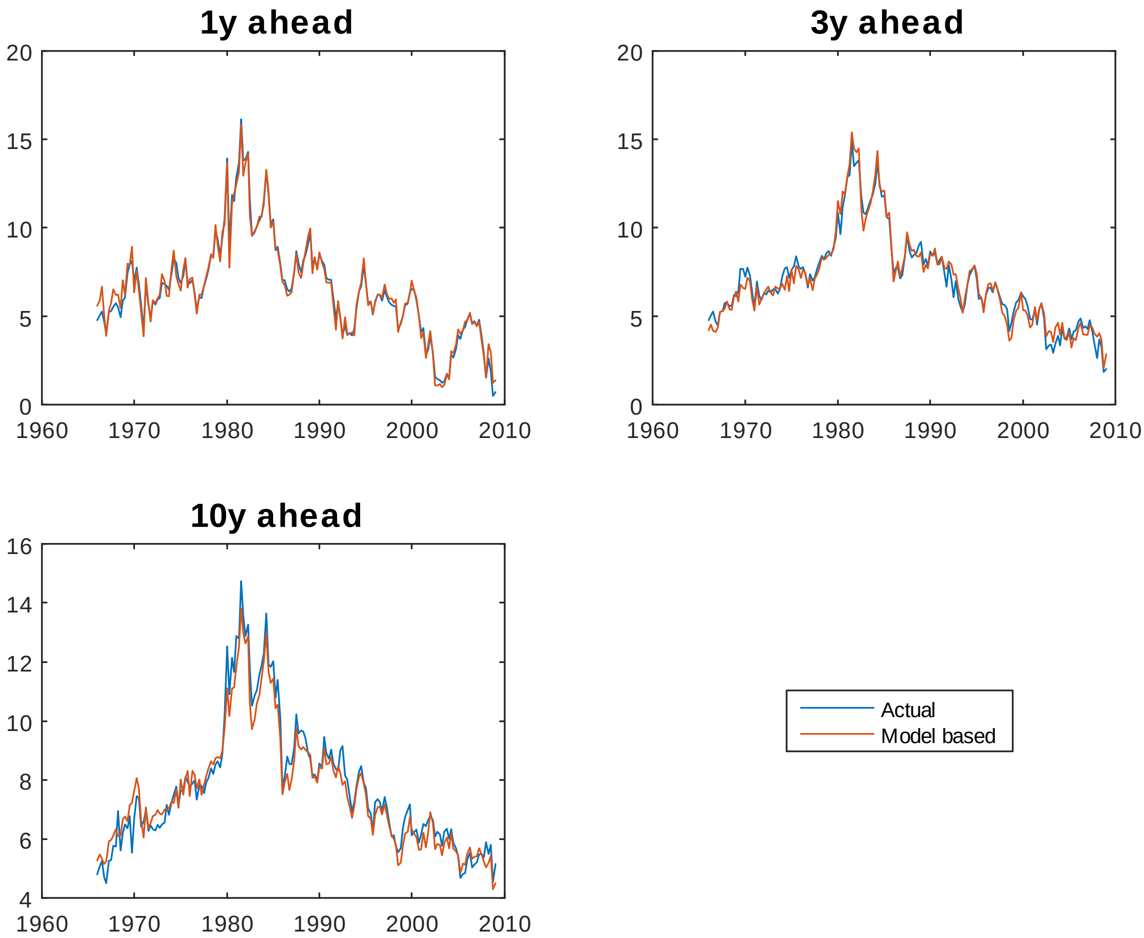
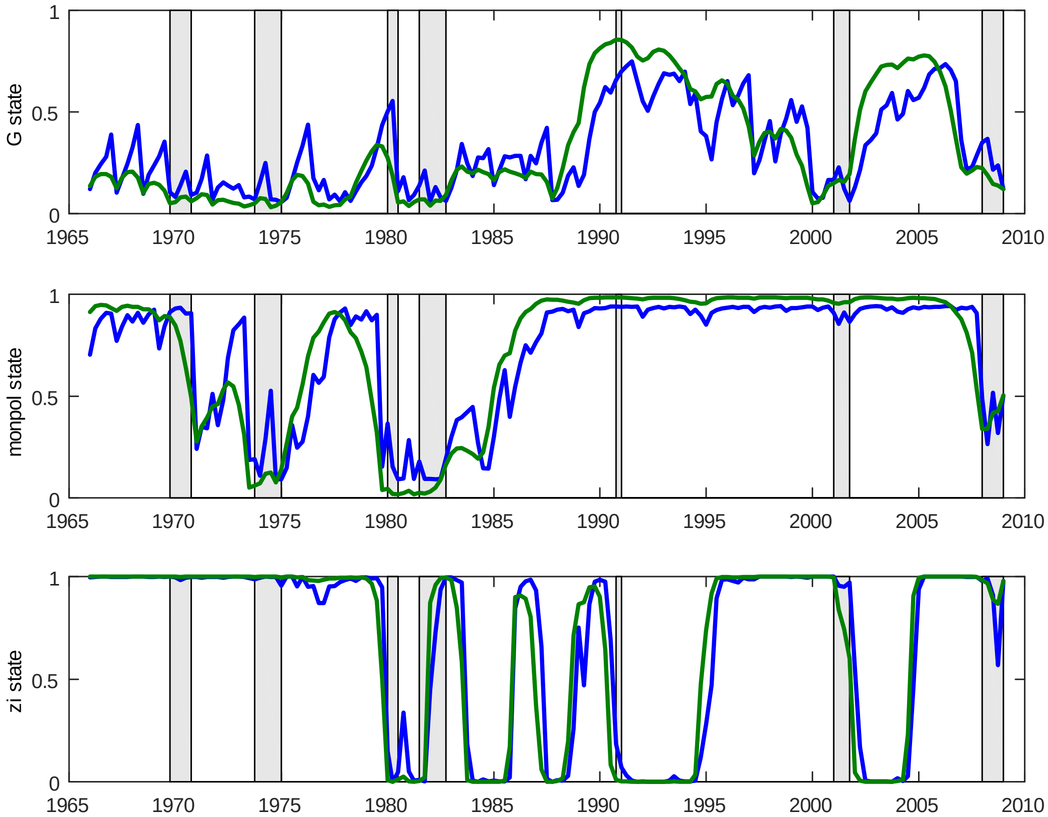


Figure 3: Marginal probability of a low-variance regime



Legend: filtered values in blue; smoothed values in green.

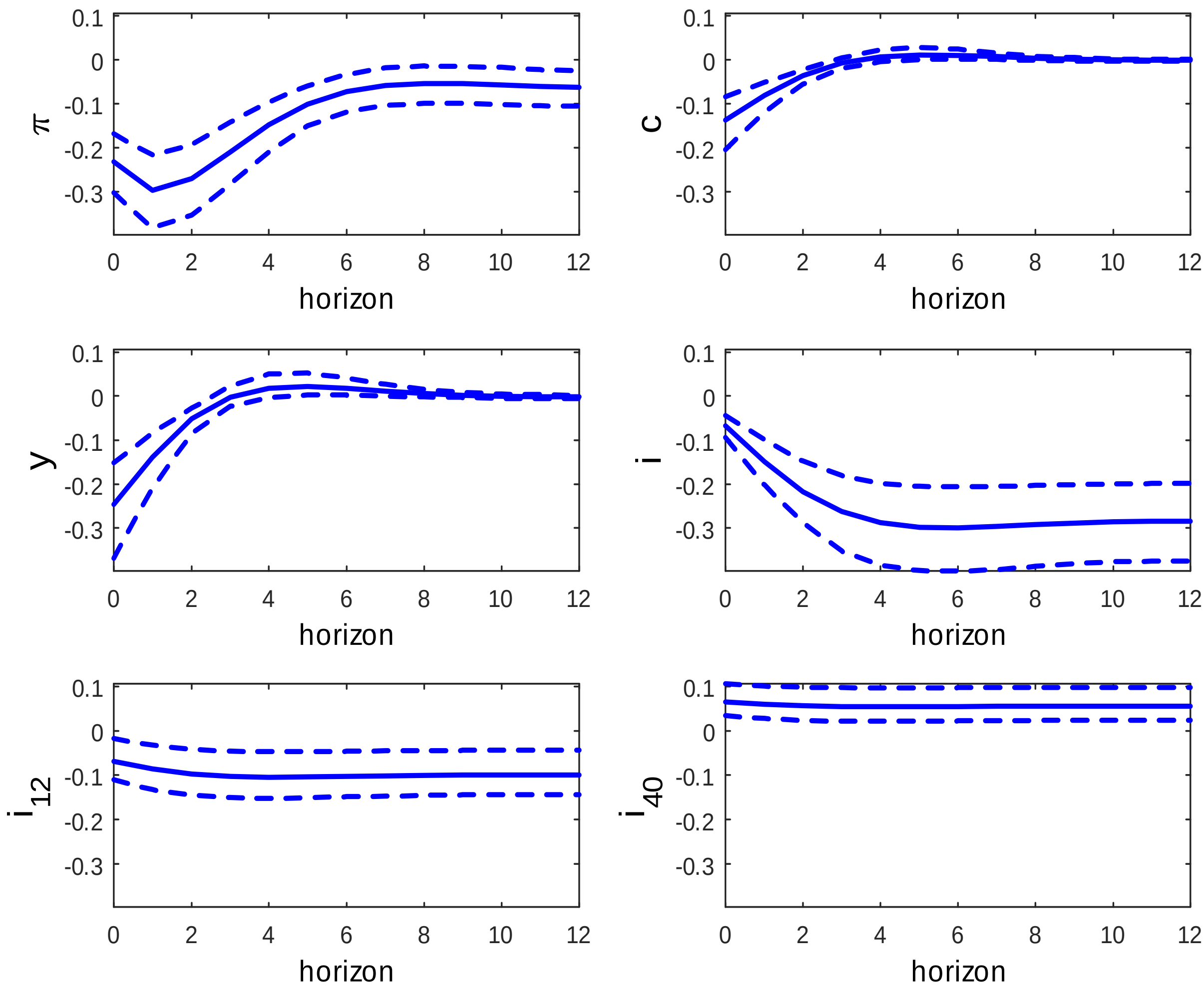


Figure 4: Impulse responses to an increase in the variance of technology shocks

Figure 5: Filtered expected excess holding period returns on 10-year and 3-year bonds (annualised percentage points)

