Monetary Policy and Bubbles in a New Keynesian Model with Overlapping Generations

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Motivation

- Asset price bubbles: present in the policy debate...

  - key source of macro instability
  - monetary policy as cause and cure

...but absent in modern monetary models

  - no room for bubbles in the New Keynesian model
  - no discussion of possible role of monetary policy

- Present paper: modification of the NK model to allow for bubbles
- Key ingredients:

(i) finitely-lived consumers (Blanchard (1984), Yaari (1965))
(ii) stochastic retirement (Gertler (1996))

Related Literature

- *Real* models of rational bubbles: Tirole (1985),..., Martín-Ventura (2012)
- Monetary models with bubbles: Samuelson (1958),..., Asriyan et al. (2016)

  $\Rightarrow$ flexible prices

- New Keynesian models with overlapping-generations à la Blanchard-Yaari: Piergallini (2018), Nisticò (2012), Del Negro et al. (2015)

  $\Rightarrow$ no discussion of bubbles

- Monetary policy, sticky prices and bubbles:

  - Bernanke and Gertler (1999,2001): ad-hoc bubble specification
  - Galí (2014). Main differences here:
    - variable employment and output
    - many-period, stochastic lifetimes (Blanchard-Yaari)
    - nests standard NK model as a limiting case

A New Keynesian Model with Overlapping Generations

- Survival probability: γ
- Size of cohort born in period s: $(1-\gamma)\gamma^{t-s}$
- Total population size: 1
- Two types of individuals:
  - "Active": manages own firm, works for others.
  - "Retired": consume financial wealth
- Probability of remaining active: υ
- Labor force (and measure of firms): $\alpha \equiv \frac{1-\gamma}{1-\upsilon\gamma} \in (0,1]$

Consumers

- Consumer's problem:

$$\max E_0 \sum_{t=0}^{\infty} (\beta\gamma)^t \log C_{t|s}$$

$$\frac{1}{P_t} \int_0^{\alpha} P_t(i) C_{t|s}(i) di + E_t\{\Lambda_{t,t+1} Z_{t+1|s}\} = A_{t|s} + W_t N_{t|s}$$

$$A_{t|s} = Z_{t|s} / \gamma$$

- Optimality conditions:

$$C_{t|s}(i) = \frac{1}{\alpha} \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_{t|s}$$

$$\Lambda_{t,t+1} = \beta \frac{C_{t|s}}{C_{t+1|s}}$$

$$\lim_{T\to\infty} \gamma^T E_t \left\{ \Lambda_{t,t+T} A_{t+T|s} \right\} = 0$$

Firms (I)

- Technology

$$Y_t(i) = \Gamma^t N_t(i)$$

where $\Gamma \equiv 1 + g$

- Calvo price setting: a fraction $\upsilon\gamma\theta$ of firms keeps prices unchanged
- Law of motion for the price level

$$p_t = \upsilon\gamma\theta p_{t-1} + (1 - \upsilon\gamma\theta) p_t^*$$

- Optimal price setting

$$p_t^* = \mu + (1 - \Lambda\Gamma\upsilon\gamma\theta) \sum_{k=0}^{\infty} (\Lambda\Gamma\upsilon\gamma\theta)^k E_t\{p_{t+k} + w_{t+k}\}$$

where $w_t \equiv \log(W_t/\Gamma^t)$ and $\Lambda \equiv \frac{1}{1+r}$. Assumption: $\Lambda\Gamma\upsilon\gamma\theta < 1$.

Firms (II)

- Implied inflation equation

$$\pi_t = \Lambda\Gamma E_t\{\pi_{t+1}\} + \lambda(w_t - w)$$

where $\lambda \equiv \frac{(1-\upsilon\gamma\theta)(1-\Lambda\Gamma\upsilon\gamma\theta)}{\upsilon\gamma\theta}$.

- *Remark*: in the standard NK model, $\Lambda\Gamma = \beta$

(i.e. $r = (1+\rho)(1+g) - 1 \simeq \rho + g$).

Asset Markets (I)

- Nominally riskless bond

$$\frac{1}{1+i_t} = E_t \left\{ \Lambda_{t,t+1} \frac{P_t}{P_{t+1}} \right\}$$

- Valuation of individual stocks

$$Q_t^F(i) = \sum_{k=0}^{\infty} (\upsilon\gamma)^k E_t\{\Lambda_{t,t+k} D_{t+k}(i)\}$$

where $D_t(i) \equiv Y_t(i) \left(\frac{P_t(i)}{P_t} - W_t \right)$

- Aggregate stock market

$$Q_t^F \equiv \int_0^{\alpha} Q_t^F(i) di$$
$$= \sum_{k=0}^{\infty} (\upsilon\gamma)^k E_t\{\Lambda_{t,t+k} D_{t+k}\}$$

Asset Markets (II)

- Bubbly asset

$$Q_t^B(j) = E_t\{\Lambda_{t,t+1} Q_{t+1}^B(j)\}$$

with $Q_t^B(j) \geq 0$ for all t. Recursively:

$$Q_t^B(j) = E_t\{\Lambda_{t,t+T} Q_{t+T}^B(j)\}$$

for $T = 1, 2, 3, \ldots$

- *Remark:* in the standard NK model

$$0 = \lim_{T\to\infty} E_t\{\Lambda_{t,t+T} A_{t+T}\} \geq \lim_{T\to\infty} E_t\left\{\Lambda_{t,t+T} Q_{t+T}^B(j)\right\} = Q_t^B(j)$$

implying $Q_t^B(j) = 0$.

Asset Markets (III)

- Aggregate bubble:

$$Q_t^B = U_t + Q_{t|t-1}^B$$

where $Q_{t|t-k}^B \equiv \int_{j \in \mathcal{B}_{t-k}} Q_t^B(j)dj$

- Equilibrium condition:

$$Q_t^B = E_t\{\Lambda_{t,t+1} Q_{t+1|t}^B\}$$

- Financial wealth "at birth":

$$A_{t|t} = Q_{t|t}^F + U_t/(1-\gamma)$$

- *Remark*: in the absence of bubble creation

$$Q_t^B = E_t\{\Lambda_{t,t+1} Q_{t+1}^B\}$$

$$A_{t|t} = Q_{t|t}^F$$

since $U_t = 0$ and $Q_{t+1|t}^B = Q_t^B$ for all t

Labor Markets and Monetary Policy

- Wage equation:

$$\mathcal{W}_t = \left(\frac{N_t}{\alpha}\right)^{\varphi}$$

where $\mathcal{W}_t \equiv W_t/\Gamma^t$ and $N_t \equiv \int_0^{\alpha} N_t(i)di$.

