Sustainable Forward Guidance at the ELB*

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Abstract

In this paper, I ask whether policy makers lacking the ability to commit may, nevertheless, still make credible announcements about future policy. In the standard analytical frameworks often used to study the effective lower bound (ELB) on nominal interest rates, this can never be the case. These frameworks assume that once the ELB episode is over, it never occurs again. Consequently, there is a cost to fulfilling the promise once the ELB episode is over, while there is no future benefit to fulfilling the promise. However, once there is a positive probability of hitting the ELB in the future, the central bank has an incentive to honor its promises. I find that a promise to keep the nominal interest rate at zero after the ELB episode is sustainable, as long as the promise is not for too many periods. In contrast, announcing an inflation rate for the period after the ELB episode is only sustainable for some values of the likelihood and expected duration of future ELB episodes.

1 Introduction

The current era of very low interest rates have raised troubling questions for all central banks, but particularly for those that target inflation. Do the dangers of hitting the effective lower bound (ELB) for short-term interest rates call for increasing inflation targets as insurance against returning to the ELB? Does inflation targeting still provide an inadequate framework for monetary policy? Or does the presence of the ELB imply

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inflation targeting should be replaced by some other policy framework, such as price-level targeting?

The discussion of the issues surrounding these questions – and on the consequences of the ELB more generally – have reached two conclusions. First, in an environment in which the central bank is able to credibly commit to future actions, the costs of the ELB are small. For example, this is the conclusion of the work by Jung, Teranishi, and Watanabe (2005), Adam and Billi (2006) and Nakov (2008).[1] A central bank able to commit to future actions is not unduly constrained when its current policy rate is at its lower bound; making promises about the future path of the policy rate is sufficient to allow policymakers to influence economic activity effectively. If commitment is the appropriate way to understand the monetary policy environment, then the ELB does not call for any reform of inflation targeting or for raising the average inflation target.

Second, if a central bank is able to commit to a policy framework such as inflation targeting but implements policy within that regime in a discretionary fashion, then the ELB can be very costly, as shown for example by Adam and Billi (2007). This conclusion leads naturally to the proposal of Blanchard, Dell'Ariccia, and Mauro (2010) to raise the average inflation target, making it less likely that the ELB will be encountered. It also leads to proposals to replace inflation targeting with alternatives policy regimes, such as price-level targeting, in which discretionary policy is able to mimic some of the advantages of commitment, as shown by Vestin (2006).

Finding policy regimes that can limit the adverse effects of the ELB is important, as episodes of very low interest rates cannot, as they once were, be viewed as extremely rare events. Figure 1 shows histograms of U.S. short term interest rates. The top panel is based on the monthly effective federal funds rate from January 1960 to July 2016, while the lower panel is for the 3-month Treasury bill rate since 1934. Both show that a large fraction of months have seen rates below 25 basis points. For the shorter sample based on the funds rate, 13% of months since January 1960 have seen the funds rate at or below 25 basis points. For the longer period, the 3-month T-bill rate fell below 25 basis points in 17% of all months.[2]

[1] Reifschneider (2016) demonstrates the effectiveness of credible forward guidance (together with balance sheet policies) using the FRB/US model. Levin, López-Salido, Nelson, and Yun (2010) argue that forward guidance may be less effective in the face of large and persistent shocks that drive the economy to the ELB.

[2] This histogram is misleading in the sense that, to take the top panel, all the months at or below 25 basis points occurred consecutively between December 2008 and December 2015.

Most of the literature that has focused on the monetary policy consequences of the ELB has treated the credibility of the central bank as either complete, as in commitment equilibria, or totally absent, as in analyses of discretion. In the one case, future promises are fully believed and subsequently delivered on. In the latter case, the public places no weight on any promises the central bank might make. Such promises – forward guidance – are thus either extremely powerful, as in work on the forward guidance puzzle by DelNegro, Giannoni, and Patterson (2012), Cochrane (2013), and McKay, Nakamura, and Steinsson (2016b), or completely powerless in a discretionary environment.[3]

Forward guidance has frequently been analyzed using simple analytical frameworks that have helped provide insights into the consequences of the ELB and the role of forward guidance. For example, Eggertsson and Woodford (2003) introduced the assumption that each period there is a fixed probability of exiting the ELB. This approach has been used by Eggertsson (2011), Christiano, Eichenbaum, and Rebelo (2011), Braun, Körber, and Waki (2012), and McKay, Nakamura, and Steinsson (2016b), among others. Alternatively, several authors have considered perfect foresight equilibria in which the ELB will bind for a known number of periods. For example, Werning (2011), Carlstrom, Fuerst, and Paustian (2012), Cochrane (2013), and Kiley (2016) use such a framework. Under either approach, the assumption has been that the ELB is a one-off occurrence. Once the economy exits from the ELB, it never returns. In this case, announcements can never be credible absent a commitment technology. Under discretion, there is no benefit to fulfilling promises made during an ELB episode; any credibility gained from fulfilling promises is of no future use.

The situation changes if the economy may encounter the ELB again. This, of course, is the presumption of work examining the role of the inflation target or the policy regime in reducing the probability of or mitigating the effects of future ELB episodes. But if the economy may return to the ELB, a rational central bank may have an incentive to fulfill past promises, even under discretion. Doing so brings a future benefit of credibility should the ELB again bind.

Of course, if promises made during an ELB period are extreme enough, it is unlikely under discretion that a central bank will fulfill them even if the economy may someday return to the ELB. However, as others have noted (Carlstrom, Fuerst, and Paustian (2012), Kiley (2016), McKay, Nakamura, and Steinsson (2016b)), forward guidance is

[3] Exceptions include Bodenstein, Hebden, and Nunes (2012) and Nakata (2014) which are discussed below.

very powerful in standard new Keynesian models. This suggests that the central bank may need to make only modest promises at the ELB. If so, the costs of fulfilling them may be correspondingly small. Thus, the power of forward guidance, combined with the possibility of a return to the ELB, may lead even a discretionary policymaker to make and keep promises. Forward guidance may be sustainable.

In this paper, I investigate forward guidance and ask whether promises can be sustained in the absence of an ability to commit. That is, can a policymaker who is unable to commit still make promises about future policy that it will in fact be rational for the policymaker to fulfill? If so, the stark contrast between the consequences of the ELB under discretion and under commitment may be too exaggerated. And if this is true, the case against inflation targeting and the arguments for raising the inflation target or switching to price-level targeting are weakened. Effective and sustainable forward guidance would reduce the need for these alternatives. Their merits would need to be based on considerations other then their effects in reducing the probability of encountering the ELB or their superior performance (relative to discretion) at the ELB.

Pure dscretionary and optimal commitment are extreme alternatives. One implies a complete absence of credibility to fulfill promises; the other involves complete credibility. If future promises are credible even in a discretionary environment, the sharp distinction between discretion and commitment is blurred and credibility is no longer an all or nothing property of policy actions. Two literatures have developed approaches that allow for partial credibility. The first follows the stochastic planning problem analyzed by Roberds (1987), and includes the work by Schaumburg and Tambalotti (2007), Debortoli and Nunes (2010), Bodenstein, Hebden, and Nunes (2012), and Debortoli, Maih, and Nunes (2014). The second builds on notion of sustainable plans developed by Chari and Kehoe (1990) and Stokey (1991) and employed by Ireland (1997), Kurozumi (2008), Kurozumi (2009) and Nakata (2014).[4]

The stochastic planning approach of Roberds (1987) and Schaumburg and Tambalotti (2007) assumes a policymaker is able to commit to future policies, but each period there

[4]In the presence of endogenous state variables, current policy choices can affect the incentives faced by future policymakers, thereby generating a channel through which the policymaker can effectively influence expectations about future policy. For example, Jeanne and Svensson (2007) have investigated how generating a large increase in the government's nominal debt can create an incentive for future inflation. Thus, a government's concerns about its balance sheet can provide a mechanism for current policy to influence future policy choices. This channel is absent in the present paper which employs a basic new Keynesian model in which there are no endogenous state variables.

is an exogenous probability a new policymaker will be appointed. Future policymakers are not constrained by the promises made by their predecessors, so promises are discounted to reflect the likelihood that the current policymaker will be replaced.[5] If the current policymaker will, with certainty, not be around to implement any promises, pure discretion emerges. At the other extreme, if the current policymaker remains in office forever with certainty, promises are completely credible.

Closely related to the imperfect credibility that arises with stochastic changes in the policymaker is the notion of loose commitment developed by Debortoli and Nunes (2010) in analyzing fiscal policy and that has been applied to monetary policy issues in Bodenstein, Hebden, and Nunes (2012), Dennis (2014) and Debortoli, Maih, and Nunes (2014). Under loose commitment, there is a fixed probability each period that the policymaker reoptimizes. Because the policymaker may reoptimize in the future, any promises made are discounted, as past promises are ignored when the policymaker reoptimizes.

In contrast to this literature, I follow Chari and Kehoe (1990), Stokey (1991) and the work by Ireland (1997), Kurozumi (2008) and Kurozumi (2009) in focusing on sustainable plans under discretion.[6] That is, I assume the absence of any commitment technology. A past promise might be honored, but only if doing so is the best strategy for the policymaker at the time the promise needs to be honored. Kurozumi (2008) has investigated whether the optimal commitment policy in the basic new Keynesian model is sustainable under discretion. He shows that the optimal sustainable policy falls between that of optimal discretion and optimal commitment, but it converges over time to the optimal commitment policy if the policymaker's discount rate is not too large. Kurozumi (2009) shows that a regime of flexible inflation targeting is sustainable, but only if the central banker places more weight – but not too much weight – on inflation stability than is reflected in social welfare. That is, the central banker must be a Rogoff (1985) conservative, but not too conservative. The framework I use is similar to that employed by Nakata (2014), whose paper is closely related to the approach I adopt but whose focus differs somewhat. I discussion Nakata's contributions below.

What has not been examined is whether announcements of the type associated with

[5] An early example of a model in which equilibrium was affected by the probability of a future change in policy maker was provided by Ball (1995). In his model, however, the new policy maker was drawn from a distribution of policy makers who differed in their preferences.

[6] This literature builds on Abreu (1988). See also Levine, McAdam, and Pearlman (2008).

forward guidance can form part of a sustainable policy plan. This gap in the existing literature is one this paper hopes to fill. The rest of the paper is organized as follows. Section 2 reviews the basic framework of Eggertsson and Woodford (2003) to highlight how, under discretion, promises made at the ELB are not sustainable. Section 3 modifies the framework to allow for a positive probability that after exiting an ELB episode the economy may again enter a period during which the ELB constraint is binding. The analysis then considers alternative forms of forward guidance. In section 4, the effects of a promise to keep the nominal rate at zero for one period after an ELB episode ends are studied, while promises to keep the nominal rate at zero for several periods after an ELB episode ends are considered in section 5. In section 6, forward guidance is interpreted as a promise to deliver a specific inflation rate when the ELB period ends, and the sustainability of the optimal inflation announcement is investigated in section 7. The robustness of the results to some modifications of the model are discussed in section 8, while conclusions are summarized in section 9.

2 An isolated ELB episode

In the interests of tractability, I work with the simple Markov structure of Eggertsson and Woodford (2003). This model is briefly reviewed before extending it, in section 3 to allow for recurring episodes at the ELB. [7]

Because the focus is on sustainable forward guidance, it is worth clarifying what is meant by sustainability before presenting the details of the model. I define a sustainable policy as follows. Let L_j^o be the present value of losses when the economy is in state j under an arbitrary policy o. Let L_j^d denote the present value of losses in state j under the optimal discretionary policy. In the present context, by optimal discretionary policy I mean the policy that, in each period, minimizes the policymaker's loss function, taking expectations and future policy as given. The policy o may involve promises made in the past about policy actions in the current state. The policy o is sustainable if $L_j^o <$

[7] A number of authors (Jung, Teranishi, and Watanabe (2005), Adam and Billi (2006), Adam and Billi (2007), Nakov (2008), Levin, López-Salido, Nelson, and Yun (2010), Billi (2015)) have examined stochastic equilibria in new Keynesian models subject to occasionally binding lower bounds on the nominal interest rate. In these models, the economy can pass into, out of, and back into periods during which the lower bound constraint is binding. However, this literature has not investigated specific examples of forward guidance. Work on forward guidance in stochastic models or on assessing the empirical effects of such policies include Campbell, Evans, Fisher, and Justiniano (2012) and Campbell (2016).

L_j^d for each j. That is, continuing to implement policy o, including any promises made in the past, constitutes a sustainable plan if the present value of losses obtained by implementing the policy is, in every state, less than that obtained by reverting to the policy d. A sustainable policy is time-consistent; the policymaker has no incentive to switch from the policy and adopt the discretionary policy.[8]

To be more specific, consider a simple new Keynesian model, given by

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \mathrm{E}_t \pi_{t+1} - r_t) \tag{1}$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t \tag{2}$$

$$i_t \geq 0, \tag{3}$$

together with a specification of monetary policy, where x_t is the output gap, π_t the inflation rate, i_t is the nominal interest rate, and r_t is an exogenous stochastic process.[9] For convenience the ELB on the nominal interest rate is taken to be zero. Any contingent sequence of inflation, the output gap, and the nominal interest rate that satisfies (1) - (3) for every $t \geq 0$ is sustainable if for each $t \geq 0$ the present discounted value of losses is less than the present value of losses under the optimal, time-consistent discretionary policy. Thus, policies for which the current period's loss exceeds that obtained under the discretionary policy may still be sustainable if future losses under the policy are less than those under discretion.