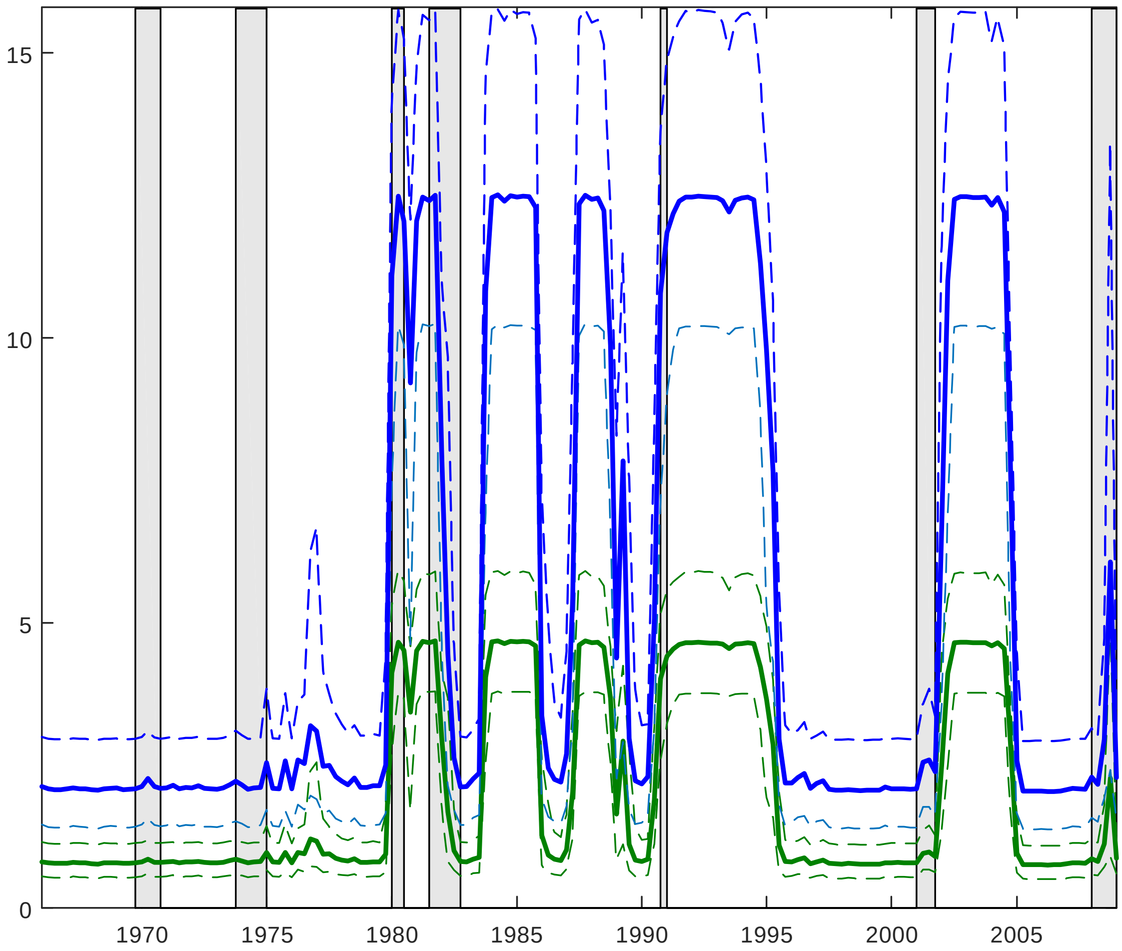


Figure 6: Long-term forward and expected interest rates and risk premia during the conundrum period (annualised percentage points)

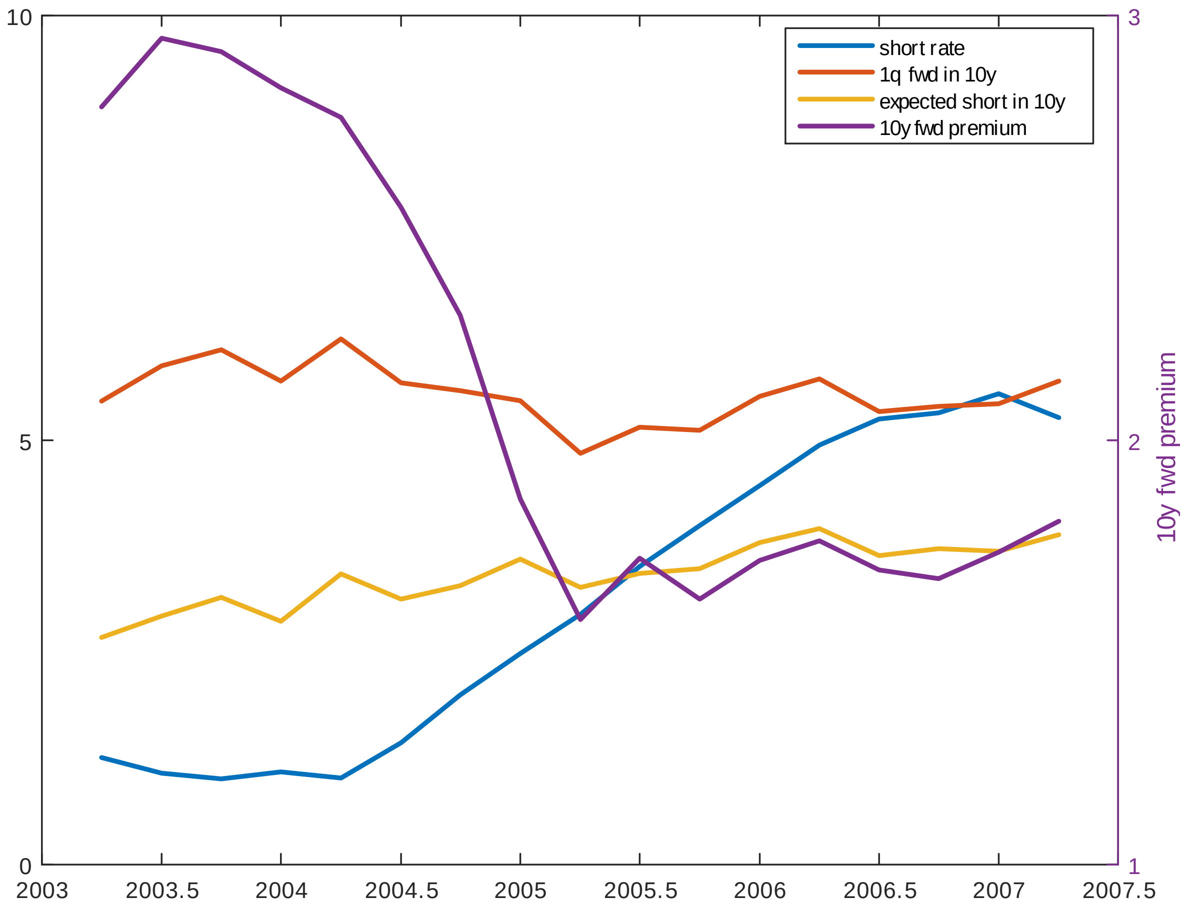
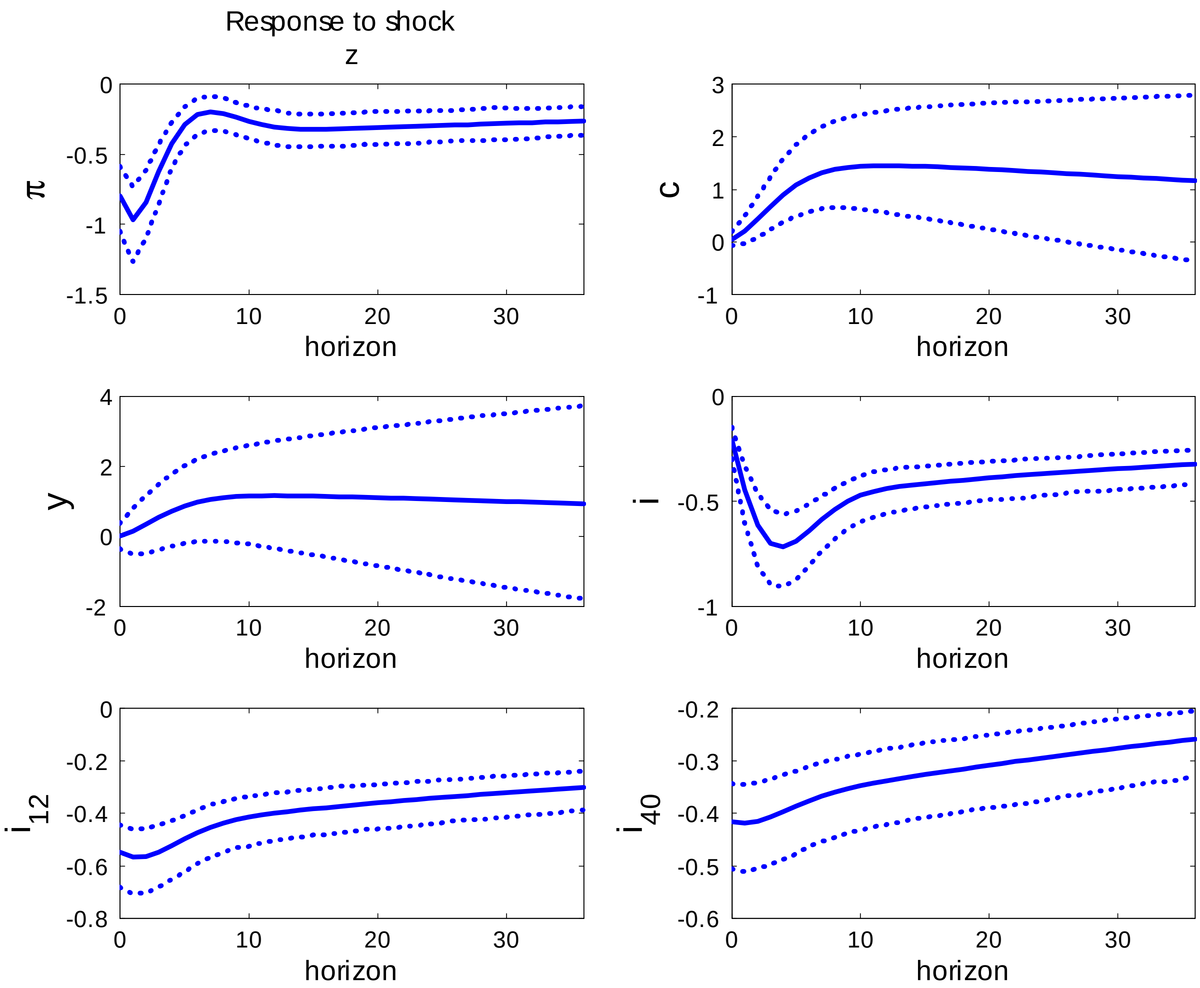


Figure 7: Impulse responses to a technology shock



Note: this impulse response is drawn abstracting from regime switches.

