Breaking the Feedback Loop: Macroprudential Regulation of Banks' Sovereign Exposures*

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Abstract

This paper develops a dynamic general equilibrium model which features both endogenous bank failure risk and sovereign default risk to study the feedback loop between sovereign and banking crises. In the model, an initial shock to the banking sector contributes to an increase in public debt and sovereign risk as a result of the government guarantees over the failed banks. Investing in risky sovereign debt may be attractive to surviving banks protected by limited liability, which profit from high returns as long as the government does not default and suffer limited losses otherwise. These risk shifting incentives may result in excessive exposure to the sovereign default, which represents an important source of systemic risk and feeds back into further financial and economic distress not internalized by the banking sector. A counterfactual exercise is performed to assess the macroprudential implications of modifying the current regulatory framework by introducing regulatory capital requirements for banks' sovereign exposures. The results suggest they can help mitigating the negative effects of the feedback loop on financial stability and economic activity.

Keywords: feedback loop, financial crises, sovereign default, macroprudential policy, systemic risk, capital requirements.

JEL codes: E44, F34, G01, G21, G28

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1 Introduction

The *negative feedback loop* between banks, sovereigns and aggregate economic activity has drawn considerable attention since the onset of the European debt crisis and it has been documented in several recent papers.[1] In a nutshell, it is argued that governments' assistance to their domestic banking systems in order to avoid their collapse increased the level of sovereign debt, raising concerns regarding its sustainability. The distress in the banking system also caused a downturn in the economic activity which put even more pressure on public finances. At the same time, banks' exposure to domestic sovereign debt increased, translating the doubts about governments' solvency into further financial instability.

In this context, several voices called for changes in the regulatory treatment of banks' exposure to (domestic) sovereign debt.[2] The current regulatory framework imposes that at least a fraction of the banks' risk-weighted assets has to be financed with bank equity capital. However, as of now, it assigns zero risk weights to Euro-area sovereign debt. Furthermore, domestic government debt is exempt from existing concentration limits to single counterparties, and it is even encouraged by current liquidity regulation.[3] In a recent report on the regulatory treatment of sovereign exposures in the books of banks and insurance companies, the European Systemic Risk Board stated: *"If sovereign exposures are in fact subject to default risk, consistency with a risk-focused approach to prudential regulation and supervision requires that this default risk is taken into account"* (ESRB, 2015).

This paper develops a dynamic general equilibrium model able to address some of the elements in the discussion introduced above. The model features both bank endogenous bank failure risk and sovereign default risk. The interplay of these two, via the government's bailout of the banking sector and the banks' exposures to risky sovereign debt, generates a *negative feedback loop* between sovereign risk and financial instability. The model allows to perform counterfactual exercises regarding the current regulatory framework. In particular, this paper addresses the potential macroprudential implications of introducing regulatory

[1] Lane (2012) provides a narrative of the European sovereign debt crisis. Reinhart and Rogoff (2011) document the recurrent link between sovereign and banking crises using long historical time series for a wide range of countries. Further evidence is provided in Section 2.

[2] See for example Gros (2013), Weidmann (2013), Enria, Farkas and Overby (2016), and BIS (2016).

[3] Nouy (2012) provides a comprehensive review of the current regulatory treatment of sovereign exposures for banks and insurance companies.

capital requirements for banks' sovereign debt exposures.

In the model, bank failure risk stems from the exposure to both idiosyncratic and aggregate shocks, as well as from banks' exposure to sovereign debt subject to default risk. Distortions arise from banks' limited liability and the existence of deposit insurance, which induce the mispricing of risk by banks. Investing in risky sovereign debt may be attractive to banks protected by limited liability, who enjoy high profits insofar as the government does not default and suffer losses limited to their initial equity contributions otherwise. These risk shifting incentives at the expense of the deposit insurance scheme may result in excessive exposure to sovereign risk, compared to what a social planner would find optimal when internalizing the deadweight losses associated to the resolution of failed banks. Additionally, the possibility that the government defaults not only on its outstanding stock of debt but also on its deposit insurance liabilities translates into higher funding costs for the banks when they increase their exposure to the sovereign risk. If depositors cannot observe the balance sheet composition of the banks, these do not internalize the effect of their individual risk taking choices on the funding costs of the whole banking system.

As a result, initial shocks to a small fraction of banks translate into an increase in public debt and sovereign risk via government guarantees over the failed banks. Surviving banks respond by endogenously increasing their exposure to risky government debt, which transmits the instability to the whole banking system. This way, the possibility of a sovereign default becomes an important source of systemic risk, even if the default does not materialize ex-post.

The model is relatively parsimonious, compared to standard dynamic general equilibrium models in the literature. The reason for this is twofold. First, keeping the model simple allows to isolate the key mechanisms behind the feedback loop and to analyze possible policy changes in a tractable framework. Second, the use of computationally intensive global solution methods restricts the size of the models that can be feasibly solved, since numerical approximation procedures in high-dimensional spaces can easily suffer from the so called curse of dimensionality.[4] Despite of this, the model is rich enough to capture and (potentially) quantify many of the relevant of elements analyzed in the theoretical literature about

[4]In order to overcome these problems, state of the art computational techniques are used. Maliar and Maliar (2014) and Fernandez-Villaverde, Rubio-Ramirez and Schorfheide (2016) provide a comprehensive survey of those techniques. Further details are provided in the Technical Appendix.

the feedback loop and documented in the recent empirical work, and to provide novel insights about possible modifications of the current regulatory treatment of banks' sovereign exposures, which could potentially guide the design of macroprudential policy tools in the future.

A tentative parameterization is presented, constituting a first attempt to illustrate the qualitative and potential quantitative properties of the model under a set of reasonable parameter values. The results suggest important amplification effects resulting from the presence of the feedback loop, which could be (at least) partially mitigated by introducing positive risk weights for sovereign exposures in the calculation of the regulatory capital requirements for banks.

The model environment provides a rationale for macroprudential policies aimed to reduce banks' incentives to excessively expose themselves to sovereign risk. However, potential drawbacks of these policies, such as higher funding costs for the government and other macroprudential considerations, would need to be taken into account as well. In these sense, the preliminary results in this paper should be understood as a first step towards assessing the effects of modifying the current regulatory treatment of banks' sovereign exposures, particularly on mitigating the (potentially) excessive risk-taking incentives of banks.

The remaining of the paper is organized as follows. Section 2 discusses how the paper relates to the existing literature. Section 3 describes the model setup. Section 4 introduces the numerical solution method, presents the baseline n, describes the main quantitative properties of the model, and provides a counterfactual exercise about the potential effects of introducing a positive risk weight for sovereign debt in the calculations of the regulatory capital requirements. Finally, Section 6 concludes. A Technical Appendix provides a detailed description of the numerical solution method and assess its accuracy.

2 Related literature

This paper connects with several strands on the literature that study the feedback effects between banks and sovereign crises, as well as with the literature on macro-financial linkages and macroprudential policies.

The existing literature identifies at least three different reasons why banks may have incentives to increase their exposure to sovereign risk during times of financial distress. First, creditor discrimination by defaulting governments may create a difference between the expected return on sovereign bonds for domestic banks and foreign investors. This difference increases during times of stress, which leads to a re-nationalization of domestic sovereign debt (see Broner, Erce, Martin and Ventura, 2014). Second, financial repression by the government in the form of moral suasion may force or incentivize banks to invest in their domestic sovereign debt. Acharya and Rajan (2013) and Chari, Dovis and Kehoe (2014) analyze this phenomenon in a theoretical framework, while Becker and Ivashina (2016) and Altavilla, Pagano and Simonelli (2017) find evidence along these lines in the context of the European sovereign debt crisis. Third, limited liability and deposit insurance may distort banks' incentives by encouraging them to take excessive exposures to sovereign risk. Evidence of this risk shifting behavior is documented in Acharya and Steffen (2015) and Altavilla, Pagano and Simonelli (2017).[5] This paper focuses on the third of these frictions.