[8] The concept of a sustainable policy plans was first introduced by Chari and Kehoe (1990). Stokey (1991) defines a pair of strategies (for the government and private sector) that is compatible with a competitive equilibium in the private sector, given the government's strategy, and for which the government has no incentive to alter its strategy as a *credible policy*. See Nakata (2014) for a formal treatment of sustainability in the context of the Markov structure I employ.

[9] The underlying nonlinear model that leads to the reduced form equations employed here is so well known that providing details on it seems unnecessary. See, for example, chapter 8 of Walsh (2010); chapter 11 of the forthcoming fourth edition provides an extended discussion of the ELB. Braun, Körber, and Waki (2012) discuss how, at least for some issues, the log linearized version used here may give misleading answers to some questions. Some of the properties of the model that they emphasize as problematic are absent in a consumption only version of the model such as the one I use. McKay, Nakamura, and Steinsson (2016a) argue that more plausible results on the power of forward guidance are obtained using a discounted Euler equation; see section 8.

2.1 The basic Eggertsson-Woodford model

Following Eggertsson and Woodford (2003), assume there are two states: in one, the ELB is binding; in the other, it is not. The basic model is given by (1) - (3), and the shock r_t in (1) follows a two-state Markov process. If $r_t = r_z < 0$, then $r_{t+1} = r_z$ with probability q and $r_{t+1} = \beta^{-1} - 1 \equiv \rho > 0$ with probability $1 - q$; if $r_t = \rho$, then $r_{t+j} = \rho$ for all $j \geq 0$. In the state denoted by a subscript z, $r_t = r_z$ and $i_t = 0$.

When the ELB constraint is nonbinding, denoted by subscript n, $r_t = \rho$, and I assume policy is set under pure discretion to minimize

$$L_t = \frac{1}{2}\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left(\pi_{t+j}^2 + \lambda x_{t+j}^2\right), \tag{4}$$

given (1) and (2). This implies the central bank follows a targeting rule that takes the form

$$\kappa\pi_t + \lambda x_t = 0. \tag{5}$$

Thus, when the ELB is not binding, inflation π_n and the output gap x_n solve the following two equations:

$$\pi_n = \beta\pi_n + \kappa x_n$$

$$\kappa\pi_n + \lambda x_n = 0,$$

and $\pi_n = x_n = 0$ constitutes an equilibrium when the ELB is non-binding.[10]

Given that $x_n = \pi_n = 0$, the output gap and inflation at the ELB are given by the solutions to

$$\pi_z = \beta q \pi_z + \kappa x_z$$

and

$$x_z = q x_z + \left(\frac{1}{\sigma}\right)\left(q\pi_z + r_z\right).$$

[10] In most of the literature using this model, the assumption is that after the ELB episode ends, policy is characterized by a simple instrument rule rather than by optimal discretion. In the present context, $\pi_n = x_n = 0$ is also the locally unique stationary equilibrium if the nominal rate is given by $i_n = \rho + \phi\pi_n$ once the ELB constrain no longer binds, with $\phi > 1$. The choice of ϕ, as long as it exceeds 1, plays no role in affecting equilibrium at the ELB or away from the ELB when the ELB episode is a one-off event. The issue of multiple equilibria will be the focus of section 6.

Jointing solving these two equations yields

$$x_z = \Delta \left(1 - \beta q\right) r_z \tag{6}$$

and

$$\pi_z = \left(\frac{\kappa}{1 - \beta q}\right) x_z = \Delta \kappa r_z, \tag{7}$$

where

$$\Delta \equiv \frac{1}{\sigma(1-q)(1-\beta q) - q\kappa}.$$

Employing Eggertsson and Woodford's calibration ($\beta = 0.99$, $\sigma = 2$, $\kappa = 0.02$, and $q = 0.9$) yields $\Delta = 263$.[11] Assume that $r_z = -2\%$ (expressed at an annual percentage rate), the equilibrium output gap and inflation rate at the ELB are $x_z = -0.1434$ and $\pi_z = -0.0263$ (-14.34% and -10.53% respectively, when inflation is expressed at an annual rate).

Define L_k as the present discounted value of the loss function (4) in state k, where $k = z$, n. Then

$$L_z = \frac{1}{2}\left(\pi_z^2 + \lambda x_z^2\right) + \beta q L_z + \beta(1-q) L_n$$

and

$$L_n = \frac{1}{2}\left(\pi_n^2 + \lambda x_n^2\right) + \beta L_n.$$

Because $x_n = \pi_n = 0$, $L_n = 0$. Therefore

$$L_z = \frac{1}{2}\frac{\pi_z^2 + \lambda x_z^2}{1 - \beta q} = \frac{1}{2}\frac{\Delta^2\left[\kappa^2 + \lambda\left(1 - \beta q\right)^2\right]}{1 - \beta q} r_z^2.$$

Following Woodford (2003), the loss function (4) can interpreted as derived from a second-order approximation of the welfare of the representative household around the economy's efficient equilibrium, In this case, $\lambda = \kappa/\theta$, where θ is the price elasticity of

[11] These values are also used by McKay, Nakamura, and Steinsson (2016b). The large value of Δ implies the negative value of r_z has a large effect on x_z and π_z. Eggertsson (2011) limits attention to cases in which the denominator of Δ is positive; this is his condition C1 (p. 70). Braun, Körber, and Waki (2012) discuss the equilibrium when the denominator of Δ is negative.

demand faced by individual firms, and

$$\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j U_{t+j} = U^* - \frac{U_c \bar{C}}{2} \left[\frac{\omega\theta\,(1+\eta\theta)}{(1-\omega)(1-\omega\beta)} \right] L_z,$$

where U_t is the time t utility of the representative household, U^* is steady-state utility, $\bar{C}$ is steady-state consumption, and U_c is the marginal utility of consumption evaluated at steady-state consumption. A fall in steady-state consumption by μ percent leads to a decline in utility of

$$\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j U_c \bar{C}\,(1-\mu) = \left(\frac{1}{1-\beta} \right) U_c \bar{C}\,(1-\mu)\,.$$

Therefore, the consumption-equivalent loss at the ELB associated with $L_z > 0$ is given by

$$\mu_z = \frac{1-\beta}{2} \left[\frac{\omega\theta\,(1+\eta\theta)}{(1-\omega)(1-\omega\beta)} \right] L_z.$$

Thus, a loss of L_z is equivalent to a $100\mu_z$ percent reduction in steady-state consumption.[12] Eggertsson and Woodford set $\lambda = 0.003$.[13] For this value of λ, $\mu_z = 31.78\%$.

2.2 Sustainability of forward guidance

Now consider the case in which the central bank promises to keep $i = 0$ in the first period in which the ELB no longer binds. In subsequent periods, it sets $\kappa\pi + \lambda x = 0$ as called for by optimal discretion. Denote the equilibrium when promises are made by a superscript p. In the exit period, denoted by subscript e, the equilibrium is given by

$$x_e^p = x_n^p + \left(\frac{1}{\sigma} \right) (\pi_n^p + \rho) = \left(\frac{1}{\sigma} \right) \rho > 0,$$

and

$$\pi_e^p = \kappa x_e^p = \left(\frac{\kappa}{\sigma} \right) \rho > 0,$$

where x_n^p and π_n^p now denote the equilibrium after the exit period (and $x_n^p = \pi_n^p = 0$).

With x_e^p and π_e^p both positive, expected inflation and the output gap are higher

[12] See also Billi (2015) who uses this measure to evaluation nominal GDP targeting and price-level targeting.

[13] Eggertsson and Woodford set θ equal to 7.66.

when the economy is at the ELB. This boosts the output gap and inflation at the ELB. Promising a positive output gap and inflation rate in the exit period improves outcomes while at the ELB.

Evaluating the loss functions, we have two valuation equations:

$$L_z^p = \frac{1}{2}\left[(\pi_z^p)^2 + \lambda\,(x_z^p)^2\right] + \beta\left[qL_z^p + (1-q)\,L_e^p\right]$$

$$L_e^p = \frac{1}{2}\left[(\pi_e^p)^2 + \lambda\,(x_e^p)^2\right] + \beta L_n^p.$$

With $L_n^p = 0$ as before, so

$$L_e^p = \frac{1}{2}\left[(\pi_e^p)^2 + \lambda\,(x_e^p)^2\right]$$

and

$$L_z^p = \frac{1}{2}\frac{\left[(\pi_z^p)^2 + \lambda\,(x_z^p)^2\right] + \beta\,(1-q)\left[(\pi_e^p)^2 + \lambda\,(x_e^p)^2\right]}{1-\beta q}.$$

For the Eggertsson-Woodford calibration, $\mu_z^p = 12.81\%$ when there is a credible promise to keep $i_e = 0$, compared to a PDV loss of 31.78% when optimal discretion is implemented immediately upon exiting the ELB. Promising to keep the nominal rate at zero for one period after the ELB constraint is relaxed reduces the present value of losses at the ELB by 60%.

But under discretion, a promise to keep the nominal rate at zero in the exit period lacks credibility because $L_e^p > L_n^d = 0$. Forward guidance in the basic Eggertsson-Woodford model is not sustainable. Once the economy exits the ELB, the gains from promising to keep the nominal rate at zero are sunk; nothing further can be gained by fulfilling the promise. It is better to revert to the optimal discretionary equilibrium in which $x_e = x_n = 0$. It will never be optimal to honor past promises as there is no need to maintain a reputation for fulfilling pledges if the need for credibility never rises again.

3 Recurring episodes at the ELB

If the possibility exists that an ELB period will occur again in the future, it may be optimal for a policymaker to deliver on past promises. In so doing, the policymaker is able to influence future expectations during the next ELB episode. The benefit of credibility in the future may be sufficient to outweigh the costs of fulfilling promises made in the past.

I modify the basic structure to allow for a return to the ELB. The transition probabilities are now specified as follows. If $r_t = r_z < 0$, then $r_{t+1} = r_z$ with probability q and $r_{t+1} = \rho > 0$ with probability $1 - q$ (as before). If $r_t = \rho$, then with positive probability $0 >< s \leq 1$, $r_{t+1} = \rho$ and with probability $1 - s$, $r_{t+1} = r_z$. Thus, $1 - s$ is the probability of reverting to the ELB. The previous literature building on the analytical structure of Eggertsson and Woodford (2003) assumed $s = 1$, as has the literature that treats the ELB as binding for a fixed number of periods after which it never binds again (see, for example, Cochrane (2013), Kiley (2014)). If the economy never returns to the ELB, promises made at the ELB will never be honored by a policymaker acting with discretion. But if $s < 1$, such a policymaker may find it optimal to honor past promises. Nakata (2014) was the first to analyze a similar Markov structure. He provides a more formal treatment of optimal policy and focuses on whether reputation can support the optimal Ramsey policy. He shows that with even a quite small chance that an ELB episode will occur in the future, the optimal Ramsey policy is sustainable.

When $s < 1$, it is no longer feasible to achieve $\pi_n = x_n = 0$, as neither expected inflation nor the expected output gap will equal zero. As long as some probability is assigned to the possibility of returning to the ELB, expected inflation and the expected output gap when not at the ELB will depend on x_z and π_z. Further, when $s = 1$, as in the previous section, it did not matter whether one assumed policy followed a Taylor-type instrument rule or implemented optimal discretion; in either case, $\pi_n = x_n = 0$. When $s < 1$, this is no longer the case, and the assumption made about policy when the economy is away from the ELB matters. One assumption, employed for example by McKay, Nakamura, and Steinsson (2016b), is that post-ELB policy is governed by a simple Taylor-type instrument rule. After briefly considering this case, I turn to the case of optimal policy under discretion before considering forms of forward guidance.

For the case of recurring ELB episodes, I solve the model for a range of values for s and q. However, for the baseline exercises, I jointly choose s and q to match the observed frequency of periods at the ELB based on U.S. data. Interpreting the ELB as quarters in which the federal funds rate is 25 basis points or less, the U.S. economy has been away from the ELB 88% of the time since 1960. To match this observed frequency, I calibrate $s = 0.975$ and $q = 0.83$.[14] The probability of reverting to the ELB is thus calibrated

[14] These values imply a steady-state distribution of time of 12.4% at the ELB and 87.6% away from the ELB. The observed values are 12.5% and 87.6%. There are other combinations of s and q that match the

at 2% per quarter, while the probability of exiting the ELB, at 17%, is higher than the 10% (i.e., $q = 0.9$) employed in Eggertsson and Woodford's calibration. As an alternative calibration, I counted the number of quarters since 1934 that the 3-month Treasury bill rate was less than or equal to 25 basis points. This frequency was matched with $s = 0.96$ and $q = 0.81$.