Figure 8: Survey and model-implied inflation expectations in the 1980s

Appendix

A The household problem

The optimization problem is:

$$\max U\left[u_t, \mathrm{E}_t V_{t+1}\right] = \left\{(1-\beta)\,u_t^{1-\psi} + \beta\left(\mathrm{E}_t V_{t+1}^{1-\gamma}\right)^{\frac{1-\psi}{1-\gamma}}\right\}^{\frac{1}{1-\psi}}$$

where u_t is shorthand for $u\left\{C_t\left(j\right) - h\Xi_t C_{t-1}, 1 - N_t\left(j\right)\right\}$, subject to

$$P_t C_t\left(j\right) + \mathrm{E}_t Q_{t,t+1} W_{t+1}\left(j\right) \leq W_t\left(j\right) + w_t\left(j\right) N_t\left(j\right) + \int_0^1 \Psi_t\left(i\right) di - T_t$$

and

$$N_t\left(j\right) = L_t\left(\frac{w_t\left(j\right)}{w_t}\right)^{-\theta_{w,t}}$$

where the choice variables are w_s and c_s

Bellman equation is

$$\begin{aligned} J\left(W_t\right) &= \max\left\{(1-\beta)\,u_t^{1-\psi} + \beta\left[\mathrm{E}_t J^{1-\gamma}\left(W_{t+1}\right)\right]^{\frac{1-\psi}{1-\gamma}}\right\}^{\frac{1}{1-\psi}} \\ &\quad -\Lambda_t\left[P_t C_t + \mathrm{E}_t Q_{t,t+1} W_{t+1} - W_t - w_t N_t - \int_0^1 \Psi_t\left(i\right) di + T_t\right] \end{aligned}$$

where

$$N_t\left(j\right) = L_t\left(\frac{w_t\left(j\right)}{w_t}\right)^{-\theta_{w,t}}$$

and

$$\frac{\partial N_t\left(j\right)}{\partial w_t\left(j\right)} = -\theta_{w,t}\frac{N_t\left(j\right)}{w_t\left(j\right)}$$

Using the aggregator function $U = \left\{(1-\beta)\,u_t^{1-\psi} + \beta v_t^{1-\psi}\right\}^{\frac{1}{1-\psi}}$, where $v_t \equiv \left[\mathrm{E}_t J^{1-\gamma}\left(W_{t+1}, C_t\right)\right]^{\frac{1}{1-\gamma}}$ define

$$U_{u,t} = (1-\beta)\left\{(1-\beta)\,u_t^{1-\psi} + \beta v_t^{1-\psi}\right\}^{\frac{\psi}{1-\psi}} u_t^{-\psi}$$

$$U_{v,t} = \beta\left\{(1-\beta)\,u_t^{1-\psi} + \beta v_t^{1-\psi}\right\}^{\frac{\psi}{1-\psi}} v_t^{-\psi}.$$

The FOCs include

$$U_{u,t} u_{c,t} = \Lambda_t P_t$$

$$u_{N,t} U_{u,t}\frac{\partial N_t\left(j\right)}{\partial w_t\left(j\right)} = -\Lambda_t\left[N_t\left(j\right) + w_t\left(j\right)\frac{\partial N_t\left(j\right)}{\partial w_t\left(j\right)}\right]$$

and state-by-state

$$U_{v,t}\left[\mathrm{E}_t J^{1-\gamma}\left(W_{t+1}\right)\right]^{\frac{\gamma}{1-\gamma}} J^{-\gamma}\left(W_{t+1}\right) J_W\left(W_{t+1}\right) = \Lambda_t Q_{t,t+1}$$

plus envelope

$$J_W\left(W_t\right) = \Lambda_t$$

The FOCs can be rewritten as

$$
\begin{aligned}
\frac{\Lambda_t P_t}{u_{c,t}} &= U_{u,t} \\
\frac{u_{N,t}}{u_{c,t}} &= \frac{1-\theta_{w,t}}{\theta_{w,t}} \frac{w_t(j)}{P_t} \\
Q_{t,t+1} &= U_{v,t} \left[\mathrm{E}_t J_{t+1}^{1-\gamma} \right]^{\frac{\gamma}{1-\gamma}} J_{t+1}^{-\gamma} \frac{\Lambda_{t+1}}{\Lambda_t}
\end{aligned}
$$

or

$$
Q_{t,t+1} = \beta \left(\frac{\left[\mathrm{E}_t J_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}}{J_{t+1}} \right)^{\gamma-\psi} \frac{u_{t+1}^{-\psi}}{u_t^{-\psi}} \frac{u_{c,t+1}}{u_{c,t}} \frac{1}{\Pi_{t+1}}
$$

Using the definition of $\mu_{w,t}$, we obtain, as in the text,

$$
-\frac{u_{N,t}}{u_{c,t}} = \mu_{w,t} \frac{w_t(j)}{P_t}
$$

and

$$
Q_{t,t+1} = \beta \left[\mathrm{E}_t \left(\frac{J_{t+1}}{J_t} \right)^{1-\gamma} \right]^{\frac{\gamma-\psi}{1-\gamma}} \left(\frac{J_{t+1}}{J_t} \right)^{-(\gamma-\psi)} \left(\frac{u_{t+1}}{u_t} \right)^{-\psi} \frac{u_{c,t+1}}{u_{c,t}} \frac{1}{\Pi_{t+1}}
$$

A.1 Detrending

Given the stochastic trend B_t, define a detrended variable as $\widetilde{x}_t \equiv x_t / B_t$. It follows that we can rewrite the conditions above as

$$
\begin{aligned}
-\frac{\widetilde{u}_{N,t}}{u_{c,t}} &= \frac{\theta_{w,t}-1}{\theta_{w,t}} \frac{\widetilde{w}_t(j)}{P_t} \\
\widetilde{J}_t^{1-\psi} &= (1-\beta)\, \widetilde{u}_t^{1-\psi} + \beta \left[\mathrm{E}_t \Xi_{t+1}^{1-\gamma} \widetilde{J}_{t+1}^{1-\gamma} \right]^{\frac{1-\psi}{1-\gamma}} \\
\widetilde{u}_t &= u\left(\widetilde{C}_t(j) - h\widetilde{C}_{t-1}, 1 - N_t(j) \right) \\
Q_{t,t+1} &= \beta \left(\frac{\left[\mathrm{E}_t \widetilde{J}_{t+1}^{1-\gamma} \Xi_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}}{\widetilde{J}_{t+1} \Xi_{t+1}} \right)^{\gamma-\psi} \left(\frac{\widetilde{u}_{t+1}}{\widetilde{u}_t} \right)^{-\psi} \frac{u_{c,t+1}}{u_{c,t}} \frac{1}{\Pi_{t+1} \Xi_{t+1}^{\psi}}
\end{aligned}
$$

A.2 Consumption

To second order, the Euler equation can be written as

$$
-\widehat{i}_t + \frac{1}{2} \widehat{i}_t^2 = \mathrm{E}_t \widehat{q}_{t,t+1} + \frac{1}{2} \mathrm{E}_t \widehat{q}_{t,t+1}^2
$$

where $\widehat{i}_t^2$ can be derived using only first order terms to obtain

$$
\widehat{i}_t = -\mathrm{E}_t \widehat{q}_{t,t+1} - \frac{1}{2} \mathrm{Var}_t \widehat{q}_{t,t+1}
$$

Rewrite the stochastic discount factor $Q_{t,t+1}$ as

$$\begin{aligned}
\widetilde{\Lambda}_t &\equiv \widetilde{u}_t^{-\psi} u_{c,t} \\
D_t &\equiv \mathrm{E}_t \widetilde{J}_{t+1}^{1-\gamma} \Xi_{t+1}^{1-\gamma} \\
Q_{t,t+1} &= \beta \frac{D_t^{\frac{\gamma-\psi}{1-\gamma}}}{\widetilde{J}_{t+1}^{\gamma-\psi}} \frac{\widetilde{\Lambda}_{t+1}}{\widetilde{\Lambda}_t} \frac{1}{\Pi_{t+1} \Xi_{t+1}^{\gamma}}
\end{aligned}$$

so that

$$\widehat{q}_{t,t+1} = \Delta \widehat{\widetilde{\lambda}}_{t+1} - \widehat{\pi}_{t+1} - \psi \widehat{\xi}_{t+1} + \frac{\gamma-\psi}{1-\gamma} \widehat{d}_t - (\gamma-\psi) \widehat{\widetilde{j}}_{t+1}$$

Now expand $\widehat{d}_t$ to second order (again using only first order terms to evaluate $\widehat{d}_t^2$) to find

$$\widehat{d}_t = (1-\gamma) \mathrm{E}_t \widehat{\xi}_{t+1} + (1-\gamma) \mathrm{E}_t \widehat{\widetilde{j}}_{t+1} + \frac{1}{2} (1-\gamma)^2 \mathrm{Var}_t \widehat{\xi}_{t+1} + \frac{1}{2} (1-\gamma)^2 \mathrm{Var}_t \widehat{\widetilde{j}}_{t+1} + (1-\gamma)^2 \mathrm{Cov}_t \widehat{\xi}_{t+1} \widehat{\widetilde{j}}_{t+1}$$