Previous theoretical literature analyzes the negative feedback loop between banks and sovereigns in partial equilibrium or finite time models.[6] These theoretical works shed light on the mechanisms behind the feedback loop, but cannot speak about the dynamic effects and are not suitable for quantitative analysis. In this regard, the contribution of this paper is to embed some of the relevant elements discussed in the theoretical literature into a dynamic general equilibrium model, in order to assess the quantitative importance of the feedback loop and the macroprudential implications of modifying the current regulatory framework by introducing regulatory capital requirements for banks' sovereign exposures.

To this end, this paper builds upon the literature on macroeconomic models with financial frictions as well as the literature on sovereign default. It is most closely related to the theoretical and quantitative literature of dynamic general equilibrium models that study the role of macroprudential regulation on the risk-taking incentives of financial intermediaries and, in particular, the macroprudential implications of regulatory capital requirements.

Early work by Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Carlstrom and

[5]Furthermore, Kirschenmann, Korte and Steffen (2017) argue that the current regulatory treatment of sovereign exposures within the European Union further encourages banks' risk shifting behavior, resulting in risk spill overs from risky periphery sovereigns to safer core countries.

[6]See for example Cooper and Nikolov (2013), Acharya, Dreschsler and Schnabl (2014), Farhi and Tirole (2016), Brunnermeier el al (2016), and Leonello (forthcoming)

Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999) underscored the amplification effects that financial frictions have in business cycles fluctuations. Since the onset of the global financial crisis, several recent papers further extended these frameworks by incorporating financial intermediaries into otherwise standard dynamic general equilibrium models. Some of these papers (for example, Gertler and Karadi, 2011; Gertler and Kiyotaki, 2011; Gertler, Kiyotaki and Queralto, 2011) emphasize the role of bankers' net worth and its dynamics as a financing constraint that financial intermediaries face, arising from the fact that bankers can divert a fraction of the funds under their management.

Other recent papers, like Martinez-Miera and Suarez (2014), Clerc et al (2015), and Mendicino, Nikolov, Supera and Suarez (2016), which are closely related to this paper in the way banks are modelled, attribute these financing constraints to the existence of regulatory capital requirements that try to reduce the excessive risk taking caused by limited liability and deposit insurance, and explore the effects on social welfare and other macroeconomic aggregates of increasing these capital requirements, among other macroprudential policies. Again, these frictions make the aggregate net worth of the banking sector a relevant variable that will determine the performance of the aggregate economy. Compared to the previous literature, these papers explicitly model the possibility of bank failure, which proved to be an important element of the global financial crisis.

This paper adds to the existing literature by explicitly modelling the sovereign default risk and studies its amplification effects and its interaction with the riskiness of the banking sector. The sovereign risk channel is absent in most of the existing literature analyzing the quantitative implications of macroprudential regulation of banks. Some exceptions are the works by Bocola (2016) and Ari (2017). The former analyzes the pass-through of sovereign risk to the banking sector in an environment in which the banking sector is modelled as in Gertler and Karadi (2011). Its model, however, abstracts from limited liability and the possibility of bank failure. As a result, banks' incentives to increase their exposure to sovereign risk are not present. The latter, on the contrary, does analyze banks' risk shifting incentives in the presence of sovereign default risk. Nevertheless, none of the two are able to capture the side of the feedback loop by which financial instability in the banking sector translates into higher sovereign risk, as the probability of default of the government in their models only responds to exogenous shocks, and not to the endogenous state of the economy as in the model presented here. Furthermore, none of them explicitly analyze the role of capital requirements for sovereign exposures. Thus, the contribution of this paper with respect to the previous literature is the introduction of the two-way feedback loop in a dynamic general equilibrium model, as well as the explicit analysis of the role of capital requirements for banks' sovereign exposures.

Regarding the sovereign default literature, quantitative studies following the seminal contribution of Eaton and Gersovitz (1981), such as Aguiar and Gopinath (2006), Arellano (2008) and Mendoza and Yue (2012), analyze sovereign debt dynamics and business cycle properties of emerging economies, by incorporating the possibility of a sovereign default as the outcome of the strategic behavior of a benevolent, social welfare maximizer government. In a recent paper, Perez (2015) extends this framework and proposes a mechanism by which the domestic banking sector exposure to sovereign debt acts as a determinant of the government's incentives to default. Although the sovereign default has a disruptive effect on the balance sheets of the domestic banking sector, however, bank failure and the risk taking of banks is absent in the model. This paper departs from this strand of the literature in that it does not model sovereign default as an optimal decision of a social welfare maximizer government. As in Bi (2012) and Bi and Traum (2012), in the model presented here the government faces a stochastic limit to the debt it can issue, which causes its default if exceeded.

Lastly, this paper relates to recent efforts to solve quantitative models of financial crises using global solution methods.[7] These papers highlight the importance of non-linear dynamics and risk premia variation, which traditional local solution methods are not able to capture and needs to be taken into account in quantitative policy work. These features are particularly relevant in the context of this paper, as sovereign default episodes are inherently non-linear events and default risk causes large variations in risk premia with important consequences for macroeconomic outcomes, as shown below.

[7] See for example Brunnermeier and Sannikov (2014). Martinez-Miera and Suarez (2014), He and Krishnamurthy (2015), Martinez-Miera and Repullo (2017), Elenev, Landvoigt and Van Nieuwerburgh (2017), and Bianchi and Mendoza (forthcoming).

3 The Model

This section presents the model economy and each of the agents that populate it. Time is discrete and runs infinitely. The domestic economy is populated by: (i) a risk-averse infinitely-lived representative household formed by a continuum of members; (ii) a continuum of (potentially) short-lived bankers who are part of the representative household; (iii) a continuum of ex-ante identical banks; (iv) a representative firm; and (v) a government. There is a single non-durable consumption good, which is also used as the numeraire and can be transformed into physical capital used for production.

(i) The representative household takes consumption and savings decisions to maximize its intertemporal expected utility. It can save in the form of government-guaranteed deposits issued by the bank or by directly holding physical capital.

(ii) Bankers are a special class of agents with exclusive access to the opportunity of investing their net worth as banks' inside equity capital. They accumulate wealth until they retire, when they transfer it to the representative household.

(iii) Banks are perfectly competitive and subject to limited liability. They borrow from households and issue equity among the bankers in order to comply with a regulatory capital requirement, which effectively constrains their intermediation ability. They invest both in physical capital and in risky sovereign debt. Under the baseline parameterization, sovereign debt is not subject to capital requirements.

(iv) A perfectly competitive firm rents capital produced by the banks and the representative household, and hires labor in order to produce consumption good.

(v) The government issues short-term debt to finance its deficits and the cost of the deposit insurance. It is subject to default risk which, when it materializes, imposes a write-off on the outstanding government obligations (its stock of debt and the deposit insurance liabilities). The sovereign faces a segmented credit market in that it can only borrow from its domestic banks and from a set of potential foreign lenders with limited wealth.

The following subsections describe each of the elements of the model in detail.

3.1 Production environment

A continuum of members of the representative household and a continuum of banks invest in physical capital using a technology that transforms one unit of consumption good into ω effective units of capital. The idiosyncratic shock ω is independent across time and across households members and banks. It is log-normally distributed with mean one and standard deviation σ.

A period after investment decisions take place, the available physical capital is rented by a perfectly competitive representative firm, which combines it with labor supplied by the household to produce consumption good. After production takes place, undepreciated capital is transformed back into consumption good and recovered by the household members and banks which produced it.

Investment in physical capital is subject to aggregate risk, which takes the form of a large and infrequent iid depreciation shock denoted by $\psi_t \in \{0, 1\}$, which is unknown when the investment decisions are taken.[8] If $\psi_t = 1$, which occurs with probability π, a fraction λ of the continuum of households and the continuum of banks sees its stock of capital fully depreciate after production takes place.

Similarly to Gertler and Kiyotaki (2015), it is further assumed that households must incur in a management cost per unit of consumption good invested in the physical capital production technology, which could reflect their comparative disadvantage with respect to banks in screening and monitoring investment projects. As a consequence, as in the cited paper, to the extent that the constraints on banks tighten in a recession, the share of capital held by households will increase, resulting in a decrease of net output produced.