3.1 An instrument rule

Suppose the nominal interest rate satisfies

$$i_t = \max\left(0,\, \rho + \phi\pi_t\right),\ \phi > 1. \tag{8}$$

Under the policy given by (8), equilibrium when the ELB does not bind satisfies the two equations

$$x_n = \left[s x_n + (1-s)x_z\right] - \left(\frac{1}{\sigma}\right)\left[\left(\phi - s\right)\pi_n - (1-s)\pi_z\right] \tag{9}$$

$$\pi_n = \beta\left[s\pi_n + (1-s)\,\pi_z\right] + \kappa x_n. \tag{10}$$

At the ELB, x_z and π_z must satisfy

$$x_z = \left[q x_z + (1-q)\,x_n\right] + \left(\frac{1}{\sigma}\right)\left[q\pi_z + (1-q)\pi_n + r_z\right] \tag{11}$$

$$\pi_z = \beta\left[q\pi_z + (1-q)\,\pi_n\right] + \kappa x_z. \tag{12}$$

These four equations can be solved jointly for x_z, π_z, x_n and π_n. The system cannot be solved separately for equilibrium at the ELB and equilibrium away from the ELB as can be done when $s = 1$.

Equilibrium both when the ELB binds and when it doesn't now depends on the value of the policy response to inflation, ϕ. This response coefficient was irrelevant when $s = 1$, as $\pi_n = x_n = 0$.[15] This is no longer the case when $s < 1$. I pick a conventional value and

observed frequencies. For example, $s = 0.985$ and $q = 0.9$ does so. However, I adopt a lower value of q to avoid equilibria in which the nominal interest rate consistent with optimal discretion when not at the ELB would be negative. See Braun, Körber, and Waki (2012) for a characterization of equilibria when $s = 1$ and the technical appendix of Nakata (2014) for the case of $s < 1$.

[15] It is relevant in ensuring $\pi_n = x_n = 0$ is the local unique, stationary equilibrium when $s = 1$, but as long as $\phi > 1$, its specific value was irrelevant.

set $\phi = 1.5$.

Figure 2 shows the equilibrium values of inflation (upper panel) and the output gap (lower panel) both at the ELB (solid lines) and away from the ELB (dashed lines) as a function of s for $q = 0.83$ (circles) and $q = 0.81$ (no marker). The bullets indicate the baseline calibration of $s = 0.975$ and $q = 0.83$. Both inflation and output deteriorate as the probability of recurring episodes at the ELB increases (s declines). With a greater likelihood of returning to the ELB, expected inflation when the economy is away from the ELB puts more weight on π_z and expected output puts more weight on x_z. By lowering expected future inflation and the output gap, both π_n and x_n decline as s falls. The decline in inflation and the output gap when away from the ELB then acts to further reduce inflation and the output gap when the economy is at the ELB. For $s < 0.968$, the rule (8) calls for a negative nominal interest rate even when the ELB is not binding.

Both inflation and the output gap rise as the probability of exiting a ELB episode rises (q declines). A smaller q, implying ELB episodes of shorter expected duration, dampens the negative inflation and output gap effects of the ELB.

When not at the ELB, inflation, which is zero under the standard case of $s = 1$, remains negative. When $s = 0.975$ and $q = 0.83$, $\pi_n = -1.55\%$, compared to -3.00% at the ELB (expressed at annual rates). The output gap at the ELB is -3.43%, while it is small but positive (0.26%) when away from the ELB. This positive output gap when $s < 1$ and the economy is away from the ELB reflects the fact that $\pi_n < 0$ and the Euler equation has the "normal" negative slope: a fall in π_n increases x_n as $\phi - s > 0$. The decline in π_n induces a greater than one-for-one decline in i_n that reduces the real interest rate and boosts aggregate demand and the output gap.[16]

For the alternative calibration of $s = 0.96$ and $q = 0.81$, episodes at the ELB are of shorter expected duration but occur more frequently. The first effect (more frequent episodes) will tend to worsen outcomes when not at the ELB and so also worsen them at the ELB, while the second effect (shorted expected duration) works in the opposite direction. The first effect dominates and to counteract the lower inflation, the instrument rule calls for cutting the nominal interest rate below zero. Figure 2 only shows outcomes for which $i_n > 0$.

[16] I am ignoring here a potentially important issue. The Phillips curve given by (2) is obtained by linearizing the Calvo model around a zero steady-state rate of inflation. Yet under the policy rule given by (8), average inflation is less than zero. For a survey on non-zero trend inflation, see Ascari and Sbordone (2013). If π is interpreted as the deviation of inflation from the central bank's target, inflation remains consistently below target when $s < 1$, even when the ELB is not a binding constraint.

3.2 Discretion

The policy given by $i_n = \rho + 1.5\pi_n$ leads to a situation in which $\pi_n < 0$ and $x_n > 0$. However, this policy rule need not be consistent with an optimal balance between non-zero inflation and output gap. In this section, I assume instead that policy is consistent with optimal discretion whenever the ELB constraint is nonbinding. The situation in which the policymaker acts under discretion also provides the relevant benchmark for assessing the sustainability of forward guidance.

The policymaker's period loss is represented by

$$l_t = \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right). \tag{13}$$

Under pure discretion when the ELB is not binding, the policymaker takes expectations as given, so the first-order condition for the policy problem is given by (5). Equilibrium is obtained by jointly solving (2) and (5) for π_n and x_n, recognizing that in equilibrium $\mathrm{E}_t\pi_{t+1} = s\pi_n + (1-s)\pi_z$. Substituting out for expected inflation, the two equilibrium conditions for π_n and x_n are

$$\pi_n = \beta\left[s\pi_n + (1-s)\pi_z\right] + \kappa x_n \tag{14}$$

$$\kappa\pi_n + \lambda x_n = 0. \tag{15}$$

In the standard case, $s = 1$ and the equilibrium under discretion is $\pi_n^d = x_n^d = 0$, where the superscript denotes equilibrium under discretion. However, if $s < 1$, this will not be the case as agents in forming expectations of future inflation take into account the possibility that the economy will lapse into the ELB equilibrium.

When away from the ELB, equilibrium also requires that $i_n \geq 0$, as it could be that the outcomes under the optimal discretionary policy would require the nominal interest rate to be negative. This is found to be the case under the rule given by (8) for $s < 0.986$. Under discretion, it can also be the case that (5) would imply $i_n^d < 0$ if s is small (high probability of returning to the ELB) and q is large (high probability of remaining at the ELB). If s is small, then the probability weigh $1-s$ placed on π_z and x_z in forming future expectations when the economy is not at the ELB is large, putting contractionary pressure on π_n and x_n and calling for a lower nominal interest rate. If this weight is large enough, the nominal rate called for by the optimal discretionary policy is negative. Similarly, if q is large, the ELB episode is expected to be of long duration, and this reduces π_z and x_z, contributing to a fall in π_n and x_n and reducing the nominal interest rate when the economy is not at the ELB. Figure 3 shows the level of the nominal rate consistent with optimal discretion for ranges of s and q. Only for $q \geq 0.89$ is the non-negativity constraint on i_n binding, and then only for $s < 0.975$. Thus, for both the benchmark calibrations used for s and q, $i_n > 0$. In the subsequent analysis, I restrict attention to values of s and q such that $i_n^d > 0$.

Figure 4, which can be compared to figure 2, shows equilibrium inflation (upper panel) and the output gap (lower panel) under discretion when the ELB is binding and when it isn't. Note the difference in scales between figures 2 and 4; optimal discretion mitigates the contractionary effects of the ELB significantly.[17] In addition, under discretion the equilibrium outcomes as s falls do not display the nonlinearities seen under the instrument rule; and the output gap under discretion actually rises as the likelihood of returning to the ELB rises (s falls). To understand this phenomenon, consider what happens under the instrument rule as s falls. With reversion to the ELB more likely, expected inflation when the ELB does not bind falls, and in response, i_n is reduced according to the policy rule (8). This acts to boost x_n, but not enough to counteract the effects of the fall in the expected future output gap. In addition,as x_n and π_n decline, this worsens x_z and π_z, further depressing x_n and π_n. In contrast, under optimal discretion, the weight on x_n is small so policy puts a large weight on attempting to stabilize π_n. This requires a more aggressive expansion. The resulting rise in x_n and the smaller decline in π_n relative to the instrument rule, prevent x_z from being adversely affected by the decline in s.

Table 1 provides a comparison of the outcomes at the ELB and away from the ELB for the baseline calibrations of s and q. Also shown are the outcomes under discretion for the alternative calibration. Recall that the simple instrument rule and discretion both yield the same equilibrium when $s = 1$ (with $\pi_n = x_n = 0$). When $s < 1$, they perform quite differently, with optimal discretion providing much better outcomes both at the ELB and away from it.

[17] As a result, $i_n^d > 0$ for smaller values of s than occurred with the instrument rule.

Table 1: Instrument rule versus discretion*

	$s = 0.975$		$q = 0.83$	
Policy	π_z	x_z	π_n	x_n
Rule	-3.003	-3.434	-1.549	0.256
Discretion	-0.958	-1.838	-0.141	0.235
	$s = 0.96$		$q = 0.81$	
	π_z	x_z	π_n	x_n
Rule†	–	–	–	–
Discretion	-0.744	-1.464	-0.161	0.269

* Inflation at annual rates; output gap in percent.

† The value of i_n implied by the rule is negative.

3.3 Welfare costs of the ELB

In examining the sustainability of forward guidance, the loss achieved under optimal discretion provides an important benchmark as the policymaker can, at any time, revert to this optimal time-consistent discretionary policy. Thus, the losses that are achieved under discretion are central to determining the temptation to ignore past promises.

Let L_k^d for $k = z, n$ be the present discounted value of losses in state k under pure discretion. Then L_z^d and L_n^d satisfy the two valuation equations given by

$$L_z^d = \frac{1}{2}\left[\left(\pi_z^d\right)^2 + \lambda\left(x_z^d\right)^2\right] + \beta q L_z^d + \beta(1-q)L_n^d$$

and

$$L_n^d = \frac{1}{2}\left[\left(\pi_n^d\right)^2 + \lambda\left(x_n^d\right)^2\right] + \beta s L_n^d + \beta(1-s)L_z^d.$$

As discussed previously, these will be expressed in terms of the consumption equivalent welfare loss.

The importance of the calibration of q for these welfare losses is apparent in Figure 5, which shows L_z^d as a function of s and q.[18] Loss increases with q, given s. An increase in s, in contrast, lowers the loss as a larger s means the economy reverts less frequently to the ELB. When $s = 1$, $L_n^d = 0$ for all q.

Not surprisingly, given the results shown in Table 1, the instrument rule (8) with

[18] The figure shows loss for $q \leq 0.89$ to avoid values of q that imply $i_n < 0$.

$\phi = 1.5$ does significantly worse than what is achieved under optimal discretion. Thus, if forward guidance is interpreted as a promise to adopt the simple rule given by (8) whenever an ELB episode ends, the promise is not sustainable. The temptation to defect from such a promise is always positive in that loss when the ELB episode ends can be reduced by defecting to the discretionary policy.[19]

4 Keeping the nominal rate at zero

We are now in a position to evaluate the sustainability of forward guidance policies. In this section, the focus is on a promise to keep the nominal rate at zero in the first period in which the ELB constraint no longer binds. Keeping the nominal rate at zero after an ELB episode has been shown to be part of an optimal commitment policy by Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005), Nakov (2008) and Werning (2011). In the following section, this type of guidance is then generalized to allow for promises to keep the nominal rate at zero for several periods. Then, in section 6, forward guidance in the form of an announced rate of inflation for the first period after an ELB episode ends is investigated.

Suppose the policymaker promises to keep the nominal rate equal to zero for one period after the ELB constraint is relaxed. If the ELB remains nonbinding (which happens with probability s), I assume the policymaker implements the optimal discretionary policy given by (5). To evaluate this form of forward guidance, it is necessary to evaluate the present value of losses when the ELB binds, in the first period in which the constraint is relaxed (the exit period), and in future periods when the economy remains away from the ELB. Using the superscript fg to indicate outcomes under the forward guidance policy, denote these three values by L_z^{fg}, L_e^{fg}, and L_n^{fg}.

No forward guidance policy would be adopted if it led to a larger loss at the ELB, so $L_z^{fg} \leq L_z^d$ is a necessary condition for a welfare improving policy of forward guidance. However, such a policy will not be sustainable if the present value of the loss obtained by actually implementing the promised policy in the exit period exceeds the present value of the loss under discretion, i.e., if $L_e^{fg} > L_n^d$. If this condition held, then as soon as the economy exited from the ELB, the policymaker would have an incentive to defect and

[19] This finding is consistent with that of Nakov (2008) who found that simple instrument rules performed much worse than optimal rules in a stochastic environment with an occassionally binding non-negativity constraint on the nominal interest rate.

adopt the optimal time-consistant policy. Private agents, understanding the incentives faced by the policymaker would attach no credibility to the forward guidance provided at the ELB.