It follows that

$$\begin{aligned}
\widehat{q}_{t,t+1} &= \Delta \widehat{\widetilde{\lambda}}_{t+1} - \psi \widehat{\xi}_{t+1} - \widehat{\pi}_{t+1} - (\gamma-\psi) \left(\widehat{\xi}_{t+1} + \widehat{\widetilde{j}}_{t+1} - \mathrm{E}_t \left[\widehat{\xi}_{t+1} + \widehat{\widetilde{j}}_{t+1} \right] \right) \\
&\quad + \frac{1}{2} (1-\gamma)(\gamma-\psi) \mathrm{Var}_t \left[\widehat{\xi}_{t+1} + \widehat{\widetilde{j}}_{t+1} \right]
\end{aligned}$$

We now evaluate $\mathrm{E}_t \widehat{q}_{t,t+1}$ and $\mathrm{Var}_t \widehat{q}_{t,t+1}$ to obtain

$$\mathrm{E}_t \widehat{q}_{t,t+1} = \mathrm{E}_t \Delta \widehat{\widetilde{\lambda}}_{t+1} - \psi \mathrm{E}_t \widehat{\xi}_{t+1} - \mathrm{E}_t \widehat{\pi}_{t+1} + \frac{1}{2} (1-\gamma)(\gamma-\psi) \mathrm{Var}_t \left[\widehat{\xi}_{t+1} + \widehat{\widetilde{j}}_{t+1} \right]$$

and (using first order terms to evaluate $\widehat{q}_{t,t+1}^2$)

$$\begin{aligned}
\mathrm{E}_t \widehat{q}_{t,t+1}^2 &= \mathrm{Var}_t \left[\Delta \widehat{\widetilde{\lambda}}_{t+1} - \psi \widehat{\xi}_{t+1} - \widehat{\pi}_{t+1} \right] + (\gamma-\psi)^2 \mathrm{Var}_t \left[\widehat{\xi}_{t+1} + \widehat{\widetilde{j}}_{t+1} \right] \\
&\quad - 2 (\gamma-\psi) \mathrm{Cov}_t \left[\Delta \widehat{\widetilde{\lambda}}_{t+1} - \psi \widehat{\xi}_{t+1} - \widehat{\pi}_{t+1}, \widehat{\xi}_{t+1} + \widehat{\widetilde{j}}_{t+1} \right]
\end{aligned}$$

Hence

$$\begin{aligned}
\widehat{i}_t &= \mathrm{E}_t \left[-\Delta \widehat{\widetilde{\lambda}}_{t+1} + \psi \widehat{\xi}_{t+1} + \widehat{\pi}_{t+1} \right] + \frac{1}{2} (\gamma-\psi)(\psi-1) \mathrm{Var}_t \left[\widehat{\xi}_{t+1} + \widehat{\widetilde{j}}_{t+1} \right] \\
&\quad - \frac{1}{2} \mathrm{Var}_t \left[\Delta \widehat{\widetilde{\lambda}}_{t+1} - \psi \widehat{\xi}_{t+1} - \widehat{\pi}_{t+1} \right] + (\gamma-\psi) \mathrm{Cov}_t \left[\Delta \widehat{\widetilde{\lambda}}_{t+1} - \psi \widehat{\xi}_{t+1} - \widehat{\pi}_{t+1}, \widehat{\xi}_{t+1} + \widehat{\widetilde{j}}_{t+1} \right]
\end{aligned}$$

We now expand $\widehat{\widetilde{\lambda}}_{t+1}$ and $\widehat{\widetilde{j}}_{t+1}$ for the specific case of the Trabandt and Uhlig (2011) form for temporary utility, which we use in the numerical application

$$\widetilde{u}_t = \left(\widetilde{C}_t - h \widetilde{C}_{t-1} \right) \left(1 - \eta (1-\psi) N_t^{1+\frac{1}{\phi}} \right)^{\frac{\psi}{1-\psi}}$$

and which, after defining surplus consumption $\overleftrightarrow{c}_t = \widetilde{C}_t - h \widetilde{C}_{t-1}$, implies

$$\begin{aligned}
\widetilde{\Lambda}_t &= \overleftrightarrow{c}_t^{-\psi} \left(1 - \eta (1-\psi) N_t^{1+\frac{1}{\phi}} \right)^{\psi} \\
\widetilde{J}_t^{1-\psi} &= (1-\beta) \overleftrightarrow{c}_t \widetilde{\overleftrightarrow{\Lambda}}_t + \beta \left[\mathrm{E}_t \Xi_{t+1}^{1-\gamma} \widetilde{J}_{t+1}^{1-\gamma} \right]^{\frac{1-\psi}{1-\gamma}}
\end{aligned}$$

A.2.1 Expanding Λ_t

To second order

$$\begin{aligned}
\hat{\hat{\lambda}}_t + \frac{1}{2}\hat{\hat{\lambda}}_t^2 &= -\psi \widehat{\overleftrightarrow{c}}_t - \psi \left(1+\frac{1}{\phi}\right) \frac{\eta (1-\psi) N_t^{1+\frac{1}{\phi}}}{1-\eta (1-\psi) N_t^{1+\frac{1}{\phi}}} \widehat{l}_t + \frac{1}{2}\psi^2 \widehat{\overleftrightarrow{c}}_t^2 \\
&\quad -\frac{1}{2}\psi \left(1+\frac{1}{\phi}\right)^2 \frac{\eta (1-\psi) N_t^{1+\frac{1}{\phi}} \left(1-\eta\sigma (1-\psi) N_t^{1+\frac{1}{\phi}}\right)}{\left(1-\eta (1-\psi) N_t^{1+\frac{1}{\phi}}\right)^2} \widehat{l}_t^2 \\
&\quad +\psi^2 \left(1+\frac{1}{\phi}\right) \frac{\eta (1-\psi) N_t^{1+\frac{1}{\phi}}}{1-\eta (1-\psi) N_t^{1+\frac{1}{\phi}}} \widehat{\overleftrightarrow{c}}_t \widehat{l}_t
\end{aligned}$$

and using first order terms to evaluate $\hat{\hat{\lambda}}_t^2$

$$\hat{\hat{\lambda}}_t = -\psi \widehat{\overleftrightarrow{c}}_t - \psi \left(1+\frac{1}{\phi}\right) \frac{\eta (1-\psi) N_t^{1+\frac{1}{\phi}}}{1-\eta (1-\psi) N_t^{1+\frac{1}{\phi}}} \widehat{l}_t - \frac{1}{2}\psi \left(1+\frac{1}{\phi}\right)^2 \frac{\eta (1-\psi) N_t^{1+\frac{1}{\phi}}}{\left(1-\eta (1-\psi) N_t^{1+\frac{1}{\phi}}\right)^2} \widehat{l}_t^2$$

It follows that

$$\Delta \hat{\hat{\lambda}}_{t+1} = -\sigma \Delta \widehat{\overleftrightarrow{c}}_{t+1} - \psi \left(1+\frac{1}{\phi}\right) \frac{\overline{n}}{1-\overline{n}} \Delta \widehat{l}_{t+1} - \frac{1}{2}\psi \left(1+\frac{1}{\phi}\right)^2 \frac{\overline{n}}{(1-\overline{n})^2} \left(\widehat{l}_{t+1}^2 - \widehat{l}_t^2\right)$$

for

$$\overline{n} \equiv \eta (1-\psi) N^{1+\frac{1}{\phi}}$$

Surplus consumption $\overleftrightarrow{c}_t$ can be expanded as

$$\widehat{\overleftrightarrow{c}}_t = \frac{1}{1-h}\left(\hat{\hat{c}}_t - h\hat{\hat{c}}_{t-1}\right) - \frac{1}{2}\frac{h}{(1-h)^2}\left(\hat{\hat{c}}_t - \hat{\hat{c}}_{t-1}\right)^2$$

so that

$$\begin{aligned}
\Delta \hat{\hat{\lambda}}_{t+1} &= -\psi \frac{1}{1-h}\left(\Delta \hat{\hat{c}}_{t+1} - h\Delta \hat{\hat{c}}_t\right) - \psi \left(1+\frac{1}{\phi}\right) \frac{\overline{n}}{1-\overline{n}} \Delta \widehat{l}_{t+1} \\
&\quad + \frac{1}{2}\psi \frac{h}{(1-h)^2}\left[\left(\Delta \hat{\hat{c}}_{t+1}\right)^2 - \left(\Delta \hat{\hat{c}}_t\right)^2\right] - \frac{1}{2}\psi \left(1+\frac{1}{\phi}\right)^2 \frac{\overline{n}}{(1-\overline{n})^2} \left(\widehat{l}_{t+1}^2 - \widehat{l}_t^2\right)
\end{aligned}$$

A.2.2 Expanding $\widetilde{J}$

Note that $\hat{\hat{J}}_{t+1}$ only enters the interest rate in terms of second order. It can therefore be evaluated to first order. We obtain

$$\widetilde{J}_t^{1-\psi} = (1-\beta) \overleftrightarrow{c}_t \widetilde{\Lambda}_t + \beta D_t^{\frac{1-\psi}{1-\gamma}}$$

so that

$$(1-\psi) \hat{\hat{j}}_t = \frac{(1-\beta) \overleftrightarrow{c} \widetilde{\Lambda}}{(1-\beta) \overleftrightarrow{c} \widetilde{\Lambda} + \beta D^{\frac{1-\psi}{1-\gamma}}} \widehat{\overleftrightarrow{c}}_t + \frac{(1-\beta) \overleftrightarrow{c} \widetilde{\Lambda}}{(1-\beta) \overleftrightarrow{c} \widetilde{\Lambda} + \beta D^{\frac{1-\psi}{1-\gamma}}} \hat{\hat{\lambda}}_t + \frac{1-\psi}{1-\gamma} \frac{\beta D^{\frac{1-\psi}{1-\gamma}}}{(1-\beta) \overleftrightarrow{c} \widetilde{\Lambda} + \beta D^{\frac{1-\psi}{1-\gamma}}} \hat{\hat{d}}_t$$

and using $\widehat{d}_t \equiv (1-\gamma)\,\mathrm{E}_t\widehat{\xi}_{t+1} + (1-\gamma)\,\mathrm{E}_t\widehat{\widehat{j}}_{t+1}$

$$
\begin{aligned}
(1-\psi)\,\widehat{\widehat{j}}_t &= \frac{(1-\beta)\overleftrightarrow{c}\widetilde{\Lambda}}{(1-\beta)\overleftrightarrow{c}\widetilde{\Lambda}+\beta D^{\frac{1-\psi}{1-\gamma}}}\widehat{\overleftrightarrow{c}}_t + \frac{(1-\beta)\overleftrightarrow{c}\widetilde{\Lambda}}{(1-\beta)\overleftrightarrow{c}\widetilde{\Lambda}+\beta D^{\frac{1-\psi}{1-\gamma}}}\widehat{\widehat{\lambda}}_t \\
&\quad + (1-\psi)\frac{\beta D^{\frac{1-\psi}{1-\gamma}}}{(1-\beta)\overleftrightarrow{c}\widetilde{\Lambda}+\beta D^{\frac{1-\psi}{1-\gamma}}}\left(\mathrm{E}_t\widehat{\xi}_{t+1}+\mathrm{E}_t\widehat{\widehat{j}}_{t+1}\right)
\end{aligned}
$$

