Monetary Policy and Bubbles
in a New Keynesian Model
with Overlapping Generations

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Motivation

- Asset price bubbles: present in the policy debate...
  - key source of macro instability
  - monetary policy as cause and cure

...but absent in modern monetary models
  - no room for bubbles in the New Keynesian model
  - no discussion of possible role of monetary policy

- Present paper: modification of the NK model to allow for bubbles

- Key ingredients:
  (i) finitely-lived consumers (Blanchard (1984), Yaari (1965))
  (ii) stochastic retirement (Gertler (1996))
Real models of rational bubbles: Tirole (1985),..., Martín-Ventura (2012)

Monetary models with bubbles: Samuelson (1958),..., Asriyan et al. (2016)

⇒ flexible prices


⇒ no discussion of bubbles

Monetary policy, sticky prices and bubbles:

- Galí (2014). Main differences here:
  - variable employment and output
  - many-period, stochastic lifetimes (Blanchard-Yaari)
  - nests standard NK model as a limiting case
A New Keynesian Model with Overlapping Generations

- Survival probability: $\gamma$
- Size of cohort born in period $s$: $(1 - \gamma)\gamma^{t-s}$
- Total population size: 1

- Two types of individuals:
  - "Active": manages own firm, works for others.
  - "Retired": consume financial wealth

- Probability of remaining active: $\nu$

- Labor force (and measure of firms): $\alpha \equiv \frac{1 - \gamma}{1 - \nu \gamma} \in (0, 1]$
Consumers

- **Consumer’s problem:**

\[
\max E_0 \sum_{t=0}^{\infty} (\beta \gamma)^t \log C_{t|s} \\
\frac{1}{P_t} \int_0^{\alpha} P_t(i) C_{t|s}(i) \, di + E_t\{\Lambda_{t,t+1} Z_{t+1|s}\} = A_{t|s} + W_t N_{t|s} \\
A_{t|s} = Z_{t|s} / \gamma
\]

- **Optimality conditions:**

\[
C_{t|s}(i) = \frac{1}{\alpha} \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_{t|s} \\
\Lambda_{t,t+1} = \beta \frac{C_{t|s}}{C_{t+1|s}} \\
\lim_{T \to \infty} \gamma^T E_t \{\Lambda_{t,t+T} A_{t+T|s}\} = 0
\]
Firms (I)

- Technology

\[ Y_t(i) = \Gamma^t N_t(i) \]

where \( \Gamma \equiv 1 + g \)

- Calvo price setting: a fraction \( \nu \gamma \theta \) of firms keeps prices unchanged

- Law of motion for the price level

\[ p_t = \nu \gamma \theta p_{t-1} + (1 - \nu \gamma \theta) p^*_t \]

- Optimal price setting

\[ p^*_t = \mu + (1 - \Lambda \Gamma \nu \gamma \theta) \sum_{k=0}^{\infty} (\Lambda \Gamma \nu \gamma \theta)^k E_t \left\{ p_{t+k} + w_{t+k} \right\} \]

where \( w_t \equiv \log(W_t / \Gamma^t) \) and \( \Lambda \equiv \frac{1}{1+r} \). Assumption: \( \Lambda \Gamma \nu \gamma \theta < 1 \).
Firms (II)

- Implied inflation equation

\[ \pi_t = \Lambda \Gamma E_t\{\pi_{t+1}\} + \lambda (w_t - w) \]

where \( \lambda \equiv \frac{(1-v\gamma \theta)(1-\Lambda \Gamma v \gamma \theta)}{v \gamma \theta} \).

- Remark: in the standard NK model, \( \Lambda \Gamma = \beta \) (i.e. \( r = (1 + \rho)(1 + g) - 1 \approx \rho + g \)).
Asset Markets (I)

- Nominally riskless bond

\[ \frac{1}{1 + i_t} = E_t \left\{ \Lambda_{t,t+1} \frac{P_t}{P_{t+1}} \right\} \]

- Valuation of individual stocks

\[ Q_t^F (i) = \sum_{k=0}^{\infty} (\nu \gamma)^k E_t \{ \Lambda_{t,t+k} D_{t+k}(i) \} \]

where \( D_t(i) \equiv Y_t(i) \left( \frac{P_t(i)}{P_t} - W_t \right) \)

- Aggregate stock market

\[ Q_t^F \equiv \int_0^\alpha Q_t^F (i) \, di \]

\[ = \sum_{k=0}^{\infty} (\nu \gamma)^k E_t \{ \Lambda_{t,t+k} D_{t+k} \} \]
Bubbly asset

\[ Q^B_t(j) = E_t\{\Lambda_{t,t+1} Q^B_{t+1}(j)\} \]

with \( Q^B_t(j) \geq 0 \) for all \( t \). Recursively:

\[ Q^B_t(j) = E_t\{\Lambda_{t,t+T} Q^B_{t+T}(j)\} \]

for \( T = 1, 2, 3, \ldots \)