- Natural level of output

$$Y_t^n = \Gamma^t \alpha \mathcal{M}^{-\frac{1}{\varphi}} \equiv \Gamma^t \mathcal{Y}$$

- New Keynesian Phillips curve

$$\pi_t = \Lambda\Gamma E_t\{\pi_{t+1}\} + \kappa \widehat{y}_t$$

where $\kappa \equiv \lambda\varphi$, and $\widehat{y}_t \equiv \log(Y_t/Y_t^n)$.

- Monetary Policy

$$\widehat{i}_t = \phi_\pi \pi_t + \phi_q \widehat{q}_t^B$$

where $\widehat{i}_t \equiv \log \frac{1+i_t}{1+r}$, $q_t^B \equiv \frac{Q_t^B}{\Gamma^t \mathcal{Y}}$

Market Clearing

- Goods market

$$Y_t(i) = (1-\gamma)\sum_{s=-\infty}^{t} \gamma^{t-s} C_{t|s}(i)$$

for all $i \in [0, \alpha]$, implying

$$Y_t = (1-\gamma)\sum_{s=-\infty}^{t} \gamma^{t-s} C_{t|s} = C_t$$

- Labor market

$$N_t = \int_0^{\alpha} N_t(i)di = \Delta_t^p \mathcal{Y}_t \simeq \mathcal{Y}_t$$

where $\mathcal{Y}_t \equiv Y_t/\Gamma^t$

- Asset markets

$$(1-\gamma)\sum_{s=-\infty}^{t} \gamma^{t-s} A_{t|s} = Q_t^F + Q_t^B$$

Balanced Growth Paths

- Consumption function (age j, normalized by productivity)

(i) active individuals:

$$\mathcal{C}_j = (1-\beta\gamma)\left[\mathcal{A}_j^a + \frac{1}{1-\Lambda\Gamma\upsilon\gamma}\left(\frac{\mathcal{W}N}{\alpha}\right)\right]$$

(ii) retired individuals

$$\mathcal{C}_j = (1-\beta\gamma)\,\mathcal{A}_j^r$$

- Aggregate consumption function

$$\begin{aligned}\mathcal{C} &= (1-\beta\gamma)\left[\mathcal{Q}^F + \mathcal{Q}^B + \frac{\mathcal{W}N}{1-\Lambda\Gamma\upsilon\gamma}\right] \\ &= (1-\beta\gamma)\left[\mathcal{Q}^B + \frac{\mathcal{Y}}{1-\Lambda\Gamma\upsilon\gamma}\right]\end{aligned}$$

using $\mathcal{Q}^F = \mathcal{D}/(1-\Lambda\Gamma\upsilon\gamma)$ and $\mathcal{Y} = \mathcal{W}N + \mathcal{D}$.

Balanced Growth Paths

- Bubbleless BGP ($\mathcal{Q}^B = 0$)

$$\Lambda\Gamma\upsilon = \beta$$

or, equivalently,

$$r = (1+\rho)(1+g)\upsilon - 1$$

Remark #1: $\upsilon = 1 \Rightarrow r = (1+\rho)(1+g) - 1 > g$

Remark #2: $\upsilon < \beta \Leftrightarrow r < g$

Balanced Growth Paths

- Recall:

$$Q_t^B = E_t\{\Lambda_{t,t+1} Q_{t+1|t}^B\}$$

or, letting $q_t^B \equiv \frac{Q_t^B}{\Gamma^t \mathcal{Y}}$ and $u_t \equiv \frac{U_t}{\Gamma^t \mathcal{Y}}$

$$\begin{aligned} q_t^B &= E_t\{\Lambda_{t,t+1}\Gamma q_{t+1|t}^B\} \\ &= E_t\{\Lambda_{t,t+1}\Gamma(q_{t+1}^B - u_{t+1})\} \end{aligned}$$

- Bubbly BGP with no bubble creation ($\mathcal{Q}^B > 0$, $u_t = 0$ all t):

$$\Lambda\Gamma = 1$$

or, equivalently,

$$r = g$$

Implied bubble size:

$$q^B = \frac{\gamma(\beta - \upsilon)}{(1-\beta\gamma)(1-\upsilon\gamma)} \equiv \overline{q}^B$$

Balanced Growth Paths

- Bubbly BGP with bubble creation ($\mathcal{Q}^B > 0$, $u_t = u > 0$ all t):

$$q^B = \frac{\gamma(\beta - \Lambda\Gamma\upsilon)}{(1-\beta\gamma)(1-\Lambda\Gamma\upsilon\gamma)}$$

$$u = \left(1 - \frac{1}{\Lambda\Gamma}\right) q^B$$

where

$$\Lambda\Gamma > 1 \Leftrightarrow r < g$$

$$\Lambda\Gamma < \frac{\beta}{\upsilon} \Leftrightarrow r > (1+\rho)(1+g)\upsilon - 1$$

Remark #1: necessary and sufficient condition for existence: $\upsilon < \beta$

Remark #2: continuum of bubbly BGPs $\{q^B, u\}$ indexed by $r \in ((1+\rho)(1+g)\upsilon - 1, g)$