The policy would also not be sustainable if $L_n^{fg} > L_n^d$. However, this cannot be the case if $L_e^{fg} < L_n^d$. The reason is that if the economy remains away from the ELB, the forward guidance policy and the optimal discretionary policy both implement the targeting criterion given by the first order condition $\kappa\pi_n + \lambda x_n = 0$. Since expected future inflation and the output gap are closer to their optimal values of zero under forward guidance (as π_z^{fg} and x_z^{fg} are smaller in absolute value than π_z^d and x_z^d), a better outcome is achieved under the forward guidance policy. Thus, $L_e^{fg} < L_n^d$ implies $L_n^{fg} < L_n^d$. Only a comparison of the present value of losses in the exit period needs to be made to determine the policy's sustainability.[20]

Equilibrium now involves three inflation rates and three output gaps, corresponding to the situation at the ELB, during the exit period, and when the economy remains away from the ELB. It is also necessary to solve for the nominal interest rate when away from the ELB to ensure it is non-negative. The seven equilibrium conditions are as follows:

$$\pi_z^{fg} = \beta q \pi_z^{fg} + \beta(1-q)\pi_e^{fg} + \kappa x_z$$

$$x_z^{fg} = \left[q x_z^{fg} + (1-q)x_e^{fg}\right] + \left(\frac{1}{\sigma}\right)\left[q\pi_z^{fg} + (1-q)\pi_e^{fg} + r_z\right]$$

$$\pi_e^{fg} = \beta s \pi_n^{fg} + \beta(1-s)\pi_z^{fg} + \kappa x_e^{fg}$$

$$x_e^{fg} = \left[s x_n^{fg} + (1-s)x_z^{fg}\right] + \left(\frac{1}{\sigma}\right)\left[s\pi_n^{fg} + (1-s)\pi_z^{fg} + \rho\right]$$

$$\pi_n^{fg} = \beta s \pi_n^{fg} + \beta(1-s)\pi_z^{fg} + \kappa x_n^{fg}$$

[20] Let $\Delta L_j = L_j^{fg} - L_j^d$ for state j. Then

$$\Delta L_n = l_n^{fg} - l_n^d + \beta s \Delta L_n + \beta(1-s)\Delta L_z.$$

Thus,

$$\Delta L_n = \frac{l_n^{fg} - l_n^d + \beta(1-s)\Delta L_z}{1-\beta s}.$$

If the gain is positive ($\Delta L_z < 0$), then $l_n^{fg} - l_n^d$ is also negative as expected inflation and the output gap are closer to zero in state n under the forward guidance policy. Hence, $\Delta L_n = L_n^{fg} - L_n^d < 0$.

$$x_n^{fg} = \left[s x_n^{fg} + (1-s) x_z^{fg}\right] - \left(\frac{1}{\sigma}\right)\left[i_n - s\pi_n^{fg} - (1-s)\pi_z^{fg} - \rho\right]$$

$$\kappa \pi_n^{fg} + \lambda x_n^{fg} = 0,$$

together with the constraint that $i_n > 0$. The last equation reflects the assumption that if the economy remains away from the ELB, the optimal time-consistent policy is implemented.

Figures 6 and 7 show inflation and the output gap, respectively, at the ELB, in the exit period, and after the exit period under the $i_e = 0$ policy (solid lines) and under discretion (dashed lines) as a function of s for $q = 0.83$. Keeping i_e at zero boosts inflation and the output gap relative to the outcomes under pure discretion during the exit period. As a consequence, setting $i_e = 0$ in the exit period increases both inflation and the output gap at the ELB, as expected. This ensures both inflation and the output gap are closer to their desired values of zero when the economy remains away from the ELB, as shown in the bottom panel of each figure.[21]

The forward guidance policy will be sustainable if the gains at the ELB and the improved performance once away from the ELB outweigh any costs during the exit period. From the middle panel of figure 6, inflation is actually closer to its desired value (zero) under the forward guidance policy than under discretion except for s close to 1. As a result, the cost of keeping $i_e = 0$ arise only from the stronger expansion experienced in the exit period, reflected in the larger, positive value of the output gap.

To assess how much the promise to keep $i_e = 0$ improves over discretion from the perspective of an economy at the ELB, the present value of losses at the ELB, in the exit period, and in subsequent periods if the economy remains away from the ELB must be calculated. Denote the present value of losses in each of these states as L_z^{fg}, L_e^{fg}, and L_n^{fg}. These valuations must satisfy the following three conditions:

$$L_z^{fg} = \frac{1}{2}\left[\left(\pi_z^{fg}\right)^2 + \lambda\left(x_z^{fg}\right)^2\right] + \beta q L_z^{fg} + \beta(1-q) L_e^{fg}$$

$$L_e^{fg} = \frac{1}{2}\left[\left(\pi_e^{fg}\right)^2 + \lambda\left(x_e^{fg}\right)^2\right] + \beta s L_n^{fg} + \beta(1-s) L_z^{fg}$$

[21] The dashed lines for discretion are the same in the middle and bottom panels of both figures.

$$L_n^{fg} = \frac{1}{2}\left[\left(\pi_n^{fg}\right)^2 + \lambda\left(x_n^{fg}\right)^2\right] + \beta s L_n^{fg} + \beta(1-s)L_z^{fg}.$$

The gain from credible forward guidance is defined as

$$G \equiv L_z^d - L_z^{fg},$$

where L_z^d is the loss at the ELB under optimal discretion. If $G > 0$, then the loss is larger under discretion than with forward guidance. Figure 8 shows G; it is positive through the range of s and q such that $i_n^d > 0$, indicating that losses are smaller with forward guidance. Not surprisingly, the gain increases with q, that is, the lower the probability of exiting the ELB, and therefore the longer the expected duration of an episode at the ELB, the greater is the gain from forward guidance. In contrast, the gain decreases with s, as more frequent returns to the ELB (a lower s) increases the gain from forward guidance.

To assess the sustainability of a promise to keep the nominal interest at zero during the exit period, the present value of losses in the exit period must be less than that obtained by switching to the optimal discretionary policy. That is, sustainability requires that $L_e^{fg} \leq L_n^d$. Define the temptation to defect as

$$T \equiv L_e^{fd} - L_n^d.$$

If $T > 0$, the policy of forward guidance is not sustainable. Figure 9 shows temptation as a function of s and q. Previously, it was verified that temptation is positive for $s = 1$, in which case forward guidance is unsustainble. To highlight the consequences of even a very small chance of returning to the ELB, figure 10 shows cross-sections of the surfaces in figures 8 and 9 corresponding to $s = 1$ and $s = 0.999$ as a function of q. The top panel shows that forward guidance reduces the present value of loss at the ELB, regardless of q. Temptation to defect from this promise when $s = 1$ is shown in the middle panel and the case when $s = 0.999$ is shown in the lower panel. Notice the difference in scales for the middle and lower panel. When $s = 1$, temptation is small but positive for all q; when $s = 1$, forward guidance is not sustainable absent a commitment technology. When $s = 0.999$, however, temptation is negative for all q. Thus, if there is even a remote probability of a future ELB episode, a promise to maintain the nominal interest rate at zero for one period after the ELB constraint is relaxed is a sustainable policy.

This is an important result. It implies that the standard comparison of pure discretionary policies at the ELB with commitment policies is too limited. Even in the absence of an ability to commit to future actions, the promises of a discretionary policymaker can be credible. Forward guidance in the form of a pledge to keep the nominal interest rate at zero for one period after exiting from an episode at the ELB is a sustainable policy in an otherwise discretionary regime as long as s is strictly less than 1.[22]

Forward guidance is sustainable because the output and inflation costs of deviating from pure discretion in the exit period are small in the sense that the deviation of π_e^{fg} and x_e^{fg} from their counterparts under discretion turn out to be small. Hence, the cost of fulfilling the promised forward guidance is also small and is dominated by the benefit of improved performance at the ELB.

5 Multi-period promises

The previous section consider forward guidance that involved keeping the nominal interest rate at zero for one period after the ELB constraint is relaxed. Suppose the central bank promises to keep the nominal rate at zero for k periods after exiting an ELB episode. I assume that if the economy has remained away from the ELB for the full k periods, policy reverts to the optimal discretion targeting criterion given by (5). The previous section limited attention to the cases of $k = 0$ (pure discretion) and $k = 1$.

Results for $k = 0$ to $k = 4$ are presented in Table 2, which shows the present value of losses at the ELB, during the first period after an ELB episode ends, and when the economy has remained away from the ELB for $k + 1$ periods. Also reported is the gain from forward guidance and the temptation to defect from the policy promised in the forward guidance.

[22] Nakata (2014) finds that the Ramsey policy is sustainable for values of $s < 0.999$ and values of q in the range I consider (see his figure 3). Only a tiny probability of reverting to the ELB leads to credible policy of policies involving future promises.

Table 2: Multi-period Guidance

$s = 0.975,\ q = 0.83$					
k	L_z	L_e	L_n	Gain	Temp
0	0.584	0.434	0.434	0	0
1	0.240	0.178	0.178	0.344	−0.256
2	0.027	0.022	0.020	0.556	−0.412
3	0.127	0.102	0.094	0.457	−0.332
4	0.761	0.589	0.567	−0.178	0.155

Losses expressed as percent of steady-state consumption.

The column labeled L_z shows the present value of losses at the ELB. A credible promise to keep the nominal interest rate at zero for up to three periods after exiting the ELB episode significantly improves over discretion. The lowest loss is achieved with a promise to keep the nominal rate at zero for two periods beyond the end of the binding ELB constraint. This is also true for the alternative calibration of $s = 0.96$ and $q = 0.81$ (not shown). A promise to extend the period of a zero nominal interest rate to four periods is worse than the outcome without any promise. The final column provides evidence on the sustainability of forward guidance. For $k \leq 3$, forward guidance is sustainable. For $k = 4$, the gain is negative and so a promise of $k = 4$ would clearly not be sustainable.

For $k \leq 3$, forward guidance is sustainable because of the significant effect forward guidance has in raising inflation and the output gap at the ELB. As a consequence, it also leads both inflation and the output gap to be closer to zero when the economy is away from the ELB than is achieved by discretion. The equilibrium outcomes for inflation and the output gap for $k = 0$ (discretion) and for $k = 2$ are shown in Table 3. Even though the output gap is much larger during the exit period under forward guidance, Table 2 showed that L_e^{fg} is only half the loss experienced in the absence of forward guidance.

Table 3: Outcomes under discretion and for $k = 2$

$s = 0.975,\ q = 0.83$						
k	π_z	π_e	π_n	x_z	x_e	x_n
0	−0.958	−0.141	−0.141	−1.838	0.235	0.235
2	−0.607	−0.053	−0.089	−1.241	0.606	0.149

* Inflation at annual rates; output gap in percent.

A promise to keep the nominal interest rate at zero for two periods after exiting the ELB constraint is optimal under the baseline calibration of the simple model. Importantly, such a multi-period promise improves outcomes significantly relative to pure discretion and is sustainable. Even though it has been assumed that there is no commitment mechanism and that the central bank will renege on past promises whenever the expected present value of losses exceeds that obtained under discretion, the optimal forward guidance is sustainable. A central bank that cannot commit can still credibly promise to keep interest rates at zero beyond the end of the ELB episode.

6 Announcing future inflation

A promise to keep the nominal interest rate at zero past the end of the ELB period is not the only form of forward guidance. As Cochrane (2013) emphasizes, what is important is the inflation rate to which the policymaker commits. In this section therefore, forward guidance is interpreted as a promise of a future rate of inflation. I assume the policymaker announces an inflation rate for the period immediately following exit from the ELB. When the economy is away from the ELB, the policymaker sets $i = \rho$. Thus, regardless of the inflation rate the central bank announces for the exit period, the path of the nominal interest rate is assumed to be the same: while at the ELB, $i = 0$ and when not at the ELB, $i = \rho > 0$. It is well known that such exogenous paths for the nominal interest rate are consistent with multiple equilibria.[23] Following Cochrane (2013), these equilibria can be indexed by the promised inflation rate during the period when the ELB becomes nonbinding. By announcement an inflation rate for the exit period, the policymaker is selecting a particular equilibrium.[24]

6.1 Equilibrium with an arbitrary exit inflation rate

Denote the exit period by $t = e$ and the announced inflation rate by π_e^a. Given π_e^a, I construct an equilibrium for π_z and x_z, as well as for π_{e+j} for $j > 0$ and for x_{e+j} for $j \geq 0$. The resulting paths for inflation and the output gap will be denoted by a superscript a.

[23] On solving models with exogenous policy paths, see Laséen and Svensson (2011).

[24] Equivalently, consider an instrument rule of the form

$$i_t = \max\left[0,\, \rho + \phi\left(\pi_t - \pi_t^a\right)\right],$$

where π_t^a is the announced path for inflation when the economy remains away from the ELB and $\phi > 1$. In equilibrium, $\pi_t = \pi_t^a$ when the economy is not at the ELB, so $i_t = \rho$.