3.2 Households

The representative household is infinitely lived and risk averse, and it chooses consumption and savings in order to maximize lifetime utility. It is formed by a continuum of members and it provides consumption insurance to them.[9] It can save in the form of one-period,

[8]The depreciation shock could be modelled as a persistent variable at a small cost (for example, following a two-state Markov chain). However, as shown in the numerical results of Section 4, even non-persistent depreciation shocks can have very persistent effects on the model economy.

[9]Note that, since the idiosyncratic shock ω has unit mean and household members are subject to consumption insurance, idiosyncratic risk is completely diversified away within the household.

government-guaranteed deposits, which are remunerated with a (gross) risky return $\widetilde{R}^d_{t+1}$ (which equals the promised return R^d_t minus potential losses Ψ_{t+1}, which can be positive if the government defaults and equal zero otherwise), and by investing in a risky technology that transforms $K^h_t + h(K^h_t)$ units of consumption good into ^h_t units of physical capital, which then it rents the following period to a perfect competitive firm that combines it with labor to produce consumption good. Its gross realized return is $\widetilde{R}^h_t = r^k_t + (1-\delta)(1-\lambda\psi_t)$, which is the sum of the rental rate of capital plus the undepreciated physical capital recovered after production takes place.

As described above, investment in physical capital requires the representative household to incur in a management cost $h(K^h_t)$. As in Gertler and Kiyotaki (2015), the capital management cost is assumed to be increasing and convex in the total amount of capital, given by the function $h(K^h_t) = \kappa(K^h_t)^2$.

The representative household discounts future utility at a rate β and obtains utility from consumption of non-durable goods under a CRRA utility function with risk aversion parameter ν. It inelastically supplies one unit of labor remunerated with a wage W_t, receives dividend payments from bankers Π_t, and pays lump-sum taxes T_t.

The problem of the representative household involves choosing consumption C_t, deposit holdings D_t, and investment in physical capital K^h_t so as to maximize its expected discounted lifetime utility

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \frac{(C_{t+i})^{1-\nu}}{1-\nu}, \tag{1}$$

subject to the budget constraint:

$$C_t + D_t + K^h_t + h(K^h_t) = W_t + \widetilde{R}^d_t D_{t-1} + \widetilde{R}^k_t K^h_{t-1} + \Pi_t - T_t. \tag{2}$$

The representative household problem results in the following first-order conditions:

$$\mathbb{E}_t[\Lambda_{t+1}\widetilde{R}^d_{t+1}] = 1, \tag{3}$$

$$\mathbb{E}_t\left[\Lambda_{t+1}\widetilde{R}^k_{t+1}\right] = 1 + h'(K^h_t), \tag{4}$$

where $\mathbb{E}_t$ is the expectation operator conditional on information available at time t and $\Lambda_{t+1} \equiv \beta\left(\frac{C_t}{C_{t+1}}\right)^{\nu}$ is the household's stochastic discount factor.

It will be useful to define the household's net worth N_t as the relevant state variable for the household problem at the beginning of period t:

$$N_t = W_t + \widetilde{R}_t^d D_{t-1} + \widetilde{R}_t^k K_{t-1}^h + \Pi_t - T_t. \tag{5}$$

3.3 Bankers

There is a continuum of measure one of bankers with accumulated net worth E_t. As in Gertler and Kiyotaki (2011), the bankers are a special class of agents that belong to the household and have exclusive access to the opportunity of investing their net worth as banks' inside equity capital. They are (potentially) short-lived, with an iid probability of retiring denoted by $1-\varphi$. When they do so, they transfer their terminal net worth to the household and are replaced by new bankers that start with an exogenous level of net worth denoted by ϖ.

At the beginning of every period, after bankers learn whether they will continue for at least one more period, they have the possibility of transferring a fraction $1-x_t$ of their net worth to the household. As in Gertler and Kiyotaki (2010), the value function of the bankers is linear in the level of net worth (since, as shown below, the returns of the bank are constant returns to scale), so the marginal value of one unit of net worth can be written as:

$$v_t = 1 - x_t + x_t \mathbb{E}_t \left[\Lambda_{t+1}(1-\varphi+\varphi v_{t+1})\widetilde{R}_{t+1}^E \right]. \tag{6}$$

The problem of the banker consists of choosing the fraction x_t of its net worth reinvested as bank equity, taking returns $\widetilde{R}_{t+1}^E$ as given.[10] Note that, from the expression above, a banker will always choose to invest all of its net worth as bank equity ($x_t = 1$) as long as $v_t \geq 1$, in which case they optimally choose to postpone any dividend payments until retirement. The numerical exercise below will focus on a parameterization where $v_t \geq 1$ (and thus $x_t = 1$) holds for every period.

The dividend payments transferred to households can be described as

$$\Pi_t = (1-x_{t-1})E_{t-1} + x_{t-1}(1-\varphi)\widetilde{R}_t^E E_{t-1}, \tag{7}$$

where the first term represents the dividends paid before retirement and the second term represents the transfers of the terminal net worth of retiring bakers. The aggregate level of

[10] $\widetilde{R}_{t+1}^E$ denotes the aggregate realized return on equity (the average return across the continuum of banks along which bankers allocate their net worth as inside equity), while $\widetilde{R}_{t+1}^e$, which is presented below, denotes the return on equity of an individual bank.

bankers' net worth evolves according to the following law of motion:

$$E_t = x_{t-1}\varphi \widetilde{R}_t^E E_{t-1} + (1-\varphi)\varpi, \tag{8}$$

where the first term represents the returns of the net worth of surviving bankers invested as bank equity and the second term represents the exogenous endowment of new bankers.

3.4 Banks

There is a continuum of measure one of perfectly competitive ex-ante identical banks. A bank lasts for one period only: it is an investment project created at t and liquidated at $t+1$. They raise deposit funds d_t from households with a promised return R_t^d and equity capital e_t from bankers, and can invest both in a capital production technology k_t and in government bonds b_t.

The banks in this economy represent a consolidation of financial intermediaries and capital producing firms. Banks' returns are heterogeneous ex-post, since the bank-specific capital production technology is subject to idiosyncratic shocks, as described in subsection 3.1. They can represent exposure to sources of risk resulting from geographic or sectoral specialization, which might, in turn, stem from specific knowledge of bankers on certain regions or sectors that are subject to idiosyncratic shocks.

Banks face liquidity management costs $m(d_t, b_t)$ which are increasing in the amount of deposits issued and decreasing in the amount of the government bonds they hold.[11,12] A functional form compatible with these assumptions which will be used in the numerical exercise is $m(d_t, b_t) = \phi\left(\frac{d_t}{b_t}\right) d_t$.

Banks are subject to limited liability, which means that the equity payoffs generated by a bank at time $t+1$ are given by the positive part of the difference between the returns from

[11] The role of government bonds in reducing banks' liquidity management costs could be justified in a model in which banks receive a random stream of intra-period liquidity shocks. Having access to a liquid asset (government bonds) would allow banks to meet deposit withdrawals without having to sell other less liquid assets under fire-sale prices. The liquidity role of public debt has been analyzed in the theoretical literature, for instance in Woodford (1990) and Holmstrom and Tirole (1998).

[12] The reason for introducing such a cost comes from the result derived in Repullo and Suarez (2004) which states that one-period lived perfectly competitive banks subject to limited liability that could invest in two different risky assets would optimally specialize in one of them, unless there exist intermediation costs that imply some complementarity between the two assets. In the logic of the model presented here, the complementarity comes from the different degrees of liquidity of each asset, as discussed above.

its assets (net of liquidity management costs) and the repayments due to its deposits:

$$\widetilde{R}^e_{t+1} e_t = \max \left\{ \widetilde{R}^k_{t+1} \omega k_t + \widetilde{R}^b_{t+1} b_t - R^d_t d_t - m(d_t, b_t), 0 \right\}. \tag{9}$$

If the returns from its assets (net of liquidity management costs) are greater than the repayments due to its deposits, the difference is paid back to the bank's equity holders. Otherwise, the bank equity is written down to zero and its assets are taken over by the deposit insurance scheme, who repays the principal and interests in full to the deposit holders (as long as the government does not default, in which case the bank's assets are seized directly by the depositors).