Remark #3: q^B increasing in r, with $\lim_{r \to g} q^B = \overline{q}^B$

Figure 1. Balanced Growth Paths

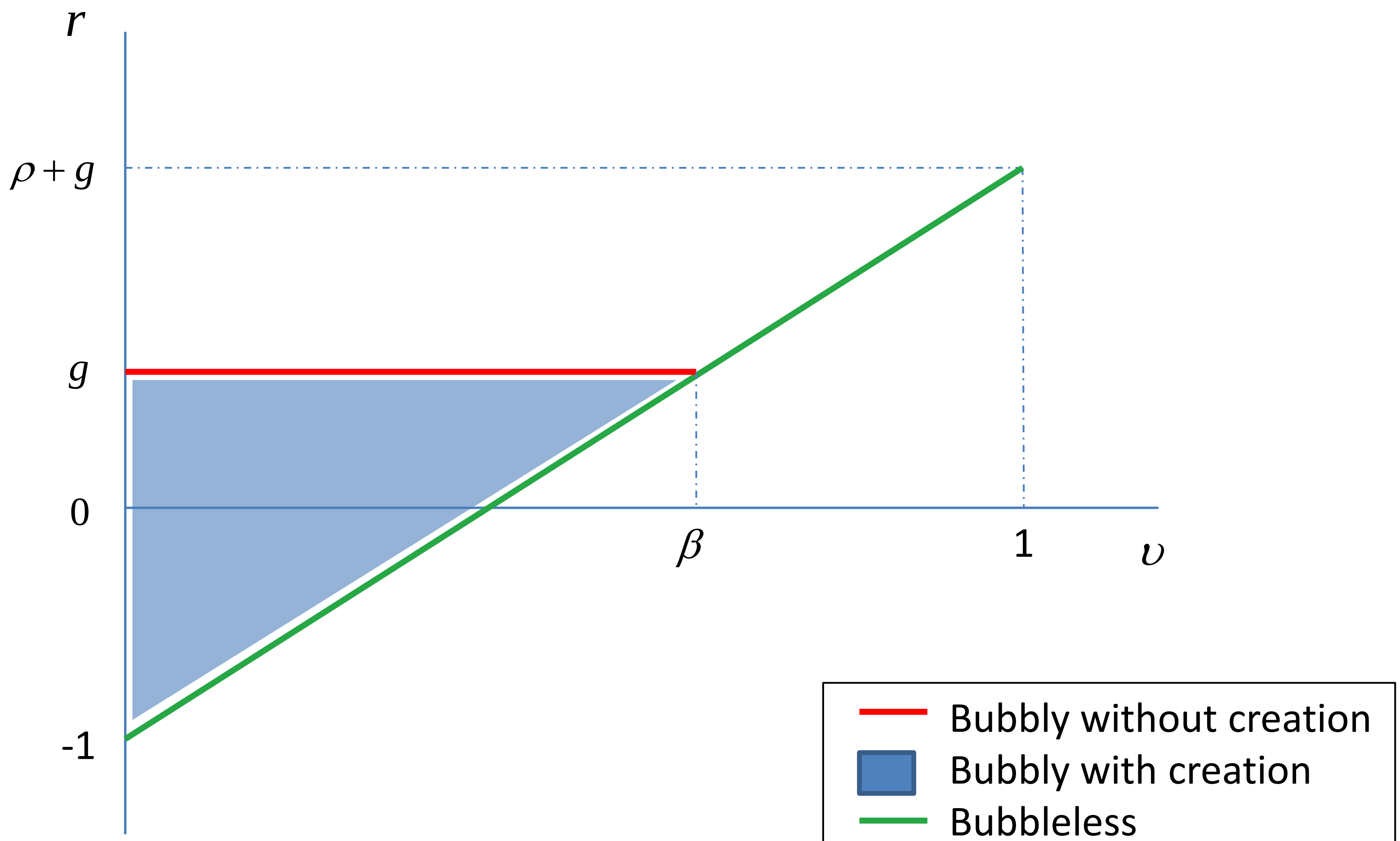
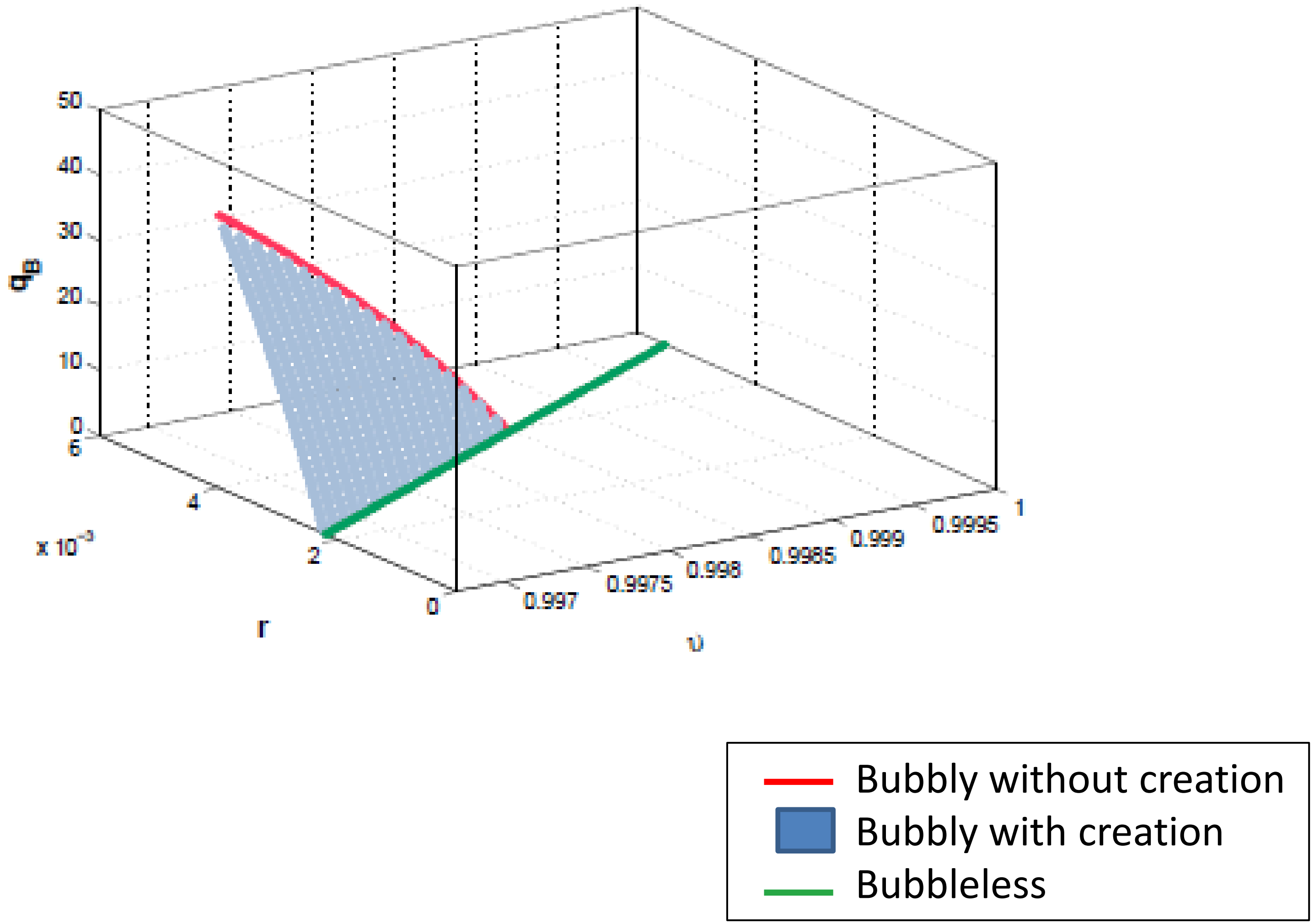