Remark: in the standard NK model

\[ 0 = \lim_{T \to \infty} E_t\{\Lambda_{t,t+T} A_{t+T}\} \geq \lim_{T \to \infty} E_t\{\Lambda_{t,t+T} Q^B_{t+T}(j)\} = Q^B_t(j) \]

implying \( Q^B_t(j) = 0 \).
Aggregate bubble:

\[ Q_t^B = U_t + Q_{t|t-1}^B \]

where \( Q_{t|t-k}^B \equiv \int_{j \in B_{t-k}} Q_t^B(j) \, dj \)

Equilibrium condition:

\[ Q_t^B = E_t\{ \Lambda_{t,t+1} Q_{t+1|t}^B \} \]

Financial wealth "at birth":

\[ A_{t|t} = Q_{t|t}^F + U_t / (1 - \gamma) \]

Remark: in the absence of bubble creation

\[ Q_t^B = E_t\{ \Lambda_{t,t+1} Q_{t+1}^B \} \]

\[ A_{t|t} = Q_{t|t}^F \]

since \( U_t = 0 \) and \( Q_{t+1|t}^B = Q_t^B \) for all \( t \)
Labor Markets and Monetary Policy

- Wage equation:
  \[ \mathcal{W}_t = \left( \frac{N_t}{\alpha} \right)^\varphi \]
  where \( \mathcal{W}_t \equiv W_t / \Gamma^t \) and \( N_t \equiv \int_0^\alpha N_t(i) di. \)

- Natural level of output
  \[ Y_t^n = \Gamma^t \alpha \mathcal{M}^{-\frac{1}{\varphi}} \equiv \Gamma^t \Upsilon \]

- New Keynesian Phillips curve
  \[ \pi_t = \Lambda \Gamma E_t \{ \pi_{t+1} \} + \kappa \hat{y}_t \]
  where \( \kappa \equiv \lambda \varphi \), and \( \hat{y}_t \equiv \log(Y_t / Y_t^n) \).

- Monetary Policy
  \[ \hat{i}_t = \phi_\pi \pi_t + \phi_q \hat{q}_t^B \]
  where \( \hat{i}_t \equiv \log \frac{1+i_t}{1+r} \), \( q_t^B \equiv \frac{Q_t^B}{\Gamma^t \Upsilon} \).
Market Clearing

- **Goods market**

\[ Y_t(i) = (1 - \gamma) \sum_{s=-\infty}^{t} \gamma^{t-s} C_{t|s}(i) \]

for all \( i \in [0, \alpha] \), implying

\[ Y_t = (1 - \gamma) \sum_{s=-\infty}^{t} \gamma^{t-s} C_{t|s} = C_t \]

- **Labor market**

\[ N_t = \int_{0}^{\alpha} N_t(i) \, di = \Delta_t^p Y_t \sim Y_t \]

where \( Y_t \equiv Y_t / \Gamma^t \)

- **Asset markets**

\[ (1 - \gamma) \sum_{s=-\infty}^{t} \gamma^{t-s} A_{t|s} = Q_t^F + Q_t^B \]
Balanced Growth Paths

- Consumption function (age $j$, normalized by productivity)
  
  (i) active individuals:
  
  $$C_j = (1 - \beta \gamma) \left[ A_j^a + \frac{1}{1 - \Lambda \Gamma \nu \gamma} \left( \frac{\mathcal{W}N}{\alpha} \right) \right]$$

  (ii) retired individuals
  
  $$C_j = (1 - \beta \gamma) A_j^r$$

- Aggregate consumption function
  
  $$C = (1 - \beta \gamma) \left[ Q^F + Q^B + \frac{\mathcal{W}N}{1 - \Lambda \Gamma \nu \gamma} \right]$$
  
  $$= (1 - \beta \gamma) \left[ Q^B + \frac{\mathcal{V}}{1 - \Lambda \Gamma \nu \gamma} \right]$$

  using $Q^F = \mathcal{D} / (1 - \Lambda \Gamma \nu \gamma)$ and $\mathcal{V} = \mathcal{W}N + \mathcal{D}$. 