The idiosyncratic return of the investment technology implies the banks which draw a value of ω below a (stochastic) threshold will default every period. In particular, this threshold will take two different values in case the depreciation shock realizes ($\psi_{t+1} = 1$), since the realized return on the investment in physical capital will be equal to $R^k_{t+1} = 1 + r^k_{t+1} - \delta$ (this is, the rental rate of capital plus the undepreciated physical capital recovered after production) for a fraction $1 - \lambda$ of banks, and equal to r^k_{t+1} for a fraction λ (this is, the fraction of banks hit by the depreciation shock do not recover any capital after production takes place and thus only receive the rental rate). The thresholds are given by

$$\overline{\omega}_{t+1} = \frac{R^d_t d_t + m(d_t, b_t) - \widetilde{R}^b_{t+1} b_t}{R^k_{t+1} k_t}, \tag{10}$$

for the fraction $1 - \lambda$ of banks not affected by the depreciation shock, and

$$\hat{\omega}_{t+1} = \frac{R^d_t d_t + m(d_t, b_t) - \widetilde{R}^b_{t+1} b_t}{r^k_{t+1} k_t}, \tag{11}$$

for the fraction λ that suffers the full depreciation of its capital after production takes place.

Taking as given the marginal value of one unit of the bankers' wealth v_t and the bankers' stochastic discount factor $\Omega_{t+1} \equiv \Lambda_{t+1}(1 - \varphi + \varphi v_{t+1})$, as well as the promised return of deposits R^d_t and the stochastic return of its assets, the representative bank chooses its portfolio allocation (k_t, b_t) and liability structure (d_t, e_t) that solve the following problem:

$$\underset{(k_t, b_t, d_t, e_t)}{\text{Max}} \quad \mathbb{E}_t \Omega_{t+1} \widetilde{R}^e_{t+1} e_t - v_t e_t, \tag{12}$$

subject to the balance sheet constraint

$$k_t + b_t = d_t + e_t. \tag{13}$$

Furthermore, banks are subject to a regulatory capital requirement, which imposes that at least a fraction γ of the banks' risk-weighted assets has to be financed with bank capital. Government bond holdings are subject to a risk weight of ι, while investment in physical capital is subject to a risk weight normalized to one:

$$e_t \geq \gamma(k_t + \iota b_t). \tag{14}$$

If deposits are cheaper than equity financing, which always happens in equilibrium under parameterization presented in Section 4, the capital requirement is binding and the problem of the representative bank can be rewritten as an unconstrained optimization problem in two variables (k_t, b_t) by substituting (9), (13) and (14) with equality into the objective function of the problem of the representative bank (12).

The first-order conditions resulting from the representative bank's problem are:

$$\begin{aligned} \mathbb{E}_t\Omega_{t+1} \big\{ &(1-\lambda\psi_{t+1}) \left[R^k_{t+1} \left(1-\Gamma(\overline{\omega}_{t+1})\right) - \left(m^k_t + R^d_t(1-\gamma)\right)\left(1-F(\overline{\omega}_{t+1})\right)\right] \\ &+\lambda\psi_{t+1}\left[r^k_{t+1}\left(1-\Gamma(\hat{\omega}_{t+1})\right) - \left(m^k_t + R^d_t(1-\gamma)\right)\left(1-F(\hat{\omega}_{t+1})\right)\right]\big\} = \upsilon_t\gamma, \end{aligned} \tag{15}$$

$$\begin{aligned} \mathbb{E}_t\Omega_{t+1} \Big\{ &(1-\lambda\psi_{t+1})(\widetilde{R}^b_{t+1} - m^b_t - R^d_t(1-\gamma\iota))\left(1-F(\overline{\omega}_{t+1})\right) \\ &+\lambda\psi_{t+1}(\widetilde{R}^b_{t+1} - m^b_t - R^d_t(1-\gamma\iota))\left(1-F(\hat{\omega}_{t+1})\right)\Big\} = \upsilon_t\gamma\iota, \end{aligned} \tag{16}$$

where

$$m^k_t \equiv \frac{\partial m(d_t, b_t)}{\partial k_t} = \phi\left[2(1-\gamma)^2(k_t/b_t) + 2(1-\gamma)(1-\gamma\iota)\right],$$

$$m^b_t \equiv \frac{\partial m(d_t, b_t)}{\partial b_t} = \phi\left[(1-\gamma\iota)^2 - (1-\gamma)^2(k_t/b_t)^2\right],$$

are the derivatives of the liquidity management cost with respect to the investment in physical capital and in sovereign bonds, respectively,

$$\Gamma(x) = \int_0^x \omega f(\omega)\mathrm{d}\omega = \Phi\left(\frac{\log(x) - \sigma^2/2}{\sigma}\right),$$

$$F(x) = \int_0^x f(\omega)\mathrm{d}\omega = \Phi\left(\frac{\log(x) + \sigma^2/2}{\sigma}\right),$$

and $f(\omega)$ is the probability density function of the idiosyncratic shock ω and $\Phi(\cdot)$ is the cumulative distribution function of the standard normal.

3.5 Firms

A perfectly competitive representative firm rents physical capital K_t (remunerated at a rate r_t^k) and hires labor L_t (remunerated at a rate W_t) in order to produce consumption good Y_t. Its profit-maximization problem is:

$$\underset{(K_t,L_t)}{\text{Max}} \quad Y_t - r_t^k K_t - W_t L_t. \tag{17}$$

Production of the consumption good follows a Cobb-Douglas function $Y_t = K_t^\alpha L_t^{1-\alpha}$ where α is the elasticity of capital.

The resulting first-order conditions of the firm's problem are:

$$r_t^k = \frac{\alpha Y_t}{K_t}, \tag{18}$$

$$W_t = \frac{(1-\alpha)Y_t}{L_t}. \tag{19}$$

3.6 Government

A government issues short-term debt to finance its deficits. There is a stochastic limit to government debt that follows a logistic distribution, similarly to Bi and Traum (2012) and Bocola (2016). This stochastic limit depends on the level of debt outstanding so that, when such limit is exceeded, the government defaults. The government default event at the end of period t is represented by the binary variable $s_{t+1} \in \{0, 1\}$ and the probability of the government default in each period is

$$p_t \equiv \text{Prob}(s_{t+1} = 1|B_t) = \frac{\exp(\eta_1 + \eta_2 B_t)}{1 + \exp(\eta_1 + \eta_2 B_t)}. \tag{20}$$

If the government does not default ($s_{t+1} = 0$), it pays back the promised (gross) return R_t^b per unit of debt to its creditors and the deposit insurance liabilities DI_t to the banks' depositors. If it defaults ($s_{t+1} = 1$), it writes off a fraction $\theta_b \in [0, 1]$ and $\theta_d \in [0, 1]$ of its outstanding stock of debt and of its deposit insurance liabilities, respectively. Thus, the realized return of the government bonds can be expressed as

$$\widetilde{R}_{t+1}^b = (1 - \theta_b s_{t+1}) R_t^b. \tag{21}$$

The budget constraint of the government states that, each period, the issuance of one-period bonds B_t has to be equal to the debt service $(1-\theta_b s_t)R^b_{t-1}B_{t-1}$, the cost of the deposit insurance scheme $(1-\theta_d s_t)DI_t$, and public spending G_t, minus tax revenues T_t:

$$B_t = (1-\theta_b s_t)R^b_{t-1}B_{t-1} + (1-\theta_d s_t)DI_t + G_t - T_t, \tag{22}$$

Tax revenues behave according to a fiscal rule

$$T_t = \tau_y Y_t + \tau_b B_{t-1}, \tag{23}$$

where the first term can be interpreted as the automatic-stabilizer component and the second term can be interpreted as the debt-stabilizer component of tax revenues. Furthermore, government spending is assumed to be constant, $G_t = \overline{G}$.