Figure 2. Balanced Growth Paths: Bubble Size



Some Numbers

- Life expectancy (at 20): $(80 - 20) \times 4 = 240$ quarters $\Rightarrow \gamma = 0.9958$
- Average retirement age: $(63 - 20) \times 4 = 172$ quarters $\Rightarrow \upsilon = 0.9983$ (conditional on survival)
- Condition for existence of bubbles: $\beta > 0.9983$
- Average real interest rate (1960-2015): $r = 1.4\% \div 4 = 0.35\%$
- Average growth rate (1960-2015): $g = 1.6\% \div 4 = 0.4\%$
- Consumers' discount factor on future income: $\Lambda\Gamma\upsilon\gamma \simeq 0.995 < 1$

Equilibrium Dynamics (I)

- Aggregate consumption function:

$$\widehat{c}_t = (1-\beta\gamma)(\widehat{q}_t^B + \widehat{x}_t)$$

where

$$\begin{aligned}\widehat{x}_t &= \sum_{k=0}^{\infty}(\Lambda\Gamma\upsilon\gamma)^k E_t\{\widehat{y}_{t+k}\} - \frac{\Lambda\Gamma\upsilon\gamma}{1-\Lambda\Gamma\upsilon\gamma}\sum_{k=0}^{\infty}(\Lambda\Gamma\upsilon\gamma)^k E_t\{\widehat{i}_{t+k} - \pi_{t+k+1}\} \\ &= \Lambda\Gamma\upsilon\gamma E_t\{\widehat{x}_{t+1}\} + \widehat{y}_t - \frac{\Lambda\Gamma\upsilon\gamma}{1-\Lambda\Gamma\upsilon\gamma}(\widehat{i}_t - E_t\{\pi_{t+1}\})\end{aligned}$$

$\Rightarrow$ solution to the forward guidance puzzle? (Del Negro et al. (2016))

- Aggregate bubble dynamics:

$$\widehat{q}_t^B = \Lambda\Gamma E_t\{\widehat{q}_{t+1}^B\} - q^B(\widehat{i}_t - E_t\{\pi_{t+1}\})$$

$\Rightarrow$ role of monetary policy (Galí (2014)):

$$E_t\{\Delta\widehat{q}_{t+1}^B\} = -\left(1 - \frac{1}{\Lambda\Gamma}\right)\widehat{q}_t^B + \frac{q^B}{\Lambda\Gamma}(\widehat{i}_t - E_t\{\pi_{t+1}\})$$

Equilibrium Dynamics (II)

- New Keynesian Phillips curve

$$\pi_t = \Lambda\Gamma E_t\{\pi_{t+1}\} + \kappa\widehat{y}_t$$

- Monetary Policy

$$\widehat{i}_t = \phi_\pi \pi_t + \phi_q \widehat{q}_t^B$$

- Goods market clearing

$$\widehat{c}_t = \widehat{y}_t$$

Equilibrium Fluctuations: The Bubbleless Case

- Equilibrium dynamics

$$\widehat{y}_t = E_t\{\widehat{y}_{t+1}\} - (\widehat{i}_t - E_t\{\pi_{t+1}\})$$

$$\pi_t = \frac{\beta}{\upsilon} E_t\{\pi_{t+1}\} + \kappa \widehat{y}_t$$

$$\widehat{i}_t = \phi_\pi \pi_t$$

- Local uniqueness

$$\phi_\pi > \max\left[1, \frac{1}{\kappa}\left(\frac{\beta}{\upsilon} - 1\right)\right]$$

$$\upsilon < \tfrac{\beta}{1+\kappa} \Rightarrow \text{"reinforced Taylor principle"}$$