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Balanced Growth Paths

- Bubbleless BGP ($Q^B = 0$)
  \[
  \Lambda \Gamma \nu = \beta 
  \]
  or, equivalently,
  \[
  r = (1 + \rho)(1 + g)\nu - 1 
  \]

**Remark #1**: $\nu = 1 \implies r = (1 + \rho)(1 + g) - 1 > g$

**Remark #2**: $\nu < \beta \iff r < g$
Balanced Growth Paths

- Recall:

\[ Q_t^B = E_t \{ \Lambda_{t,t+1} Q_{t+1}^B | t \} \]

or, letting \( q_t^B \equiv \frac{Q_t^B}{\Gamma t Y} \) and \( u_t \equiv \frac{U_t}{\Gamma t Y} \)

\[ q_t^B = E_t \{ \Lambda_{t,t+1} \Gamma q_{t+1}^B | t \} \]

\[ = E_t \{ \Lambda_{t,t+1} \Gamma (q_{t+1}^B - u_{t+1}) \} \]

- Bubbly BGP with no bubble creation \( (Q^B > 0, u_t = 0 \text{ all } t) \):

\[ \Lambda \Gamma = 1 \]

or, equivalently,

\[ r = g \]

Implied bubble size:

\[ q^B = \frac{\gamma (\beta - \nu)}{(1 - \beta \gamma)(1 - \nu \gamma)} \equiv \bar{q}^B \]
Balanced Growth Paths

- **Bubbly BGP with bubble creation** ($Q^B > 0, u_t = u > 0$ all $t$):

$$q^B = \frac{\gamma(\beta - \Lambda \Gamma \nu)}{(1 - \beta \gamma)(1 - \Lambda \Gamma \nu \gamma)}$$

$$u = \left(1 - \frac{1}{\Lambda \Gamma}\right) q^B$$

where

$$\Lambda \Gamma > 1 \iff r < g$$

$$\Lambda \Gamma < \frac{\beta}{\nu} \iff r > (1 + \rho)(1 + g)\nu - 1$$

**Remark #1**: necessary and sufficient condition for existence: $\nu < \beta$

**Remark #2**: continuum of bubbly BGPs $\{q^B, u\}$ indexed by $r \in ((1 + \rho)(1 + g)\nu - 1, g)$

**Remark #3**: $q^B$ increasing in $r$, with $\lim_{r \to g} q^B = \overline{q}^B$
Figure 1. Balanced Growth Paths

- Bubbly without creation
- Bubbly with creation
- Bubbleless
Figure 2. Balanced Growth Paths: Bubble Size

- Bubbly without creation
- Bubbly with creation
- Bubbleless
Some Numbers

- Life expectancy (at 20): \((80 - 20) \times 4 = 240\) quarters \(\Rightarrow \gamma = 0.9958\)
- Average retirement age: \((63 - 20) \times 4 = 172\) quarters \(\Rightarrow \nu = 0.9983\) (conditional on survival)
- Condition for existence of bubbles: \(\beta > 0.9983\)
- Average real interest rate (1960-2015): \(r = 1.4\% \div 4 = 0.35\%\)
- Average growth rate (1960-2015): \(g = 1.6\% \div 4 = 0.4\%\)
- Consumers’ discount factor on future income: \(\Delta \Gamma \nu \gamma \approx 0.995 < 1\)
Aggregate consumption function:

\[ \hat{c}_t = (1 - \beta \gamma)(\hat{q}_t^B + \hat{x}_t) \]

where

\[ \hat{x}_t = \sum_{k=0}^{\infty} (\Lambda \Gamma^n \nu)^k E_t \{ \hat{y}_{t+k} \} - \frac{\Lambda \Gamma^n \nu}{1 - \Lambda \Gamma^n \nu} \sum_{k=0}^{\infty} (\Lambda \Gamma^n \nu)^k E_t \{ \hat{i}_{t+k} - \pi_{t+k+1} \} \]

\[ = \Lambda \Gamma^n \nu E_t \{ \hat{x}_{t+1} \} + \hat{y}_t - \frac{\Lambda \Gamma^n \nu}{1 - \Lambda \Gamma^n \nu} (\hat{i}_t - E_t \{ \pi_{t+1} \}) \]

⇒ solution to the forward guidance puzzle? (Del Negro et al. (2016))

Aggregate bubble dynamics:

\[ \hat{q}_t^B = \Lambda \Gamma E_t \{ \hat{q}_{t+1}^B \} - q^B (\hat{i}_t - E_t \{ \pi_{t+1} \}) \]