3.7 Deposit insurance

When a bank fails, its equity capital is written down to zero and the deposits become a liability for the government, which has to repay principal and interests in full to the depositors. The deposit insurance scheme takes over the failed bank's assets minus resolution costs which are assumed to be a fraction μ of the bank's return on investment in physical capital, resulting in a deadweight loss every time a bank defaults. The deposit insurance liabilities can be expressed as:

$$\begin{aligned} DI_{t+1} = (1-\lambda\psi_{t+1})&\left[\left(R^d_t d_t - \widetilde{R}^b_{t+1}b_t + m(d_t,b_t)\right)F(\overline{\omega}_{t+1}) - \mu R^k_{t+1}k_t\Gamma(\overline{\omega}_{t+1})\right] \\ +\lambda\psi_{t+1}&\left[\left(R^d_t d_t - \widetilde{R}^b_{t+1}b_t + m(d_t,b_t)\right)F(\hat{\omega}_{t+1}) - \mu r^k_{t+1}k_t\Gamma(\hat{\omega}_{t+1})\right]. \end{aligned} \tag{24}$$

From this expression, the loss for depositors due to banks' failure per unit of deposits is

$$\Psi_{t+1} = \frac{\theta_d s_{t+1} DI_{t+1}}{D_t}. \tag{25}$$

3.8 International investors

As in Aguiar, Chatterjee, Cole and Stangebye (2016), international financial markets are segmented, such that only a subset of foreign investors participates in the sovereign debt market. For simplicity, all period t lenders participate in the sovereign bond market for one period and are replaced by a new set of lenders in the following period, with an exogenous endowment W^f. They derive utility from final consumption C^f_{t+1} under a CRRA utility function with risk aversion parameter ν and solve a conventional one-period portfolio problem. They can choose between investing in government bonds or in a international risk-free asset which offers them a gross return R (or they can borrow at the same rate).

The problem each international investor solves is:

$$\underset{B^f_t}{\text{Max}} \quad \mathbb{E}_t \frac{{C^f_{t+1}}^{1-\nu}}{1-\nu}, \tag{26}$$

subject to the budget constraint:

$$C^f_{t+1} = \widetilde{R}^b_{t+1} B^f_t + R\left(W^f - B^f_t\right), \tag{27}$$

which results in the following first-order condition:

$$\mathbb{E}_t\left[(\widetilde{R}^b_{t+1} - R)\left[\widetilde{R}^b_{t+1} B^f_t + R\left(W^f - B^f_t\right)\right]^{-\nu}\right] = 0 \tag{28}$$

3.9 Market clearing

Every period, the aggregate level of bankers' net worth must equal the bank equity issued by the banks:

$$E_t = e_t, \tag{29}$$

the level of deposits supplied by the household must equal the deposits issued by the banks:

$$D_t = d_t, \tag{30}$$

the supply of government bonds must equal the bonds held by the banks and the international investors:

$$B_t = b_t + B^f_t, \tag{31}$$

the physical capital rented by the consumption good producing firm must equal the stock of capital held by the household and by the banks:

$$K_t = K^h_{t-1} + k_{t-1}, \tag{32}$$

and the labor hired by the firm must equal the unit of labor inelastically supplied by the household:

$$L_t = 1. \tag{33}$$

3.10 Equilibrium

In equilibrium, the state of the economy at any date t can be summarized by three state variables collected in the vector $\mathbf{S} = \{N, E, B\}$: the aggregate net worth of the representative household N_t, the aggregate net worth available to the active bankers E_t, and the level of sovereign debt outstanding B_t. Formally:

Definition 1. *A competitive equilibrium is given by the policy functions for the representative bank ($k(\mathbf{S}), b(\mathbf{S}), d(\mathbf{S}), e(\mathbf{S})$), the representative household ($C(\mathbf{S}), D(\mathbf{S}), K^h(\mathbf{S})$), the representative firm ($K(\mathbf{S}), L(\mathbf{S})$) and the representative international investor ($B^f(\mathbf{S})$), which determine the actions of each of the agents for each triple $\mathbf{S} = \{N, E, B\}$ that summarizes the aggregate state of the economy, such that, given prices ($v(\mathbf{S}), R^d(\mathbf{S}), R^b(\mathbf{S}), r^k(\mathbf{S}), w(\mathbf{S})$) and the realization of the shocks:*

1. *The sequence of consumption and saving decisions $\{C_t, D_t, K_t^h\}_{t=0,1,\ldots}$ solves the problem of the representative household, ie eq. (2)–(4).*

2. *The sequence of portfolio choices $\{k_t, b_t\}_{t=0,1,\ldots}$ and liability structure $\{d_t, e_t\}_{t=0,1,\ldots}$ solves the problem of the representative bank, ie eq. (13)–(16).*

3. *The sequence of input choices $\{K_t, L_t\}_{t=0,1,\ldots}$ solves the problem of the representative firm, ie eq. (18)–(19).*

4. *The sequence of portfolio choices $\{B_t^f\}_{t=0,1,\ldots}$ solves the problem of the representative international investor, ie eq. (28).*

5. *The sequence of prices $\{v_t, R_t^d, R_t^b, r_t^k, W_t\}_{t=0,1,\ldots}$ clears the equity market, the deposits market, the physical capital market and the labor market, ie eq. (29)–(33).*

6. *The sequence of endogenous state variables $\{N_{t+1}, E_{t+1}, B_{t+1}\}_{t=0,1,\ldots}$ satisfies the respective laws of motion, ie eq. (5), (8) and (22).*

4 Numerical results

This section outlines the numerical solution method used, presents the baseline parameterization of the model, its main quantitative properties and provides a counterfactual exercise about the potential effects of introducing a positive risk weight for sovereign debt in the calculation of regulatory capital requirements. In particular, it analyzes the dynamic response of the model to a depreciation shock under different parameterizations, in order to assess the importance of the bank-sovereign feedback loop mechanisms and the effect of different values for the risk weight applied to banks' sovereign bond holdings.

4.1 Solution method

The model is solved using global solution methods. In particular, the method used is policy function iteration (Coleman, 1990), also known as time iteration (Judd, 1998). Functions are approximated using piecewise linear interpolation, as advocated in Richter, Throckmorton and Walker (2014). A detailed description of the numerical solution method and some measures of its accuracy are provided in the Technical Appendix.

Using global solution methods is important given the inherent non-linearities present in sovereign default models. Traditional log-linearisation methods are not able to capture the variation in risk premia (due to the certainty equivalence), which represents an important source of amplification in this model, as shown below, while higher order perturbation methods provide accurate approximations only locally, failing to capture the dynamics of models with large deviations from the steady state as the one presented here.[13] The main drawback of using global solution methods is that they are very computationally intensive, which restricts the size of the models that can be feasibly solved. This is because additional state variables increase exponentially the size of the steady state, rendering the so called curse of dimensionality. Recent improvements in computational power and numerical solution procedures, as surveyed in Maliar and Maliar (2014) and Fernandez-Villaverde, Rubio-Ramirez and Schorfheide (2016), allow to solve increasingly complex models, but still pose a constraint that is not easily overcome.

[13] Arouba, Fernandez-Villaverde and Rubio-Ramirez (2006), Maliar and Maliar (2014), Richter, Throckmorton and Walker (2014) or Fernandez-Villaverde, Rubio-Ramirez and Schorfheide (2016) provide a comprehensive comparison of existing solution methods for dynamic general equilibrium models.

4.2 Calibration

The model parameter values are summarized in Table 1. The baseline parameterization should be considered as a first attempt to illustrate the qualitative and potential quantitative properties of the model, by assigning standard values used in the literature when available and reasonable parameter values otherwise.

The subjective discount rate of the representative household β is set equal to 0.99, a standard value in the literature, which implies a deposits rate around 4% in annual terms (one period in the model represents one quarter). The capital management cost for households κ is equal to 0.00012, which implies that, in equilibrium, households directly hold around 20% of the physical capital in the economy.