Since to first order $\widehat{\widehat{\lambda}}_t = -\psi\widehat{\overleftrightarrow{c}}_t - \psi\left(1+\frac{1}{\phi}\right)\frac{\eta(1-\psi)N^{1+\frac{1}{\phi}}}{1-\eta(1-\psi)N^{1+\frac{1}{\phi}}}\widehat{l}_t$ we further obtain

$$
\begin{aligned}
\widehat{\widehat{j}}_t &= \frac{(1-\beta)\overleftrightarrow{c}\widetilde{\Lambda}}{(1-\beta)\overleftrightarrow{c}\widetilde{\Lambda}+\beta D^{\frac{1-\psi}{1-\gamma}}}\widehat{\overleftrightarrow{c}}_t - \frac{\psi}{1-\psi}\left(1+\frac{1}{\phi}\right)\frac{\eta(1-\psi)N^{1+\frac{1}{\phi}}}{1-\eta(1-\psi)N^{1+\frac{1}{\phi}}}\frac{(1-\beta)\overleftrightarrow{c}\widetilde{\Lambda}}{(1-\beta)\overleftrightarrow{c}\widetilde{\Lambda}+\beta D^{\frac{1-\psi}{1-\gamma}}}\widehat{l}_t \\
&\quad + \frac{\beta D^{\frac{1-\psi}{1-\gamma}}}{(1-\beta)\overleftrightarrow{c}\widetilde{\Lambda}+\beta D^{\frac{1-\psi}{1-\gamma}}}\left(\mathrm{E}_t\widehat{\xi}_{t+1}+\mathrm{E}_t\widehat{\widehat{j}}_{t+1}\right)
\end{aligned}
$$

Recall that in steady state

$$
\begin{aligned}
\widetilde{J}^{1-\psi} &= \frac{1-\beta}{1-\beta\Xi^{1-\psi}}\overleftrightarrow{c}\widetilde{\Lambda} \\
\widetilde{\Lambda} &= \overleftrightarrow{c}^{-\psi}\left(1-\eta(1-\psi)N^{1+\frac{1}{\phi}}\right)^{\psi} \\
D &\equiv \Xi^{1-\gamma}\widetilde{J}^{1-\gamma}
\end{aligned}
$$

to obtain

$$
\widehat{\widehat{j}}_t = \left(1-\beta\Xi^{1-\psi}\right)\left(\widehat{\overleftrightarrow{c}}_t - \frac{\psi}{1-\psi}\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}\widehat{l}_t\right) + \beta\Xi^{1-\psi}\left(\mathrm{E}_t\widehat{\xi}_{t+1}+\mathrm{E}_t\widehat{\widehat{j}}_{t+1}\right)
$$

This can be solved forward to obtain

$$
\begin{aligned}
\widehat{\widehat{j}}_t + \widehat{\xi}_t &= \lim_{n\to\infty}\sum_{i=0}^{n}\left[\beta\Xi^{1-\psi}\right]^i \mathrm{E}_t\left[\widehat{\xi}_{t+i} + \left(1-\beta\Xi^{1-\psi}\right)\left(\widehat{\overleftrightarrow{c}}_{t+i} - \frac{\psi}{1-\psi}\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}\widehat{l}_{t+i}\right)\right] \\
&\quad + \lim_{n\to\infty}\left[\beta\Xi^{1-\psi}\right]^n\left(\mathrm{E}_t\widehat{\xi}_{t+n}+\mathrm{E}_t\widehat{\widehat{j}}_{t+n}\right)
\end{aligned}
$$

Assuming that $\lim_{n\to\infty}\left[\beta\Xi^{1-\psi}\right]^n\left(\mathrm{E}_t\widehat{\xi}_{t+n}+\mathrm{E}_t\widehat{\widehat{j}}_{t+n}\right) = 0$ and that the other sums converge, we obtain

$$
\widehat{\widehat{j}}_t + \widehat{\xi}_t = \sum_{i=0}^{\infty}\left(\beta\Xi^{1-\psi}\right)^i \mathrm{E}_t\left[\widehat{\xi}_{t+i} + \left(1-\beta\Xi^{1-\psi}\right)\left(\widehat{\overleftrightarrow{c}}_{t+i} - \frac{\psi}{1-\psi}\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}\widehat{l}_{t+i}\right)\right]
$$

or using the first order expansion of $\widehat{\overleftrightarrow{c}}_t$

$$
\widehat{\widehat{j}}_t + \widehat{\xi}_t = \sum_{i=0}^{\infty}\left(\beta\Xi^{1-\psi}\right)^i \mathrm{E}_t\left[\widehat{\xi}_{t+i} + \left(1-\beta\Xi^{1-\psi}\right)\left(\frac{1}{1-h}\left(\widehat{\widehat{c}}_{t+i} - h\widehat{\widehat{c}}_{t+i-1}\right) - \frac{\psi}{1-\psi}\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}\widehat{l}_{t+i}\right)\right]
$$

Note that the first order approximation of temporary utility is

$$
\widehat{\widehat{u}}_t = \frac{1}{1-h}\left(\widehat{\widehat{c}}_t - h\widehat{\widehat{c}}_{t-1}\right) - \frac{\psi}{1-\psi}\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}\widehat{l}_t
$$

Hence

$$\widehat{\widehat{j}}_t + \widehat{\xi}_t = \sum_{i=0}^{\infty} \left(\beta \Xi^{1-\psi}\right)^i \mathrm{E}_t \left[\widehat{\xi}_{t+i} + \left(1 - \beta \Xi^{1-\psi}\right) \widehat{\widehat{u}}_{t+i}\right]$$

A.2.3 Second order approximation to the Euler equation

It follows that the second order approximation to the Euler equation, using also the assumption $\mathrm{E}_t \left[\widehat{\xi}_{t+1}\right] = 0$, can be written as

$$\begin{aligned}
\widehat{\widehat{c}}_t \quad = \quad & \frac{1}{1+h} \mathrm{E}_t \widehat{\widehat{c}}_{t+1} + \frac{h}{1+h} \widehat{\widehat{c}}_{t-1} - \frac{1}{\psi} \frac{1-h}{1+h} \left(\widehat{i}_t - \mathrm{E}_t \left[\widehat{\pi}_{t+1}\right]\right) + \frac{1-h}{1+h} \left(1 + \frac{1}{\phi}\right) \frac{\overline{n}}{1-\overline{n}} \mathrm{E}_t \Delta \widehat{l}_{t+1} - \frac{1}{2} \mathrm{Var}_t \Omega_{t+1} \\
& + \frac{1}{2} (\gamma - \psi) \frac{\psi - 1}{\psi} \frac{1-h}{1+h} \mathrm{Var}_t \left[\widehat{\xi}_{t+1} + \widehat{\widehat{j}}_{t+1}\right] - (\gamma - \psi) \frac{1}{\psi} \frac{1-h}{1+h} \mathrm{Cov}_t \left[\psi \widehat{\xi}_{t+1} + \widehat{\pi}_{t+1}, \widehat{\xi}_{t+1} + \widehat{\widehat{j}}_{t+1}\right] \\
& - (\gamma - \psi) \frac{1-h}{1+h} \mathrm{Cov}_t \left[\frac{1}{1-h} \left(\Delta \widehat{\widehat{c}}_{t+1} - h \Delta \widehat{\widehat{c}}_t\right) + \left(1 + \frac{1}{\phi}\right) \frac{\overline{n}}{1-\overline{n}} \Delta \widehat{l}_{t+1}, \widehat{\xi}_{t+1} + \widehat{\widehat{j}}_{t+1}\right]
\end{aligned}$$

for

$$\begin{aligned}
\mathrm{Var}_t \Omega_{t+1} \quad = \quad & \frac{h}{1-h^2} \left[\mathrm{E}_t \left(\Delta \widehat{\widehat{c}}_{t+1}\right)^2 - \left(\Delta \widehat{\widehat{c}}_t\right)^2\right] - \frac{1-h}{1+h} \left(1 + \frac{1}{\phi}\right)^2 \frac{\overline{n}}{(1-\overline{n})^2} \left(\mathrm{E}_t \widehat{l}_{t+1}^2 - \widehat{l}_t^2\right) \\
& + \frac{1}{\psi} \frac{1-h}{1+h} \mathrm{Var}_t \left[\psi \frac{1}{1-h} \left(\Delta \widehat{\widehat{c}}_{t+1} - h \Delta \widehat{\widehat{c}}_t\right) + \psi \left(1 + \frac{1}{\phi}\right) \frac{\overline{n}}{1-\overline{n}} \Delta \widehat{l}_{t+1} + \psi \widehat{\xi}_{t+1} + \widehat{\pi}_{t+1}\right]
\end{aligned}$$