- Forward guidance puzzle remains

Figure 3a
Monetary Policy and Equilibrium Uniqueness around the Bubbleless BGP

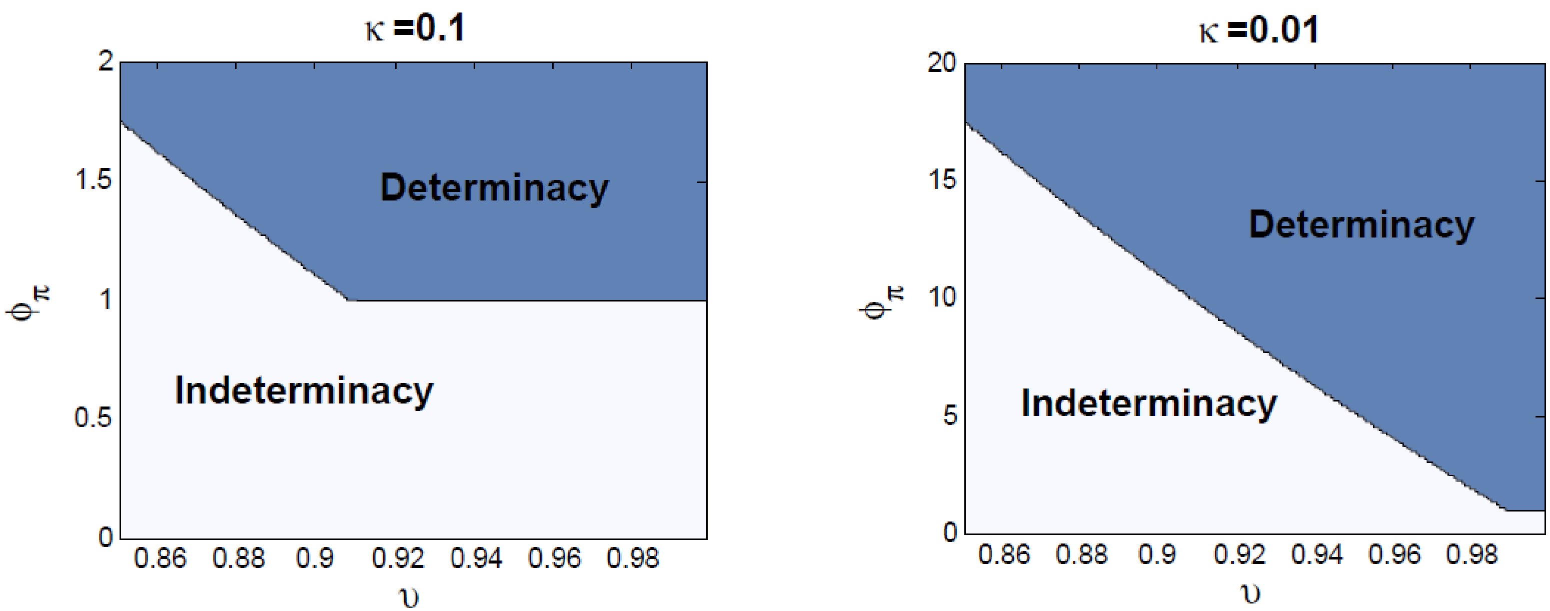
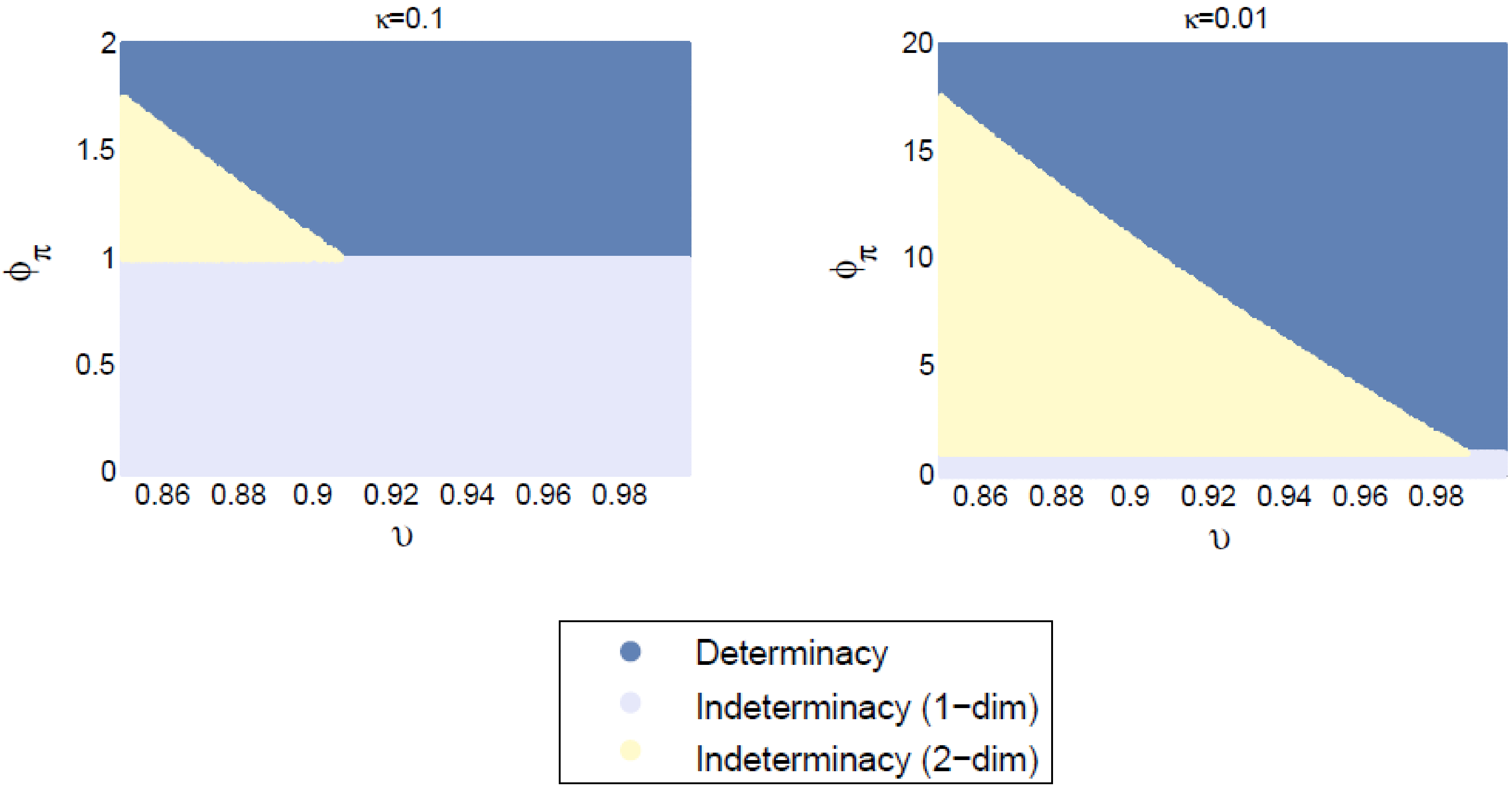


Figure 3b
Monetary Policy and Equilibrium Uniqueness around the Bubbleless BGP



Bubbly Equilibrium Fluctuations

- Equilibrium dynamics

$$\widehat{y}_t = \frac{\Lambda\Gamma\upsilon}{\beta} E_t\{\widehat{y}_{t+1}\} + \Phi \widehat{q}_t^B - \frac{\Upsilon\upsilon}{\beta}(\widehat{i}_t - E_t\{\pi_{t+1}\})$$

$$\pi_t = \Lambda\Gamma E_t\{\pi_{t+1}\} + \kappa \widehat{y}_t$$

$$\widehat{q}_t^B = \Lambda\Gamma E_t\{\widehat{q}_{t+1}^B\} - q^B(\widehat{i}_t - E_t\{\pi_{t+1}\})$$

$$\widehat{i}_t = \phi_\pi \pi_t + \phi_q \widehat{q}_t^B$$

where $\Phi \equiv \frac{(1-\beta\gamma)(1-\upsilon\gamma)}{\beta\gamma}$, $\Upsilon \equiv \left(1 + \frac{(1-\beta\gamma)(\Lambda\Gamma-1)}{1-\Lambda\Gamma\upsilon\gamma}\right)$, $q^B = \frac{\gamma(\beta-\Lambda\Gamma\upsilon)}{(1-\beta\gamma)(1-\Lambda\Gamma\upsilon\gamma)}$

- *Particular case* #1 (no bubble creation): $\Lambda\Gamma = \Upsilon = 1$; $q^B = \overline{q}^B$
- *Particular case* #2 (about bubbleless BGP): $\Lambda\Gamma = \Upsilon = \frac{\beta}{\upsilon}$; $q^B = 0$
- Intermediate cases: $\Lambda\Gamma \in \left(1, \frac{\beta}{\upsilon}\right)$, $q^B \in (0, \overline{q}^B)$

Figure 4
Monetary Policy and Equilibrium Uniqueness:
The Case of No Bubble Creation (r=g=0.004)