Table 1: Baseline parameterization

Description	Par.	Val.	Description	Par.	Val.
Subjective discount rate	β	0.99	Elasticity of physical capital	α	0.33
Capital mgmt. cost	κ	0.00012	Depreciation rate	δ	0.02
Bankers' survival rate	φ	0.975	Govt. spending	G	0.75
New bankers' endowment	ϖ	0.005	Automatic stabilizer	τ_y	0.20
Capital requirement	γ	0.08	Debt stabilizer	τ_b	0.05
Risk weight of sov. bonds	ι	0.0	Write-off parameter (B)	θ_b	0.6
Liquidity mgmt. cost	ϕ	1e-6	Write-off parameter (DI)	θ_d	0.0
Repossession cost	μ	0.3	Sovereign default dist.	η_1	-11.6
			Sovereign default dist.	η_2	1.0
Dispersion of iid shock	σ	0.03	Intl. risk-free rate	R	1.01
Fraction affected if ψ_t=1	λ	0.09	Intl. investors' endowment	W^f	3
Prob(ψ_t=1)	π	0.0076	Intl. investors' risk aversion	ν	2

The bankers' survival rate φ is equal to 0.975, as in Mendicino et al (2016), implying a divind payout rate of 2.5%, and the new bankers' endowment ϖ is equal to 0.005. In all, these values imply a return on equity around 10% in the steady state.

The capital requirement γ is set to 0.08, as in Clerc et al (2015), compatible with the full weight level of Basel I and the treatment of not rated corporate loans in Basel II and III. The risk weight of government bonds is set to zero in the baseline case, resembling the current regulatory treatment of banks' sovereign exposures. This parameter takes several different values in the counterfactual exercises performed below.

The liquidity management cost is set to 1e-6, a value that guarantees an interior solution in the banks' portfolio problem and implies that sovereign bond holdings represent around 8% of banks' total assets.

Again as in Clerc et al (2015) and Mendicino et al (2016), the bank bankruptcy cost (the fraction of the banks' assets that the government cannot recover in case of bankruptcy) is set to 0.3.

The standard deviation σ of the distribution of idiosyncratic shocks ω is equal to 0.03, which implies a bank failure rate equal to 1% in steady state. The probability π that the aggregate depreciation shock realizes is equal to 0.0076, which means that it occurs once each 33 years, on average, a frequency close to the systemic shock in Martinez-Miera and Suarez (2014). The fraction λ of banks and household members affected when the shock realizes is equal to 0.09, which implies a realized bank failure rate around 9-10% the period when the shock hits.

The elasticity of physical capital α and its depreciation rate δ are set to standard values in the literature of 0.33 and 0.02, respectively, which imply a capital-to-GDP ratio around 2.7.

The constant level of government spending, $\overline{G}$, is set to 0.75, which represents a 23% of GDP. The parameters governing the tax revenues τ_y and τ_b are set to 0.25 and 0.05, respectively, which implies that tax revenues equal 26% of GDP and guarantees that they are sufficiently large so that debt does not explode when it reaches high levels. These values imply a steady state ratio of debt-to-GDP around 32%.

The write-off parameter for sovereign debt, θ_b, is set to 0.6, so that the government only repays less than half of its outstanding debt when it defaults. The write-off parameter for deposit insurance liabilities, θ_d, is set to 0, so that the full losses from bank failure when the government defaults are absorbed by the depositors. The parameters of the fiscal limit distribution are set so that the annualized unconditional probability of the sovereign default is around 0.66% (so that, on average, the default event occurs once in 150 years), while the probability of default in the steady state is around 0.22%.

Table 2: Selected endogenous variables at the steady state

Variable	**Value**
Annualized deposit rate R^d	3.93%
Annualized return on sov. bonds R^b	4.26%
Annualized return on equity R^e	10.45%
Capital-to-GDP ratio	2.67
Sovereign debt (% of GDP)	31.57%
Fraction of sov. debt held abroad	27.6%
Banks' sovereign exposure (% of bank assets)	8.28%
Annualized default rate of banks	1.00%
Annualized sov. default probability	0.21%

The choice for international investors' parameters is quite tentative. The international risk-free rate R is equal to $1/\beta$, which implies the same degree of impatience for domestic households and the rest of the world. The endowment W^f and the risk aversion parameter ν of the international investors are set equal to 3 and 2, respectively, implying that a fraction of sovereign debt held abroad close to 30%.

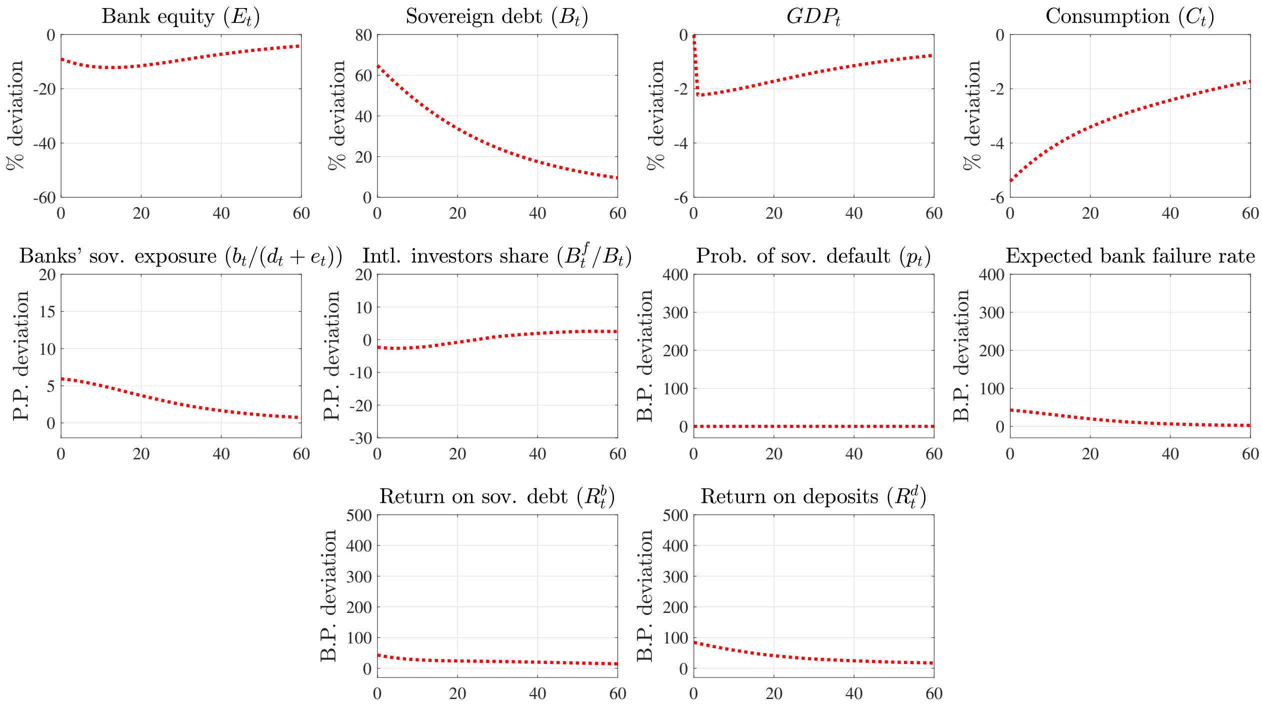
The value for some selected endogenous variables at the model's steady state under the baseline parameterization is reported in Table 2.

4.3 Main results

In order to assess the amplification effects of the feedback loop between banks and the sovereign, this section first presents the dynamic response to an aggregate depreciation shock when sovereign default risk does not react to increases in the outstanding amount of debt by remaining constant for all periods. To achieve this, the parameters governing the probability of default, η_1 and η_2, are set equal to -6.4 and 0.0, respectively, so that p_t is equal to 0.66% for all periods, which corresponds to the unconditional probability of default under the baseline parameterization.

Figure 1 presents the impulse response functions to an aggregate quality shock under the alternative parameterization described above. The realization of the aggregate depreciation shock ψ_t is set equal to 1 for $t = 0$ and equal to 0 for all other t from there on. The realization of the sovereign default event s_t is equal to 0 for all t, meaning that the default event never

Figure 1: Impulse-response functions to an aggregate depreciation shock



materializes ex-post in the simulated paths presented here.[14] Each panel represents the dynamic responses of one of the selected endogenous variables, in (relative) differences with respect to the steady state values in $t = -1$.