and

$$\widehat{\widehat{j}}_{t+1} + \widehat{\xi}_{t+1} = \sum_{i=0}^{\infty} \left(\beta \Xi^{1-\psi}\right)^i \mathrm{E}_t \left[\widehat{\xi}_{t+1+i} + \left(1 - \beta \Xi^{1-\psi}\right) \widehat{\widehat{u}}_{t+1+i}\right]$$

and

$$\overline{n} = \eta (1 - \psi) N^{1 + \frac{1}{\phi}}$$

A.3 Expected excess holding period returns

Recall that

$$HPR_{n,t} = \frac{\mathrm{E}_t B_{n-1,t+1}}{B_{n,t}}$$

and

$$\widehat{i}_{t,t+n} = -\frac{1}{n} \widehat{b}_{t,t+n}$$

so that

$$HPR_{2,t} = \frac{\mathrm{E}_t B_{1,t+1}}{B_{2,t}}$$

To second order, holding period return are

$$\widehat{h}_{n,t} + \frac{1}{2} \widehat{h}_{n,t}^2 = \mathrm{E}_t \widehat{b}_{n-1,t+1} - \widehat{b}_{n,t} + \frac{1}{2} \mathrm{E}_t \widehat{b}_{n-1,t+1}^2 + \frac{1}{2} \widehat{b}_{n,t}^2 - \widehat{b}_{n,t} \mathrm{E}_t \widehat{b}_{n-1,t+1}$$

with

$$\frac{1}{2} \widehat{h}_{n,t}^2 = \frac{1}{2} \left(\mathrm{E}_t \widehat{b}_{n-1,t+1}\right)^2 + \frac{1}{2} \widehat{b}_{n,t}^2 - \widehat{b}_{n,t} \mathrm{E}_t \widehat{b}_{n-1,t+1}$$

so that

$$\widehat{h}_{n,t} = -\widehat{b}_{n,t} + \mathrm{E}_t \widehat{b}_{n-1,t+1} + \frac{1}{2} \mathrm{E}_t \widehat{b}_{n-1,t+1}^2 - \frac{1}{2} \left(\mathrm{E}_t \widehat{b}_{n-1,t+1}\right)^2$$

or

$$\widehat{h}_{n,t} = -\widehat{b}_{n,t} + \mathrm{E}_t\widehat{b}_{n-1,t+1} + \frac{1}{2}\mathrm{Var}_t\widehat{b}_{n-1,t+1}$$

and since bond prices are

$$\widehat{b}_{t,n} = \widehat{b}_{t,1} + \mathrm{E}_t\widehat{b}_{t+1,n-1} + \frac{1}{2}\mathrm{Var}_t\widehat{b}_{t+1,n-1} + \mathrm{Cov}_t\left[\widehat{b}_{t+1,n-1}, \widehat{q}_{t,t+1}\right]$$

we can in general rewrite expected holding period returns as

$$\widehat{h}_{n,t} = \widehat{i}_t - \mathrm{Cov}_t\left[\widehat{b}_{t+1,n-1}, \widehat{q}_{t,t+1}\right]$$

Excess holding period returns are therefore

$$\widehat{h}_{n,t} - \widehat{i}_t = -\mathrm{Cov}_t\left[\widehat{b}_{t+1,n-1}, \widehat{q}_{t,t+1}\right]$$

Using the approximated stochastic discount factor, we obtain

$$\widehat{h}_{n,t} - \widehat{i}_t = -\mathrm{Cov}_t\left[\widehat{b}_{t+1,n-1}, \Delta\widehat{\widehat{\lambda}}_{t+1} - \psi\widehat{\xi}_{t+1} - \widehat{\pi}_{t+1} - (\gamma-\psi)\left(\widehat{\xi}_{t+1} + \widehat{\widehat{j}}_{t+1}\right)\right]$$

Now use the first order expansion of $\widehat{\widehat{\lambda}}_t$ to write

$$\widehat{h}_{n,t} - \widehat{i}_t = \mathrm{Cov}_t\left[\widehat{b}_{t+1,n-1}, -\frac{\psi}{1-h}\widehat{\widehat{c}}_{t+1} - \psi\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}\widehat{l}_{t+1} - \psi\widehat{\xi}_{t+1} - \widehat{\pi}_{t+1} - (\gamma-\psi)\left(\widehat{\xi}_{t+1} + \widehat{\widehat{j}}_{t+1}\right)\right]$$

Define the first order approximation of variable v as $F_v\widehat{\mathbf{x}}_t$. Then (note that we use F_j to denote the first order approximation of the infinite sum $\widehat{\xi}_{t+1} + \widehat{\widehat{j}}_{t+1}$)

$$\widehat{h}_{n,t} - \widehat{i}_t = \mathrm{Cov}_t\left[F_{B_{n-1}}\widehat{\mathbf{x}}_{t+1}, \left(\psi\frac{1}{1-h}F_c + \psi\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}F_l + \psi F_\xi + F_\pi + (\gamma-\psi)F_j\right)\widehat{\mathbf{x}}_{t+1}\right]$$

It follows that

$$\begin{aligned}\widehat{h}_{n,t} - \widehat{i}_t &= \mathrm{E}_t\left[F_{B_{n-1}}\widehat{\mathbf{x}}_{t+1}\widehat{\mathbf{x}}'_{t+1}\left(\psi\frac{1}{1-h}F_c + \psi\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}F_l + \psi F_\xi + F_\pi + (\gamma-\psi)F_j\right)'\right] \\ &\quad -\mathrm{E}_tF_{B_{n-1}}\widehat{\mathbf{x}}'_{t+1}\mathrm{E}_t\left[\widehat{\mathbf{x}}'_{t+1}\left(\psi\frac{1}{1-h}F_c + \psi\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}F_l + \psi F_\xi + F_\pi + (\gamma-\psi)F_j\right)'\right]\end{aligned}$$

and using the law of motion for $\widehat{\mathbf{x}}_{t+1}$

$$\widehat{h}_{n,t} - \widehat{i}_t = \widetilde{\sigma}^2 F_{B_{n-1}}\mathrm{E}_t\left[\mathbf{u}_{t+1}\mathbf{u}'_{t+1}\right]\left(\psi\frac{1}{1-h}F_c + \psi\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}F_l + \psi F_\xi + F_\pi + (\gamma-\psi)F_j\right)'$$

B Firms' optimization problem

Under Rotemberg prices, firm j maximizes real profits

$$\max_{P_t^j}\mathrm{E}_t\sum_{s=t}^{\infty}Q_{t,s}\left[\frac{P_s^jY_s^j}{P_s} - \frac{w_s}{P_s}\left(\frac{Y_s^j}{A_s}\right)^{\frac{1}{\alpha}} - \frac{\zeta}{2}\left(\frac{P_s^j}{P_{s-1}^j} - (\Pi^*)^{1-\iota}\Pi_{s-1}^{\iota}\right)^2 Y_s\right]$$

subject to the total demand for its output

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} Y_t$$

and to the production function

$$Y_t(j) = A_t L_t^{\alpha}(j)$$

where L_t is the labour index defined above.

The FOC is

$$\begin{aligned} 0 &= (1-\theta)\left(\frac{P_t^j}{P_t}\right)^{-\theta} Y_t \frac{1}{P_t} + \frac{\theta}{\alpha}\frac{w_t}{P_t}\left(\frac{Y_t}{A_t}\right)^{\frac{1}{\alpha}}\left(\frac{P_t^j}{P_t}\right)^{-\frac{\theta}{\alpha}-1}\frac{1}{P_t} - \zeta\left(\frac{P_t^j}{P_{t-1}^j} - (\Pi^*)^{1-\iota}\Pi_{t-1}^{\iota}\right) Y_t \frac{1}{P_{t-1}^j} \\ &\quad + \mathrm{E}_t Q_{t,t+1}\zeta\left(\frac{P_{t+1}^j}{P_t^j} - (\Pi^*)^{1-\iota}\Pi_t^{\iota}\right) Y_{t+1}\frac{P_{t+1}^j}{P_t^j}\frac{1}{P_t^j} \end{aligned}$$

or, noting that all firms will set the same price and expressing variables in detrended form,

$$(\theta-1)\widetilde{Y}_t + \zeta\left(\Pi_t - (\Pi^*)^{1-\iota}\Pi_{t-1}^{\iota}\right)\widetilde{Y}_t\Pi_t = \frac{\theta}{\alpha}\frac{\widetilde{w}_t}{P_t}\frac{1}{Z_t^{\frac{1}{\alpha}}}\widetilde{Y}_t^{\frac{1}{\alpha}} + \mathrm{E}_t Q_{t,t+1}\zeta\left(\Pi_{t+1} - (\Pi^*)^{1-\iota}\Pi_t^{\iota}\right)\widetilde{Y}_{t+1}\Xi_{t+1}\Pi_{t+1}$$

C Equilibrium

Equilibrium is described by the following system:

- households

$$\begin{aligned} \frac{\Lambda_t P_t}{u_{c,t}} &= (1-\beta)\widetilde{u}_t^{-\psi}\widetilde{J}_t^{\psi} \\ -\frac{\widetilde{u}_{N,t}}{u_{c,t}} &= \frac{\theta_{w,t}-1}{\theta_{w,t}}\frac{\widetilde{w}_t}{P_t} \\ \widetilde{J}_t^{1-\psi} &= (1-\beta)\widetilde{u}_t^{1-\psi} + \beta\left[\mathrm{E}_t\Xi_{t+1}^{1-\gamma}\widetilde{J}_{t+1}^{1-\gamma}\right]^{\frac{1-\psi}{1-\gamma}} \\ \widetilde{u}_t &= u\left(\widetilde{C}_t - h\widetilde{C}_{t-1}, 1-N_t\right) \\ Q_{t,t+1} &= \beta\left[\mathrm{E}_t\widetilde{J}_{t+1}^{1-\gamma}\Xi_{t+1}^{1-\gamma}\right]^{\frac{\gamma-\psi}{1-\gamma}}\frac{\widetilde{J}_t^{\psi}}{\widetilde{J}_{t+1}^{\gamma}\Xi_{t+1}^{\gamma}}\frac{\Lambda_{t+1}}{\Lambda_t} \end{aligned}$$