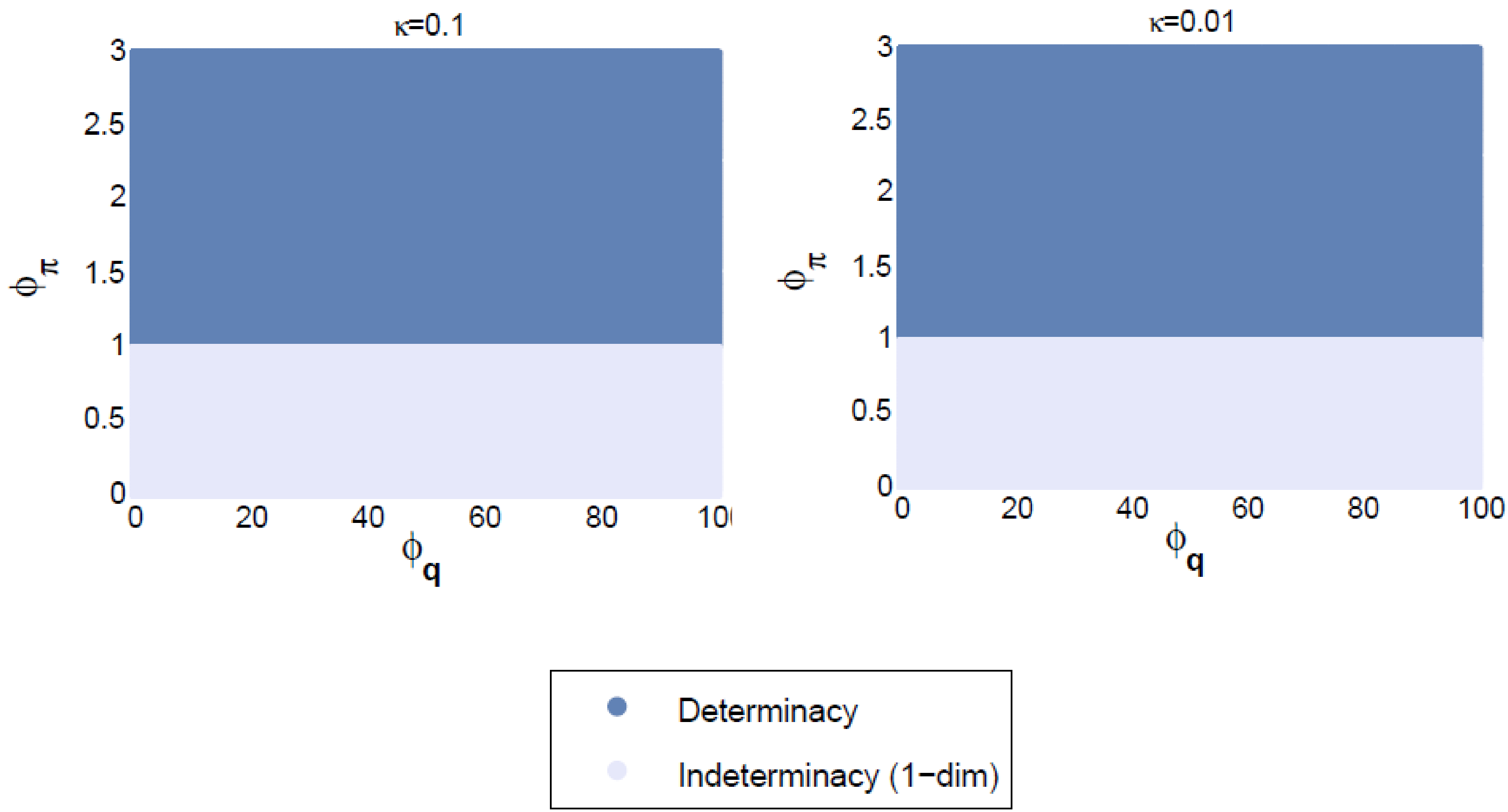


Figure 6a
Monetary Policy and Equilibrium Uniqueness
around a Bubbly BGP with Bubble Creation (r=0.003935)

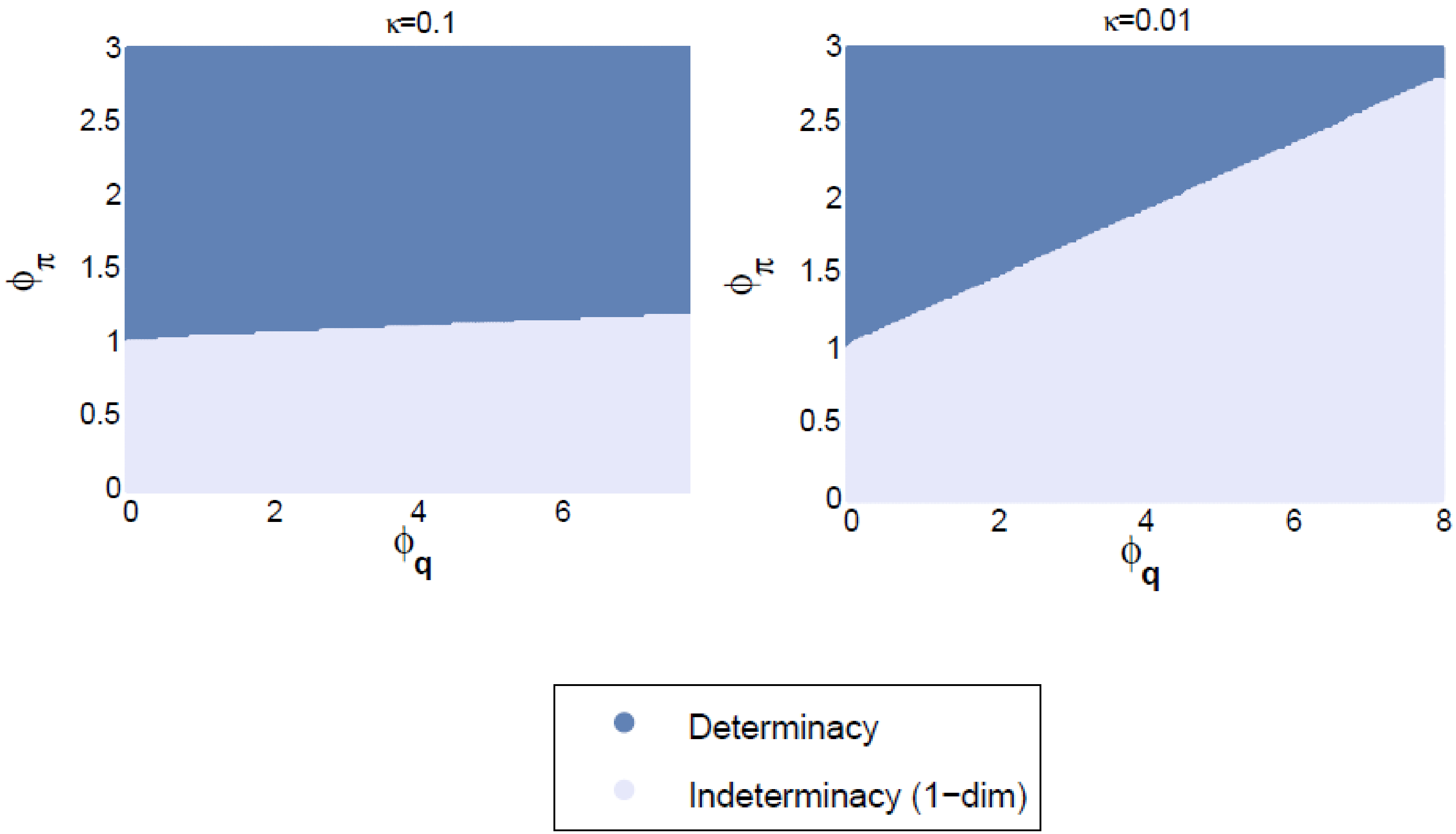


Figure 6b
Monetary Policy and Equilibrium Uniqueness
around a Bubbly BGP with Bubble Creation (r=0.003931)