The initial shock drives up the realized bank failure rate to around 10%, which translates into a decrease of aggregate bank equity of around the same magnitude and an increase in the outstanding sovereign debt of more than 60% from its initial level, due to the increase in repayments of the deposit insurance liabilities of the government. The household's consumption goes down almost 6% due to the decrease in their net worth (caused by a combination of the decreased returns to physical capital and labor, and the increase in lump-sum taxes). The fall in GDP is caused by shrinkage of the banks' balance sheets and the change in the composition of the owners of physical capital: since the decrease in aggregate bank equity constrains the ability of banks to invest in physical capital, the fraction of the aggregate stock that is in hands on the household increases, resulting in a decrease in net output given their relative inefficiency in managing it (in other words, the amount of resources dedicated

[14]Nevertheless, all of the agents form their expectactions taking into account the possibility that the government defaults on its obligations.

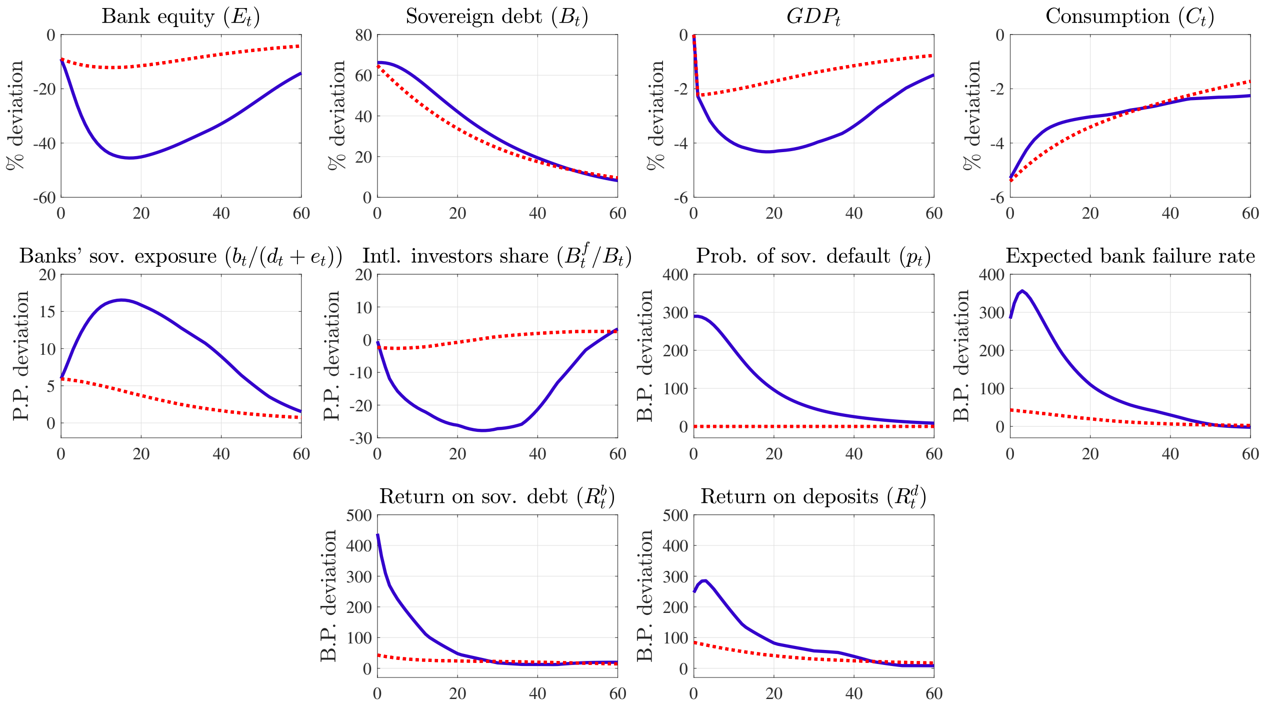
by the household to manage the stock of physical capital increases, limiting the amount of output that remains for consumption and investment).

The increase in the stock of debt is absorbed by the banks, who increase their exposure relative to the size of their balance sheet, and by the international investors, who increase their bond holdings as well, leaving the fraction of the total in their hands barely unchanged. The riskiness of the sovereign bonds under this alternative parameterization, as described above, remains constant, making their promised return increase only slightly because of the increase in their supply. The expected bank failure rate also increases by around 50 basis points (bps), due to the increase in banks' leverage, translating into an increase in the promised return on deposits of almost 100 bps. This increase in the banks' funding costs diminishes the return on equity of the banks, which results in a slow recovery of the aggregate bank equity and thus of the economic activity.

Figure 2 presents, with solid blue lines, the dynamic response to the same shock under the baseline parameterization described in Table 1, where sovereign risk does react to increases in the outstanding level of debt. The dashed red lines depict the same impulse response functions as in Figure 1, when sovereign risk is kept constant.

Following the initial 60% increase in the level of sovereign debt, the annualized probability of default goes up by 300 bps, from an initial 0.21% (see Table 2). This sudden increase translates into an spike of the interest rate paid by the government of 400 bps. The higher leverage of banks and their increased exposure to sovereign risk makes the expected bank failure rate go up by more than 300 bps. As a result, the depositors, anticipating that a sovereign default, which is now much more likely, would mean the failure of the deposit insurance scheme, demand a promised return on deposits up to 250 bps higher. The increase in the banks' funding costs have a large impact on their profitability, making the aggregate level of bank equity go further down up to a -40% after ten quarters, much lower than under the constant sovereign risk parameterization. This has also consequences for net output since, as explained above, tighter constraints on banks' intermediation ability force the relatively inefficient households to manage an increasing share of the stock of physical capital. In all, these results shed light on the huge amplification effects that sovereign default risk has on the baking sector, representing an important source of systemic risk. As shown in Figure 2,

Figure 2: Impulse-response functions to an aggregate depreciation shock

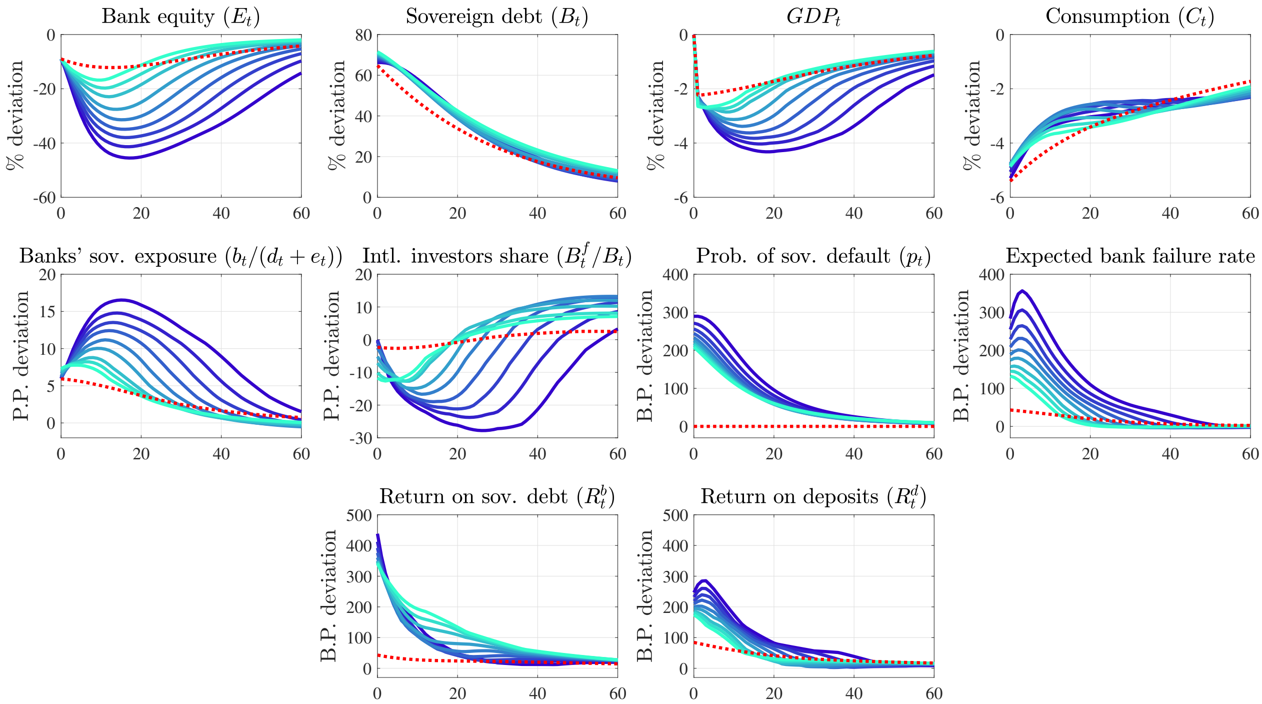


an initial shock that affects a relatively small fraction of banks translates into system-wide instability through the endogenous contagion effect that sovereign risk has on bank failure risk, even if the default of the government does not materialize ex-post.