- firms

$$\begin{aligned} (\theta-1)\widetilde{Y}_t &= -\zeta\left(\Pi_t - (\Pi^*)^{1-\iota}\Pi_{t-1}^{\iota}\right)\widetilde{Y}_t\Pi_t + \frac{\theta}{\alpha}\frac{\widetilde{w}_t}{P_t}\frac{1}{Z_t^{\frac{1}{\alpha}}}\widetilde{Y}_t^{\frac{1}{\alpha}} \\ &\quad + \mathrm{E}_t Q_{t,t+1}\zeta\left(\Pi_{t+1} - (\Pi^*)^{1-\iota}\Pi_t^{\iota}\right)\widetilde{Y}_{t+1}\Xi_{t+1}\Pi_{t+1} \end{aligned}$$

- market clearing

$$\begin{aligned} \widetilde{Y}_t &= \widetilde{C}_t + \widetilde{G}_t + \frac{\zeta}{2}\left(\Pi_t - (\Pi^*)^{1-\iota}\Pi_{t-1}^{\iota}\right)^2\widetilde{Y}_t \\ N_t &= \widetilde{Y}_t^{\frac{1}{\alpha}} Z_t^{-\frac{1}{\alpha}} \end{aligned}$$

- policy rule

$$I_t = \left(\frac{\Pi^* \Xi_{t+1}^{\psi}}{\beta}\right)^{1-\rho_I} \left(\frac{\Pi_t}{\Pi_t^*}\right)^{\psi_\Pi} \left(\frac{\widetilde{Y}_t}{\widetilde{Y}}\right)^{\psi_Y} I_{t-1}^{\rho_I} e^{\varepsilon_{t+1}^{I}}$$

- shocks

$$\Xi_t = \Xi^{1-\rho_\xi} \Xi_{t-1}^{\rho_\xi} e^{\varepsilon_t^{\xi}}, \qquad \varepsilon_{t+1}^{\xi} \approx N\left(0, \sigma_\xi\right)$$

$$\widetilde{G}_t = \left(g\widetilde{Y}\right)^{1-\rho_g} \widetilde{G}_{t-1}^{\rho_g} e^{\varepsilon_t^{g}}, \qquad \varepsilon_{t+1}^{g} \approx N\left(0, \sigma_g\right)$$

$$\mu_{w,t+1} = \mu_w^{1-\rho_\mu} \left(\mu_{w,t}\right)^{\rho_\mu} e^{\varepsilon_{t+1}^{\mu}}, \qquad \varepsilon_{t+1}^{\mu} \approx N\left(0, \sigma_\mu\right)$$

$$Z_t = Z_{t-1}^{\rho_z} e^{\varepsilon_t^{z}}, \qquad \varepsilon_{t+1}^{z} \approx N\left(0, \sigma_{z,s_{z,t}}\right)$$

$$\eta_{t+1} = e^{\varepsilon_{t+1}^{\eta}}, \qquad \varepsilon_{t+1}^{\eta} \approx N\left(0, \sigma_{\eta,s_{\eta,t}}\right).$$

- standard deviations

$$\begin{aligned} \sigma_{z,s_{z,t}} &= \sigma_{z,0} s_{z,t} + \sigma_{z,1}\left(1 - s_{z,t}\right) \\ \sigma_{\eta,s_{\eta,t}} &= \sigma_{\eta,0} s_{\eta,t} + \sigma_{\eta,1}\left(1 - s_{\eta,t}\right) \end{aligned}$$

- C_{-1}, I_{-1}, Π_{-1} given.

D Numerical implementation

For the numerical implementation of the model, we scale the maximum value function by a constant κ to increase accuracy. Define a dummy variable $\widetilde{D}_t = \mathrm{E}_t \Xi_{t+1}^{1-\gamma} \widetilde{J}_{t+1}^{1-\gamma} / \kappa^{1-\gamma}$. It follows that $\kappa^{1-\gamma} \widetilde{D}_t = \mathrm{E}_t \Xi_{t+1}^{1-\gamma} \widetilde{J}_{t+1}^{1-\gamma}$. This implies

$$\widetilde{D}_t = \frac{\mathrm{E}_t \Xi_{t+1}^{1-\gamma} \widetilde{J}_{t+1}^{1-\gamma}}{\kappa^{1-\gamma}}$$

$$\widetilde{J}_t^{1-\psi} = (1-\beta)\, \widetilde{u}_t^{1-\psi} + \beta \kappa^{1-\psi} \widetilde{D}_t^{\frac{1-\psi}{1-\gamma}}$$

$$Q_{t,t+1} = \beta \left(\frac{\kappa \widetilde{D}_t^{\frac{1}{1-\gamma}}}{\widetilde{J}_{t+1}}\right)^{\gamma-\psi} \left(\frac{\widetilde{u}_{t+1}}{\widetilde{u}_t}\right)^{-\psi} \frac{u_{c,t+1}}{u_{c,t}} \frac{1}{\Xi_{t+1}^{\gamma}} \frac{1}{\Pi_{t+1}}$$

D.1 Functional forms

We rely on the Trabandt and Uhlig (2011) form for temporary utility, i.e.

$$u_t = \left(C_t - h\Xi_t C_{t-1}\right) \left(1 - \eta\,(1-\psi)\, N_t^{1+\frac{1}{\phi}}\right)^{\frac{\psi}{1-\psi}}$$

As a result

$$\frac{\widetilde{w}_t}{P_t} = \frac{\eta\psi\left(1 + \frac{1}{\phi}\right)\left(\widetilde{C}_t - h\widetilde{C}_{t-1}\right) N_t^{\frac{1}{\phi}}}{1 - \eta\,(1-\psi)\, N_t^{1+\frac{1}{\phi}}} \frac{\theta_{w,t}}{\theta_{w,t} - 1}$$

$$\widetilde{J}_t^{1-\psi} = (1-\beta)\left(\widetilde{C}_t - h\widetilde{C}_{t-1}\right)^{1-\psi}\left(1-\eta(1-\psi)N_t^{1+\frac{1}{\phi}}\right)^{\psi} + \beta\kappa^{1-\psi}\widetilde{D}_t^{\frac{1-\psi}{1-\gamma}}$$

$$Q_{t,t+1} = \beta\left(\frac{\kappa\widetilde{D}_t^{\frac{1}{1-\gamma}}}{\widetilde{J}_{t+1}}\right)^{\gamma-\psi}\left(\frac{\widetilde{C}_{t+1}-h\widetilde{C}_t}{\widetilde{C}_t-h\widetilde{C}_{t-1}}\right)^{-\psi}\left(\frac{1-\eta(1-\psi)N_{t+1}^{1+\frac{1}{\phi}}}{1-\eta(1-\psi)N_t^{1+\frac{1}{\phi}}}\right)^{\psi}\frac{1}{\Xi_{t+1}^{\gamma}}\frac{1}{\Pi_{t+1}}$$

$$\begin{aligned}(\theta-1)\widetilde{Y}_t &= -\zeta\left(\Pi_t - (\Pi_t^*)^{1-\iota}\Pi_{t-1}^{\iota}\right)\widetilde{Y}_t\Pi_t + \frac{\theta}{\alpha}\frac{\widetilde{w}_t}{P_t}\left(\frac{\widetilde{Y}_t}{Z_t}\right)^{\frac{1}{\alpha}} + ...\\ &\quad + \mathrm{E}_t Q_{t,t+1}\zeta\left(\Pi_{t+1} - \left(\Pi_{t+1}^*\right)^{1-\iota}\Pi_t^{\iota}\right)\widetilde{Y}_{t+1}\Xi_{t+1}\Pi_{t+1}\end{aligned}$$

E Elasticity of intertemporal substitution

We compute the elasticity of intertemporal substitution of consumption as the elasticity of consumption to a change in the real interest rate holding labour supply constant.

Define the "consumption surplus" $\overleftrightarrow{c}_t \equiv \widetilde{C}_t - h\widetilde{C}_{t-1}$. The first order approximation to the nominal stochastic discount factor

$$Q_{t,t+1} = \beta\left(\frac{\kappa\widetilde{D}_t^{\frac{1}{1-\gamma}}}{\widetilde{J}_{t+1}}\right)^{\gamma-\psi}\left(\frac{\overleftrightarrow{c}_{t+1}}{\overleftrightarrow{c}_t}\right)^{-\psi}\left(\frac{1-\eta(1-\psi)N_{t+1}^{1+\frac{1}{\phi}}}{1-\eta(1-\psi)N_t^{1+\frac{1}{\phi}}}\right)^{\psi}\frac{1}{\Xi_{t+1}^{\gamma}}\frac{1}{\Pi_{t+1}}$$

can be written as[1]

$$\widehat{q}_{t,t+1} = -\psi\Delta\widehat{\overleftrightarrow{c}}_{t+1} - \psi\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}\Delta\widehat{N}_{t+1} - \psi\widehat{\xi}_{t+1} - \widehat{\pi}_{t+1} - (\gamma-\psi)\left(\widehat{\xi}_{t+1} + \widehat{\widehat{j}}_{t+1} - \mathrm{E}_t\left[\widehat{\xi}_{t+1} + \widehat{\widehat{j}}_{t+1}\right]\right)$$

where

$$\widehat{\widehat{j}}_t + \widehat{\xi}_t = \sum_{i=0}^{\infty}\left(\beta\Xi^{1-\psi}\right)^i \mathrm{E}_t\left[\widehat{\xi}_{t+i} + \left(1-\beta\Xi^{1-\psi}\right)\left(\widehat{\overleftrightarrow{c}}_{t+i} - \frac{\psi}{1-\psi}\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}\widehat{N}_{t+i}\right)\right]$$

As a result,

$$\widehat{q}_{t,t+1} = -\psi\Delta\widehat{\overleftrightarrow{c}}_{t+1} - \psi\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}\Delta\widehat{N}_{t+1} - \psi\widehat{\xi}_{t+1} - \widehat{\pi}_{t+1}$$

and the real rate is

$$\widehat{r}_t = \psi\mathrm{E}_t\Delta\widehat{\overleftrightarrow{c}}_{t+1} + \psi\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}\mathrm{E}_t\Delta\widehat{N}_{t+1} + \psi\mathrm{E}_t\widehat{\xi}_{t+1}$$

Rearranging terms

$$\widehat{\overleftrightarrow{c}}_t = -\frac{1}{\psi}\widehat{r}_t + \mathrm{E}_t\widehat{\overleftrightarrow{c}}_{t+1} + \frac{1}{\psi}\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}\mathrm{E}_t\Delta\widehat{N}_{t+1} + \mathrm{E}_t\widehat{\xi}_{t+1}$$

so that the long-run elasticity of substitution $\overline{EIS}$, i.e. the elasticity which is obtained after households have adjusted their consumption habits, takes the usual value

$$\overline{EIS} = \frac{1}{\psi}$$

[1] In these derivations, $\kappa = 1$.