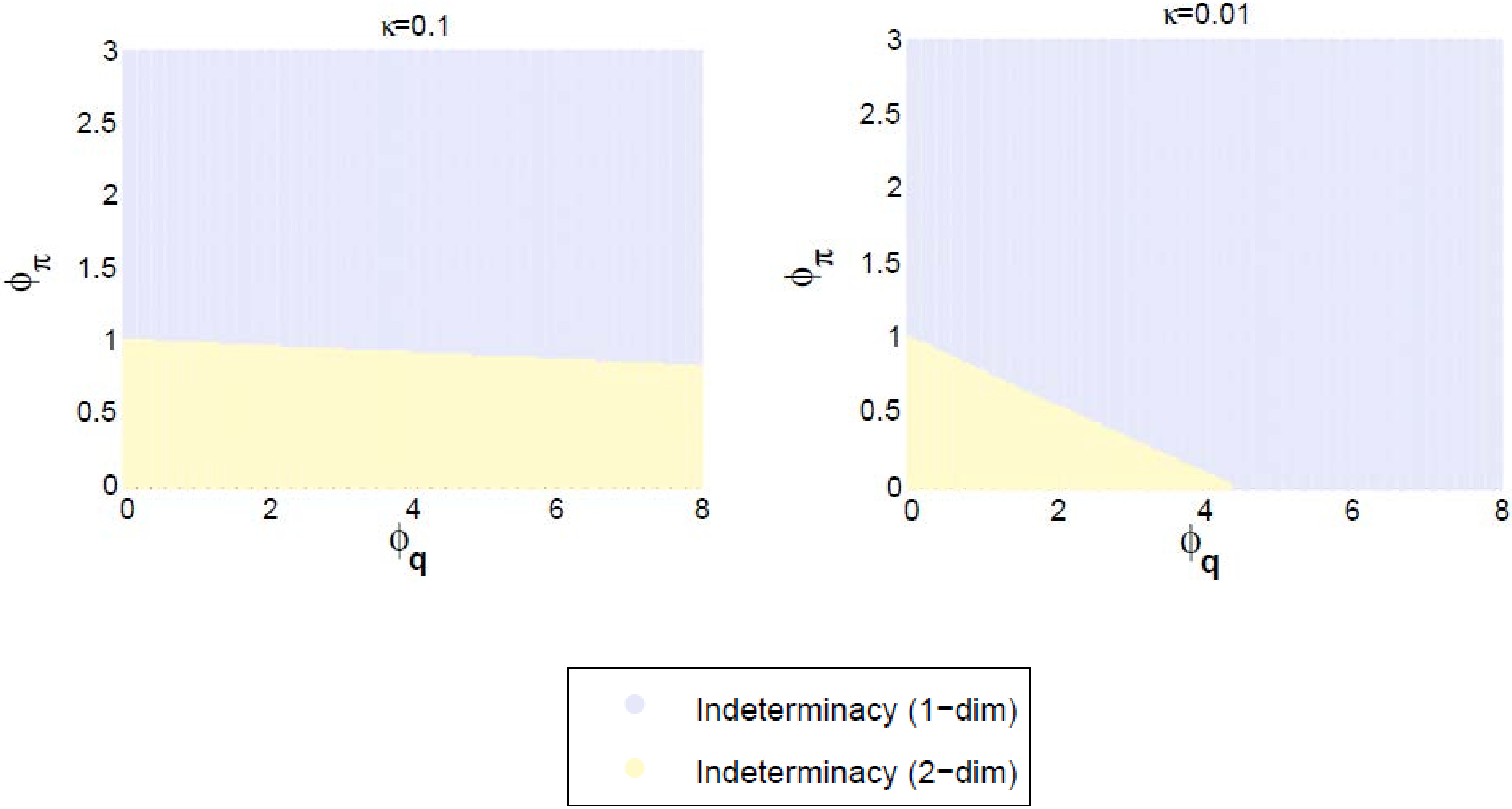


Figure 5
Monetary Policy and Equilibrium Uniqueness around the Bubbleless BGP

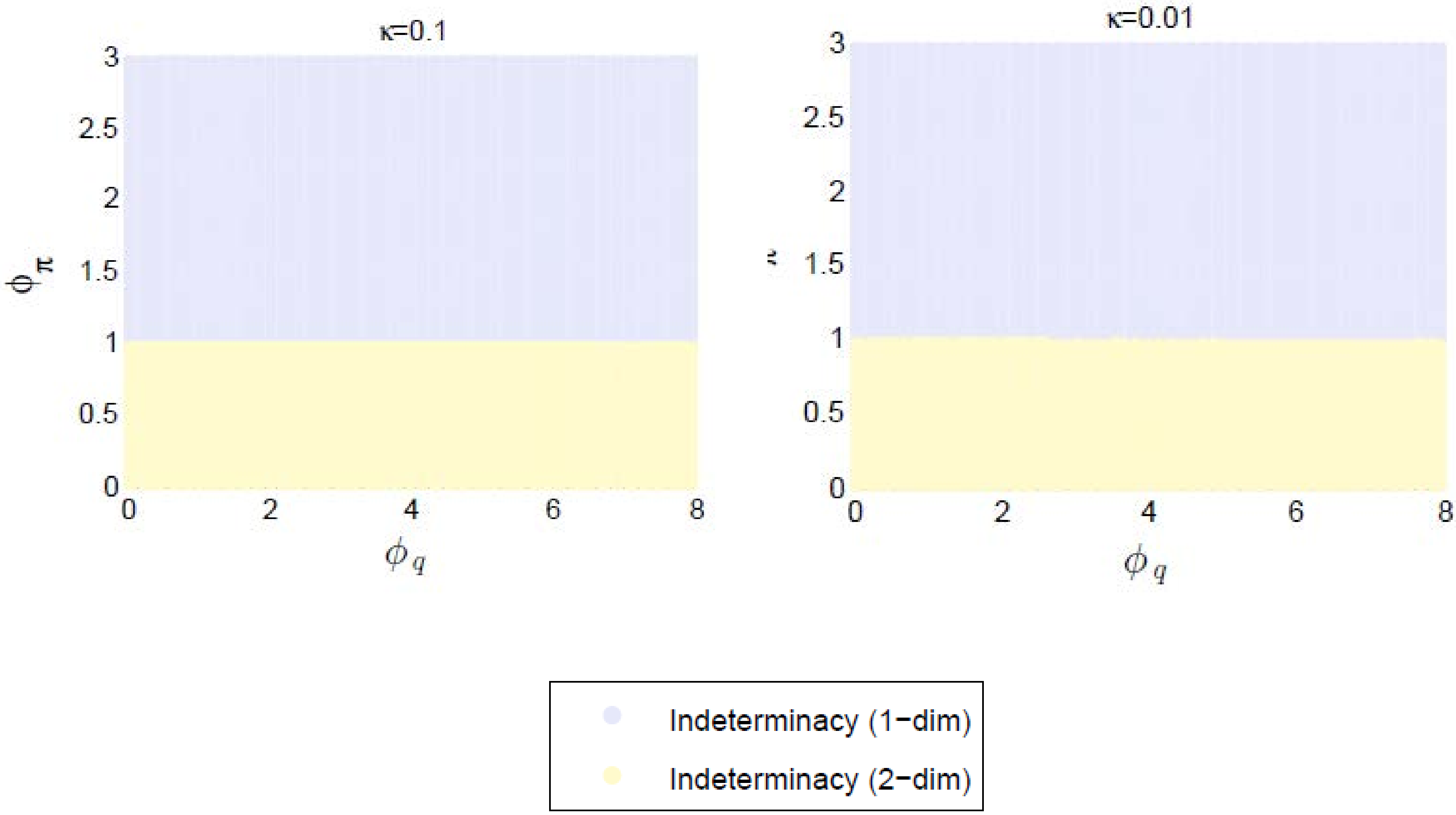
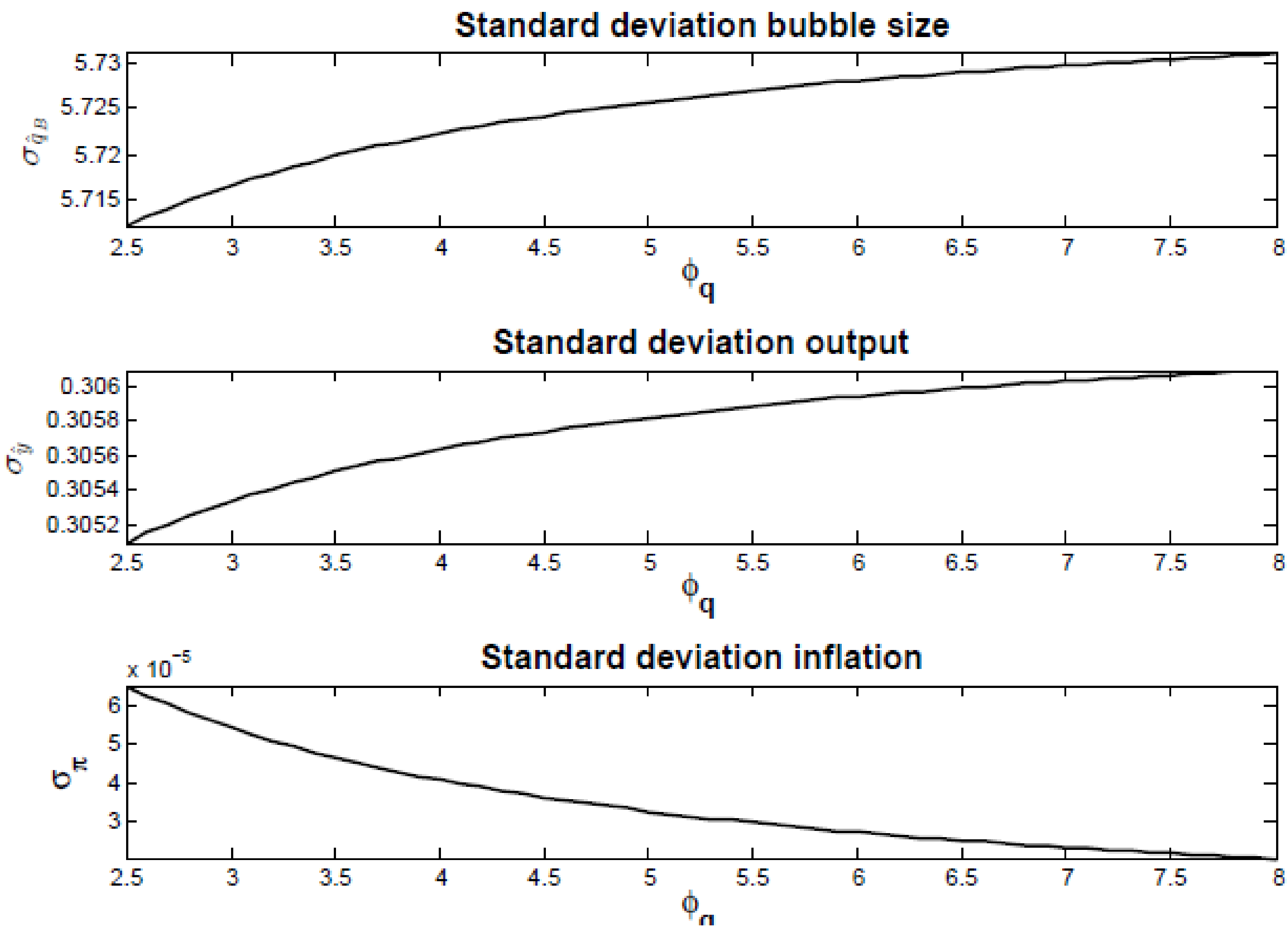


Figure 8
Macro Volatility and Leaning against the Bubble Policies
(type II bubbles, r=0.39%)



Main Messages and Next Steps

- Reminder of the possibility of bubbly equilibria once we depart from the infinite-lived representative consumer framework

  - more likely in an environment of low natural interest rates

- Perils of using interest rate policy to tame asset price bubbles

  - indeterminacy more likely
  - risk of larger fluctuations

- Caveats

  - *rational* bubbles
  - no role for credit supply factors

- Next steps:

  - Welfare and role of monetary policy
  - Global equilibrium dynamics (nonlinearities, switching equilibria)