When Cochrane solved for the equilibrium associated with an announced inflation rate, he assumed the economy never reverts to the ELB. In that case, the equilibrium for $j > 0$ is independent of the equilibrium at the ELB, as one can obtain x_e^a as the unique value consistent with a stationary equilibrium and then solve the model forward for the paths of inflation and the output gap in the post-ELB periods. Given π_e^a and x_e^a as terminal conditions, one can also solve backward for the equilibrium during the ELB period. This separation, in which the post-ELB equilibrium is independent of the equilibrium during the ELB, is no longer possible when $s < 1$. Knowledge of π_z^a and x_z^a is required to determine x_e^a, because expectations for $j > 0$ put some weight on reverting to the ELB. Hence, the equilibrium after the exit period depends on π_z^a and x_z^a during the ELB period as well as on π_e^a and x_e^a.

Equilibrium involves solving

$$\begin{bmatrix} \pi_{e+1}^a \\ x_{e+1}^a \end{bmatrix} = P \begin{bmatrix} \pi_e^a \\ x_e^a \end{bmatrix} + Q \begin{bmatrix} \pi_z^a \\ x_z^a \end{bmatrix} \tag{16}$$

for the post-exit equilibrium and

$$\begin{bmatrix} \pi_z^a \\ x_z^a \end{bmatrix} = S \begin{bmatrix} \pi_e^a \\ x_e^a \end{bmatrix} + Tr_z \tag{17}$$

for the ELB period. The matrices appearing in these two systems of equations are given in the appendix, which also provides the details on how (16) and (17) are solved. If $s = 1$, then $Q = 0$, and (16) for the post-exit equilibrium can be solved independently of the equilibrium while the ELB binds. This corresponds to the case considered by Cochrane.[25] If $s < 1$, then $Q \neq 0$, and it is necessary to solve (16) and (17) jointly. And this means the equilibrium when away from the ELB is affected not just by the expected time until another ELB episode, as determined by s, but also by the expected duration of ELB episodes, as determined by q.

Valuation equations are also somewhat more complicated in the case of an announced inflation rate, as inflation and the output gap are not constant once the exit period is over. Let l_z^a, l_e^a, and l_{e+i}^a denote the period losses at the ELB, in the exit period, and

[25] Cochrane assumed the ELB period lasted for a fixed number of periods rather than that the duration is stochastic as employed here. However, the points made apply in either case.

after $i \geq 1$ periods during which the ELB constraint has been nonbinding. Then

$$L_z^a = l_z^a + \beta q L_z^a + \beta(1-q)L_e^a$$

$$L_e^a = l_e^a + \beta s L_{e+1}^a + \beta(1-s)L_z^a$$

$$L_{e+1}^a = l_{e+1}^a + \beta s L_{e+2}^a + \beta(1-s)L_z^a$$

$$L_{e+2}^a = l_{e+2}^a + \beta s L_{e+3}^a + \beta(1-s)L_z^a$$

and so on. Given the assumption of stationarity,

$$\lim_{T\to\infty} L_{e+T}^a = \frac{1}{1-\beta s}\left[\lim_{T\to\infty} l_{e+T}^a + \beta(1-s)L_z^a\right].$$

6.2 Results for announced inflation

Figure 11 shows the gain, measured by $L_z^d - L_z^a$, from making an inflation announcement as a function of s and π_e^a for $q = 0.83$. For all s, there is a range of inflation announcements that reduce the PDV of losses at the ELB relative to optimal discretion and produce a positive gain. Both the upper and lower boundaries of this range increase as s decreases. A decrease in s implies a higher probability of reverting to the ELB. This reduces expected inflation and the output gap when not at the ELB, thereby worsening outcomes when not at the ELB as well as when at the ELB. To offset the decline in welfare induced by a fall in s, a higher inflation rate must be announced for the exit period if outcomes are to be improved over discretion. A rise in q, implying a longer expected duration of ELB episodes, increases the maximum announced inflation rate that still delivers a welfare gain.

Are inflation announcements sustainable? To answer this questions requires an examination of the temptation to revert to the discretionary equilibrium rather than deliver the promised inflation rate in the exit period. This temptation for $q = 0.83$ it is shown in figure 12. An announced inflation rate π_e^a is sustainable if it yields a welfare gain and the temptation to defect is negative. The announced inflation rates for which temptation is negative is a subset of the rates for which the gain shown in figure 11 is positive. Thus, some announced rates that would improve welfare are not sustainable. Figure 13 illustrates this by plotting gain and temptation as a function of π_e^a for various combinations of s and q. For example, if $s = 1$, there is no inflation announcement that is sustainable (top panel). The set of sustainable announced inflation rates shrinks as q declines. A fall in q reduces the gain and increases the temptation to default on promises; a rise in q, by leading to longer ELB episodes, increases the gain from promises and also reduces the temptation to defect.

7 Optimal inflation announcements

The previous section considered equilibria indexed by the inflation rate promised once the ELB episode ends. Different choices for π_e^a lead to different losses at the ELB. Figure 14 shows L_z^a as a function of π_e^a for different values of s and q. The lines marked by circles and diamonds correspond to the two baseline calibrations for s and q; the dashed lines show other combinations of s and q. The optimal inflation rate to announce, for a given s and q, is the value that minimizes L_z^a. Because L_z^d is independent of any announcement, minimizing L_z^a is the same as maximizing the gain $L_z^d - L_z^a$. Figure 15 shows the optimal inflation rate to promise for the exit period as a function of s and q. The value of π_e^a at which L_z^a reaches a minimum decreases with s and with q. Results are intuitive: given s, π_z^a rises with the expected persistence of an ELB episode (a rise in q); given q, it falls as the probability of reverting to the ELB declines (a rise in s).

Is the optimal inflation rate to announce for the exit period sustainable? For the equilibrium in which the central bank promises to deliver the inflation rate in the exit period that minimizes L_z^a, it must be that (a) the gain is positive and (b) the temptation to defect once the exit period arrives is negative.

Table 4 shows the optimal inflation rate, denoted by π_e^{a*} and the sign of the gain and of the temptation. In all cases, the optimal inflation rate to promise for the exit period is quite small. The two benchmark combinations of s and q are highlighted in bold. For the calibration based on the longer sample ($s = 0.96$ and $q = 0.81$), the gain from the optimal inflation announcement is positive, but the temptation to defect once the exit period arrives is also positive. Thus, the optimal announcement is not sustainable. The expected duration of ELB episodes is critical in determining sustainability; holding $s = 0.96$, the optimal inflation increases and is sustainable when q is increased from 0.81 to 0.83 (second row). For the calibration based on the 1960-2016 experience ($s = 0.975$ and $q = 0.83$), the optimal announced inflation rate improves welfare at the ELB and is sustainable; the present value of the loss in the exit period from fulfilling the promise is less than the loss obtained under discretion. The period loss is initially larger when the announcement is carried out, but the expected gain should the economy again encounter the ELB outweighs this one-period loss, and the central bank has an incentive to fulfill its promise. Of course, when $s = 1$, no forward guidance is sustainable.

Table 4: Sustainability of optimal π_e^a

s	q	π_e^{a*}	L_z	L_e	L_n	L_z^d	L_n^d	Gain	Temp	Sustainable
0.96	**0.81**	0.500	0.419	0.433	0.403	0.460	0.384	+	+	No
0.96	0.83	0.588	0.515	0.534	0.486	0.840	0.699	+	-	Yes
0.975	0.81	0.413	0.222	0.228	0.202	0.315	0.235	+	-	Yes
0.975	**0.83**	0.500	0.282	0.291	0.250	0.584	0.434	+	-	Yes
1.0	0.88	0.713	0.098	0.090	0.005	2.262	0.0	+	+	No

Figure 16 divides the area defined by $s \in [0.95\ 1]$ and $q \in [0.75\ 0.9]$ into two regions based on the sustainability of the optimal inflation announcement. The two baseline calibrations for s and q are indicated in the figure. For $s = 1$, no announcement is sustainable. As s falls below 1, the optimal inflation announcement policy becomes sustainable, but only if the expected duration of ELB episodes is sufficiently long (i.e, for sufficiently high values of q). If s is large, the potential future benefit of credibility declines as this credibility is unlikely to be needed. But if q is high enough, the value of credibility increases as ELB episodes are likely to be long lasting. However, once s falls more, then a policy may become unsustainable. This was illustrated in Table 4; for $q = 0.81$, the optimal inflation announcement was sustainable if $s = 0.975$ but it wasn't if $s = 0.96$.

Ceteris paribus, a fall in s makes ELB episodes more frequent and worsens outcomes while at the ELB. This increases the optimal inflation rate to announce for the exit period which, in turn, increases equilibrium inflation both at the ELB and for periods beyond the exit period. Both π_z^a and π_n^a deviate more from zero when $s = 0.96$ than when $s = 0.975$ (for the same q). The effects on the output gap for the two values of s are less pronnunced than the inflation differences, and with λ small, it is the differences in inflation that dominate the welfare comparison. A fall in s from 0.975 to 0.96 worsens outcomes away from the ZLB under discretion, but the deterioration during the exit period under the inflation announcement policy is even greater, leading the optimal announcement policy to be unsustainable.

Table 5 shows the equilibrium outcomes for different states under discretion (top row) and with the optimal inflation announcement (bottom row) for $s = 0.975$ and $q = 0.83$. It is informative to compare table 5 to table 3 which showed the outcomes when forward guidance takes the form of a promise to keep the nominal rate at zero for two periods after the ELB episode ends.

Table 5: Outcomes under discretion and the optimal π_e^a

$s = 0.975,\ q = 0.83$						
	π_z	π_e	π_n	x_z	x_e	x_n
discretion	-0.958	-0.141	-0.141	-1.838	0.235	0.235
π_e^a	0.053	0.500	0.205	-0.935	0.441	0.073

* Inflation at annual rates; output gap in percent.

The results in tables 2 and 4 allow us to assess the relative performance of sustainable promises to maintain the nominal rate at zero for multi-periods versus promising an optimal inflation rate for the exit period. For $s = 0.975$ and $q = 0.83$, the present value of losses at the ELB under discretion is 0.584% of steady-state consumption. Promising to keep the nominal rate at zero for two periods after the end of the ELB period reduces the loss to 0.027%. Promising an exit inflation rate of 0.500% leads to a loss of 0.282%. Forward guidance in the form of a promise to maintain the nominal rate at zero clearly dominates. While the promised inflation rate significantly boosts π_z (actually making it positive, see table 5), the cost comes in the form of a much higher inflation rate in the exit period (0.500% versus -0.053% with the interest rate promise, see table 3) and higher inflation for periods after the exit period. The optimal inflation announcement stabilizes the output gap more successfully than the zero nominal rate guidance. In each state, the output gap is closer to zero with the inflation announcement relative to policy of keeping the nominal rate at zero for two periods after existing the ELB.

8 Robustness (incomplete)

McKay, Nakamura, and Steinsson (2016b) have argued that the basic Euler equation given by (1) implies implausibly large effects of forward guidance and these large effects arise because expected future output has a one-to-one effect on current output. Based on an incomplete markets model that leads to precautionary savings on the part of households, they propose a discounted Euler equation that takes the form

$$x_t = \delta \mathrm{E}_t x_{t+1} - \left(\frac{\chi}{\sigma}\right)(i_t - \mathrm{E}_t \pi_{t+1} - r_t), \tag{18}$$

with $0 < \delta \leq 1$ and $0 < \chi \leq 1$. In their base calibration, they set $\delta = 0.97$ and $\chi = 0.75$. With these values, together with the same parameter values used by Eggertsson and Woodford (2003), they find the output gap is -2.88% at the ELB, significantly less that the -14.43% obtained with the standard Euler equation. While they consider only the case in which the economy never returns to the ELB once it exits, similar effects carry over to the case in which $s < 1$. Because both inflation and the output are not as negative at the ELB as with the standard Euler equation, expected inflation and output are higher for any $s < 1$ when the economy is not at the ELB. This means, in turn, that under discretion the nominal interest rate is not as low when the economy is away from the ELB. As a consequence, $i_n > 0$ for even small s and large q, unlike the case depicted in figure 3 for the standard Euler equation.

The discounted Euler equation implies the consequences of the ELB and the strength of forward guidance policies are muted. However, the basic findings are robust to replacing (1) with (18). Table 6 shows that the optimal number of periods to promise to keep the nominal interest rate at zero after exiting an ELB episode is still two, and this promise is sustainable.

Table 6: Multi-period Guidance: Discounted Euler

$s = 0.975$, $q = 0.83$					
k	L_z	L_e	L_n	Gain	Temp
0	0.200	0.149	0.149	0	0
1	0.081	0.061	0.061	0.112	−0.094
2	0.012	0.010	0.009	0.189	−0.140
3	0.040	0.034	0.030	0.160	−0.074
4	0.227	0.180	0.169	−0.027	0.031

9 Summary and conclusions

Recent research has emphasized the adverse consequences for the economy when the central bank's policy instrument is constrained by an effective lower bound on the short-term nominal interest rate and policy is implemented in a time-consistent, discretionary manner. These adverse effects stand in contrast to the situation in which the central bank is able to implement the optimal but time-inconsistent commitment policy. Under the presumption that discretion is the more realistic assumption about policy, proposals for reforming inflation targeting policy frameworks have emphasized changes that either make it less likely the ELB will be encountered or that establish alternative regimes, such as price-level targeting, that can cause expectations to move in a manner that promotes stabilization and mimics a commitment policy regime.