Note that, in the absence of habits, this expression boils down to the usual value $1/\psi$.

To compute the short-run elasticity, we rewrite the consumption surplus in terms of the underlying consumption levels to obtain

$$\widehat{\widehat{c}}_t = -\frac{1}{\psi}\frac{1-h}{1+h}\widehat{r}_t + \frac{1}{1+h}\mathrm{E}_t\widehat{\widehat{c}}_{t+1} + \frac{h}{1+h}\widehat{\widehat{c}}_{t-1} + \frac{1-h}{1+h}\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}\mathrm{E}_t\Delta\widehat{N}_{t+1} + \frac{1-h}{1+h}\mathrm{E}_t\widehat{\xi}_{t+1}$$

The short-run elasticity of substitution EIS is therefore

$$EIS = \frac{1}{\psi}\frac{1-h}{1+h}$$

which again boils down to $1/\psi$ when $h = 0$. Note that, since $h > 0$, it is always the case that $EIS < \overline{EIS}$.

F Approximate likelihood

Consider the reduced form system of equations

$$\begin{aligned}
\mathbf{y}^o_{t+1} &= k_{y,j} + F\hat{\mathbf{x}}_{t+1} + \frac{1}{2}\left(I_{n_y} \otimes \hat{\mathbf{x}}'_{t+1}\right) E\hat{\mathbf{x}}_{t+1} + D\mathbf{v}_{t+1} \\
\mathbf{x}_{t+1} &= k_{x,i} + P\hat{\mathbf{x}}_t + \frac{1}{2}\left(I_{n_x} \otimes \hat{\mathbf{x}}'_t\right) G\hat{\mathbf{x}}_t + \widetilde{\sigma}\Sigma_i\mathbf{w}_{t+1} \\
\mathbf{s}_t &\backsim \text{Markov switching}
\end{aligned}$$

where

$$\begin{aligned}
k_{y,j} &= k_{y,s_{t+1}=j} \\
k_{x,i} &= k_{x,s_t=i} \\
\Sigma_i &= \Sigma(s_t = i).
\end{aligned}$$

The vector $\mathbf{y}^o_t$ includes all observable variables, and $\mathbf{v}_{t+1}$ and $\mathbf{w}_{t+1}$ are measurement and structural shocks, respectively. In this representation, the regime switching variables affect the system by changing the intercepts $k_{y,j}$, $k_{x,i}$ and the loadings of the structural innovations Σ_i (we indicate here with i the value of the discrete state variables at t and with j the value of the discrete state variables at $t+1$).

To compute the approximate likelihood, at any point in time we first linearise the two quadratic terms around the conditional mean of the continuous state variables conditional on the prevailing regime. As a result, the two equations above can be rewritten as

$$\begin{aligned}
\mathbf{y}^o_{t+1} &= \widetilde{k}^{(i,j)}_{y,t+1} + \widetilde{F}^{(i,j)}_{t+1}\hat{\mathbf{x}}_{t+1} + Dv_{t+1} \\
\widehat{\mathbf{x}}_{t+1} &= \widetilde{k}^{(i)}_{x,t} + \widetilde{P}^{(i)}_t\widehat{\mathbf{x}}_t + \Sigma_i\mathbf{w}_{t+1}
\end{aligned}$$

where

$$
\begin{aligned}
\widetilde{k}^{(i,j)}_{y,t+1} &= \widetilde{k}_{y,j} + \frac{1}{2}\left(I_{n_y} \otimes \hat{\mathbf{x}}^{(i)\prime}_{t+1|t}\right) E\hat{\mathbf{x}}^{(i)}_{t+1|t} - \mathbf{\Delta}_{i,t+1}\hat{\mathbf{x}}^{(i)}_{t+1|t} \\
\widetilde{F}^{(i,j)}_{t+1} &= F + \mathbf{\Delta}_{i,t+1}\hat{\mathbf{x}}^{(i)}_{t+1|t} = E(\mathbf{x}_{t+1}|\underline{\mathbf{y}}^{o}_{1:t}, s_t = i, \boldsymbol{\theta}) \\
\mathbf{\Delta}_{i,t+1} &= \left[\frac{\partial\left(\frac{1}{2}\left(I_{n_y} \otimes \hat{\mathbf{x}}^{\prime}_{t+1}\right) E\hat{\mathbf{x}}_{t+1}\right)}{\partial \hat{\mathbf{x}}_{t+1}}\right]_{\hat{\mathbf{x}}_{t+1} = \hat{\mathbf{x}}^{(i)}_{t+1|t}} \\
\widetilde{k}^{(i)}_{x,t} &= \widetilde{k}_{x,i} + \frac{1}{2}\left(I_{n_x} \otimes \hat{\mathbf{x}}^{(i)\prime}_{t|t}\right) G\hat{\mathbf{x}}^{(i)}_{t|t} - \mathbf{\Delta}_{i,t}\hat{\mathbf{x}}^{(i)}_{t|t} \\
\widetilde{P}^{(i)}_{t} &= P + \mathbf{\Delta}_{i,t}\hat{\mathbf{x}}^{(i)}_{t|t} = E(\hat{\mathbf{x}}_{t}|\underline{\mathbf{y}}^{o}_{1:t}, s_t = i, \boldsymbol{\theta}) \\
\mathbf{\Delta}_{i,t} &= \left[\frac{\partial\left(\frac{1}{2}\left(I_{n_x} \otimes \hat{\mathbf{x}}^{\prime}_{t}\right) G\hat{\mathbf{x}}_{t}\right)}{\partial \hat{\mathbf{x}}_{t}}\right]_{\hat{\mathbf{x}}_{t} = \hat{\mathbf{x}}^{(i)}_{t|t}}
\end{aligned}
$$

for regime-dependent intercepts $\widetilde{k}^{(i,j)}_{y,t+1}$, $\widetilde{k}^{(i)}_{x,t}$ and slope coefficients $\widetilde{F}^{(i,j)}_{t+1}$, $\widetilde{P}^{(i)}_{t}$.We then apply Kim's (1994) approximate filter to forecast

$$
\begin{aligned}
\hat{\mathbf{x}}^{(i,j)}_{t+1|t} &= \widetilde{k}^{(i)}_{x,t} + \widetilde{P}^{(i)}_{t}\hat{\mathbf{x}}^{(i)}_{t|t} = \hat{\mathbf{x}}^{(i)}_{t+1|t} \\
\mathbf{Q}^{(i,j)}_{t+1|t} &= \widetilde{P}^{(i)}_{t}\mathbf{Q}^{(i,j)}_{t|t}\widetilde{P}^{(i)\prime}_{t} + \Sigma_i\Sigma_i^{\prime} = \mathbf{Q}^{(i)}_{t+1|t}
\end{aligned}
$$

and update the vector of continuous state variables

$$
\hat{\mathbf{x}}^{(j)}_{t+1|t+1} = \sum_{i=1}^{m} \hat{\mathbf{x}}^{(i,j)}_{t+1|t+1} \times p(s_t = i|s_{t+1} = j, \underline{\mathbf{y}}_{1:t+1})
$$

$$
\begin{aligned}
\mathbf{Q}^{(j)}_{t+1|t+1} = \sum_{i=1}^{m} &\left[\left(\hat{\mathbf{x}}^{(i,j)}_{t+1|t+1} - \hat{\mathbf{x}}^{(j)}_{t+1|t+1}\right)\left(\hat{\mathbf{x}}^{(i,j)}_{t+1|t+1} - \hat{\mathbf{x}}^{(j)}_{t+1|t+1}\right)^{\prime} + \mathbf{Q}^{(i,j)}_{t+1|t+1}\right] \times \\
&\times p(s_t = i|s_{t+1} = j, \underline{\mathbf{y}}_{1:t+1})
\end{aligned}
$$

and then update the regime probabilities

$$
p(s_{t+1} = j, s_t = i|\underline{\mathbf{y}}_{1:t}) = p_{ij,t+1|t} = p_{ij} \times p(s_t = i|\underline{\mathbf{y}}_{1:t})
$$

and

$$
\begin{aligned}
p(s_{t+1} = j, s_t = i|\underline{\mathbf{y}}_{t+1}) &= p_{ij,t+1|t} \times \frac{p(\mathbf{y}_{t+1}|\underline{\mathbf{y}}_{t}, s_{t+1} = j, s_t = i)}{p(\mathbf{y}_{t+1}|\underline{\mathbf{y}}_{t})} \\
p(s_{t+1} = j|\underline{\mathbf{y}}_{1:t+1}) &= \sum_{i=1}^{m} p(s_{t+1} = j, s_t = i|\underline{\mathbf{y}}_{1:t+1}) \\
p(s_t = i|s_{t+1} = j, \underline{\mathbf{y}}_{1:t+1}) &= \frac{p(s_{t+1} = j, s_t = i|\underline{\mathbf{y}}_{1:t+1})}{p(s_{t+1} = j|\underline{\mathbf{y}}_{1:t+1})}
\end{aligned}
$$

where $p(\mathbf{y}_{t+1}|\underline{\mathbf{y}}_{1:t}) = \sum_{i=1}^{m}\sum_{j=1}^{m} p(\mathbf{y}_{t+1}|\underline{\mathbf{y}}_{1:t}, s_{t+1} = j, s_t = i) \times p(s_{t+1} = j, s_t = i|\underline{\mathbf{y}}_{1:t})$

The conditional log-likelihood islog $L = \sum_{t=1}^{T} \log p(\mathbf{y}_{t+1}|\underline{\mathbf{y}}_{1:t})$