The increase in banks' funding costs and the resulting decrease in their profitability, in addition to the high yield paid by the government bonds, encourages banks to increase their exposure to sovereign risk. Given the opacity of their balance sheets, individual banks do not internalize the effect of their increased riskiness on the funding costs of the whole banking sector. Furthermore, because of limited liability, they can enjoy the high returns from holding sovereign bonds as long as the government does not default, while suffering limited losses in case the default materializes, effectively shifting the risk to their depositors. Thus, the results seem to point to a potential role of macroprudential regulation in making banks internalize the effects of their sovereign risk taking.

Figure 3 presents the dynamic response to the same shock under a number of parameterizations where the risk weight ι applied to banks' sovereign bond holdings is increased from its initial level of zero. Each of the light blue lines depict the impulse response functions under a different risk weight ι, following 5% increments, with lighter blues representing higher values,

Figure 3: Impulse-response functions to an aggregate depreciation shock



from 5% to 40%.

Increasing capital requirements for banks' sovereign exposures has two effects: first, for the same promised return, it makes investing in sovereign debt less attractive, since the cost of equity with which they have to finance it is higher than the cost of deposits, and also the equity losses that banks suffer in case of default are higher; second, it reduces banks' leverage, making them effectively safer and decreasing the depositors losses in case of default. As a result, banks reduce their exposure to sovereign risk and they are safer than in the baseline case, which translates into lower funding costs, less amplification effects and quicker recoveries from an initial shock. Each increase in the risk weight ι brings the impulse response functions closer to the alternative parameterization with constant sovereign risk presented in Figure 1, depicted by the red dashed lines, suggesting that capital requirements are effective in mitigating the effects of the bank-sovereign feedback loop.

However, the benefits of increasing the risk weight for sovereign exposures need to be weighed against the potential costs of such policy. For instance, Figure 3 shows that imposing capital requirements for domestic banks' debt holdings increases the funding costs for the government for a longer period, which slows down the return of its debt to the steady state level. Therefore, a formal welfare analysis would need to be performed before having a final word on the desirability of a change in the regulatory treatment of sovereign exposures.

5 Concluding remarks

This paper examines the *negative feedback loop* between sovereign and banking crises, and the potential effects of capital requirements for banks' sovereign exposures on mitigating it by discouraging banks' endogenous exposure to sovereign risk. To this purpose, it develops a dynamic general equilibrium model in which banks decide on their exposure to sovereign debt issued by a government subject to default risk.

One of the contributions of the model presented in this paper is that it features both endogenous bank failure risk and sovereign default risk, which have reinforcing effects on each other (what has been called the *negative feedback loop* between banks and sovereigns). The model allows to study the macroeconomic consequences of such feedback effects: the impact of an increase in bank failure on the probability of a sovereign default via the government guarantees over the banking system, the endogenous increase in banks' exposure to sovereign risk, and the feedback effects that an increase in the sovereign default risk have on banks' solvency and their funding costs. In this sense, the possibility of a sovereign default acts as an important source of systemic risk, by which an initial shock to a small fraction of banks transform into system-wide instability.

The distortions coming from banks' limited liability and government guarantees, which induce the mispricing of risk by banks, make investing in risky sovereign debt attractive for banks, who enjoy high profits insofar as the government does not default and suffer losses limited to their initial equity contributions otherwise. These risk shifting incentives at the expense of the deposit insurance scheme result in excessive exposure to sovereign risk. Additionally, the possibility that the government defaults not only on its outstanding stock of debt but also on its deposit insurance liabilities translates into higher funding costs for the banks when they increase their exposure to the risky sovereign. If depositors cannot observe the balance sheet composition of the banks protected by limited liability, banks do not internalize the effect of their individual risk-taking choices on the funding costs of the whole banking system.

The model is used to address some central issues in recent discussions on the way banks' exposure to (domestic) sovereign debt is currently regulated. In particular, the paper analyzes the potential macroprudential role of capital requirements for sovereign debt. The main finding is that a positive risk weight for sovereign debt in the calculation of capital requirements reduce banks' endogenous exposure to sovereign risk and, consequently, the two-way feedback effects between banking and sovereign crises.

The calibration of the model constitutes a first attempt to explore the qualitative and (potential) quantitative implications of the model. Because of this reason, the magnitudes of the quantitative results discussed above should be understood as an illustration of the conclusions that a more detailed and fully calibrated model could deliver. The model could be used to assess other sets of macroprudential policies, such as time-varying capital requirements, concentration limits to the exposure of banks to sovereign debt, or different combinations of the general regulatory capital requirement and the risk weights for sovereign debt exposures, among others.

The model could also be used to analyze the international dimension of the feedback loop. This would be particularly interesting in the context of a monetary union and could shed light on issues such as common deposit insurance mechanisms, common resolution and supervisory regimes, and their effect on international risk spill overs. Conceptually, this would only require embedding the elements of the model like the one presented here into a multi-country setup. The main difficulty, however, would come from the computationally intensive solution methods that would be needed to solve it. Notwithstanding this, these appear to be interesting topics for a future research agenda.

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Technical Appendix

A Solution method

The model is solved using global solution methods. In particular, the method used is policy function iteration (Coleman, 1990), also known as time iteration (Judd, 1998). Functions are approximated using piecewise linear interpolation, as advocated in Richter, Throckmorton and Walker (2014).

A sketch of the numerical solution procedure is as follows:

1. Discretize the state variables by creating an evenly space grid, covering the relevant range of values each of them can take.

2. Select the set of policy functions. In this case, the variables chosen are $C(\mathbf{S})$, $b(\mathbf{S})$, $v(\mathbf{S})$, $R^d(\mathbf{S})$, $R^b(\mathbf{S})$.

3. Specify an initial guess for the policy functions at each point i of the state space (note that the size of the state space equals the product of all the state variable grids' sizes) and use them as candidate policy functions.

4. For each point i of the state space, plug the candidate policy functions into the equilibrium equations and calculate the value of the endogenous state variables at $t+1$.

5. Using the value of the endogenous state variables at $t+1$, use linear interpolation to obtain the value of the policy variables at $t+1$ for each possible realization of the exogenous state variables.

6. Using the value of the endogenous state variables and the policy variables at $t+1$, obtain the value at $t+1$ of the remaining variables necessary to calculate time t expectations, for each possible realization of the aggregate shocks.

7. Use a numerical root-finder to solve for the zeros of the residual equations, subject to each of the remaining equilibrium conditions. Numerical integration is needed at this step to compute expectations in the equilibrium equations. The result is a set of policy values in each point i of the state space that satisfies the equilibrium system of equations up to a specified tolerance level, which characterizes the updated policy function for the next step.

8. If the distance between the candidate policy function and the updated policy values obtained in the previous step is less than the convergence criterion for all i, then the policies have converged to their equilibrium values. Otherwise, use the updated policy functions as the new candidate and go back to step 5.

B Accuracy of the numerical solution

It is possible to assess the accuracy of the numerical solution by computing the residual errors of the equilibrium equations after simulating the model for a given sequence of the aggregate shocks using the approximated policy functions obtained by the numerical procedure described above, as proposed by Judd (1992). To this end, the model is simulated for 200,000 periods. Following standard practice, the decimal log of the absolute value of these residual errors is reported here. Figure B.1 reports the density (histogram) of these errors.

Figure B.1: Equilibrium equations' residual errors