Forward guidance is powerful in models, like the new Keynesian model, in which agents are rational and forward looking. However, if rational agents believe the policymaker will never honor promises about policy in the post-ELB environment, forward guidance lacks credibility. Consequently, if the central bank cannot commit, ELB episodes are likely to be costly, and policy reform should seek to reform flexible inflation targeting to ensure better outcomes at the ELB.

Proposed reforms presume that the ELB will be encountered again in the future. Yet analytical analysis of policy at the ELB typically assumes that once the economy exits the ELB, it never again encounters the ELB.[26] If this is the case, then any promises about future policy – that is, forward guidance – lack credibility. Once the economy is out of the ELB period, there is no incentive for the policymaker to implement the policies that were promised in the past.

But if the economy may revert to the ELB, then promises made during an ELB episode may be credible even in the absence of a commitment mechanism. If the promised policy actions improve outcomes when at the ELB, then it may be rational for the central bank to fully implement those promises because, while doing so generates a cost, it also brings an expected future benefit. Future promises may be sustainable.

I modify the basic model of Eggertsson and Woodford (2003) to allow for both a constant probability of exiting the ELB and a constant probability of returning to the ELB. Unlike the standard analysis, the economy does not achieve zero inflation and a zero output gap once it exits the ELB. With a positive probability of reverting to the ELB, expected inflation and the output gap are no longer zero as in the Eggertsson-Woodford analysis. The equilibrium also depends on the assumption made about post-ELB policy, so a simple instrument rule does not lead to the same equilibrium as obtained optimal

[26] As noted earlier, the exception is Nakata (2014).

discretion. Outcomes at the ELB are much worse if the central bank is expected to adopt a simple but commonly employed instrument rule than if it follows the optimal one-period policy upon exiting.

The main focus of the paper, though, is on the sustainability of forward guidance. Three forms of such guidance were considered: a pledge to keep the nominal interest rate at zero for the initial post-ELB period; a promise to keep the nominal rate at zero for multiple periods after an ELB episode ends; and a promise to deliver a specific inflation rate on exiting the ELB. When there is no chance of returning to the ELB, none of these policies are sustainable. However, if there is even the slightest chance of returning to the ELB, forward guidance policies may be sustainable. For example, the promise to keep the nominal rate at zero for one period after the ELB constraint is relaxed is sustainable if the probability of another ELB episode is as little as 0.1%. For multiple-period forward guidance, policies that promise to keep the nominal rate at zero for too long are unsustainable. However, the optimal number of periods in the calibrated model was only two periods, and this policy is sustainable.

I also investigate the effects of promising a specific inflation rate on exiting the ELB, with the nominal rate held to zero during the ELB and set equal the natural rate when not at the ELB. Here the results depended on the calibration. For values calibrated to the 1934-2016 period, the optimal inflation rate to promise led to a welfare gain at the ELB, but such a promise was not sustainable. Once the ELB period ended, the central bank would have an incentive to revert to the policy under pure discretion. For values calibrated to the 1960-2016 period, however, the optimal inflation rate to promise was lower, still led to a welfare gain, and was sustainable.

The results obtained here were derived using a very stylized model. The basic model does, however, generalize the framework that has been employed widely in analyzes of the ELB. There are many directions in which the basic model could be extended to determine how robust the reported findings are. The key implication, a result consistent with the findings of Nakata (2014), is likely to be robust: if future episodes at the ELB are likely, then promises made during the ELB period may be credible despite the absence of any mechanism to ensure commitment.

10 Appendix

This appendix outlines the method used to solve for the equilibrium with an exogenous interest rate path and an announced rate of inflation for the exit period. To solve the equilibrium involves several steps. Note that for e,

$$\pi_e^a = \beta s \pi_{e+1}^a + \beta(1-s)\pi_z^a + \kappa x_e^a$$

and

$$\begin{aligned} x_e^a &= \left[s x_{e+1}^a + (1-s) x_z^a \right] - \left(\frac{1}{\sigma} \right) \left[i_e - s\pi_{e+1}^a - (1-s)\pi_z^a - \rho \right] \\ &= \left[s x_{e+1}^a + (1-s) x_z^a \right] + \left(\frac{1}{\sigma} \right) \left[s\pi_{e+1}^a + (1-s)\pi_z^a \right], \end{aligned}$$

where terms such as π_{e+1}^a and x_{e+1}^a denote equilibrium values along the path that remains out of the ELB and $i_e = \rho$ has been used.[27]

Rewrite these two equations as

$$\begin{bmatrix} \pi_{e+1}^a \\ x_{e+1}^a \end{bmatrix} = P \begin{bmatrix} \pi_e^a \\ x_e^a \end{bmatrix} + Q \begin{bmatrix} \pi_z^a \\ x_z^a \end{bmatrix} \tag{19}$$

where

$$P = \begin{bmatrix} \beta s & 0 \\ s & \sigma s \end{bmatrix}^{-1} \begin{bmatrix} 1 & -\kappa \\ 0 & \sigma \end{bmatrix}$$

and

$$Q = \begin{bmatrix} \beta s & 0 \\ s & \sigma s \end{bmatrix}^{-1} \begin{bmatrix} -\beta(1-s) & 0 \\ -(1-s) & -\sigma(1-s) \end{bmatrix}.$$

The dependence of the post-ELB inflation rate and output gap on equilibrium at the ELB is illustrated clearly in (19). If $s = 1$, $Q = 0$ and the post-exit equilibrium is independent of the pre-exit equilibrium.

Define $\bar{\pi}$ and $\bar{x}$ as the steady-state, stationary equilibrium values of inflation and the

[27] That is, if the economy has remained away from the ELB for $j+1$ periods, then inflation is equal to π_{e+j}^a. In the following period, inflation will equal π_{e+j+1}^a with probability s and π_z^a with probability $1-s$.

output gap if the economy were to remain away from the ELB. Then from (19),

$$\begin{bmatrix} \bar{\pi} \\ \bar{x} \end{bmatrix} = P \begin{bmatrix} \bar{\pi} \\ \bar{x} \end{bmatrix} + Q \begin{bmatrix} \pi_z^a \\ x_z^a \end{bmatrix}. \tag{20}$$

Subtracting this from (19) results in

$$\begin{bmatrix} \hat{\pi}_{e+1}^a \\ \hat{x}_{e+1}^a \end{bmatrix} = P \begin{bmatrix} \hat{\pi}_e^a \\ \hat{x}_e^a \end{bmatrix}, \tag{21}$$

where $\hat{\pi} = \pi^a - \bar{\pi}$ and $\hat{x} = x^a - \bar{x}$ are now expressed in deviation form. The matrix P has one eigenvalue outside the unit circle and one inside, so multiple equilibria are feasible; a locally unique, stationary equilibrium will be selected by the policymaker's announcement of the inflation rate in the exit period.

Note that (20) consists of the following two equations:

$$(1 - \beta s)\,\bar{\pi} = \beta(1 - s)\pi_z + \kappa\bar{x}$$

$$(1 - s)\bar{x} = (1 - s)x_z + \left(\frac{1}{\sigma}\right)[s\bar{\pi} + (1 - s)\pi_z.]$$

We can solve these two equation:

$$\bar{x} = \frac{(1 - s)\left[\sigma\,(1 - \beta s)\,x_z + \pi_z\right]}{\left[\sigma(1 - s)\,(1 - \beta s) - s\kappa\right]}.$$

$$\bar{\pi} = \left[\frac{\beta(1 - s)\pi_z + \kappa\bar{x}}{1 - \beta s}\right]$$

The denominator in the expression for $\bar{x}$ is negative for the benchmark value $s = 0.975$. Since the numerator is also negative, $\bar{x} > 0$ but it is decreasing in π_z – i.e., since increasing π^a increases π_z and x_z, it decreases $\bar{x}$.

The matrix P in (21) can be written as $P = VDV^{-1}$, where D is a diagonal matrix with elements equal to the eigenvalues of P and V consists of the eigenvectors of P. P has one eigenvalue greater than one and one less than one. Assume D is ordered such that the largest eigenvalue is ordered first. Premultiplying (21) by V^{-1} yields

$$V^{-1} \begin{bmatrix} \hat{\pi}_{e+1}^a \\ \hat{x}_{e+1}^a \end{bmatrix} = DV^{-1} \begin{bmatrix} \hat{\pi}_e^a \\ \hat{x}_e^a \end{bmatrix}$$

which can be written as

$$\begin{bmatrix} z^a_{1,e+1} \\ z^a_{2,e+1} \end{bmatrix} = D \begin{bmatrix} z^a_{1,e} \\ z^a_{2,e} \end{bmatrix} \tag{22}$$

where

$$\begin{bmatrix} z^a_{1,e} \\ z^a_{2,e} \end{bmatrix} \equiv V^{-1} \begin{bmatrix} \hat{\pi}^a_e \\ \hat{x}^a_e \end{bmatrix}.$$

The system in (22) consists of two equations:

$$z_{1,e+1} = \lambda_1 z_{1,e}\colon\ \lambda_1 \geq 1;$$

$$z_{2,e+1} = \lambda_1 z_{2,e}\colon\ \lambda_2 < 1.$$

Since $\lambda_1 \geq 1$, the first of these equations is nonstationary. Hence, if attention is restricted to stationary equilibria, we require that

$$z_{1,e} = 0.$$

Let

$$V^{-1} \equiv \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}.$$

Then $z_{1,e} = 0$ if and only if

$$v_{11}\hat{\pi}^a_e + v_{12}\hat{x}^a_e = 0,$$

or

$$\hat{x}^a_e = -\left(\frac{v_{11}}{v_{12}}\right)\hat{\pi}^a_e. \tag{23}$$

This determines $\hat{x}^a_e$ once $\hat{\pi}^a_e$ is fixed (but fixing $\hat{\pi}^a_e$ requires knowing the announced inflation rate π^a_e and $\bar{\pi}$, the mean post-exit inflation rate which will depend on π^a_z and x^a_z).

Given $\hat{\pi}_e$ and $\hat{x}_e$, $z_{2,e}$ is given by

$$z_{2,e} = v_{21}\hat{\pi}^a_e + v_{22}\hat{x}^a_e.$$

Since $z_{2,e+1} = \lambda_2 z_{2,e}$, knowledge of $z_{2,e}$ determines $z_{2,e+1}$. Future values of π^a_{e+j} and x^a_{e+j}

for $j \geq 1$, if the economy remains away from the ELB are obtained by jointly solving

$$0 = v_{11}\hat{\pi}^a_{e+j} + v_{12}\hat{x}^a_{e+j}$$

$$z_{2,e+j} = v_{21}\hat{\pi}^a_{e+j} + v_{22}\hat{x}^a_{e+j} = \lambda_2 z_{2,e+j-1} = \lambda_2 \left(v_{21}\hat{\pi}^a_{e+j-1} + v_{22}\hat{x}^a_{e+j-1}\right).$$

for the $e+j$ equilibrium, and so on. More compactly, once we have $\hat{\pi}^a_e$ and $\hat{x}^a_e$, then future values of inflation and the output in the non-ELB equilibrium can be obtained by

$$\begin{bmatrix} v_{11}\hat{\pi}^a_{e+1} + v_{12}\hat{x}^a_{e+1} \\ v_{21}\hat{\pi}^a_{e+1} + v_{22}\hat{x}^a_{e+1} \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda_2 \left(v_{21}\hat{\pi}^a_e + v_{22}\hat{x}^a_e\right) \end{bmatrix},$$

or

$$\begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} \begin{bmatrix} \hat{\pi}^a_{e+1} \\ \hat{x}^a_{e+1} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} \begin{bmatrix} \hat{\pi}^a_e \\ \hat{x}^a_e \end{bmatrix}$$

as $v_{11}\hat{\pi}^a_e + v_{12}\hat{x}^a_e = 0$. Pre-multiplying both sides by V (recalling that the matrix of the $v'_{ij}s$ is V^{-1} yields

$$\begin{aligned} \begin{bmatrix} \hat{\pi}^a_{e+1} \\ \hat{x}^a_{e+1} \end{bmatrix} &= V \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} V^{-1} \begin{bmatrix} \hat{\pi}^a_e \\ \hat{x}^a_e \end{bmatrix} \\ &= VDV^{-1} \begin{bmatrix} \hat{\pi}^a_e \\ \hat{x}^a_e \end{bmatrix} = P \begin{bmatrix} \hat{\pi}^a_e \\ \hat{x}^a_e \end{bmatrix}, \end{aligned} \tag{24}$$

as required.