⇒ role of monetary policy (Galí (2014)):

\[ E_t \{ \Delta \hat{q}_{t+1}^B \} = - \left( 1 - \frac{1}{\Lambda \Gamma} \right) \hat{q}_t^B + \frac{q^B}{\Lambda \Gamma} (\hat{i}_t - E_t \{ \pi_{t+1} \}) \]
New Keynesian Phillips curve

\[ \pi_t = \Lambda \Gamma E_t \{ \pi_{t+1} \} + \kappa \hat{y}_t \]

Monetary Policy

\[ \hat{i}_t = \phi_{\pi} \pi_t + \phi_q \hat{q}^B_t \]

Goods market clearing

\[ \hat{c}_t = \hat{y}_t \]
Equilibrium Fluctuations: The Bubbleless Case

- Equilibrium dynamics

\[ \hat{y}_t = E_t\{\hat{y}_{t+1}\} - (i_t - E_t\{\pi_{t+1}\}) \]

\[ \pi_t = \frac{\beta}{\nu} E_t\{\pi_{t+1}\} + \kappa \hat{y}_t \]

\[ \hat{i}_t = \phi_\pi \pi_t \]

- Local uniqueness

\[ \phi_\pi > \max \left[ 1, \frac{1}{\kappa} \left( \frac{\beta}{\nu} - 1 \right) \right] \]

\[ \nu < \frac{\beta}{1+\kappa} \Rightarrow \text{"reinforced Taylor principle"} \]

- Forward guidance puzzle remains
Figure 3a
Monetary Policy and Equilibrium Uniqueness around the Bubbleless BGP

\( \kappa = 0.1 \)

\[ \begin{array}{c}
\phi_\pi \\
\kappa = 0.1 \\
\end{array} \]

\( \kappa = 0.01 \)

\[ \begin{array}{c}
\phi_\pi \\
\kappa = 0.01 \\
\end{array} \]
Figure 3b
Monetary Policy and Equilibrium Uniqueness around the Bubbleless BGP
Bubbly Equilibrium Fluctuations

- Equilibrium dynamics

\[
\hat{y}_t = \frac{\Lambda \Gamma \nu}{\beta} E_t \{\hat{y}_{t+1}\} + \Phi \hat{q}_t^B - \frac{\nu}{\beta} (\hat{i}_t - E_t \{\pi_{t+1}\})
\]

\[
\pi_t = \Lambda \Gamma E_t \{\pi_{t+1}\} + \kappa \hat{y}_t
\]

\[
\hat{q}_t^B = \Lambda \Gamma E_t \{\hat{q}_{t+1}^B\} - q^B (\hat{i}_t - E_t \{\pi_{t+1}\})
\]

\[
\hat{i}_t = \phi_\pi \pi_t + \phi_q \hat{q}_t^B
\]

where \( \Phi \equiv \frac{(1-\beta \gamma)(1-\nu \gamma)}{\beta \gamma} \), \( Y \equiv \left( 1 + \frac{(1-\beta \gamma)(\Lambda \Gamma - 1)}{1-\Lambda \Gamma \nu \gamma} \right) \), \( q^B = \frac{\gamma (\beta - \Lambda \Gamma \nu \gamma)}{(1-\beta \gamma)(1-\Lambda \Gamma \nu \gamma)} \)

- **Particular case #1** (no bubble creation): \( \Lambda \Gamma = Y = 1 \); \( q^B = \bar{q}^B \)
- **Particular case #2** (about bubbleless BGP): \( \Lambda \Gamma = Y = \frac{\beta}{\nu} \); \( q^B = 0 \)
- Intermediate cases: \( \Lambda \Gamma \in \left( 1, \frac{\beta}{\nu} \right) \), \( q^B \in (0, \bar{q}^B) \)
Figure 4
Monetary Policy and Equilibrium Uniqueness:
The Case of No Bubble Creation ($r=g=0.004$)
Figure 6a
Monetary Policy and Equilibrium Uniqueness around a Bubbly BGP with Bubble Creation ($r=0.003935$)
Figure 6b
Monetary Policy and Equilibrium Uniqueness around a Bubbly BGP with Bubble Creation ($r=0.003931$)
Figure 5
Monetary Policy and Equilibrium Uniqueness around the Bubbleless BGP
Figure 8
Macro Volatility and Leaning against the Bubble Policies
(type II bubbles, r=0.39%)
Main Messages and Next Steps

- Reminder of the possibility of bubbly equilibria once we depart from the infinite-lived representative consumer framework
  - more likely in an environment of low natural interest rates
- Perils of using interest rate policy to tame asset price bubbles
  - indeterminacy more likely
  - risk of larger fluctuations
- Caveats
  - *rational* bubbles
  - no role for credit supply factors
- Next steps:
  - Welfare and role of monetary policy
  - Global equilibrium dynamics (nonlinearities, switching equilibria)