It remains to determine $\bar{\pi}$ and $\bar{x}$. Recall we started with an announced inflation rate π^a_e. We need $\bar{\pi}$ to map π^a_e into $\hat{\pi}^\alpha_e$, and we need $\bar{x}$ to map $\hat{x}^a_e$ into x^a_e. Once x^a_e is known, it, together with π^a_e, allow π^a_z and x^a_z to be obtained. At the ELB, π^a_x and x^a_z must satisfy

$$(1 - \beta q)\,\pi_z = \beta(1-q)\pi^a_e + \kappa x_z$$

and

$$\sigma\,(1-q)\,x_z = \sigma(1-q)x^a_e + \left[q\pi_z + (1-q)\pi^a_e + r_z\right],$$

or

$$\begin{bmatrix} \pi^a_z \\ x^a_z \end{bmatrix} = S \begin{bmatrix} \pi^a_e \\ x^a_e \end{bmatrix} + Tr_z \tag{25}$$

where

$$S=\left[\begin{array}{cc}1-\beta q & -\kappa \\ -q & \sigma(1-q)\end{array}\right]^{-1}\left[\begin{array}{cc}\beta(1-q) & 0 \\ (1-q) & \sigma(1-q)\end{array}\right]$$

$$T=\left[\begin{array}{cc}1-\beta q & -\kappa \\ -q & \sigma(1-q)\end{array}\right]^{-1}\left[\begin{array}{c}0 \\ 1\end{array}\right].$$

From (23),

$$\left[\begin{array}{c}\pi_e^a \\ x_e^a\end{array}\right]=\left[\begin{array}{cc}0 & 0 \\ \frac{v_{11}}{v_{12}} & 1\end{array}\right]\left[\begin{array}{c}\bar{\pi} \\ \bar{x}\end{array}\right]+\left[\begin{array}{c}1 \\ -\frac{v_{11}}{v_{12}}\end{array}\right]\pi_e^a.$$

This allows us to rewrite (25) as

$$\left[\begin{array}{c}\pi_z^a \\ x_z^a\end{array}\right]=S\left[\begin{array}{cc}0 & 0 \\ \frac{v_{11}}{v_{12}} & 1\end{array}\right]\left[\begin{array}{c}\bar{\pi} \\ \bar{x}\end{array}\right]+S\left[\begin{array}{c}1 \\ -\frac{v_{11}}{v_{12}}\end{array}\right]\pi_e^a+Tr_z.$$

Using this in (20) yields

$$\left[\begin{array}{c}\bar{\pi} \\ \bar{x}\end{array}\right]=P_2\left[\begin{array}{c}\bar{\pi} \\ \bar{x}\end{array}\right]+P_3\pi_e^a+QTr_z,$$

where

$$P_2\equiv P+QS\left[\begin{array}{cc}0 & 0 \\ \frac{v_{11}}{v_{12}} & 1\end{array}\right],$$

and

$$P_3\equiv QS\left[\begin{array}{c}1 \\ -\frac{v_{11}}{v_{12}}\end{array}\right].$$

Since r_z is exogenous and π_e^a is a policy choice, we can then solve for $\bar{\pi}$ and $\bar{x}$ as

$$\left[\begin{array}{c}\bar{\pi} \\ \bar{x}\end{array}\right]=(I_2-P_2)^{-1}\left(P_3\pi_e^a+QTr_z\right), \tag{26}$$

where I_2 is the 2×2 identity matrix. From $\bar{\pi}$, $\hat{\pi}_e^a=\pi_e^a-\bar{\pi}$. From (23),

$$x_e^a=\bar{x}-\left(\frac{v_{11}}{v_{12}}\right)\left(\pi_e^a-\bar{\pi}\right). \tag{27}$$

This completes the derivation of the solution. Given the announced inflation rate

π_e^a for the exit period $t = e$, (26) can be solved for the post-exit average inflation and output gap. Equation (27) then pins down x_e^a consistent with a stationary equilibrium. Equilibrium when the ELB is binding is given by (25), while (24) can be solved for future inflation rates and output gaps should the economy remain away from the ELB.

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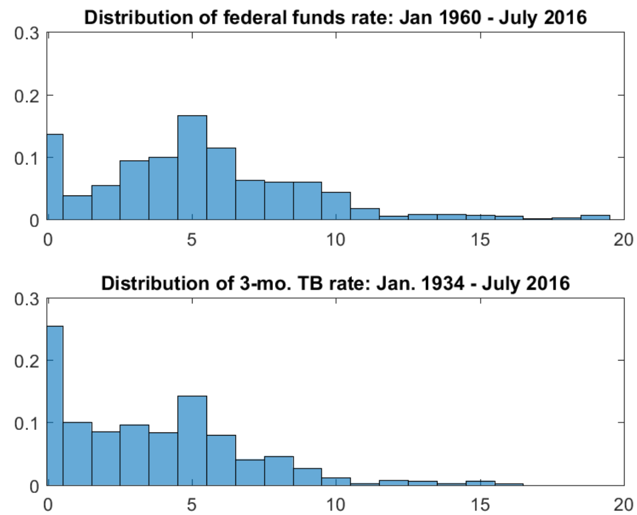

Figure 1: Histogram of U.S. interest rates. Upper panel: federal funds rate. Lower panel: 3-month T-Bill rate.

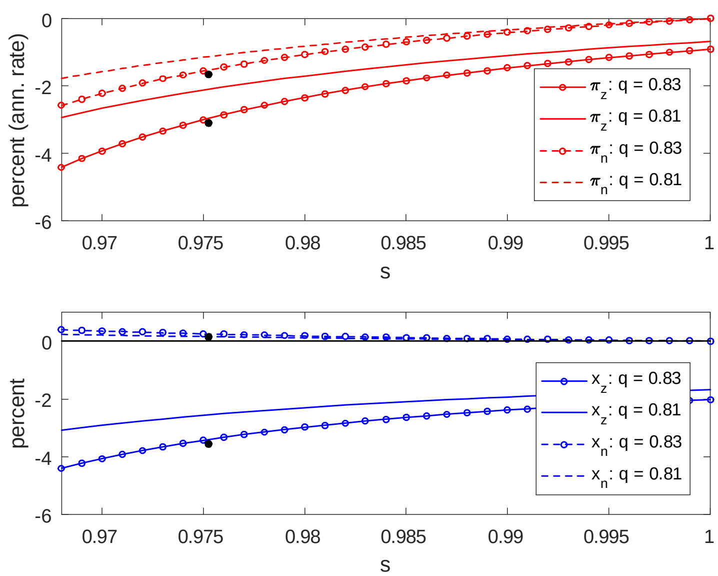

Figure 2: Equilibrium inflation (upper panel) and the output gap (lower panel) under policy rule (8). Base calibration indicated by bullet, alternative by x.

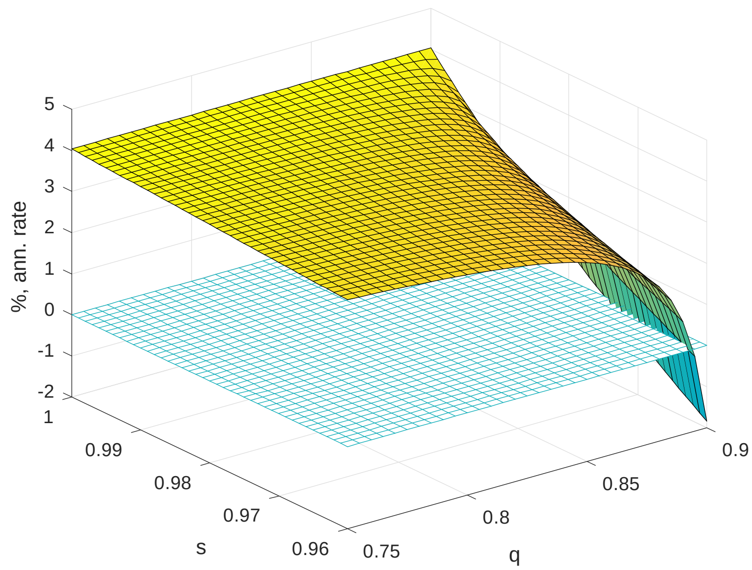

Figure 3: Nominal interest rate away from the ELB under optimal discretion.

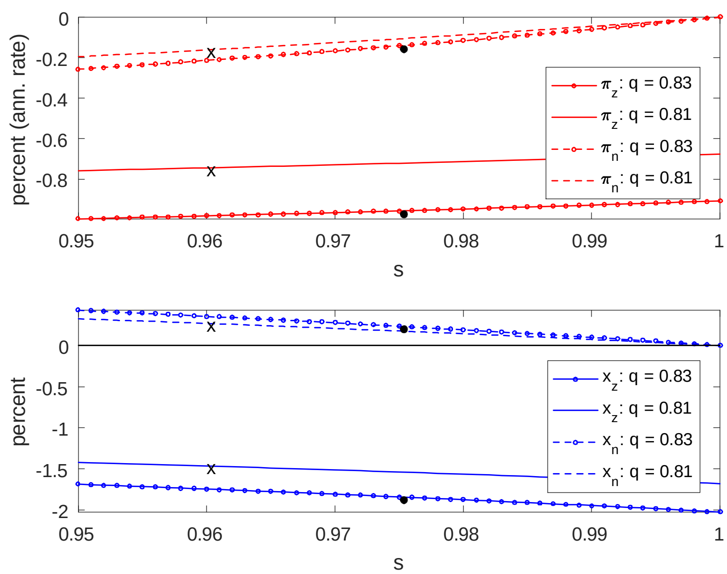

Figure 4: Equilibrium inflation (upper panel) and the output gap (lower panel) under optimal discretion. Base calibration indicated by bullet, alternative by x.

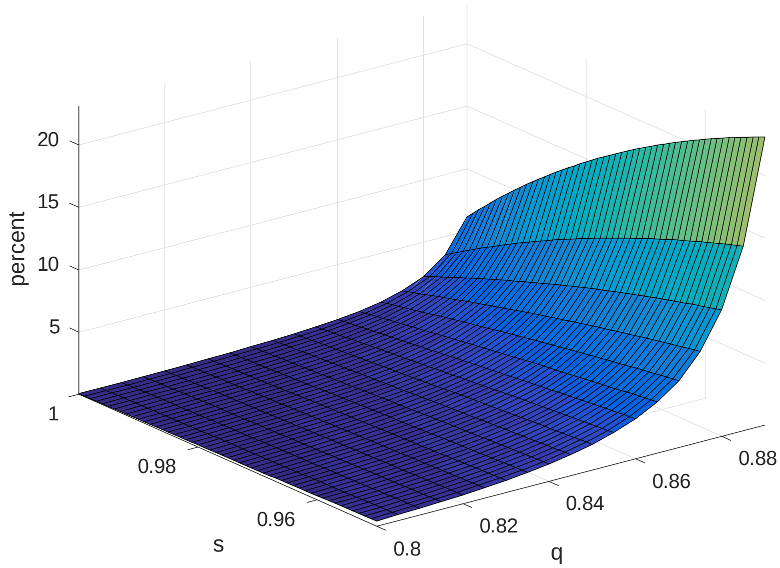

Figure 5: Present value of losses at the ELB: optimal discretion.

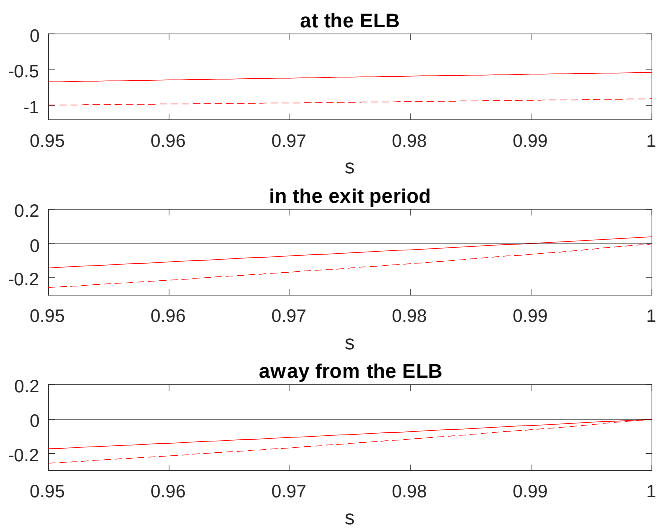

Figure 6: Inflation with forward guidance: solid line, $i_e = 0$; dashed line, discretion.

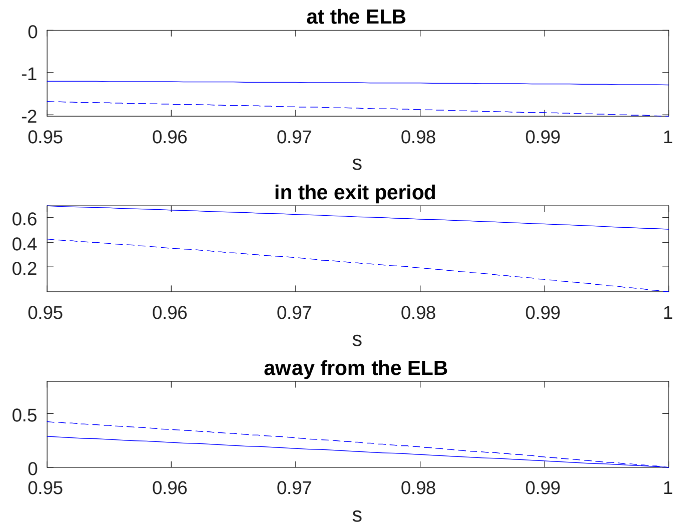

Figure 7: Output gap with forward guidance: solid line, i_e= 0; dashed line, discretion.

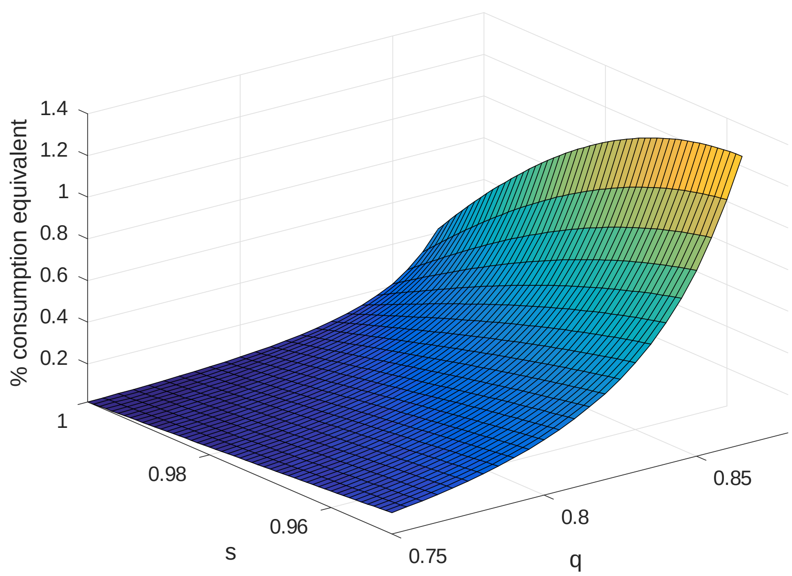

Figure 8: The gain from promising $i_e = 0$.

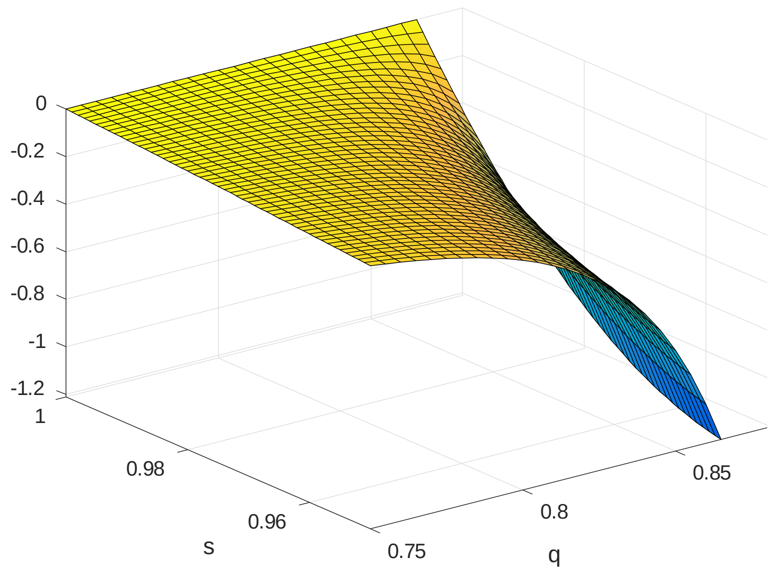

Figure 9: The temptation to renege on the promise to keep $i_e = 0$.

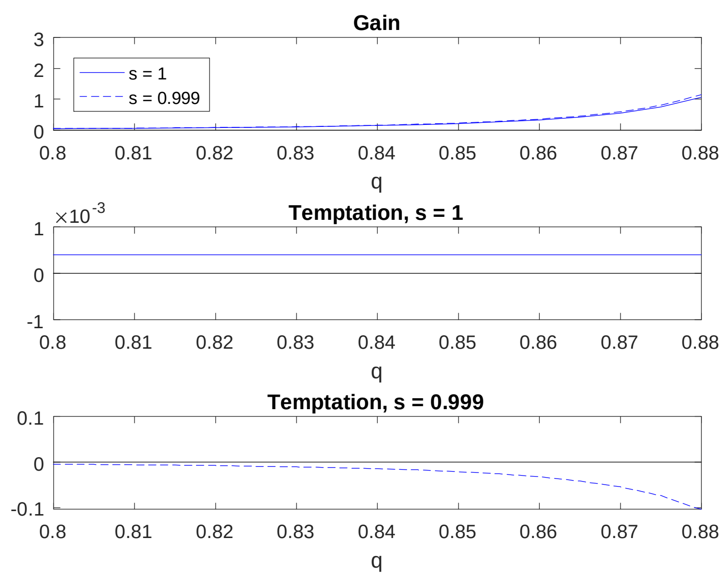

Figure 10: The gain from setting $i_e = 0$ for $s = 1$ and $s = 0.999$ as a function of q (upper panel), the temptation to defect for $s = 1$ (middle panel) and for $s = 0.999$ (lower panel).

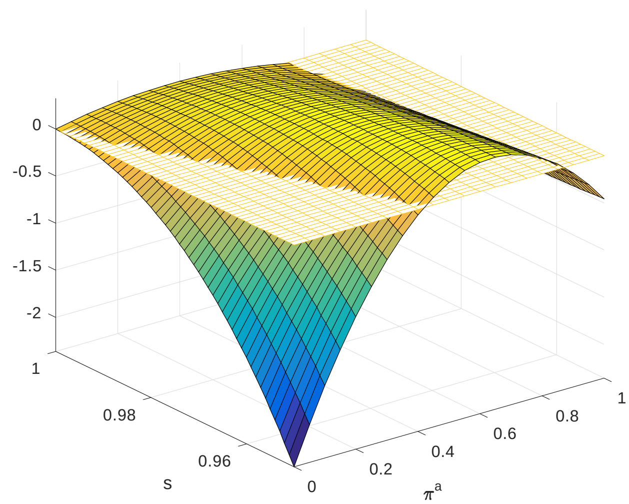

Figure 11: Gain for $q = 0.83$ as a function of s and π_e^a.

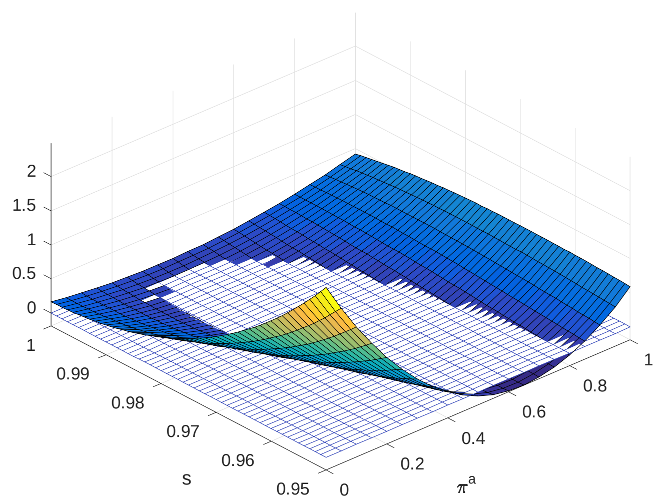

Figure 12: Temptation for $q = 0.83$.

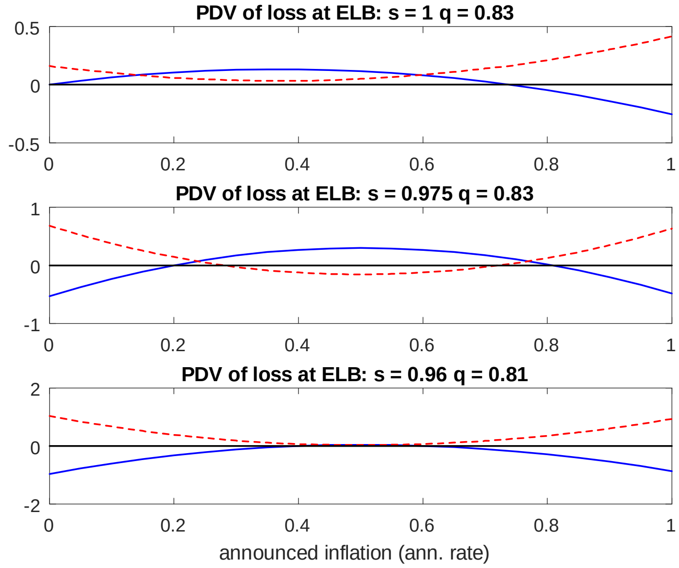

Figure 13: Gain (solid line) and temptation (dashed line) as a function of the announced inflation rate.

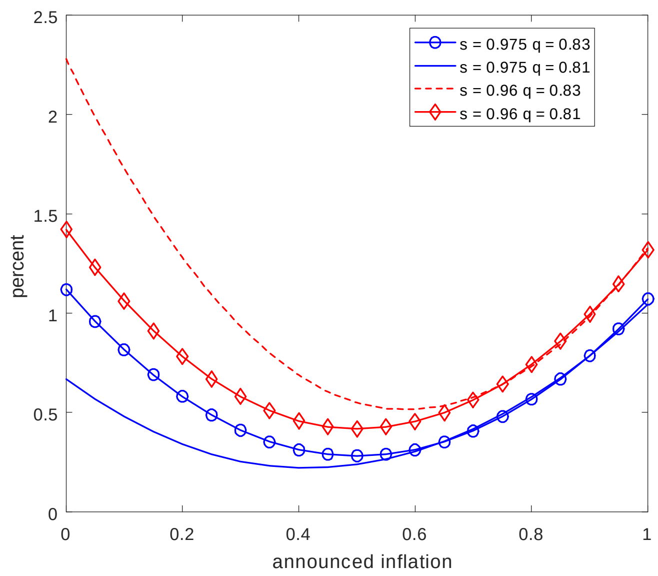

Figure 14: The present value of losses at the ELB as a function of π_e^a for different values of s and q. Benchmark calibration: blue circles. Alternative calibration: red circles.

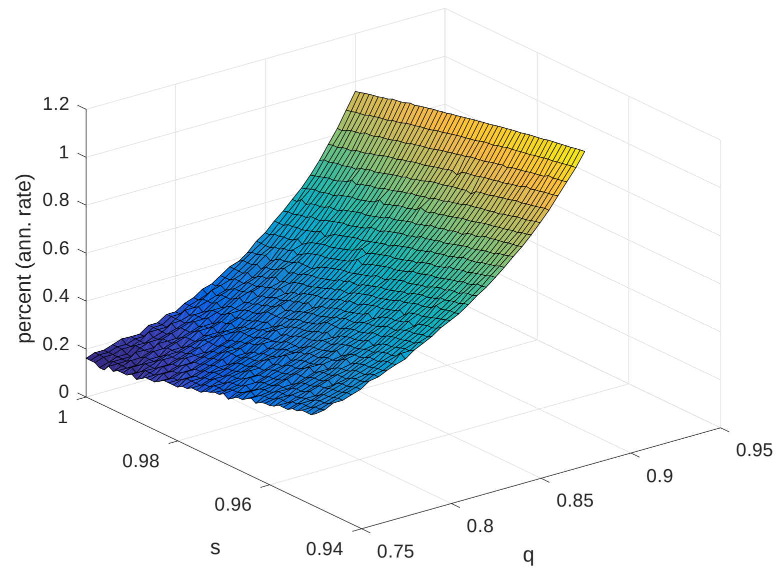

Figure 15: The optimal rate of inflation to promise for the exit period as a function of s and q.

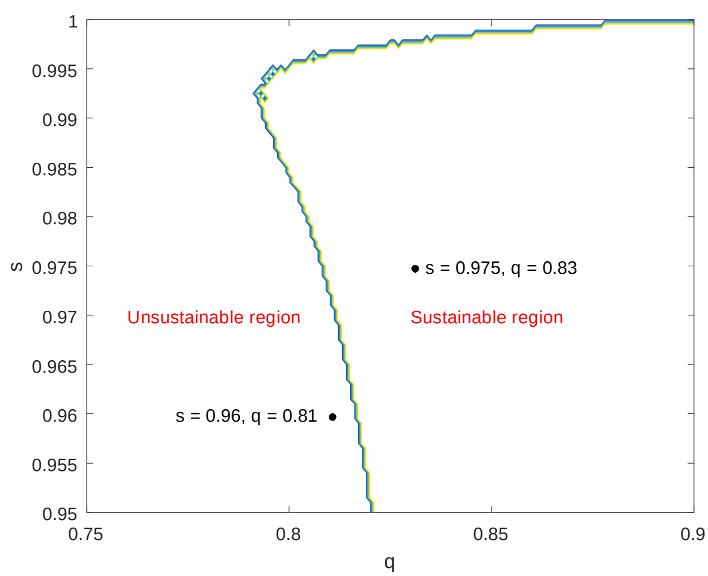

Figure 16: Region of sustainable optimal inflation announcements.